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Contribution of mathematical modeling to lake ecosystem understanding: Lake Bourget (Savoy, France)

Brigitte Vinçon-Leite, Bruno Tassin¹ & Jean-Michel Jaquet²

¹*CERGRENE, Centre d'Enseignement et de Recherche pour la Gestion des Ressources Naturelles et de l'Environnement, Ecole Nationale des Ponts et Chaussées, Ecole Nationale du Génie Rural, des Eaux et des Forêts, F-93167 Noisy-le-Grand, France;*

²*Unité de Télédétection (UTED-S), Faculté des Sciences, Université de Genève, 13 Rue des Maraîchers, CH-1211 Genève 4, Switzerland*

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Abstract

This paper demonstrates how mathematical modeling can contribute to improve understanding of lake behavior. Since the 60's Lake Bourget, one of the largest in France, had been suffering from eutrophication which was checked in 1980 by the diversion of the main sewers entering the lake. A research program was implemented between 1987 and 1990, including an on-site sampling campaign conducted concurrently with thermal and biogeochemical modeling of lake behavior. The model helped provide a better understanding of the ecosystem, displaying some processes hitherto misunderstood: (1) Winter overturn does not reach the bottom of the water column when the weather is mild. This leads to a incomplete reoxygenation of the hypolimnion and to redox conditions inducing the release of orthophosphate from the sediment, (2) Grazing by herbivorous zooplankton is getting more important in the control of spring algal growth as eutrophication of the lake regresses, (3) Settling of particulate phosphorus seems a complex and very important process in Lake Bourget, showing high sedimentation rates for particulate mineral phosphorus.

Introduction

Preventing or decreasing eutrophication in lake waters remains one of the main concerns in aquatic ecosystem management. Elements for decision making when starting costly water quality protection or restoration programs are usually scattered and will not produce a comprehensive vision of the lake behavior.

Mathematical modeling of a lake ecosystem allows for synthesis of available knowledge by offering a conceptual representation of the main interactions between the different elements of the system and then by formalizing them by equations.

Modeling does not improve knowledge of a given ecosystem. However, available data can be structured coherently, information linked and relations occurring between data quantified. Modeling has to be based on a sufficient field database, optimizing its interpretation and representing lake features synthetically.

We will use the Lake Bourget study in order to illustrate how modeling can contribute to an improved understanding of lake functioning.

The Lake Bourget

Lake Bourget, in Savoy, is one of the largest lakes in France. Its main morphometric characteristics are summarized in Table 1. Since the 1960's, this lake has been suffering from eutrophication which was checked in 1980 by the implementation of facilities to intercept waste waters from main lakeside cities (Chambéry, Aix-les-Bains, Le Bourget-du-Lac). Phosphorus and nitrogen loadings were then reduced by 50 and 70% respectively, which led to a slow improvement of some water quality indicators.

Table 1. Lake Bourget morphometric and hydrologic characteristics.

Lake surface area	42 10 ⁶ m ²
Volume	3.5 10 ⁹ m ³
Maximal depth	145 m
Mean depth	80 m
Length	18 km
Maximum width	3 km
Catchment area	560 km ²
Mean discharge	16 m ³ s ⁻¹
Mean residence time of water	7 years

Method

A research program was implemented between 1987 and 1990, including an on-site sampling campaign conducted concurrently with thermal and biogeochemical modeling of lake behavior (Vinçon-Leite, 1991).

The sampling campaign carried out in 1988 and 1989, took place at the maximal depth station and included monthly water sampling of the water column, a bottom sediment study and the laying of sediment traps. Joint analysis of this data, together with modeling results, revealed the importance of processes unknown in Lake Bourget until today.

The model

The Lake Bourget water quality model is of a vertical, unidimensional type, and represents the evolution of variables at lake maximal depth. It consists of a temperature sub-model coupled to a biogeochemical sub-model.

Temperature modeling allows the consideration of the system physical behavior. Satisfactory temperature modeling means that the influence of the main water movements (*i.e.* inflows-outflows, convective movements) is accounted for accurately. State variables of the biogeochemical sub-model were determined in terms of their importance in the characterization of water quality and the availability of data. Thus model state variables are as follows: temperature, dissolved mineral phosphorus, particulate algal phosphorus, particulate zooplankton phosphorus and oxygen.

Our model is of dispersive type, in which vertical transportation of substances is expressed by the

advection-diffusion equation integrated into the horizontal plan (equation 1):

$$\frac{\partial V(z, t)}{\partial t} = \frac{1}{A(z)} \frac{\partial}{\partial z} \left[A(z) K(z, t) \frac{\partial V(z, t)}{\partial z} \right] \quad \text{Dispersion} \quad (1)$$

$$- \frac{1}{A(z)} \frac{\partial}{\partial z} [A(z) W(z, t) V(z, t)] \quad \text{Advection}$$

$$+ SS(z, t) \quad \text{Sources and sinks}$$

where $V(z, t)$ is the variable considered at depth z and time t , $A(z)$ is the cross-sectional lake area at depth z , $K(z, t)$ is the eddy diffusion coefficient and $W(z, t)$ is the vertical velocity of water.

$SS(z, t)$ is the term of sources and sinks corresponding for each variable to all the fluxes in the system. For the thermal sub-model $SS(z, t)$ expresses the heat exchange between the lake and the atmosphere. For the biogeochemical sub-model, exchanges between variables are shown in Fig. 1.

For the resolution of the equations system, the lake is divided into 1 m deep layers. On this grid, the resolution is found by a semi-implicit finite differences method. The calculation time step is 3 hours, which is the time structure of the meteorological data files given by Météo-France. More details on equations which come into play within this model can be found in Tassin (1986) and Vinçon-Leite (1991).

Results

The traditional stages of parameter adjustment of the model and validation of results were followed: (1) calibration of parameters was done by fitting model results to 1981 data and (2) verification of results was performed over a long time period (1981–1989).

The modeling results had a significant role in the detection of some five mechanisms hitherto unsuspected in Lake Bourget.

Physical behavior of the ecosystem

Until now Lake Bourget was classified among warm monomictic lakes (CEMAGREF, 1984) with a single complete mixing of the water column at the end of winter (February or March).

This in-depth study of the evolution of winter temperatures in the lake and the confrontation of the temperature model results with data have shown that when

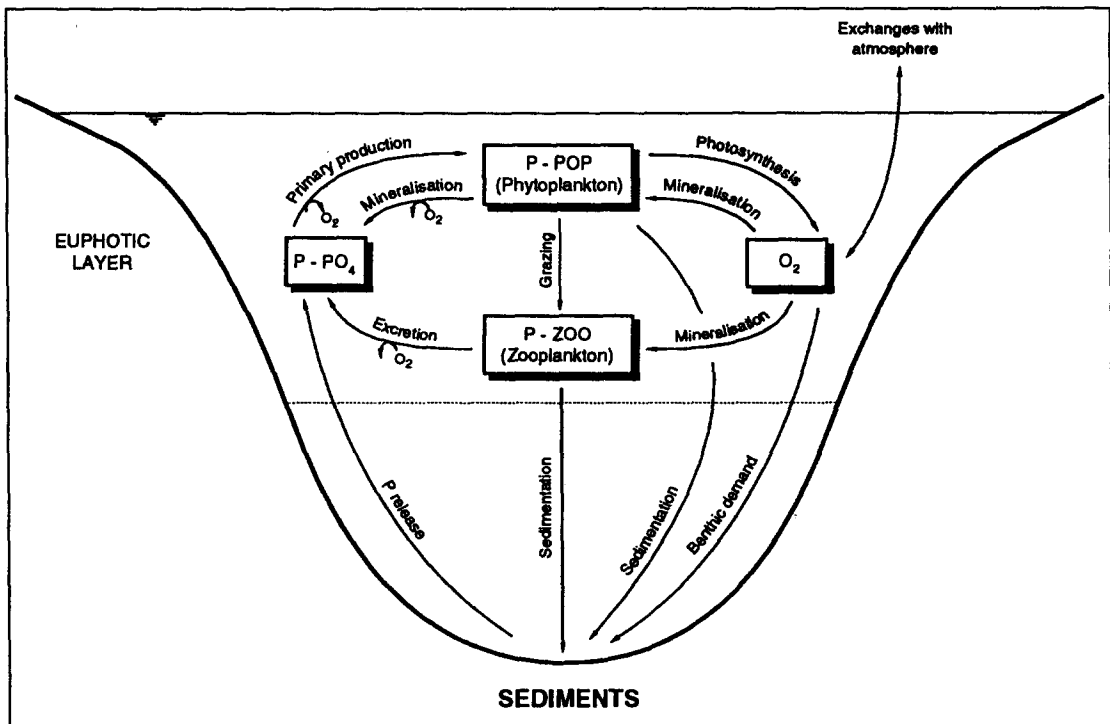


Fig. 1. Conceptual representation of Lake Bourget model.

winters are not very severe (little wind and gentle temperature – conditions which happened frequently at the end of the 80's) – the water column overturn does not occur all the way to the lake bottom.

Figure 2 shows during the winter of 1988 the evolution of thermal destratification of Lake Bourget, remaining incomplete because of the mild weather conditions, which parallels the evolution of partial hypolimnion reoxygenation.

The discovery of a winter partial mixture in the water column of Lake Bourget constitutes an important result because winter mixing strongly influences the chemical and biological behavior of a lake. In deep lakes, it is particularly important for the hypolimnion where it is responsible for carrying nutrients from sediment and deep layers to the trophogenic zone or, on the contrary, for bringing oxygen to the lake bottom.

If we consider the evolution of water quality indicators, it is particularly important to be aware of the initial conditions of vertical stratification of the lake, since they partly account for the subsequent evolution of water quality throughout the year.

Evolution of the spring plankton succession

For the years which followed the measures of water quality restoration in Lake Bourget, we have but rare qualitative and quantitative data on planktonic species present in the lake. It is therefore particularly difficult to assess whether an evolution in the biological equilibrium of the lake occurred. In addition, examination of spring Secchi transparency during these years shows a very sharp evolution, particularly from the second half of the 80's. A clear water phase in May and June is becoming more pronounced. In 1988 and 1989 transparency measured during clear water phase is about 10–11 meters.

Different hypotheses can explain the appearance of a clear water phase: (1) the limitation of nutrients in the epilimnion, (2) if phytoplankton is mostly diatoms, its growth could be checked by silica exhaustion, followed by rapid sedimentation to the lake bottom, (3) a change in temperature conditions, for example a sudden drop in water temperature, could lead to a thermal limitation of the growth of algae species which had developed when the water was warmer and (4) the grazing by herbivorous zooplankton.

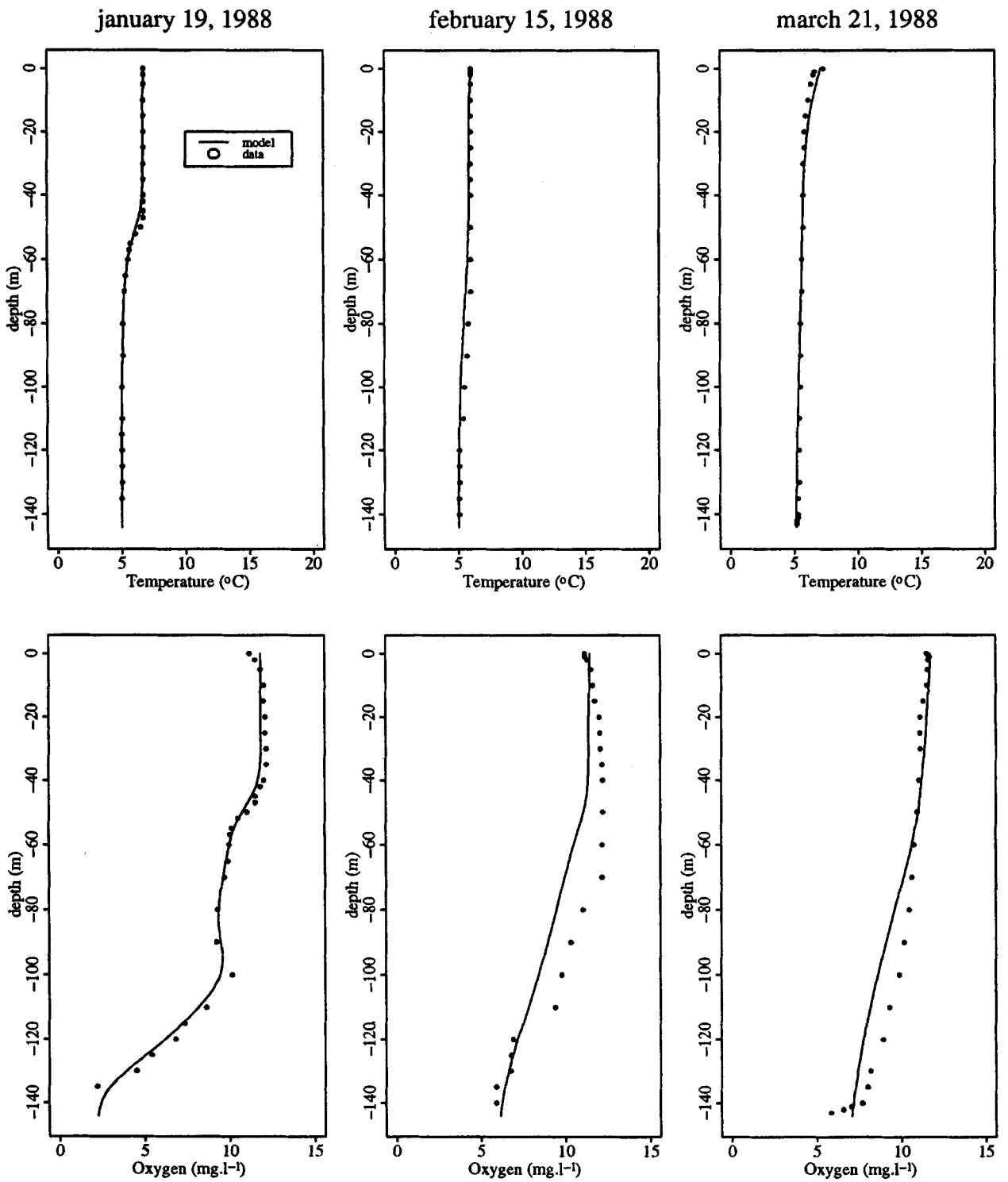


Fig. 2. Temperature and oxygen profiles during the winter 1988.

In lakes undergoing the process of oligotrophication, the clear water phase is very often associated with the presence of an abundant population of herbivorous zooplankton, in particular *Daphnia*, responsible for the intensive grazing of phytoplankton. This is the case of Lake Washington (Lehman, 1988), Lake Constance (Tilzer & Beese, 1988), Lake Geneva (Pelletier *et al.*, 1990), and Lake Lucerne (Bürgi *et al.*, 1985). In Lake Bourget, microscope observations performed on one occasion (23 June 1989), while the Secchi depth was 11 meters, indicated a very abundant *Daphnia* population.

Algal growth in the spring seems therefore under control at the end of the 1980's, through grazing by herbivorous zooplankton, while at the beginning of the 80's the role of this zooplankton seemed insignificant. The importance of zooplankton indicates that algal populations, or at least their relative abundance, have changed in the course of these years. Indeed, the development of herbivorous zooplankton, in particular *Daphnia*, depends on the size of algae, since it seems to prefer smaller algae. In Lake Bourget, the number of small size algae species has probably increased.

By simulations with or without state variables representing zooplankton, the model allowed us to corroborate the hypothesis of the importance of grazing for the control of spring phytoplankton growth. Results of simulations performed taking, or not, herbivorous zooplankton into account are presented in Fig. 3. They show a better representation of algal growth at the end of spring submitted to control by grazing by zooplankton, even though the simulated spring chlorophyll spike lags behind the measured peak.

Influence of river floods on orthophosphate concentration variations in Lake Bourget

In the course of an annual cycle, orthophosphate concentrations in the whole of the water column in Lake Bourget can vary considerably. These variations appear several times during the study of the database.

Between September 21 and October 11, 1988, sampling campaigns the mean P-PO₄ concentration below 10 meters dropped from 80 µgP l⁻¹ to 45 µgP l⁻¹ (Fig. 4), while mean total phosphorus concentration varied little (100 µgP l⁻¹ to 90 µgP l⁻¹). Between October 6–16 1988, the most important flood in that year (Q ≈ 100 m³ s⁻¹) occurred.

A possible explanation is that suspended matter brought into the lake by flooding from tributaries adsorb dissolved phosphorus which thus disappears

by sedimentation from the water column. This phenomenon was observed in other lakes, in particular Lake Constance (Stabel & Geiger, 1985). However, this remains a hypothesis for several reasons: (1) there is no precise available data on suspended matter contributions by tributaries, (2) such a decrease would correspond to very high suspended matter concentrations, with very high adsorption coefficients and (3) the phenomenon appears simultaneously throughout the entire depth of the hypolimnion.

The representation of orthophosphate concentration evolution by the model allows us to eliminate the not too realistic hypothesis of a lake orthophosphate dilution by the water of flooded tributaries. Indeed, the influence of the volume of water brought by the flood is taken into account within the model by using measured values of mean daily flows of the two tributaries.

Phosphorus sedimentation in Lake Bourget

The validation phase led by running the model over a continuous 9 years period (1981 to 1989), allowed us to pin-point a sub-estimate of the decrease of orthophosphate concentrations in Lake Bourget. To give a more global vision of orthophosphate evolution throughout the 1981–89 period, Fig. 5 shows winter orthophosphate stocks when they are at their highest as estimated from the data and as calculated by the model.

Winter orthophosphate stocks appear to decrease strongly in Lake Bourget from 1981 to 1989. The model shows this tendency for the first five years of the period but from 1985 the decrease is a lot sharper according to data than according to the model. Figure 5 shows that after 1985, the tendency of orthophosphate to decrease as calculated from measurements retains the same pace whereas with the model, the decrease is nearly non-existent.

In the lake, P-PO₄ stocks shift from a mean value of 440 tons during the winter of 1981 to 203 tons during the winter of 1989, *i.e.* a more than 50% decrease, whereas in the model the decrease is only 32%. This discrepancy between model and data prompted us to verify in detail the representation of the different processes taken into account for modeling and, in particular, the different terms which intervene in the calculation of phosphorus budget values in the model (Fig. 6).

The different terms accounted for by the model are in line with rough estimates, either from data gathered on-site (loadings, primary production, release of orthophosphate, sedimentation: (CEMAGREF, 1985;

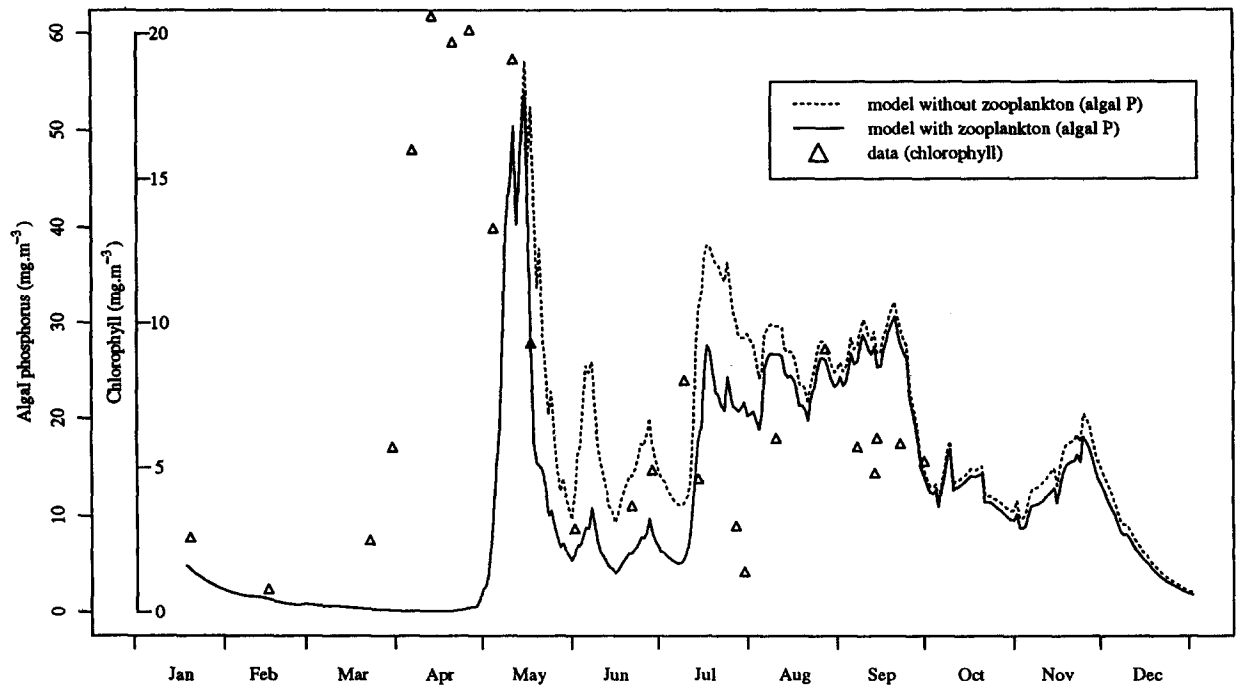


Fig. 3. Simulations of algal growth in the euphotic zone with and without herbivorous zooplankton (1988).

CEMAGREF, 1990; Stroffek & Fontvieille, 1990)) or from literature data corresponding to comparable lakes (mineralization, phosphorus turnover: (Bloesch *et al.*, 1977; Lehman, 1988; Jaquet *et al.*, 1983)). If a discrepancy, such as on Fig. 5, between model results and data exists, it is probably because an important process for the phosphorus cycle in the lake, is not taken into account by the model as this process is hitherto unknown.

One of the key terms which could explain orthophosphate decrease in the lake is sedimentation. Special attention was given to its verification, basing it on a comparison with on-site information. This data, obtained with sediment traps in the Southern basin of the lake, at three depths (10, 30, and 80 meters) and throughout several periods during the years 1988–89 (Stroffek & Fontvieille, 1990), gives values for particulate organic phosphorus (POP) and total phosphorus (P_{tot}) sedimentation fluxes.

The only fluxes which are directly comparable between the model and the data pertain to particulate organic phosphorus since the total particulate phospho-

Table 2. Flux of particulate phosphorus in 1988–89 - Model and data.

	Depth		
	10 m	30 m	80 m
Traps (1988–89)			
P _{tot} (mg m ⁻² day ⁻¹)	4.13	5.94	12.33
POP (mg m ⁻² day ⁻¹)	1.80	2.23	3.91
$\frac{POP}{P_{tot}}$	0.44	0.37	0.32
Model (1988–89)			
POP (mg m ⁻² day ⁻¹)			
1988	3.30	1.67	1.05
POP (mg m ⁻² day ⁻¹)			
1989	2.55	1.42	1.18

rus fluxes can also include allochthonous phosphorus, which the model does not represent.

POP fluxes increase with depth in traps whereas in the model they decrease with depth (Table 2). In the model, POP is produced only in the euphotic zone.

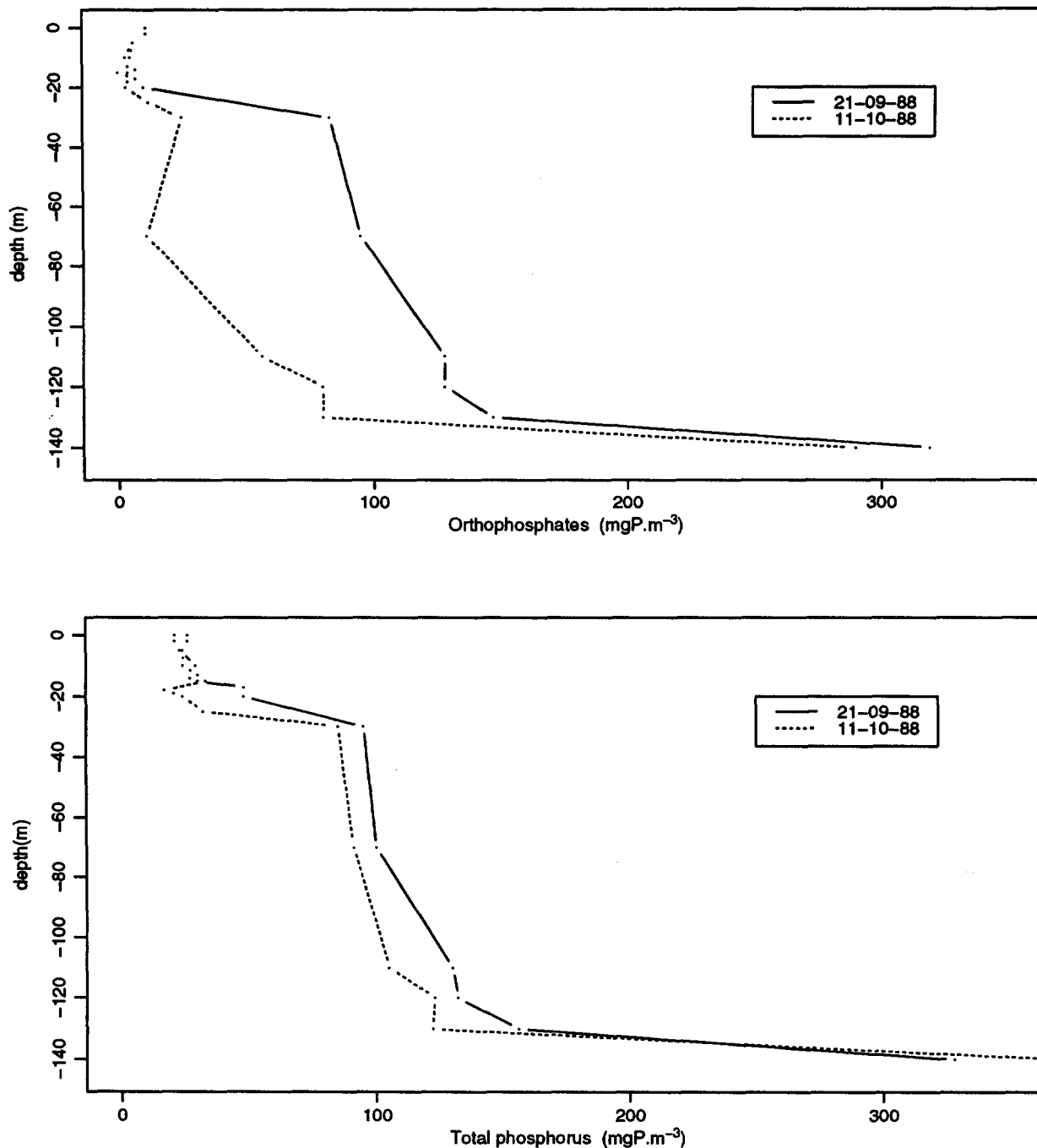


Fig. 4. Orthophosphate and total phosphorus profiles on September 21 and October 11, 1988.

It then settles to the bottom of the lake after having been submitted to grazing and mineralization. As such, flux can therefore only decrease with depth. In reality, other POP production mechanisms probably exist at

depth: *e.g.* lateral contributions, funnel effect or bacterial phosphorus incorporation.

In traps, total phosphorus sedimentation flux also increases with depth, but faster than POP, which indicates that the proportion of particulate mineral phos-

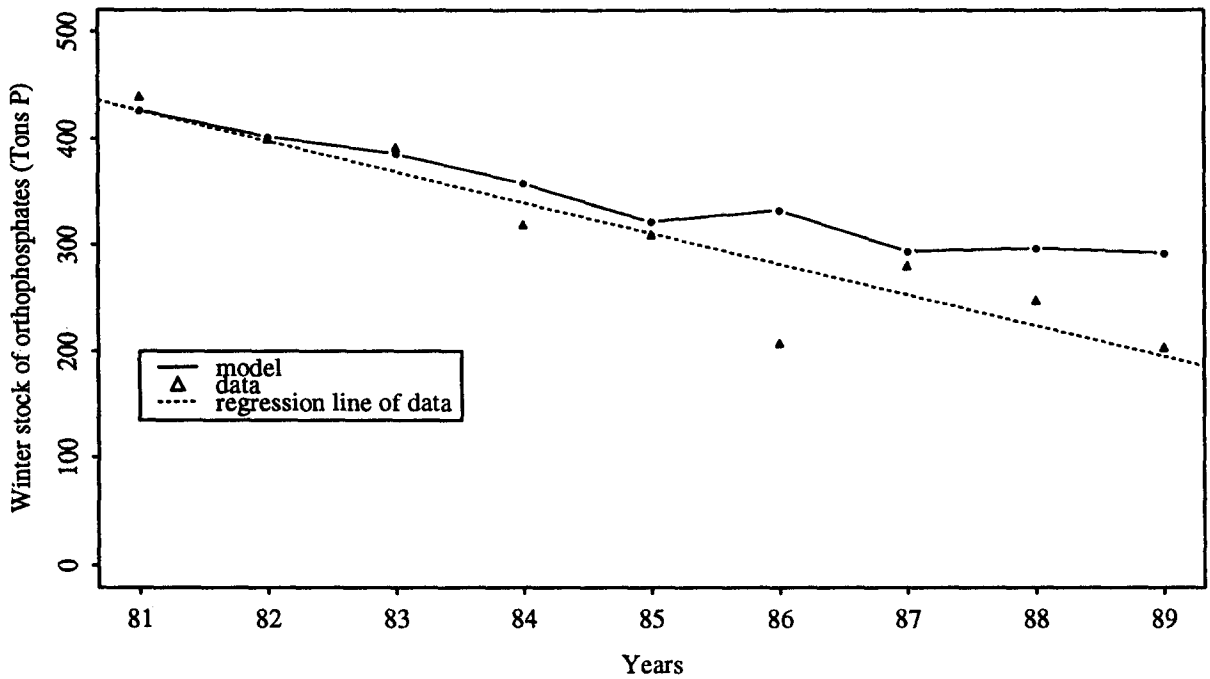


Fig. 5. Evolution of winter orthophosphate stock throughout the 1981–1989 period.

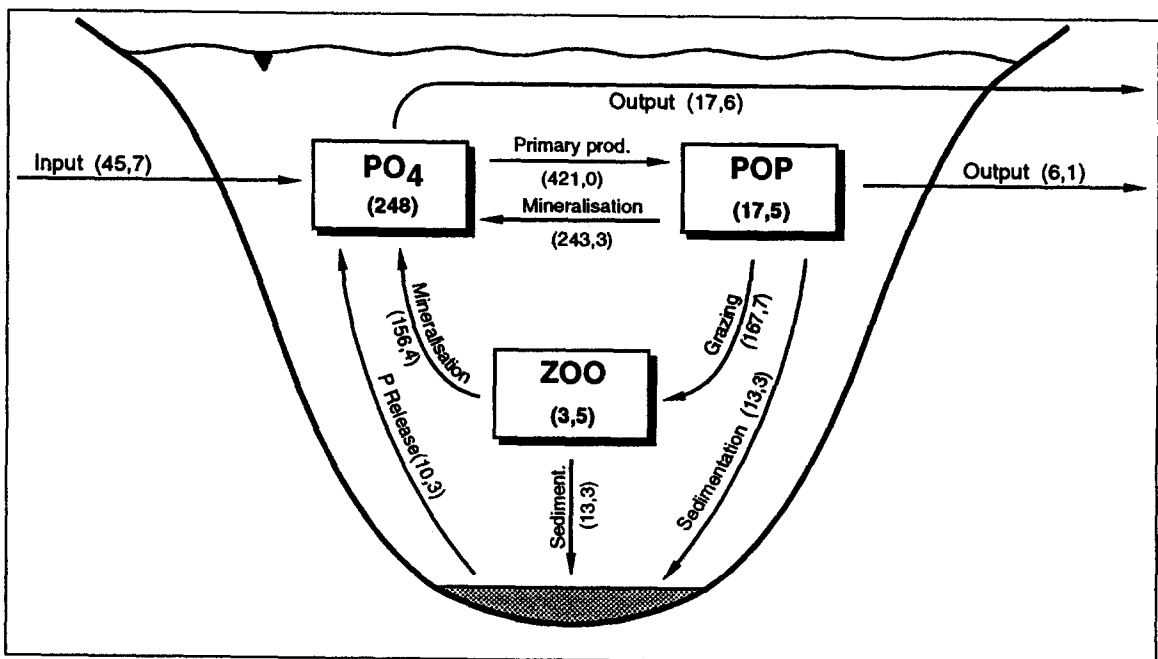


Fig. 6. Phosphorus budget computed by the model in 1988 (stocks are in tons and fluxes in tons.yr⁻¹).

phorus in total phosphorus also increases with depth. The $\frac{POP}{P_{tot}}$ ratio is 0.44 at 10 meters, 0.37 at 30 meters, and 0.32 at 80 meters.

It therefore appears that particulate organic phosphorus sedimentation is correctly represented by the model only in the epilimnion since it is in agreement

with data corresponding to traps located at 10 m. In addition, in Lake Bourget, particulate mineral phosphorus seems to make a particularly high proportion (over 60%) of the total sedimented phosphorus, but is not taken into account by the present version of the model.

Sedimentation of total phosphorus in Lake Bourget was estimated by different methods (calculations based on trap data, bottom sediment cores data, input-output phosphorus budget) at about 130 tons a year, in the same range as annual total phosphorus loading from the lake watershed. Sedimentation is thus especially important and constitutes a fundamental phosphorus elimination mechanism in the lake.

The mechanisms which intervene in mineral phosphorus sedimentation are probably varied. They may be due to: (1) adsorption phenomena of mineral dissolved phosphorus on the allochthonous particulate matter (contribution of suspended matter by flooding tributaries) (Stabel & Geiger, 1985), (2) adsorption phenomena on the autochthonous particulate matter, *e.g.* calcium carbonate crystals during high rates of primary production (Wodka *et al.*, 1985) and (3) coprecipitation of mineral dissolved phosphorus with calcium carbonate crystals (Kleiner, 1988).

These mechanisms, however, are not clearly identified and an in-depth study in order to pin-point and understand them would be necessary.

Conclusion

The goal of a mathematical model in the field of water quality management is very often to predict the future change of the aquatic ecosystem studied. But the use of models for forecasting is, in most cases, in contradiction with limitations of knowledge.

Aquatic ecosystems are made of a very high number of elements linked together by complex, non-linear relationships, which present innumerable feedback and regulation mechanisms subjected to varied and non-predictable external forces, capable of adapting to changes in external conditions (Fedra, 1983). In addition, in the case of several lakes of medium size and importance, databases are often very incomplete. It is therefore necessary to act with utmost modesty and care if forecasting the evolution of water quality or the impact of possible restoration measures is intended when using a mathematical model.

Prior to all previsions on ecosystem evolution, it is important to thoroughly understand its current way of behavior. In this understanding phase, a model is

a very useful tool as it affords a way to synthesize and formalize all available knowledge in order to build a functional 'analogue' of the lake which can then be tested. Processes which are thought predominant are quantified and values given by the model can be confronted with data collected on-site in order to verify the hypotheses. These comparisons between the site and the model also allow us to better define the protocol of lake monitoring (frequency and depth of samples, parameters to be measured, etc ..) or to determine complementary sampling campaigns in order to better understand some mechanisms.

Thus, for Lake Bourget, the primary modeling objective prompted us to conduct a complementary sampling effort, which in turn allowed us to validate the functional scheme the model had proposed for the lake. We could significantly progress in the understanding of the lake behavior, and formulate new hypotheses on mechanisms which had been little studied until now but could play an ever more important role in the evolution of water quality.

We now have a model capable to describe fairly well the main aspects of the present behavior of Lake Bourget, and which can be used as a base for the development of a 'predictive model'.

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