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(February 1, 2008)

The dephasing time in coupled mesoscopic conductors is caused by the fluctuations of the dipolar charge permitted by the long range Coulomb interaction. We relate the phase breaking time to elementary transport coefficients which describe the dynamics of this dipole: the capacitance, an equilibrium charge relaxation resistance and in the presence of transport through one of the conductors a non-equilibrium charge relaxation resistance. The discussion is illustrated for a quantum point contact in a high magnetic field in proximity to a quantum dot.

Pacs numbers: 72.70.+m, 72.10.-d, 73.23.-b

Mesoscopic systems coupled only via the long range Coulomb forces are of importance since one of the systems can be used to perform measurements on the other [1]. Despite the absence of carrier transfer between the two conductors their proximity affects the dephasing rate. Of particular interest are *which path* detectors which can provide information on the paths of a carrier in an interference experiment [2–4]. It is understood that at very low temperatures the basic processes which limit the time τ_ϕ over which a carrier preserves its quantum mechanical phase are electron-electron interaction processes [5,6]. For the zero-dimensional conductors of interest here, the basic process is a charge accumulation in one of the conductors accompanied by a charge depletion in the other conductor. The Coulomb coupling of two conductors manifests itself in the formation of a charge dipole and the fluctuations of this dipole governs the dephasing process. The dynamics of this dipole, and thus the dephasing rate, can be characterized by elementary transport coefficients: In the absence of an external bias excess charge relaxes toward its equilibrium value with an RC-time. In mesoscopic conductors [7] the RC-time is determined by an electrochemical capacitance C_μ and a charge relaxation resistance R_q. In the presence of transport through one of the conductors, the charge pile-up associated with shot noise [8], leads to a non-equilibrium charge relaxation resistance [9] R_v. Below we relate R_q and R_v to the dephasing rate.

Renewed interest in dephasing was also generated by experiments on metallic diffusive conductors and a suggested role of zero-point fluctuations [10]. We refer to the resulting discussion only with a recent item [11]. More closely related to our work are experiments by Huibers et al. [12] in which the dephasing rate in chaotic cavities is measured. At low frequencies such cavities can be treated as zero dimensional systems [13].

Consider two mesoscopic conductors coupled by long range Coulomb interactions. An example of such a sys-
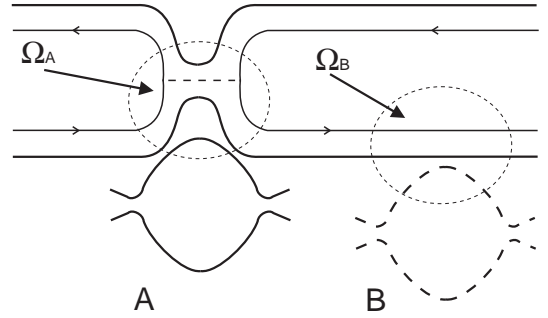

FIG. 1. Quantum point contact coupled to a quantum dot either in position A or B.

tem, suggested in Ref. [14], is shown in Fig. 1. In case A, a quantum point contact (QPC) in a high magnetic field is close to a quantum dot and in case B the QPC is some distance away from a quantum dot. First we focus on case A. To describe the charge dynamics of such a system we use two basic elements. First we characterize the long range Coulomb interaction with the help of a geometrical capacitance, much as in the literature on the Coulomb blockade. Second the electron dynamics in each conductor (i) is described with the help of its scattering matrix, $s_{\alpha\beta}^{(i)}(E, U_i)$ which relates the amplitudes of incoming currents at contact β to the amplitudes of the outgoing currents at α. The scattering matrix is a function of the energy of the carriers and is a function of the electrostatic potential U_i inside conductor i. In case A, the total excess charge on the conductor is of importance. In this case the charge dynamics of the mesoscopic conductor can be described with the help of a density of states matrix

$$\mathcal{N}_{\delta\gamma}^{(i)} = \frac{1}{2\pi i} \sum_\alpha s_{\alpha\delta}^{(i)\dagger} \frac{ds_{\alpha\gamma}^{(i)}}{dE}. \qquad (1)$$

Eq. (1) is valid in the WKB limit in which derivatives with regard to the potential can be replaced by an energy

derivative. Eq. (1) are elements of the Wigner-Smith delay-time matrix [15,16]. Later, we consider also situations in which energy derivatives are not sufficient. The diagonal elements of this matrix determine the density of states of the conductor $N_i = \sum_\gamma \text{Tr}(\mathcal{N}_{\gamma\gamma}^{(i)})$; the trace is over all quantum channels. The non-diagonal elements are essential to describe fluctuations.

At equilibrium, if all contacts of conductor i are held at the same potential, the two conductors can be viewed, as the plates of a capacitor holding a dipolar charge distribution with an electrochemical capacitance [7] $C_\mu^{-1} = C^{-1} + D_1^{-1} + D_2^{-1}$ which is the series combination of the geometrical capacitance C of the two conductors, and the quantum capacitances $D_i = e^2 N_i$ determined by their density of states. An excess charge relaxes with a resistance determined by [7]

$$R_q^{(i)} = \frac{h}{2e^2} \frac{\sum_{\gamma\delta} \text{Tr}\left(\mathcal{N}_{\gamma\delta}^{(i)} \mathcal{N}_{\gamma\delta}^{(i)\dagger}\right)}{[\sum_\gamma \text{Tr}(\mathcal{N}_{\gamma\gamma}^{(i)})]^2}. \quad (2)$$

In the presence of transport through the conductor i the role of the equilibrium charge relaxation resistance, Eq. (2), is played by the non-equilibrium resistance [9] $R_v^{(i)}$,

$$R_v^{(i)} = \frac{h}{e^2} \frac{\text{Tr}\left(\mathcal{N}_{21}^{(i)} \mathcal{N}_{21}^{(i)\dagger}\right)}{[\sum_\gamma \text{Tr}(\mathcal{N}_{\gamma\gamma}^{(i)})]^2}. \quad (3)$$

Note that the charge relaxation resistance $R_q^{(i)}$ invokes all elements of the density of states matrix with equal weight, but in the presence of transport the non-diagonal elements are singled out. Next we relate these resistances to the voltage fluctuations in the two coupled mesoscopic conductors and subsequently to the dephasing time. Here we mention only that at equilibrium, if all contacts at each conductor are at the same potential, the dynamic conductance of our capacitor is given by $G(\omega) = -i\omega C_\mu + C_\mu^2 R_q \omega^2 + O(\omega^3)$. Thus R_q determines the dissipation associated with charge relaxation on the two conductors.

Charge and potential fluctuations are related by

$$\hat{Q} = C(\hat{U}_1 - \hat{U}_2) = e\hat{\mathcal{N}}_1 - e^2 N_1 \hat{U}_1. \quad (4)$$

$$-\hat{Q} = C(\hat{U}_2 - \hat{U}_1) = e\hat{\mathcal{N}}_2 - e^2 N_2 \hat{U}_2. \quad (5)$$

$\hat{Q}$ is the charge operator of the dipole. These equations state that the dipole charge Q on conductors 1 and 2 can be written in two ways: First it can be given by the potential differences and the geometrical capacitances and second it can be expressed as sum of the bare charges $e\mathcal{N}_1, e\mathcal{N}_2$ calculated in the absence of screening and a screening charge which here is taken to be proportional to the density of states of the conductor N_i times the induced potential U_i. Using $D_i = e^2 N_i$ for the density of

states we find that the effective interaction G_{ij} between the two systems is

$$\mathbf{G} = \frac{C_\mu}{D_1 D_2 C} \begin{pmatrix} (C + D_2) & C \\ C & (C + D_1) \end{pmatrix}. \quad (6)$$

With Eq. (6) we find for the potential operators

$$\hat{U}_i = e \sum_j G_{ij} \hat{\mathcal{N}}_j. \quad (7)$$

The fluctuation spectra of the voltages $S_{U_i U_k}(\omega)\delta(\omega + \omega') = 1/2\langle \hat{U}_i(\omega)\hat{U}_k(\omega') + \hat{U}_k(\omega')\hat{U}_i(\omega)\rangle$ now follow from the fluctuation spectra of the bare charges [7,9],

$$S_{N_i N_k}(\omega) = \delta_{ik} \sum_{\delta\gamma} \int dE\, F_{\gamma\delta}(E, \omega)$$
$$\text{Tr}[\mathcal{N}_{\gamma\delta}^{(i)}(E, E + \hbar\omega)\mathcal{N}_{\gamma\delta}^{\dagger(i)}(E, E + \hbar\omega)]. \quad (8)$$

where $F_{\gamma\delta} = f_\gamma(E)(1 - f_\delta(E + \hbar\omega)) + f_\delta(E + \hbar\omega)(1 - f_\gamma(E))$ is a combination of Fermi functions. In the low frequency limit of interest here the elements of the density of states matrix $\mathcal{N}_{\gamma\delta}^{(i)}$ are specified by Eq. (1). Using Eqs. (7,9) we find that at equilibrium the low frequency fluctuations of the potential in conductor 1 are given by

$$S_{U_1 U_1} = 2(\frac{C_\mu}{C})^2 \left((\frac{C + D_2}{D_2})^2 R_q^{(1)} + (\frac{C}{D_1})^2 R_q^{(2)} \right) kT \quad (9)$$

with $R_q^{(i)}$ determined by Eq. (2). Similar results hold for $S_{U_2 U_2}$ and the correlation spectrum $S_{U_1 U_2}$. If a bias eV is applied, for instance to the conductor 2, we find the same spectrum as above, except that $R_q^{(2)} kT$ is replaced, to first order in $e|V|$, by $R_v^{(2)} e|V|$ for $e|V| > kT$.

To relate the voltage fluctuation spectra to the dephasing rate we follow Levinson [4]. A carrier in conductor 1 moves in the fluctuating potential U_1. As a consequence the phase of the carrier is not sharp but on the average determined by $\langle \exp(i(\hat{\phi}(t) - \hat{\phi}(0)))\rangle = \langle \hat{T} \exp(i \int_0^t dt' \hat{U}_1(t'))\rangle$. Assuming that the fluctuations are Gaussian this quantum mechanical average is given by $\exp(-t/\tau_\phi)$ with $\tau_\phi^{-1} = (e^2/2\hbar^2)S_{U_1 U_1}$. Since the voltage fluctuation spectrum Eq. (9) consists of two additive terms we can decompose the dephasing rate into two contributions $(1/\tau_\phi)_{(11)}$ and $(1/\tau_\phi)_{(12)}$ where the index pair (ik) indicates that we deal with the dephasing rate in conductor i generated by the presence of conductor k.

Before discussing the results it is useful to clarify the limit in which we are interested. Typically, the Coulomb charging energy $U = e^2/2C$ is large compared to the level spacing Δ in the conductors of interest. Since $\Delta_i = 1/N_i$ this has the consequence that any deviations of the electrochemical capacitance from its geometrical value are very small. We can thus take $C_\mu = C$ and $C + D_2/D_2 \approx 1$ in Eq. (9). Now we are interested in the dephasing time $\tau_\phi^{(12)}$ in conductor 1 due to the presence of conductor 2. Our discussion gives for this contribution

$$\left(\frac{1}{\tau_\phi}\right)_{(12)} = \frac{e^2}{\hbar^2}\left(\frac{C}{D_1}\right)^2 R_q^{(2)} kT. \qquad (10)$$

with R_q given by Eq. (2) if conductor 2 is at equilibrium and

$$\left(\frac{1}{\tau_\phi}\right)_{(12)} = \frac{e^2}{\hbar^2}\left(\frac{C}{D_1}\right)^2 R_v^{(2)} e|V|, \qquad (11)$$

with R_v given by Eq. (3) if it is in a transport state with $e|V| > kT$. Note that for closed $2D$-conductors $e-e$-scattering leads to a rate [6] proportional to T^2 wheras for open conductors Eq. (10) predicts a rate which is linear in T.

We now specify that transport in the conductor 1 is via a single resonant tunneling state. Thus the relevant density of states in conductor 1 is a Breit-Wigner expression. For simplicity we assume that we are at resonance and hence $N_1 = (2/\pi\Gamma)$, where Γ is the half-width of the resonance. The conductor 2 is a QPC. The resistance R_q and R_v for a QPC in the absence of a magnetic field have been discussed in Ref. [9]. $R_q^{(2)}$ is,

$$R_q^{(2)} = \frac{h}{e^2}\frac{\sum_n (d\phi_n/dE)^2}{[\sum_n(d\phi_n/dE)]^2} \qquad (12)$$

where ϕ_n is the phase accumulated by carriers in the n-th eigen channel of the QPC traversing the region in which the potential is not screened. Note that if only a single channel is open, $R_q^{(2)}$ is universal and given by $R_q^{(2)} = \frac{h}{e^2}$. Thus in the one-channel limit the dephasing caused in conductor 1 due to a QPC at equilibrium is given by $(1/\tau_\phi)_{(12)} = (\pi^4\Gamma^2)/(hU^2)kT$.

Next consider the case where a current is driven through the QPC. The non-equilibrium charge relaxation resistance of a QPC with transmission $\mathcal{T}_n$ and reflection probabilities $\mathcal{R}_n$ in the eigen channels is

$$R_v^{(2)} = \frac{h}{e^2}\frac{\sum_n \frac{1}{\mathcal{T}_n\mathcal{R}_n}\left(\frac{d\mathcal{T}_n}{dE}\right)^2}{[\sum_n(d\phi_n/dE)]^2}. \qquad (13)$$

This result depends on the detailed shape of the QPC even in the single channel limit. The similarity of this result with the one-channel (n =1) result of Buks et al [2] can be seen by identifying the effective variation $\Delta\mathcal{T}$ of the transmission coefficient with $\Delta\mathcal{T} = (d\mathcal{T}_1/dE)(d\phi_1/dE)^{-1}$. Our result provides a complete specification of the dephasing rate in terms of the scattering matrix and geometrical capacitances. We take screening into account and thus can clarify the dependence on the quantum dot properties (via Γ) and the capacitive coupling constant C. R_v has been evaluated for zero magnetic field in Ref. [9]. For the high magnetic field case, the non-equilibrium charge relaxation resistance for a QPC is shown in Fig. 2. We have used a saddle [8] point potential $V(x,y) = V_0 - (1/2)m\omega_x^2 x^2 + (1/2)m\omega_y^2 y^2$ and

as in Ref. [9] have evaluated the density of states semi-classically. Note the strong suppression of the dephasing rate at threshold of the opening of a new channel. Indeed the experiment of Buks et al. [2] shows a double peak structure in the visibility of the Ahronov-Bohm oscillations.
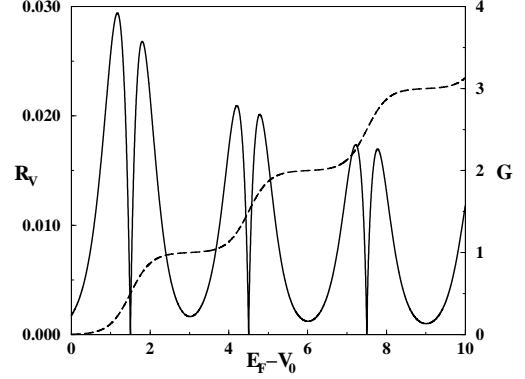

FIG. 2. R_v (solid line) in units of h/e^2 and G (dashed line) in units of e^2/h as a function of $E_F - V_0$ for a saddle QPC with $\omega_x/\omega_y = 1$ and $\omega_c/\omega_x = 4$.

So far we always assumed that the QPC and the dot are located such that the total charge piled up in the QPC matters. Thus the above results involve only the energy derivatives of the scattering matrix of the QPC and the dot. Consider now the situation B shown in Fig. 1, where the quantum dot is located away from the QPC down-stream along an edge. Clearly, now the predominant interaction effect is due to the charge fluctuations on the edge state adjacent to the quantum dot. The charge which counts is that in a region Ω_B affected by the potential of the dot.

Very importantly, the approach introduced above can now be extended to this more general situation. To generalize the above results we need to find the charge and its fluctuations in region Ω_B. This can be accomplished by taking the derivative of the scattering matrix with respect to a small potential perturbation which extends over the region of interest. Thus in general we arrive at a density of states matrix by replacing the energy derivative in Eq. (1) by a functional derivative,

$$d/dE \to -\frac{1}{V_\Omega}\int_{V_\Omega} d^3\mathbf{r}\frac{\partial}{\partial eU(\mathbf{r})} \qquad (14)$$

Let us apply this prescription to the case B of Fig. 1.

First, let us establish the scattering matrix for this system. For the QPC with transmission probability $\mathcal{T} = 1 - \mathcal{R}$ we chose $r \equiv s_{11} = s_{22} = -i\mathcal{R}^{(1/2)}$ and $t \equiv s_{21} = s_{12} = \mathcal{T}^{(1/2)}$. A carrier traversing the region Ω_B adjacent to the quantum dot acquires a phase $\phi_2(U_2)$

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where U_2 characterizes the potential of the edge state in Ω_B. We assume that only the charge pile up in the region Ω_B matters and consequently all additional phases in the scattering problem are here without relevance. The total scattering matrix of the QPC and the traversal of region Ω_B is then simply $s_{11} = r, s_{21} = t, s_{12} = t\exp(i\phi_2)$ and $s_{22} = r\exp(i\phi_2)$.

Consider next the charge operator. We have to evaluate the variation of the scattering matrix with respect to the potential U_2. Only s_{12} and s_{22} depend on this potential. We find $ds_{12}/edU_2 = (ds_{12}/d\phi_2)(d\phi_2/edU_2)$. But $(d\phi_2/edU_2) = -2\pi dN_2/dE$, where now dN_2/dE is the density of states of the edge state in region Ω_B of conductor 2. Simple algebra now gives $\mathcal{N}_{11}^{(2)} = \mathcal{T}dN_2/dE$,

$$\mathcal{N}_{12}^{(2)} = \mathcal{N}_{21}^{(2)*} = r^*t\exp(-i\phi_2)dN_2/dE, \qquad (15)$$

and $\mathcal{N}_{22}^{(2)} = \mathcal{R}dN_2/dE$. At equilibrium we find $R_q^{(2)} = h/2e^2$ as is typical for an edge state that is perfectly connected to a reservoir [17]. The non-equilibrium resistance is

$$R_v^{(2)} = (h/e^2)\mathcal{T}\mathcal{R}. \qquad (16)$$

Note that in the one-channel case both R_q and R_v are independent of the density of states N_2. The additional dephasing rate generated by the edge at equilibrium in the quantum dot at resonance is $(1/\tau_\phi)_{(12)} = (\pi^4\Gamma^2/2hU^2)kT$. Note that this rate depends on the edge state only through its geometrical capacitance. In the non-equilibrium case, the additional dephasing rate caused by the charge fluctuations on the edge state is $(1/\tau_\phi)_{(12)} = (\pi^4\Gamma^2/hU^2)\mathcal{T}\mathcal{R}e|V|$. A rate proportional to $\mathcal{T}\mathcal{R}$ is also obtained by Buks et al. [14]. Of interest is the effect of screening: While in the one channel case, the rate depends on the capacitance of the edge channel only, such a universal result does not apply as soon as additional edge states are present. Thus consider an additional edge state which is transmitted with probability 1. It generates no additional noise and leaves the dc-shot noise invariant [8]. But the additional edge channel contributes to screening. If we take the two edge channels to be close together in the region Ω_B both edge channels will see the same potential U_2. Now the total density of states of the two edge channels in region Ω_B has a contribution from both the perfectly transmitted edge state (1) and edge state (2) $N_2 = N_{21} + N_{22}$. As a consequence the dephasing rate is now reduced and given by

$$\left(\frac{1}{\tau_\phi}\right)_{(12)} = \frac{\pi^4\Gamma^2}{hU^2}\left(\frac{N_{22}}{N_{21}+N_{22}}\right)^2\mathcal{T}\mathcal{R}e|V|. \qquad (17)$$

Eq. (17) is valid if there is no population equilibration among the two edge channels between the QPC and the dot. If there is equilibration (which can be achieved by placing a voltage probe between the QPC and the dot

[14]) we will show elsewhere that the dephasing rate becomes $(1/\tau_\phi)_{(12)} = (\pi^4\Gamma^2/hU^2\nu^2)\mathcal{T}\mathcal{R}e|V|$ where ν is the number of edge states. In particular if only one edge state is present the dephasing rate is unaffected by the presence of a phase randomizing reservoir. This result provides a simple test of the theory presented here.

If the magnetic polarity is reversed the additional dephasing rate is only due the equilibrium fluctuations, independent of whether or not a bias is applied to the QPC.

In this work we have presented a discussion of the dephasing in Coulomb coupled mesoscopic conductors which is based on the fluctuations of the dipolar charge that is generated by the long range Coulomb interaction. This dipole is associated with a capacitance and its dissipative behavior is characterized by charge relaxation resistances R_q and R_v. These resistances are determined by the low frequency collective modes of the Coulomb coupled conductors.

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