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Bouncing pre-big bang on the brane

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Abstract

A regular bouncing universe is obtained in the context of a dilaton-gravity brane world scenario. The scale factor starts in a contracting inflationary phase both in the Einstein and in the string frame, it then undergoes a bounce (due to interaction with the bulk Weyl tensor), and subsequently enters into a decelerated expanding era. This *graceful exit* is obtained at low curvature and low coupling, and without violating the Null Energy Condition.

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I. INTRODUCTION

According to the singularity theorems [1], it is impossible to avoid the big bang singularity within the context of General Relativity, at least if some “reasonable energy conditions” hold. In particular, the Einstein equations tell us that in order to have a *bounce* (namely a transition from a contracting to an expanding phase) in a flat or open universe [18], the so-called Null Energy Condition (NEC) (which for a perfect fluid states $\rho + p \geq 0$) must be violated.

A violation of the NEC is usually considered unacceptable in that it generally implies a violation of causality, but this is not always the case. For example, in various extended theories of gravity it is possible that terms coming from modifications of General Relativity (e.g. close to the Planck scale) give an (apparent, and thus not troublesome) violation of the NEC if they are interpreted as sources in the modified Einstein equations; consequently, they can also drive the evolution of the scale factor through a bounce. In such cases the main theoretical problem does not come from the apparent violation of the NEC, but from our ignorance on the exact nature and form of the new terms; this fact casts some doubt on the reliability of the bouncing solutions that can be obtained in this way.

This is exactly what happens in the pre-big bang model [3], where the connection of the pre- to the post-big bang phase (henceforth, the *graceful exit*) can be actually mapped into a bounce if one uses Einstein frame variables, and thus requires the use of NEC-violating terms [4]. Such terms are already incorporated in the model (they are loop and high curvature corrections dictated by string theory, and the violation of the NEC by means of such effective sources is considered to be acceptable), but our knowledge of their exact form is far from being complete. As a consequence, all the realizations of the graceful exit presented so far [5] can not be taken as completely reliable in that they imply some assumption on the exact form of these loop and high curvature corrections [19].

In the brane world scenario the situation is quite different: due to the presence of interaction terms between the brane and the bulk, a bounce can quite easily be obtained without any NEC violation and without the need of any quantum gravity or high curvature effect. This has been obtained for example in [7] by means of a slight modification of the Randall-Sundrum model [8] (for other considerations on bouncing brane world models, see also [9]).

Generally speaking, one expects a bouncing universe in the brane world scenario to be less difficult to obtain because it does not require a dramatic change in the structure of bulk spacetime. In other words, since in the Randall-Sundrum picture the singularity can be avoided by a timelike trajectory, there is no need to introduce NEC violating sources in order to eliminate it (as is needed in standard cosmology, where the big bang singularity is spacelike): rather, it is sufficient to prevent the brane trajectory from reaching it. So, in some sense, the singularity is still there in the bulk (for which the singularity theorems are, after all, still valid), but simply a brane observer never comes close to it.

This paper is organized as follows: in section II, after having discussed the general conditions for a bounce in a brane world model, I will apply these considerations to a gravi-dilaton scenario, where an explicit solution will be found in the Einstein frame. Brane models in the presence of a scalar field have already been studied in [10], [11] and [12]; in particular some bouncing solutions have already been found in [10] for a domain wall in the presence of a Liouville potential. In the present treatment, which extends the work of [10] in that it is valid for a generic matter content on the brane, the emphasis will be put on the string frame behavior: in fact in section III the bouncing solution will be transformed in the string frame and it will be compared with what one obtains in the pre-big bang model, whose basic features will also be briefly recalled. It will be shown that such a solution can be seen as a realization of a *graceful exit* in a pre-big bang model, and that this exit can be realized without any use of quantum or high curvature corrections, and is thus under complete control. This is a concrete realization of the idea (suggested in [12]) that in brane world models there is room for dealing with problems which are difficult or intractable in more conventional scenarios.

A few comments and remarks will then conclude the paper.

II. BOUNCING UNIVERSE IN A BRANE WORLD SCENARIO

A. General conditions for a bounce

Following [13], we write the projected Einstein equations on the brane as

$$G_{\mu\nu} = 8\pi G_N \tau_{\mu\nu} + \frac{d-1}{d} \kappa^2 \left[\parallel T_{\nu\mu} + \frac{1}{d+1} (d^\perp T - \parallel T) h_{\mu\nu} \right] - \Lambda_d h_{\mu\nu} + \kappa^4 \pi_{\mu\nu} - E_{\mu\nu}, \quad (1)$$

where $\tau_{\mu\nu}$ is the energy momentum tensor on the brane, $\parallel T_{\mu\nu}$ and $\perp T$ are, respectively, the projections parallel and perpendicular to the brane of the bulk energy momentum tensor[20], Λ_d the effective cosmological constant on the brane, $E_{\mu\nu}$ the projection on the brane of the bulk Weyl tensor, $h_{\mu\nu}$ the induced metric on the $(d + 1)$ -brane, κ the bulk gravitational coupling, G_N the effective brane Newton constant and, finally, $\pi_{\mu\nu}$ a term quadratic in $\tau_{\mu\nu}$ given by the following expression:

$$\pi_{\mu\nu} = -\frac{1}{4}\tau_{\alpha\mu}\tau_{\nu}^{\alpha} + \frac{1}{4d}\tau\tau_{\mu\nu} + \frac{1}{8}\tau^{\alpha\beta}\tau_{\alpha\beta}h_{\mu\nu} - \frac{1}{8d}\tau^2h_{\mu\nu}. \quad (2)$$

By making a flat FRW ansatz for $h_{\mu\nu}$ (as already stated, curvature-driven bounces will not be considered here), and a perfect fluid one for the energy momentum tensors $\tau_{\nu}^{\mu} \equiv (-\rho, \vec{p})$, $\parallel T_{\nu}^{\mu} \equiv (-R, \vec{P})$, one can write an equation for the time derivative of the Hubble parameter on the brane:

$$\dot{H} = -\frac{1}{d}\kappa^2(R + P) - \left(\frac{8\pi G_N}{d-1} + \frac{\kappa^4}{4d}\rho\right)(\rho + p) - \frac{d+1}{d(d-1)}E_0^0. \quad (3)$$

As can be read in this equation, if the Weyl tensor is set to zero, it is impossible to have $\dot{H} > 0$, a necessary condition for a bounce, without violating the NEC either in the bulk or on the brane, or without having $\rho < 0$ (this last condition deriving from the term $\rho(\rho + p)$, which comes from the explicit expression of $\pi_{\mu\nu}$ in terms of ρ and p). Thus, if one wants to realize a bounce without resorting to “exotic” forms of matter, one has to consider the case of a nonvanishing projected Weyl tensor. More precisely:

$$E_0^0 < 0 \quad \longrightarrow \quad \dot{H} > 0. \quad (4)$$

The bouncing solution presented in [7] exploits exactly this kind of mechanism.

B. Gravi-dilaton system and relevant equations

With the previous condition in mind, let's consider the following gravi-dilaton system in the Einstein frame

$$S = \frac{1}{\kappa^2} \int d^{d+2}x \sqrt{-g} \left[\mathcal{R} - \frac{1}{2}(\nabla\Phi)^2 - 2\Lambda e^{\gamma\Phi} \right] + \int d^{d+1}y \left[\sqrt{-h} \left(\frac{2K^{\pm}}{\kappa^2} + \lambda e^{\alpha\Phi} \right) - \mathcal{L}_m(\chi_m; \tilde{h}_{\mu\nu}) \right], \quad (5)$$

where the brane matter fields χ_m are coupled to the dilaton through the conformally rescaled brane metric $\tilde{h}_{\mu\nu}$, defined as

$$\tilde{h}_{\mu\nu} = e^{2\xi\Phi} h_{\mu\nu}. \quad (6)$$

The bulk equations of motion and junction conditions are immediately derived (Z_2 symmetry is assumed):

$$\begin{cases} \mathcal{R}_{\mu\nu} = \frac{1}{2}\nabla_\mu\Phi\nabla_\nu\Phi + \frac{2}{d}g_{\mu\nu}\Lambda e^{\gamma\Phi} \\ \nabla^2\Phi = 2\Lambda\gamma e^{\gamma\Phi} \end{cases}, \quad (7)$$

$$\begin{cases} K_{\mu\nu} = -\frac{\kappa^2}{4}\left[\tau_{\mu\nu} - \frac{1}{d}\left(\tau - \lambda e^{\alpha\Phi}\right)h_{\mu\nu}\right] \\ n \cdot \nabla\Phi = \frac{\kappa^2}{2}\left(-\xi\tau + \alpha\lambda e^{\alpha\Phi}\right) \end{cases}, \quad (8)$$

being n_μ the normal to the brane pointing *into* the bulk, $K_{\mu\nu} \equiv \nabla_\mu n_\nu$, and $\tau_{\mu\nu} \equiv -\frac{2}{\sqrt{-h}}\frac{\delta\mathcal{L}_m}{\delta h^{\mu\nu}}$.

If one makes the following static ansatz for the bulk metric

$$ds_{bulk}^2 = -F(r)^2 dT^2 + B(r)^2 dr^2 + A(r)^2 d\vec{x}_d^2, \quad (9)$$

which implies

$$ds_{brane}^2 = -dt^2 + A[r(t)]^2 d\vec{x}_d^2, \quad (10)$$

then the junction conditions can be explicitly written as

$$H^2 \equiv \left(\frac{\dot{A}}{A}\right)^2 = \frac{\kappa^4}{16d^2}(\rho + \rho_\lambda)^2 - \left(\frac{A'}{AB}\right)^2, \quad (11)$$

$$\dot{\rho} + H \left[d(1+w)\rho + \alpha \left(\Phi' \frac{A}{A'}\right) \rho_\lambda + \frac{1}{2d} \left(\Phi' \frac{A}{A'}\right)^2 (\rho + \rho_\lambda) \right] = 0, \quad (12)$$

$$(\rho + \rho_\lambda)\Phi' \left(\frac{A}{A'}\right) = 2d\xi\rho(dw - 1) - 2d\alpha\rho_\lambda, \quad (13)$$

where $\dot{A} \equiv \frac{dA}{dt}$, $A' \equiv \frac{\partial A}{\partial r}$, $w \equiv \frac{p}{\rho}$, and $\rho_\lambda \equiv \lambda e^{\alpha\Phi}$. The first two equations, coming from the junction conditions for $K_{\mu\nu}$, can be seen as the effective Friedmann equation and energy conservation equation on the brane (note the anomalous terms containing Φ' in the latter, coming from the interactions of the dilaton with the brane), while the last one is the junction condition for the dilaton.

C. Bulk solution and brane equation

It is now sufficient to know an explicit bulk solution and to plug it in eqns. (11), (12), (13) in order to have an explicit set of equations for the evolution of the brane. All the static bulk solutions with the symmetry given by eq. (9) are known (see [14]); here only the following simple class of solutions is considered:

$$\left\{ \begin{array}{l} ds_{bulk}^2 = -h(r)r^{s+\frac{2}{d}}dt^2 + \frac{r^{s+\frac{2}{d}}}{h(r)}dr^2 + r^{\frac{2}{d}}d\vec{x}_d^2 \\ \left[h(r) = \left(\frac{2\Lambda}{s}r^{-s}e^{\gamma\Phi_0} + C \right), s = \frac{\gamma^2}{2} - \frac{d+1}{d} \right], \\ \Phi = \Phi_0 - \gamma \log r. \end{array} \right. \quad (14)$$

Particularly relevant for the following will be the role of the integration constant C , which can be shown to be zero *iff* the bulk Weyl tensor is vanishing.

Now one has to plug the bulk solution into the explicit junction conditions. First of all, eq. (13) gives two constraints on the parameters of the model, namely

$$\gamma = -2\xi(dw - 1), \quad \alpha = \frac{\gamma}{2}. \quad (15)$$

Then, since the dilaton behavior is known, the energy conservation equation (12) can be used to find out how the energy density depends on r , and thus on $A(r) = r^{\frac{1}{d}}$: one finds

$$\rho = R_0 A^{-d\left(1+w+\frac{\gamma^2}{2}\right)}, \quad (16)$$

with R_0 an integration constant. A similar equation for ρ_λ follows trivially from its definition in terms of the dilaton.

Finally, everything can be plugged in the equation for H , which one can rewrite as

$$H^2 = \frac{\kappa^4}{16d^2} \left(\rho^2 + 2\rho_\lambda\rho + \rho_\lambda^2 \right) - \frac{1}{d^2} \left[\frac{2\Lambda}{s}e^{\gamma\Phi_0}A^{-d\gamma^2} + CA^{-d\frac{\gamma^2}{2}-d-1} \right]. \quad (17)$$

By taking into account of eq. (16), one can see that without the contribution of the Weyl tensor, it is impossible to obtain a bouncing solution with $w \geq -1$ (as is implied by the NEC and by the positivity ρ). In fact, under these assumptions, the first term on the r.h.s. of (17) turns out always to be dominant over the others for small A , and consequently the solution can not be driven towards a bounce. Thus the only strategy for getting a bouncing solution is to consider the action of the last term, the one related to the Weyl tensor.

Moreover, the definition of ρ_λ in terms of Φ implies that ρ_λ^2 has exactly the same A -dependence of the first term in the squared brackets in eq. (17). One can take advantage of this fact by requiring an exact cancellation between these two terms in the effective Friedmann equation, cancellation that is realized if $\frac{\kappa^4}{32}\lambda^2 = \frac{\Lambda}{s}$. This condition is the counterpart of the fine tuning required in the Randall-Sundrum model between the brane tension and the bulk cosmological constant in order to have a vanishing effective brane cosmological constant.

D. Getting the bounce

Eq. (17) can be rearranged in such a way that it has the form

$$\frac{1}{2}\dot{A}^2 + V(A) = 0, \quad (18)$$

so that the evolution of the system can be easily studied as the evolution of a point particle of unit mass and zero energy which moves in the potential $V(A)$ given by

$$V(A) = \frac{1}{2d^2} \left[CA^{p_1} - \frac{\kappa^4}{16} \left(R_0^2 A^{p_2} + 2\lambda R_0 e^{\frac{\gamma}{2}\Phi_0} A^{p_3} \right) \right], \quad (19)$$

with

$$p_1 = -\frac{d\gamma^2}{2} - d + 1, \quad p_2 = -2d(1+w) - d\gamma^2 + 2, \quad p_3 = p_2 + d(1+w). \quad (20)$$

The parameters γ and ξ can be now chosen in such a way that $V(A)$ has the appropriate form (shown in figure 1(b)) for producing the desired behavior, namely:

1. bounce:

$$V(A) > 0 \text{ as } A \rightarrow 0^+ \implies p_1 < p_2, \quad C > 0, \quad (21)$$

2. correct behavior at late time (decelerated expansion):

$$V(A) \rightarrow 0^- \text{ as } A \rightarrow \infty \implies p_3 < 0, \quad (22)$$

These conditions ensure also that at late times the Friedmann equation is dominated by the term $\sim \kappa^4 \rho_\lambda \rho$, thus recovering Einstein gravity with an effective (dilaton-dependent) Newton constant proportional to $\kappa^4 \rho_\lambda$.

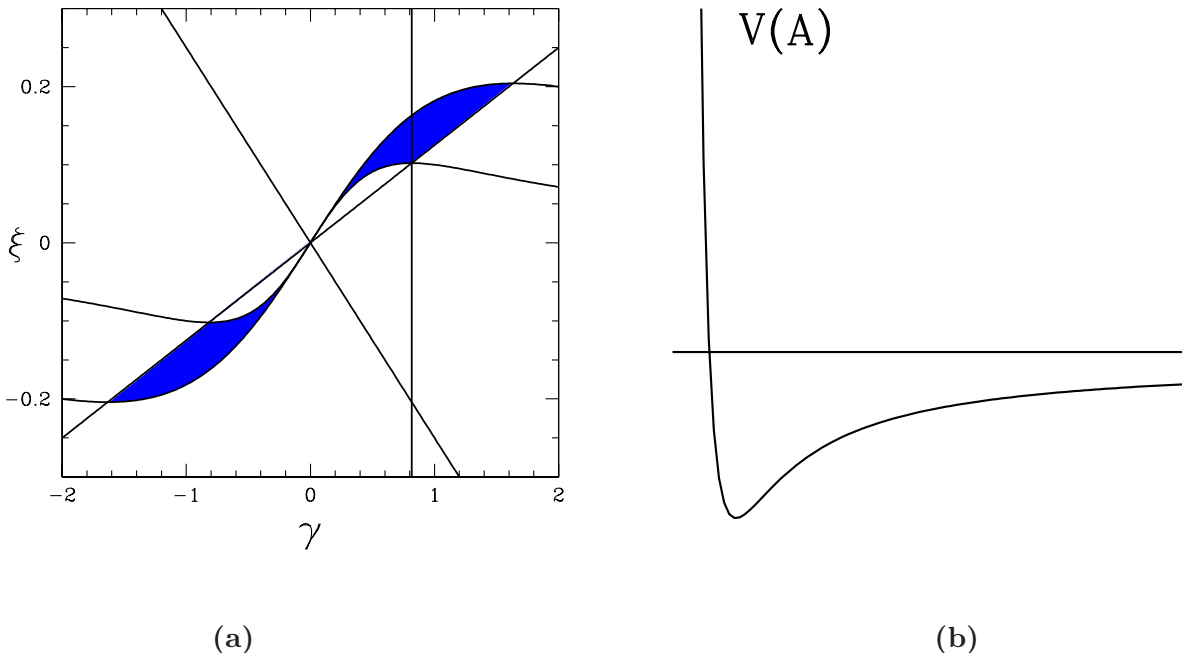


FIG. 1: On the left (a), the shaded region indicates the part of the (γ, ξ) plane for which the potential $V(A)$ has the form displayed on the right (b), with the supplemental condition $|w| \leq 1$; the vertical line is related to the string frame behavior (see paragraph III B). The case $d = 3$ has been displayed, the other cases being qualitatively analogous.

As well as eqns. (21,22), the condition $|w| \leq 1$ should also be imposed, thus giving

$$(1 - d) \leq \frac{\gamma}{2\xi} \leq (1 + d). \quad (23)$$

All the conditions (21), (22) and (23) can be simultaneously satisfied, and the region in the (γ, ξ) plane for which this happens is displayed in fig. 1(a).

It can be shown that in the allowed region one has always $s < 0$, which in turn implies $\Lambda < 0$ (this last condition coming from the fine tuning relation $\frac{\kappa^4}{32}\lambda^2 = \frac{\Lambda}{s}$). Thus, the region of parameter space of interest for the bounce is also the one for which the bulk has a negative Λ , which is expected to provide the confinement of a massless graviton mode on the brane.

It should also be noted that the conditions $C > 0$, $s < 0$ and $\Lambda < 0$ imply the existence of a naked singularity in the bulk at $r = 0$. In the present context, this should not be regarded as a problem: clearly, for small r the equations of motion are not valid anymore and should be replaced by a more complete description (like string theory with loop and curvature corrections included), which should be capable of damping the singularity, or at

least hiding it behind an horizon. Though at the present stage we do not know how this mechanisms work in detail, we will show in brief that there are solutions for which the brane never comes close to the high curvature region: thus the brane never exits the region of validity of General Relativity (or low energy string theory) and its evolution can be studied without knowing the details of the high curvature regime.

The evolution of the universe can be read directly in figure 1(b): the scale factor starts very large, it then undergoes a phase of accelerated contraction, which is an inflationary phase, being characterized by $\dot{A}\ddot{A} > 0$ (see [15]), until it reaches the minimum of the potential. Then the contraction rate slows down, until it becomes zero (bounce) and a period of accelerated expansion starts (another inflationary stage); finally, after having passed again through the minimum of the potential, the brane enters a stage of decelerated expansion, eventually dominated by the term $\sim G_N(\Phi)\rho$. More precisely, at late times the Friedmann equation approaches the following form:

$$H^2 \simeq \frac{\kappa^4}{16d^2} 2\lambda R_0 e^{\frac{\gamma}{2}\Phi_0} A^{-d(1+w)-d\gamma^2} . \quad (24)$$

Due to the running of the dilaton, the exponent of A is not yet the one which is found in standard cosmology. This means that, for instance, a fluid with $w = 1/d$ (radiation) would not induce the expected behavior $\sim A^{-(d+1)}$ in the r.h.s. of eq. (24). As is typical in dilatonic models, this can be obtained only after having achieved dilaton stabilization, a task which will not be pursued here.

Incidentally, we notice that because of the running of the dilaton it is possible to have a radiation-like behavior in the Friedmann equation *without radiation*, i.e. with $w \neq 1/d$: it is in fact sufficient to tune γ^2 in such a way that the exponent of A in eq. (24) is equal to $-(d+1)$. A quick calculation, and comparison with eq. (15), shows that this “ghost radiation” scenario is realized for

$$\gamma = \frac{1}{2d\xi} , \quad (25)$$

and that the curve defined by this equation passes through the allowed region in the (γ, ξ) plane, meaning that this condition is compatible with all the others that have been previously imposed.

III. STRING FRAME BEHAVIOR

A. Pre-big bang model

Let us now briefly recall the basic features of the graceful exit problem in pre-big bang model. The starting point is the minimal gravitational sector contained in the low energy effective action of any critical superstring theory, which in the string frame has the following form:

$$\mathcal{S} = \frac{1}{\lambda_S^{D-2}} \int d^D x \sqrt{g} e^{-\Phi} [\mathcal{R} + (\nabla\Phi)^2], \quad (26)$$

where λ_S the string length, and $10 - D$ spatial dimensions are assumed to be compactified and frozen. If one makes a flat cosmological ansatz, the equations of motion can be written as:

$$\begin{cases} \dot{H} = \pm \sqrt{D-1} H^2 \\ \dot{\Phi} = (D-1 \pm \sqrt{D-1}) H. \end{cases} \quad (27)$$

In the pre-big bang model, one has two disconnected branches of solution: the first solves the equations of motion with the (+) sign, has as initial conditions a Minkowski space with constant dilaton, and describes a pre-big bang phase of superinflationary expansion and growing dilaton (which means growing coupling constant); the second, separated from the first by the big bang singularity, solves the equations with the (-) sign, and describes a phase of decelerated expansion, which is supposed to be joined in a later time to a radiation dominated phase with stabilized dilaton.

The graceful exit problem consists in joining these two branches and thus ending with a single smooth evolution for the universe. Such a problem can be studied also in the Einstein frame, i.e. by conformally transforming the metric in such a way that the action (26), expressed in terms of the new metric, describes a General Relativity model minimally coupled to the dilaton, exactly like the bulk part of eq. (5) (with Λ set to zero).

The computation of the Einstein frame Hubble parameter is straightforward

$$g_{\mu\nu}^E = g_{\mu\nu}^S e^{-\frac{2\Phi_S}{D-2}} \implies H_E = e^{-\frac{\Phi_S}{D-2}} \left[H_S - \frac{\dot{\Phi}_S}{D-2} \right], \quad (28)$$

where the exponential prefactor comes from the fact that the cosmic times in the Einstein frame and in the string frame are related by $dt_E = e^{\frac{\Phi_S}{D-2}} dt_s$. One can immediately check

with the help of eqns. (27) that H_E is negative in the pre-big bang phase and positive in the post-big bang one, so that the graceful exit problem can be actually mapped into that of having a bouncing universe in the Einstein frame.

In the pre-big bang model, this is achieved by considering the effect of loop and high curvature corrections, which become important as the solution approaches the singularity, and which can be seen as effective NEC violating sources that provide the bounce. As already mentioned in the introduction, here one encounters a technical problem in that our knowledge of the exact form of these corrections is still too limited.

In string cosmology it is useful to represent the solution in the $(\dot{\Phi}, H)$ plane, with $\dot{\Phi} \equiv \dot{\Phi} - dH$. The origin of such a plane is Minkowski space with a constant dilaton and represents, as well as the initial condition for the pre-big bang model, also an attractor for the late time evolution. With reference to figure 2, the pre- and post-big bang branches are represented by the two lines in the upper part of the plane, and realizing the graceful exit (dashed red arrow) consists in connecting the pre-big bang line, which escapes from the origin, to the post-big bang one. The evolution described by the ekpyrotic scenario [16] can also be represented in this plane: in this case the pre-big bang stage is represented by the line in the lower part of the plane, and describes a phase of accelerating contraction for the scale factor on the brane, which corresponds also to the fact the two branes of the model are approaching each other. In the ekpyrotic model the problem of joining this line to the post-big bang one is in some sense ignored, in that the scale factor is let shrink to zero (the two branes collide as the evolution proceeds along the pre-big bang line until infinity), and then starts expanding again (the branes pass across each other, and the evolution now proceeds from infinity towards the origin along the post-big bang line).

B. Comparison with the bouncing brane

Although the variable $\dot{\Phi}$ does not have a particular dynamical meaning in the present context, the solution found in the previous section can nevertheless be mapped in the same plane, in order to conveniently compare it with the other models. In order to do that, it is necessary to transform it in the string frame, via

$$A_S = A_E e^{\frac{\Phi_E}{\sqrt{2d}}} = e^{\frac{\Phi_0}{2d}} A_E^{1-\gamma} \sqrt{\frac{d}{2}}. \quad (29)$$

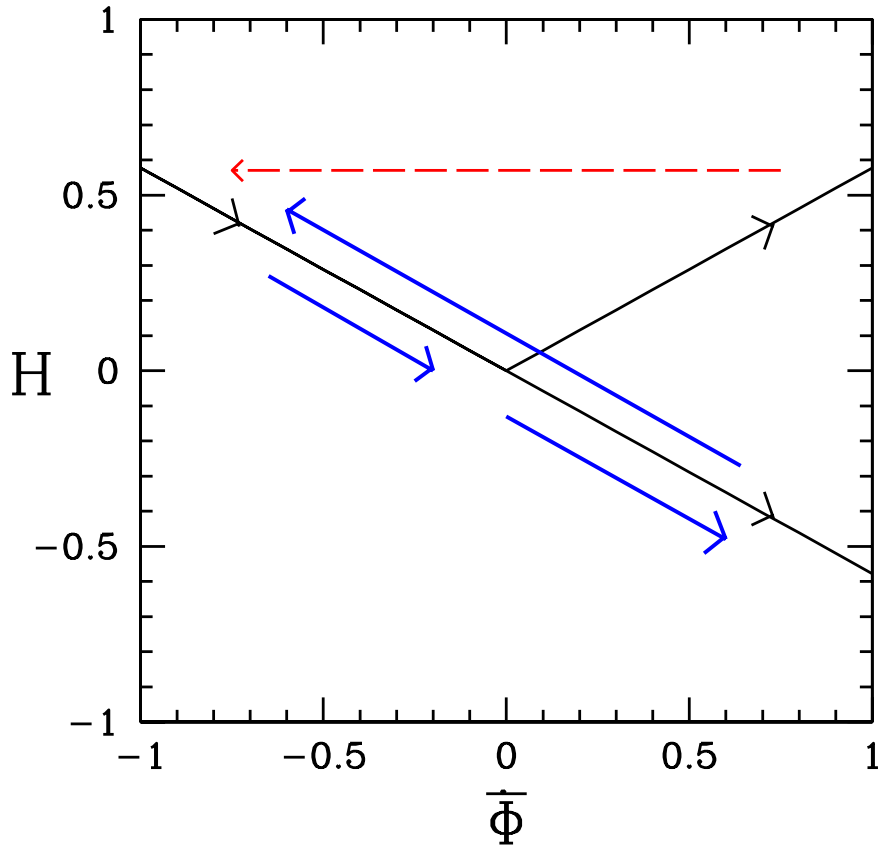


FIG. 2: Different routes towards the exit in the pre-big bang model, in the ekpyrotic one, and in the scenario analyzed in the present work.

We now restrict our attention to the case

$$\gamma < \sqrt{\frac{2}{d}} \quad (30)$$

in such a way that to an expanding Einstein frame scale factor, also an expanding string frame one corresponds. The condition (30) is satisfied at the left of the vertical line in figure 1(a); note that γ is still allowed to be either positive or negative.

Given eqns. (29), the behavior of $\dot{\Phi}_S$ can be found:

$$\Phi_S = \sqrt{\frac{d}{2}} \Phi_E \quad \Rightarrow \quad \dot{\Phi}_S = -\frac{d}{1 - \gamma\sqrt{\frac{d}{2}}} H_S \quad \left(\Rightarrow \frac{\dot{\Phi}_S}{H_S} = \text{const.} < 0 \right). \quad (31)$$

Thus, as in the ekpyrotic case, the solution still moves along a line of negative slope in the $(\dot{\Phi}, H)$ plane, but in this case it follows a somehow less drastic route to the exit, simply

reversing at some point its motion along this line, passing through the origin (bounce), and then being smoothly connected to the decelerated expanding phase. Such a behavior is represented by the blue arrows in figure 2.

Differently from what happens in the standard pre-big bang case, now the exit can be realized at low curvature and low coupling. To ensure the first point, it is sufficient that the free parameter C is big enough; as to the second request, one can take $\gamma < 0$, so that (as eq. (14) tells) if the coupling is small at large A (certainly a desirable feature, if one wants this model to be connected to our universe at late time), then it will be even smaller during the bouncing phase. Thus, the whole evolution takes place in a regime under full theoretical control.

IV. COMMENTS AND CONCLUSIONS

The solution discussed in this work is an example of bouncing universe in the context of Einstein brane gravity coupled to a scalar field. When transformed in the string frame, such a solution describes a pre-big bang model where the transition between pre- and post-big bang is realized at low curvature and weak coupling, thus providing an example of graceful exit which takes place under complete theoretical control.

The investigation focused on the graceful exit aspect, while other issues, such as the naturalness of the initial conditions, the amount of inflation, the dilaton stabilization, have not been addressed, partly because this goes beyond the purposes of this paper, and partly for a more technical reason. In fact, while from the discussion of paragraph II A it should be clear that the bouncing mechanism is not related to the particular kind of bulk background chosen for the present analysis (depending only on the sign of the bulk Weyl tensor), on the other hand the same can not be said about other important features of the cosmological behavior.

The quantity of inflation produced in this model certainly depends on the form of the potential $V(A)$, and thus on the exact form of the bulk background. Also the initial configuration does, and actually one could argue that a brane which starts falling towards a naked singularity does not look natural at all; this problem could be addressed by moving to more general, time dependent bulk backgrounds, maybe like the one discussed in [17] in the context of initial conditions for the pre-big bang model.

On the other hand, the background solution chosen in this work is simple enough for emphasizing the main point of this paper, i.e. that in brane world models it is possible to realize a graceful exit without resorting either to quantum gravity (or stringy) corrections, or to exotic forms of matter. The embedding of the mechanism studied in the present paper into more complete cosmological scenarios is left for future investigations.

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- [1] S. W. Hawking and G. F. R. Ellis, *The large scale structure of spacetime*, (Cambridge university Press, Cambridge, 1973);
 - [2] D. N. Page, *A fractal set of perpetually bouncing universes?*, *Class. Quant. Grav.* **1**, (1984) 417,
 - [3] G. Veneziano, *Scale factor duality for classical and quantum strings*, *Phys. Lett.* **B265** (1991) 287-294; M. Gasperini, and G. Veneziano, *Pre - big bang in string cosmology*, *Astropart. Phys.* **1** (1993) 317-339, hep-th/9211021; for a recent review, see M. Gasperini and G. Veneziano, *The pre-big bang scenario in string cosmology*, *Phys. Rept.* **373** (2003) 1-212, hep-th/0207130;
 - [4] R. Brustein and R. Madden, *Graceful exit and energy conditions in string cosmology*, *Phys. Lett.* **B410** (1997) 110-118, hep-th/9702043;
 - [5] See for instance R. Brustein and R. Madden, *A model of graceful exit in string cosmology*, *Phys. Rev.* **D57** (1998) 712-724, hep-th/9708046; S. Foffa, M. Maggiore and R. Sturani, *Loop corrections and graceful exit in string cosmology*, *Nucl. Phys.* **B552** (1999) 395-419, hep-th/9903008; C. Cartier, E. J. Copeland and R. Madden, *The graceful exit in string cosmology*, *JHEP* **01** (2000) 035, hep-th/9910169;
 - [6] J. C. Fabris, R. G. Furtado, N. Pinto-Neto and P. Peter, *Regular cosmological solutions in low energy effective action from string theories*, hep-th/0212312;
 - [7] S. Mukherji and M. Peloso, *Bouncing and cyclic universes from brane models*, *Phys. Lett.* **B547** (2002) 297-305, hep-th/0205180;
 - [8] L. Randall and R. Sundrum, *An alternative to compactification*, *Phys. Rev. Lett.* **83** (1999) 4690-4693, hep-th/9906064;
 - [9] S. Nojiri, S. D. Odintsov and S. Ogushi, *Friedmann-Robertson-Walker brane cosmological equations from the five-dimensional bulk (A)dS black hole*, *Int. J. Mod. Phys.* **A17** (2002) 4809-4870, hep-th/0205187; Y. S. Myung, *Bouncing and cyclic universes in the charged AdS bulk*

- background*, *Class. Quant. Grav.* **20** (2003) 935-948, hep-th/0208086; Y. Shtanov and V. Sahni, *Bouncing braneworlds*, *Phys. Lett.* **B557** (2003) 1-6, gr-qc/0208047; A. J. M. Medved, *Of bounces, branes and bounds*, hep-th/0301010; A. Biswas, S. Mukherji, and S. S. Pal, *Nonsingular cosmologies from branes*, hep-th/0301144; P. Kanti and K. Tamvakis, *Challenges and obstacles for a bouncing universe in brane models* hep-th/0303073;
- [10] H. A. Chamblin and H. S. Reall, *Dynamic dilatonic domain walls*, *Nucl. Phys.* **B562** (1999) 133-157, hep-th/9903225.
- [11] C. Csaki, J. Erlich, C. Grojean and T. J. Hollowood, *General properties of the self-tuning domain wall approach to the cosmological constant problem*, *Nucl. Phys.* **B584** (2000) 359-386, hep-th/0004133; K. Maeda and D. Wands, *Dilaton-gravity on the brane*, *Phys. Rev.* **D62** (2000) 124009, hep-th/0008188; H. Ochiai and K. Sato, *Vacuum brane and the bulk dynamics in dilatonic brane world*, *Phys. Lett.* **B503** (2001) 404-412, hep-th/0010163; A. Feinstein, K. E. Kunze and M. A. Vazquez-Mozo, *Curved dilatonic brane worlds*, *Phys. Rev.* **D64** (2001) 084015, hep-th/0105182; D. Langlois and M. Rodriguez-Martinez, *Brane cosmology with a bulk scalar field*, *Phys. Rev.* **D64** (2001) 123507, hep-th/0106245; L. E. Mendes and A. Mazumdar, *Brans-Dicke brane cosmology*, *Phys. Lett.* **B501** (2001) 249-256, gr-qc/0009017; S. Nojiri, O. Obregon, S. D. Odintsov and V. I. Tkach, *String versus Einstein frame in AdS/CFT induced quantum dilatonic brane-world universe*, *Phys. Rev.* **D64** (2001) 043505, hep-th/0101003; P. Brax, D. Langlois and M. Rodriguez-Martinez, *Fluctuating brane in a dilatonic bulk*, hep-th/0212067;
- [12] S. Foffa, *Pre-big bang on the brane*, *Phys. Rev.* **D66** (2002) 063512, hep-th/0207103;
- [13] T. Shiromizu, K. Maeda and M. Sasaki, *The Einstein equations on the 3-brane world*, *Phys. Rev.* **D62** (2000) 024012, gr-qc/9910076;
- [14] C. Charmousis, *Dilaton spacetimes with a Liouville potential*, *Class. Quant. Grav.* **19** (2002) 83-114, hep-th/0107126;
- [15] M. Gasperini, and G. Veneziano, *Inflation, deflation, and frame independence in string cosmology*, *Mod. Phys. Lett.* **A8** (1993) 3701-3714, hep-th/9309023;
- [16] J. Khoury, B. A. Ovrut, P. J. Steinhardt and N. Turok, *The ekpyrotic universe: Colliding branes and the origin of the hot big bang*, *Phys. Rev.* **D64** (2001) 123522, hep-th/0103239;
- [17] A. Buonanno, T. Damour, and G. Veneziano, *Pre-big bang bubbles from the gravitational instability of generic string vacua*, *Nucl. Phys.* **B543** (1999) 275-320, hep-th/9806230.

- [18] In the case of a closed universe it is possible to find bouncing solutions without violating the NEC. However, such solutions are somehow pathological, being them unstable, or requiring some kind of fine tuning, see for example [2]; in the present treatment curvature-driven bouncing solutions will not be considered.
- [19] In general Brans-Dicke models the situation is more variegated and a bouncing behavior can be obtained in some cases, see [6].
- [20] In the Randall-Sundrum model only a cosmological constant is allowed to be present in the bulk, according to the prescription that the matter fields should be confined on the brane. Since in this work we are going to analyze a gravi-dilaton system, the dilaton field is also allowed to propagate in the bulk, and $T_{\mu\nu}$ is meant to represent the bulk energy momentum tensor related to this field.