Solving Ordinary Differential Equations I. Nonstiff Problems

Hairer, Ernst; Norsett, Syvert Paul; Wanner, Gerhard

This publication URL: https://archive-ouverte.unige.ch/unige:12346
Publication DOI: 10.1007/978-3-540-78862-1

Springer Series in Computational Mathematics

8

E. Hairer
S. P. Nørsett
G. Wanner

Solving Ordinary Differential Equations I

Nonstiff Problems

Second Revised Edition
With 135 Figures

Springer

Ernst Hairer
Gerhard Wanner

Université de Genève
Section de Mathématiques
2–4 rue du Lièvre
1211 Genève 4
Switzerland
Ernst.Hairer@math.unige.ch
Gerhard.Wanner@math.unige.ch

Syvert P. Nørsett

Norwegian University of Science
and Technology (NTNU)
Department of Mathematical Sciences
7491 Trondheim
Norway
norsett@math.ntnu.no

Corrected 3rd printing 2008

ISBN 978-3-540-56670-0 e-ISBN 978-3-540-78862-1

DOI 10.1007/978-3-540-78862-1

Springer Series in Computational Mathematics ISSN 0179-3632

Library of Congress Control Number: 93007847

Mathematics Subject Classification (2000): 65Lxx, 34A50

Cover design: WMX Design GmbH, Heidelberg
Typesetting: by the authors
Production: LE-TEX Jelonek, Schmidt & Vöckler GbR, Leipzig

Printed on acid-free paper

9 8 7 6 5 4 3 2 1

springer.com

This edition is dedicated to

Professor John Butcher

on the occasion of his 60th birthday

His unforgettable lectures on Runge-Kutta methods, given in June 1970 at the University of Innsbruck, introduced us to this subject which, since then, we have never ceased to love and to develop with all our humble abilities.

From the Preface to the First Edition

> So far as I remember, I have never seen an Author's Preface which had any purpose but one — to furnish reasons for the publication of the Book. (Mark Twain)

> Gauss' dictum, "when a building is completed no one should be able to see any trace of the scaffolding," is often used by mathematicians as an excuse for neglecting the motivation behind their own work and the history of their field. Fortunately, the opposite sentiment is gaining strength, and numerous asides in this Essay show to which side go my sympathies. (B.B. Mandelbrot 1982)

> This gives us a good occasion to work out most of the book until the next year. (the Authors in a letter, dated Oct. 29, 1980, to Springer-Verlag)

There are two volumes, one on non-stiff equations, ..., the second on stiff equations, ... The first volume has three chapters, one on classical mathematical theory, one on Runge-Kutta and extrapolation methods, and one on multistep methods. There is an Appendix containing some Fortran codes which we have written for our numerical examples.

Each chapter is divided into sections. Numbers of formulas, theorems, tables and figures are consecutive in each section and indicate, in addition, the section number, but not the chapter number. Cross references to other chapters are rare and are stated explicitly. ... References to the Bibliography are by "Author" plus "year" in parentheses. The Bibliography makes no attempt at being complete; we have listed mainly the papers which are discussed in the text.

Finally, we want to thank all those who have helped and encouraged us to prepare this book. The marvellous "Minisymposium" which G. Dahlquist organized in Stockholm in 1979 gave us the first impulse for writing this book. J. Steinig and Chr. Lubich have read the whole manuscript very carefully and have made extremely valuable mathematical and linguistical suggestions. We also thank J.P. Eckmann for his troff software with the help of which the whole manuscript has been printed. For preliminary versions we had used textprocessing programs written by R. Menk. Thanks also to the staff of the Geneva computing center for their help. All computer plots have been done on their beautiful HP plotter. Last but not least, we would like to acknowledge the agreable collaboration with the planning and production group of Springer-Verlag.

October 29, 1986 The Authors

Preface to the Second Edition

The preparation of the second edition has presented a welcome opportunity to improve the first edition by rewriting many sections and by eliminating errors and misprints. In particular we have included new material on

- Hamiltonian systems (I.14) and symplectic Runge-Kutta methods (II.16);
- dense output for Runge-Kutta (II.6) and extrapolation methods (II.9);
- a new Dormand & Prince method of order 8 with dense output (II.5);
- parallel Runge-Kutta methods (II.11);
- numerical tests for first- and second order systems (II.10 and III.7).

Our sincere thanks go to many persons who have helped us with our work:

- all readers who kindly drew our attention to several errors and misprints in the first edition;
- those who read preliminary versions of the new parts of this edition for their invaluable suggestions: D.J. Higham, L. Jay, P. Kaps, Chr. Lubich, B. Moesli, A. Ostermann, D. Pfenniger, P.J. Prince, and J.M. Sanz-Serna.
- our colleague J. Steinig, who read the entire manuscript, for his numerous mathematical suggestions and corrections of English (and Latin!) grammar;
- our colleague J.P. Eckmann for his great skill in manipulating Apollo workstations, font tables, and the like;
- the staff of the Geneva computing center and of the mathematics library for their constant help;
- the planning and production group of Springer-Verlag for numerous suggestions on presentation and style.

This second edition now also benefits, as did Volume II, from the marvels of TEXnology. All figures have been recomputed and printed, together with the text, in Postscript. Nearly all computations and text processings were done on the Apollo DN4000 workstation of the Mathematics Department of the University of Geneva; for some long-time and high-precision runs we used a VAX 8700 computer and a Sun IPX workstation.

November 29, 1992 The Authors

Contents

Chapter I. Classical Mathematical Theory

Chapter II. Runge-Kutta and Extrapolation Methods

Chapter III. Multistep Methods and General Linear Methods