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How many properties of spin does a particle have?

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ABSTRACT

A common assumption in non-relativistic quantum mechanics is that self-adjoint operators mathematically represent properties of quantum systems. Focusing on spin, we argue that a natural view considers observables as determinable properties and their eigenvalues as their corresponding determinates. We provide a taxonomy of the different views that one can hold, once it is accepted that spin can be modelled with the determinable-determinate relation. In particular, we present the two main families of views, dubbed Spin Monism and Pluralism, and we show that the current literature does not take a stance between the two. Then we put forward two arguments in favour of the former. Finally, we present a new account of Spin Monism, that is absent in current literature; such a view is worth discussing, or so we contend, because several compelling considerations support it, and it opens new ways of thinking about the ontology of quantum mechanics.

1. Introduction

If a realist stance toward scientific theories is assumed, it is natural to think that measurements are performed on physical entities, and their outcomes reflect the value of the properties measured. Furthermore, it has been proposed (Swoyer, 1987) to understand measurements employing the determinable-determinate relation: the property measured is a determinable, whose determinates are the possible outcomes of the measurement.

The determinable-determinate relation (Wilson, 2017) is a metaphysical tool to understand how general properties are related to more specific ones: more general properties, like ‘having mass’, are called *determinable properties* while their more specific ones, such as ‘having mass 3 kg’, ‘having mass 4 kg’ and so on, are called their *determinate properties*. Moreover, the determinable-determinate relation is relative to some level of determination: a determinate of a given determinable may be, in turn, a determinable with some more specific determinates. The property of ‘being red,’ say, is a determinable of ‘being scarlet’ but a determinate of ‘being coloured’. The following definitions are instrumental for the point we wish to make. A *maximally unspecific determinable* is a determinable which is not a determinate of any other determinables; a determinate (of a given determinable) which, in turn, is not a determinable of other determinates is called a *maximally specific determinate*. In the previous example, ‘being coloured’ is the maximally unspecific determinable, and ‘being scarlet’ is one of its maximally specific determinates.

Wolff (2015) proposed a determinable-determinate based analysis of spin in non-relativistic quantum mechanics (QM hereafter). However, Wolff does not explain much about how to understand spin components, i.e. spin along different spatial directions, in terms of determinable-determinate relations.[1] The first aim of the paper is that of completing Wolff's analysis by giving a determinable-determinate account of spin components.

We argue that spin components, represented by operators like $\hat{S}_x$, $\hat{S}_z$, and so on, are, in the light of the Eigenvector-Eigenvalue Link, naturally interpreted as determinable properties whose determinates are the concrete orientations of spin, e.g. spin up along the $x-$axis. So far, the literature seems to agree that the operators above represent some properties, but no explicit stance on whether they are *maximally unspecific determinables* is taken. Hence, we propose a taxonomy which divides into two main families the possible views: **Spin Pluralism**, according to which the operators above are maximally unspecific determinables, and **Spin Monism**, according to which there is a unique maximally unspecific determinable of spin. Even if some authors might be read as implicitly assuming **Spin Monism**, nobody proposes which operator is supposed to

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[1] On this topic, Wolff (2015, p.383) only says that “To find the orientation of spin we need to look at the different spin components, and it is here that matters become difficult. For spin, like for other vectors, these components behave in some respects like determinables, but they behave differently from components for spatially oriented vectors.”

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represent such a unique determinable of spin. We propose two arguments in favour of **Spin Monism**, based on some formal properties of spin Hilbert spaces. By presenting them, we suggest that the operator $\hat{\vec{S}} \cdot \vec{n}$ is the best candidate to represent the unique maximally unspecific determinable of spin.[2]

We then turn to the second aim of the paper, that is, to present a novel account of spin monism according to which operators like $\hat{S}_x$, $\hat{S}_z$ and so forth, *do not* represent any property. Such an account is radical and countertrend yet worth discussing. Indeed, according to our account, the spin of a particle is always determinate, even before a measurement is performed. Superpositions are then understood in terms of the intrinsic indeterminacy of the dynamics of the quantum system, rather than by introducing a new metaphysical state of affairs in which a quantum system might be.[3]

Before starting, it is worth laying out some assumptions that we hold. We assume a realist attitude towards QM (McMullin, 1984). Moreover, we do not enter into the complexities of the interpretations of QM. Instead, we limit ourselves to the formalism of textbook QM.[4] This is so for two reasons. First, some of our results cannot be accommodated in some interpretations,[5] but they might plausibly be in others.[6] Hence, a more general discussion is preferable. Second, textbook QM is considered a formalism without an ontology, when not considered an anti-realist view. Our view allows one to be a realist on textbook QM[7] and to retain an always definite spin ontology without modifying the formalism, but only by refusing to consider some Hermitian operators as properties. So far, our conclusions are limited to spin and single-systems only. Yet our account suggests a general way of interpreting incompatible observables, which might lead to an explicit ontology of textbook QM.

Furthermore, our analysis is, due to length limitations, utterly simplified: we talk only about idealised free single-particle systems in pure states, neglecting experimental errors. We also bracket the controversial problem of what quantum objects are, discussing properties instantiated by quantum systems as if this concept were unproblematic. Finally, the examples we present are drawn only from the more straightforward case of spin$-\frac{1}{2}$ particles. Nonetheless, our account can be straightforwardly generalised to any spin$-\frac{n}{2}$ particle since nothing in our analysis hinges on the spin number.

[2] To be clear, the operator $\hat{\vec{S}} \cdot \vec{n}$ is know in physics (Hughes, 1989; Sakurai, 1994), and quoted in some works on the metaphysics of QM (Calosi & Wilson, 2021, fn.25). The original aspect of the present paper is the thesis that such an operator corresponds to the maximally unspecific determinable of spin.

[3] Examples can be found in Dorato (2007); Bokulich (2014); Calosi and Wilson (2018).

[4] i.e. the currently taught quantum mechanics based on the Dirac-von Neumann formulation of the theory. Sometimes called 'Orthodox' or 'standard' QM.

[5] As an example, in some versions of Bohmian mechanics the only property instantiated by a quantum object is its position. It is clear that in these frameworks, the question of how many properties of spin there are is trivial since there are not any, at least at the fundamental level. See Bell (1982); Daumer et al. (1996).

[6] For instance, Relational QM and, perhaps, GRW.

[7] Whether scientific realists must be committed to the reality of spin has been recently discussed in the literature. It has been argued that, insofar as the ontological status of spin is underdetermined by different interpretations of QM, scientific realists are not necessarily committed to spin (Saatsi, 2020; Vickers, 2020). Our attitude toward textbook QM nicely fits to Egg (2021)'s reply to (Vickers, 2020) and (Saatsi, 2020). Albeit we are explicitly sympathetic with Egg's view, we think such a debate is orthogonal to the result of our paper. As Saatsi (2020, p.50) himself notices, 'metaphysical assumptions should not need to be part of the scientific realist account of the empirical successes of quantum physics either. Such assumptions belong to the metaphysical foundations of quantum theory, which is an extremely well-motivated and important endeavour, but one that the realist need not engage with in articulating her epistemic commitments.' We owe an anonymous referee the connections of our work with such a debate.

The paper is structured as follows. In §2, we clarify what it means to consider self-adjoint operators as properties. Then (§3), we present a taxonomy of the views that accept to understand spin in terms of determinable-determinate relations, dividing the possible views into two main families dubbed **Spin Pluralism** and **Spin Monism**. We continue by presenting (§4) two arguments against **Spin Pluralism**. Finally (§5), we put forward a novel account of **Spin Monism**, and we discuss a natural objection against it. Conclusions follow (§6).

2. Spin operators as properties

In this section, we lay down some groundwork for what follows. In particular, we provide a brief introduction to the mathematics of spin (§2.1), and we show what does it mean to consider spin operators as representing determinable/determinate properties (§2.2). Readers already familiar with the formalism of QM and the notation of spin operators, eigenvalues and eigenstates, may jump directly to §2.2.

2.1. The mathematics of spin

In QM, spin is an infinitesimal generator of the rotation group $SU(2)$, described by a vector $\vec{S}$ (which in turn can be represented as a self-adjoint operator on $\mathcal{H}_s$). The fact that $\vec{S}$ is a generator of $SU(2)$ means that its components should obey the following commutation relations:

$$\left[\hat{S}_i, \hat{S}_j\right] = i\hbar\varepsilon_{ijk}\hat{S}_k, \tag{1}$$

where ε_{ijk} is the Levi-Civita symbol. The fact that the spin components do not commute and that they all are self-adjoint operators acting on the same domain jointly imply that there is an indeterminacy relation between them, namely:

$$\Delta\hat{S}_i\Delta\hat{S}_j \geq \frac{\hbar}{2}\left|\varepsilon_{ijk}\langle\hat{S}_k\rangle\right|. \tag{2}$$

Operators corresponding to spin along different directions are not the only spin operators. Indeed, it is possible to build up the operator $\hat{S}^2 = \vec{S} \cdot \vec{S}$, which is the Casimir operator of the $SU(2)$ group. An interesting feature of operators of this latter kind is that they commute with any other operator of the group. From (1), one can show that $\hat{S}^2$ commutes with every component $\hat{S}_i$, for every i, and also with $\hat{P}$ and $\hat{X}$. In other words, $\hat{S}^2$ commutes with every element of the irreducible set of observables $\left\{\hat{\vec{X}}, \hat{\vec{P}}, \hat{\vec{S}}\right\}$, and so it is a multiple of the identity. Being a multiple of the identity entails that an eigenvalue of $\hat{S}^2$ will characterise every particle. Finally, since $\hat{S}^2$ commutes with $\hat{S}_i$, it is possible to *arbitrarily* choose a component of spin to characterise the spin states of the particle as follows:[8]

$$\hat{S}2|s, s_i\rangle = \hbar^2 s(s+1)|s, s_i\rangle, \qquad s \in \mathbb{N}/2; \tag{3}$$

$$\hat{S}_i|s, s_i\rangle = \hbar\, s_i|s, s_i\rangle, \qquad s_i \in \{s,\ s-1,\ \ldots, -s\}, \tag{4}$$

where i is a generic axis upon which the spin can be measured. An operator of spin – that will behave as (4) – will be associated to every measurement of spin along given spatial directions, e.g. the $x-$axis or the $z-$axis. For instance, the eigenvalue relation of the operator corresponding to the property of having spin along the $x-$axis is:

[8] Note that when s and s_i are inside the ket, they label the eigenstate, whereas when they are outside the ket, they refer to the eigenvalues of the operators $\hat{S}^2$ and $\hat{S}_i$.

$$\hat{S}_x|s,s_x\rangle = \hbar\, s_x|s,s_x\rangle, \qquad s_x \in \{s,\ s-1,\ \ldots,-s\}; \tag{5}$$

after a measurement of spin along the x−axis, the state of the system will be $|s,s_x\rangle$, where the eigenvalue s_x will be the outcome of the measurement. For a spin−$\frac{1}{2}$ particle, for example, such a measurement has only two possible outcomes: $+1$ when the state of the particle after the measurement is spin up along the x−axis, i.e. $|\uparrow\rangle_x$, and -1 when it has spin down on that axis, i.e. $|\downarrow\rangle_x$.[9] The same happens for all the possible axes upon which spin can be measured.

2.2. *The metaphysics of spin*

The well-known empirical fact that spin cannot be measured simultaneously along different axes has its mathematical explanation in (1). This equation is usually taken in textbook QM as a forewarning that an indeterminacy relation will hold between the values of spin along different axes. Moreover, further mathematical theorems show that if a system is in a definite state of spin along an axis, the spins along different directions cannot have any definite value whatsoever, showing that these indeterminacy relations do not concern an epistemic limitation. Theorems such as those found in Kochen and Specker (1967) and Gleason (1957) show indeed that if one tries to 'cheat', so to speak, by assigning definite values to spin along all the different directions, then a logical contradiction or a violation of empirical data occurs. As such, these results are taken to suggest - if one is a realist about QM - that the indeterminacy relations describe something fundamental about the microphysical world. What precisely these indeterminacy relations tell us about the world is a matter of controversy.

In textbook QM, it is often assumed that physical properties are represented by self-adjoint[10] operators. In the case of spin, such an assumption is natural given the connection between the operators of spin along different directions ($\hat{S_x}$, $\hat{S_z}$ and so forth) and the experimental outcomes of spin measurements along those axes. How natural it is to assume that operators formally represent some physical quantities, i.e. what metaphysicians call 'properties' of the system measured, is attested by the fact that one of the principles of orthodox QM is the so-called eigenvector-eigenvalue Link - **(EEL)** hereafter:[11]

(EEL) The eigenvector-eigenvalue link: Given a physical quantity *O* represented mathematically by a self-adjoint operator $\hat{\mathscr{O}}$:

1. A system in a state $|\psi\rangle$ possesses a definite value of *O* if and only if $|\psi\rangle$ is an eigenstate of $\hat{\mathscr{O}}$, $\hat{\mathscr{O}}|\psi\rangle = o_i|\psi\rangle$.
2. In this case, the definite value is the associated eigenvalue o_i.

The role of the **(EEL)** is that of connecting the mathematical description of a quantum system (operators, eigenvectors and states) with its metaphysical description in terms of properties instantiated.

Even if we share this assumption, we argue that **(EEL)** shifts between two different understandings of the claim that 'a self-adjoint operator represents a physical property.' Such a claim may mean either that:

(i) the operator stands for a *determinate* property. This conception seems to be what is expressed in the first bullet of **(EEL)**, where it says under what condition the quantity *O* has a *definite* value. In this first sense, we postulate that a determinate property is represented in the formalism of QM by the tuple composed of the operator and one of its eigenvalues, i.e. $(\hat{\mathscr{O}}, o_1)$, $(\hat{\mathscr{O}}, o_2)\ldots(\hat{\mathscr{O}}, o\hat{i})$.[12] For instance, the property of 'having spin up along the x−axis' is represented by $(\hat{S}_x,\ +)$, where $+$ is the eigenvalue corresponding to the state of spin up;

(ii) or that the operator stands for a *determinable* property. To us, the first line of **(EEL)** - a physical quantity *O* (is) represented by a self-adjoint operator $\hat{\mathscr{O}}$ - clearly states that the operator $\hat{\mathscr{O}}$ represents a determinable property *O*. Indeed, if the first line assumed that *O* is a determinate, then the first bullet - which introduces the condition under which *O* has a definite value, i.e. is a determinate - would be redundant. In this second sense, we postulate that a determinable property is represented in the formalism of QM by the tuple made of the operator and its eigenvalues, i.e. $(\hat{\mathscr{O}}, o_i)$. For instance, the property of 'having spin along the x−axis' is represented by $(\hat{S}_x, s_x)$, where s_x stands for all the possible eigenvalues of $\hat{S}_x$. To avoid a pedantic notation, sometimes we will speak loosely of an operator representing a determinable, e.g. we say that $\hat{S}_x$ – rather than the tuple $(\hat{S}_x, s_x)$ – stands for the determinable 'having spin along the x−axis.'

The distinction between determinables and determinates is not just a piece of metaphysical theorising, useless for the philosophy of physics. As we will soon see, it helps characterise the received view of spin properties and the contrasting view that we are proposing.

At a first look, equations (3) and (4) tell us that two different quantities characterise the spin of a particle. Given **(EEL)** and the determinable-determinate relation, (3) tells us that the first determinable property of spin is the spin number, represented by $(\hat{S}^2, s)$. Such a determinable represents how much spin, so to speak, a particle has. Its determinates, i.e. $\frac{1}{2}$, 1, $\frac{3}{2}$, 2 and so on (see Fig. 1)[13], partition the spin Hilbert space in superselection sectors; as a consequence, they divide particles into kinds, that is, spin−$\frac{1}{2}$ particles, spin -1 particles, etc … As Wolff (2015) already showed, to analyse spin number as a determinable is quite straightforward and natural.

Analogously, Equation (4) may be seen as saying that spin components have the same determinable-determinate structure as the spin

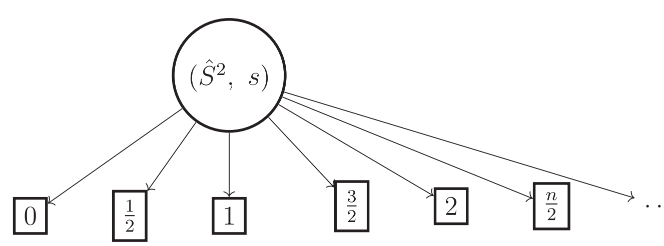

Fig. 1. The spin number as a determinable.

[9] Formally speaking, the eigenvalues of every $\hat{S}_i$ operator of a spin-$\frac{1}{2}$ particle are $+\frac{\hbar}{2}$ and $-\frac{\hbar}{2}$. In what follows, we use '+' and '−' as shorthands when it is clear that $\frac{\hbar}{2}$ is implicit.

[10] In the case of finite-dimensional Hilbert spaces, self-adjointness and symmetry coincide. For further discussions on the features of operators acting on Hilbert spaces, see (Moretti, 2018, Ch. 5) Since we deal only with this kind of Hilbert spaces in the paper, we avoid a pedantic notation in favour of the readability of the text.

[11] The definition of (EEL) used here is a slightly modified version of the one presented by Wallace (2019, p. 2). Note that Wallace (2019) himself argues that **(EEL)** is not a part of orthodox QM. See Gilton (2016) for some replies to Wallace's arguments. In what follows, we assume that **(EEL)** is a part of QM, at least as an effective principle.

[12] This is not a postulation we came up with; rather, it is the common ways in which the idea that operators stands for properties is captured in QM. See, for example, Hughes (1989, Ch. 6).

[13] In all the figures of the paper, determinables/determinates on the same level of determination are drawn with the same shape; determinables are positioned above their determinates. The arrows represent the determination relations that hold between a determinable and its determinates. Reasonably, when there are infinite determinates, we avoid drawing all of them in the figure. In Fig. 1, for example, $\frac{n}{2}$ followed by ' …' is a shortcut for the infinite determinates that one obtains by substituting to n an integer.

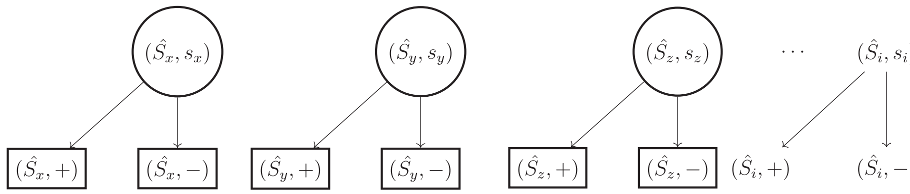

Fig. 2. Diagram of Spin pluralism for spin$-\frac{1}{2}$ particles.

number, as captured by (i) and (ii): given an arbitrary direction i, $(\hat{S}_i, s_i)$ represent the determinable property of 'having spin along the i-axis' where $(\hat{S}_i, +)$ and $(\hat{S}_i, -)$ represent, respectively, the properties of having spin up and spin down along that very axis. We now turn to how to understand more precisely the determinable-determinate relation for spin components.

3. Varieties of spin realism

Having laid down the groundwork, we enter in the hearth of the paper. In the rest of the paper, we present:

- a taxonomy, which is supposed to be exhaustive, of broad families of views that understand spin as determinable/determinate properties. We dub these broad families **Spin Monism** and **Spin Pluralism** (§3);
- reasons for preferring a broadly monistic view to forms of **Spin Pluralism** (§4);
- within the monistic camp, a new account of spin which, we contend, is worth discussing and developing (§5).

Let us start by noticing that in discussions concerning spin observables, it is not unusual to read claims like 'when an electron has a determinate value of spin along the $x-$axis, the value of the spin along the $z-$component is indeterminate.' If the eigenvalues of spin along different spatial directions are the values of properties instantiated by a system, then claims like the one above hinge on the idea that (potentially) there are an infinite number of determinable spin properties - one for every spatial axis upon which spin can be measured. The determinable-determinate machinery introduced above is helpful to characterise the possible ways of considering spin as a property. The main bipartition between the possible views is obtained once the question "Are $\hat{S}_x$, $\hat{S}_y$ and $\hat{S}_z$ maximally unspecific determinables?" is answered. Such a question separates what we dub **Spin Pluralism** from **Spin Monism**:

(SP) Spin Pluralism: a quantum system could instantiate an infinite set of maximally unspecific determinable spin properties that stand on the same level of determination. These determinables are the properties of having spin along the $x-$axis, having spin along the $z-$axis and spin along all the other continuously many possible axes. The maximally specific determinates corresponding to the determinables above are the concrete orientation of spin a particle could have, e.g. spin up along the $x-$axis.

(SM) Spin Monism: a quantum system instantiates a unique maximally unspecific determinable property of spin that is independent of any axis. Its maximally specific determinates are the concrete orientation of spin a particle could have.

We defined the determinables of spin components as the tuples consisting of an operator of spin along a given axis and its eigenvalues. For instance, the determinable corresponding to the property of having spin along the $x-$axis is defined as $(\hat{S}_x, s_x)$; the determinates of a given determinable of spin components are represented by the tuple consisting of the same operator along a given axis and one of its particular eigenvalues. For a spin$-\frac{1}{2}$ particle for instance, the determinates of $(\hat{S}_x, s_x)$ are spin up and spin down along the $x-$axis, which in our notation are respectively $(\hat{S}_x, +)$ and $(\hat{S}_x, -)$ (see Fig. 2). The $(\hat{S}_i, s_i)$ appearing in Fig. 2 stand for all the possible properties of spin one obtains by substituting for i every possible spatial direction; $(\hat{S}_x, s_x)$, $(\hat{S}_y, s_y)$ and $(\hat{S}_z, s_z)$ are pictured as an example, i.e. $i = x$, $i = y$ and $i = z$, to emphasize that all these determinables stand on the same level of determination. Finally, according to **(SP)** all of these determinables are maximally unspecific: they are not determinates of a further determinable(s).

According to **Spin Monism** the maximally specific determinates of spin are, unsurprisingly, the same accepted by spin pluralists. What does change between the two views is that according to **(SM)** there is a unique maximally unspecific determinable of spin.

Spin Pluralism and **Spin Monism**, as defined above, are families of accounts, rather than single views. Indeed, different variants of the two can be crafted once more metaphysical theses are added. For example, Glick (2017)'s **Sparse view** - according to which determinables are instantiated only if one of their determinates is - can be added to both **Spin Pluralism** and **Spin Monism**. The way of crafting some variants of **Spin Monism** which matters here is that of considering the possible answers to the following questions:

(1) Which mathematical object represents the unique maximally unspecific determinable property of spin (if any)?
(2) What is the metaphysical counterpart of the operators of spin along different components, like $\hat{S}_x$, $\hat{S}_z$, and so forth, i.e. the operators that according to spin pluralists represent the maximally unspecific determinable properties of having spin along all the possible axes?

Concerning (1), no definite answer has been presented in the literature yet. Some authors speak as if spin were a single property, but they never point to a specific operator that should represent such a maximally unspecific determinable.[14] For example Calosi and Mariani (2020, Fn. 38), implicitly claim that operators like $\hat{S}_x$, $\hat{S}_z$ and the like, are not maximally unspecific determinables; at the same time though, they do not explicitly assess what the maximally unspecific determinable(s) is (are). One of the tasks of the next sections is to suggest a definitive answer to (1).

Concerning (2), it seems to be accepted unanimously by the literature that $\hat{S}_x$, $\hat{S}_z$, etc ..., represent determinable properties. Sometimes spins along different axes are not explicitly called 'properties.' Nonetheless, it is quite clear from the context that they are often implicitly considered as such.[15] Examples abound. They can be found in textbooks both on the foundations of QM – from now-classic texts (Hughes, 1989, ch. 6) to recent ones (Norsen, 2017, p. 217) – and on the philosophy of physics (Albert, 1992, p. 1).

One could say that our focus - the question 'How many spin properties are there?' - is intrinsically a metaphysical question. As such, we should

[14] See for example Bigaj (2012); Cetto et al. (2020).

[15] Even though in the original paper Einstein et al. (1935, p. 778) talk of position and momentum, rather than different components of spin (as in the well-known version of the EPR paradox presented by Bohm (1951)), it seems uncontroversial to read their 'physical quantities' as what metaphysicians call 'properties.'

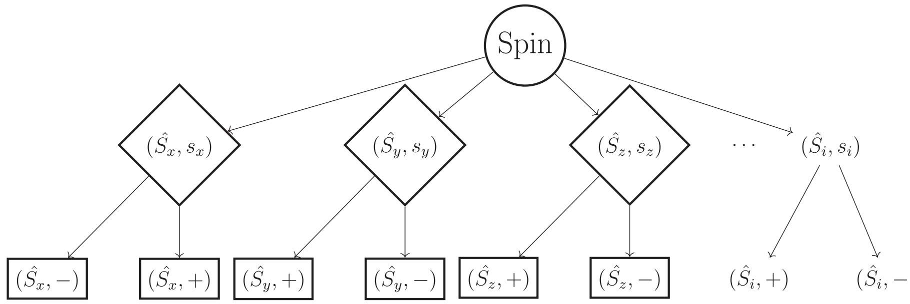

Fig. 3. Pictorial of ($\mathbf{SM_{CR}}$) for spin$-\frac{1}{2}$ particles.

be charitable and allow that physicists and less metaphysically inclined philosophers could use more freely the word 'property' without being committed to the views here under scrutiny. Nevertheless, the same way of talking about spin can be easily found in analytic metaphysics. Again, this is true from textbooks about metaphysical aspects of QM (Lewis, 2016, p. 10), to more research-oriented papers (Dorato, 2007; Bokulich, 2014; Calosi & Wilson, 2018; Darby & Pickup, 2019, pp. 1–26). Finally, some authors, like Darby (2010, p. 233), also make an explicit reference to the determinable-determinate machinery when talking about spin along different axes.

As remarked above, there is no evidence of which view - **Spin Pluralism** or **Spin Monism** - is the most accepted in the literature. Indeed, nobody explicitly states yet which is/are the maximally unspecific determinable/s of spin. Since the literature seems to agree that spins along different axes are determinables, the mainstream view must be either a form of **Spin Pluralism** or the following variant of **Spin Monism**:

($\mathbf{SM_{CR}}$) **Spin Monism - Component Realism:** a particle instantiates a unique maximally unspecific determinable property of spin that is independent from any axis. Its maximally specific determinates are the particular directions along which the spin of the system could be polarised. There is a third level of determination that stands in between the maximally unspecific determinable and the maximally specific determinates; such a level is made of the properties of 'having spin along the $x-$axis', 'having spin along the $z-$axis', and so forth, for each of the uncountably many axes upon which spin can be measured (see Fig. 3).

So far, we have shown that spin monism and spin pluralism are two broad families of views, and we argued that the current literature is ambiguous on which of these families is the correct one. Furthermore, we outlined some possible ways of crafting different forms of spin monism/pluralism, and we remarked that it has yet to be specified which is the mathematical object which represents the unique maximally unspecific determinable, if any. The aim of what follows is to provide an answer to the following questions:

- Is there an argument for preferring **Spin Pluralism** over **Spin Monism**, or vice-versa?
- Is there a particular account of **Spin Pluralism/Monism**, undiscussed in the literature, that deserves further attention?

We tackle these questions, respectively, in §4 and §5. Moreover, we argue, in §4, that the answer to the first question strongly suggests a particular mathematical structure as representing the maximally unspecific determinable of spin.

4. In favour of spin monism

In this section, we present two arguments in favour of **Spin Monism**. As we will soon see, the first argument (§4.1) holds only if a controversial assumption is accepted. Nonetheless, its presentation is instrumental in formulating a stronger argument. Section 4.2 consists of a mathematical interlude necessary for the statement of the second argument, which is presented in §4.3. Such an argument incidentally reveals that the operator $\vec{\hat{S}} \cdot \vec{n}$ is the best candidate for representing the maximally unspecific determinable of spin.

4.1. The Uniqueness Argument

When a system has a definite value of spin along a component, it is well-known that it is in a superposition on the spin along the other axes. Moreover, the symmetry group $SU(2)$ guarantees that there is no privileged basis that should be preferred. Indeed, given an arbitrary direction of space, and as a basis two orthogonal vectors that represent the possible orientations of spin along that axis, then one can describe any other possible state of spin as a linear combination of our basis. The fact that a system has a definite value of spin along a given direction does not merely *entail* that it is in a superposition on the others. Rather, having a definite value of spin along an axis and being in a superposition on another is *the very same state*. Since there is no privileged basis, we claim that descriptions on different bases cannot have a strong metaphysical import. In other words, since one does not change the *physics* of the system by performing a change of basis, then a definite state of spin on an axis and a superposition of spin on other axes must correspond to the same metaphysical structure. Spin states represented in different bases are mathematically equivalent descriptions of a single vector.

How can we better characterise such a unique spin vector? The clearest way to do so is to represent the spin of the particle as a sphere, the well-known Bloch Sphere (see Fig. 4). In this representation, once one chooses a basis, say, the eigenvectors of $\hat{S}_z$, a point in the sphere represents a generic spin state $|\psi\rangle_s$ with following coordinates:[16]

$$|\psi_s\rangle = \cos\left(\frac{\theta}{2}\right)|\downarrow\rangle_z + e^{i\varphi}\sin\left(\frac{\theta}{2}\right)|\uparrow\rangle_z. \tag{6}$$

The sphere is invariant under rotations (see Fig. 4): we can rotate the sphere so that any pair of orthonormal vectors (say, spin up and down along the $x-$axis) take the place of spin up and down along the $z-$axis. Upon performing the rotation, what changes is just the basis upon which the state $|\psi_s\rangle$ is described; and the only structures invariant under this rotation are the state $|\psi_s\rangle$ and the vector $\vec{a}$ - also called the Bloch vector - which connects the centre of the Bloch Sphere to the state $|\psi_s\rangle$ itself.[17] It is the vector $\vec{a}$ that represents, independently from the coordinates in which it is expressed, the physically relevant information concerning the spin of the system. This graphically shows why writing the spin of a system along different directions is merely changing the basis upon which we describe the unique vector of spin $\vec{a}$. Since the physics

[16] For spin$-\frac{1}{2}$ particles in a pure state, the point lies on the surface of the Bloch Sphere; for the states of composite systems the point lies inside the sphere. Note that particles with a spin number larger than $\frac{1}{2}$ are formally treated as composite system.

[17] The vector $\vec{a} = a^i\hat{e}_i$ (where Einstein's rule is intended) has components a^i that contravary with a change of basis. The vector $\vec{a}$ itself is an object that is independent of any coordinate $\hat{e}_i$ one chooses.

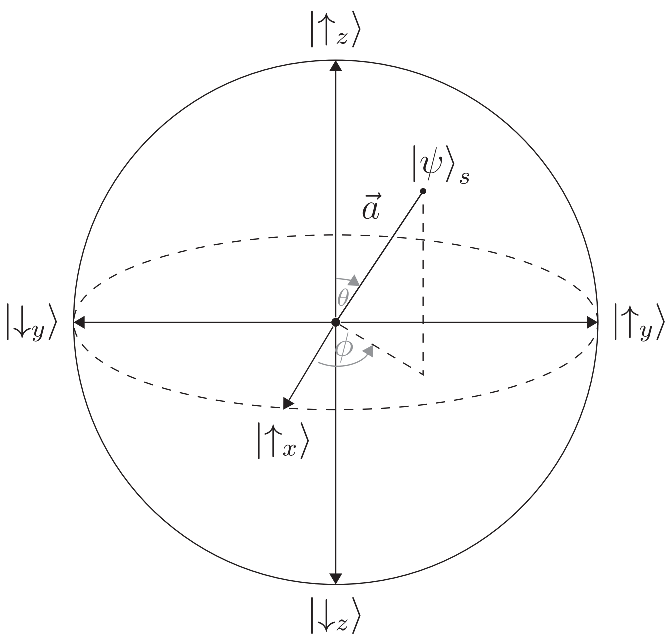

Fig. 4. The Bloch sphere.

concerning the spin of the system of the particle is captured by $\vec{a}$ which, in turn, picks a unique $|\psi_s\rangle$, the latter is the only mathematical structure to which one should give a metaphysical weight.[18] The uniqueness of the spin vector entails there is a unique property of spin.

Unfortunately, this argument - that we dub the 'Uniqueness Argument' - hinges on a controversial assumption that badly limits its scope. According to Glick (2017, pp.205–206), there are three broad ways in which the ontology could be read off QM:

(1) "one can view the quantum state non-ontologically," interpreting it as a "book-keeping" device that just informs us about the fundamental properties described in QM by self-adjoint operators; (ibid.)
(2) one can consider "the properties to be ontologically derivative and quantum states to be fundamental; " (ibid.)
(3) or "one can advocate a flat ontology for standard QM", and consider states and properties as on "equal foot ontologically". (ibid.)

Now, by itself, the Uniqueness Argument is valid only if one is willing to assign the ontological status of 'property' to the quantum state (Monton, 2004; Belot, 2012; Dorato & Esfeld, 2010). That is, the Uniqueness Argument is valid only if the ontological stance assumed is (2) or (3). Indeed, the Uniqueness Argument only shows that the spin vector, and as a consequence, the quantum *state*, is invariant under a change of basis. Hence, the uniqueness of the spin property follows from the argument only if one gives a metaphysical import to the quantum state.

True to the spirit of the **(EEL)**, we accepted above that the values of a property are represented by self-adjoint operators and eigenvalues, rather than by eigenvectors (as discussed in §2), i.e. our ontological stance towards QM is (1).[19] Therefore, we reckon that this argument is not conclusive in favour of **Spin Monism** but for those that read ontologically quantum states. Yet, it provides a basis to formulate a more convincing argument in favour of **(SM)**, that we present in §4.3.

4.2. *Intermezzo - the spin-polarisation principle*

It is essential now to pause our discussion to briefly discuss a theorem of QM. Given a spin state, there exists a unique polarisationation vector $\vec{n}$, which is determinate at all times. This result is called the **Spin-polarisation Principle**,[20] and its statement is the following:

Spin-polarisation principle: Any state of a single spin is an eigenvector of some component of the spin. In other words, given a generic state $|\psi\rangle \in \mathbb{C}^2$, there exists some direction $\vec{n}$ such that:

$$\hat{\vec{S}} \cdot \vec{n}\,|\psi\rangle = \frac{\hbar}{2}|\psi\rangle \tag{7}$$

where $\hat{\vec{S}}$ is the spin vector:

$$\hat{\vec{S}} = \frac{\hbar}{2}(\sigma_x, \sigma_y, \sigma_z), \tag{8}$$

constructed with the Pauli matrices σ_x, σ_y and σ_z, i.e. a possible representation of the $SU(2)$ group.

Now, what does the theorem entail? Equation (7) means that for a generic spin state $|\psi\rangle$, a unique vector $\vec{n}$ always exists – the so-called 'polarisation vector' – along which the component of spin predictably is $+\frac{\hbar}{2}$.

The relevance of the theorem - for our purposes - is that the polarisation vector is nothing but the Bloch vector $\vec{a}$ itself. To see why, take an eigenvector $|\psi\rangle$ of the operator $\hat{\vec{S}} \cdot \vec{n}$ which is aligned along the direction $\hat{n}$. How can we write this state in terms of the eigenvectors of, say, $\hat{S_z}$? We rotate the state $|\psi\rangle$ by two angles to make $\hat{n}$ and $\hat{z}$ coincide. The mathematically correct way of doing so is employing (6). As a consequence, the eigenstates of $\hat{\vec{S}} \cdot \vec{n}$ and the states individuated by the Bloch vector are the same. Since the polarisation vector and the Bloch vector always individuate the very same spin state, we conclude that they are nothing but two names attached to the very same vector.

Even though the **Spin-polarisation principle** is trivial from a mathematical point of view, it is not so from a metaphysical one. This theorem guarantees that a system always has a defined polarisation vector $\vec{n}$ even before any spin measurement is performed. In other words, a particle with spin necessarily has a definite state of spin along a given axis even before a measurement is performed. However, even if the property of spin is definite at all times along a given axis, it does not follow that it is always possible to *know* in which direction the spin is aligned. To put it explicitly, the spin of a (free) particle is always metaphysically determinate in one direction, even if such a direction could be epistemically indeterminate.

4.3. *The Invariance Argument*

The Uniqueness Argument above (§4.1) suggests that the descriptions of spin along particular directions are, somehow, arbitrary. In other words, not only the very same vector can be written in two different ways using different bases, but none of the spin components is privileged. Insofar as the unique basis-independent vector of spin does not change

[18] Alternatively, one may give an ontological weight only to $\vec{a}$, rather than to the state it selects. We think that such a view would be interesting, and it was discussed in previous drafts. Nonetheless, we think there are good reasons to be dissatisfied with such a view which, unfortunately, lay outside the scope of the paper.

[19] Stance (1) has also been defended in the context of different interpretations of QM, e.g., Bohmian mechanics (Suárez, 2015), Relational (Rovelli, 2018; Calosi & Mariani, 2020) and Modal QM (Lombardi, 2019).

[20] The "Spin-polarisation Principle," is the name used by Susskind and Friedman (2014, p.90). The polarisation vector is seldom presented in textbooks as explicitly as we do in the paper. A pedagogical (yet quite technical) reconstruction of the polarisation vector, a proof of the uniqueness of $\vec{n}$ and a detailed discussion of $\hat{\vec{S}} \cdot \vec{n}$, can be found in (Sakurai, 1994, pp. 165–168).

when we perform a change of basis, one should conclude that we must give some metaphysical weight to this very vector rather than to its description in different bases. Indeed, a common practice in reading the metaphysics out of physics is that of looking for invariant structures.

If one accepts that the values of a property are the eigenvalues of a self-adjoint operator, rather than the vectors themselves, one may start wondering whether there is some relationship between the unique spin vector and the eigenvalues of some particular operator. If such an operator exists, then one should give metaphysical import to this very operator rather than to those associated with the descriptions of spin along a particular axis. We argue that such an operator is the $\hat{\vec{S}}\cdot\vec{n}$ that appeared in (7).

Let us start by noticing that the polarisation vector $\vec{n}$ can be parametrised through three angles (α, β and γ). Selecting three directions in space, say $\hat{i}, \hat{j}$ and $\hat{k}$, one can define the following angles:

$$\cos\alpha = \vec{n}\cdot\frac{\hat{i}}{|\vec{n}|}, \quad \cos\beta = \vec{n}\cdot\frac{\hat{j}}{|\vec{n}|}, \quad \cos\gamma = \vec{n}\cdot\frac{\hat{k}}{|\vec{n}|}, \tag{9}$$

where $|\vec{n}|$ is the modulus of the polarisation vector. The polarisation vector itself will be then fully characterised by:

$$\vec{n} = |\vec{n}|\begin{pmatrix}\cos\alpha\\ \cos\beta\\ \cos\gamma\end{pmatrix} \tag{10}$$

Specifying the values of α, β and γ, one characterises the polarisation vector.

We can now see how $\hat{\vec{S}}\cdot\vec{n}$ captures all the possible states of spin in which a particle could be. As we have seen, the spin states are captured univocally by the polarisation vector $\vec{n}$. If we take a unit polarisation vector $\vec{n} = \{\cos(\alpha), \cos(\beta), \cos(\gamma)\}$,[21] the explicit form of this spin operator $\hat{\vec{S}}\cdot\vec{n}$ is given by:

$$\hat{\vec{S}}\cdot\vec{n} = \frac{\hbar}{2}\begin{pmatrix} cos(\gamma) & cos(\alpha) - icos(\beta)\\ cos(\alpha) + icos(\beta) & -cos(\gamma)\end{pmatrix}. \tag{11}$$

Once the values of the three angles α, β and γ that characterise the polarisation vector are given (together with the spin number), they fix a particular value of $\hat{\vec{S}}\cdot\vec{n}$. For instance, if we take $\left(\alpha = 0, \beta = \frac{\pi}{2}, \gamma = \frac{\pi}{2}\right)$, $\hat{\vec{S}}\cdot\vec{n}$ will take the form of $S\cdot\hat{x}$; or if we choose instead $\left(\alpha = \pi, \beta = \frac{\pi}{2}, \gamma = \frac{\pi}{2}\right)$, $\hat{\vec{S}}\cdot\vec{n}$ will be $\hat{\vec{S}}\cdot(-\hat{x})$.[22] And so on for all the possible measures of α, β and γ. That is, all the possible spin states are eigenstates of $\hat{\vec{S}}\cdot\vec{n}$. Moreover, the operator $\hat{\vec{S}}\cdot\vec{n}$ is the only spin operator that is invariant under rotations. Indeed, a change of basis modifies both the Pauli matrices, i.e. $\hat{\vec{S}}$, and the components of $\vec{n}$, while, crucially, preserving the form of the operator $\hat{\vec{S}}\cdot\vec{n}$. In other words, $\hat{\vec{S}}\cdot\vec{n}$ captures the fact that when we rotate the Bloch sphere, the spin vector (and the state connected to it) does not change. Since this operator is a general and invariant spin operator representing spin independently from the choice of basis, this very operator must represent a determinable property of having spin. Since every spin state is an eigenvector of such an operator, and so there are no determinate properties of spin that are not determinates of this determinable, we claim that $\hat{\vec{S}}\cdot\vec{n}$ must be a *maximally unspecific determinable.* The uniqueness of $\hat{\vec{S}}\cdot\vec{n}$ straightforwardly entails that **Spin Monism** is the correct way of understanding the spin property. This concludes what we dub the "Invariance Argument" in favour of **Spin Monism**.

5. A new account of spin monism

Section 3 ended with two main questions which we aimed to answer. So far, we have addressed just the first by arguing that **Spin Monism** has to be preferred to **Spin Pluralism**. Moreover, the considerations above showed that $\hat{\vec{S}}\cdot\vec{n}$ is the mathematical counterpart of the maximally unspecific determinable of spin. We now turn to the second question by investigating whether there is any spin monist view – different from **(SM_{CR})** – worth exploring. We answer affirmatively by presenting a novel account of **Spin Monism**. A full-fledged defence of such a view is left for future works. Here, we limit ourselves to present the view, offer some *prima facie* motivations in favour of its plausibility and defend it from a straightforward objection.

In the literature to date, only one version of **(SM)** different from **(SM_{CR})** has been presented. Namely, the view cursorily sketched by Funkhouser (2006) and heavily criticised by Wolff (2015). We argue now that there is a new variant of **Spin Monism** - which we dub "Component Nihilism" - worth discussing:

(SM_{CN}) Spin Monism - Component Nihilism: a particle instantiates a unique maximally unspecific determinable of spin, represented by $(\hat{\vec{S}}\cdot\vec{n}, +)$. Here, $\vec{n}$ must be understood as a shorthand for all the possible directions along which the polarisation vector could be aligned.[23] The maximally specific determinates of $(\hat{\vec{S}}\cdot\vec{n}, +)$ are the particular directions along which the spin of the system could be polarised, i.e. what we obtain by substituting $\vec{n}$ with a precise spatial direction. For example, having spin up and spin down along the $x-$axis will be respectively represented by $(\hat{\vec{S}}\cdot\hat{x}, +)$ and $(\hat{\vec{S}}\cdot -\hat{x}, +)$ (cf. fn. 23). There are no further levels of determination for the property of spin, e.g. there are no spin properties such that they are both determinables of maximally specific determinates and determinates of the unique maximally unspecific determinable (see Fig. 5).

We think there are several motivations for seriously considering **(SM_{CN})**. To start, note that there are two main ways in which **(SM_{CR})** could be read: (a) one could accept that the determinables are instantiated even when none (or more than one) of its determinates are, as the friends of metaphysical indeterminacy contend (Calosi & Wilson, 2018); or (b) endorse the **Sparse view**, and deny that determinables are instantiated when none of their determinates is (Glick, 2017). A full comparison of the three views is, for length reasons, reserved for future work. Briefly, **(SM_{CN})** seems to be, *prima faciae*, a better alternative than the ones above. First, it retains what we take to be the intuition that underlies the **Sparse View**, without claiming that determinable properties are miraculously created when a measurement is performed or denying that determinables are instantiated only when their determinates are. Moreover, **(SM_{CN})** crucially avoids some problematic features of the **Sparse view** (Calosi & Wilson, 2021). Second, **(SM_{CN})** is not committed either to the problematic idea that a particle instantiates infinitely many determinables of spin or that the determinable-determinate relation has to be modified - as the friends of metaphysical indeterminacy contend.

Moreover, the arguments against **(SP)** hinge on the meta-metaphysical principle that one has to give an ontological weight, preferably, to invariant structures only.[24] It is true that **(SM_{CN})** is quite

[21] Since in the examples that follows, we consider only spin$-\frac{1}{2}$ particles, we take $|\vec{n}| = 1$.

[22] This notation may seems puzzling at first. But note that the determinate $(\hat{\vec{S}}\cdot(-\hat{x}), +)$ corresponds to what was previously defined as $(\hat{S_x}, -)$, as $(\hat{\vec{S}}\cdot(\hat{x}), +)$ is equal to $(\hat{S_x}, +)$. Ditto for all the other possible axes.

[23] In the same way in which in our general characterizations of determinables (cf. §2), the o_i in $(\hat{\mathscr{O}}, o_i)$ stands for all the possible eigenvalues of $\hat{\mathscr{O}}$, rather than for one in particular.

[24] Such an assumption remained implicit below, insofar as it is widely shared among philosophers of physics. A classic reference on this point is van Fraassen (1989).

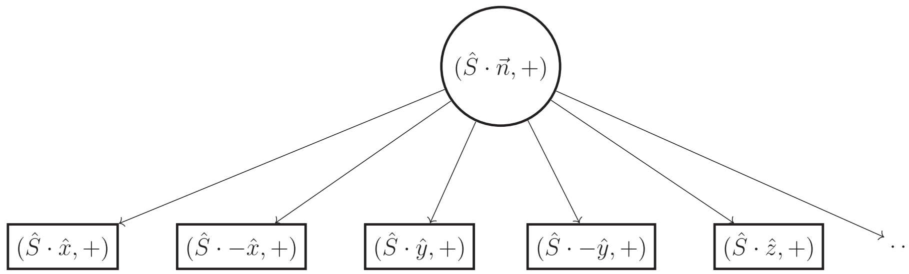

Fig. 5. Diagram of ($\mathbf{SM_{CN}}$) for spin$-\frac{1}{2}$ particles.

radical, insofar as it denies what seems to be unanimously endorsed in the literature, namely, that $\hat{S_x}$, $\hat{S_z}$ and the like, stand for determinable spin properties. As we have seen, though, these operators are non-invariant descriptions of spin. Hence, if the assumption that motivates **Spin Monism** in the first place is seriously embraced, it naturally follows that no metaphysical role has to be assigned to these operators. Surely, one can contend that invariant structures may be as real as non-invariants ones and that the only difference between the two is a question of fundamentality.[25] Even if we understand why in some theories one would want to give some derivative reality to some non-invariant structures, it seems problematic to argue that $\hat{S}_x, \hat{S}_y, \hat{S}_z$ somehow 'emerge' from a more fundamental property.

Furthermore, once it is accepted that $(\hat{\vec{S}}\cdot\vec{n},+)$ represents a maximally unspecific determinable, one could see that $\hat{S}_x$, $\hat{S}_y$, $\hat{S}_z$ are redundant, and they arbitrarily partition the maximally specific determinates of spin. If one agrees that adding redundant and/or arbitrary metaphysical structures to a theory should be avoided at all costs, then these operators should not be understood ontologically. It remains to be argued that the properties which should correspond to these operators are redundant and arbitrary. $\hat{S}_x$, $\hat{S}_y$, $\hat{S}_z$, and the like, are *redundant* because they do not explain anything more than what could be explained in terms of the maximally unspecific determinable of spin and its maximally specific determinates. The reader may object here that $\hat{S}_x$, $\hat{S}_z$ and so on, are necessary to explain experimental results. A reply to such an objection is provided below (pp. 23 *ff.*). An argument for their *arbitrariness* is the following. From the point of view of the polarisation vector, pairing two aligned vectors is no better than pairing, say, vectors rotated by 90° clockwise. As we argue in §4.3, the concrete orientations of the polarisation vectors correspond to the maximally specific determinates of spin. From the perspective of the three-dimensional space, there is no privileged way of pairing these maximally specific determinates: 'having spin up along the $x-$axis' is not more similar to 'having spin down along the $x-$axis' than it is to, say, 'having spin up along the $z-$axis.' Considering $\hat{S}_x$, $\hat{S}_y$, $\hat{S}_z$ as representing determinable properties implies that, instead, there is a privileged way of pairing maximally specific determinates.

Furthermore, our account of **Spin Monism** fully captures the physicists' intuition about the meaninglessness of asking the value of spin along a direction when the system is not in one of the eigenstates of the operator connected to it. According to ($\mathbf{SM_{CN}}$), it is meaningless to ask, but the reason why is not the empiricists' dogma that one cannot ask about properties before measurements. Indeed, according to ($\mathbf{SM_{CN}}$) the spin of a (free) quantum system is definite even before any measurement. Instead, it is meaningless in the same way in which it is meaningless to ask "Which determinate of the determinable 'being red' is instantiated?" - when referring to a blue object.

($\mathbf{SM_{CN}}$) is, in a way, an orthodox but *realist* account of spin. Indeed, it can still provide an answer to many paradoxes in an orthodox way. Take, for instance, Bohm (1951)'s retelling of the EPR paradox, where we have a couple of particles entangled on their spin, and we simultaneously measure their spin along different axes. The conclusion of the argument is a disjunction: either the quantum description before the measurement is incomplete or spin along different axes do not correspond to different 'elements of reality,' to put it in Einstein et al. (1935)'s terminology. Friends of ($\mathbf{SM_{CN}}$) explicitly say that components of spin are not properties of the quantum systems under investigation, mimicking the orthodox way of answering that paradox.[26] Historically, the answers to these paradoxes presented by defenders of orthodox QM ended either by accepting a form of antirealism, or by assuming that it is meaningless to ask questions before measurements, or by claiming that QM signals an epistemic limitation intrinsic of human beings' condition (Dorato, 2020). **Spin Monism** allows one to avoid both by presenting a realist understanding of spin in QM. The usual strategies for building a realistic interpretation of QM have been to modify the formalism of orthodox QM.[27] Our account shows instead - as far spin goes - that one can retain a realist and always determinate ontology of quantum properties by *removing* part of its *metaphysical* structure, that is, by refusing to consider some operators as properties.

The reasons above, we contend, clearly show that ($\mathbf{SM_{CN}}$) is worth considering as a serious contender among the accounts of spin. Such an account is, admittedly, quite radical. Indeed, the fact that the whole literature endorses what ($\mathbf{SM_{CN}}$) denies – i.e. that $\hat{S_x}$, $\hat{S_z}$, and the like, are properties – does not come out of the blue. We are aware of how controversial such a view might sound: if one accepts that every observable is a determinable property that can be measured,[28] then ($\mathbf{SM_{CN}}$) can be understood as denying that $\hat{S}_x$, $\hat{S}_z$ and $\hat{S}_y$ are different observables. We suspect that the reason why so many accept that $\hat{S}_x$, $\hat{S}_y$, $\hat{S}_z$ stand for some property has to do with their empirical value: the result of measuring the spin along an axis is that the system measured ends up in an eigenstate of the operator associated to that same axis. Since the state of a system is an eigenstate of some operator after a measurement is performed upon it, given **(EEL)**, it is natural to assume that the operators above of spin along different axes stand for as many measurable determinable properties. Therefore, to be taken seriously, friends of ($\mathbf{SM_{CN}}$) must provide a convincing answer to the following:

The Experimental Challenge: If the operators of spin along different axes are not properties, how can spin monists explain their empirical role, i.e. that they are used in concrete experimental situations to obtain reliable predictions?

[25] Frame of references in relativity are a paradigmatic example; see: Lipman (2020). For arguments in favour of giving ontological weight preferably to invariants only in relativity, see: Gilmore et al. (2016).

[26] For example, see Hughes (1989, p. 159).

[27] For example, GRW modifies Schrödinger's equation, Bohmian mechanics adds the pilot wave, Everettian QM rejects the collapse principle (and some old formulations of it add to each term infinite sets of worlds and a measure given by the Born's rule (Deutsch, 1985)), and so forth. A remarkable exception is Relational QM.

[28] See for instance: Heisenberg (1958).

The following - we contend - is a plausible way of defending (**SM$_{CN}$**). It is true that in the light of **(EEL)** it is natural to read $\hat{S}_x$, $\hat{S}_y$ and $\hat{S}_z$ as properties. However, **(EEL)** by itself does not tell us *which* operators must be understood as determinable properties. The correlation between operators and properties must be argued on other grounds. (**SM$_{CN}$**) can explain experimental results as well as **Spin Pluralism** and (**SM$_{CR}$**), without rejecting **(EEL)**. Eigenstates of $\hat{S}_x$ (or any other operator of spin along a particular axis) are also eigenstates of $\hat{\vec{S}} \cdot \vec{n}$ (because any spin state is an eigenstate of it). Therefore, after a spin measurement, the system is always also in an eigenstate of $\hat{\vec{S}} \cdot \vec{n}$. Hence, friends of (**SM$_{CN}$**) must not renounce to **(EEL)**.

Furthermore, our way of articulating Spin Monism does not deny that $\hat{S}_x$, $\hat{S}_y$ and $\hat{S}_z$ are important from pragmatical point of view.[29] Instead, our view is that these criteria are not by themselves reasons enough to give the operators above an ontological weight. Indeed, physical interactions of a particle and another system, like an inhomogeneous magnetic field, can be explained as an evolution of the polarisation vector only. That is, adding the properties of having spin along different axes to our ontology does not explain more than what is already explained by the determinable represented by $\hat{\vec{S}} \cdot \vec{n}$ and its determinates, i.e. the concrete orientations of the polarisation vector. Roughly, according to (**SM$_{CN}$**), all the metaphysics of spin is contained in $\hat{\vec{S}} \cdot \vec{n}$ because the polarisation vector $\vec{n}$ can represent all the physics (concerning spin). Nonetheless, to fulfil pragmatical roles, one has to extract information about the evolution of the system by projecting the eigenstates of $\hat{\vec{S}} \cdot \vec{n}$ along particular axes - i.e. to use the operators of spin along a given axis. Therefore, we can say that $\hat{S}_x$, $\hat{S}_y$ and $\hat{S}_z$ play many fundamental roles, but not a metaphysical one.

It is now time to answer what we measure when performing a spin measurement along a given axis. We answer that taking some cases of experimental practices seriously clearly suggests that measurements of spin along different axes are not measures of different properties. Instead, they are measures of the direction of the polarisation vector. Consequently, measurements of spin along a definite axis always measure the unique spin property represented by $\hat{\vec{S}} \cdot \vec{n}$, rather than $\hat{S_x}$, $\hat{S_z}$, and so forth. As an example, we briefly consider a problem of quantum tomography: we have a beam of electrons, and we know that the particles in the beam all have the same state of spin, but we ignore which it is. How can we reconstruct it through an experimental procedure? Answer: we perform three measurements of spin along mutually orthogonal directions. We start, for example, by orienting the Stern-Gerlach apparatus along the $z-$axis. The experimental apparatus measures the z component of spin of every particle in the beam. The measurement's result will be that a certain number $|\alpha|^2$ of particles have spin up along the $z-$axis (those deviated upward), and a certain number $|\beta|^2$ have spin down (those deviate downward). If the number of electrons measured is sufficiently large, we can reconstruct[30] from $|\alpha|^2$ and $|\beta|^2$ (by 'reversing', so to speak, the Born's rule), the state of the electrons in the polarised beam written in the $z-$basis. In particular, such a state is a superposition of this form:

$$|\psi\rangle = \alpha|\uparrow\rangle_z + \beta|\downarrow\rangle_z, \tag{12}$$

where $|\alpha|^2 + |\beta|^2 = 1$. It is possible to compute the mathematical expression of the spin state along the $z-$axis by writing down $\vec{n}$ in polar coordinates:

$$\begin{pmatrix} |\uparrow\rangle_n \\ |\downarrow\rangle_n \end{pmatrix} = \begin{pmatrix} e^{-\frac{i\varphi}{2}} cos\left(\frac{\theta}{2}\right) & e^{\frac{i\varphi}{2}} sin\left(\frac{\theta}{2}\right) \\ e^{-\frac{i\varphi}{2}} sin\left(\frac{\theta}{2}\right) & -e^{\frac{i\varphi}{2}} cos\left(\frac{\theta}{2}\right) \end{pmatrix} \begin{pmatrix} |\uparrow\rangle_z \\ |\downarrow\rangle_z \end{pmatrix}. \tag{13}$$

From α (or β) of (12), one can determine a first angle in (13), i.e. θ or φ, along which the spin polarisation vector is pointing. Repeating the same process on two other axes, say the x and $y-$axis, using identically prepared particles, we can fully reconstruct the initial spin state of the system. The measure on the second axis, the $x-$axis say, fixes the angle not fixed by the first measurement, i.e. θ or φ of (13). Finally, the third measure on the $y-$axis fixes the direction of the initial state of spin of the system, i.e. it selects between $|\uparrow\rangle_n$ and $|\downarrow\rangle_n$.

In this case, measurements of spin upon a particular axis just fix the *angle* between the polarisation vector and the spatial axes upon which the Stern-Gerlach apparatuses are aligned. In this example, it seems indubitable that the measurements of spin along different axes are not measurements of different properties, but measures of the direction of the polarisation vector. We take the example above as a paradigmatic case of what happens, according to (**SM$_{CN}$**), whenever we measure the spin along a particular axis: rather than measuring a different spin determinable that the system could instantiate, we measure the angle between the directions of the magnetic field and its currently instantiated maximally specific determinate, represented by the spatial direction along which the polarisation vector is aligned to. Since measurements of spin along different axes correspond to different spatial angles of $\vec{n}$, three of them are necessary to gather enough *information* to reconstruct the initial spin of the system. Nevertheless, one must not be led astray by the pragmatical necessity of these measurements and be convinced that, given their empirical role, then they must have some metaphysical counterparts.

Finally, a further reason supports the idea – that is a direct consequence of (**SM$_{CN}$**) – that every measurement of spin is a measure of the angle between the polarisation vector and the axis along with the measurement is performed. The direction of the polarisation vector alone pictorially illustrates why the evolution of the system is intrinsically indeterministic. Consider the simplest example of an electron with spin up along a particular axes $\hat{n}$ that is measured using a Stern-Gerlach with its magnetic field aligned along the $x-$axis. The polarisation vector will form an angle α with the $x-$axis, as illustrated in Fig. 6. There are two ways in which the vector can align itself to x, i.e. two directions in which it can rotate: it can go over angle α or angle β, and thus end up in state spin up or spin down along the $x-$axis. The evolution is indeterministic because there are no physical reasons why spin should rotate in one direction or the other. Even if the evolution is intrinsically indeterministic, a 'principle of least action', so to speak, seems to hold: the vector has higher chances to rotate through the shortest angle - in Fig. 6, through α rather than β. Indeed, to predict the probability with which the vector will rotate in one sense or the other, one should write the state of the system in the $x-$basis, which means:

$$|\uparrow\rangle_n = \cos\left(\frac{\alpha}{2}\right)|\uparrow\rangle_x + \sin\left(\frac{\alpha}{2}\right)|\downarrow\rangle_x. \tag{14}$$

When $\cos^2\left(\frac{\alpha}{2}\right) > \sin^2\left(\frac{\alpha}{2}\right)$, i.e. when $\alpha < \pi/2$, it is more probable that the polarisation vector will align itself along $|\uparrow\rangle_x$. It is more likely that it ends up in $|\downarrow\rangle_x$ instead when $\alpha > \pi/2$. Pictorially, when the polarisation vector is more tilted towards the direction $\hat{x}$, it is more likely that the outcome of the experiment will be spin up along the $x-$axis. Vice versa, when it is more tilted towards $-\hat{x}$, spin down will be the more probable

[29] With 'pragmatical role,' we mean that, from a mathematical point of view, these operators are useful to reconstruct, e.g., the polarisation vector or the probabilities of the possible outcomes of a measurement (when the polarisation vector is unknown; when it is known instead, $\hat{\vec{S}} \cdot \vec{n}$ suffices).

[30] Clearly, such an operation is not devoid of experimental errors. The accuracy of the reconstructed superposition, shown in (12), is proportional to the number of measured electrons. Given the theoretical focus of the paper, we work here under the idealistic assumption that the number of electrons tends to infinite, thus minimizing any experimental error.

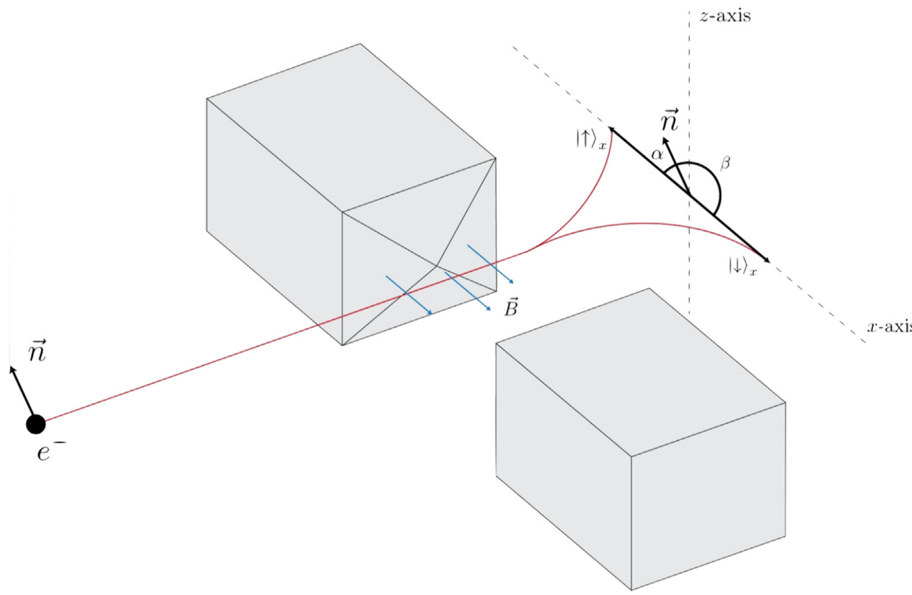

Fig. 6. A particle with polarisation vector $\vec{n}$ passing through a Stern-Gerlach magnet aligned along the $x-$axis.

result. When $\vec{n}$ is perpendicular to the magnetic field, spin up and down are equally probable, as it should be.[31]

6. Conclusions

We presented two ways of interpreting spin operators as properties: **Spin Pluralism**, according to which $\hat{S}_x$, $\hat{S}_z$ and $\hat{S}_y$ stand for maximally unspecific determinables of spin and **Spin Monism**, according to which there is a unique maximally unspecific determinable of spin. We proceeded by presenting two arguments in favour of **Spin Monism** and by pinpointing which operator should be considered as representing the unique maximally unspecific determinable of spin. Then we presented a new account of **Spin Monism**, dubbed **Component Nihilism**, according to which the operators of spin along different axes are just mathematical tools helpful from pragmatic viewpoints but devoid of any ontological meaning. We presented several reasons in favour of such a radical view, and we provide an answer to a compelling question: if these operators are not properties, what do we measure when we perform measurements of spin along different axes? Our answer has been: the angle between the polarisation vector and the direction upon which the measurement is performed. We want to conclude by highlighting some interesting consequences that follow naturally from the work here presented. There have been presented many accounts of superposition in the literature of metaphysics of science (Dorato, 2007; Calosi & Wilson, 2018; Darby & Pickup, 2019, pp. 1–26; Simon, 2018). *Prima facie*, some of these accounts are incompatible with (**SM$_{\mathbf{CN}}$**). The upshot of our view is that superpositions of spin must not be read in metaphysical terms, insofar as the property of spin is always determinate. According to (**SM$_{\mathbf{CN}}$**), spin superposition reflects just the intrinsically indeterministic evolution of quantum properties. Hence, it denies that superposition signals either that the properties above might be indeterminate or that asking about properties before measurements is meaningless.

Finally, the topic faced in the present paper could be interesting to philosophers of physics for a couple of reasons. Both the points are limited to the case of spin, given the scope of the paper; that being said, we think they point to new ways of thinking about quantum properties in general that are worth exploring. Firstly, quantum indeterminacy has been considered for ages as the litmus test of the impossibility of interpreting QM realistically. Almost every realist interpretation of QM, indeed, has tried to 'wash away' such an indeterminacy by modifying the orthodox formulation. (**SM$_{\mathbf{CN}}$**) exemplifies a way of arguing that a quantum property is always determinate - while still exhibiting indeterministic dynamics (see Fig. 6) - without adding any mathematical or metaphysical structure to the theory. Secondly, in the context of QM, people usually speak of observables as if they were determinable properties. If (**SM$_{\mathbf{CN}}$**) is the right view, then it suggests a different interpretation of incompatible observables from the received view: being incompatible is not due to obscure relations between different properties, but because they are mathematically equivalent ways of describing a unique underlying property. As of now, this is just a hypothesis, insofar as our paper is limited to spin only.

Nevertheless, we think that such a hypothesis is interesting and worth exploring. Indeed, if one would show that this approach could be extended to composite systems and other incompatible observables[32] different from - and more interesting than - spin itself, then one may hope to put forward a new realist understanding of QM. The present work aims to be the first step in this direction.

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Conflict of interest

The authors declare that they have no conflict of interest.

[31] We assumed here that the evolution of the state of a system after an interaction with a Stern-Gerlach is instantaneous, in line with the 'collapse principle.' We know that there are better explanations of how a quantum system evolves, i.e. decoherence. We left decoherence out the discussion, for reasons of length. However, note that decoherence would strengthen our claim: decoherence is a gradual process that could show the unitary evolution of the polarisation vector towards $\hat{x}$ or $-\hat{x}$.

[32] How Spin Monism deals with composite systems, and the application of this hypothesis to the cases of position and momentum, coordinates in non-commutative geometries and observables in quantum field theory are topics we hope to cover in the not-too-distant future.

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