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Ekström, Sylvia

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The Evolution of Fast Rotating Massive Stars at Low or Zero Metallicity

THÈSE

présentée à la Faculté des sciences de l'Université de Genève
pour obtenir le grade de Docteur ès sciences,
mention Astronomie et Astrophysique

par

Sylvia EKSTRÖM GARCÍA NOMBELA

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Thèse de *Madame Sylvia EKSTRÖM GARCIA NOMBELA*
intitulée :

**"The Evolution of Fast Rotating Massive Stars
at Low or Zero Metallicity"**

La Faculté des sciences, sur le préavis de Monsieur G. MEYNET, professeur ordinaire et directeur de thèse (Département d'astronomie), A. MAEDER, professeur ordinaire (Département d'astronomie), D. SCHAEERER, professeur adjoint (Département d'astronomie), S. UDRY, professeur ordinaire (Département d'astronomie) et de Madame P. MARIGO, docteur (Département d'astronomie, Université de Padoue, Italie), autorise l'impression de la présente thèse, sans exprimer d'opinion sur les propositions qui y sont énoncées.

Genève, le 22 septembre 2008

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Le Doyen, Jean-Marc TRISCONE

N.B. - La thèse doit porter la déclaration précédente et remplir les conditions énumérées dans les "Informations relatives aux thèses de doctorat à l'Université de Genève".

Nombre d'exemplaires à livrer par colis séparé à la Faculté : - 4 -

Ce travail de thèse a donné lieu à des publications dont la liste se trouve à la page 161.

Thesis Outline

Rotation plays a key role in stellar evolution. Its inclusion in evolutionary codes has allowed to better reproduce many observed features such as the number ratio of red supergiants to blue supergiants, the surface abundance enrichments or the variations of the Wolf-Rayet stars populations according to the metallicity. While at solar metallicity, or down to the metallicity of the Magellanic Clouds, the stellar evolution codes can be tested through numerous observational constraints, this is no longer true when the metallicity becomes very low. It is however interesting to explore the predictions given at very low-metallicity by a code that has been successfully tested at higher metallicity.

The aim of this thesis is to explore the effects of rotation on the evolution of the first generations of stars. After a general introduction (Chapter 1), Chapter 2 describes the metallicity evolution in the early Universe. The formation peculiarities of the very first generation of stars are summarised, and the role the dark matter annihilation could play is explored. The characteristics of the most metal-poor stars observed in the halo of our Galaxy are briefly summarised, and a work aimed at understanding the origin of their peculiar surface abundance patterns is presented. In Chapter 3, a consistent grid of stellar models is presented, a grid with four different masses (3, 9, 20, and 60 $M_{\odot}$), four different metallicities ($Z = 0.020, 0.002, 0.00001$, and 0) and seven different rotational rates ($\Omega/\Omega_{\text{crit}} = 0.1, 0.3, 0.5, 0.7, 0.8, 0.9$, and 0.99). These models are computed with exactly the same input parameters and the same physical ingredients, so it is possible to explore precisely the effects on the stellar evolution of varying rotation rates, metallicity, or mass. These models are then used to study the Be phenomenon in relation with the reaching of the critical velocity. In Chapter 4, we turn to strictly zero-metallicity stars, and a grid of rotating Population III models with masses ranging between 9 and 200 $M_{\odot}$ is presented. These models are followed up to the pre-supernova stage. Stellar yields are given and included in a chemical evolution model for the Galactic halo to compare them with the observed very metal-poor stars. The effects of the adjunction of a magnetic field in the evolution of a very massive Pop III star are briefly explored. Models of Pop III stars are then used as physics laboratories to test the possible variation of the fine structure constant. Chapter 5 presents a work in progress that has been undertaken by the Geneva group as a whole: the computation of new grids of stellar models. Since the publication of the previous grids between 1992 and 1999, the physical ingredients included in the Geneva code has been much improved, and an accurate treatment of rotation has been implemented. The time is ripe to provide the astronomical community with a new set of theoretical tracks, both rotating and non rotating. Conclusion and perspectives are given in Chapter 6.

Résumé

L'évolution des étoiles massives en rotation rapide à métallicité très basse ou nulle

Introduction

Les étoiles massives sont des objets rares (lors d'un épisode de formation stellaire, il se forme environ mille fois plus d'étoiles de type solaire que d'étoiles massives), mais ce sont des acteurs essentiels dans l'histoire de l'Univers. En effet, elles sont responsables de la formation des éléments lourds qui ont permis l'existence du monde dans lequel nous vivons, ce sont des objets visibles à grandes distances qui nous renseignent sur les galaxies lointaines, et qui sont à l'origine des événements les plus énergétiques après le Big Bang, les supernovae.

La connaissance de leur évolution est un ingrédient fondamental de la compréhension de l'Univers, et dans ce cadre, la modélisation numérique est devenu un outil indispensable. Les modèles stellaires sont très sensibles à notre connaissance des processus physiques impliqués. Certains de ces processus ont fait des pas de géants, comme la détermination des taux de réactions nucléaires (décrivant la génération d'énergie) et le calcul des opacités (décrivant l'interaction de la radiation avec la matière). D'autres nous échappent encore, comme une bonne théorie de la convection, qui fait encore actuellement intervenir des paramètres libres.

Lors de la confrontation avec les observations, les premiers modèles calculés présentaient de nombreuses inadéquations. La situation s'est considérablement améliorée avec l'inclusion du traitement de l'overshoot du coeur (l'accroissement de sa taille due aux mouvements convectifs à sa frontière) vers 1975, puis celle de la perte de masse dans les années 1980. La nécessité d'inclure un mécanisme de mélange interne s'est rapidement fait sentir lors de la détermination observationnelle des abondances à la surface de certaines étoiles (voir par exemple Liubimkov 1984). La rotation offre naturellement un tel mécanisme de mélange, et son inclusion dans les codes d'évolution stellaire a bien amélioré l'accord entre les abondances de surface prédites par les modèles et les enrichissements observés (Heger & Langer 2000; Meynet & Maeder 2000).

Les effets de la rotation sur l'évolution stellaire

Une étoile est une boule de gaz maintenue en équilibre par la compétition entre la gravitation (qui entraîne la contraction) et la pression (qui entraîne l'expansion). Si l'étoile tourne sur elle-même, elle bénéficie d'une nouvelle force pour la soutenir contre la gravitation: la force centrifuge. Au tout début de son évolution, une étoile en rotation va présenter les caractéristiques de

luminosité et de température d'une étoile de plus petite masse qui ne tournerait pas. Au cours de l'évolution, la rotation va entraîner des instabilités, parmi lesquelles on peut mentionner:

- l'instabilité de **cisaillement**: l'étoile ne tournant pas comme un corps solide, la rotation différentielle induit un cisaillement entre les couches qui tournent à des vitesses angulaires différentes. Ce cisaillement produit un mélange turbulent entre les différentes couches;
- la **circulation d'Eddington-Sweet**: excepté à l'équateur, la force centrifuge n'est pas parallèle à la force de gravité, et donc la gravité effective, résultant de ces deux forces, n'est plus radiale comme dans le cas sans rotation. Les surfaces équipotentielles ne sont plus des sphères. Le flux radiatif, qui est proportionnel à la gravité effective locale, n'est plus constant le long d'une équipotentielle. Selon von Zeipel (1924), l'étoile ne peut plus être à la fois en équilibre hydrostatique et radiatif au même moment. Cela déclenche une circulation à grande échelle sur le plan méridien (d'où son autre nom de *circulation méridienne*). Cette circulation transporte principalement le moment cinétique à l'intérieur de l'étoile.

Le mélange interne ramène de l'hydrogène frais dans le coeur de l'étoile qui le consume, ce qui fait grossir le coeur et augmenter la luminosité de l'étoile. Comme l'étoile dispose de plus de carburant, la durée de la séquence principale¹ est prolongée. La rotation augmente la perte de masse par deux mécanismes: premièrement, la gravité effective étant plus grande aux pôles, le flux radiatif est plus fort dans cette direction (théorème de von Zeipel); deuxièmement, l'équateur subit une diminution de sa température, ce qui induit une augmentation de l'opacité (dans le cas où cette dernière n'est pas uniquement due à la diffusion électronique).

Après la séquence principale, le mélange rotationnel amène progressivement les éléments nouvellement synthétisés vers la surface, ce qui augmente l'opacité des couches externes. La température en surface diminue, la perte de masse augmente, ainsi que la métallicité en surface.

Il faut noter que l'inclusion des effets de la rotation dans les codes d'évolution stellaire n'a pas seulement amélioré l'adéquation avec les observations en ce qui concerne les abondances de surface. Cela a aussi amélioré d'autres caractéristiques observationnelles, comme la variation avec la métallicité du rapport des supergéantes bleues sur les rouges (Maeder & Meynet 2001), ou des étoiles de Wolf-Rayet sur les étoiles de type O (Meynet & Maeder 2003, 2005).

Rotation et basse métallicité

Si la composition chimique de l'étoile est différente, l'évolution est modifiée. Ainsi, si la fraction de masse des éléments avec masse atomique $A \geq 12$ (la *métallicité*) est plus basse, l'opacité va diminuer. L'enveloppe sera plus transparente, et l'étoile sera donc plus compacte, plus chaude. La radiation interagira peu avec la matière, et les vents radiatifs seront plus faibles. L'étoile finira sa vie avec un coeur plus massif. Alors qu'à métallicité standard ($Z = 0.020$) la perte de masse élevée des étoiles massives les condamne toutes à finir leur vie dans l'explosion d'une supernova à l'origine d'une étoile à neutron ou d'un trou noir, à basse métallicité il existe toute une variété de sorts possibles, de l'effondrement direct en trou noir sans supernova, à la disruption totale dans une explosion cataclysmique ne laissant aucun rémanent (Heger et al. 2003).

Les caractéristiques de l'évolution stellaire à basse métallicité rendent les effets de la rotation plus importants. Comme l'étoile est plus compacte, on attend une vitesse équatoriale plus élevée pour un moment cinétique similaire. La perte de masse diminuée ne permet pas l'évacuation du moment cinétique, ce qui conduit à une accélération de la vitesse de rotation de l'étoile, au point qu'elle peut atteindre la vitesse critique, à laquelle la force centrifuge contrebalance exactement la force gravifique. Dans ce cas l'étoile va perdre de la masse par un processus mécanique. L'efficacité du mélange est également augmentée à basse métallicité. En effet, le gradient de la vitesse angulaire est plus fort et le rayon de l'étoile est plus petit, ce qui raccourcit le temps de diffusion des

¹La séquence principale correspond à la première phase de fusion de l'étoile, la fusion de l'hydrogène.

espèces chimiques. Lors de la phase de combustion de l'hélium, le mélange entre le coeur et la coquille de fusion de l'hydrogène entraîne une forte synthèse d'azote primaire. Les enrichissements de surface peuvent devenir très importants, ce qui augmente la force des vents radiatifs. Comme le transport du moment cinétique au sein de l'étoile est lui moins efficace, l'étoile termine son évolution avec un coeur à haute vitesse de rotation, ce qui induit une grande déviation à la sphéricité lors de l'effondrement final.

Cette thèse s'inscrit dans ce contexte, son but étant d'explorer les effets de la rotation sur les toutes premières générations d'étoiles. Ces étoiles ont disparu depuis fort longtemps, mais on peut observer leur signature radiative dans les caractéristiques de la ré-ionisation de l'Univers primordial, ainsi que leur signature chimique dans les abondances de surface des étoiles les plus déficientes en métaux observées dans le halo de notre Galaxie.

Modification apportées au code de Genève

Le code de Genève est un code d'évolution stellaire développé à l'Observatoire Astronomique de l'Université de Genève depuis plusieurs décennies. Il permet de modéliser des étoiles depuis le début de la séquence principale (Zero-Age Main Sequence, ZAMS en anglais) jusqu'à la phase pré-supernova. Il inclut tous les processus standards de l'évolution stellaire. Depuis les 15 dernières années, il a été modifié pour prendre en compte les effets de la rotation, tant au niveau de la structure même de l'étoile que des diverses instabilités induites.

Afin de modéliser les étoiles primordiales, quelques modifications ont dû être apportées à ce code. En effet, les étoiles massives brûlent leur hydrogène principalement par une chaîne de réactions nucléaires appelée cycle CNO, où le carbone, l'azote et l'oxygène sont utilisés comme catalyseurs. Or les étoiles primordiales (étoiles de population III, ci-après Pop III) ne contiennent pas de carbone et ne peuvent donc pas enclencher ce cycle. La gravité n'est que faiblement contre-carrée, ce qui fait que la contraction initiale continue jusqu'à ce que la température centrale ait atteint des valeurs si élevées qu'une faible combustion de l'hélium devient possible. Un peu de carbone est produit par la réaction 3α , et à ce moment le cycle CNO peut s'enclencher. Ainsi, lors de la séquence principale, hydrogène et hélium brûlent simultanément, quoiqu'à des taux bien différents, au lieu que les réseaux de combustion nucléaire soient bien séparés comme dans le cas des métallicités plus élevées. Il s'est donc agi de joindre ces deux réseaux. Une loi de perte de masse adéquate pour les objets très massifs à basse métallicité a été ajoutée en option.

Evolution stellaire en rotation à différentes métallicités

Les effets de la rotation ont été largement étudiés sur des modèles à différentes métallicités au fil des années (cf. Meynet & Maeder 2000; Maeder & Meynet 2001; Meynet & Maeder 2002, 2003, 2005). Cependant entre les différentes séries de modèles, certaines caractéristiques physique de la simulation sont très différentes, ce qui peut entraîner une incertitude quant à la cause réelle de certaines tendances obtenues. Il était donc intéressant d'effectuer une étude systématique couvrant une gamme de masses, métallicités et taux de rotation. Ainsi une grille consistante de modèles a été calculée, pour cinq masses différentes ($M = 1, 3, 9, 20$ et $60 M_{\odot}$), quatre métallicités ($Z = 0.020, 0.002, 10^{-5}$ et 0), et sept taux de rotation ($\Omega/\Omega_{\text{crit}} = 0.1, 0.3, 0.5, 0.7, 0.8, 0.9$ et 0.99).

Paramètres de base au début de l'évolution

On l'a mentionné, les étoiles à basse métallicité sont plus compactes qu'à métallicité standard. La figure 1 (*gauche*), à la page vi, montre que la variation peut atteindre un facteur 4 entre la métallicité $Z = 0.020$ et 0 . Sur la même figure (*centre*), on voit que la rotation déforme l'étoile, l'aplatissant aux pôles (l'étoile devient *oblate*). Cet aplatissement devient marqué seulement pour

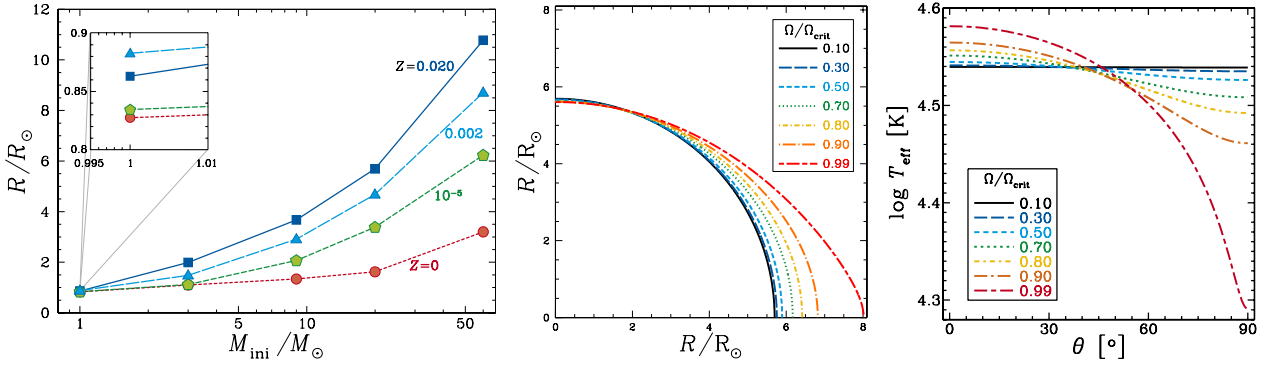


Figure 1: *Gauche*: variation du rayon en fonction de la masse initiale sur la ZAMS dans le cas sans rotation pour les quatre métallicités considérées. *Centre*: variation de la forme de l'étoile en fonction du taux de rotation (légende sur la figure). *Droite*: variation de la température effective en fonction de la colatitude avec le taux de rotation.

des taux de rotation supérieurs à 80% de la rotation critique. Lorsqu'on est tout proche de celle-ci, le rayon équatorial est 1.5 fois plus long que le rayon polaire. À droite, on voit la variation de la température effective (T_{eff}) en fonction de la colatitude. À 99% de la rotation critique, la différence de T_{eff} entre le pôle et l'équateur se monte à 0.3 dex, soit un facteur 2.

La vitesse critique dépend de la masse et du rayon comme $v_{\text{crit}} \propto \sqrt{M/R}$, donc à masse donnée, une étoile à basse métallicité (plus compacte) va devoir tourner beaucoup plus vite pour atteindre un même rapport v/v_{crit} qu'une étoile à métallicité plus élevée. Sur la figure 2, on voit que pour une étoile de 20 ou de 60 $M_\odot$, la vitesse critique est deux fois plus élevée à $Z=0$ qu'à $Z=0.020$. Ainsi un rapport $\Omega/\Omega_{\text{crit}} = 0.70$ pour une 20 $M_\odot$ à $Z=0.020$ correspondra à une vitesse équatoriale $v_{\text{eq}} = 344 \text{ km s}^{-1}$, alors que ce même rapport à $Z=0$ correspondra à $v_{\text{eq}} = 644 \text{ km s}^{-1}$. Si la distribution initiale du rapport $\Omega/\Omega_{\text{crit}}$ ne dépend pas de la métallicité, on s'attend donc naturellement à rencontrer des vitesses équatoriales plus élevées dans les milieux à basse métallicité.

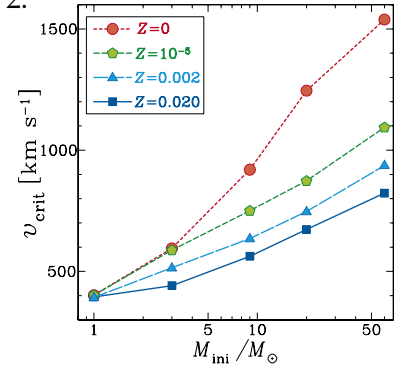


Figure 2: Variation de la vitesse critique en fonction de la masse initiale pour différentes métallicités.

Evolution pendant la séquence principale

On représente habituellement l'évolution des étoiles sur un diagramme de la luminosité en fonction de la température effective (diagramme de Hertzsprung-Russell). Sur un tel diagramme (figure 3), on voit que la métallicité décale les tracés vers des T_{eff} plus élevées, et pour $M_{\text{ini}} \leq 9 M_\odot$, également vers des luminosités plus élevées. L'effet en température, on l'a vu plus haut, est dû à la compacité et à la transparence des étoiles. L'effet en luminosité n'apparaît que pour les étoiles dont l'opacité n'est pas dominée par la diffusion électronique. La rotation, quant à elle, décale les tracés vers une T_{eff} moyenne et une luminosité plus basse au tout début de l'évolution, car ce sont les effets hydrostatiques qui dominent à ce moment-là (cf. plus haut) et l'étoile se comporte comme une étoile de plus faible masse. Lorsque l'évolution se poursuit, la rotation augmente la luminosité de l'étoile par l'augmentation de la taille de son cœur.

Ce qui nous intéresse particulièrement dans ce travail est l'évolution de la vitesse équatoriale. Cette évolution est gouvernée par deux processus, la circulation méridienne (qui apporte du moment cinétique du cœur vers l'enveloppe et accélère la surface), et la perte de masse (qui emporte du moment cinétique et ralentit la surface). On a mentionné plus haut que la faible métallicité réduit les vents radiatifs, donc l'étoile se débarrassera moins facilement de son moment cinétique. De son côté l'amplitude de la circulation méridienne est sensible à la densité de l'étoile. La figure 4

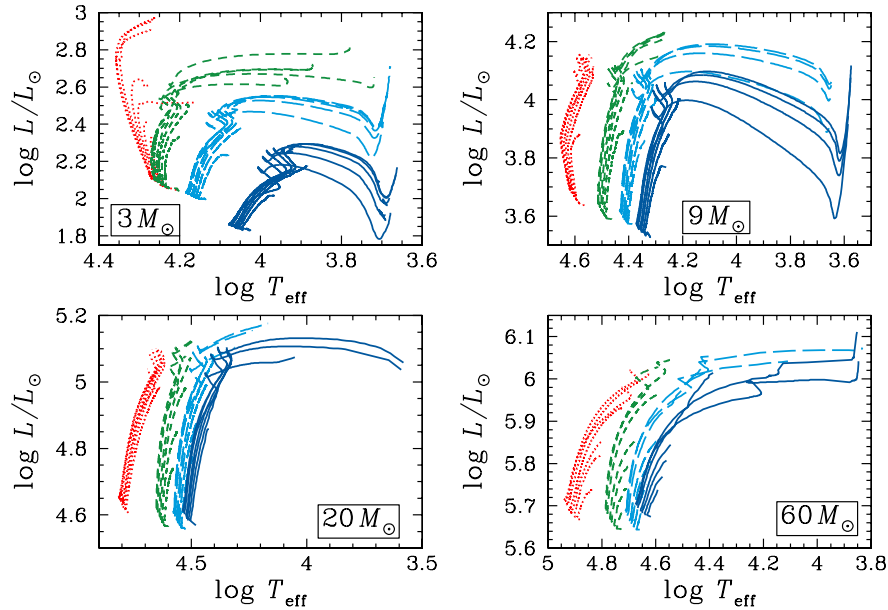


Figure 3: Tracés évolutifs dans le diagramme de Hertzsprung-Russell pour tous les modèles calculés. Lignes bleues continues: $Z = 0.020$; lignes cyan à longs traits: $Z = 0.002$; lignes vertes à traits courts: $Z = 10^{-5}$; lignes rouges pointillées: $Z = 0$.

(gauche) montre l'amplitude de la circulation méridienne à un moment donné de l'évolution sur la séquence principale à l'intérieur de modèles similaires à différentes métallicités. On voit que l'amplitude diminue fortement avec la métallicité. Ceci est dû à la densité bien plus élevée des étoiles à basse métallicité. En passant de $Z = 0.020$ à $Z = 0.002$, 10^{-5} puis 0, l'amplitude maximale diminue d'un facteur 6, 25 puis 100 respectivement. Ainsi, moins de moment cinétique est transporté du cœur vers l'enveloppe, et la surface sera plus difficilement accélérée.

Ainsi, les deux processus déterminant pour l'évolution de la vitesse équatoriale diminuent tous deux avec la métallicité. Le résultat net dépend donc de l'influence respective de ces deux processus. Sur la figure 4 (droite) on voit que dans le cas d'un modèle de $20 M_{\odot}$ à métallicité standard, la circulation méridienne est forte, mais la perte de masse l'emporte et la vitesse équatoriale diminue lentement mais sûrement au cours de l'évolution. Ce n'est qu'à la toute fin de la séquence principale que la contraction naturelle de l'étoile permet l'accélération de la surface. Quand la métallicité descend à $Z = 0.002$ ou 10^{-5} , la circulation méridienne est moins efficace, mais la perte de masse est également diminuée et c'est ce dernier effet qui l'emporte: la vitesse équatoriale augmente nettement au cours de la séquence principale. Lorsque la métallicité tombe à zéro, la perte de masse est nulle, mais cette fois la circulation méridienne est si faible que l'étoile évolue pratiquement en régime de conservation locale de moment cinétique: l'inflation naturelle progressive du rayon fait diminuer très nettement la vitesse de rotation. Il est important de noter ici que même une vitesse équatoriale qui diminue peut s'approcher néanmoins de la vitesse critique. Cette dernière en effet évolue au cours du temps, car d'une part la masse de l'étoile diminue avec la perte de masse, et d'autre part son rayon augmente régulièrement. La figure 5 montre que par exemple dans le cas d'un modèle de $20 M_{\odot}$ à $Z = 0.020$ avec $v_{\text{ini}}/v_{\text{crit}} = 0.50$, la vitesse critique à la fin de la séquence principale ne vaut plus que la moitié de sa valeur sur la ZAMS.

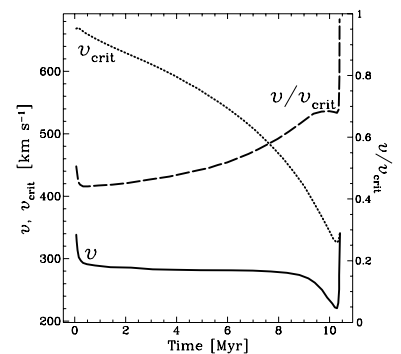


Figure 5: Evolution de v_{eq} , v_{crit} et v/v_{crit} à la surface d'un modèle de $20 M_{\odot}$ avec $v_{\text{ini}}/v_{\text{crit}} = 0.50$ à $Z = 0.020$.

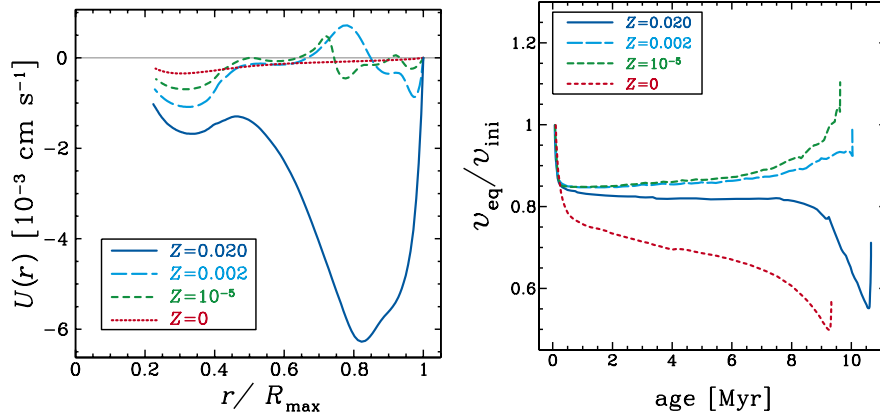


Figure 4: Étoiles de $20 M_{\odot}$ avec $\Omega_{\text{ini}}/\Omega_{\text{crit}} = 0.80$, à différentes métallicités. *Gauche*: variation de l’amplitude de la circulation méridienne à l’intérieur de modèles sur la séquence principale lorsque l’abondance centrale d’hydrogène est $X_c = 0.40$. *Droite*: évolution de la vitesse équatoriale, normalisée à sa valeur sur la ZAMS.

La figure 6 montre l’évolution du rapport $\Omega/\Omega_{\text{crit}}$ pour tous les modèles de $20 M_{\odot}$ aux différentes métallicités. On voit que dans le cas $Z \neq 0$, tous les modèles ayant commencé leur évolution avec $\Omega_{\text{ini}}/\Omega_{\text{crit}} \geq 0.70$ atteignent la vitesse critique au cours de la séquence principale, et ceci d’autant plus tôt que la métallicité est basse. Ceci est cohérent avec le comportement observé à la figure 4.

Application au phénomène des étoiles Be

Une application intéressante des résultats de ce travail est la compréhension du phénomène des étoiles Be. Ces étoiles sont des étoiles de type B dont le spectre présente des raies $H\alpha$ en émission, raies qui ont été attribuées à la présence d’un disque équatorial. L’origine de ce disque a très tôt été associée à la rotation rapide de l’étoile centrale (Dachs et al. 1986), mais il existe encore maintenant une controverse quant au fait que l’étoile soit réellement en rotation critique et que le disque soit le résultat d’éjection de matière équatoriale par l’étoile. Par exemple, Porter (1996), Chauville et al. (2001) ou Tycner et al. (2005) obtiennent que les étoiles Be tournent à 70-80% de la rotation critique seulement. Il faut noter que selon le théorème de von Zeipel, le flux d’une étoile proche de la vitesse critique va être dominé par les régions polaires, qui présentent des vitesses de rotation lentes, ce qui va entraîner une sous-estimation de la vitesse réelle des rotateurs extrêmes (Collins 2004; Townsend et al. 2004).

Dans les dernières années, des observations par interférométrie sont venues apporter de l’eau au moulin de la rotation critique, soit en permettant la détermination de la forme oblate de l’étoile (Domiciano de Souza et al. 2003; Carciofi et al. 2008), soit en mettant en évidence une forte émission polaire (Kervella & Domiciano de Souza 2006; Meilland et al. 2007). La cause de la rotation critique est elle aussi controversée, et deux scénarios sont proposés, l’accélération de la surface par l’accrétion de la matière d’un compagnon dans un système binaire, ou l’accélération naturelle d’une étoile au cours de son évolution. La binarité n’a pas pu être détectée dans la plupart des cas d’étoile Be, mais il faut noter que parfois des années d’observations attentives ont été nécessaires avant qu’un compagnon puisse être mis en évidence (Gies et al. 1998; Rivinius et al. 2004; Maintz et al. 2005; Kervella & Domiciano de Souza 2007).

Dans le cadre du scénario de l’évolution naturelle d’une étoile en rotation, on peut faire ici deux remarques. D’une part, le phénomène Be est transitoire et récurrent, présentant une variabilité non périodique sur des échelles de temps de l’ordre de quelques mois à quelques dizaines d’années. Si on suppose une éjection équatoriale due à la rotation critique, celle-ci évacue du moment ciné-

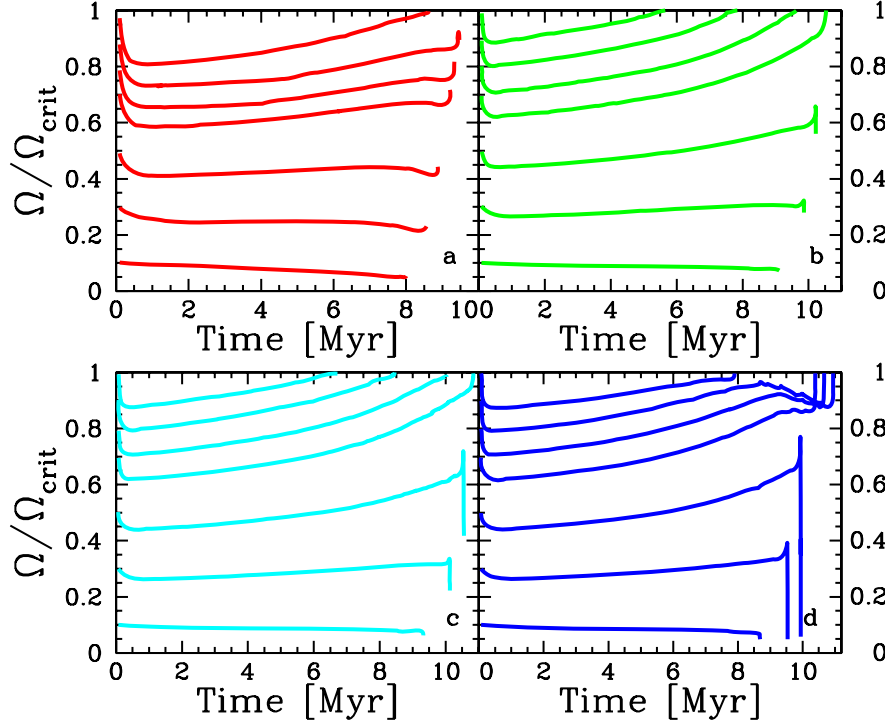


Figure 6: Evolution du rapport $\Omega/\Omega_{\text{crit}}$ pour tous les modèles de $20 M_{\odot}$ à $Z = 0$ (a), $Z = 10^{-5}$ (b), $Z = 0.002$ (c) et $Z = 0.020$ (d).

tique en même temps que de la masse, et la surface redevient sous-critique pour quelques temps. Lorsque l'évolution naturelle ramène du moment cinétique en surface, l'étoile s'accélère à nouveau et entre dans une nouvelle phase d'éjection. La probabilité d'observer cette étoile durant une phase quiescente est bien plus grande que celle de l'observer pendant la phase d'éjection, ce qui tendrait à expliquer pourquoi la plupart des observations montrent des vitesses inférieures à celle permettant l'éjection de matière. D'autre part, la fraction des étoiles Be ($\text{Be}/(\text{B} + \text{Be})$) tend à croître lorsque la métallicité diminue (Maeder et al. 1999; Wisniewski & Bjorkman 2006; Martayan et al. 2007a). Tout scénario valable doit pouvoir rendre compte de cette tendance. Or on a vu (figure 6) que la basse métallicité favorise l'accélération vers la vitesse critique, donc la tendance avec la métallicité apparaît tout naturellement dans ce contexte.

Nous avons utilisé nos modèles pour déterminer la fraction des étoiles tournant plus rapidement qu'une valeur donnée de v/v_{crit} ($v/v_{\text{crit}} \geq u$) dans un amas d'un âge donné. Pour cela nous avons aussi utilisé une fonction initial des masses (IMF de Salpeter) et la distribution de vitesse déterminée par Huang & Gies (2006a). Le résultat est présenté en figure 7. On remarque que les courbes pour 0.99 et 0.90 sont souvent confondues, ce qui montre qu'une fois que la valeur de 0.90 atteinte, l'accélération vers la vitesse critique se fait rapidement. À $Z = 0.020$, la bosse vers 20-30 millions d'années qu'on remarque sur les courbes pour $v/v_{\text{crit}} \geq 0.80$ est due au mécanisme suivant: pour des âges plus petits que 20 millions d'années, ce sont les étoiles massives qui contribuent le plus aux populations concernées, et la perte de masse plus importante à ces masses-là empêche l'accélération vers la vitesse critique. Pour les amas plus âgés, ce sont les étoiles moins massives, dont la faible circulation méridienne n'arrive pas à les accélérer suffisamment. Aux métallicités $Z = 0.002$ et 10^{-5} , la bosse est décalée à des âges plus jeunes, et concerne un plus grand nombre d'étoiles, ce qui illustre la faible perte de masse et la plus grande facilité à atteindre la vitesse critique. À $Z = 0$ en revanche, on voit que les modèles de 20 et $9 M_{\odot}$ ont beaucoup de peine à atteindre la vitesse critique. Curieusement, ce sont les modèles de $3 M_{\odot}$ qui arrivent à s'accélérer suffisamment. Ceci est dû au fait qu'ils passent la plupart de la séquence principale en

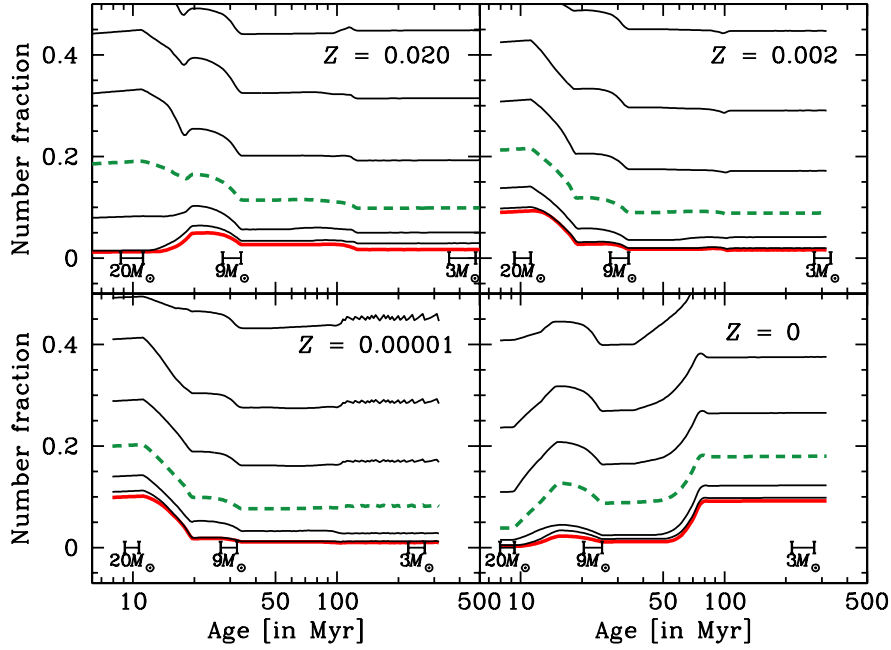
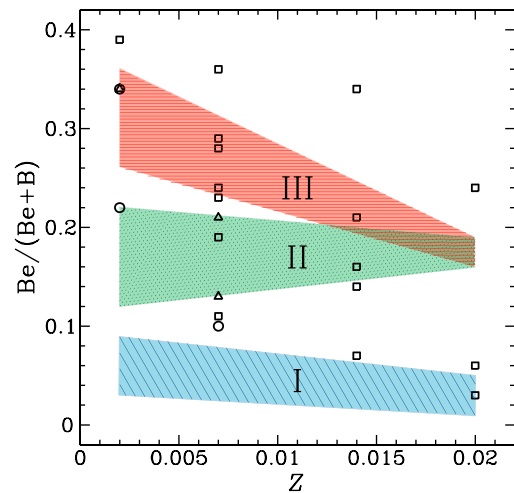


Figure 7: Fraction des étoiles qui tournent avec un rapport v/v_{crit} supérieur à 0.99 (ligne inférieure rouge épaisse), 0.9, 0.8, 0.7 (ligne verte traitillée), 0.6, 0.5, 0.4 (ligne supérieure) dans un amas d'âge donné, à différentes métallicités. L'intervalle de temps pendant lequel les étoiles de 3, 9 et 20 $M_{\odot}$ quittent la séquence principale est indiqué en bas de chaque fenêtre.

contraction, faute d'avoir réussi à produire suffisamment de carbone pour allumer le cycle CNO, et cette contraction permet l'accélération de la surface.

En utilisant les valeurs ainsi obtenues, on peut prédire la fraction $\text{Be}/(\text{B} + \text{Be})$ attendue pour une population d'étoiles de type B dans des amas de notre Galaxie ($Z = 0.020$) ou du Petit Nuage de Magellan ($Z = 0.002$). La figure 8 montre la comparaison entre nos prédictions et la fraction $\text{Be}/(\text{B} + \text{Be})$ obtenue par les observations de Wisniewski & Bjorkman (2006), Keller et al. (1999) et Maeder et al. (1999). Si l'on ne prend en compte que les modèles qui atteignent strictement la

Figure 8: Fraction des étoiles Be dans des amas âgés de 10 à 25 millions d'années en fonction de la métallicité. Les symboles ouverts sont les valeurs obtenues pour les étoiles de type B0-B3 observées par Wisniewski & Bjorkman (2006, cercles), Keller et al. (1999, triangles) et Maeder et al. (1999, carrés). Les zones hachurées correspondent aux prédictions de nos modèles. La zone I est obtenue en ne considérant que les étoiles réellement à la vitesse critique; la zone II est obtenue en comptant toutes les étoiles avec $v/v_{\text{crit}} \geq 0.70$; la zone III est obtenue avec les étoiles ayant $v/v_{\text{crit}} \geq 0.70$ et en supposant qu'il y a un plus grand nombre de rotateurs rapides à basse métallicité (voir texte).



vitesse critique (zone I), on n'arrive pas à reproduire la fraction observée, qui est nettement plus élevée. Même en prenant en compte le fait que nos modèles ne représentent pas la fraction de Be formée par la voie binaire, les résultats obtenus restent trop bas. Un meilleur accord est obtenu si l'on considère tous les modèles qui arrivent à $v/v_{\text{crit}} \geq 0.70$ (zone II). Cependant on ne retrouve

pas la tendance avec la métallicité suggérée par les observations. Si en revanche on considère que la distribution des vitesses n'est pas la même à basse métallicité et qu'on utilise une distribution plate à $Z = 0.002$ (zone III), l'accord entre les observations et les prédictions des modèles devient bien meilleur.

Ainsi, nos résultats suggèrent non seulement que le phénomène Be apparaît déjà lorsqu'une étoile tourne à 70% de sa vitesse critique, mais également que la distribution de vitesses à basse métallicité est décalée vers les rotateurs rapides. La première assertion semble confirmée par les observations, comme nous l'avons vu plus haut. La rotation extrême offrirait donc un contexte favorable, mais d'autres processus devraient ensuite entrer en jeu pour que la matière soit finalement éjectée. Parmi ces processus, on peut évoquer les pulsations non radiales (Owocki 2004; McSwain et al. 2008) ou les mouvements convectifs (Maeder et al. 2008). La deuxième assertion semble également suggérée par les observations, en tout cas dans le domaine des étoiles de type B. Les vitesses moyennes de ces étoiles mesurées dans les amas jeunes de la Galaxie ou des Nuages de Magellan montrent une tendance aux vitesses plus rapides dans ces derniers (Keller 2004; Hunter et al. 2008). Martayan et al. (2007b) montre la même tendance pour le rapport $\Omega/\Omega_{\text{crit}}$ moyen. Il faut noter que la population des étoiles B est principalement composée d'étoiles entre 3 et 9 $M_{\odot}$. Or dans ce domaine de masse, la perte de masse est de toute façon faible, donc l'évolution de la vitesse équatoriale est dominée par l'effet de la circulation méridienne. Comme cette dernière est plus faible à basse métallicité, on s'attendrait plutôt à un $\Omega/\Omega_{\text{crit}}$ moyen inférieur dans les Nuages de Magellan que dans la Galaxie. Ce résultat observationnel peut donc être considéré comme une forte indication que les milieux à basse métallicités contiennent une plus grande proportion de rotateurs rapides à la naissance, ce qui corrobore nos résultats.

Evolution des étoiles primordiales

La toute première génération d'étoiles nées après le Big Bang est un cas extrême de basse métallicité. En effet, pendant la nucléosynthèse cosmologique, aucun élément plus lourd que le lithium ($A = 6 - 7$) n'est produit en quantité qui ne soit pas négligeable (cf. par exemple Iocco et al. 2007). Ces étoiles naissent donc avec une métallicité nulle, et commencent à produire les premiers éléments lourds de l'Univers. Ces étoiles occupent ainsi une position clé, à la transition entre un Univers composé purement d'hydrogène et d'hélium neutres et celui que nous observons de nos jours. Ces étoiles sont éteintes depuis longtemps, mais elles ont laissé leur signature radiative dans la ré-ionisation de l'Univers, et leur signature chimique dans les abondances des étoiles les plus déficientes en métaux observées dans le halo de notre Galaxie. Depuis la première modélisation par Ezer & Cameron (1971), les étoiles de Pop III ont fait l'objet de nombreuses études. Une seule étude publiée présente des modèles en rotation (Marigo et al. 2003), mais celle-ci y est traitée comme un corps solide et ne prend pas en compte les effets de mélange.

Rotation à métallicité nulle

Afin d'étudier les effets de la rotation différentielle à métallicité nulle, nous avons calculé sept modèles entre 9 et 200 $M_{\odot}$ avec une vitesse de rotation initiale de 800 km s⁻¹ (sauf pour le modèle de 9 $M_{\odot}$ pour lequel cette vitesse était sur-critique et qui a été calculé avec $v_{\text{ini}} = 500$ km s⁻¹). Nous avons également calculé une série de modèles similaires sans rotation. L'évolution des modèles a été poursuivie jusqu'à la phase pré-supernova. La figure 9 présente le diagramme HR (*gauche*) et l'évolution de la température centrale en fonction de la densité (*droite*) pour l'ensemble des modèles. Comme le coeur est déjà assez chaud pour brûler un peu d'hélium pendant la séquence principale, la transition entre la fusion de l'hydrogène et celle de l'hélium se fait de manière continue, sans réajustement de la structure, l'étoile restant compact même durant la plus grande partie de la phase de combustion centrale de l'hélium. La coquille de combustion de l'hydrogène est

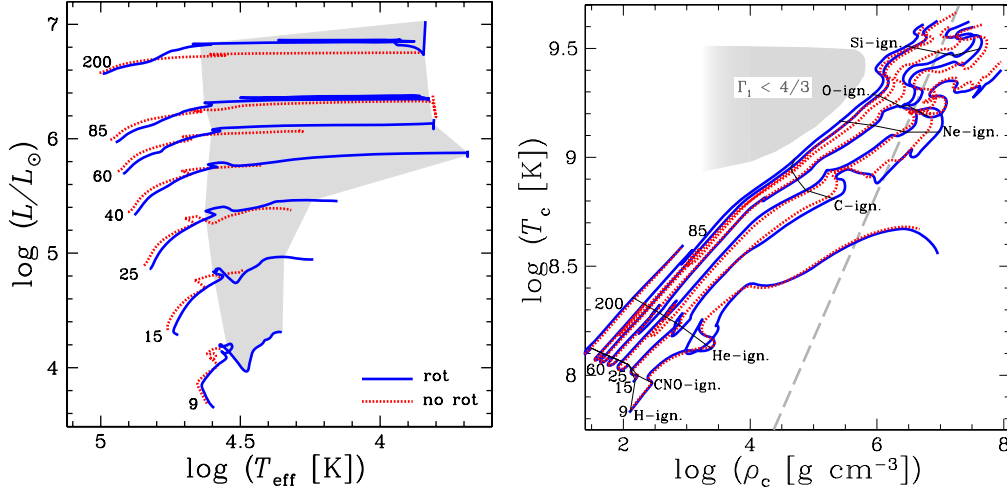


Figure 9: Modèles à $Z = 0$ en rotation (ligne bleue continue) et sans rotation (ligne rouge pointillée). *Gauche*: diagramme HR. La zone grisée indique la zone où la combustion centrale de l'hélium a lieu. *Droite*: diagramme $\log T_c - \log \rho_c$. Le début des différentes phases de combustion est indiquée. La zone grisée indique la zone où l'instabilité par création de paires a lieu. La ligne traitillée indique la limite entre le gaz non-dégénéré et le gaz dégénéré non-relativiste.

peu active et la luminosité provient principalement du coeur. Dans les modèles avec rotation, le mélange rotationnel permet que du carbone et de l'oxygène produits par le coeur soient diffusés vers la coquille de combustion de l'hydrogène. Le cycle CNO devient soudain très actif, ce qui entraîne un flash dans la coquille et une diminution brusque du coeur convectif. La coquille devient convective et l'étoile part vers le rouge dans le diagramme HR. Cet épisode permet la production de grandes quantités d'azote primaire à toutes les masses. Dans le cas sans rotation, on n'assiste à un tel événement que dans certains domaines de masse bien déterminés (25 et 85 $M_\odot$, voir aussi Chieffi & Limongi 2004).

On a vu plus haut que la circulation méridienne est extrêmement faible à métallicité nulle, donc il n'y a pratiquement pas de couplage entre le coeur et l'enveloppe. La perte de masse est quasiment nulle elle aussi, donc les modèles évoluent pratiquement en régime de conservation locale du moment cinétique $\Omega r^2 = \text{constante}$. Malgré tout, comme on l'a vu, la diminution de la vitesse critique au cours de l'évolution permet à tous les modèles de l'atteindre, à part celui de 9 $M_\odot$. Cependant, la perte de masse due à ce processus mécanique reste faible (moins de 10% de la masse initiale) car les couches extérieures sont peu denses (figure 10, *gauche*). À métallicité non nulle, la fin de la séquence principale entraîne un fort changement de structure: le coeur se contracte et l'enveloppe s'expand, ce qui étire le gradient de Ω et permet un fort mélange à la frontière du coeur. À métallicité nulle, cela ne se passe pas et le mélange reste du même ordre que durant la séquence principale. De plus l'étoile reste longtemps du côté bleu du diagramme HR, donc le développement d'une zone convective externe est tardif et peu profond. Ainsi la plus grande partie de l'évolution post-séquence principale a lieu avec une métallicité de surface qui reste inférieure à 10^{-6} (figure 10, *droite*).

Rendements stellaires et évolution chimique de la Galaxie

À la fin de l'évolution, on peut calculer les abondances nouvellement produites par l'étoile pour certains éléments. Comme nos modèles sont suivis jusqu'à la phase pré-supernova, on peut s'intéresser aux éléments qui ne seront pas ou peu modifiés par le passage de l'onde de choc au cours de l'explosion finale (cf. Limongi & Chieffi 2002). La figure 11 montre ces résultats pour les modèles avec (*gauche*) et sans (*droite*) rotation. La différence amenée par la rotation peut atteindre

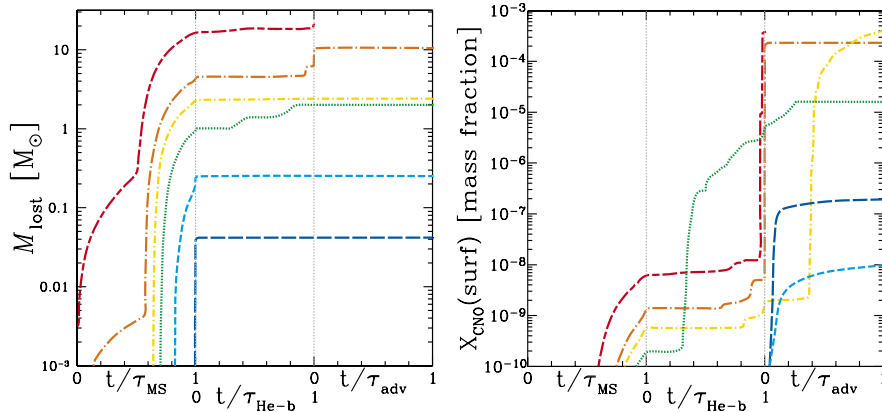


Figure 10: *Gauche*: évolution de la perte de masse pour tous les modèles en rotation. L'axe des x est un axe temporel divisé en trois parties: la fraction de la séquence principale, celle de la phase de combustion centrale de l'hélium, et celle des phases avancées. *Droite*: évolution de l'abondance en surface des éléments C+N+O pour les mêmes modèles.

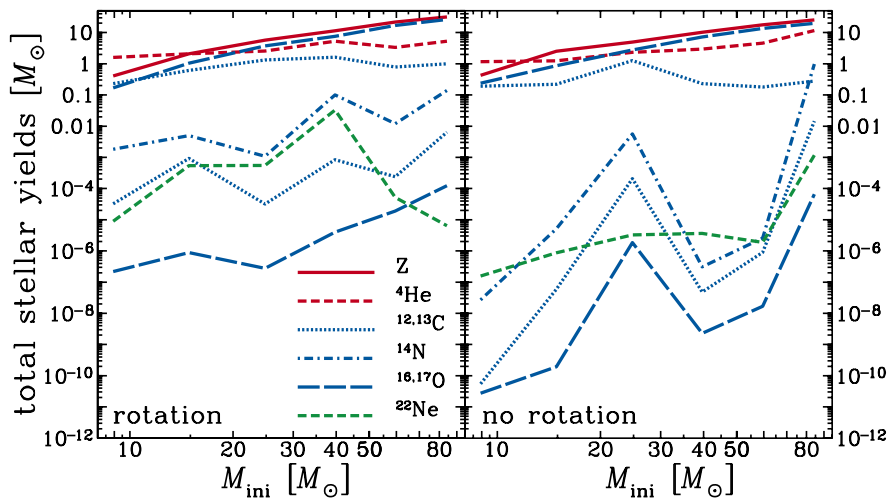


Figure 11: Rendements stellaires à $Z = 0$ pour les éléments légers: *gauche*: modèles avec rotation; *droite*: modèles sans rotation.

des ordres de grandeur pour certains éléments (comme le ^{13}C , le ^{14}N , le ^{17}O ou le ^{22}Ne).

Heger et al. (2003) ont déterminé le sort final des étoiles en fonction de la masse du cœur d'hélium (M_α): supernova de type II ($M_\alpha < 9 M_\odot$), trou noir sans supernova ($9 \leq M_\alpha < 40 M_\odot$), trou noir faisant suite à des instabilités pulsationnelles dues à la création de paires $e^+ e^-$ ($40 \leq M_\alpha < 64 M_\odot$), supernova par instabilité de paires avec disruption complète ($64 \leq M_\alpha \leq 133 M_\odot$) ou trou noir direct ($M_\alpha > 133 M_\odot$). Avec ces limites, on voit que la rotation change le sort réservé à certains modèles, car elle augmente généralement la masse du cœur d'hélium. Ainsi, le modèle de $25 M_\odot$ sans rotation est sensé exploser en supernova de type II alors que celui en rotation termine en trou noir sans supernova. De même, le modèle de $85 M_\odot$ sans rotation doit finir en trou noir sans explosion, alors que le modèle avec rotation va subir les instabilités pulsationnelles par création de paires.

Il faut noter que ces limites ont été obtenues par des modèles sans rotation. En l'occurrence, on a vu que la faible perte de masse et le transport inefficace du moment cinétique au travers de l'étoile permet au cœur de garder une quantité importante de moment cinétique à la fin de l'explosion. Selon Nomoto et al. (2003), il y a des indications que les supernovae les plus énergétiques (les hypernovae) soient dues à une forte rotation stellaire. L'asymétrie engendrée par un

effondrement en rotation permet d'augmenter significativement l'énergie de l'explosion (Shimizu et al. 2001; Kotake et al. 2003; Yamasaki & Yamada 2005; Burrows et al. 2005). On s'attend donc à ce que nos modèles subissent une explosion bien plus forte que l'image courante associée aux étoiles de Pop III. La non-sphéricité de l'effondrement est également supposée favoriser l'émission d'ondes gravitationnelles, qui pourraient être détectées à l'avenir par des détecteurs comme DECIGO, le successeur de LIGO (Suwa et al. 2007).

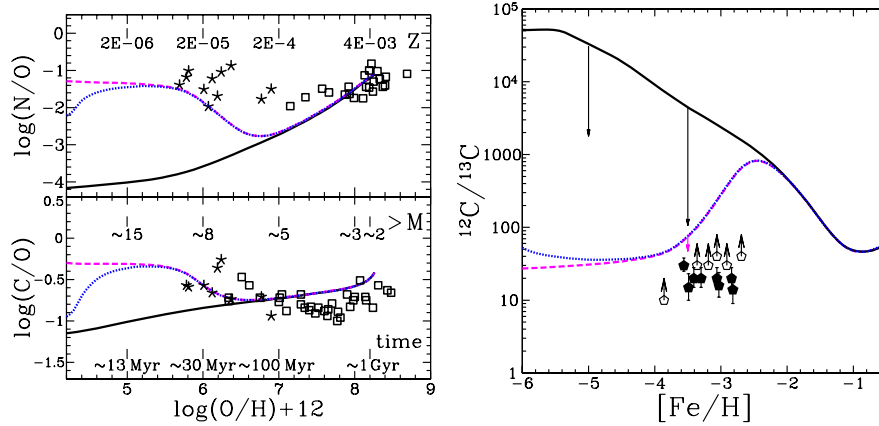


Figure 12: Comparaison entre l'évolution chimique prédite par les modèles et les observations. La ligne noire continue est obtenue avec les rendements stellaires des modèles à $Z = 10^{-5}$ en rotation lente (300 km s^{-1}) de Meynet & Maeder (2002) et ceux de Hirschi et al. (2004). La ligne magenta traitillée est obtenue en rajoutant les modèles à $Z = 10^{-8}$ de Hirschi (2007) à très basse métallicité. La ligne bleue pointillée est obtenue en utilisant les modèles présentés ici pour les métallicités extrêmes. *Gauche*: rapport N/O et C/O. Observations: Israelian et al. (2004, carrés ouverts) et Spite et al. (2005, étoiles). *Droite*: évolution du rapport $^{12}C/^{13}C$. Observation: étoiles non-mélangées de Spite et al. (2006). Les symboles ouverts représentent une limite inférieure. Les flèches noires descendant de la courbe supérieure indiquent la diminution du rapport $^{12}C/^{13}C$ théoriquement attendue dans les géantes à la fin de la première plongée de la zone convective externe.

Une fois les rendements stellaires calculés, on peut les intégrer dans un modèle d'évolution chimique de la Galaxie, ce que nous avons fait (figure 12). On voit que la courbe obtenue par l'adjonction de nos modèles prédit une diminution des rapport N/O et C/O et une augmentation du rapport $^{12}C/^{13}C$ à très basse métallicité ($Z \leq 6 \cdot 10^{-7}$).

Des modèles de Pop III, sans rotation cette fois, ont été utilisés pour poser des contraintes stellaires sur l'éventuelle variation de la constante de structure fine. Un domaine de variations possibles a été déterminé sur des critères nucléosynthétiques.

Evolution avec champ magnétique

Les modèles présentés ci-dessus ont été calculés sans l'adjonction des effets d'un champ magnétique. La présence d'un tel champ dans l'Univers primordial est encore débattu (Giovannini 2004), mais s'il est présent, il va considérablement modifier l'évolution d'une étoile de Pop III puisqu'il va fournir le couplage coeur-enveloppe qui fait défaut à métallicité nulle. On peut s'attendre à une perte de masse nettement plus élevée, et à des phénomènes de mélange bien plus importants (Maeder & Meynet 2004; Heger et al. 2005; Yoon & Langer 2005).

La figure 13 montre l'évolution d'un modèle de $150 M_{\odot}$ pour lequel on a inclus le traitement du champ magnétique selon Spruit (1999, 2002) modifié par Maeder & Meynet (2005). Le champ magnétique entraîne effectivement une très forte perte de masse, puisque l'étoile termine la fusion centrale de l'hélium en ayant perdu près des deux tiers de sa masse initiale. Ce résultat peut

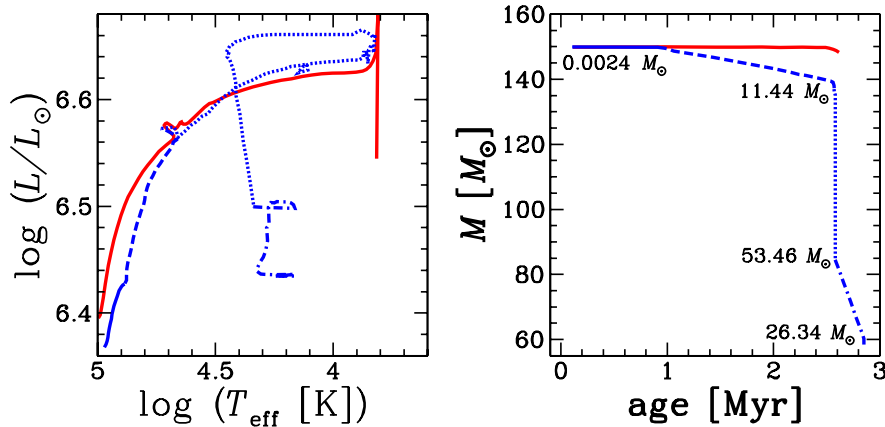


Figure 13: Evolution d'un modèle de $150 M_\odot$ à $Z = 0$ (ligne rouge: modèle sans rotation; ligne bleue: modèle avec rotation et champ magnétique). Les différentes étapes de l'évolution du modèle en rotation sont indiquées par des lignes différentes: début de la séquence principale, la perte de masse est uniquement radiative (ligne continue au coin à gauche); l'étoile est à la vitesse critique, la perte de masse est dominée par la perte de masse mécanique (ligne traitillée); début de la phase de fusion de l'hélium, le modèle est à la vitesse critique et en plus à la limite d'Eddington en luminosité (ligne pointillée); l'étoile est devenue Wolf-Rayet (ligne continue). *Gauche*: diagramme HR. *Droite*: évolution de la masse. Les chiffres indiquent la masse perdue à chacune des étapes.

avoir une grande importance dans le domaine de masse particulier qui normalement conduit aux supernovae par création de paires. Ces supernovae sont supposées laisser une empreinte chimique très particulière (Heger & Woosley 2002), or celle-ci n'est pas observée dans les étoiles les plus déficientes en métaux (Tumlinson et al. 2004). La raison de cette absence est encore inconnue, mais nous proposons ici un élément de réponse qui méritera d'être approfondi.

Nouvelles grilles de modèles stellaires

Les grilles de modèles stellaires sont des outils importants mis à disposition de la communauté astronomique. Les grilles publiées par le groupe de Genève entre 1992 et 1999 totalisent à l'heure actuelle plus de 3'000 citations. Depuis l'époque de la publication de ces grilles, de nombreuses améliorations ont été apportées au code d'évolution stellaire de Genève, et le traitement de la rotation y a été implémenté. Le groupe de Genève s'est donc lancé dans le projet de nouvelles grilles, couvrant des masses comprises entre 0.4 et $120 M_\odot$ et les métallicités $Z = 0.030, 0.014, 0.006, 0.002$ et 10^{-5} .

Dans le cadre de ce travail de thèse, le code a été remis à jour en vue de cet important projet. Il a également été réorganisé afin de fusionner trois versions qui avaient divergé au cours de leur développement. De nombreux tests et calibrations ont dû être effectués. Les calculs ont commencé pour deux métallicités, $Z = 0.014$ et $Z = 0.002$. Les résultats sont encore partiels à ce jour, ils seront complétés dans les mois prochains.

Conclusions et perspectives

Durant ce travail de thèse, le code d'évolution stellaire de Genève a été modifié pour pouvoir modéliser les toutes premières générations d'étoiles. Il a également été réorganisé en profondeur et remis à jour dans le projet de calculer des grilles extensives de modèles stellaires. Le prochain gros travail sur le code consistera à étendre le réseau de réactions nucléaires afin de suivre le processus s de nucléosynthèse par capture de neutrons.

Une étude systématique des effets de la rotation différentielle sur l'évolution des étoiles à différentes métallicités a été effectuée. Cette étude a été utilisée pour apporter un éclairage au phénomène des étoiles Be dans le cadre du scénario de l'évolution d'une étoile simple. Elle montre que le phénomène Be doit apparaître déjà lorsque la vitesse équatoriale atteint 70% de la vitesse critique. Elle indique également que pour rendre compte de l'augmentation de la fraction d'étoiles Be à basse métallicité, on doit supposer que la distribution de taux de rotation contient plus de rotateurs rapides à basse métallicité.

La première étude des effets de la rotation différentielle sur l'évolution des étoiles primordiales a été effectuée. Les modèles ont été suivis jusqu'à la phase de pré-supernova, permettant le calcul de rendements stellaires pour les éléments légers. Ces rendements stellaires ont été inclus dans un modèle d'évolution chimique de la Galaxie. La perte de masse totale des modèles reste extrêmement faible, ce qui permet au cœur de conserver une grande quantité de moment cinétique lors de l'effondrement final ce qui pourrait changer fondamentalement le sort réservé à ces étoiles. Le projet d'utiliser les structures finales de ces modèles dans un code hydrodynamique 3D afin de simuler le collapse et l'explosion sur la base de modèles réels est en cours. Il a été montré que dans le cas où les effets d'un champ magnétique sont pris en compte, la perte de masse devient considérable et pourrait permettre aux étoiles extrêmement massives d'éviter de finir en supernova par création de paires. Un premier résultat dans ce sens a été obtenu, et ce point sera approfondi dans une étude future. Un travail consistant à étudier l'impact de la rotation extrême sur la formation des étoiles primordiales est actuellement en cours. Dans l'étude présente, nous avons laissé de côté l'aspect de la ré-ionisation par les étoiles de Pop III. Cet aspect sera abordé dans une étude prochaine.

De nombreux tests et calibrations ont été effectués dans le but de calculer des grilles extensives de modèles stellaires. Les calculs des modèles à deux métallicités ont été entrepris, calculs qui seront achevés dans les mois prochains.

Remerciements

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Introduction

Quoi de plus ressemblant aux insensés que ceux
Qui, voyant les secrets d'en haut venir vers eux,
Marchent à leur rencontre et donnent aux algèbres
L'ordre de prendre un peu de lumière aux ténèbres

V. HUGO, *La Comète*

1.1 General context

It is probably an understatement to say that without massive stars, our world would look completely different from what it looks now. Without them, we would be nothing else than diffuse clouds in a pure hydrogen and helium world. Massive stars created the iron and silicon in the rocks we walk upon, the calcium in our bones, a great part of the oxygen in the air we breathe. For us astronomers, they are the far-seen objects that help us study the distant galaxies, they are the beacons of star-forming regions, they are the progenitors of the most energetic events after the Big Bang: supernovae. They are also fascinating objects by themselves, evolving from collapsing gas clouds to exploding gas clouds with, in between, a lifetime dedicated to the elaboration of heavy elements.

1.1.1 Stellar evolution

For though the stars seem to be eternal viewed by human eyes bound to human timescales, they are indeed evolving. Scientists have started to understand their true nature at the very beginning of the 20th century, after the two first equations governing a sphere of gas (the hydrostatic equilibrium and the mass continuity) had been established by J. H. Lane and A. Ritter, complemented in 1907 by R. Emden: the stars could then be considered as spheres of gas ruled by gravity and following the laws of gas thermodynamics. These equations gave clues about the stellar structure in hydrostatic configuration, but the notion of evolution had not yet entered the scene. In 1911 and 1914 respectively, E. Hertzsprung and H. Russell, working independently, plotted a graph of the luminosity of the observed stars versus their temperature (see Fig. 1.1). They were able to show that the stars were not randomly distributed on this graph, but gathered in distinct regions. H. Russell proposed an evolutionary scheme in which the red giants were an early phase of stellar evolution, and gravitational contraction would heat them up and drive them on the main sequence. The comprehension of the internal structure of stars progressed between 1916 and 1924,

when Sir A. Eddington completed the equations of stellar structure by the equations of energy generation and transport, and thus elaborated a coherent theory of the internal structure of stars. It is only in 1938 that the source of the energy sustaining our Sun was really understood by H. Bethe and C.-F. von Weizsacker independently, i.e. the fusion of hydrogen that produces helium. The evolution of the chemical composition of the gas was then related to an evolution of the star itself. After this essential step, E. Öpik showed that the red giants were the outcome of a main sequence star having exhausted hydrogen in its core: the evolutionary sequence of the Hertzsprung-Russell (HR) diagram was corrected.

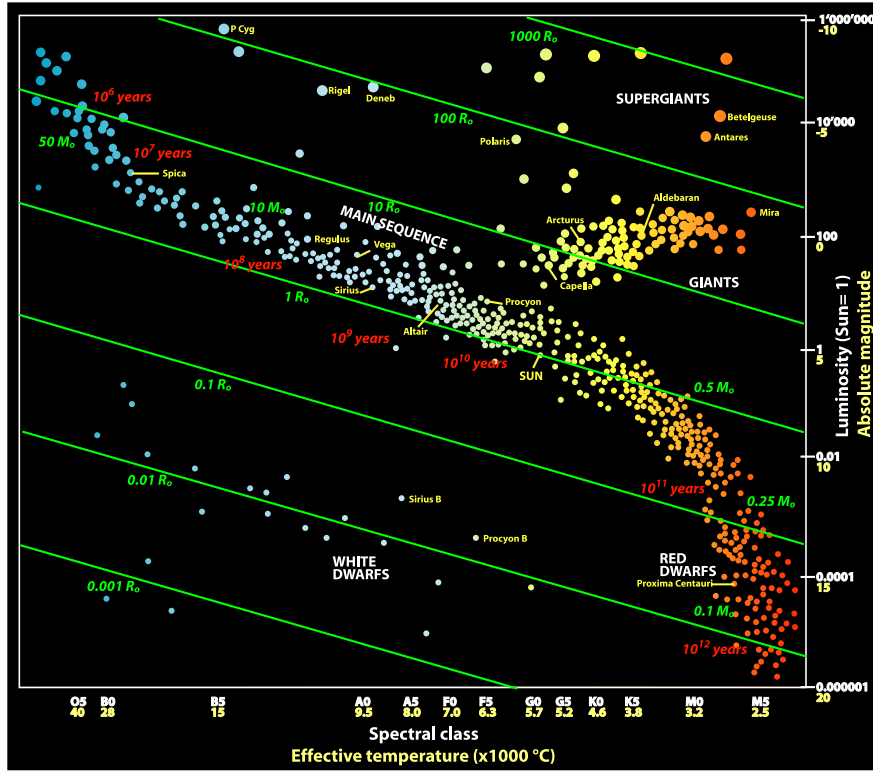


Figure 1.1: Hertzsprung-Russell diagram. Some identified stars are indicated. The green lines are lines of equal radius.

The development of nuclear astrophysics in the following decades revealed that hydrogen burning is only the first stage of a star life. It is followed by the fusion of helium into carbon and oxygen, these two first burning stages representing 99% of the lifetime of the star. For stars with a mass smaller than about nine times our Sun's mass ($9 M_{\odot}$), the end of helium burning marks the end of the whole story, but if the star is more massive it can go through advanced burning stages: carbon burning (producing mainly neon, sodium and magnesium), neon photodisintegration (producing oxygen and magnesium), oxygen burning (producing silicon, phosphorus, sulphur, argon and calcium) and eventually silicon burning (producing iron, cobalt, nickel and other iron-group elements). At the end of this last stage, the star is left with an iron core which cannot burn further to produce energy by fusion, iron being the most tightly bound nucleus. Gravity is no more counteracted, the core collapses and the star explodes as a supernova. During the explosion, heavier nuclei are produced by rapid neutron capture (*r*-process), filling the periodic table of elements.

Until the middle of the 20th century, the integrations of the stellar structure equations were done by hand, demanding days or sometimes months of laborious work. We must acknowledge here the great achievements that have been accomplished by the patient and devoted astrophysicists of those times! A giant step was made when computers entered the scientific institutes, allowing for the rapid modelling of stellar evolution. After core helium burning, the evolution

accelerates and most of the energy produced in the core is released in the form of neutrinos instead of photons as in the previous stages. Neutrinos do not interact with the gas, so the core is decoupled from the surface, and the star does not travel anymore in the HR diagram. We rely only on the numerical models to understand what is going on behind the curtain of the stellar surface.

Stellar evolution codes are very sensitive to our knowledge and understanding of the invoked processes. Some fundamental parameters for stellar evolution such as the opacities (describing the interaction of the radiation with the gas) or the nuclear reaction rates (describing the energy generation), have been much improved in the latest years. Others still evade us, such as a good convection theory (describing the energy transfer by matter buoyancy when the radiative transfer is impeded), which is nowadays still being parametrised.

Confronted to observations, the first computed stellar models showed many discrepancies. The adequation was much improved with the inclusion in the evolutionary codes of core overshooting around 1975, and of mass loss around 1980. The need for internal mixing was soon raised by the abundance determinations at the surface of the stars (see for example Liubimkov 1984). Rotation provides a natural mechanism for mixing, and its inclusion in stellar evolution codes has much improved the agreement between the models and the observed enrichment levels (Heger & Langer 2000; Meynet & Maeder 2000).

1.1.2 Evolution with rotation

Why would stars rotate? From a molecular cloud with a typical radius of the order of 0.1 pc (about 10^{12} km) to a newborn star with a typical radius of the order of twice the solar radius (about 10^6 km), there is a contraction over 6 orders of magnitude. The physical law of conservation of angular momentum ($\Omega r^2 = \text{const.}$) implies that even if the cloud had only the slightest rotation movement at the beginning of its collapse, the star formed from it should present an extremely high rotation rate at birth. Thus the question which arises is not whether stars rotate or not, but how come they are not all fast rotators? We shall leave this stringent question to the star-formation specialists. I will just briefly enounce here the changes brought by rotation to the stellar evolution.

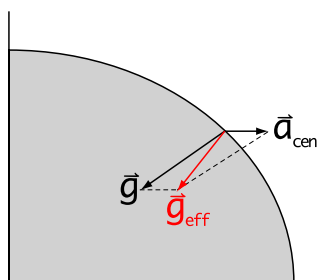


Figure 1.2: Effective gravity $\vec{g}_{\text{eff}}$ as the result of the gravity $\vec{g}$ counterbalanced by the centrifugal acceleration $\vec{a}_{\text{cen}}$.

A star is roughly a sphere of gas maintained in equilibrium by the competition between the gravity, which leads to contraction, and the pressure gradient, which leads to expansion. When rotation comes into play, it adds a sustaining force against gravity, so at the beginning of its evolution, the star behaves as if its mass were smaller: its luminosity and effective temperature are shifted to lower values. As the evolution proceeds, rotation triggers various instabilities inside the star, among which:

- the **shear instability**: as the star does not rotate like a rigid body, the differential rotation induces a shear between layers which have different angular velocities. This shear drives a turbulent mixing of the chemical species between the different layers.
- the **Eddington-Sweet circulation**: except at the equator, the centrifugal force is not parallel to the gravity, thus the resultant effective gravity is not radial anymore (see Fig. 1.2). The

equipotential surfaces are no longer spheres. Moreover, the radiative flux, which depends on the local effective gravity, is not constant on equipotentials anymore. According to von Zeipel (1924), the star cannot be in hydrostatic and radiative equilibrium at the same time. This drives a large-scale circulation, known also as the *meridional circulation*, which mainly transports angular momentum inside the star.

The shear mixing brings fresh hydrogen to the burning core which grows, enhancing the luminosity of the star. Since more fuel is available, the duration of the main sequence is also increased. Rotation enhances the mass loss by radiatively driven winds through two mechanisms: first, the stronger effective gravity at the pole induces a stronger flux in that direction (von Zeipel theorem); second, the lower effective temperature at the equator increases the opacity (if this latter is not only due to electron scattering), which increases the mass-loss rates.

After the main sequence, the rotational mixing brings freshly synthesised elements towards the surface, enhancing the opacity of the outer layers. The evolution towards the coolest part of the HR diagram is favoured, the mass loss is enhanced, and the surface metallicity increases.

The inclusion of the effects of rotation in stellar evolution codes not only improved the adequation between models and observations in the case of the surface abundances. It also improved other observed characteristics, such as the variation with the metallicity of the blue to red supergiants ratio (Maeder & Meynet 2001) and of the WR to O-type stars ratio (Meynet & Maeder 2003, 2005).

1.1.3 Evolution at low metallicity

The quest for the origins is a powerful driving force for the scientists in general and the astronomers in particular. In this context, the search for the most metal-poor stars benefits from vast observation campaigns, as the HK survey (Beers et al. 1992), the Hamburg/ESO survey (Christlieb 2003), the SDSS/SEGUE survey (York et al. 2000), or forthcoming dedicated facilities as LAMOST¹ or SSS². The record holders to date are HE 1327-2326 and HE 0107-5240, with an iron abundance that is about a factor of 1'000'000 lower than that of the Sun. These extreme cases, but also a great proportion of the slightly less metal-poor stars of the Galactic halo, present very peculiar surface abundances, and many questions are raised by these relics of the early Universe. What kind of precursor lies at the origin of these peculiarities? The question is not settled yet, and improvements in the modelling are needed to better understand the stellar evolution in the early Universe.

Stellar models at low metallicity reveal that a low content in heavy elements modifies substantially the evolution. The opacity is lower, so the stars are more compact, and therefore hotter. They spend their life in a bluer part of the HR diagram because their envelope is more transparent. Their radiative winds are extremely weak. The stars end their evolution with large cores. While the massive stars at standard metallicity in the whole mass range undergo a supernova explosion, the low-metallicity ones may undergo a whole variety of fates depending on their core masses: some may collapse directly into a black hole, without any supernova; others may be entirely disrupted by a cataclysmic explosion.

When rotation is taken into account, the modifications are even larger (Meynet et al. 2006; Hirschi 2007). The compactness leads naturally to a high equatorial velocity. The low mass-loss rates do not allow the star to lose significant amounts of angular momentum, so the stellar surface accelerates and might even reach the critical velocity, at which the centrifugal force counterbalances the gravitational force. In this case, the star loses mass by a mechanical process. Also the mixing efficiency is larger: the gradient of angular velocity is steeper and the radius is smaller, so the diffusion time is shorter. Mixing between the He-burning core and the H-burning shell drives a strong primary nitrogen production. The enrichment of the surface may be very strong, and the

¹<http://www.lamost.org>

²<http://www.mso.anu.edu.au/skymapper/>

radiative winds may be significantly enhanced. The transport of angular momentum inside the star is less efficient, so that at the end of the evolution, the core is spinning fast. The final collapse is expected to be highly aspherical.

1.2 Overview of the thesis

So rotation does play a key role in stellar evolution, and all the more so at low metallicity. At solar metallicity, or down to the metallicity of the Magellanic Clouds, the stellar evolution codes can be tested by numerous observational constraints, and the Geneva code has passed these tests. When the metallicity becomes very low, there are no observational constraints anymore. It is however interesting to explore the predictions given at very low metallicity by a code that has been successfully tested at higher metallicity.

The aim of this thesis is to explore the effects of rotation on the evolution of the first generations of stars. The next chapter (Chapter 2) describes the metallicity evolution in the early Universe. The formation peculiarities of the very first generation of stars are summarised, and the role the dark matter annihilation could play is explored. The characteristics of the most metal-poor stars observed in the halo of our Galaxy are briefly summarised, and a work aimed at understanding the origin of their peculiar surface abundance patterns is presented.

In Chapter 3, a consistent grid of stellar models is presented, a grid with four different masses (3, 9, 20, and $60 M_{\odot}$), four different metallicities ($Z = 0.020, 0.002, 0.00001$, and 0) and seven different rotational rates ($\Omega/\Omega_{\text{crit}} = 0.1, 0.3, 0.5, 0.7, 0.8, 0.9$, and 0.99). These models are computed with exactly the same input parameters and the same physical ingredients, so it is possible to explore precisely the effects on the stellar evolution of varying rotation rates, metallicity, or mass. These models are then used to study the Be phenomenon in relation with the reaching of the critical velocity.

In Chapter 4, we turn to strictly zero-metallicity stars, and a grid of rotating Population III models with masses ranging between 9 and $200 M_{\odot}$ is presented. These models are followed up to the pre-supernova stage. Stellar yields are given and included in a chemical evolution model for the Galactic halo to compare them with the observed very metal-poor stars. The effects of the adjunction of a magnetic field in the evolution of a very massive Pop III star is briefly explored. Models of Pop III stars are then used as physics laboratories to test the possible variation of the fine structure constant.

Chapter 5 presents a work in progress that has been undertaken by the Geneva group as a whole: the computation of new grids of stellar models. Since the publication of the previous grids between 1992 and 1999, the physical ingredients included in the Geneva code has been much improved, and an accurate treatment of rotation has been implemented. The time is ripe to provide the astronomical community with a new set of theoretical tracks, both rotating and non rotating.

Conclusion and perspectives are given in Chapter 6.

Early Universe

2.1 Zero-metallicity Universe

In the Big Bang theory, the Universe begins as a singularity of infinite density and temperature and expands immediately in all directions. We find in Boesgaard & Steigman (1985) and Steigman (2007) a summary of the different steps marking the composition evolution of the primordial Universe (see also Table 2.1):

- when the Universe is only **10 ms** old, the temperature is so high that the weak interactions are in equilibrium. The reaction $e^+ + e^- \rightleftharpoons \nu + \bar{\nu}$ keeps all the leptons in interactions through neutral current, while protons and neutrons are kept in equilibrium by the charged current reactions $p + e^- \rightleftharpoons n + \nu_e$ or $n + e^+ \rightleftharpoons p + \bar{\nu}_e$;
- after **100 ms**, the temperature has dropped to about $3 \cdot 10^{10}$ K, and the neutral current becomes slower than the expansion rate, so the neutrinos decouple. Actually, only the ν_μ and ν_τ fully decouple from the electrons at that time, while the ν_e keep interacting through charged-current interactions;
- when the Universe is about **1 s** old, its temperature is about 10^{10} K (0.8 MeV), and the charged current freezes out as well: the ratio of protons to neutrons is given by the Boltzmann equation:

$$\left(\frac{n}{p}\right)_* \approx \exp\left(-\frac{\Delta m}{T_*}\right) \simeq \frac{1}{6}$$

where $\Delta m = 1.293$ MeV is the mass difference between proton and neutron in energy units and the subscript $*$ indicates the time of freeze out. From then on, the density of neutrons is ruled by β -decay $n \rightarrow p + e^- + \bar{\nu}$, and decreases exponentially: the density of neutrons at a time $t > t_*$ is

$$n_n(t) = n_n(t_*) \exp\left(-\frac{t - t_*}{\tau_n}\right)$$

where τ_n is the lifetime of neutrons (885.7 ± 0.8 s). Protons and neutrons keep interacting together to form deuterium (D) by the reaction $p(n, \gamma)D$, but the photons are so energetic that the inverse reaction is quickly realised, so D is continuously destroyed (this is known as the *D bottleneck*);

- at the age of **10 s**, the Universe has cooled down to a temperature of about $3 \cdot 10^9$ K which is below the electron mass in energy. The production of e^+e^- pairs by the reaction $\gamma + \gamma \rightleftharpoons e^+ + e^-$ is no more possible, while the pair annihilation continues, producing photons;

- it is only after **100 s** that the Universe has cooled enough so the D photodissociation is no more possible and D survives. This represents the first step in the Big Bang nucleosynthesis (BBN). Heavier elements can be built from D nuclei through various reactions: tritium comes from $D(n,\gamma)^3\text{H}$ and $D(D,p)^3\text{H}$, ^3He is built through $D(p,\gamma)^3\text{He}$ or $D(D,n)^3\text{He}$. Then ^4He (α -particle) is formed from tritium by the reactions $^3\text{H}(p,\gamma)^4\text{He}$ and $^3\text{H}(D,n)^4\text{He}$, or from ^3He by the reactions $^3\text{He}(n,\gamma)^4\text{He}$, $^3\text{He}(D,p)^4\text{He}$ and $^3\text{He}(^3\text{He},2p)^4\text{He}$. Traces of ^7Li and ^7Be are produced by $^3\text{H}(\alpha,\gamma)^7\text{Li}$ and $^3\text{He}(\alpha,\gamma)^7\text{Be}$. Fig. 2.1 (left) shows the evolution of the nuclides with the temperature during the phase of BBN.

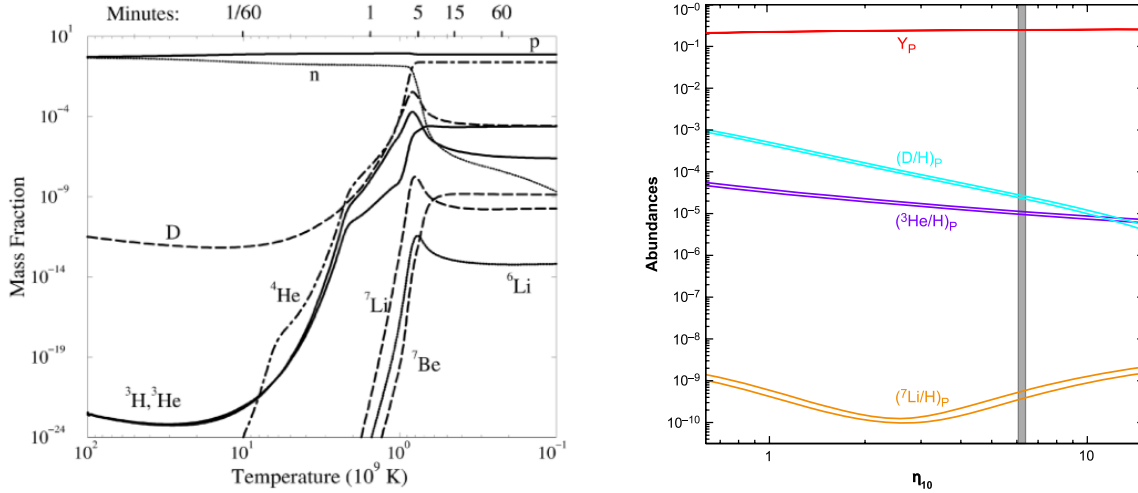


Figure 2.1: *Left*: evolution of the nuclides during the BBN (taken from Tytler et al. 2000). A value $\eta_{10} = 5.1$ was assumed for this calculation. *Right*: predicted primordial number abundances as a function of η_{10} (adapted from Steigman 2007). The grey area represent the WMAP (five years) value: $\eta_{10} = 6.20 \pm 0.16$ (Komatsu et al. 2009).

The Universe encounters then another bottleneck: there is no stable nucleus with $A = 5$ or $A = 8$. In stars, Salpeter (1952) shows that this is overcome by a three-body reaction known as 3α , which converts three ^4He nuclei in ^{12}C . In the BBN, this reaction is not possible because the density is quickly decreasing. After less than 1000 s, the primordial nucleosynthesis can be considered as terminated, and no metals ($A \geq 12$) have yet been synthesised.

Recently, Iocco et al. (2007) have revisited the standard BBN with an extended nuclides network and the inclusion of previously neglected reactions. They show that even so, the maximum abundance of CNO isotopes produced during the BBN is completely negligible: at most a few 10^{-16} (in number) compared to the H abundance.

2.1.1 Predicted primordial abundances

In the standard BBN model, when the number of the neutrinos family equals 3, and no deviation to standard cosmology is considered, the BBN predictions depend on only one free parameter: the ratio of baryons to photons η_{10} . It is defined as:

$$\eta_{10} = 10^{10} (n_b/n_\gamma) = 274 \Omega_b h^2$$

where $\Omega_b = \rho_b/\rho_{\text{crit}}$ is the baryons contribution to the critical mass density $\rho_{\text{crit}} = 3 H_0^2/(8\pi G)$, and h is the present value of the Hubble expansion factor, expressed in $100 \text{ km s}^{-1} \text{Mpc}^{-1}$. In the context of the Λ CDM model, the latest determination by WMAP gives $\eta_{10} = 6.23 \pm 0.17$ if only the WMAP data are considered, and $\eta_{10} = 6.20 \pm 0.16$ when the baryons acoustic oscillations (BAO) and the Type Ia SNe results for the distance scales are included (Komatsu et al. 2009).

Table 2.1: Main steps in the history of the early Universe (Durrer 2006; Loeb 2006; Komatsu et al. 2009).

	time	Temperature	redshift
Planck time	10^{-43} s	10^{32} K	
Inflation epoch	10^{-36} - 10^{-32} s		
Quarks confinement into nucleons	$2 \cdot 10^{-4}$ s	10^{12} K	
Neutrinos ν_μ and ν_τ decoupling	0.1 s	$3 \cdot 10^{10}$ K	
Neutrinos ν_e decoupling (neutrons freeze out)	1 s	10^{10} K	
e^+e^- annihilation	10 s	$3 \cdot 10^9$ K	
BBN	100-1000 s	$10^9 - 3 \cdot 10^8$ K	$z = 10^8$
Recombination	380'000 yr	3100 K	$z = 1100$
First stars formation	~ 200 Myr		$z \sim 15 - 20$
Reionisation	~ 400 Myr		$z \simeq 11$
Today	13.7 Gyr	2.73 K	$z = 0$

The abundance of D depends strongly on η_{10} . If there are a lot of baryons (high η_{10}), the D burning occurs quicker and its final abundance is lower (see Fig. 2.1, *right*). The same trend is valid for ^3He .

On the contrary, the abundance of ^4He is almost insensitive to η_{10} . It is constrained by the neutron abundance, so it depends on the competition between the charged-current weak interaction rate and the Universe expansion rate, and also on the number of neutrinos families. These dependences rule directly the value of the ratio n/p . η_{10} acts only indirectly on this ratio in the following way: if η_{10} is high, the D bottleneck is overcome earlier, i.e. at a higher temperature, so the value of n/p is larger. The neutron lifetime is of the same timescale as the duration of the primordial nucleosynthesis and ^4He is the more robustly bound nucleus, so we can consider that about all the neutrons are safely locked inside ^4He nuclei at the end of the BBN. The number fraction of He atoms (mostly ^4He) is thus about half the number fraction of neutrons at the time the photodissociation of D stops ($(n/p)_D \simeq 1/7$), and the primordial He mass fraction may thus be estimated as

$$Y_P \simeq 4 \frac{(n_n/2)_D}{n_n + n_p} = 2 \frac{(n/p)_D}{1 + (n/p)_D} \simeq \frac{1}{4}.$$

Note that in this estimation, we have taken $m_{\text{He}} = 4 m_{\text{H}}$, which is not accurate.

At low η_{10} , the reaction $^3\text{H}(\alpha, \gamma)^7\text{Li}$ dominates, so the abundance of ^7Li is favoured against that of ^7Be , while at high η_{10} values, it is the $^3\text{He}(\alpha, \gamma)^7\text{Be}$ which dominates. It explains why in Fig. 2.1 (*right*) the abundance curve of ^7Li starts decreasing with increasing η_{10} . For high values of η_{10} , i.e. large baryons abundances, the reaction $^7\text{Be}(n, p)^7\text{Li}$ increases the final ^7Li abundance again.

In the range $5.7 \lesssim \eta_{10} \lesssim 6.5$, Steigman (2007) proposes a fit to determine the primordial abundances of D, ^3H , $^3,^4\text{He}$, and ^7Li which is consistent with several BBN codes results (Kernan & Krauss 1994; Copi et al. 1995; Coc et al. 2004; Cyburt et al. 2004; Serpico et al. 2004):

$$\begin{aligned}
10^5(\text{D}/\text{H}) &= 2.68 (1 \pm 0.03) (6/\eta_{10})^{1.6} \\
10^5(^3\text{He}/\text{H}) &= 1.06 (1 \pm 0.03) (6/\eta_{10})^{0.6} \\
Y_P &= 0.2483 \pm 0.0005 + 0.0016 (\eta_{10} - 6) \\
10^{10}(^7\text{Li}/\text{H}) &= 4.30 (1 \pm 0.1) (\eta_{10}/6)^2.
\end{aligned} \tag{2.1}$$

With the WMAP value for η_{10} , this yields:

$$\begin{aligned}
 \text{D/H} &= (2.54 \pm 0.18) \cdot 10^{-5} \rightarrow \epsilon(\text{D}) = 7.40 \pm 0.03 \\
 {}^3\text{He/H} &= (1.04 \pm 0.04) \cdot 10^{-5} \rightarrow \epsilon({}^3\text{He}) = 7.02 \pm 0.02 \\
 Y_{\text{p}} &= 0.2486 \pm 0.0007 \\
 {}^7\text{Li/H} &= (4.6 \pm 0.6) \cdot 10^{-10} \rightarrow \epsilon({}^7\text{Li}) = 2.66 \pm 0.06
 \end{aligned} \tag{2.2}$$

where $\epsilon(X) = 12 + \log(X/H)$.

2.1.2 Observations of the primordial abundances

To date, even the deepest observations cannot reach objects that are composed of primordial abundances. Even the most metal-poor environments or the higher redshift observations bear the signature of some stellar processing. The difficulty to extrapolate from those observations to the primordial values depends strongly on the post-BBN evolution of each nuclide.

Deuterium

Deuterium can only be destroyed in stars. Its post-BBN evolution is thus very simple. The higher measured value will bring a lower limit to its primordial abundance. The preferred targets to measure D abundance are the absorption systems in the line of sight of distant QSOs. However, the isotopic shift between HI and DI is only 81.6 km s^{-1} (Pettini et al. 2008a), and the HI is largely overabundant, so only a few QSOs absorption systems have given reliable results (seven systems so far).

Pettini et al. (2008a) review the results for these seven systems, which have an absorption redshift ranging between $z = 2.07$ and 3.57 . The observed D abundance ranges between $\log(\text{D/H}) = -4.78$ and -4.40 , which leads to the following estimation for the primordial value:

$$\text{D/H} = (2.82 \pm 0.20) \cdot 10^{-5} \rightarrow \epsilon(\text{D}) = 7.45 \pm 0.03.$$

This value is well in agreement with the theoretical one.

${}^3\text{He}$

The post-BBN evolution of ${}^3\text{He}$ is much more complicated. It is produced during the pre-stellar evolution, but then it is destroyed in hot stellar interiors. However, in low-mass cool stars, it is produced by the pp -chains reactions during the main sequence (MS), and the first dredge-up of the red giant phase brings it to the cool envelope where it is preserved. The general picture is that there is a net production of ${}^3\text{He}$ (Bania et al. 2007). However, observations do not show a trend with the metallicity gradient, as would be expected if stars were net ${}^3\text{He}$ producers. Charbonnel & Do Nascimento (1998) show that an extra-mixing process occurs in some red giants after the bump, which brings ${}^3\text{He}$ down inside the star where it is destroyed. The fact that 90-95% of the RGBs undergo such a mixing seems to account for the observed values. Charbonnel & Zahn (2007) show that this mixing is most probably thermohaline mixing. They also show that thermohaline mixing is inhibited in the presence of a magnetic field, and that the proportion of 5-10% of red giants that show no sign of mixing corresponds to the proportion of magnetic Ap stars among the A type star population.

${}^3\text{He}$ is detected in emission in HII regions or in planetary nebulae. Bania et al. (2002) analyse twenty-one Galactic HII regions and find a plateau at ${}^3\text{He/H} = (1.9 \pm 0.2) \cdot 10^{-5}$. However, for the estimation of the primordial value, they prefer to rely on the single HII region S209 which is

the most metal-poor of their sample and which shows a high helium-ionisation level. Following them, we get:

$${}^3\text{He}/\text{H} = (1.1 \pm 0.2) \cdot 10^{-5} \rightarrow \epsilon({}^3\text{He}) = 7.04 \pm 0.08.$$

This value is well in agreement with the theoretical one.

${}^4\text{He}$

The stellar processing transforms H into He, so the post-BBN evolution of ${}^4\text{He}$ is a systematic increase. ${}^4\text{He}$ is detected in emission in the HII regions, so the search for its primordial abundance targets the most metal-poor Blue Compact Galaxies (BCGs) HII regions. Since massive stars are short-lived and both He and oxygen producers, the studies usually look for a $[\text{He}/\text{H}] - [\text{O}/\text{H}]$ correlation and extrapolate it to $[\text{O}/\text{H}] = 0$. Most recent studies analyse the 82 HII regions observed by Izotov & Thuan (2004).

Many sources of systematic errors have been identified (among them, collisional excitation of Balmer lines, stellar absorption, temperature or ionisation structure, blindness to neutral atoms, see Peimbert et al. 2003). Often a well-chosen subset of the Izotov & Thuan sample is used to get a better estimation.

Recently, Peimbert et al. (2007) re-analyse a subset of five BCGs HII regions with updated atomic data and stellar population synthesis, and get:

$$Y_{\text{P}} = 0.2477 \pm 0.0029.$$

This value is well in agreement with the theoretical one.

${}^7\text{Li}$

As for the ${}^3\text{He}$, although ${}^7\text{Li}$ is destroyed in the hot stellar interiors, the general trend for the post-BBN evolution of ${}^7\text{Li}$ is a net production. The discovery of a plateau in the abundance of ${}^7\text{Li}$ in metal-poor halo stars by Spite & Spite (1982) has been interpreted as the observation of the primordial abundance at $({}^7\text{Li}/\text{H}) = (1.12 \pm 0.38) \cdot 10^{-10}$ (i.e. $\epsilon({}^7\text{Li}) = 2.05 \pm 0.15$).

Later studies have remain close to this value, but amongst the latest ones, Asplund et al. (2006) and Bonifacio et al. (2007) find a slight trend with $[\text{Fe}/\text{H}]$ instead of a true plateau. For the primordial abundance, Bonifacio et al. (2007) find:

$${}^7\text{Li}/\text{H} = 1.26 \cdot 10^{-10} \rightarrow \epsilon({}^7\text{Li}) = 2.1 \pm 0.1.$$

This value is a factor of 3-4 lower than the predicted one. Many attempts to solve this so-called *Li problem* have been unsuccessful. We will address this point in more details in Sect. 4.5.1 on page 91.

2.2 Formation of the first stars

At the end of the BBN, the heaviest stable nuclide is ${}^7\text{Li}$. About 380'000 years later, the temperature has cooled down enough so the protons and the electrons recombine to form neutral hydrogen atoms. The photons can now propagate freely in the transparent Universe. The expansion lowers their frequency below the optical one, and since there are no other sources of light, the Universe enters the so-called *Dark Ages*. Its composition is about 75% hydrogen and 25% helium, and no nucleosynthesis is possible anymore. A simple look around us tells us however that something did happen since then, that somehow heavy elements were produced... Only stars can produce them, so there must be a very first generation of stars that inherit the BBN composition and start enriching the Universe. It is during the Dark Ages that the conditions for star formation set in.

To understand what happened at that time, we have to go back to the very beginning of the Universe. Loeb (2006) describes how primordial density fluctuations are generated during the inflation epoch (between 10^{-36} and 10^{-32} s after the Big Bang) by quantum mechanical fluctuation of the vacuum. From the WMAP image of the recombination time (380'000 yr after the Big Bang), we know that the spatial fluctuations in energy density and gravitational potential are not larger than about 10^{-5} at that time. During the Dark Ages, which will last for a few hundreds million years, these small fluctuations are accentuated by gravitational instability, and the dark matter (DM) concentrates into growing halos. The baryons fall into the gravitational well of the DM, and if the halo mass is around $10^6 M_\odot$, the gas collapses to form the first stars.

Star formation in the present days is a complex process, including complicated physics and many feedbacks. At the time the very first stars started to form, the process was much simpler: the initial conditions are well specified by the cosmology, and the stars form in an isolated medium, without any feedback process. However, some properties of a metal-free gas make the formation of the first generation stars quite peculiar. The first stage of a cloud's collapse into stars is isothermal: the heating created by the compression of the gas is radiated away by dust and molecules lines. The Jeans mass $M_J \propto T^{3/2} \rho^{-1/2}$ gets lower during that phase, so the cloud is able to fragment. In a metal-free gas though, the only molecule which can act as a cooling agent is H_2 . In absence of dust grains, there are two channels for H_2 formation in a gas phase:

1. the *H^- channel*: the reaction $H + e^- \rightarrow H^- + \gamma$ followed by $H^- + H \rightarrow H_2 + e^-$. The free electrons which play the role of catalyst come from the recombination epoch or from collision-induced ionisation.
2. a *3-body interaction*: at high density ($n \gtrsim 10^8 \text{ cm}^{-3}$) three-body interactions become possible and H_2 can form through $3 H \rightarrow H_2 + H$.

The H_2 molecule offers only rotational-vibrational transitions, so its cooling efficiency is low. 3D cosmological simulations, as Abel et al. 2002 (hereafter ABN02) or Bromm et al. 2002 (hereafter BCL02) have shown that the gas is able to cool down to 200 K with a typical density of 10^4 cm^{-3} . It is interesting to note that these simulations have been performed with different methods (adaptive mesh refinement, AMR, for ABN02, smoothed particle hydrodynamics, SPH, for BCL02) and different initial conditions, but converge to similar results. With the above values of temperature and density, the typical mass of a clump is in the order of 10^2 - $10^3 M_\odot$, or more precisely, according to Clarke & Bromm (2003): $700 M_\odot (T/200 \text{ K})^{3/2} (n/10^4 \text{ cm}^{-3})^{-1/2}$.

In ABN02 and BCL02 studies as well as in more recent ones (Yoshida et al. 2006; O'Shea & Norman 2007), a single protostar is formed in each clump, because it is not able to fragment further. Omukai & Nishi (1998) determine the mass of the protostellar hydrostatic core to be $\sim 0.005 M_\odot$, a value not very different than the protostellar cores in today's star formation. Then this protostellar core will grow by accretion. Generally, the accretion rate can be expressed as the ratio of the Jeans mass to the free-fall time. It is related to the sound speed and thus to the temperature: $\dot{M}_{\text{accr}} \propto c_s^3/G \propto T^{3/2}$. In today's star formation, the typical temperature is around 10 K, while at $z \sim 20$ we saw that it is around 200 K, so we expect a higher accretion rate in the primordial star formation process. Omukai & Palla (2003) define a critical accretion rate:

$$\dot{M}_{\text{crit}} = \frac{4\pi c R_{\text{ZAMS}}}{\kappa_{\text{es}}} \left(1 - \frac{L_{\text{ZAMS}}}{L_{\text{Edd}}} \right) \simeq 4 \cdot 10^{-3} M_\odot \text{ yr}^{-1}$$

with κ_{es} being the electron-scattering opacity. It is the accretion rate for which the star arriving on the zero-age main sequence (ZAMS) will have a luminosity exceeding the Eddington luminosity. Below this value, the nascent star accretes the surrounding gas at a slow pace, the structure has time to readjust, and thus the final mass is only limited by the mass of the clump and the lifetime of the star. Above $\dot{M}_{\text{crit}}$, the reaching of the Eddington limit induces a rapid expansion of the radius, reversing the flow and limiting the final mass to less than $100 M_\odot$. Note that these results

have been obtained in a spherical geometry. Tan & McKee (2004) and more recently McKee & Tan (2008) show that when rotation is implemented, the geometry of the collapse is modified: the accretion occurs through an equatorial disc, while the radiation flows mainly in the polar direction, unable to counteract the accretion. An aspherical HII region is formed, wider towards the poles. In a second stage, the radiation photodissociates the surface of the disc, so the accretion is limited to the shadowed part of the disc. Bromm & Loeb (2004) show that the accretion rate is varying with time: very strong at the beginning ($0.1 M_{\odot} \text{ yr}^{-1}$), it then declines in a complex way. They propose a double power-law fit to express the growth of the protostellar core: $M_{\star} = \int \dot{M}_{\text{accr}} dt \approx 20 M_{\odot} (t/10^3 \text{ yr})^{\xi}$, with $\xi = 0.75$ for $t < 10^3 \text{ yr}$ and $\xi = 0.4$ after.

There is a large consensus that the very first stars to form in the early Universe were very massive objects (VMOs), around $150\text{--}1000 M_{\odot}$. While at solar metallicity, such mass domain (if ever formed) would be highly unstable to pulsations and would lose enormous amounts of mass before stabilising, Baraffe et al. (2001) show that at $Z = 0$, the upper mass limit is much higher. In fact, metal-free VMOs are as unstable to pulsations on the ZAMS as their metal-rich counterparts, but due to their high central temperature and compact structure, the efficiency of the pulsation mechanism is reduced and the characteristic time for the growth of the amplitude is much longer. When evolution proceeds, the increase of the mean molecular weight at the centre is a stabilising factor (Maeder 1985), so after some H is burnt in the core, the star becomes stable.

2.2.1 Two distinct populations of metal-free stars

Were there only VMOs in the first generation of stars? On the basis of the work of Nakamura & Umemura (1999), who determine the characteristic mass of Pop III stars to be $\sim 3 M_{\odot}$, growing to $\sim 16 M_{\odot}$ by accretion, Nakamura & Umemura (2001) find that if the initial density of the cloud is lower than 10^5 cm^{-3} , the characteristic mass of the star will be in the order of $10^2 M_{\odot}$. If the initial density is higher than 10^5 cm^{-3} , more H_2 is produced by three-body interactions, so the characteristic mass is lowered to $\sim 1\text{--}2 M_{\odot}$, leading to a bi-modal IMF. While it is not clear whether these conditions are realistic (Bromm & Larson 2004), there are mounting evidences that after a first star-formation episode, the radiative feedback can lead to a second metal-free star formation with lower masses.

Once a VMO is formed inside a DM halo, its strong UV radiation exerts a negative feedback on the molecular hydrogen. This molecule is fragile and can easily be destroyed by UV photons in a two-step mechanism: $\text{H}_2 + \gamma \rightarrow \text{H}_2^* \rightarrow 2 \text{ H}$. This is supposed to suppress subsequent star formation. However, if the central object does not explode but collapses directly into a black hole (BH), as is expected for the mass range $40 M_{\odot} \leq M \leq 140 M_{\odot}$ or above $260 M_{\odot}$ (see Sect. 4.3.4 on page 81), it leaves behind a metal-free HII region which will later recombine. In the relic HII region, the number of free electrons favours the formation of H_2 molecules, and more interestingly of HD molecules. A small fraction of HD molecules can enhance significantly the cooling efficiency, because of a permanent dipole moment (Uehara & Inutsuka 2000; Flower et al. 2000; Galli & Palla 2002). With HD cooling, the temperature can be lowered to a floor temperature set by the CMB photons, under which no radiative cooling can occur ($T_{\text{CMB}} \approx 2.7(1+z) \text{ K}$, which gives $\sim 50 \text{ K}$ at a redshift $z \simeq 20$). With such a cooling, the peak mass of the IMF drops from $100 M_{\odot}$ for the very first generation of Pop III (Pop III.1) to about $10 M_{\odot}$ for this second generation of metal-free population (Pop III.2) (Nagakura & Omukai 2005; O’Shea et al. 2005; Johnson & Bromm 2006; Yoshida et al. 2007).

2.2.2 Primordial star formation with dark matter annihilation

Very recently, there has been renewed interest in the role the DM could play as an external source of energy during the primordial star formation. When the baryons collapse inside a DM halo, they exert a pull on the DM, which in turn concentrates at the centre of the forming stellar

core. An increase in the DM density leads to a high self-annihilation rate with a significant energy release. About 1/3 of this energy is emitted in the form of neutrinos, which will not interact with the gas, but 2/3 of the energy is emitted in the form of particles that can be thermalised by the gas (Bertone et al. 2005).

The best DM candidates to date are the weakly-interacting massive particles (WIMPs): these are particles that arise from a supersymmetric extension of the standard model of particle physics. The mass of the most popular WIMPs, the *neutralinos*, is of the order of 100 GeV. DM particles only very weakly interact with the baryons, but they self-annihilate with an energy release per unit volume which is:

$$\begin{aligned} Q_{\text{ann}} &= \langle \sigma v \rangle \frac{\rho_\chi}{m_\chi} \\ &= 1.2 \cdot 10^{-29} \frac{\text{erg}}{\text{cm}^3 \text{s}} \left(\frac{\langle \sigma v \rangle}{3 \cdot 10^{-26} \text{cm}^3 \text{s}^{-1}} \right) \left(\frac{n}{\text{cm}^{-3}} \right)^{1.26} \left(\frac{100 \text{ GeV}}{m_\chi} \right). \end{aligned}$$

$\langle \sigma v \rangle$ is the thermally-averaged annihilation rate, with $3 \cdot 10^{-26} \text{cm}^3 \text{s}^{-1}$ the best fit for the actual value of the DM relic density, see Bertone et al. (2005). ρ_χ is the WIMPs density, related to the gas density n by $\rho_\chi = 5 \frac{\text{GeV}}{\text{cm}^3} \left(\frac{n}{\text{cm}^{-3}} \right)^{0.81}$, and m_χ is the WIMPs mass.

Spolyar et al. (2008) describe the primordial star formation with DM interaction. The DM annihilation has two direct results: first, it releases free electrons that contribute to form H_2 , so the cooling rate is high. Second, as we mentioned above, it brings a new source of energy to heat the gas. When the gas concentration becomes $n \gtrsim 10^{13} \text{cm}^{-3}$ (for $m_\chi = 100 \text{ GeV}$), the DM heating becomes higher than the cooling rate: the collapse stalls. Spolyar et al. (2008) propose to call *dark star* such an object whose gravity is not sustained by nuclear burning but by DM annihilation. This new phase of stellar evolution is supposed to last as long as the DM annihilation timescale:

$$\begin{aligned} \tau_{\text{ann}} &= \frac{m_\chi}{\rho_\chi \langle \sigma v \rangle} \\ &= 0.6 \text{ Gyr} \left(\frac{m_\chi}{100 \text{ GeV}} \right) \left(\frac{10^{13} \text{cm}^{-3}}{n} \right)^{0.8} \left(\frac{3 \cdot 10^{-26} \text{cm}^3 \text{s}^{-1}}{\langle \sigma v \rangle} \right) \end{aligned}$$

which is around 600 Myr for the numerical values adopted here.

Note that Natarajan et al. (2008) show that the DM concentration by baryons collapse could be altered by the geometry of the collapse: if the accretion on the protostar occurs through a disc, the DM response could be delayed and the star would be left with only the initial amount of DM.

Iocco et al. (2008) find that during the stalling phase, an equilibrium is established: the stalling of the collapse also inhibits the DM contraction, then DM annihilation reduces ρ_χ , so the annihilation luminosity is reduced, and thus the baryons slightly contracts, with the result of again condensing the DM, which annihilates again with a higher rate and stops the collapse, and so on. During that phase, the star remains cool ($\log T_{\text{eff}} \approx 3.7$). Eventually, the annihilation timescale becomes longer than the Kelvin-Helmholtz timescale, and the star sets on the pre-MS track, however at a slower pace than a “normal” star. They determine that a $100 M_\odot$ dark star stalls for about $5 \cdot 10^3$ yr. Lower mass stars are more affected and this phase is longer ($1.5 \cdot 10^4$ yr for a $7 M_\odot$ dark star).

The WIMPs have a non-vanishing elastic-scattering cross section with the baryons. Some DM particles are expected to be scattered off the stellar core, reducing ρ_χ , but also to be captured and sink at the centre of the star. After a characteristic time for thermalisation

$$\tau_{\text{th}} = \frac{4\pi}{3\sqrt{2}G} \frac{m_\chi}{\sigma_0} \frac{R_\star^{7/2}}{M_\star^{3/2}}$$

where σ_0 is the DM-baryons elastic-scattering cross section, the WIMPs get re-distributed inside the star following a Maxwell-Boltzmann distribution:

$$n_\chi(r) = n_0 e^{-r^2/r_\chi^2} \quad \text{with} \quad r_\chi = \sqrt{\frac{3kT_c}{2\pi G \rho_c m_\chi}}$$

where T_c and ρ_c are the core temperature and density. While the star evolves on the pre-MS track, the capture rate increases. Considering a continuous DM flux through the star, an equilibrium is reached between capture and annihilation after a time

$$\tau_\chi = \sqrt{\frac{\pi^{3/2} r_\chi^3}{C \langle \sigma v \rangle}}$$

where C is the capture rate (Gould 1987). The DM sustains the star with a luminosity $L_\chi(r) = 4\pi \langle \sigma v \rangle m_\chi c^2 n_\chi^2(r)$. According to Iocco et al. (2008), a DM luminosity of 10^{41} erg s $^{-1}$ can be maintained for about 10^{12} yr in a $10^6 M_\odot$ DM halo. Although this luminosity is clearly dominant inside the star, they find that in stars more massive than $40 M_\odot$, a slow nuclear burning is still possible: the star evolves to the ZAMS and then slowly burns its hydrogen. The MS lifetime is increased by 88% for a $100 M_\odot$ star and by a factor of 5 for a $40 M_\odot$ star. The stars with $M < 40 M_\odot$, on the contrary, are frozen on the pre-MS phase.

In collaboration with Marco Taoso (Istituto Nazionale di Fisica Nucleare Sezione di Padova, Italy; Institut d'Astrophysique de Paris (IAP), France) and Gianfranco Bertone (IAP), we have studied the impact of DM capture and annihilation on the evolution of Pop III stars. The results of this study have been published in the following article (that can be found on page 163):

DARK MATTER ANNIHILATION IN POP III STARS
 M. Taoso, G. Bertone, G. Meynet & S. Ekström
 Phys. Rev. D, **78**, id. 123510

The Geneva code in its version `origin2006` (see Appendix A.3 on page 132) was adapted to compute the additional energy source, and to get the suitable discretisation of the central parts of the star so that the DM distribution is accurately resolved. Models of 20 and $200 M_\odot$ were followed from the ZAMS to the end of the MS with various values of the DM density ρ_χ . Selected models were followed to the end of central He burning (CHeB). For the WIMP model, we adopted $m_\chi = 100$ GeV, and the DM-baryons cross section $\sigma_0 = 10^{-38}$ cm ^{2} .

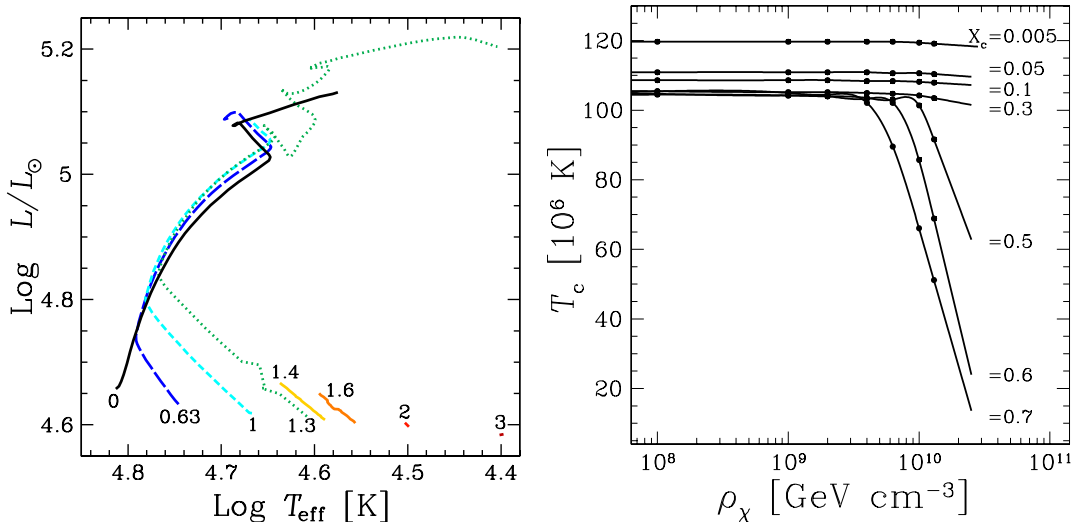


Figure 2.2: *Left*: HR diagram of $20 M_\odot$ Pop III models for various WIMP densities (labelled on the plot in units of $10^{10} \text{ GeV cm}^{-3}$). The tracks with $\rho_\chi = 1.4$ and $1.6 \cdot 10^{10}$ follow the same trend as the tracks with $\rho_\chi < 1.4 \cdot 10^{10}$, so only the beginning of the MS is shown for the sake of clarity. *Right*: Central temperature as a function of the WIMP density ρ_χ for the $20 M_\odot$ models at different stages of the MS.

The evolutionary tracks in the Hertzsprung-Russell diagram (HRD) are shown in Fig. 2.2 on previous page (*left*). The ZAMS is shifted to lower T_{eff} for higher ρ_χ . The energy released by the WIMPs can be evaluated as $E_\chi \approx L_\chi \tau_{\text{KH}}$. On the ZAMS, for the $20 M_\odot$ model with $\rho_\chi = 10^{10}$, it amounts to ~ 0.8 times the gravitational potential energy: the star is not completely sustained by DM energy, so it contracts. The T_c increases and so do the nuclear reactions, while the DM luminosity remains the same because it does not depend on the temperature. When the energy released by nuclear burning becomes comparable to the DM energy, the situation is very close to the “normal” situation and the tracks join in the HRD.

Figure 2.2 on previous page (*right*) shows the evolution of the T_c as a function of ρ_χ , at different H-burning stages. For $\rho_\chi \gtrsim 6.3 \cdot 10^9 \text{ GeV cm}^{-3}$, the H burning occurs at a lower T_c at the beginning of the MS. The junction with the standard case occurs when $X_c = 0.3$ in the case of the $20 M_\odot$ model with $\rho_\chi = 10^{10} \text{ GeV cm}^{-3}$. The nuclear burning at a low T_c implies a longer MS lifetime for the models that include DM interaction than for the standard model. Figure 2.3 shows the MS lifetime as a function of ρ_χ for the 20 and $200 M_\odot$ models. In the case of the $20 M_\odot$ models, below $\rho_\chi = 10^{10} \text{ GeV cm}^{-3}$ the delay amounts at most to 10%, but it rapidly increases for higher DM densities. If we extrapolate the curve, we can determine a critical DM density $\rho_{\chi,c} = 2.5 \cdot 10^{10} \text{ GeV cm}^{-3}$ above which the MS duration becomes longer than the age of the Universe and the evolution of the star “freezes”.

For the $200 M_\odot$ models, the critical density must be higher to stop the evolution: $\rho_\chi = 5.3 \cdot 10^{10} \text{ GeV cm}^{-3}$. We have checked the influence of our choice of the WIMP mass m_χ on these results. If we take a lower mass, $m_\chi = 10 \text{ GeV}$, the enhancement of the MS lifetime for $\rho_\chi = 10^{10} \text{ GeV cm}^{-3}$ is 0.2%, while it is 5% in the case of $m_\chi = 100 \text{ GeV}$.

After the MS, the impact of DM interaction fades away because the annihilation luminosity becomes lower than the nuclear luminosity. For a $20 M_\odot$ model with $\rho_\chi = 1.6 \cdot 10^{10} \text{ GeV cm}^{-3}$, the CHeB phase is increased by a factor of 1.2, while its MS lifetime is increased by a factor of 37. Our results are in good agreement with the work of Yoon et al. (2008), who find that after H exhaustion, the decrease of the number of H atoms reduces the DM capture rate and thus its luminosity. In the same work, they explore the impact of DM annihilation on rotating stars. They show that while the nuclear timescale is increased, the mixing timescale remains unchanged, so quasi chemically-homogeneous evolution is expected even for slowly rotating models ($v_{\text{ini}}/v_{\text{Kepler}} = 0.1$). This may have important consequences on the fate of the star (pair-instability SN with very little H expelled, GRB production).

Could we still observe “frozen stars” nowadays? They would appear abnormally inflated and cool for their mass and metallicity. Figure 2.4 on next page presents the ZAMS location in the $\log g$ vs $\log T_{\text{eff}}$ diagram. Of course, the frozen star could remain so only if the external conditions do not change. Would the DM density change for any reason (halo merging, star’s displacement, ...), the DM luminosity would be modified and the stellar evolution might proceed. We can also wonder how long a frozen metal-free star can remain metal-free in the chemically evolving Universe. If its surface accretes gas that has been enriched by other stars, the thermo-haline mixing will bring the heavy elements inside the star, which may modify the reaction rates and thus the equilibrium between DM and nuclear luminosities.

Note that we have here only addressed the role DM plays in the formation of primordial stars,

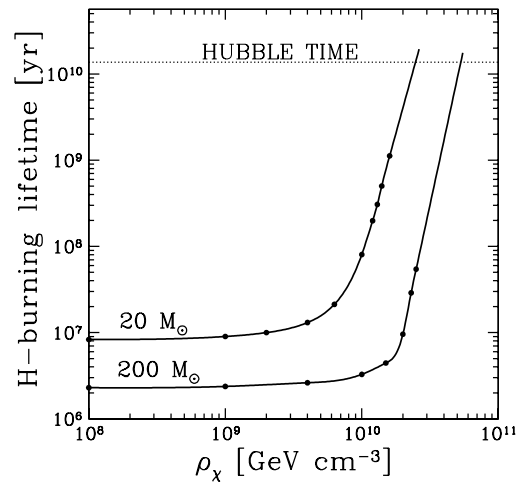


Figure 2.3: MS lifetime of $20 M_\odot$ Pop III models as a function of the WIMP density ρ_χ .

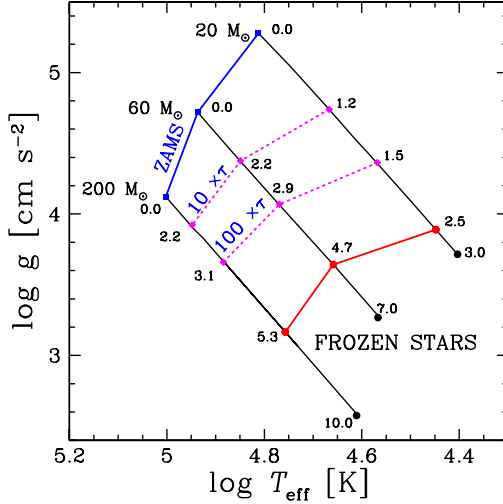


Figure 2.4: ZAMS location in the $\log g$ vs $\log T_{\text{eff}}$ for Pop III models of 20, 60, and 200 $M_{\odot}$ with various ρ_{χ} (labelled in units of $10^{10} \text{ GeV cm}^{-3}$). The ZAMS of the standard case is marked by the solid blue line, the ZAMS for the critical value of ρ_{χ} which leads to frozen stars is marked by the solid red line. Intermediate cases where the MS lifetime is increased by a factor of 10 or of 100 are marked by dotted magenta lines.

but it is supposed to be present in the local Universe as well. Fairbairn et al. (2008) show however that the densities needed for the DM to have a significant role are met only in the central 10^{-2} pc of our Galaxy.

2.3 Peculiarity of the evolution at $Z = 0$

Before we develop the effects of rotation on the evolution of Pop III stars (Chapters 3 and 4), we will recall here the basics of metal-free stellar evolution. The abundance peculiarity has consequences on two main aspects:

1. the **opacity** is reduced, the radiation flows more easily through the stellar plasma. The reduction of the interaction between radiation and matter will be manifested in the following characteristics (compared to higher metallicity stars):
 - the luminosity is larger and the radius is smaller;
 - there is less convective zones, and they are less extended;
 - the radiative winds are strongly reduced.
2. with only H and He present, the **reaction network** is reduced, the star can only rely on *pp*-chains to burn its hydrogen, at least at the very beginning of the evolution. Since *pp*-chains are not much sensitive to temperature, they have more difficulties to counteract the gravitation, so the stars contract for a longer time during their formation.

The impact of these characteristics are different in the different mass domains, as shown by Maeder (2008), who summarised the works of Marigo et al. (2001) and Feijóo (1999).

2.3.1 Low-mass domain

Below 3 $M_{\odot}$, the stars burn their H through the *pp*-chains during most of the MS phase. They evolve along the ZAMS line until the central H abundance X_c is almost exhausted and the core partially degenerate. The subsequent contraction eventually brings the core to high enough central temperature (T_c) to synthesise some ^{12}C through the 3α reaction. The CNO cycle starts, the energy production increases substantially, so the core becomes convective and expands. Some fresh H is engulfed, creating a small CNO flash in the partially degenerate core. In the HR diagram, this flash is visible as a loop in the tracks of a 1 or 1.5 $M_{\odot}$ model (see Marigo et al. 2001, their Fig. 2). The He-burning phase is marked by short-lived loops in the HR diagram.

2.3.2 Intermediate-mass domain

Between 3 and 10 $M_{\odot}$, the CNO ignition occurs during the MS, earlier for higher masses. Before the ignition, the star evolves along the ZAMS line and its convective core recedes until it becomes radiative. At the CNO ignition, the core becomes convective again and the star expands rapidly until X_c becomes so low that the star has to contract again to compensate for the energy loss. At H exhaustion, the star has two H-burning shells: a peaked internal one, powered by the CNO cycle, and a broad external one powered by pp -chains. Without much delay, the He-burning starts, so the star expands slightly again. When after a while the core becomes convective, the central density decreases, which leads to a slight contraction of the envelope by mirror effect. At about half the CHeB phase, the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction becomes significant and the star starts a redwards journey in the HR diagram. Note that in this mass range, the He exhaustion occurs while $\log T_{\text{eff}}$ is still higher than 4.0.

2.3.3 Massive-stars domain

Above 10 $M_{\odot}$, the initial contraction cannot be stopped by pp -chains, so the star goes on contracting until the 3α reaction can start and produce some ^{12}C . For stars with $M \approx 20 M_{\odot}$, the CNO cycle is able to ignite already on the ZAMS. The stars are hot and compact, but otherwise the MS phase proceeds normally. At H exhaustion, the stars are already hot enough to burn some He, so the transition from H burning to He burning is continuous, without any structural readjustment. The stars remain in the blue side of the HR diagram, except for the most massive stars ($M > 70 M_{\odot}$) which become red supergiants at the end of CHeB.

2.4 From Pop III to Pop II

As we will see in Sect. 4.3.4 on page 81, some Pop III stars will end their life in a direct collapse into a BH, engulfing all the heavy elements they produced during their life, but others will explode (as pair-instability SNe or Type II SNe) and enrich their surroundings with newly produced metals. The critical metallicity allowing the transition from Pop III to Pop II stars has been determined to be $Z_{\text{trans}} \approx 10^{-4} Z_{\odot}$ (Bromm et al. 2001; Schneider et al. 2002; Tumlinson 2006). More precisely, the determinant process is supposed to be atomic fine-structure line cooling on [C II] (157.74 μm) and [O I] (63.18 μm , 145.5 μm) (Bromm & Loeb 2003) and also on [Si II] (34.8 μm), and [Fe II] (25.99 μm , 35.35 μm) (Santoro & Shull 2006). Bromm & Loeb (2003) determine the critical abundances to be $[\text{C}/\text{H}] = -3.5 \pm 0.1$ and $[\text{O}/\text{H}] = -3.05 \pm 0.2$. Note that according to Schneider et al. (2006), if the carbon is mostly depleted into dust grains and CO molecules, the main cooling will come from the dust itself, with a critical metallicity $Z_{\text{crit}} = 10^{-6} Z_{\odot}$, two orders of magnitude lower than the case of fine-structure line cooling.

While Pop III stars are since long extinct, low-mass Pop II stars are expected to still be observable provided their initial mass is $M_{\text{ini}} \lesssim 0.8 M_{\odot}$.

2.4.1 Observations of low-metallicity halo stars

The most metal-poor stars observed to date are found in the halo of our Galaxy. A nomenclature has been proposed for these objects (Beers & Christlieb 2005):

Metal-poor	MP	$[\text{Fe}/\text{H}] = < -1.0$
Very metal-poor	VMP	< -2.0
Extremely metal-poor	EMP	< -3.0
Ultra metal-poor	UMP	< -4.0
Hyper metal-poor	HMP	< -5.0
Mega metal-poor	MMP	< -6.0

Although for the time being no MMP star has yet been found, the discovery of two HMP stars (HE 0107-5240 by Christlieb et al. 2002, and HE 1327-2326 by Frebel et al. 2005) opens the possibility of their existence.

Table 2.2: CNO abundances of the most metal-poor stars observed in the halo of the Galaxy.

Star	Status	[Fe/H]	[C/Fe]	[N/Fe]	[O/Fe]	$^{12}\text{C}/^{13}\text{C}$	$\odot^a$	Refs ^b
HE 1327-2326	subgiant	-5.90	3.78	4.28	3.42		ASG05	3
HE 0107-5240	giant	-5.28	3.7	2.57	2.3	> 50	ASG05	1, 2
HE 0557-4840	giant	-4.75	1.65	< 1.47	< 3.1		ASG05	4
CD -38°245	giant	-4.19	< -0.33	1.07			GS98	6
					0.9		ASG05	2
G77-61	dwarf	-4.03	3.2	2.2	2.2	5 ± 1	ASG05	5, 6
CS 22949-037	giant	-3.97	1.25	2.25	1.98	4 ± 2	GS98	7, 8
CS 22172-002	unmixed giant	-3.86	0.0	0.24	1.04	> 10	GS98	7, 8
CS 22885-096	unmixed giant	-3.78	0.26	0.26			GS98	8
BS 16467-062	unmixed giant	-3.77	0.25	< 0.45			GS98	8
CS 22968-014	unmixed giant	-3.56	0.26	0.24	0.9	30^{+8}_{-5}	GS98	7, 8
CS 22189-009	unmixed giant	-3.49	0.34	0.27		15^{+8}_{-5}	GS98	8
CS 29498-043		-3.49	2.2	2.26	2.45	6	ASG05	5

^aReference solar values: GS98 stands for Grevesse & Sauval (1998) and ASG05 stands for Asplund et al. (2005)

^bReferences: 1–Bessell et al. (2004); 2–Christlieb et al. (2004); 3–Frebel et al. (2008); 4–Norris et al. (2007); 5–Plez & Cohen (2005); 6–Plez et al. (2005); 7–Spite et al. (2005); 8–Spite et al. (2006)

Table 2.2 presents the record holders with their CNO abundances. It must be noted that while all these objects are extremely Fe poor, their CNO abundances are enhanced compared to solar (following A. Loeb, they could be called *anaemic stars*). Frebel et al. (2007) propose the observational index $D_{\text{trans}} = \log(10^{[\text{C}/\text{H}]} + 0.3 \times 10^{[\text{O}/\text{H}]})$ to compare metal-deficient stars with theoretical prediction of the primordial low-mass stars. According to the critical C and O abundances defined by Bromm & Loeb (2003), no observed star could have $D_{\text{trans}} \lesssim D_{\text{trans,crit}} = -3.5$. The two HMP stars have $D_{\text{trans}} = -2.1$ (HE 1327-2326) and -1.6 (HE 0107-5240), well above $D_{\text{trans,crit}}$. According to Frebel et al. (2007), all UMPs should be C and/or O rich.

Christlieb (2003) shows that among the stars with $[\text{Fe}/\text{H}] < -2.5$, 25% are carbon rich (CEMP stars), while the proportion drops down to 1% among more metal-rich Pop II stars. The origin of the peculiar abundance pattern of the CEMP stars is still an open question. Several scenarios have been proposed, divided in two categories:

1. **The polluted Pop III star:** CEMP stars are low-mass Pop III stars that underwent a surface pollution due to various mechanisms:
 - (a) *ISM accretion:* Shigeyama et al. (2003) propose that HE 0107-5240 was born metal-free and accreted polluted ISM matter during its life. They show that an accretion rate of $10^{-13} M_{\odot} \text{ yr}^{-1}$ is possible if the star has a slow velocity relative to the gas cloud ($\sim 10 \text{ km s}^{-1}$). In such a scenario, the metals are accreted at the surface, and mixed in the outer convective zone. In this case, one should expect a much lower heavy elements content in the giant HE 0107-5240 that mixes them in a large convective zone than in the subgiant HE 1327-2326 that have a much thinner convective zone (Beers & Christlieb 2005).
 - (b) *Pollution from an AGB companion:* Suda et al. (2004) propose a scenario where the CEMPs are the surviving secondaries of a binary system. The C and N enhancement comes from a phase called “He-flash driven deep mixing” (He-FDDM) described by Fujimoto et al. (1990), which brings C and N to the surface of the primary companion

and which is then accreted by the secondary during a mass-transfer episode. Note that this scenario implies also an accretion of the ISM gas to account for the Fe-group elements. A variation on this scenario is the binarity of an already slightly metal-enriched star ($Z = 10^{-7}$). Lau et al. (2007) show that the abundances of HE 0107-5240 can be modelled by the wind accretion from a $7 M_{\odot}$ primary. However, since so far no radial velocity variations have been detected for the two HMPs (Frebel et al. 2008), the possible separation of the binaries would be so large that no Roche-lobe overflow can be expected, so Lau et al. (2007) must invoke a tidally-enhanced mass transfer.

(c) *self-enrichment*: Picardi et al. (2004) explore the possibility of surface self-enrichment proposed by Christlieb et al. (2002). They firmly discard this scenario in the same paper.

2. **The second generation star**: the abundances observed at the surface of CEMP stars are those of the star-forming cloud. At such low Z , the protostellar cloud is supposed to be enriched by 1 to 10 SNe (Tumlinson 2006). Nomoto et al. (2003) show that HE 0107-5240 can be well fitted by the yields of a $25 M_{\odot}$ SN with a low explosion energy $E_{51} = E_{\text{expl}}/10^{51} [\text{erg}] = 0.3$. For the same star, Limongi et al. (2003) propose the explosion of two distinct SNe: a $15 M_{\odot}$ normal SN, which is required to account for the heavy elements of HE 0107-5240, and a $35 M_{\odot}$ weak SN, which is required for the light elements. By adding the possibility of some matter to fallback on the remnant, Iwamoto et al. (2005) revise the work of Nomoto et al. (2003) and fit both HE 0107-5240 and HE 1327-2326 with the single SN of a $25 M_{\odot}$ Pop III star with $E_{51} = 0.7$ and slightly different mass cut values. With a SN model where the explosion is induced by relativistic jets, Tominaga et al. (2007a) explains HE 0107-5240 and HE 1327-2326 as the results of a $40 M_{\odot}$ explosion in GRB without bright SN, with $E_{51} = 1.5$ and 0.5 respectively.

Note that when we take a more general look to the population of EMPs, it must be underlined that the observed abundances of many elements show a very small scatter for the stars with metallicities as low as $[\text{Fe}/\text{H}] \sim -4.0$ (Cayrel et al. 2004). This would indicate that the gas was already well mixed at these early times of our Galaxy.

2.4.2 Fast rotation and CNO yields at very low metallicity

The peculiar abundance pattern of CEMPs seems to be a mixture of H- and He-burning products. Since rotation allows these products to coexist in the outer layers of the stars, we found interesting to explore the chemical imprint of fast rotating massive ($60 M_{\odot}$) and intermediate-mass ($7 M_{\odot}$) stars at $Z = 10^{-5}$ and 10^{-8} . The results of this study have been published in the following article (see on page 209):

THE EARLY STAR GENERATIONS: THE DOMINANT EFFECT OF ROTATION ON THE CNO YIELDS

G. Meynet, S. Ekström & A. Maeder

2006 A&A **447**, 623

Mass loss processes

As shown by Meynet & Maeder (2002), since the radiative mass loss is very low at low metallicity ($\dot{M} \propto (Z/Z_{\odot})^{0.5-0.8}$), the star is not able to lose angular momentum and is prone to reach the critical velocity¹ during the MS evolution (see the description of the detailed mechanism in

¹The critical velocity v_{crit} is reached when the gravitational acceleration is counterbalanced by the centrifugal force: $v_{\text{crit}} = \sqrt{\frac{2}{3} \frac{GM}{R_{\text{p,c}}}}$, where $R_{\text{p,c}}$ is the polar radius at the critical limit.

Sect. 3.4.2 on page 43). Both the $60 M_{\odot}$ models at $Z = 10^{-5}$ and 10^{-8} (with initial velocity² $v_{\text{ini}} = 800 \text{ km s}^{-1}$) indeed reach the critical limit when their central H abundance is $X_c \approx 0.56$ and 0.40 respectively. At the critical limit, the surface layer is no longer bound to the star and is expelled by the mechanical pull of the centrifugal acceleration, bringing the surface back to sub-critical velocity. An equilibrium is reached between the meridional circulation that continues to accelerate the surface and the mass loss that removes the unbound layers and slows down the surface. The mass loss is thus enhanced during this stage, but not much because only the outermost layers are removed and those contain little mass (2.4 and $6.2 M_{\odot}$ for the $Z = 10^{-8}$ and 10^{-5} models respectively).

Rotational mixing is stronger at low metallicity than at the solar one because of steeper Ω -gradients (Maeder & Meynet 2001; Meynet & Maeder 2002). Our models are consistent with this and the $Z = 10^{-8}$ model shows a higher mixing signature at the surface than the $Z = 10^{-5}$ one ($\text{N/C} = 103$ and 40 respectively). However, during the MS, while rotational mixing brings nitrogen to the surface, it depletes carbon and oxygen. The sum of the CNO elements remains constant, and so does the total metallicity. The enhancement in the mass lost at this stage is thus only due to the mechanical process.

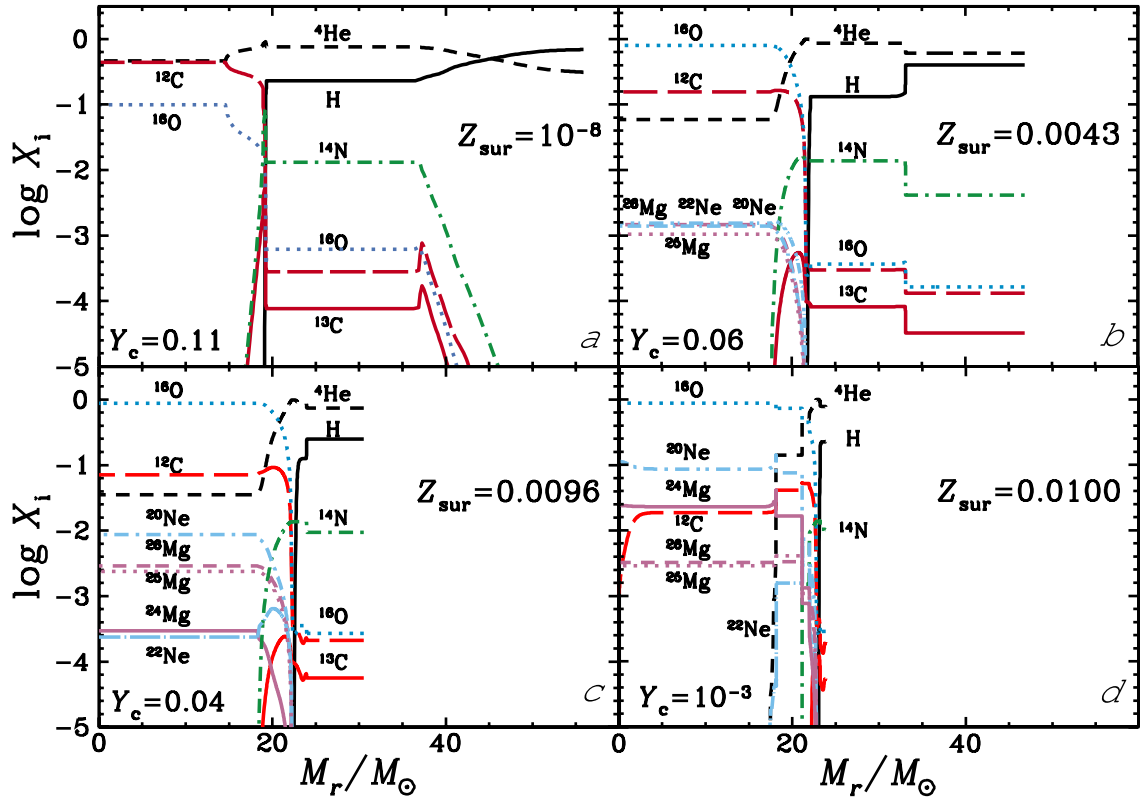


Figure 2.5: Abundance profiles (in mass fraction) inside a $60 M_{\odot}$ model with $v_{\text{ini}} = 800 \text{ km s}^{-1}$ at $Z = 10^{-8}$. The four panels show different stages at the end of the CHyB phase: **a**) the helium mass fraction at the centre is $Y_c = 0.11$ and the total mass of the model is $M = 54.8 M_{\odot}$ - **b**) $Y_c = 0.06$, $M = 48.3 M_{\odot}$ - **c**) $Y_c = 0.04$, $M = 31.5 M_{\odot}$ - **d**) $Y_c = 10^{-3}$, $M = 23.8 M_{\odot}$. The surface metallicity Z_{sur} is indicated in each panel.

After the MS, the effect of rotational mixing becomes very strong. A redwards evolution in

²For the choice of the initial equatorial velocity, we consider that stars arrive on the ZAMS with the same amount of angular momentum whatever the metallicity. We consider the amount that can reproduce the mean observed rotational velocity on the MS at solar metallicity ($\bar{v}_{\text{MS}} \approx 200 \text{ km s}^{-1}$, corresponding to an angular momentum content $\mathcal{L} \approx 2.2\text{--}2.5 \cdot 10^{53} \text{ g cm}^2 \text{ s}^{-1}$ for a $60 M_{\odot}$ model) and find that at $Z = 10^{-8}$, this amount corresponds to an initial velocity $v_{\text{ini}} \approx 800 \text{ km s}^{-1}$. The same v_{ini} was adopted for both metallicities.

the HRD is favoured (Maeder & Meynet 2001), which significantly enhances the mass loss. The Ω -gradient between the core and the expanding envelope is stretched, increasing the shear mixing. Some C and O are diffused from the He-burning core to the H-burning shell, and large amounts of primary nitrogen are produced. Figure 2.5 on previous page presents the abundances profile inside the $60 M_{\odot}$ model at $Z = 10^{-8}$ at the end of the CHeB phase: freshly synthesised CNO elements are brought to the surface, so the opacity of the envelope increases to the point that it becomes convective. The outer convective zone deepens and dredges up more heavy elements, so the surface metallicity increases in a spectacular way: the $Z = 10^{-8}$ model ends the CHeB with a surface metallicity $Z_{\text{sur}} = 10^{-2}$, i.e. similar to that of the Large Magellanic Cloud!

At the late stage of CHeB, as a result of mass loss and mixing, the model evolves along a blue loop in the HRD: the stellar contraction brings the surface velocity to the critical limit (in fact to the $\Omega\Gamma$ -limit, see Sect. 4.2.4 on page 66 for a description of this limit), so that the mass loss is multiplied by very large factors.

Eventually, the total mass lost during the evolution of the $60 M_{\odot}$ model at $Z = 10^{-8}$ amounts to a little more than $36 M_{\odot}$, i.e. about 60 % of its initial mass. The model at $Z = 10^{-5}$ loses a little less mass ($\sim 23 M_{\odot}$, i.e. 38%). This can be understood by the weaker mixing efficiency at higher Z : less C and O are diffused towards the H-burning shell, so less primary ^{14}N is produced. The surface is less enriched, so the outer convective zone does not dive so deep. Also the blue loop does not extend as much towards the blue side of the HRD and the model does not reach the $\Omega\Gamma$ -limit.

“Wind-only” chemical enrichment

The wind ejected by rotating models is enriched by strongly CNO-cycle processed material. It has thus a low $^{12}\text{C}/^{13}\text{C}$ ratio (4-5 versus 73 for the initial composition) and high N/C and N/O ratios (41 and 32 respectively for the $Z = 10^{-8}$ model, 29 and 36 for the $Z = 10^{-5}$ model). At the end of the CHeB phase, the models have large He cores ($M_{\alpha} = 23.83$ and $36.90 M_{\odot}$ for the $Z = 10^{-8}$ and 10^{-5} models respectively). For such large values, the star is expected to be entirely engulfed in a direct BH, without any SN explosion (Heger et al. 2003, see also Sect. 4.3.4 on page 81 for details). If this is the case, only the wind will contribute to the chemical enrichment of the medium.

Let us suppose that the CEMP stars observed in the Galactic halo are born from the interstellar medium enriched by the wind of fast rotating massive stars. Its Fe content would be the same as that of the massive stars, because the short lifetime of the latter does not allow for a strong enrichment of the medium, it would also show the chemical signature of the massive stars’ wind. Figure 2.6 on next page shows a comparison between the chemical composition of the wind of the models at $Z = 10^{-8}$ ($[\text{Fe}/\text{H}] = -6.6$) and 10^{-5} ($[\text{Fe}/\text{H}] = -3.6$), and that of some CEMP stars. The actual $[X/\text{Fe}]$ values obtained by our models vary when we change the initial metallicity (we see a shift of ~ 3.6 dex between the CNO abundances of the two models). They would also vary if we add a dilution factor: element X would be diluted, but the mass fraction of iron would remain the same, so the $[X/\text{Fe}]$ would be lower. More robust are the abundance ratios between the elements. The abundance pattern of the wind shows similarities with that of HE 1327-2326. There seems to be some CNO-processing in the abundances observed at the surface of some of the CEMP giants, like CS 22949-037 with $^{12}\text{C}/^{13}\text{C} \approx 4$, $\text{N}/\text{C} = 10$ and $\text{N}/\text{O} = 1.8$. However this processing seems to be weaker than that of the wind of our models.

“Wind+SN” chemical enrichment

While stars of this mass range are supposed to collapse directly into a BH, we will see in Sect. 4.3.4 on page 83 that if they are fast rotators, they may explode after all. We have also explored the case of an enrichment by both wind and SN contribution. To do so, we need to determine the

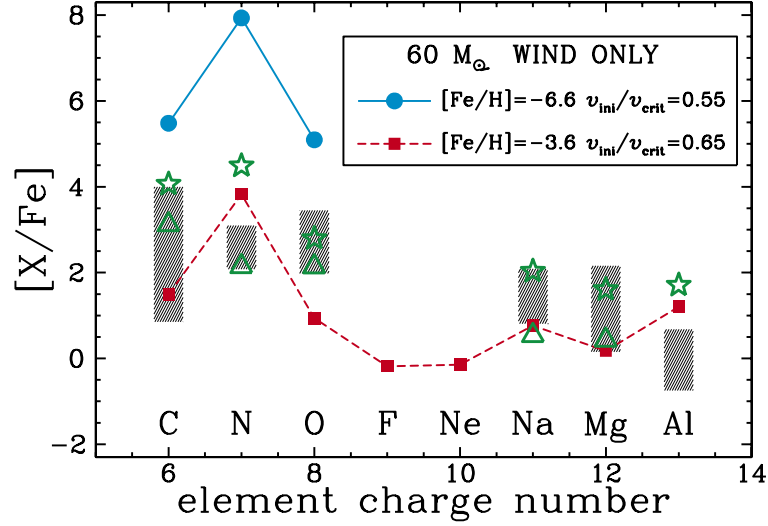


Figure 2.6: Chemical composition of the wind of rotating $60 M_{\odot}$ models at $Z = 10^{-8}$ ($[\text{Fe}/\text{H}] = -6.6$, blue filled circle) and $Z = 10^{-5}$ ($[\text{Fe}/\text{H}] = -3.6$, red filled squares). The hatched areas are the range of values measured at the surface of several CEMP giants: HE 0107-5240 (Christlieb et al. 2004), CS 22949-037 (Norris et al. 2001; Depagne et al. 2002), and CS 29498-043 (Aoki et al. 2004). Green empty triangles are for the dwarf G77-61 (Plez & Cohen 2005; Plez et al. 2005) and green empty stars are for HE 1327-2326 (Frebel et al. 2005, 2006, subgiant solution)

mass fraction of element X in the total ejectas:

$$X_{\text{ej}} = \frac{X_{\text{wind}}m_{\text{wind}} + X_{\text{SN}}m_{\text{SN}}}{m_{\text{wind}} + m_{\text{SN}}} = \frac{X_{\text{wind}}m_{\text{wind}} + X_{\text{SN}}m_{\text{SN}}}{m_{\text{ej}}}$$

with obvious meaning of these quantities. The way we obtain the wind and SN contributions is explained in details in Sect. 4.3.1 on page 73. The next step is to determine the mass of the interstellar medium (ISM) with which the ejectas will be mixed, so that we can determine the new abundance of element X in the medium:

$$X = \frac{X_{\text{ej}}m_{\text{ej}} + X_{\text{ini}}m_{\text{ISM}}}{m_{\text{ej}} + m_{\text{ISM}}} = \frac{X_{\text{ej}}\frac{m_{\text{ej}}}{m_{\text{ISM}}} + X_{\text{ini}}}{\frac{m_{\text{ej}}}{m_{\text{ISM}}} + 1}.$$

Since the ISM is very metal-poor, we consider $X_{\text{ini}} \approx 0$ for the heavy elements synthesised in large amounts by the star. We also suppose that $m_{\text{ej}} \ll m_{\text{ISM}}$, so we get $X = X_{\text{ej}}m_{\text{ej}}/m_{\text{ISM}}$, where $m_{\text{ej}}/m_{\text{ISM}}$ is simply the dilution factor. Starting from our $Z = 10^{-8}$ model, which corresponds to $[\text{Fe}/\text{H}] = -6.6$, we need to determine the ejected mass of Fe and the dilution factor. The mass of ejected Fe depends on the mass cut, the energetic of the SN and the geometry of the explosion. The variation can amount to orders of magnitude (Maeda & Nomoto 2003), so it is impossible to predict this quantity *ex nihilo*. We will thus impose the new mixture to have $[\text{Fe}/\text{H}] = -5.4$ and $[\text{O}/\text{Fe}] = +3.5$ (the metallicity of the dwarf solution for HE 1327-2326, and a lower O value than the upper limit of Frebel et al. 2005).

The mass fraction of ejected Fe can be determined from:

$$[\text{O}/\text{Fe}] = \log\left(\frac{\text{O}_{\text{ej}}}{\text{O}_{\odot}}\right) - \log\left(\frac{\text{Fe}_{\text{ej}}}{\text{Fe}_{\odot}}\right) = 3.5$$

and the dilution factor from:

$$\begin{aligned} [\text{Fe}/\text{H}] &= \log\left(\frac{\text{Fe}}{\text{H}}\right) - \log\left(\frac{\text{Fe}_{\odot}}{\text{H}_{\odot}}\right) \\ &= \log\left(\frac{\text{Fe}_{\text{ej}}}{\text{Fe}_{\odot}}\right) - \log\left(\frac{\text{H}_{\text{ej}}}{\text{H}_{\odot}}\right) + \log\left(\frac{m_{\text{ej}}}{m_{\text{ISM}}}\right) = -5.4. \end{aligned}$$

We consider that the mass fraction in H varies very little with the metallicity, so $\log(H_{\text{ej}}/H_{\odot}) \approx$

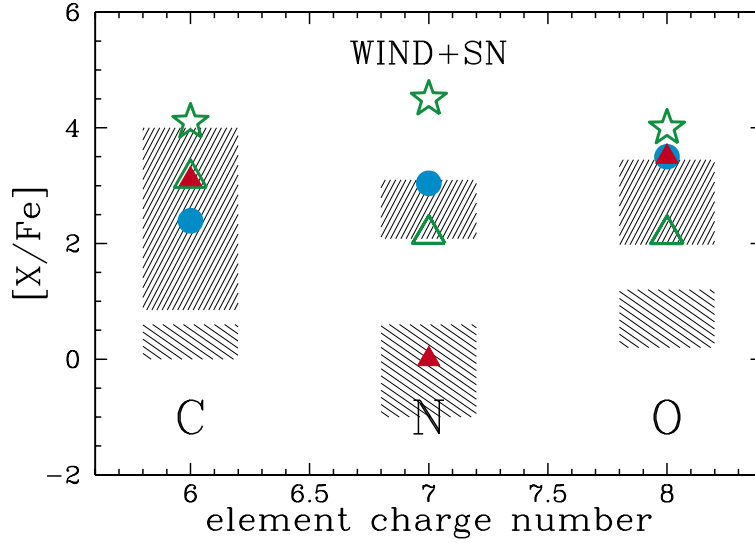


Figure 2.7: CNO composition of the ejecta (wind + SN) of rotating (blue filled circles) and non-rotating (red filled triangles) $60 M_{\odot}$ models at $Z = 10^{-8}$ ($[\text{Fe}/\text{H}] = -6.6$). The thick hatched area is the range of values measured at the surface of the same CEMP giants than in Fig. 2.6 on previous page. The thin hatched zone is the range of values measured at the surface of normal EMP giants by Cayrel et al. (2004) and Spite et al. (2005, unmixed sample only). Green empty triangles and stars stand for G77-61 and HE 1327-2326 respectively (see Fig. 2.6 for details).

0. With the imposed values $[\text{O}/\text{Fe}] = +3.5$ and $[\text{Fe}/\text{H}] = -5.4$, the mass of ejected Fe is of the order of $10^{-3} M_{\odot}$, and mixed with a mass of interstellar medium of about $2 \cdot 10^5 M_{\odot}$. The classical value for the mass of Fe ejected (in the form of ^{56}Ni) is $0.07\text{--}0.1 M_{\odot}$, quite larger than the one we inferred. However, if the remnant mass is large (which is expected for a large CO core at the end of evolution), a large part of the mass can fallback onto the remnant (Umeda & Nomoto 2003), so the mass ejected in Fe is much lower than usual. The mass of the ISM that is enriched by a SN explosion can be related to the explosion energy (Shigeyama & Tsujimoto 1998):

$$m_{\text{ISM}} = 5.1 \cdot 10^4 M_{\odot} \left(\frac{E_{\text{expl}}}{10^{51} \text{erg}} \right).$$

The explosion energy we infer from our value of $2 \cdot 10^5 M_{\odot}$ is $E_{\text{expl}} = 4 \cdot 10^{51}$ erg, i.e. a value that is not unrealistic for a SN explosion.

To determine the abundance of the other elements, we simply write:

$$\begin{aligned} [X/\text{Fe}] &= [X/\text{O}] + [\text{O}/\text{Fe}] \\ &= \log \left(\frac{X_{\text{ej}}}{X_{\odot}} \right) - \log \left(\frac{\text{O}_{\text{ej}}}{\text{O}_{\odot}} \right) + 3.5. \end{aligned}$$

Figure 2.7 presents the results on the CNO elements. The oxygen value is obtained by construction, so we cannot say anything about it. Both the non-rotating and the rotating models might account for a level of C-enrichment that is compatible with the range of values observed at the surface of CEMP stars. However, only the rotating model produces N-rich material that is compatible with the observed values. As expected, the addition of the SN contribution reduces the N/C and N/O values. The abundance pattern of the rotating $60 M_{\odot}$ at $Z = 10^{-8}$ is less in agreement with the abundance pattern of HE 1327-2326 than it was the case with the “wind only” contribution (Fig. 2.6 on previous page). With their almost solar values, the normal EMP stars seem to be a very different population than the CEMP stars, and are probably made from a reservoir that has undergone a very different chemical history.

Chemical enrichment by the envelope of E-AGB stars

We saw that Suda et al. (2004) propose that CEMP stars might have acquired their peculiar abundance pattern from a binary companion. We explore this scenario with the computation of a $7 M_{\odot}$ model at $Z = 10^{-5}$ and with $v_{\text{ini}} = 800 \text{ km s}^{-1}$.

This model loses almost no mass during H and He burning: although its initial velocity is high, it does not reach the critical limit (see Sect. 3.5.2 on page 52 for explanations), and the surface enrichment occurs late, because the star evolves towards the red part of the HRD only after the CHeB phase. At that stage, the enrichment of the envelope by primary CNO elements reaches about 10^3 times the initial one and the envelope expands. If such a star is in a close binary system, there is a possibility that the secondary accretes matter from the envelope and gains the peculiar chemical abundances.

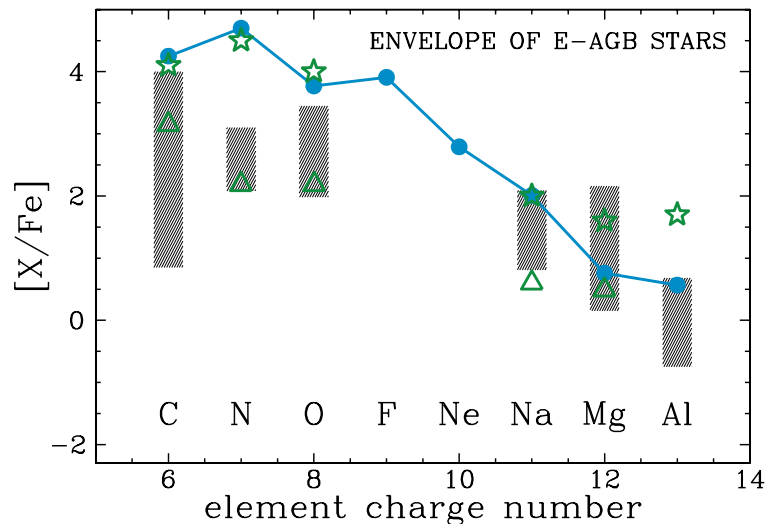


Figure 2.8: Chemical composition of the envelope of a $7 M_{\odot}$ rotating E-AGB model (blue solid line). The hatched zones correspond to the observed range in CEMP giants (thick hatched area of Fig. 2.7 on previous page). Green empty triangles and stars stand for G77-61 and HE 1327-2326 respectively (see Fig. 2.6 on page 23 for details).

Figure 2.8 presents the resulting envelope composition at the E-AGB stage for the $7 M_{\odot}$ model, compared with CEMP stars observed abundances. The CNO abundances are in good agreement with those observed at the surface of HE 1327-2326, with a lower C/N and C/O than the massive star wind-only composition. The heavier elements diverge from this star, but are in good agreement with the CEMP giants abundance range. Note that when large amounts of ^{14}N are synthesised, we expect also a significant production of ^{13}C and ^{22}Ne , which are both neutrons suppliers for the s -process. In the present $7 M_{\odot}$ model, excesses of Na, Mg, and Al are produced.

Discussion

In all the computations presented here, we considered only one set of initial parameters (M_{ini} , Z , v_{ini}). We may find a better fit with a different combination of parameters. The enrichment of the medium may also be the result of several contributions with a distribution of initial masses and velocities, if not of metallicities. An important point to underline is that there has been no fine-tuning for any parameters of the modelling to obtain the best possible agreement. The results are obtained with the basic physics of the models.

The three scenarios we presented here can be ordered by the degree of richness in CNO-processed material. The richest one is the massive-star “wind-only” contribution, then the E-AGB

envelope material, and finally the massive-star “wind+SN” contribution. A very sensitive indicator is the $^{12}\text{C}/^{13}\text{C}$ ratio: its value for the wind of massive stars is 4-5, it increases to ~ 20 in the envelope of the fast rotating E-AGB, and to $\sim 300\text{-}4000$ in the total ejecta of massive stars. Observations of the $^{12}\text{C}/^{13}\text{C}$ ratio in CEMP stars are starting to be available, so we might in the future distinguish the different contributions.

Note that the scenario of the wind of fast-rotating massive stars presented in this section has been proposed by Maeder & Meynet (2006) to explain the blue MS in ω Cen (Anderson 1997): a solution is a high He content in the bluer, higher metallicity branch (Piotto et al. 2005), which implies a high $\Delta Y/\Delta Z \approx 70$. In the wind ejecta of the $60 M_{\odot}$ model at $Z = 10^{-5}$ a $\Delta Y/\Delta Z$ ratio of 63 is achieved.

Stellar evolution with rotation at various Z

The results presented in this chapter have been published in the following article (see on page 186):

EVOLUTION TOWARDS THE CRITICAL LIMIT AND THE ORIGIN OF BE STARS
S. Ekström, G. Meynet, A. Maeder & F. Barblan
2008 A&A 478, 467

Electronic tables of the models (ZAMS and evolutionary sequence files) are available at
<http://obswww.unige.ch/Recherche/evol/Critical-limit-and-Be-stars>.

3.1 Motivations for a systematic study

The effects of differential rotation on solar-metallicity massive stars have been extensively studied (Langer 1998; Meynet & Maeder 2000; Heger et al. 2000a; Heger & Langer 2000; Meynet & Maeder 2003). The changes brought to those effects by lowering the metallicity Z have been also the subject of many papers (Maeder & Meynet 2001; Meynet & Maeder 2002, 2005; Meynet et al. 2006). However, these studies present each different physical inputs. For example, between Meynet & Maeder (2000, models at $Z = 0.020$) and Maeder & Meynet (2001, models at $Z = 0.004$), the opacities and the reaction rates are different, the prescription for rotational mixing has changed, and the mass loss prescription too. Moreover, the masses computed by different studies do not always match. All this prevents us to compare the results directly.

There are interesting astrophysical contexts that demand a good comprehension of the physics of rotation at various Z . We can mention among them: the variation with Z of the different massive star populations (supergiants, Wolf-Rayet stars), the computation of chemical yields, the nature of the SNe events (Type II, Type Ib, Type Ic) and of the compact remnant, the gamma-ray bursts progenitors. Here we focus on the Be phenomenon, and the early generations of stars. The Be phenomenon is related to nearly critical rotation during the main sequence (MS), and will be treated in Sect. 3.5. The low-metallicity environment seems to favour fast rotation. Low- Z stars are much more compact than the metal-rich ones, so low- Z stars having a similar initial rotational rate $\Omega/\Omega_{\text{crit}}$ will already have a higher equatorial velocity (v_{eq}) on the ZAMS. Since they do not lose much mass, the radiatively-driven winds being very low, they do not lose angular momentum, so they might be accelerated during their evolution and are more prone to reach the critical velocity, as shown by Maeder & Meynet (2001). The same study shows also that the diffusion timescale $\tau_{\text{diff}} \propto R^2/D$ (with D the diffusion coefficient) is shorter at low Z , which means that we expect a stronger surface enrichment at lower metallicity.

It is thus interesting to undertake a systematic study aimed at answering the following questions:

- How does rotation act in different mass ranges?
- How does the metallicity affect the physics of rotation?
- What are the main processes governing the evolution of the equatorial velocity?
- When is very fast rotation encountered in the course of the evolution of massive single stars in the MS phase?
- What are the expected chemical enrichments at the surface of these very fast rotating stars?

3.2 Physical inputs of the models

We have computed the evolution of five different masses, $M = 1, 3, 9, 20$, and $60 M_{\odot}$, at four different metallicities, $Z = 0.020, 0.002, 10^{-5}$, and 0 . For each (M, Z) combination, we have computed models with seven different initial rotational rates, $\Omega/\Omega_{\text{crit}} = 0.10, 0.30, 0.50, 0.70, 0.80, 0.90$, and 0.99 . The 3 to $60 M_{\odot}$ models were evolved until either the beginning of core helium burning or reaching the break-up limit. For the $1 M_{\odot}$ models, only the ZAMS models are presented.

The initial composition of the models for the metallicities $Z = 0.020, 0.002, 10^{-5}$, and 0 are respectively $X = 0.7050, 0.7545, 0.7599$, and 0.76 and $Y = 0.275, 0.2435, 0.24002$, and 0.24 . For the heavy elements, we used the same mixture as the one used in the OPAL opacity tables¹: the solar mixture of Grevesse & Noels (1993) for $Z = 0.020$ and 0.002 and the α -enhanced mixture of Weiss (1995) for $Z = 10^{-5}$. The nuclear reaction rates are from the NACRE database² (Angulo et al. 1999).

The convection is treated according to the Schwarzschild criterion. The size of the convective core is increased by a moderate overshooting of $0.1 H_P$.

The $3 M_{\odot}$ models were computed without line-driven mass loss, and the $9 M_{\odot}$ models were computed with de Jager et al. (1988) prescription. For the higher mass models we used the prescription proposed by Kudritzki & Puls (2000) and de Jager et al. (1988) in the temperature domain not covered by the first prescription. As described in Appendix A.1.7 on page 129, the line-driven mass loss is multiplied by the factor F_{Ω} due to rotation (Eq. (A.23), page 129).

The rotational instabilities taken into account are the meridional circulation, the secular and dynamical shear (see Appendix A.1 on page 125. Note that in these calculations, the D_h is determined with Eq. (A.16)). The transport of the angular momentum is treated as an advective process (with both terms in Eq. (A.20)), while that of chemical species is treated as a diffusion only (see Chaboyer & Zahn 1992) with Eq. (A.22), page 129.

3.3 Basic parameters on the zero-age main sequence

The models are considered as settled on the zero-age main sequence (ZAMS) when 3‰ of hydrogen (in mass fraction) is burned at the centre. The rotating models are started on the ZAMS with a flat Ω profile. Table 3.2 (page 37) presents an overview of the characteristics of our models on the ZAMS.

¹see <http://www-phys.llnl.gov/Research/OPAL/opal.html> for exact references

²http://pntpm.ulb.ac.be/Nacre/barre_database.htm

3.3.1 Radii, luminosities and effective temperatures

Radii

Metallicity effect

We will first look at the metallicity effect on the radius, without mingling it with rotation effects: Figure 3.1 (*left*) shows how the radius varies with the initial mass at various metallicities for non-rotating models³. While we have $R \propto M^{0.6}$ at $Z = 0.020$, the relation becomes shallower at lower Z : we have $R \propto M^{0.57}$ at $Z = 0.002$, $R \propto M^{0.5}$ at $Z = 10^{-5}$, and only $R \propto M^{0.3}$ at $Z = 0$.

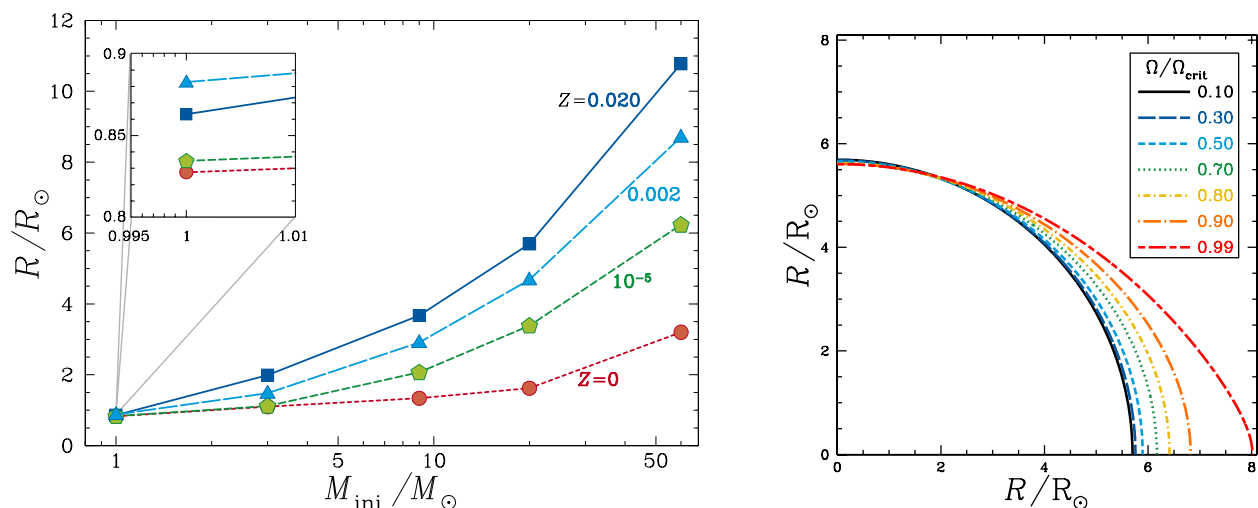


Figure 3.1: *Left*: variations of the radius on the ZAMS as a function of the mass at various metallicities for non-rotating models. *Right*: shape of a $20 M_\odot$ model at $Z = 0.020$ when $\Omega/\Omega_{\text{crit}}$ varies from 0.1 to 0.99.

There are two main causes to this:

1. a low (or absent) metal content means a weak (or absent) CNO-cycle, so at the beginning of its evolution, the star relies mostly on pp -chains to generate its energy. Since pp -chains are much less sensitive to temperature than CNO-cycle, the star must contract more during the pre-MS to get a higher central temperature (the $Z = 0$ models must even contract until the T_c allows some ^4He to be burnt into ^{12}C): it is more compact when it arrives on the ZAMS;
2. at low metallicity, the opacities are lower, so the envelope is transparent: the star remains compact.

Rotation effect

How does rotation affect the radius? The centrifugal force acts stronger at the equator than at the pole, so the star becomes oblate. Figure 3.1 (*right*) shows the shape of a rotating star. We see that even at $\Omega/\Omega_{\text{crit}} = 0.80$, R_{eq} is only 10% longer than R_{pol} . At $\Omega/\Omega_{\text{crit}} = 0.90$ the increase amounts almost to 20%, and at 0.99 to 40%. As established in the framework of the Roche model, the maximal increase is fixed at $R_{\text{eq}} = 1.5 R_{\text{pol}}$ when the critical limit is reached (see Appendix A.1.1). We also see that at a colatitude $\theta \simeq 20^\circ$, the radii are the same for all the rotation rates.

This oblateness is not only a theoretical result: it has been observed in some stars known to be fast rotators. The development of optical interferometry has allowed to determine the shape

³In this chapter, we will call “non-rotating models” the models with $\Omega/\Omega_{\text{crit}} = 0.1$. It is reasonable because at this low rotation rate, the models do not present any rotational effects. Moreover, it is interesting because in this way, the structures of the non-rotating models are determined with exactly the same treatment as rotating models.

of Altair (van Belle et al. 2001; Ohishi et al. 2004; Domiciano de Souza et al. 2005; Peterson et al. 2006), of Regulus (McAlister et al. 2005), and of Alderamin (van Belle et al. 2006), which all show a marked oblateness. Note that in a first study of Achernar, Domiciano de Souza et al. (2003) find the ratio of the major axis on the minor axis to be $a/b = 1.56 \pm 0.5$, higher than the maximal ratio allowed in the Roche model. Since most of the stellar evolution models are based on the Roche model, this finding has stirred a great interest in the community. 2D computations by Jackson et al. (2004) show that this high a/b ratio could be reproduced, but at the cost of some discrepancies in the T_{eff} and v_{eq} inferred. In a later study, however, Carciofi et al. (2008) show that the inclusion of a small equatorial disk in the analysis of the interferometric data restores the agreement with the Roche model.

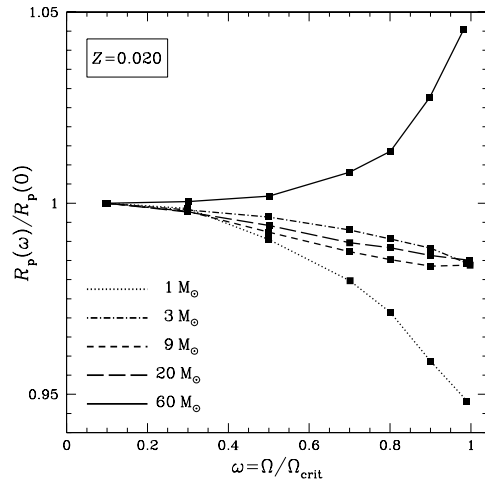


Figure 3.2: Variation of the polar radius on the ZAMS as a function of the rotation rate at various masses for $Z = 0.020$ models.

Lets go back to our models. While the radii at a colatitude $\theta \neq 0$ are affected by the centrifugal force, the polar radius is only weakly affected. It can be considered as the “true” radius of the star, the one that gives insights on the internal conditions. Figure 3.2 shows the variations of the polar radius (normalised to its non-rotating value) as a function of the rotation rate $\Omega/\Omega_{\text{crit}}$. We show here only the result at $Z = 0.020$, but it is representative of the behaviour at all metallicities. In the case of the 1 to 20 $M_{\odot}$, an increase of $\Omega/\Omega_{\text{crit}}$ leads to a decrease of the polar radius. It can be understood as the result of the sustaining effect of the centrifugal force: the star feels less gravity and behaves like a non-rotating star of a slightly lower mass. The central temperature and density are lower, which leads to a decrease in both luminosity and radius. The 3, 9, and 20 $M_{\odot}$ models deviate at most by 1.5% from the assumption $R_{\text{pol,crit}}/R_{\text{pol,0}} = 1$, which means that in this mass range, the assumption is absolutely valid.

Mass effect

The deviation of the 1 $M_{\odot}$ model is larger, amounting to a little more than 5%. This model is nearly completely radiative, contrarily to the heavier ones, which have a substantial convective core. In convective zones, the density gradient is shallower, so more mass is contained in a given volume. This makes gravity stronger and thus the star is better protected against deformation.

The 60 $M_{\odot}$ model shows also a variation of its polar radius with the rotation rate of about the same order of magnitude as the 1 $M_{\odot}$ model but in the opposite direction. It illustrates the high contribution of the radiative pressure P_{rad} to the total pressure in high mass stars. In rotating stars, the effective gravity is maximal at the pole, so the radiative flux is favoured in that direction and it inflates the polar radius (von Zeipel theorem).

Luminosities

As we mentioned above, rotation provides a sustaining force against gravity, so the star shows the surface characteristics of a lower-mass non-rotating one. The luminosity decreases with increasing $\Omega/\Omega_{\text{crit}}$, as shown by Fig. 3.3 (left) for the $60 M_{\odot}$ models. This effects is the strongest at very low metallicity: while at $Z = 0.020$ the decrease amounts to less than 5%, it reaches more than 11% at $Z = 10^{-5}$ and $Z = 0$. When the star is more compact, as is the case at low Z , the regions having a temperature sufficient to undergo nuclear burning are more extended inside the star, so they reach layers that are more affected by rotation effects.

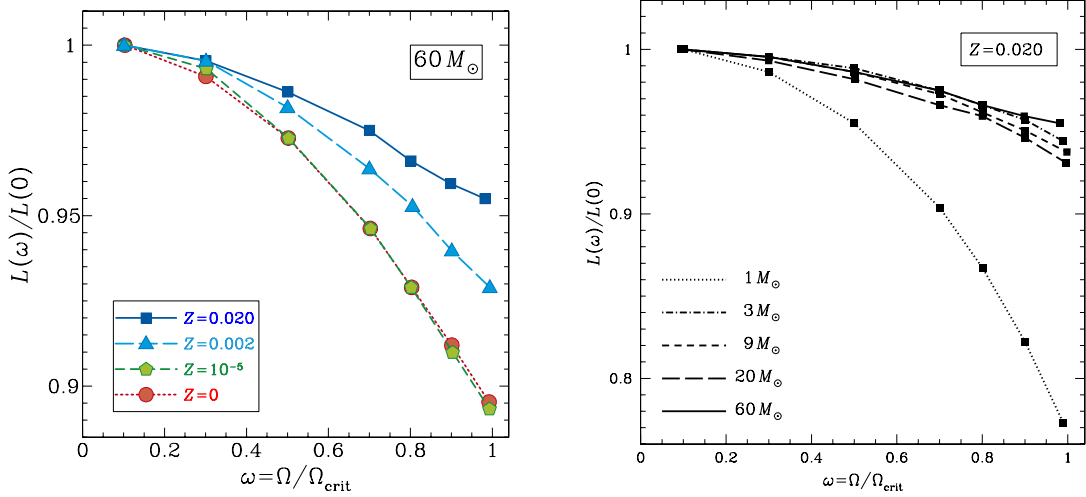


Figure 3.3: *Left*: variation of the luminosity on the ZAMS as a function of the rotation rate in $60 M_{\odot}$ models at various metallicities. The luminosity is normalised to the non-rotating value. *Right*: variation of the luminosity on the ZAMS as a function of the rotation rate in $Z = 0.020$ models at various masses. The luminosity is normalised to the non-rotating value.

All the masses present the same trend, but the amplitude is much stronger in the low-mass domain. Figure 3.3 (right) shows the $Z = 0.020$ models of different masses. We see that the $1 M_{\odot}$ model is strongly affected by the increase of $\Omega/\Omega_{\text{crit}}$. The decrease in luminosity amounts to almost 22%, while it is at most 7% for the more massive models. As above, we relate this behaviour to the extension of the burning zone: the pp -chains reactions are less dependent in temperature than the CNO-cycle, so it is less peaked inside the star and may extend toward regions that are more affected by rotation.

Effective temperatures

Metallicity effect

Here again, we will first look at the metallicity effects without any rotation. Figure 3.4 (left) shows the relation between T_{eff} and M . We have $T_{\text{eff}} \propto M^{0.5-0.6}$ at all metallicities, but there is a systematical shift towards higher T_{eff} at low Z . Low Z implies low opacities and high temperature, so the radiative gradient $\nabla_{\text{rad}} \propto \frac{\kappa P}{T^4}$ is lower. The temperature gradient in the star is thus lower, which leads to a higher T_{eff} . It is amazing to see the difference that still exists between $Z = 10^{-5}$ and $Z = 0$ for the highest masses.

Rotation effect

The oblateness of a rotating star leads to the cooling of the equatorial regions while the polar ones become hotter. This is illustrated in the right panel of Fig. 3.4, which shows this effect at the surface of a $20 M_{\odot}$ models at $Z = 0.020$. At 99% of the critical limit, the ratio $(T_{\text{eff}}^{\text{pol}} - T_{\text{eff}}^{\text{eq}})/T_{\text{eff}}^{\text{eq}}$

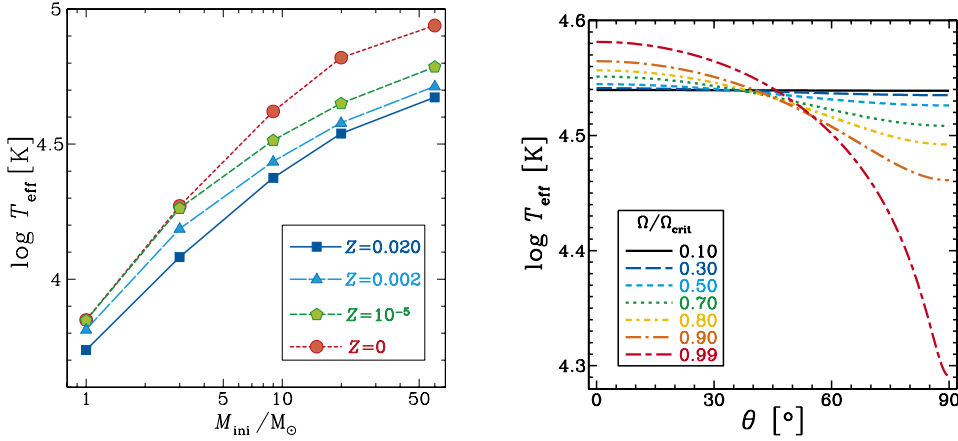


Figure 3.4: *Left*: variation of T_{eff} as a function of the initial mass at various metallicities (models without rotation). *Right*: variation of T_{eff} as a function of the colatitude θ at the surface of a $20 M_{\odot}$ model at $Z = 0.020$ for various rotation rates.

amounts to 0.3 dex, *i.e.* a factor of 2. We can compare this result with the recent work of Espinosa Lara & Rieutord (2007) who study the structure and dynamics of rapidly rotating stellar models with a 2D code. They compute a model with $\Omega = 0.82$ for which the ratio $(T_{\text{eff}}^{\text{pol}} - T_{\text{eff}}^{\text{eq}})/T_{\text{eff}}^{\text{eq}}$ is 14%. Our closest model ($\Omega = 0.80$) shows a slightly higher ratio of 16%, though we should expect a lower value due to the lower Ω . Let us note that the 2D simulation has been carried out within a spherical box, which according to the authors, may soften the latitudinal variations. It would be interesting to check how much of the discrepancy remains were the 2D computation done without such a container.

The cooling of the equatorial regions have an effect on the opacity, and Maeder et al. (2008) show that it may trigger convective instabilities close to the critical limit. This is important, because convective motions are a source of acceleration for the gas, and it could play a important role in mass ejection (see Sect. 3.5.4 on page 55).

Note that in what will follow, when we talk about T_{eff} we refer to a mean T_{eff} obtained by averaging over the equipotential surface (see Meynet & Maeder 1997, their Appendix A.3): $T_{\text{eff}} = (L/\sigma S_P)^{1/4}$, where S_P is a true surface accounting for rotational deformation.

3.3.2 Inertia, angular momentum, rotational energies

Inertia

The moment of inertia is obtained by summing the contributions of each shell i of a model: $I = \sum_i (2/3) r_i^2 dm_i$, where r_i is the mean radius of the shell, *i.e.* the radius where the second-order Legendre polynomial⁴ cancels: $P_2(\cos \theta) = 0$. Figure 3.5 shows how the moment of inertia varies with the initial mass for various metallicities on the ZAMS. Note that I presents almost no dependence on $\Omega/\Omega_{\text{crit}}$, so only non-rotating models are presented. This could seem surprising at first. In our way of calculating I , the effects of rotation are included through r_i , which accounts for the rotational distortion, and this approximation is accurate up to the second-order Legendre polynomial. The physical interpretation of this is that rotation deforms only the outermost layers of the model, which contain very little mass. An illustration of this is given in Fig. 3.21 on page 49. It explains why the moment of inertia does not change much with or without rotation.

⁴The Legendre polynomials are a set of orthogonal functions that are used to solve Laplace's equation $\nabla^2 \Phi = 0$ in spherical coordinates when there is an azimuthal symmetry $\partial \varphi = 0$. The second-order Legendre polynomial is $P_2(x) = \frac{1}{2} (3x^2 - 1)$

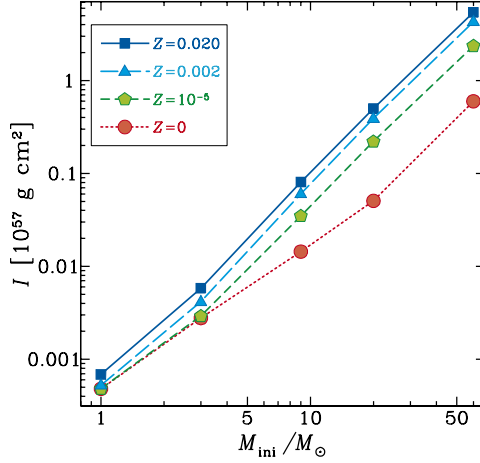


Figure 3.5: Variation of the moment of inertia on the ZAMS as a function of the initial mass for various metallicities (non-rotating models).

The moment of inertia increases with the initial mass as $I \propto M^{1.7}$ at $Z = 0$, $I \propto M^{2.0}$ at $Z = 10^{-5}$, and $I \propto M^{2.2}$ for the highest metallicities. This was expected, given the relation between R and M we found in the previous section: $I \propto R^2 M \propto M^{2\xi+1}$, with $\xi = 0.3, 0.5$, and 0.6 for $Z = 0$, $Z = 10^{-5}$, and $Z = 0.020$ respectively.

For a given mass, the moment of inertia decreases with the metallicity. This is again expected, since the low metallicity stars are more compact. As in Fig. 3.4 on previous page, we see that the difference between $Z = 10^{-5}$ and $Z = 0$ is large.

Angular momentum

The total angular momentum is calculated as $\mathcal{L} = \sum_i I_i \Omega_i$. Since I does not present any marked dependence on $\Omega/\Omega_{\text{crit}}$ and we supposed a solid-body rotation on the ZAMS, we expect a quasi linear relation between $\mathcal{L}$ and $\Omega/\Omega_{\text{crit}}$. Figure 3.6 shows that it is indeed the case.

The way the total angular momentum content of a star varies with metallicity depends on the mass considered. In the case of the $1 M_{\odot}$ models, the $\mathcal{L}$ values at all metallicities are neatly stacked, except at $Z = 0.020$, where the values are systematically higher. At $\Omega/\Omega_{\text{crit}} = 0.80$, the difference between the values at $Z = 0.020$ and at lower metallicities amounts to 36%. At higher mass, the trend changes. In the case of the $3 M_{\odot}$ models, the highest values of $\mathcal{L}$ are found at $Z = 10^{-5}$ and $Z = 0$, and the lowest at $Z = 0.020$. In the more massive models ($9-60 M_{\odot}$), the $\mathcal{L}$ values for a given $\Omega/\Omega_{\text{crit}}$ at $Z = 0$ are much lower than at higher Z . For the $9 M_{\odot}$ models with $\Omega/\Omega_{\text{crit}} = 0.80$, there is a difference of 23% between $Z = 0.020$ and $Z = 0$. It amounts to 41% for the corresponding $60 M_{\odot}$ models, and to 53% for the $20 M_{\odot}$ models. In this mass range, the highest $\mathcal{L}$ values are found in the $Z = 0.002$ models.

What is important to keep in mind here is that when Z varies, the resulting $\Omega/\Omega_{\text{crit}}$ for a given $\mathcal{L}$ changes. At $Z = 0.002$, a $20 M_{\odot}$ model having $\mathcal{L} \simeq 3 \cdot 10^{52} \text{ g cm}^2 \text{ s}^{-1}$ would have $\Omega/\Omega_{\text{crit}} = 0.50$ while at $Z = 0$ it would have $\Omega/\Omega_{\text{crit}} = 0.80$.

The upper-right point of the curves in Fig. 3.6 can be considered as the maximal angular momentum content allowed on the ZAMS (for rigid-body rotation) at a given mass and metallicity, since it corresponds to nearly critical rotation.

Rotational energies

The rotational kinetic energy $E_{\text{rot}} = (1/2) \sum_i I_i \Omega_i^2$ represents only a very small fraction of the binding energy $|E_{\text{pot}}| = G \sum_i (M_i/r_i) dm_i$, even when the star is close to critical rotation. Figure 3.7 shows the ratio $E_{\text{rot}}/|E_{\text{pot}}|$ for the $1, 3$, and $20 M_{\odot}$ models. In the mass interval $9-60 M_{\odot}$, the curves are similar to that of the $20 M_{\odot}$ models. Table 3.2 on page 37 provides the values of E_{rot} and $|E_{\text{pot}}|$ for all the models.

3.3. Basic parameters on the zero-age main sequence

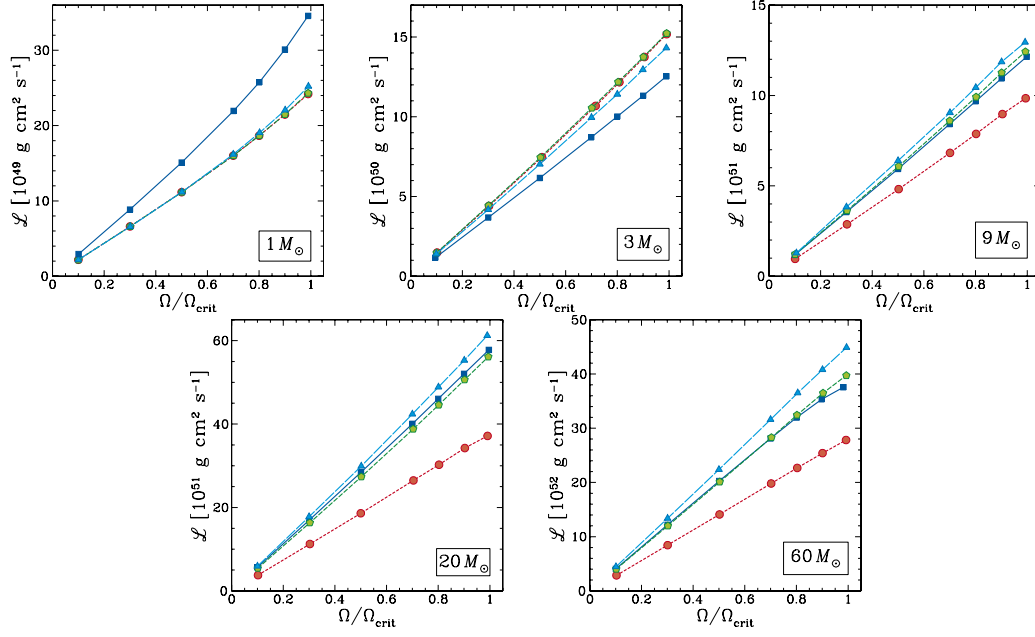


Figure 3.6: Variation of the total angular momentum as a function of $\Omega/\Omega_{\text{crit}}$, for all the models. The colour code is the same as Fig. 3.5

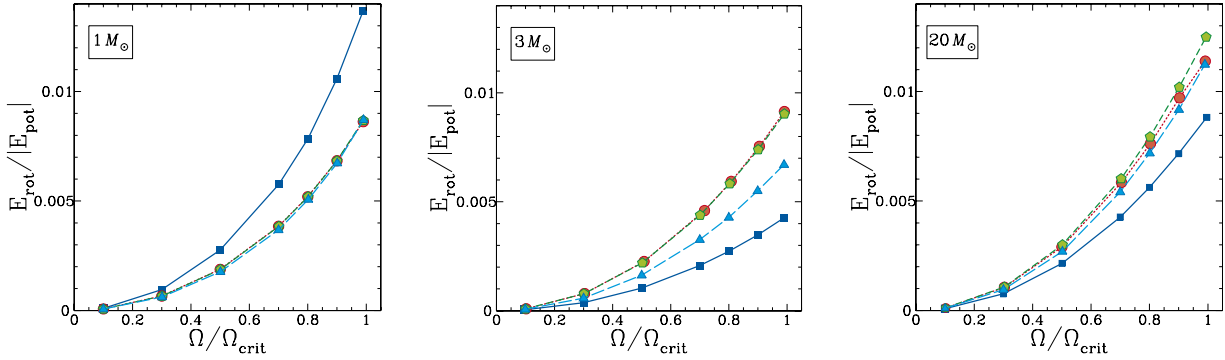


Figure 3.7: Variation of the rotational energy (normalised by the binding energy) as a function of $\Omega/\Omega_{\text{crit}}$. The colour code is the same as Fig. 3.5

We see that the rotational energy is at most a few percents of the binding energy. This confirms that rotation plays only little role on the hydrostatic structure of a star (see the results on the moment of inertia on page 32).

3.3.3 Velocities on the ZAMS: v_{crit} , v

Figure 3.8 presents the critical velocity v_{crit} on the ZAMS as a function of the initial mass for various metallicities. Referring to Eq. (A.6), we have $v_{\text{crit}} \propto \sqrt{M/R}$. A larger mass allows a larger v_{crit} , and so does a shorter radius. This explains why low Z stars, being more compact, have higher v_{crit} . The difference between $Z = 10^{-5}$ and $Z = 0$ starts to be noticeable from $9 M_{\odot}$ and above, i.e. in stars that rely strongly on CNO-cycle to counterbalance their gravity. In the highest mass range, v_{crit} is almost twice at $Z = 0$ as at $Z = 0.020$. Note that we show here the critical velocity *on the ZAMS*. As we will see in Sect. 3.4.2 on page 46, during the star evolution, v_{crit} changes because of changes in mass (through mass loss) and radius (through the evolution of L and T_{eff}), so the values given here are valid only at the very beginning of the MS.

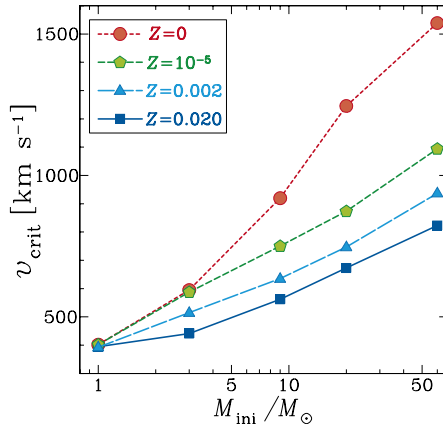


Figure 3.8: Variation of the critical velocity on the ZAMS as a function of the initial mass for various metallicities.

Table 3.1 gives the rotational characteristics at the critical limit for the masses and metallicities considered. In the context of the Roche model (presented in Appendix A.1.1), we could determine Ω_{crit} from whatever $\Omega < \Omega_{\text{crit}}$ model with Eq. (A.8) or (A.9), see page 126. However, the closer from critical limit the more accurate the determination. The Ω_{crit} values presented in Table 3.1 have been obtained from the fastest rotating model of each M and Z . This reduces the error to less than 0.001%, even when using the simplified Eq. (A.9). The determination of $P_{\text{crit}} = 2\pi/\Omega_{\text{crit}}$ follows directly. $R_{\text{pol,crit}}$ has been obtained from Eq. (A.5), and then $v_{\text{crit}} = 1.5 R_{\text{pol,crit}} \Omega_{\text{crit}}$.

Figure 3.9 presents the equatorial velocities v_{eq} on the ZAMS⁵ for all the models (the values are also given in Table 3.2). Except in the case of the $1 M_{\odot}$ models, which show almost no metallicity effect, for a given $\Omega/\Omega_{\text{crit}}$ there is an increase of v_{eq} when Z decreases. This is expected, since we saw that $v_{\text{crit}} \propto \sqrt{M/R}$: v_{crit} increases with decreasing Z (because of the shorter radius), so a higher v_{eq} is needed to get a given v/v_{crit} . The more massive the model, the stronger the difference between v_{eq} at $Z = 0$ and $Z = 0.020$: merely 2% for the $1 M_{\odot}$ models, 35% for the $3 M_{\odot}$ models, almost 60% for the $9 M_{\odot}$ models, 80% for the $20 M_{\odot}$ models and 95% for the $60 M_{\odot}$ models.

Once again, what is important to keep in mind here is that when Z varies, the resulting v_{eq} for a given $\Omega/\Omega_{\text{crit}}$ changes: at $Z = 0.020$, a $20 M_{\odot}$ model with $\Omega/\Omega_{\text{crit}} = 0.70$ will have $v_{\text{eq}} = 344 \text{ km s}^{-1}$, while the same $\Omega/\Omega_{\text{crit}}$ at $Z = 0$ will lead to $v_{\text{eq}} = 644 \text{ km s}^{-1}$. We expect thus naturally faster equatorial velocities at low Z , if the initial distribution of $\Omega/\Omega_{\text{crit}}$ does not depend on the metallicity.

⁵We recall that on the ZAMS, the rotation is treated with the solid-body assumption (see Sect. 3.4.2 on page 43 for a more complete picture, and the discussion in Sect. 3.4.4 on page 50).

3.3. Basic parameters on the zero-age main sequence

Table 3.1: Ω , P_{rot} , R_{pol} , and v_{eq} at the critical limit on the ZAMS (solid body rotation).

M [$M_{\odot}$]	Z [mass frac.]	Ω_{crit} [s^{-1}]	P_{crit} [hours]	$R_{\text{pol,crit}}$ [$R_{\odot}$]	v_{crit} [km s^{-1}]
1	0	4.9e-04	3.6	0.8	400
1	10^{-5}	4.9e-04	3.6	0.8	400
1	0.002	4.6e-04	3.8	0.8	390
1	0.020	4.6e-04	3.8	0.8	395
3	0	5.3e-04	3.3	1.1	595
3	10^{-5}	5.1e-04	3.4	1.1	585
3	0.002	3.4e-04	5.1	1.4	515
3	0.020	2.2e-04	8.1	2.0	440
9	0	6.5e-04	2.7	1.4	920
9	10^{-5}	3.5e-04	4.9	2.0	750
9	0.002	2.2e-04	8.1	2.8	635
9	0.020	1.5e-04	11.7	3.6	560
20	0	7.3e-04	2.4	1.6	1245
20	10^{-5}	2.5e-04	7.0	3.3	870
20	0.002	1.6e-04	11.1	4.6	745
20	0.020	1.2e-04	15.2	5.6	670
60	0	4.6e-04	3.8	3.2	1540
60	10^{-5}	1.7e-04	10.6	6.4	1095
60	0.002	1.0e-04	16.9	8.7	935
60	0.020	7.0e-05	24.9	11.3	820

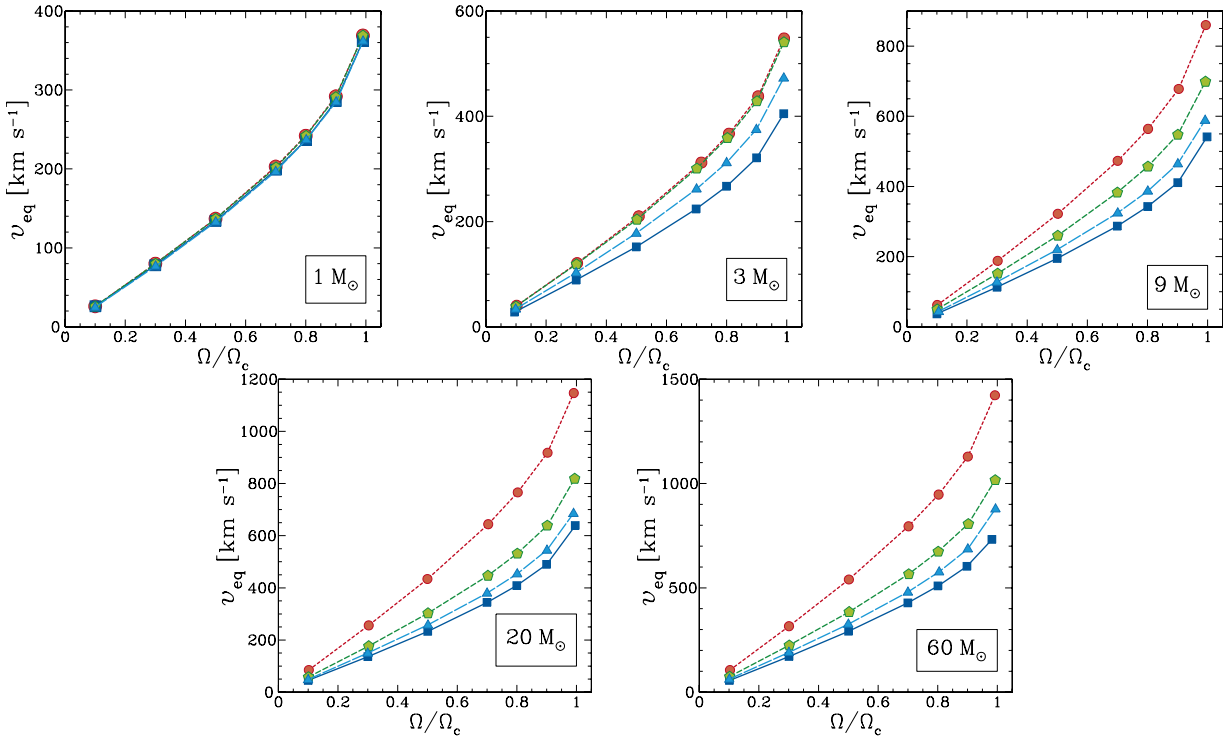


Figure 3.9: Variation of the equatorial velocity on the ZAMS as a function of $\Omega/\Omega_{\text{crit}}$, for all the models. The colour code is the same as Fig. 3.5

Table 3.2: Characteristics of the models on the ZAMS.

Framed cells: models attaining v_{crit} when the central H fraction is: $X_c < 0.01$, $0.01 < X_c < 0.1$, $0.1 < X_c < 0.2$, $0.2 < X_c < 0.3$, $0.3 < X_c < 0.4$, $0.4 < X_c < 0.5$, $0.5 < X_c < 0.6$, $0.6 < X_c < 0.7$

$\Omega/\Omega_{\text{crit}}$	M_{ini} Z	1					3					9					20					60				
		0	10^{-5}	0.002	0.020	0.020	0	10^{-5}	0.002	0.020	0.020	0	10^{-5}	0.002	0.020	0.020	0	10^{-5}	0.002	0.020	0.020	0	10^{-5}	0.002	0.020	0.020
0.10	$R [R_\odot]$	0.829	0.830	0.883	0.867	0.867	1.090	1.120	1.472	1.990	1.990	1.345	2.079	2.890	3.672	1.636	3.407	4.682	5.713	3.191	6.293	8.718	10.787	10.787	10.787	10.787
	$v_{\text{ini}} [\text{km s}^{-1}]$	26	26	25	26	26	40	39	34	28	28	62	50	44	37	85	58	49	45	106	75	63	56	56	56	56
	$\mathcal{L} [\text{erg cm}^2 \text{ s}^{-1}]$	2.19e49	2.17e49	2.18e49	2.94e49	2.94e49	1.48e50	1.47e50	1.40e50	1.16e50	1.16e50	9.59e50	1.21e51	1.28e51	1.18e51	3.77e51	5.44e51	5.93e51	5.65e51	2.87e52	4.03e52	4.45e52	4.07e52	4.07e52	4.07e52	4.07e52
	$E_{\text{rot}} [\text{erg}]$	4.98e44	4.88e44	4.50e44	6.29e44	6.29e44	3.94e45	3.67e45	2.34e45	1.16e45	1.16e45	3.18e46	2.09e46	1.36e46	8.61e45	1.40e47	6.70e46	4.49e46	3.19e46	6.87e47	3.44e47	2.30e47	1.53e47	1.53e47	1.53e47	1.53e47
0.30	$E_{\text{pot}} [\text{erg}]$	6.85e48	6.84e48	6.64e48	5.92e48	5.92e48	4.40e49	4.29e49	3.61e49	3.13e49	3.13e49	3.00e50	1.92e50	1.46e50	1.27e50	1.17e51	5.62e50	4.22e50	3.75e50	5.33e51	2.68e51	1.98e51	1.77e51	1.77e51	1.77e51	1.77e51
	$ E_{\text{pot}} [\text{erg}]$	0.832	0.834	0.887	0.869	0.869	1.095	1.127	1.480	2.004	2.004	1.356	2.089	2.917	3.697	1.651	3.423	4.714	5.720	3.205	6.359	8.778	10.912	10.912	10.912	10.912
	$v_{\text{ini}} [\text{km s}^{-1}]$	80	80	77	78	78	121	119	103	89	89	188	152	128	113	256	177	150	136	316	225	191	171	171	171	171
	$\mathcal{L} [\text{erg cm}^2 \text{ s}^{-1}]$	6.61e49	6.61e49	6.59e49	8.85e49	8.85e49	4.40e50	4.40e50	4.18e50	3.68e50	3.68e50	2.87e51	3.63e51	3.83e51	3.55e51	1.12e52	1.63e52	1.78e52	1.69e52	8.43e52	1.20e53	1.34e53	1.22e53	1.22e53	1.22e53	1.22e53
0.50	$E_{\text{rot}} [\text{erg}]$	4.53e45	4.52e45	4.11e45	5.66e45	5.66e45	3.47e46	3.34e46	2.08e46	1.16e46	1.16e46	2.85e47	1.88e47	1.21e47	7.79e46	1.24e48	6.04e47	4.04e47	2.86e47	5.93e48	3.04e48	2.07e48	1.37e48	1.37e48	1.37e48	1.37e48
	$E_{\text{pot}} [\text{erg}]$	6.85e48	6.83e48	6.63e48	5.91e48	5.91e48	4.40e49	4.29e49	3.61e49	3.13e49	3.13e49	3.00e50	1.92e50	1.46e50	1.27e50	1.17e51	5.62e50	4.22e50	3.75e50	5.33e51	2.68e51	1.98e51	1.77e51	1.77e51	1.77e51	1.77e51
	$ E_{\text{pot}} [\text{erg}]$	0.840	0.846	0.895	0.879	0.879	1.115	1.142	1.499	2.043	2.043	1.383	2.118	2.947	3.753	1.674	3.466	4.769	5.820	3.265	6.440	8.778	10.912	10.912	10.912	10.912
	$\mathcal{L} [\text{erg cm}^2 \text{ s}^{-1}]$	1.11e50	1.12e50	1.12e50	1.51e50	1.51e50	7.46e50	7.45e50	7.02e50	6.16e50	6.16e50	4.82e51	6.08e51	6.39e51	5.94e51	1.86e52	2.73e52	2.99e52	2.84e52	1.41e53	2.01e53	2.24e53	2.03e53	2.03e53	2.03e53	2.03e53
0.70	$E_{\text{rot}} [\text{erg}]$	1.28e46	1.28e46	1.17e46	1.63e46	1.63e46	9.91e46	9.42e46	5.89e46	3.25e46	3.25e46	7.95e47	5.27e47	3.37e47	2.17e46	3.41e48	1.69e48	1.14e48	8.07e47	1.65e49	8.51e48	5.78e48	3.78e48	3.78e48	3.78e48	3.78e48
	$E_{\text{pot}} [\text{erg}]$	6.83e48	6.82e48	6.62e48	5.89e48	5.89e48	4.39e49	4.28e49	3.61e49	3.13e49	3.13e49	2.99e50	1.92e50	1.46e50	1.28e50	1.18e51	5.62e50	4.22e50	3.75e50	5.33e51	2.68e51	1.98e51	1.77e51	1.77e51	1.77e51	1.77e51
	$ E_{\text{pot}} [\text{erg}]$	0.834	0.834	0.887	0.869	0.869	1.095	1.127	1.480	2.004	2.004	1.356	2.089	2.917	3.697	1.651	3.423	4.714	5.720	3.205	6.359	8.778	10.912	10.912	10.912	10.912
	$\mathcal{L} [\text{erg cm}^2 \text{ s}^{-1}]$	1.11e50	1.12e50	1.12e50	1.51e50	1.51e50	7.46e50	7.45e50	7.02e50	6.16e50	6.16e50	4.82e51	6.08e51	6.39e51	5.94e51	1.86e52	2.73e52	2.99e52	2.84e52	1.41e53	2.01e53	2.24e53	2.03e53	2.03e53	2.03e53	2.03e53
0.80	$E_{\text{rot}} [\text{erg}]$	1.28e46	1.28e46	1.17e46	1.63e46	1.63e46	9.91e46	9.42e46	5.89e46	3.25e46	3.25e46	7.95e47	5.27e47	3.37e47	2.17e46	3.41e48	1.69e48	1.14e48	8.07e47	1.65e49	8.51e48	5.78e48	3.78e48	3.78e48	3.78e48	3.78e48
	$E_{\text{pot}} [\text{erg}]$	6.83e48	6.82e48	6.62e48	5.89e48	5.89e48	4.39e49	4.28e49	3.61e49	3.13e49	3.13e49	2.99e50	1.92e50	1.46e50	1.28e50	1.18e51	5.62e50	4.22e50	3.75e50	5.33e51	2.68e51	1.98e51	1.77e51	1.77e51	1.77e51	1.77e51
	$ E_{\text{pot}} [\text{erg}]$	0.834	0.834	0.887	0.869	0.869	1.095	1.127	1.480	2.004	2.004	1.356	2.089	2.917	3.697	1.651	3.423	4.714	5.720	3.205	6.359	8.778	10.912	10.912	10.912	10.912
	$\mathcal{L} [\text{erg cm}^2 \text{ s}^{-1}]$	1.11e50	1.12e50	1.12e50	1.51e50	1.51e50	7.46e50	7.45e50	7.02e50	6.16e50	6.16e50	4.82e51	6.08e51	6.39e51	5.94e51	1.86e52	2.73e52	2.99e52	2.84e52	1.41e53	2.01e53	2.24e53	2.03e53	2.03e53	2.03e53	2.03e53
0.90	$E_{\text{rot}} [\text{erg}]$	1.28e46	1.28e46	1.17e46	1.63e46	1.63e46	9.91e46	9.42e46	5.89e46	3.25e46	3.25e46	7.95e47	5.27e47	3.37e47	2.17e46	3.41e48	1.69e48	1.14e48	8.07e47	1.65e49	8.51e48	5.78e48	3.78e48	3.78e48	3.78e48	3.78e48
	$E_{\text{pot}} [\text{erg}]$	6.83e48	6.82e48	6.62e48	5.89e48	5.89e48	4.39e49	4.28e49	3.61e49	3.13e49	3.13e49	2.99e50	1.92e50	1.46e50	1.28e50	1.18e51	5.62e50	4.22e50	3.75e50	5.33e51	2.68e51	1.98e51	1.77e51	1.77e51	1.77e51	1.77e51
	$ E_{\text{pot}} [\text{erg}]$	0.834	0.834	0.887	0.869	0.869	1.095	1.127	1.480	2.004	2.004	1.356	2.089	2.917	3.697	1.651	3.423	4.714	5.720	3.205	6.359	8.778	10.912	10.912	10.912	10.912
	$\mathcal{L} [\text{erg cm}^2 \text{ s}^{-1}]$	1.11e50	1.12e50	1.12e50	1.51e50	1.51e50	7.46e50	7.45e50	7.02e50	6.16e50	6.16e50	4.82e51	6.08e51	6.39e51	5.94e51	1.86e52	2.73e52	2.99e52	2.84e52	1.41e53	2.01e53	2.24e53	2.03e53	2.03e53	2.03e53	2.03e53
0.99	$E_{\text{rot}} [\text{erg}]$	1.28e46	1.28e46	1.17e46	1.63e46	1.63e46	9.91e46	9.42e46	5.89e46	3.25e46	3.25e46	7.95e47	5.27e47	3.37e47	2.17e46	3.41e48	1.69e48	1.14e48	8.07e47	1.65e49	8.51e48	5.78e48	3.78e48	3.78e48	3.78e48	3.78e48
	$E_{\text{pot}} [\text{erg}]$	6.83e48	6.82e48	6.62e48	5.89e48	5.89e48	4.39e49	4.28e49	3.61e49	3.13e49	3.13e49	2.99e50	1.92e50	1.46e50	1.28e50	1.18e51	5.62e50	4.22e50	3.75e50	5.33e51	2.68e51	1.98e51	1.77e51	1.77e51	1.77e51	1.77e51
	$ E_{\text{pot}} [\text{erg}]$	0.834	0.834	0.887	0.869	0.869	1.095	1.127	1.480	2.004	2.004	1.356	2.089	2.917	3.697	1.651	3.423	4.714	5.720	3.205	6.359	8.778	10.912	10.912	10.912	10.912
	$\mathcal{L} [\text{erg cm}^2 \text{ s}^{-1}]$	1.11e50	1.12e50	1.12e50	1.51e50	1.51e50	7.46e50	7.45e50	7.02e50	6.16e50	6.16e50	4.82e51	6.08e51	6.39e51	5.94e51	1.86e52	2.73e52	2.99e52	2.84e52	1.41e53	2.01e53	2.24e53	2.03e53	2.03e53	2.03e53	2.03e53

3.4 Evolution on the main sequence

To follow the main sequence (MS) evolution of the $1 M_{\odot}$ models, we need a careful treatment of the magnetic braking at the surface, as well as tabulated equations of state that are not included in the code used for this study, so we will leave them on the ZAMS. We will now proceed with the 3 to $60 M_{\odot}$ models to study their evolution through the MS. This study was aimed mainly at determining the conditions needed to reach the critical limit, so the models were stopped when they reached it, if they did, or just before the beginning of the core helium burning (CHeB) if they did not.

3.4.1 Hertzsprung-Russell diagram, central conditions and lifetimes

HR diagram

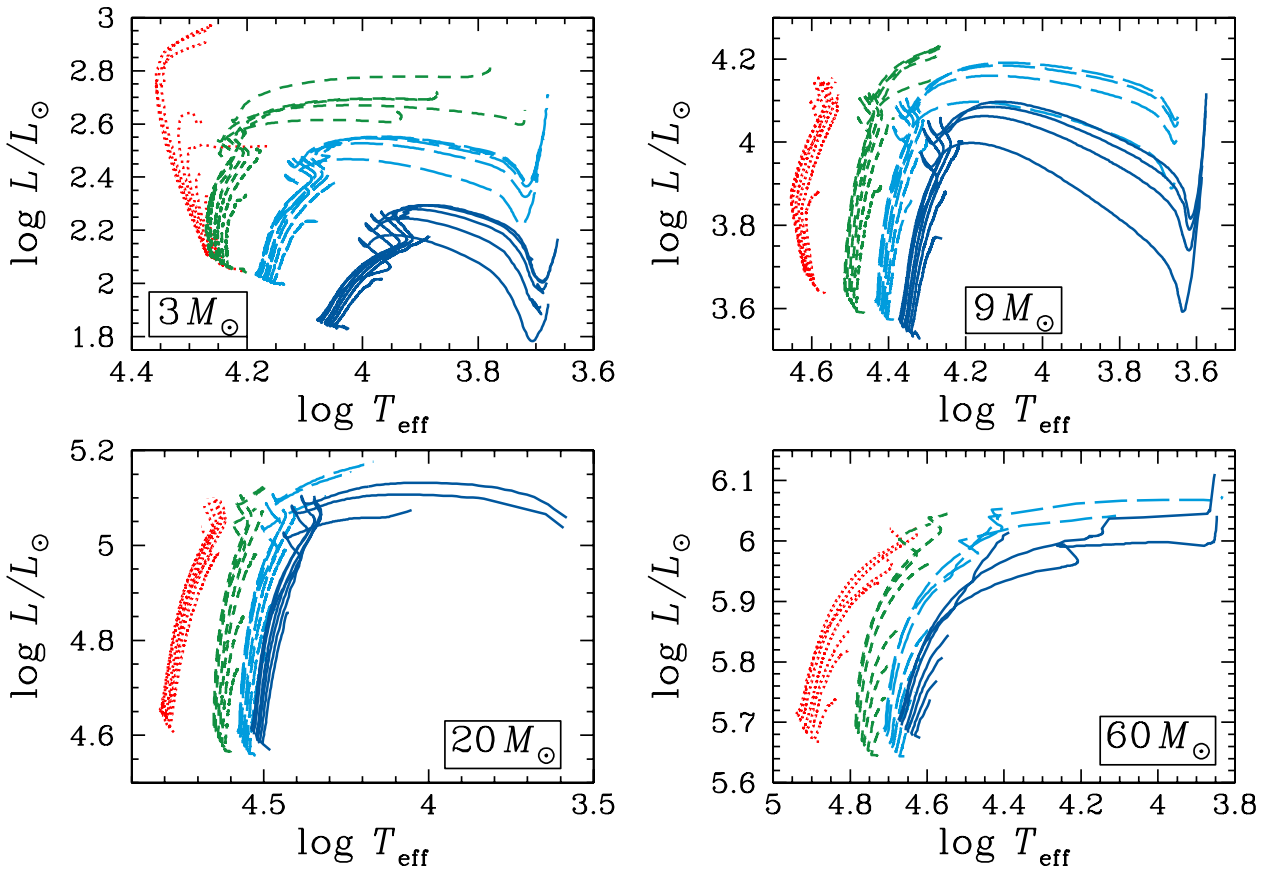


Figure 3.10: Evolutionary tracks in the theoretical Hertzsprung-Russell diagram for all the models computed beyond the ZAMS: Blue solid line: $Z = 0.020$; cyan long-dashed line: $Z = 0.002$; green short-dashed line: $Z = 10^{-5}$; red dotted line: $Z = 0$. The effective temperature corresponds to a surface-averaged effective temperature. The computations are stopped when the star reaches the critical limit or before the beginning of the core He-burning phase.

Metallicity effect

Figure 3.10 presents the Hertzsprung-Russell diagram (HRD) for all our models, grouped by mass. The metallicity effect is a translation toward higher T_{eff} (and higher L for the 3 and $9 M_{\odot}$ models) when Z decreases. This is due to the greater compactness and the lower opacities (see Sect. 3.3.1). For the models with $Z \neq 0$, the lower the mass, the greater the relative shift $T_{\text{eff}}(M, Z)/T_{\text{eff}}(M, Z = 0.020)$. For the $60 M_{\odot}$ models at $Z = 0.002$ and 10^{-5} , the shift amounts to

less than 10% and 30% respectively, while for the $3 M_{\odot}$ models of the same Z the shift amounts to 27% and 52% respectively. At $Z = 0$, the situation is inverted up to $20 M_{\odot}$: the higher the mass, the larger is the shift (54% for the $3 M_{\odot}$, 91% for the $20 M_{\odot}$).

At $Z = 0$, the tracks of the 3 and $9 M_{\odot}$ start bent towards the blue in the HRD. As mentioned earlier, at the beginning of their evolution the metal-free models rely only on pp -chains to sustain their gravity. In the high mass domain, the central temperature is quickly high enough to produce some ^{12}C by 3α reaction even before reaching the ZAMS. At lower mass, this is not the case. The MS evolution starts with only pp -chains, so the star goes on contracting: the CNO-cycle ignition occurs later on, when the star has produce a ^{12}C mass fraction of about 10^{-12} . In the HRD, the CNO-cycle ignition is marked by the tracks changing direction towards the red side of the diagram.

Figure 3.10 shows another important difference in the mass domain $M \geq 9 M_{\odot}$ between the models at $Z \leq 10^{-5}$ and those at higher Z : looking only at the models which do not reach the critical limit, we see that the low- Z ones remain in the blue part of the HRD after H exhaustion, while the high- Z ones cross the HRD and start CHeB in the red part. This particularity of low- Z models and its many consequences will be studied in more details in Chapter 4 in the $Z = 0$ case, but we will just mention here that it is a result of the compactness of the low- Z models: H burning occurs at much higher T_c , so there is almost no structure readjustment at H exhaustion. In the high- Z models, the core is left without any sustaining energy source and has to contract abruptly to start CHeB. In response, the envelope expands rapidly, driving the HRD crossing. In the $Z = 0$ models, the core is already hot enough to burn some He, and at $Z = 10^{-5}$ it is almost hot enough. The transition between H- and He-burning is smooth, and the stars remain in the blue side.

Rotation effect

The rotation effect begins on the ZAMS by a translation toward lower L and T_{eff} when $\Omega/\Omega_{\text{crit}}$ increases, as seen in Sect. 3.3.1 on page 29. Then when the evolution proceeds, the luminosity increases, the MS ends at higher L (and still lower T_{eff}), and the crossing of the HRD occurs at higher L when $\Omega/\Omega_{\text{crit}}$ is higher. The evolution of the luminosity is related to that of the convective core, and we will study this aspect below; let's just mention that a larger core leads to a higher luminosity.

The time at which the CNO-cycle ignition occurs in the 9 and $3 M_{\odot}$ models at $Z = 0$ differs with the mass and the rotation rate. Table 3.3 shows the results obtained by the models. Rotation delays systematically the time of ignition: the central temperature and density are lowered by rotation (see below), so it takes longer to produce the necessary amount of ^{12}C .

Table 3.3: CNO-cycle ignition time for the 3 and $9 M_{\odot}$ models. The age is given in Myr; X_c is the central abundance of hydrogen in mass fraction. The $3 M_{\odot}$ models with $\Omega/\Omega_{\text{crit}} \geq 0.70$ reach the critical limit before the CNO-cycle ignition.

time	$\Omega/\Omega_{\text{crit}}$						
	0.10	0.30	0.50	0.70	0.80	0.90	0.99
9 $M_{\odot}$							
age	9.888	11.514	11.995	12.427	12.403	12.634	12.774
X_c	0.483	0.448	0.438	0.433	0.435	0.434	0.429
3 $M_{\odot}$							
age	210.762	221.044	231.274	—	—	—	—
X_c	0.037	0.043	0.045	—	—	—	—

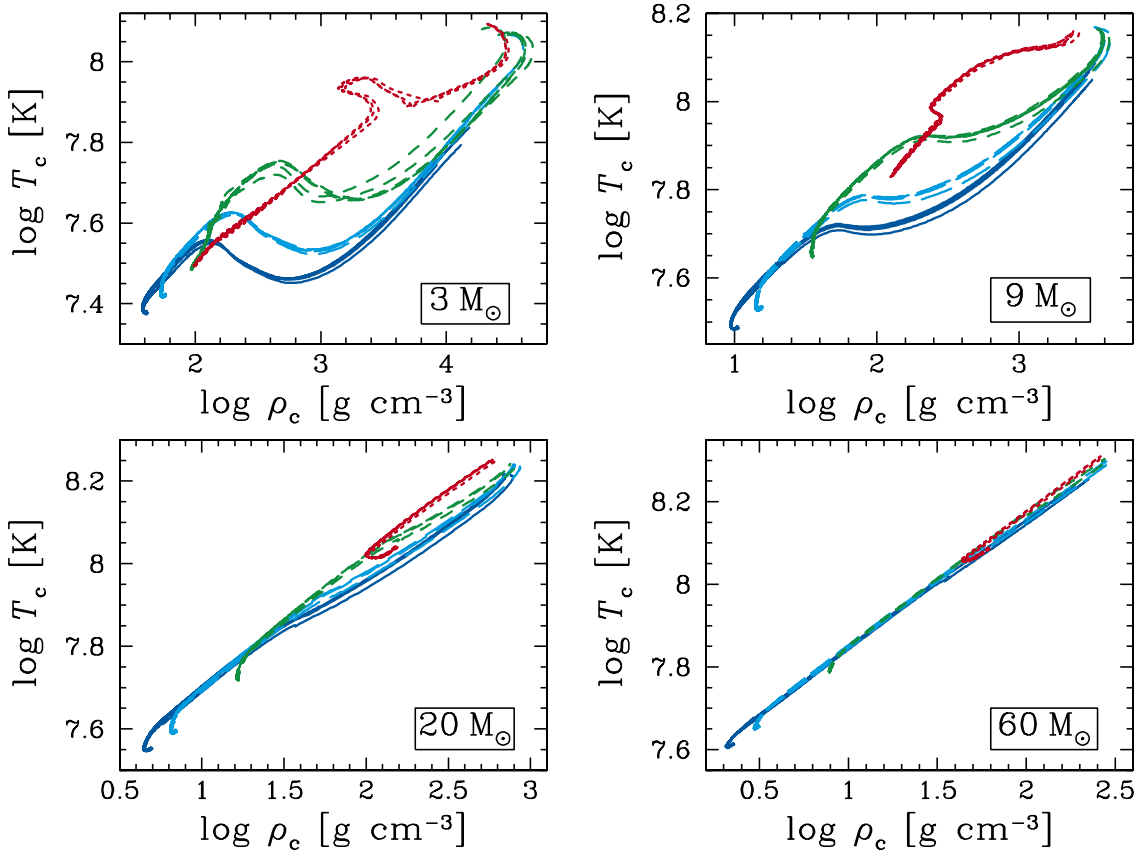
$\log T_c - \log \rho_c$ diagram

Figure 3.11: Evolutionary tracks in the $\log T_c - \log \rho_c$ diagram for all the models computed beyond the ZAMS (same colour code as Fig. 3.10). The H ignition is located in the lower left point of the tracks, the He ignition in the upper-right point. The CNO-cycle ignition appears as a hook in the tracks of the 3 and 9 $M_\odot$ at $Z = 0$.

Metallicity effect

In the $\log T_c - \log \rho_c$ diagram (Fig. 3.11), the metallicity effects is a translation toward higher T_c and ρ_c . The 9 to 60 $M_\odot$ models at $Z = 0$ start their evolution with a T_c which is higher than the $Z = 0.020$ ones by a factor of 2.5 to 3 times. In the course of the MS evolution, they reach the temperature at which the higher Z models start the CHeB. The density at the centre of a 9 $M_\odot$ model at $Z = 0$ is 14 times higher than at $Z = 0.020$, while it is 30 times higher in the 20 and 60 $M_\odot$ models.

The CNO-cycle ignition which we have mentioned above leaves also an imprint in the $\log T_c - \log \rho_c$ diagram of the 3 and 9 $M_\odot$ models: the tracks start with the usual 1/3 slope, but suddenly they form a hook towards lower densities thanks to this new release of energy. After that, the evolution proceeds with tracks similar as the ones at higher metallicity, except they occupy a higher (T_c, ρ_c) position.

Rotation effect

The rotation effects are almost unnoticeable in the $\log T_c - \log \rho_c$ diagram.

Core mass

As we mentioned above, the evolution of the luminosity is closely related to that of the convective core. The convective core in turn is very sensitive to the innermost conditions pictured in the $\log T_c - \log \rho_c$ diagram. So only by looking at the evolution of the convective core, we could predict many features seen both in the HRD and the $\log T_c - \log \rho_c$ diagram.

Metallicity effect

Let's first look at the metallicity effects on the core mass evolution, shown in Fig. 3.12 (non-rotating models). Generally, the lower the metallicity, the larger the convective core (and thus the higher the luminosity). This is due to the higher central temperature at low Z . The $Z = 0$ curve deviates from this trend: though the central temperature is much higher, the energy release is much weaker. The $9M_\odot$ model at $Z = 0$ shows a big bump when the CNO cycle is ignited. In the $3M_\odot$ model at $Z = 0$, the core even becomes radiative before the CNO-cycle ignition makes it convective again, for a short period of time.

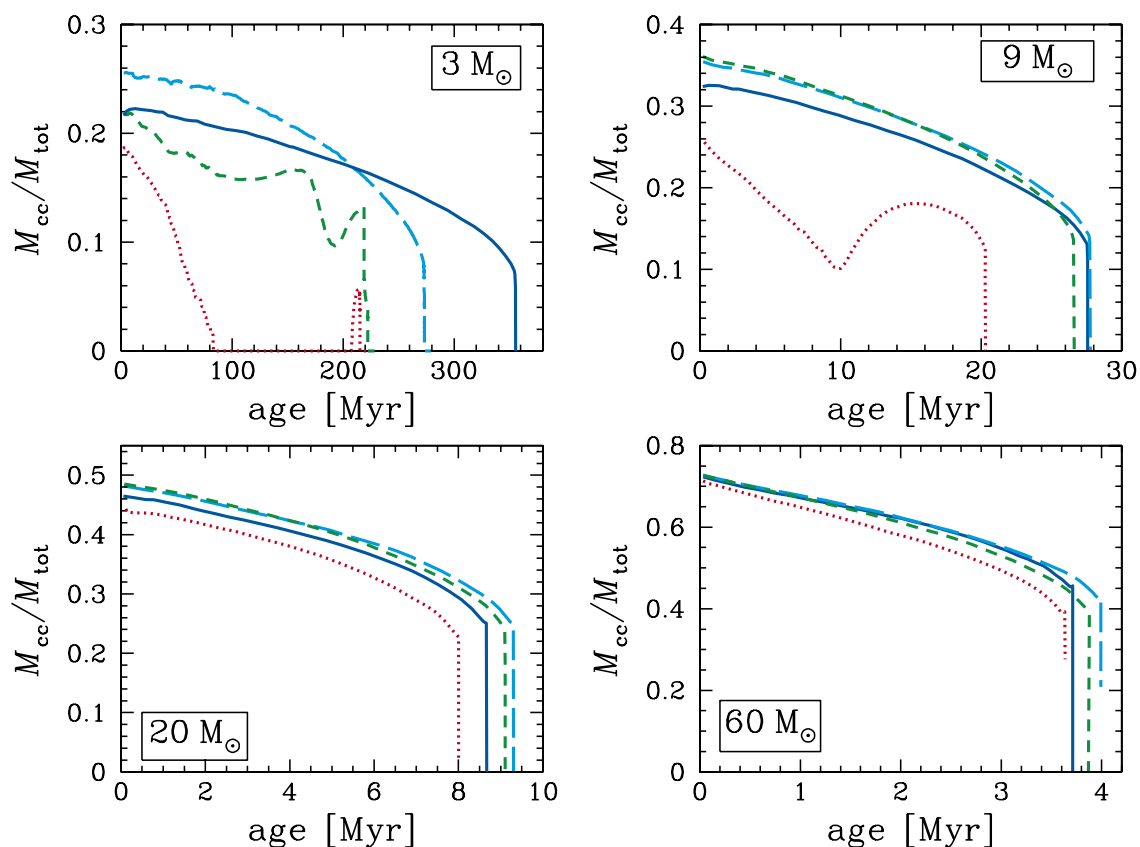


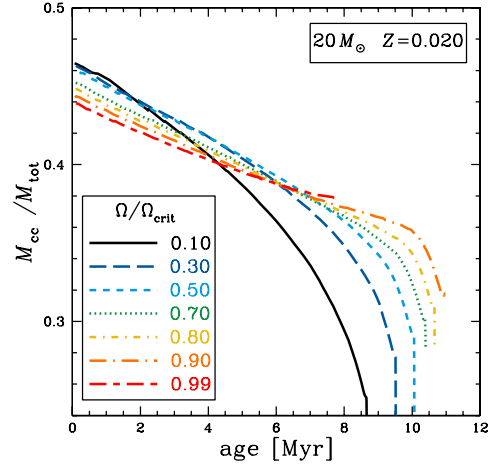
Figure 3.12: Evolution of the size of the convective core during the MS for the non-rotating models (same colour code as Fig. 3.10).

Rotation effect

At the beginning of the evolution, rotation sustains the star against its own gravity, so it behaves like a lower-mass non-rotating one, and the mass of the convective core is lower, leading to a lower luminosity in the HRD.

When the evolution proceeds, the rotationally-induced mixing refuels the core in fresh hydrogen. This delays the reduction of the convective core during MS, as illustrated in Fig. 3.13, so the luminosity is enhanced. Another consequence we will address later is an increase of the MS lifetime (see Table 3.4).

Figure 3.13: Convective core evolution inside a $20 M_{\odot}$ model at $Z = 0.020$ when $\Omega/\Omega_{\text{crit}}$ varies from 0.1 to 0.99.



Lifetimes

Table 3.4 presents the lifetime on the MS for all the models. For the models that reach the critical velocity, we have estimated the lifetime from the evolution of the central abundance of hydrogen (the estimated values are marked by an asterisk). In that case, we give in parenthesis the time at which the critical limit was reached and the model stopped. For the $60 M_{\odot}$ models, the critical velocity is reached early in the MS evolution, so we did not estimate the value, we only give in parenthesis the time at which the model was stopped.

Table 3.4: Lifetimes (in Myr) during the MS phase. An asterisk means that the value was estimated (see text).

Z	$\Omega/\Omega_{\text{crit}}$						
	0.1	0.3	0.5	0.7	0.8	0.9	0.99
$3 M_{\odot}$							
0	215.6	227.8	237.8	249* (223.7)	255* (206.2)	264* (189.0)	275* (170.6)
10^{-5}	224.9	243.6		261.5	265.6	271* (270.6)	271* (213.4)
0.002	274.2	301.3	312.3	320.9	325.4	331* (316.4)	331* (237.9)
0.020	357.0	399.6	424.4	439.2	444.4	454* (446.7)	484* (302.9)
$9 M_{\odot}$							
0	20.31	22.11	23.24	24.31	24.74	25* (24.77)	25* (16.12)
10^{-5}	26.62	28.70	29.98	30.85	31.29	31.8* (31.30)	32.4* (25.03)
0.002	27.77	30.11	31.32	32.26	32.87	33.5* (31.84)	34.0* (26.00)
0.020	27.59	30.33	31.62	32.62	33.0* (32.91)	34.0* (28.89)	34.0* (23.05)
$20 M_{\odot}$							
0	8.003	8.554	8.868	9.199	9.324	9.434	9.44* (8.616)
10^{-5}	9.099	9.853	10.213	10.54* (10.533)	10.68* (9.626)	10.73* (7.843)	10.73* (5.651)
0.002	9.303	10.117	10.527	10.85* (10.840)	11.00* (10.033)	11.06* (8.444)	11.15* (6.712)
0.020	8.663	9.521	9.928	10.39* (10.389)	10.66* (10.662)	10.96* (10.937)	10.96* (7.892)
$60 M_{\odot}$							
0	3.633	3.802	3.924	(3.386)	(2.538)	(2.259)	(1.173)
10^{-5}	3.874	4.112	(3.696)	(2.746)	(2.200)	(1.525)	(0.783)
0.002	3.994	4.251	(4.432)	(2.818)	(2.336)	(1.717)	(1.087)
0.020	3.714	3.978	(3.960)	(2.426)	(2.031)	(1.595)	(1.179)

Metallicity effect

For the $3 M_{\odot}$ models, lowering the metallicity induces an increase of the MS lifetime. For the higher masses, this is still true between $Z = 0.002$ and 0, but the standard metallicity models live shorter than the $Z = 0.002$ ones, and the 20 and $60 M_{\odot}$ models at $Z = 0.020$ live even shorter than the $Z = 10^{-5}$ ones. This is directly linked with the behaviour of the cores with metallicity shown in Fig. 3.12.

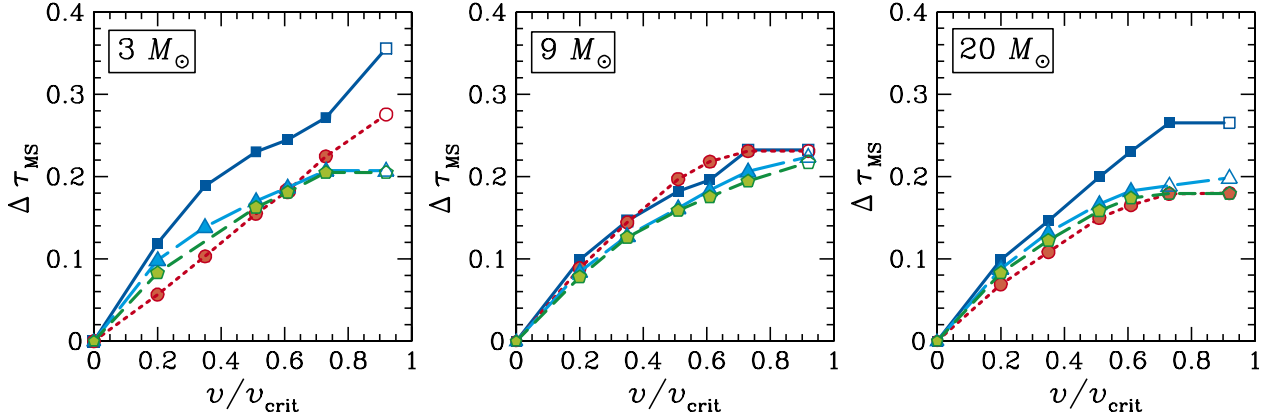


Figure 3.14: MS lifetime enhancement $\Delta \tau_{\text{MS}}(v/v_{\text{crit}}) = (\tau_{\text{MS}}(v/v_{\text{crit}}) - \tau_{\text{MS}}(0))/\tau_{\text{MS}}(0)$ as a function of v/v_{crit} for the 3, 9, and $20 M_{\odot}$ models at various Z (same colour code as Fig. 3.10). Open symbols mean that the value was estimated for a model stopped with $X_c > 0.20$.

Rotation effect

As mentioned above, rotation induces an increase in the MS lifetime because the internal mixing refuels the core in fresh hydrogen. Figure 3.14 shows the relative enhancement $\Delta \tau_{\text{MS}}(v/v_{\text{crit}}) = \frac{\tau_{\text{MS}}(v/v_{\text{crit}}) - \tau_{\text{MS}}(0)}{\tau_{\text{MS}}(0)}$. We see that generally, the curve flattens at high v/v_{crit} . Note that most of the models with $v/v_{\text{crit}} > 0.80$ reached the critical limit early and were stopped with $X_c > 0.20$, so part of this trend may be due to our estimation of the value of τ_{MS} . However the trend is already sketched at lower v/v_{crit} , where the values are the real MS lifetimes. A look at Fig. 3.13 shows that the growth of the convective core is large between $\Omega/\Omega_{\text{crit}} = 0.10$ and 0.30 , but then the difference becomes smaller and smaller when $\Omega/\Omega_{\text{crit}}$ increases, which explains the trend seen in Fig. 3.14. In the case of the models at $Z \neq 0$, for a given v/v_{crit} , $\Delta \tau_{\text{MS}}$ is larger at higher metallicity. This could be surprising for two reasons: first, we have seen in Fig. 3.12 on page 41 that the lower the metallicity, the larger the convective core; second, as will be shown in Sect. 3.4.3 on page 49, the shear mixing is more efficient with the steeper rotation profiles inside lower Z stars, so the refuelling of the core in H should be larger. However, the lifetime depends of course on the amount of H available, but also on the efficiency to burn it. Low Z stars are more compact, their T_c is higher, so H is burnt quicker and the difference due to rotation becomes smaller.

3.4.2 Evolution of the equatorial velocity

As mentioned in Sect. 3.3.3, our models start on the ZAMS with a solid-body rotation. At the very beginning of the evolution, there is a short adjustment period during which there is only one big cell of meridional circulation having $U(r) > 0$. This means that the matter dives inward at the equator and rises outward along the polar axis, building a gradient of Ω inside the star (with the core rotating faster than the surface) until an equilibrium profile is reached between the meridional circulation, which builds the gradient, and the shear, which erodes it. During that period, v_{eq} decreases quickly before the equilibrium profile is reached. The higher the rotation rate, the larger is the decrease. The adjustment period in our models takes a few 10^5 yr (1-2% of

the MS duration), so a probably more relevant value than the true v_{ini} would be a time-averaged value on this adjustment period. In Table 3.5, we give for each model first this time-averaged initial velocity and then the time-averaged equatorial velocity during MS ($\bar{v}_{\text{MS}}$).

Table 3.5: For each model, the first column indicates $\bar{v}_{\text{ini}}$, the time-averaged value of the velocity over the adjustment period from solid to non-solid rotation at the beginning of the MS phase (see Sect. 3.4.2 on previous page; the true v_{ini} is given in Table 3.2 on page 37). The second column gives $\bar{v}_{\text{MS}}$ the time-averaged equatorial velocity during the MS: the average is taken either on the whole MS phase or on the part of the evolution before the critical limit is reached. All the velocities are in km s^{-1} .

Z	0.10		0.30		0.50		$\Omega/\Omega_{\text{crit}}$ 0.70		0.80		0.90		0.99	
	$\bar{v}_{\text{ini}}$	$\bar{v}_{\text{MS}}$	$\bar{v}_{\text{ini}}$	$\bar{v}_{\text{MS}}$	$\bar{v}_{\text{ini}}$	$\bar{v}_{\text{MS}}$	$\bar{v}_{\text{ini}}$	$\bar{v}_{\text{MS}}$	$\bar{v}_{\text{ini}}$	$\bar{v}_{\text{MS}}$	$\bar{v}_{\text{ini}}$	$\bar{v}_{\text{MS}}$	$\bar{v}_{\text{ini}}$	$\bar{v}_{\text{MS}}$
3 $M_{\odot}$														
0	39	38	110	108	179	198	258	294	299	333	342	373	397	423
10^{-5}	38	33	107	95	174	167	249	247	293	297	340	353	391	416
0.002	32	27	92	80	151	139	218	207	255	246	300	298	348	352
0.020	29	23	83	69	137	120	199	181	231	215	269	258	312	311
9 $M_{\odot}$														
0	62	56	171	151	279	265	393	394	462	477	532	569	607	666
10^{-5}	48	39	132	117	220	204	322	305	378	363	444	435	511	515
0.002	40	33	111	99	186	173	269	256	316	305	371	369	421	425
0.020	36	30	102	90	169	157	247	236	291	284	340	341	384	387
20 $M_{\odot}$														
0	77	54	214	169	349	295	512	446	584	507	672	588	769	721
10^{-5}	55	45	156	144	263	254	386	389	454	469	535	557	626	651
0.002	45	37	129	120	221	213	323	323	379	387	444	456	515	528
0.020	41	32	118	107	200	189	291	279	342	325	397	369	459	447
60 $M_{\odot}$														
0	97	63	276	248	476	456	688	645	827	830	952	930	1140	1167
10^{-5}	71	63	213	221	364	408	533	587	632	687	751	797	880	890
0.002	57	48	175	179	300	323	440	474	517	552	609	641	708	727
0.020	50	37	157	140	268	258	389	400	458	472	534	547	600	609

Once the equilibrium Ω profile is built after the adjustment period, the evolution of the equatorial velocity at the surface of a star is mainly governed by two competing processes that transport angular momentum:

1. the meridional circulation, which brings angular momentum toward the surface,
2. the mass loss, which removes some mass, and thus angular momentum.

The two processes are strongly dependent on the metallicity, so we expect the evolution of v_{eq} to vary with Z . We will see how below.

Meridional circulation

The prescription for the radial amplitude $U(r)$ is presented in Appendix A.1.2 on page 127. After the adjustment period at the very beginning of the MS, an outer cell appears, with $U(r) < 0$, bringing angular momentum outwards at the equator and thus accelerating the surface. Eq. (A.15) page 127 shows that $U(r) \propto \left[1 - \frac{\Omega^2}{2\pi g \bar{\rho}}\right]$, and the dependence on $1/\bar{\rho}$ makes it largely negative in the outer layers where the density is low. However, we have seen that low- Z stars are much more compact, so we expect the outer cell to be of less amplitude at lower Z . Figure 3.15 shows the profile of meridional circulation amplitude inside 20 $M_{\odot}$ models at various metallicities taken

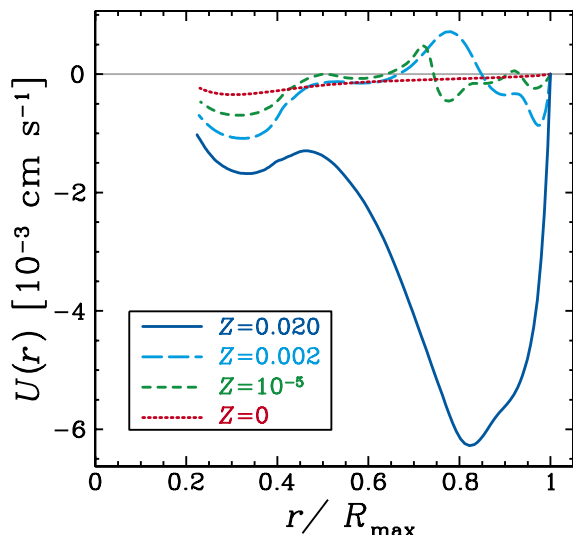


Figure 3.15: Variation of the amplitude of the meridional circulation inside $20 M_{\odot}$ models at various Z during the MS ($X_c = 0.40$).

at the same evolutionary stage ($X_c = 0.40$). Compared to $Z = 0.020$, the amplitude of $U(r)$ at $Z = 0.002$, 10^{-5} , and 0 is lower by a factor of 6, 25, and 100 respectively. This means that at low Z , the meridional current will be much less efficient at feeding the surface with angular momentum than at higher Z .

Mass loss

Castor et al. (1975) identified the mechanism driving the mass loss: the interaction between the radiation and the spectral lines. The mass loss rate is thus expected to depend on the chemical composition of the atmosphere as $\dot{M} \propto (Z/Z_{\odot})^m$, with $m = 0.50$ for Kudritzki et al. (1987), 0.80 for Leitherer et al. (1992), or 0.64 - 0.69 for Vink et al. (2001). More recently, Mokiem et al. (2007) find m to be 0.83 ± 0.16 for O-type stars. This means that at low Z , there will be less mass lost than at higher Z , so more angular momentum will be kept in the star and at the surface. Of course the result on the equatorial velocity will strongly depend on the mass loss prescription we use: this aspect will be addressed later (Sect. 3.4.2 on page 47).

Effects of metallicity on the evolution of the equatorial velocity

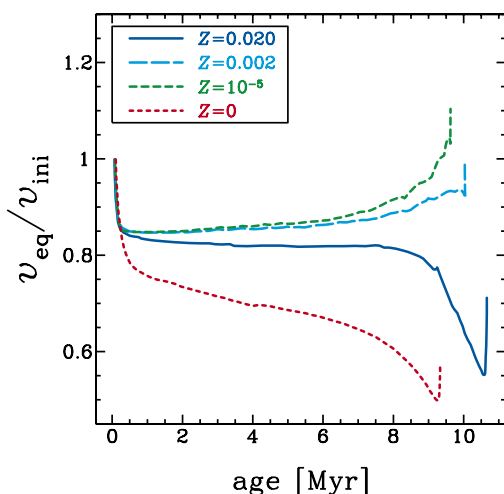


Figure 3.16: Evolution of v_{eq} at the surface of $20 M_{\odot}$ models with $\Omega_{ini}/\Omega_{crit} = 0.80$ at various Z . The velocity is normalised to its initial value.

Since metallicity makes both meridional circulation and mass loss weaker, we cannot guess which will be the net effect on the evolution of the equatorial velocity, we have to check it on our models. Figure 3.16 shows the evolution of v_{eq} at the surface of $20 M_{\odot}$ models at various Z , starting with $\Omega/\Omega_{crit} = 0.80$. As already known, we see that mass loss is strong at $Z = 0.020$, so the surface is spun down during MS evolution, whatever the strength of the meridional circulation. This finding is in agreement with Huang & Gies (2006b) who find a net spin-down with evolution. The $Z = 0.020$ model reaches the critical limit only at the very end of the MS, when the tracks makes a blue hook before turn off. At lower metallicity (but non zero), the meridional current is weaker, but the reduction in mass loss is the most important effect, so the models spin up and reach the critical limit when there is still about 15% of hydrogen in the centre.

The lower the metallicity, the faster this acceleration occurs. At $Z = 0$ strictly, the situation changes drastically. In this case, there is almost no mass loss at all, but the meridional circulation is so weak that the star evolves almost in a regime of local conservation of angular momentum $\Omega r^2 = \text{const}$: the natural inflation of the outer radius during the MS evolution spins down the surface.

Evolution of v/v_{crit}

An important quantity for stellar evolution is v/v_{crit} . The evolution of this ratio depends of course on the evolution of v_{eq} we have detailed above, but also on that of v_{crit} itself. In the frame of the Roche model, we have $v_{\text{crit}} \propto \sqrt{M/R}$ (see Appendix A.1.1, Eq. (A.6) on page 126). During the MS, the radius inflates and the mass decreases because of mass loss. v_{crit} is thus expected to decrease, so even if v_{eq} decreases also, we can expect v/v_{crit} to increase in some circumstances. This is shown in Fig. 3.17. At the end of the MS, the value of v_{crit} is only half its value on the ZAMS. While v_{eq} is slowly decreasing, the ratio v/v_{crit} increases steadily.

In Table 3.2 on page 37, the models that reach the critical limit are marked by a frame, the colour of which indicates roughly the central mass fraction of hydrogen (X_c) left in the core when the model arrives at the limit (the exact value of X_c is also given in the table).

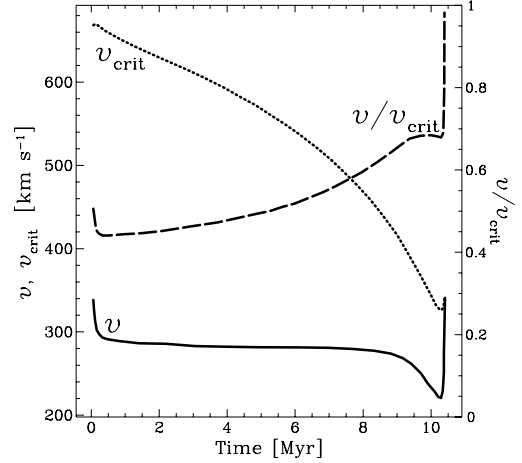


Figure 3.17: Evolution of v_{eq} , v_{crit} , and v/v_{crit} at the surface of a $20 M_{\odot}$ model with $v_{\text{ini}}/v_{\text{crit}} = 0.50$ at $Z = 0.020$.

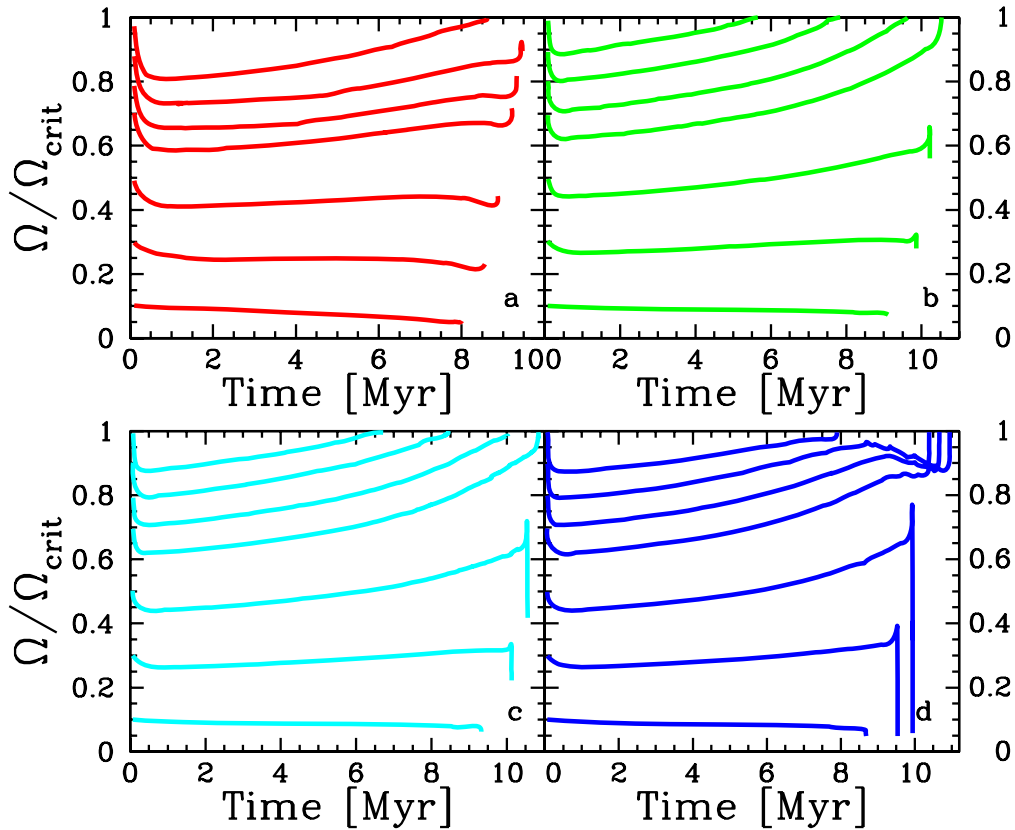


Figure 3.18: Evolution of $\Omega/\Omega_{\text{crit}}$ for all the $20 M_{\odot}$ models at $Z = 0$ (panel a), $Z = 10^{-5}$ (panel b), $Z = 0.002$ (panel c), and $Z = 0.020$ (panel d).

Figure 3.18 shows the evolution of $\Omega/\Omega_{\text{crit}}$ for all the $20 M_{\odot}$ models. At $Z = 0$, the model with $\Omega_{\text{ini}}/\Omega_{\text{crit}} = 0.99$ is the only one to reach the critical limit, but in spite of its high initial ratio, it has to wait until the central abundance of hydrogen has dropped down to 13% before it reaches the limit. This could seem surprising at first, but we recall that $\Omega/\Omega_{\text{crit}} = 0.99$ is the value on the ZAMS for solid-body rotation: in fact, after the adjustment period (see Sect. 3.4.2 on page 43), the model spend the beginning of the MS further away from the critical limit. The models at $Z = 0.020$ with $\Omega_{\text{ini}}/\Omega_{\text{crit}} \geq 0.70$ show an interesting feature: there is a bump around 8 Myr, when the surface velocity gets close to the critical limit but the enhancement of mass loss prevents the models from reaching it really. The surface velocity evolves thus away from the limit, until the very end of the MS evolution, when the contraction of the star accelerates the surface back to the critical limit. This illustrates the importance of the mass loss rates in the evolution of v/v_{crit} . We recall that we computed this set of models with the mass loss prescription from Kudritzki & Puls 2000 (hereafter KP00):

$$\dot{M} = \frac{D_{\text{mom}}}{v_{\infty} (R_{\star}/R_{\odot})^{0.5}} \cdot \left(\frac{Z}{Z_{\odot}} \right)^{0.5} = \frac{D_0 (L/L_{\odot})^{\chi}}{C_{T_{\text{eff}}} v_{\text{esc}} (R_{\star}/R_{\odot})^{0.5}} \cdot \left(\frac{Z}{Z_{\odot}} \right)^{0.5}$$

where $v_{\text{esc}} = (2g_{\star}R_{\star}(1 - \Gamma))^{0.5}$ is the escape velocity (with $g_{\star}$ the photospheric gravity and Γ the ratio of radiative Thomson to gravitational acceleration), $C_{T_{\text{eff}}}$ a parameter depending on T_{eff} :

$$C_{T_{\text{eff}}} = \begin{cases} 2.65 & \text{for } T_{\text{eff}} \geq 21'000 \text{ K} \\ 1.4 & \text{for } 10'000 \text{ K} < T_{\text{eff}} < 21'000 \text{ K} \\ 1.0 & \text{for } T_{\text{eff}} \leq 10'000 \text{ K} \end{cases}$$

and D_0 and χ parameters depending on the spectral type:

Spectral type	$\log D_0$	χ
A I	14.22	2.64
mid B I	17.07	1.95
early B I	21.24	1.34
O I	20.69	1.51
O III, V	19.87	1.57

Would the use of another prescription give different results?

Influence of the choice of the mass loss prescription

In order to check the influence of this choice on our results, we have computed a similar set of 20 and $60 M_{\odot}$ models at $Z = 0.020$, but using this time the mass loss prescription from Vink et al. 2000 (hereafter VdKL00). Their recipe includes the so-called *bi-stability jump*: an abrupt change of mass loss rate due to a shift in the ionisation balance of the Fe atoms around $T_{\text{eff}} \simeq 25'000 \text{ K}$. Below the bi-stability jump, the mass loss is given by:

$$\begin{aligned} \log \dot{M} = & -6.688 + 2.21 \log \left(\frac{L/L_{\odot}}{10^5} \right) - 1.339 \log \left(\frac{M/M_{\odot}}{30} \right) \\ & - 1.601 \log \left(\frac{v_{\infty}/v_{\text{esc}}}{2.0} \right) + 1.07 \log \left(\frac{T_{\text{eff}}}{20'000} \right) \end{aligned}$$

Above the bi-stability jump, the mass loss rate becomes:

$$\begin{aligned} \log \dot{M} = & -6.697 + 2.194 \log \left(\frac{L/L_{\odot}}{10^5} \right) - 1.313 \log \left(\frac{M/M_{\odot}}{30} \right) \\ & - 1.226 \log \left(\frac{v_{\infty}/v_{\text{esc}}}{2.0} \right) + 0.933 \log \left(\frac{T_{\text{eff}}}{40'000} \right) - 10.92 \left[\log \left(\frac{T_{\text{eff}}}{40'000} \right) \right]^2 \end{aligned}$$

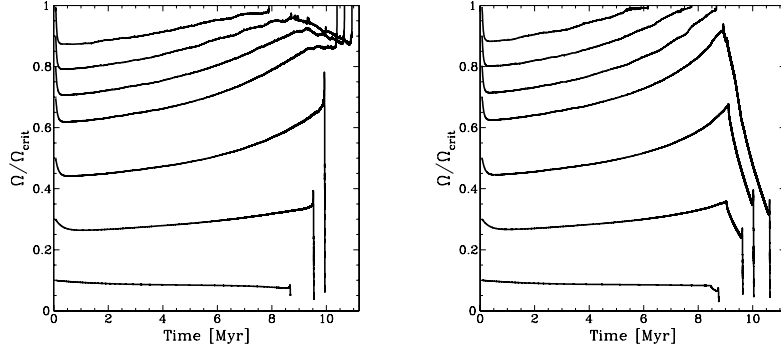


Figure 3.19: Evolution of $\Omega/\Omega_{\text{crit}}$ for the $20 M_{\odot}$ models at $Z = 0.020$. *Left*: with Kudritzki & Puls (2000) mass loss prescription; *right*: with Vink et al. (2000) prescription.

In the case of the $20 M_{\odot}$ models, the VdKL00 rates are slightly lower than the KP00 rates. The models with $\Omega/\Omega_{\text{crit}} \geq 0.80$ reach the critical limit sooner than is the case with the KP00 rates (see Fig. 3.19). The model with $\Omega/\Omega_{\text{crit}} = 0.70$ shows also a bump in $\Omega/\Omega_{\text{crit}}$ in the course of its evolution, but the mass loss enhancement is so strong that it drives the model away from the critical limit even at the very end of the MS. This difference is linked to the bi-stability jump in the VdKL00 formula. We see that in the case of the $20 M_{\odot}$ models, using VdKL00 rates instead of KP00 rates changes the lower $\Omega/\Omega_{\text{crit}}$ necessary to reach the critical limit, and also increases the time during which the models will remain at the limit. Otherwise, the results are qualitatively similar to the one obtained with KP00 prescription.

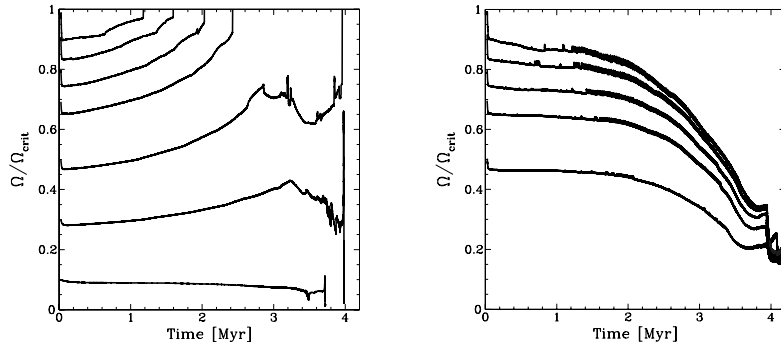


Figure 3.20: Evolution of $\Omega/\Omega_{\text{crit}}$ for the $60 M_{\odot}$ models at $Z = 0.020$. *Left*: with Kudritzki & Puls (2000) mass loss prescription; *right*: with Vink et al. (2000) prescription.

In the case of the $60 M_{\odot}$ models, the situation is striking: the VdKL00 rates are so much higher that none of the new models reach the critical limit anymore, even the one starting with $\Omega/\Omega_{\text{crit}} = 0.99$ (see Fig. 3.20). This illustrates the dominant role played by the mass loss in the evolution of the most massive stars. It shows that we must be very cautious when comparing high mass theoretical models with observations: in this mass domain, the comparison constitutes rather a test for the mass loss prescription than anything else.

Internal v/v_{crit} profile

We mentioned in Sect. 3.3.2 on page 32 that the effects of rotation are strong only in the outermost layers of the star. Figure 3.21 illustrates this: it presents the profile of the v/v_{crit} ratio inside a $20 M_{\odot}$ model at $Z = 0.020$, when the surface has just arrived at the critical limit.

It is striking to see that although the surface is at the critical limit, 95% of the mass is at $v/v_{\text{crit}} \leq 0.3$.

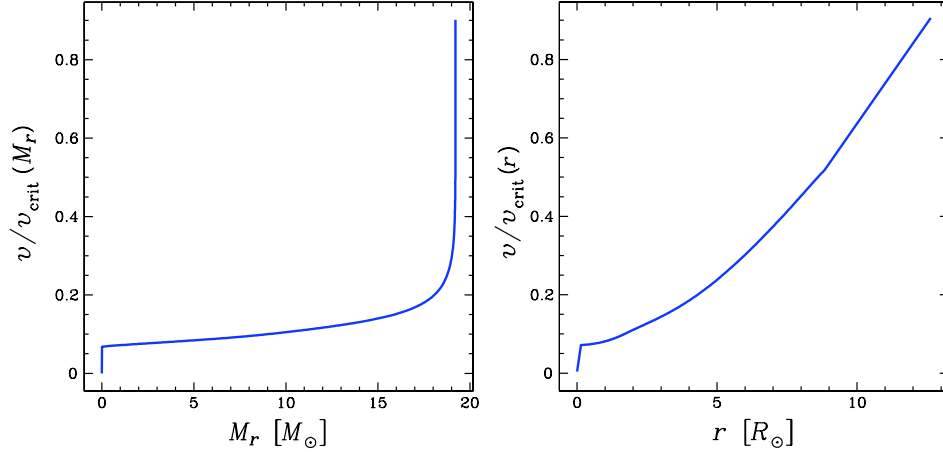


Figure 3.21: Profile of v/v_{crit} inside a $20 M_\odot$ model at $Z = 0.020$ when the surface is at the critical limit. *Left*: as a function of the enclosed mass; *right*: as a function of the internal radius.

3.4.3 Surface enrichments

An extremely important effect of rotation is the internal mixing it drives inside the star. The nuclear reactions in the core produce new elements, and these elements are brought toward the surface by the mixing. During core H burning, the signature of this enrichment is a modification

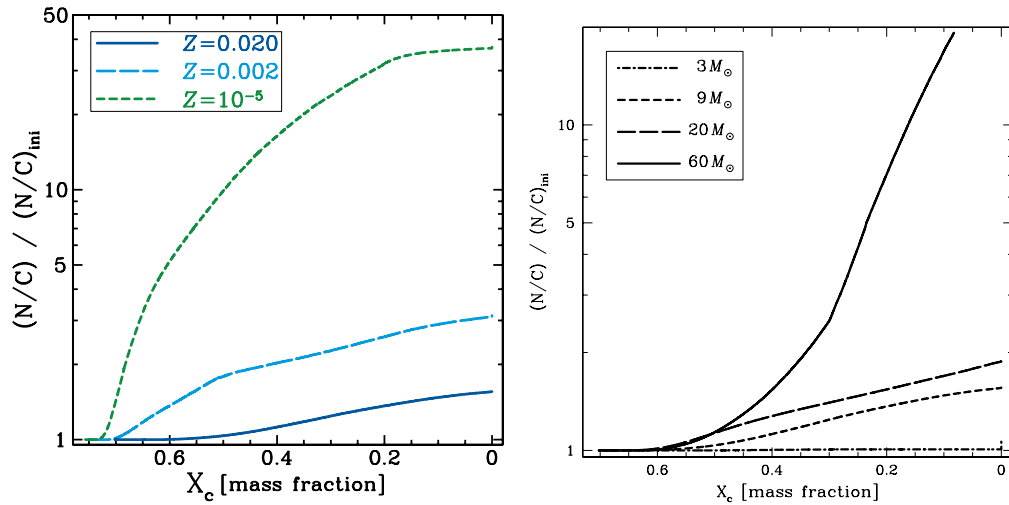


Figure 3.22: Evolution of the N/C abundances ratio, normalised to its initial value, as a function of the central H abundance. Note that the CNO equilibrium value would correspond to an enrichment of 160. *Left*: $9 M_\odot$ models with $\Omega_{\text{ini}}/\Omega_{\text{crit}} = 0.50$ at various Z ; *right*: models of various masses with $\Omega_{\text{ini}}/\Omega_{\text{crit}} = 0.50$ at $Z = 0.020$.

of the abundances ratio of the CNO elements at the surface, their sum C+N+O remaining constant. Figure 3.22 (*left*) shows the metallicity effect on the N/C surface ratio. At a given mass ($9 M_\odot$) and rotation rate ($\Omega_{\text{ini}}/\Omega_{\text{crit}} = 0.50$), the lower the metallicity, the sooner the surface enrichment appears. This is due to two mechanisms:

1. the profile of Ω are steeper at low Z , so the shear is stronger (Eq. (A.19));
2. the star is more compact, so the characteristic time of mixing is shorter ($\tau_{\text{mix}} \propto R^2/D$).

At the end of the MS, the stars cross the HRD, developing an outer convective zone which mixes the surface with deeper layers (*first dredge-up*). At $Z = 0.020$, the dredge-up dives more

deeply than at lower metallicity, so the surface enrichment is suddenly enhanced. If we now keep the metallicity constant ($Z = 0.020$) and let the mass vary, we see (Fig. 3.22, *right*) that the higher the mass, the stronger the surface enrichment. This is also due to the higher efficiency of the shear in high mass stars.

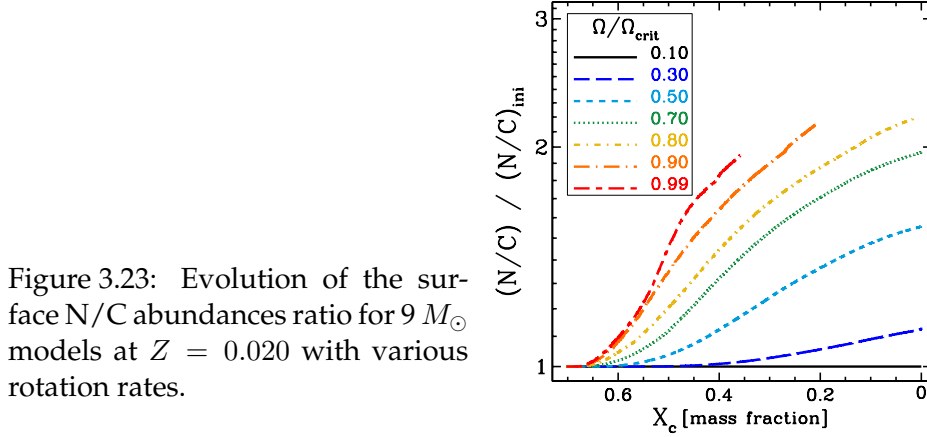


Figure 3.23: Evolution of the surface N/C abundances ratio for $9 M_{\odot}$ models at $Z = 0.020$ with various rotation rates.

Figure 3.23 shows the evolution of the surface N/C abundances ratio in $9 M_{\odot}$ models at $Z = 0.020$ for various rotation rates. As expected, the higher the rotation rate, the sooner the surface enrichment appears. The models with $\Omega_{\text{ini}}/\Omega_{\text{crit}} \geq 0.80$ reach the critical limit before the end of the MS, so they were stopped with a lower surface enrichment than that of the models that have been followed until the first dredge-up.

Here we followed the models only during the MS, so the consequences of the rotation-induced mixing are only a change of composition at the surface. We will see in Chapter 4 that during the later burning stages, the rotational mixing has extremely important impacts on the nucleosynthesis.

3.4.4 Discussion

To which extent can we be confident in the results presented above? Would they be modified with different physical inputs or assumptions?

Our models are started on the ZAMS with a solid-body rotation. We saw (Sect. 3.4.2 on page 43) that at the very beginning of the MS (during a few 10^5 yr), v_{eq} decreases while an equilibrium Ω profile is built inside the star, so the true $\Omega_{\text{ini}}/\Omega_{\text{crit}}$ of our models are actually lower than the one found on the ZAMS. We do not consider this as very important, given that the decrease becomes non negligible only at the highest $\Omega/\Omega_{\text{crit}}$: it would concern only the fast end of a velocity distribution, so only very rare objects. Note that recently, Martayan et al. (2008) show that a decrease between pre-MS and MS rotation velocities of 18% is indeed observed. This feature, which is artificial in our models, seems to agree with an observed feature.

We saw (Sect. 3.4.2 on page 47) that the highest mass models are extremely sensitive to the mass loss rates used. We expect the $60 M_{\odot}$ stars to be much rarer objects than 9 or $20 M_{\odot}$ stars: most of our description of the effects of rotation at various Z have been drawn from the 9 and $20 M_{\odot}$ models, which are only marginally affected by the mass loss prescription.

The models presented here do not include magnetic fields. They would impose an almost flat Ω profile inside the models during the MS phase, so the reaching of the critical limit would occur earlier. They lead also to a stronger mixing of the chemical species. However, the only prescription available to date, the Tayler-Spruit dynamo (Spruit 1999, 2002) is still much debated (see Zahn et al. 2007, and the discussion in Sect. 4.4.5 on page 90). We thus consider there is still some room for non-magnetic models, and for the time being the magnetic ones would also be subject to caution.

We have also neglected the winds anisotropy, due to von Zeipel's theorem: the poles, which become hotter, experience a higher mass loss than the equator, which becomes cooler. When mass

is lost preferentially at the poles, it removes less angular momentum than would be the case in the usual spherical configuration. This effect becomes substantial only at $\Omega/\Omega_{\text{crit}} \geq 0.75$, so only in a phase where the acceleration of v_{eq} is already rapid. It could however change the situation for the $20 M_{\odot}$ models at $Z = 0.020$. It is not sure that the effect would be strong enough, but it could allow the models with $\Omega/\Omega_{\text{crit}}$ between 0.70 and 0.90 to reach the critical limit sooner, without the bump seen in Fig. 3.18 on page 46. They would also retain more angular momentum in the interior, which could have important repercussions on the later phases of the evolution.

3.5 Application: Be stars

3.5.1 Motivations

As mentioned in Sect. 3.1 on page 27, an interesting domain to confront our computations to is the Be phenomenon.

The Be phenomenon appears in some B type stars, which exhibit the $H\alpha$ line in emission in their spectrum. This emission has been interpreted as arising from an equatorial disc, probably ejected by a star close to critical rotation (e.g. Dachs et al. 1986). While there is a consensus on Be stars being fast rotating B stars, there is quite a controversy on whether they are really critically rotating or not (see Porter 1996, Chauville et al. 2001 or Tycner et al. 2005, for example, who find a mean ratio $v/v_{\text{crit}} \approx 0.70 - 0.80$). According to Townsend et al. (2004), when $v/v_{\text{crit}} = 0.70$ the surface gravity is only reduced by a factor of 2, while when $v/v_{\text{crit}} \approx 0.95$, $\vec{g}_{\text{eff}}$ is reduced by an order of magnitude, and the velocity needed to launch matter into orbit is only of the order of the sound speed. In this case, many instabilities, like non-radial pulsations, become sufficient for orbital ejection (Owocki 2004; McSwain et al. 2008). However, the determination of the equatorial velocity may be difficult: because of von Zeipel's theorem, most of the flux of a fast rotating star is emitted by the polar regions. These regions, which present slow rotational velocities, will dominate the observed spectrum, leading to an underestimate of the true rotation rate of the very fast rotators (Collins 2004; Townsend et al. 2004). Recently, interferometric observations have brought a strong support to the idea that Be stars are very fast rotators:

- Domiciano de Souza et al. (2003) show that the Be star Achernar is strongly oblate. The $R_{\text{eq}}/R_{\text{pol}}$ obtained in this first study is even larger than that allowed by the Roche model ($R_{\text{eq}}/R_{\text{pol}} \leq 1.5$), but a recent re-analysis, which now includes the disc in the model, revises downward this ratio and shows it to be compatible with the Roche approximation (Carciofi et al. 2008);
- Kervella & Domiciano de Souza (2006) show that Achernar has also a enhanced polar emission;
- Meilland et al. (2007) obtain direct evidence that the disc around the Be star α Arae is in Keplerian rotation. In this case too, they show that the geometry of the wind is compatible with a thin disc wind and an enhanced polar emission.

What is the cause of the critical rotation? There are two proposed scenarios:

1. the spin-up of a binary star by matter accretion from its companion;
2. the natural evolution of the equatorial velocity of a single star.

Some Be stars are known to be in a binary system, but in most cases, no binarity has yet been clearly assessed. We must stress here that it may be very difficult to detect companions because of the long orbital period and the very small orbital motions. Some Be stars have undergone decades of careful observations before revealing their binary nature (see Gies et al. 1998 for ϕ Per, Rivinius

et al. 2004 for FY CMa, Maintz et al. 2005 for 59 Cyg or lately Kervella & Domiciano de Souza 2007 for Achernar). McSwain & Gies (2005) consider that about 75% of Be stars could have been spun-up by binary mass transfer. However, theoretical models of case B mass transfer⁶ predict that probably only 40% (surely no more than 60%) of the Be stars may form through the binarity channel (Pols et al. 1991). Recently, Kervella et al. (2008) propose a combination of binarity and fast rotation in the case of Achernar: Achernar A is indeed nearly critically rotating, so the gravity at the equator is very low, and the gravitational pull from the newly discovered Achernar B when it passes at the periastron helps removing the matter.

In the context of the single-star evolution scenario, let us make a few generic remarks. The Be phenomenon is transient and recursive, showing aperiodic variability on timescales of months or decades. If indeed the matter is launched by the natural surface evolution towards critical rotation, this variability can easily be explained: when the unbound layers of a critically-rotating star are expelled, angular momentum is lost, so the surface slows down and evolves away from the critical limit. Then secular evolution brings the star back to this limit and to a new ejection phase. Moreover, we can suppose that the phases between the outbursts are longer than the ejection phases. The probability of observing a star during the quiet phase is much higher than observing it during the outburst, which could explain why the velocities observed are lower than those needed to launch the matter into orbit.

Another point is that the number fraction of Be stars ($\text{Be}/(\text{B} + \text{Be})$) seems to be increasing with decreasing metallicity, as shown by Maeder et al. (1999), Wisniewski & Bjorkman (2006) or Martayan et al. (2007a), so the scenario for Be formation must explain this Z -dependency. We have seen that single low-metallicity stars are more prone to reach the critical velocity during the MS evolution than higher Z stars, so a Z -dependency seems to occur naturally in this context.

It is therefore interesting to confront the coherent set of models we have computed here with the observational constraints of Be stars. We will use them to answer the following questions:

- What are the initial conditions required for single stars to reach the critical velocity limit?
- Do these conditions depend on the mass and metallicity, and if so, how?
- What is the percentage of stars rotating faster than a given value of v/v_{crit} in stellar clusters of various ages and metallicities?

3.5.2 Initial conditions for reaching nearly critical velocities

From our computations, we can determine the minimum $\Omega/\Omega_{\text{crit}}$ value on the ZAMS ($\Omega_{\text{ini}}/\Omega_{\text{crit}}$) that is needed to reach a given value v/v_{crit} during the MS evolution. This is plotted in Fig. 3.24. In general, the $\Omega_{\text{ini}}/\Omega_{\text{crit}}$ value needed decreases with increasing mass. This translates the greater efficiency of the meridional current in the less dense envelopes of the more massive stars. The 20 $M_{\odot}$ models at $Z = 0.020$ are an exception to that trend, because they undergo a strong mass loss episode at the end of the MS, which keeps them away from the highest values of v/v_{crit} . When Z decreases, the $\Omega_{\text{ini}}/\Omega_{\text{crit}}$ value at a given mass and for a given v/v_{crit} value increases for the 3 and 9 $M_{\odot}$ models which mostly suffer from the loss of efficiency of the meridional circulation. It decreases for the 20 and 60 $M_{\odot}$ models which mostly benefit from the reduction of the mass loss. At $Z = 0$, the $\Omega_{\text{ini}}/\Omega_{\text{crit}}$ values for the 9 to 60 $M_{\odot}$ models are always higher than at $Z = 10^{-5}$, which is easily understood as the consequence of passing from a weak core-envelope coupling to nearly no coupling at all. In contrast, the metal-free 3 $M_{\odot}$ models show lower minimum $\Omega_{\text{ini}}/\Omega_{\text{crit}}$ values, which could be surprising at first. It should however be expected: we recall that the 3 $M_{\odot}$ models are contracting most of their MS evolution (see Sect. 3.4), because the CNO cycle ignites very late in these models. We have seen that they almost evolve in a regime of $\Omega r^2 = \text{const.}$, so their surface is accelerating during the MS phase.

⁶mass transfer during shell hydrogen burning

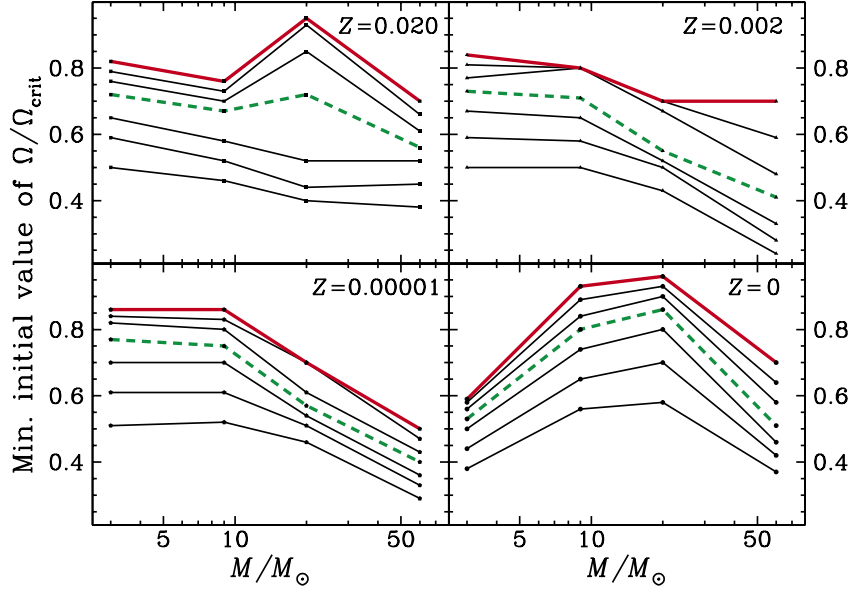


Figure 3.24: Minimum value of $\Omega/\Omega_{\text{crit}}$ on the ZAMS needed by a model of a given mass to reach values of v/v_{crit} superior to 0.99 (top thick red line) 0.9, 0.8, 0.7 (dotted green line), 0.6, 0.5, and 0.4 (bottom line) respectively during the MS evolution.

3.5.3 Rotation of stars in clusters of various ages and metallicities

We want to determine the fraction of MS stars rotating faster than a given v/v_{crit} in a cluster of a given age, considering only the stars that are brighter than 2 magnitudes below the turnoff. We must thus determine

- M_{max} , the mass of the stars that are at the turnoff at the time t
- M_{min} , the mass of the stars that are 2 magnitudes below the turnoff at the time t
- $\bar{n}(M)$, the number of stars born with the mass M
- $f(M, t)$, the fraction of these stars that are still on the MS at time t
- $p(M, t, u)$, the fraction of $\bar{n}(M)$ that are still on the MS and that have $v/v_{\text{crit}} \geq u$ at time t

so that the fraction of MS stars rotating with $v/v_{\text{crit}} \geq u$ in a cluster is:

$$\frac{n(t, u)}{N(t)} = \frac{\int_{M_{\text{min}}}^{M_{\text{max}}} \bar{n}(M) p(M, t, u) dM}{\int_{M_{\text{min}}}^{M_{\text{max}}} \bar{n}(M) f(M, t) dM} \quad (3.1)$$

Can we determine these quantities from our models? In clusters, we observe stars born at a given time and from a given initial composition, but with a distribution of masses and velocities. The mass distribution commonly used is the Salpeter IMF ($dN/dM = A M^{-2.35}$ with A a normalisation constant), and this is what we used for $\bar{n}(M)$.

From observations of 496 OB stars, Huang & Gies (2006a) provide a distribution of equatorial velocities corrected for the $\sin i$ effect (see Fig. 3.25). This distribution

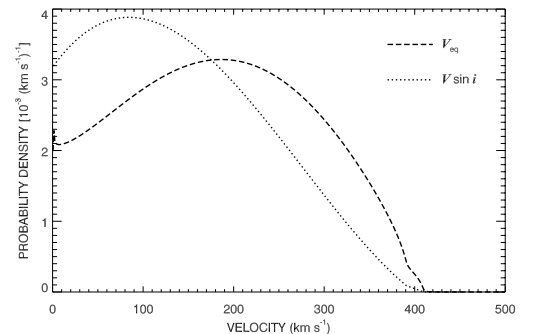


Figure 3.25: $v \sin i$ and v_{eq} distributions from Huang & Gies (2006a).

is obtained from stars with different masses, ages and rotation rates, but the statistics are sufficient for considering it as a distribution of average velocities during the MS ($\bar{v}_{\text{MS}}$). From our 9 $M_{\odot}$ models, we can find a relation between $\bar{v}_{\text{MS}}$ and $\omega = \Omega/\Omega_{\text{crit}}$ on the ZAMS, and so obtain a distribution of ω , which we apply then to all our models. Note that going from a probability density for v to a probability density for ω has been done through proper normalisation so that $\int_0^{420} D_p(v) dv = \int_0^1 d_p(\omega) d\omega = 1$.

We have seen that rotation enhances the MS lifetime. Therefore, a rotating star of a given mass will remain longer on the MS, just like a lower-mass non-rotating one. M_{max} will thus correspond to the mass of the star with $\Omega_{\text{ini}}/\Omega_{\text{crit}} = 0.99$ having a MS lifetime equal to t . From our models, we can determine for each mass M the minimum ω that allows the star to be still on the MS at age t , $\omega_{\text{min}}(M, t)$, so that we will have $f(M, t) = \int_{\omega_{\text{min}}(M, t)}^{\omega=1} d_p(\omega) d\omega$. From figures similar to Fig. 3.18 on page 46, we can determine for each initial mass and rotation rate the time at which v/v_{crit} becomes superior to u ⁷. We determine $\omega_{\text{min}}(M, t, u)$ the minimum ω that allows the star to be still on the MS and to have $v/v_{\text{crit}} \geq u$ at age t , so we will have $p(M, t, u) = \int_{\omega_{\text{min}}(M, t, u)}^{\omega=1} d_p(\omega) d\omega$. We now have all the ingredients needed to solve Equation (3.1).

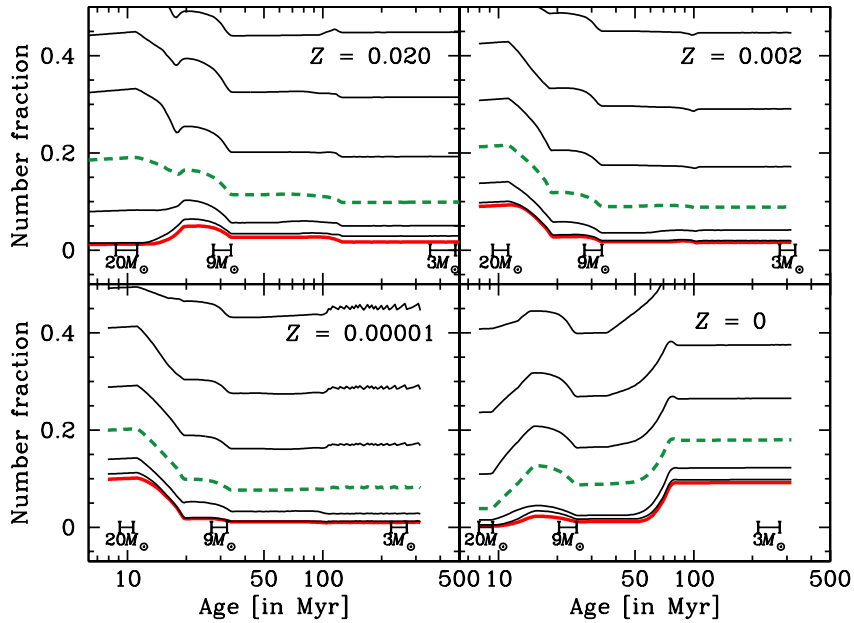


Figure 3.26: Fraction of stars rotating with values of v/v_{crit} superior to 0.99 (bottom thick red line) 0.9, 0.8, 0.7 (dashed green line), 0.6, 0.5, and 0.4 (top line) in clusters of various ages and metallicities. The time interval when the 3, 9 and 20 $M_{\odot}$ models leave the MS is indicated in each plot.

The results of these calculations are given in Fig. 3.26. We computed a coarse grid in masses, so only qualitative results can be obtained from our models, but it is nevertheless interesting to see what they predict. From Fig. 3.26 we can make a few remarks:

- In general, the curve for $v/v_{\text{crit}} > 0.90$ overlaps the curve for $v/v_{\text{crit}} > 0.99$. This means that once 0.90 is reached, the surface evolves rapidly towards the critical limit.
- At $Z = 0.020$, the behaviour of the curves for $v/v_{\text{crit}} > 0.80 - 0.99$ are different than those for $v/v_{\text{crit}} > 0.40 - 0.70$ in clusters with ages smaller than 20 Myr. This is a consequence of

⁷Here we suppose that once $v/v_{\text{crit}} \geq u$, it remains so for the rest of the MS evolution. Note that this is true for all our models except for the 20 $M_{\odot}$ models with $\Omega_{\text{ini}}/\Omega_{\text{crit}} \geq 0.80$ when the mass loss prescription from Kudritzki & Puls (2000) is used.

the behaviour of the $\Omega/\Omega_{\text{crit}}$ ratio for the $20 M_{\odot}$ models illustrated in Fig. 3.18 on page 46: the enhanced mass loss of the fastest rotators impedes the surface from reaching the critical limit before the very end of the MS evolution, reducing drastically the fraction of stars that are at the critical limit.

- The bump seen at $Z = 0.020$ in the curves for $v/v_{\text{crit}} > 0.80$ and higher at ages between 20 and 30 Myr can be explained in the following way: at younger ages than the bump, mass loss in higher-mass stars hampers the surface from reaching the critical limit, while at ages older than the bump, the lower amplitude of the meridional circulation in lower-mass stars is not sufficient to bring enough angular momentum so as to reach the critical limit.
- At $Z = 0.002$ and 10^{-5} , the curves are very similar in shape. The bump is shifted at younger ages and reaches higher values. This comes from the lower mass loss at lower Z , which favours the reaching of the critical limit.
- At $Z = 0$, the shape of the curves are totally different. Only clusters of ages $\gtrsim 70$ Myr have a fraction of critical rotators higher than 5%. We have seen that the total absence of metals reduces drastically the efficiency of the meridional circulation, so the 9 and $20 M_{\odot}$ models have great difficulties to reach the critical limit. On the contrary, the $3 M_{\odot}$ models are contracting during most of the MS phase, so their surface accelerates and easily reaches the critical limit.

Note that these results are obtained upon the assumption that the same distribution of $\Omega/\Omega_{\text{crit}}$ on the ZAMS applies at all metallicity, which might not be the case (see below).

3.5.4 Link with Be stars

The results of the previous section can be used to predict the number fraction of Be stars among a population of B-type stars ($\text{Be}/(\text{B} + \text{Be})$) in Galactic ($Z = 0.020$) or SMC ($Z = 0.002$) clusters. Figure 3.27 shows a comparison between the prediction of our models and observed $\text{Be}/(\text{B} + \text{Be})$ fraction (Wisniewski & Bjorkman 2006; Keller et al. 1999; Maeder et al. 1999).

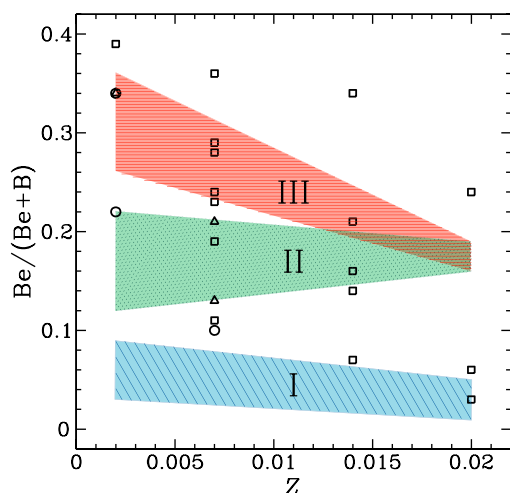


Figure 3.27: Number fraction of Be stars in clusters with ages between 10 and 25 Myr as a function of metallicity. The open symbols are the values among B0-B3 type stars observed by Wisniewski & Bjorkman (2006, circles), Keller et al. (1999, triangles) and Maeder et al. (1999, squares). The hatched areas correspond to the predictions of our models. The area labelled I is obtained when we consider only stars that are at the critical limit; the area labelled II is obtained when we count all stars with $v/v_{\text{crit}} \geq 0.70$; the area labelled III is obtained with stars having $v/v_{\text{crit}} \geq 0.70$ and assuming that there are more fast rotators at low Z (see text).

If we consider only stars that are strictly at the critical limit, the prediction of our models falls well below the observed points, especially at $Z = 0.002$ (hatched area labelled I). Of course, our models stand only for the single-star channel of Be formation. But even if we add the 40% of Be stars that would form through binary mass transfer (see Sect. 3.5.1 on page 51), we still fall too short.

If we assume that stars with $v/v_{\text{crit}} \geq 0.70$ can show Be properties, we obtain the hatched area labelled II, which improves slightly the results. However, there seems to be no trend with metallicity, contrarily to what is suggested by the observations. Could this trend be due to an observation bias? It is not excluded. Be stars are usually found among the brighter B-type stars (McSwain & Gies 2005; Keller et al. 2001). Since low Z clusters are usually more distant, observations will be biased towards brighter stars, where the Be fraction may be higher. However, if the observed sample is complete, and if the counts are performed in the same absolute magnitude for all the clusters, this observation bias should not have a strong impact on the results.

To get a metallicity trend, we need to assume that the velocity distribution is not the same at low Z than at $Z = 0.020$, containing more fast rotators. The area labelled III is obtained upon the use of a flat velocity distribution at $Z = 0.002$. We see that the agreement with the data points at $Z = 0.002$ is much improved.

The implications of these findings are the following:

1. the Be phenomenon should occur already at $v/v_{\text{crit}} \approx 0.70$;
2. the low-metallicity environment should have a greater fraction of fast rotators at birth than the standard metallicity.

The first assessment seems to be confirmed by observations (Chauville et al. 2001; Cranmer 2005; Martayan et al. 2006, 2007b). It means that fast rotation sets a favourable context, but then some other processes must enter into play to build the ejection disc. We have already mention the possibility of non-radial pulsations (see Sect. 3.5.1 on page 51). There is also another possibility: Maeder et al. (2008) show that rotation favours convective instabilities in the stellar envelope. Convective motions could play a significant role in the ejection of the matter into the disc.

The second assessment seems also to be observed, at least in the B-type stars mass range. Keller (2004) finds that the mean $v \sin i$ ($\langle v \sin i \rangle$) of Galactic young clusters B-type stars is 116 km s^{-1} while $\langle v \sin i \rangle = 146 \text{ km s}^{-1}$ in the Large Magellanic Cloud (LMC) young clusters. It is not clear in this study whether this increase is due to evolutionary differences or to an initial velocity distribution that favours fast rotators. The same trend is found by Hunter et al. (2008), who find the peak of the velocity distribution to be 100 km s^{-1} in the LMC clusters B-type stars, and 175 km s^{-1} in the Small Magellanic Cloud (SMC) clusters. They find the difference between Galactic and LMC clusters to be much less significant. According to Martayan et al. (2007b), the observed values of the mean $\Omega/\Omega_{\text{crit}}$ ratio in the Milky Way (MW) and the two Magellanic Clouds are the following:

$\langle \Omega/\Omega_{\text{crit}} \rangle$	B	Be
MW	0.35	0.83
LMC	0.37	0.85
SMC	0.58	0.95

In this study too, while the difference is not significant between the Galaxy and the LMC, there is a net increase of $\langle \Omega/\Omega_{\text{crit}} \rangle$ when the metallicity decreases to the SMC value. Note that the B stars population is mainly composed of stars with masses ranging between 3 and $9 M_{\odot}$. These stars experience an almost negligible mass loss rate, so the evolution of their surface velocity is mainly governed by the ability of the meridional circulation to accelerate them. We saw (Sect. 3.4.2 on page 44) that the amplitude of the meridional circulation decreases at low Z : would the $\Omega_{\text{ini}}/\Omega_{\text{crit}}$ distribution be the same at all metallicities, we would expect this population to show *lower* values of $\langle \Omega/\Omega_{\text{crit}} \rangle$. We thus take this observational result as a strong indication for a larger fraction of fast rotators at birth in low Z environment.

Let us note that for more massive stars, the work by Penny et al. (2004), based on *IUE* UV spectra, finds no significant differences between the rotational velocities of O-type stars in the

MW, LMC and SMC. However, we saw in Sect. 3.4.2 on page 47 how the most massive stars are sensitive to the choice of the mass loss rates, so any result involving O-type stars must be taken with extreme caution. Anyway, the question is not yet settled and more surveys are needed.

3.5.5 Discussion

The discussion about the physical inputs of our models has already been done in Sect. 3.4.4 on page 50. Two new ingredients have appeared in this section: the IMF and the velocity distribution. These two ingredients have an impact on two points we discussed above.

First, the choice of the mass loss prescription: we saw that the $60 M_{\odot}$ models are very sensitive to this choice, but the use of an IMF reduces the impact of the most massive objects. The choice of the mass loss rates becomes less crucial. We could discuss the choice of the IMF, but since we computed only four different masses, it does not make sense to test this point: we already warned the reader that only qualitative results can be obtained here.

Second, the flat rotation profile on the ZAMS: we saw that the models have in fact a lower true initial velocity since there is a short adjustment period just after the ZAMS. However, the link between our models and the velocity distribution is done through their mean MS velocity, which is a robust quantity, and the velocity distribution accounts already for faster rotating objects.

The velocity distribution proposed by Huang & Gies (2006a) has been established upon observations of stars with different masses and different evolutionary ages. Moreover, Townsend et al. (2004) show that above $v/v_{\text{crit}} = 0.70$, the relation between the FWHM of the HeI 4471 line and the equatorial velocity is no more linear, so the FWHM becomes a poor indicator of the true surface velocity: a star rotating at the critical limit presenting the same line broadening as a star rotating at 80% of the critical velocity. In other words, this technique may underestimate the real velocity of the star by 20-25% for very fast rotators (typically for $v/v_{\text{crit}} > 0.75$, *i.e.* for velocities superior to about 340 km s^{-1} , for a $9 M_{\odot}$ model at standard metallicity). To study this effect, we did the following experiment. We modified the velocity distribution given by Huang & Gies (2006a) by assuming that between 300 and 420 km s^{-1} , all the velocities should be enhanced by a factor equal to $1 + (v - 300)/120 \times 0.3$ (*i.e.* a linear enhancement amounting at most to a factor 1.3 at 420 km/s). Using such a modified velocity distribution, we find that the fraction of stars at the critical limit for an age equal to 25 Myr at $Z = 0.020$ would be equal to 0.055 instead of 0.049 with the original velocity distribution of Huang & Gies (2006a, Fig. 3.25 on page 53). Thus this effect would not significantly change the results.

We have also tested the results that would be obtained had we used a uniform velocity distribution: we obtain that the fraction of stars with values of $\Omega/\Omega_{\text{crit}}$ on the ZAMS superior to 0.6, 0.7, 0.8, and 0.9, passes from 37.5, 26.5, 15.5, and 6.5% to 40, 30, 20, and 10% respectively. We find also that the fraction of stars at the critical limit equal 12% at $Z = 0.020$ and for an age equal to 25 Myr (instead of 5% with the original velocity distribution).

Evolution of Population III stars

The results presented in this chapter have been published in the following article (see on page 168):

EFFECTS OF ROTATION ON THE EVOLUTION OF PRIMORDIAL STARS
S. Ekström, G. Meynet, C. Chiappini, R. Hirschi & A. Maeder
2008 *A&A* **489**, 685

Electronic tables of the models (evolutionary sequence and final structure files) are available at <http://obswww.unige.ch/Recherche/evol/Rotating-primordial-stars>.

4.1 Motivations

We have seen in Sect. 2.2 on page 11 that Population III (Pop III) stars occupy a key position in the history of the Universe, as a link between the pure H-He Universe of the beginning of times, and the metal-rich one we observe nowadays around us. Their life is important, for this is when they start forming the first heavy elements, but their death is also important, for it is when those elements may be released in the surrounding medium, starting the chemical enrichment of the Universe.

4.1.1 Existing studies

Since the first modelling by Ezer & Cameron (1971), Pop III stars have been the subjects of many studies. Although the peculiarities of star formation in a metal-free Universe suggest that Pop III stars were very massive objects (VMOs, Bromm et al. 2002; Abel et al. 2002), the studies have not limited themselves to the high mass range. They span an extremely large variety of mass domains, from 0.8 to 1000 $M_{\odot}$, sometimes preceding some later findings which show that the Pop III formation could indeed not be limited to VMOs (Omukai & Yoshii 2003; O’Shea et al. 2005; Greif & Bromm 2006).

Amongst the papers published during the last decade, most are studying the evolution until AGB phase or C ignition (Chieffi et al. 2001; Marigo et al. 2001, 2003; Siess et al. 2002; Gil-Pons et al. 2005, 2007; Lau et al. 2008). Others are focused on the final stages and nuclear yields (Chieffi & Limongi 2002; Heger & Woosley 2002; Umeda & Nomoto 2002; Umeda & Nomoto 2005; Tominaga et al. 2007b). There are actually only two studies that include rotating models: one by Marigo et al. (2003) and one by Heger et al. (2000b). The former study computes VMOs with masses ranging between 120 and 1000 $M_{\odot}$. Rotating stars are treated as rigid bodies, and the effects of rotation intervene mainly through the correction factor applied to the radiative mass loss (Eq. (A.23), page

129). In these models, no rotational mixing is accounted for. The latter study follows the evolution of differentially-rotating stellar models of 15-250 $M_{\odot}$, from the ZAMS to core collapse, but the detailed results of these models have not been published. There is thus room for a new study on differentially-rotating Pop III stars.

4.1.2 Effects of rotation at very low metallicity

Differential rotation has been shown to strongly modify the stellar evolution at very low metallicity (Meynet et al. 2006). In Chapter 3 on page 27, we limited our study to the MS evolution. The main effects at that stage are the reaching of the critical limit and the beginning of the surface enrichment. After the MS, the rotational mixing becomes much stronger because rotation favours a redwards evolution: the Ω -gradients are stretched, so shear diffusion is extremely efficient at the border of the core, and the outer convective zone brings freshly synthesised elements towards the surface. The effective metallicity increases drastically, and the line-driven winds are enhanced. Meynet et al. (2006) show that a 60 $M_{\odot}$ model at $Z = 10^{-8}$ with an initial velocity $v_{\text{ini}} = 800 \text{ km s}^{-1}$ loses more than half its initial mass during CHeB.

Another interesting effect of rotation concerns the nucleosynthesis. Rotational mixing connects different burning zones and thus modifies the nucleosynthetic products. This is how primary nitrogen can be produced in massive stars during CHeB: some freshly synthesised ^{12}C is diffused out of the He-burning core and reaches the H-burning shell, where it is engulfed in the CNO cycle and transformed into ^{14}N .

These effects become very strong when Z decreases, so it is interesting to study if this behaviour follows the same trend down to $Z = 0$ strictly.

4.2 Evolution at $Z = 0$

4.2.1 Physical inputs of the models

The stellar models presented here have masses ranging from 9 to 200 $M_{\odot}$. The initial composition is $X = 0.76$, $Y = 0.24$ and $Z = 0$. The models with masses between 25 and 85 $M_{\odot}$ were computed until the end of the hydrostatic core Si burning. The 9 $M_{\odot}$ model developed a strongly degenerate core before carbon ignition and was thus stopped then. The 15 $M_{\odot}$ model was stopped at the end of oxygen-burning also because of a core that became too degenerate at that moment. Because of numerical difficulties, the 200 $M_{\odot}$ model was stopped shortly after central He exhaustion.

The Geneva stellar evolution code was adapted to include the necessary implementations to go on through advanced stages up to the pre-SN as developed by Hirschi (2004). It has been slightly modified to join the reaction networks of H and He burning during the MS, as required for zero-metallicity models (see Appendix A.2).

The reaction rates are taken from NACRE database¹. The opacities come from Iglesias & Rogers (1996), complemented at low temperature by the molecular opacities of Alexander & Ferguson (1994). The treatment of convective instability is done according to the Schwarzschild criterion. Hirschi (2004) shows that until core O burning, the convection can be treated as an instantaneous mixing, but during O and Si burning, the convection mixing timescale ($\tau_{\text{conv}} \approx R_{\text{core}}^2 / D_{\text{conv}} \sim 10^4 - 10^5 \text{ s}$) becomes longer than the evolution time steps ($\Delta t \sim 10^2 - 10^3 \text{ s}$). Thus, in the last burning phases, convection is treated as a diffusive process, with a diffusion coefficient D_{conv} determined with the mixing length theory. An overshooting parameter of 0.2 H_P is used for H- and He-burning cores only.

¹http://pntpm.ulb.ac.be/Nacre/barre_database.htm

Rotation

The rotational instabilities taken into account are the same as in Chapter 3: meridional circulation, dynamical and secular shear. The way these instabilities are implemented in the code is described in Appendix A.1. Note that here we use Eq. (A.16) for the horizontal turbulence coefficient.

We have seen that very low- or zero-metallicity stars are more compact than their metal-rich counterparts, so a similar v/v_{crit} ratio leads to a higher equatorial velocity. In order to explore the effects of rotation on metal-free models, we chose a high initial velocity, 800 km s^{-1} . Table 3.2 on page 37 shows that for the $60 M_{\odot}$ model, this initial velocity corresponds to a total angular momentum content that is similar to a solar metallicity model of the same mass rotating at about 300 km s^{-1} . The $9 M_{\odot}$ model was computed with a lower initial velocity (500 km s^{-1}) because 800 km s^{-1} was overcritical on the ZAMS for this model. Table 4.1 presents the initial equatorial velocities of the models, the corresponding v/v_{crit} ratio and the mean velocity during MS.

Table 4.1: Rotational properties of the models. The masses are in $M_{\odot}$ and the velocities in km s^{-1} . The last column gives the central hydrogen mass fraction when the model reaches the critical limit.

Mass	v_{ini}	v/v_{crit}	$\bar{v}_{\text{MS}}$	$X(H)_{\text{crit}}$
9	500	0.54	430	–
15	800	0.72	626	0.018
25	800	0.63	657	0.253
40	800	0.56	700	0.326
60	800	0.52	740	0.360
85	800	0.48	754	0.399
200	800	0.40	733	0.428

In order to be able to compare our results with other works, we computed, for each mass, a corresponding model without rotation.

Mass loss

We have seen that zero-metallicity stars are supposed to evolve at constant mass due to a scaling of the wind with metallicity of the form $\dot{M} \propto (Z/Z_{\odot})^m$ with $m = 0.5 - 0.83$ (see page 45). However, Kudritzki 2002 (hereafter K02) shows that H and He lines are able to remove some mass if the luminosity is sufficiently high. We thus adopted the K02 prescription (cf. Appendix A.2.2 on page 130) for the models presenting a luminosity $\log(L/L_{\odot}) > 6$. The 85 and $200 M_{\odot}$ models were computed with it from the start, while we turned it on during the MS evolution of the $60 M_{\odot}$ models. Since K02's prescription does not correspond to the strictly $Z = 0$ case, we used the same adaptations as in Marigo et al. (2003): 1) the surface metallicity is set at $Z = \max(Z_{\text{min}}, Z_{\text{eff}})$, where $Z_{\text{min}} = 10^{-4} Z_{\odot}$ is the lowest value in the validity range of K02's prescription, and $Z_{\text{eff}} = 1 - X - Y$ is the effective surface metallicity of the model (allowing for self-enrichment); and 2) the effective temperature is set at $T_{\text{eff}} = \min(T_{\text{eff}}, 60'000 \text{ K})$, i.e. we did not extrapolate K02's formula beyond its validity domain but constrained $T_{\text{eff}} \leq 60'000 \text{ K}$ for the calculation of the mass loss. The models with a mass lower than $60 M_{\odot}$ never reach $\log(L/L_{\odot}) > 6$, so they were computed without radiative mass loss.

To take into account the effects of rotation, the line-driven wind was multiplied by the factor F_{Ω} as described in Eq. (A.23, Appendix A.1.7).

There is another situation in which the models can lose mass: it occurs when the external layers of the star reach the critical velocity. There is an adjustment between two opposite effects: the overcritical layers are no longer bound to the star and are free to evade the surface; the removal of these layers brings the surface back to sub-critical velocity until the evolution of the star accelerates it

again. It is not yet clear whether the matter launched is really expelled into the interstellar medium or remains in the surroundings in the form of a disc. However, if it forms a disc, the radiation field of the star should dissipate it quickly (S. Owocki, priv. comm.), so the matter can be considered as lost by the star. In the present calculations, once the model reached the critical limit, we adapted the mass loss rate to keep the surface just under the critical velocity. This treatment was applied to all the models (except the $9 M_{\odot}$, which never reached the critical limit).

4.2.2 Hertzsprung-Russell diagram and central conditions

Figure 4.1 shows the HR and $\log T_c - \log \rho_c$ diagrams of both rotating and non-rotating models. In Chapter 3, we studied in details the characteristics of rotation at low Z during the MS, so we will not go through it again. Let us just mention that the ignition of the CNO cycle is clearly visible on the $9 M_{\odot}$ tracks (cf. Sect. 3.4.1), but it is also sketched at the very beginning of the $15 M_{\odot}$ tracks, while it is absent of the $25 M_{\odot}$ models tracks. Marigo et al. (2001) find the mass limit for CNO ignition already on the ZAMS to be $20 M_{\odot}$, and our models are consistent with that finding.

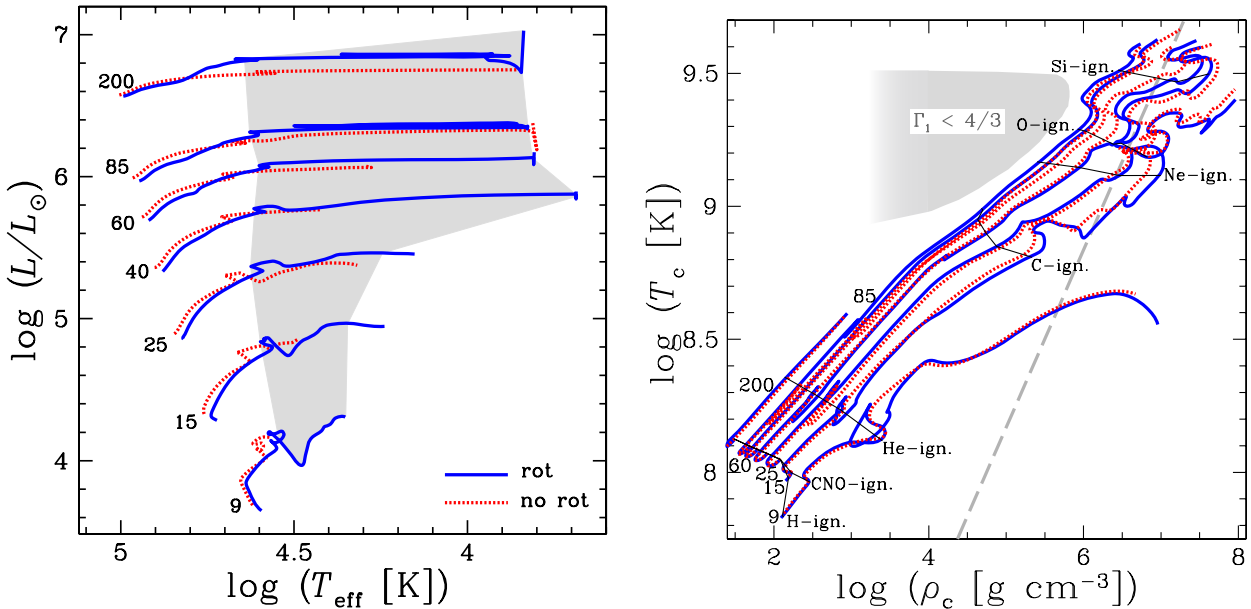


Figure 4.1: *Left*: evolutionary tracks in the theoretical Hertzsprung-Russell diagram for the rotating (solid blue line) and non-rotating (red dotted line) models at $Z = 0$. The shaded area represents the zone where He is burnt in the core. *Right*: $\log T_c - \log \rho_c$ diagram for the same models. The ignition of the different burning stages are indicated. The dashed line is the frontier between non-degenerate and degenerate (non-relativistic) gas. The shaded area is the zone where the conditions for pair-creation instability are met.

After central H exhaustion, the evolution of metal-free stars becomes really particular compared to the evolution at higher metallicity: the core does not need to rapidly contract in order to start burning helium, it is already hot enough. The transition between the two burning phases is smooth, without the usual structural readjustment (core contraction and envelope expansion). The luminosity mostly comes from the core, the H-burning shell remains quiet and radiative (see Fig. 4.2, *left*). Therefore, the models remain in the blue side of the HRD.

Then, during the core helium burning phase (CHeB), because of rotational mixing, some carbon produced in the core is diffused towards the H-burning shell, allowing a sudden ignition of the CNO cycle in the shell. This boost of the shell leads to a retraction of the convective core and a decrease of the luminosity. At the same time, it transforms the quiet radiative H-burning shell into an active convective one (see Fig. 4.2 *centre*). A similar event is described in Hirschi (2007)

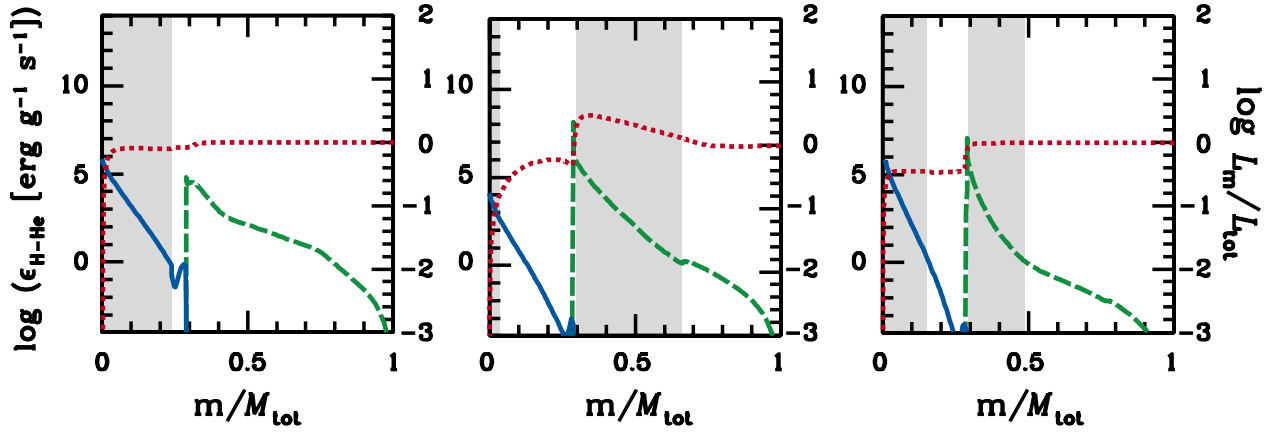


Figure 4.2: Internal structure of the $15 M_{\odot}$ rotating model at different stages of the core He burning. The blue solid line is the He-burning energy production and the green dashed line the H-burning energy production. The red dotted line is the fraction of the total luminosity (scale on the right). The grey areas show the convective zones. *Left*: before the CNO shell boost (age: 13.739 Myr; $Y_c = 0.4632$) - *centre*: at the moment of the boost (age: 13.744 Myr; $Y_c = 0.4919$) - *right*: after the boost (age: 13.757 Myr; $Y_c = 0.4840$).

for rotating models at $Z = 10^{-8}$, and is shown to be a robust feature. Later, the model takes the structure of a higher metallicity one: convective He-burning core, convective H-burning shell and expanding envelope (see Fig. 4.2 *right*). The boost appears in the HRD (Fig. 4.1, *left*) as a deep V-shaped feature in the rotating tracks of the 9 and $15 M_{\odot}$ models, and as a slight depression in the 25 and $40 M_{\odot}$ tracks: the luminosity decreases at the same time as the T_{eff} cools down. The only non-rotating models that present the same feature are the 25 and $85 M_{\odot}$ models. In this case, the structure is such that the growing convective core reaches the bottom of the quiet radiative H-burning shell, triggering the same event. Such a CNO boost leaves also an imprint in the $\log T_c - \log \rho_c$ diagram (Fig. 4.1, *right*): a second hook can be seen in the tracks, close to the He-ignition line. In the case of the rotating $9 M_{\odot}$ model, it draws a complex loop, not just a hook.

In the non-rotating set, all the models with mass between 9 and $60 M_{\odot}$ models end their life as blue supergiants (BSG), while the 85 and $200 M_{\odot}$ models become red supergiants (RSG). This compares well with the tracks obtained by Marigo et al. (2001). In their study, all the models with masses between 9 and $60 M_{\odot}$ start and finish CHeB in the blue part of the HRD. Only the highest masses models ($M \geq 70 M_{\odot}$) end the central He combustion on their Hayashi line.

Generally, the rotating models end their evolution in a cooler part of the HRD than their non-rotating counterparts. The only exception is the $85 M_{\odot}$ model, which ends its life as a BSG because of a blue loop after CHeB. The rotating 15 and $25 M_{\odot}$ models end their evolution in the blue part of the HRD, without any redward incursion (but in a cooler location than the non-rotating models of the same masses). The rest of the rotating models become RSG. The crossing of the HRD is however slow, unlike higher metallicity models: we saw that the nuclear burning is continuous inside the core, therefore the $Z = 0$ models do not go through a phase of rapid core contraction and its mirror effect of fast envelope expansion. On the HRD, we note that the crossing of the diagram takes place during the whole CHeB (grey area).

None of our models meet the conditions to be Wolf-Rayet stars ($\log T_{\text{eff}} > 4.0$, $X_{\text{surf}} < 0.30$). The rotating $85 M_{\odot}$ model ends its life with a $\log T_{\text{eff}}$ higher than 4.3, but its surface abundance of hydrogen remains too high, we will see why below (Sect. 4.2.5).

4.2.3 Lifetimes

Table 4.2 presents the lifetime on the MS (τ_{MS}) and during CHeB (τ_{CHeB}) for all the models. As expected (see Sect. 3.4.1), the MS lifetime is increased in the rotating models compared to the non-rotating ones. This increase amounts to 12.8% in the case of the 200 $M_{\odot}$ models, and up to 24.5% in the case of the 15 $M_{\odot}$ models. The models started with higher initial v/v_{crit} show larger increases of τ_{MS} .

Table 4.2: Final masses, total lifetime, MS and core He-burning durations, C/O ratio at the end of CHeB, masses of the He-, CO-, Si-, and Fe-core, and masses of the remnant for our models at the pre-SN stage. Masses are in $M_{\odot}$, velocities in km s^{-1} and times in Myr.

Mass	v_{ini}	M_{fin}	τ_{life}	τ_{MS}	τ_{CHeB}	C/O	M_{α}	M_{CO}	M_{Si}	M_{Fe}	M_{rem}
9	0	9.00	21.7	19.50	1.99	0.59	1.73	0.46	–	–	0.46
9	500	9.00	26.8	23.90	2.58	0.51	2.09	0.63	–	–	0.63
15	0	15.00	11.6	10.60	0.83	0.44	3.62	2.92	1.42	–	1.64
15	800	14.96	14.4	13.20	0.98	0.41	4.31	2.71	1.48	–	1.58
25	0	25.00	7.29	6.64	0.56	0.29	7.51	5.39	2.03	1.28	2.26
25	800	24.75	8.47	7.88	0.50	0.31	9.14	6.30	2.17	1.33	2.49
40	0	40.00	5.03	4.60	0.36	0.21	15.95	14.52	3.21	1.83	4.49
40	800	37.99	5.88	5.41	0.41	0.22	17.22	13.53	2.73	1.55	4.22
60	0	59.99	3.96	3.60	0.30	0.16	26.52	25.11	4.61	2.38	7.70
60	800	57.59	4.57	4.20	0.31	0.14	30.86	30.58	4.82	2.51	9.26
85	0	84.93	3.42	3.05	0.31	0.14	34.77	34.50	5.45	2.61	10.37
85	800	74.57	3.84	3.52	0.28	0.11	44.38	43.92	8.47	2.78	13.00
200	0	199.95	2.53	2.27	0.23	0.06	101.63	95.69	–	–	27.32
200	800	183.56	2.83	2.56	0.24	0.05	109.65	95.97	–	–	27.40

The variations brought on the duration of the CHeB phase are much less predictable. Rotation generally brings an increase in τ_{CHeB} , but it is not correlated with the initial rotation rate. Two rotating models have shorter CHeB lifetime than their non-rotating counterparts: it is the case where the non-rotating models experience a shell boost as described in Sect. 4.2.2 (the 25 and 85 $M_{\odot}$ models), so it is rather the non-rotating lifetime which is enhanced by the boost. This illustrates the sensitivity of the CHeB phase on the convection history of the internal structure. The $\tau_{\text{CHeB}}/\tau_{\text{MS}}$ ratio is about 8-10% for the non-rotating models. It is slightly decreased by rotation (6-9%), except in the case of the 9 $M_{\odot}$ models. This decrease is mainly caused by the increase of τ_{MS} . In the case of the 9 $M_{\odot}$ rotating model, the shell boost leads to an increase of τ_{CHeB} of about 30%, so the $\tau_{\text{CHeB}}/\tau_{\text{MS}}$ ratio is increased. The net effect on the total lifetime, advanced phases included, is an increase with rotation.

How does our work compare with the existing studies? We have identical non-rotating models as Gil-Pons et al. (2007, 9 $M_{\odot}$), as Marigo et al. (2001, 9 and 15 $M_{\odot}$), and as Siess et al. (2002, 15 $M_{\odot}$). Gil-Pons et al. (2007) present two models of 9 $M_{\odot}$: one without overshooting, and one with an overshooting parameter calculated with the prescription presented in Eldridge & Tout (2004). The convective boundaries calculated with this prescription would be similar to the ones found using an overshoot parameter $\alpha_{\text{over}} = 0.3 H_P$ (P. Gil-Pons, priv. comm.). Our 9 $M_{\odot}$ model have a MS lifetime that is 21% shorter than their model computed with this high overshooting, which can be simply understood. However, it is also 11% shorter than the lifetime of their model computed without any overshooting. In a previous paper (Gil-Pons et al. 2005), they show the evolution of the convective core as well as the evolution of the central H and He abundances during the MS phase. We notice that the transition from pp -chains to CNO-cycle H burning occurs earlier in their model (4.4 Myr) than in our (10 Myr), and in a much more abrupt way (see their Fig. 4).

The convective core suddenly expands, engulfing a great amount of H, and this allows a longer lifetime. The difference in the time at which the CNO cycle ignites most probably arises from differences in the reaction rates and the equation of state.

The τ_{MS} values we find for the 9 and 15 $M_{\odot}$ are shorter by 12% and 10% respectively than the value found by Marigo et al. (2001). This difference is well explained by the lower overshooting parameter we use. The convective cores of our models are about 10% smaller than theirs and this has a direct consequence on the lifetime.

Compared to the non-rotating 15 $M_{\odot}$ computed by Siess et al. (2002), our model has again a shorter lifetime by about 12%, although the overshooting parameter is the same. In this case, at least a part of the difference arises from the burning of the light elements: in the Geneva code, ${}^7\text{Be}$, ${}^7\text{Li}$ and ${}^8\text{B}$ are supposed at the equilibrium, while in the code used by Siess et al. (2002, STAREVOL), they are explicitly followed. Decressin (2007) shows that the explicit treatment of the light elements leads to longer MS lifetime: in a 3 $M_{\odot}$ model at $Z = 0.020$, this increase amounts to 4%. In this work, a similar model computed with a reaction network that supposes the light elements at the equilibrium has the same lifetime as a model computed with the Geneva code.

4.2.4 Evolution of the equatorial velocity

At the very beginning of the MS, the models experience the adjustment period described in Sect. 3.4.2 on page 43. After this, we have seen in Sect. 3.4.2 that the evolution of the equatorial velocity results from the interplay between the mass loss (which removes angular momentum at the surface) and meridional circulation (which brings angular momentum from the inner parts to the surface). We have also seen (Fig. 3.16 on page 45) that at $Z = 0$, although the mass loss is almost null, the meridional currents are so weak that the stars almost evolve in a regime of local conservation of angular momentum: the slow inflation of the radius spins down the surface. Figure 4.3 (left) shows that this is true for the models with $M < 25 M_{\odot}$. Note that the 9 $M_{\odot}$ model

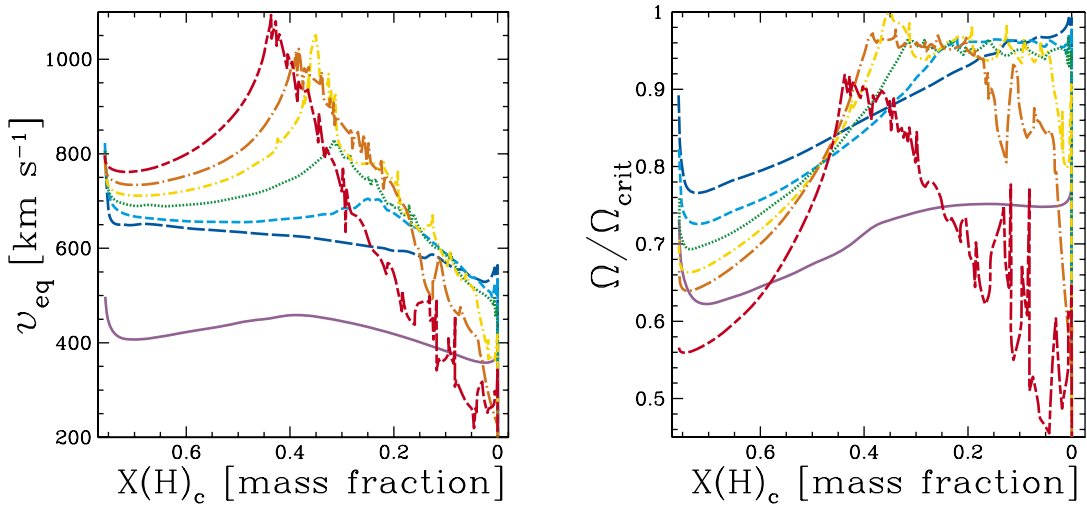


Figure 4.3: *Left*: evolution of v_{eq} normalised to its initial value during the MS for all our rotating models at $Z = 0$: 9 $M_{\odot}$, purple solid line; 15 $M_{\odot}$, blue long-dashed line; 25 $M_{\odot}$, cyan short-dashed line; 40 $M_{\odot}$, green dotted line; 60 $M_{\odot}$, yellow dot-short-dashed line; 85 $M_{\odot}$, orange dot-long-dashed line; 200 $M_{\odot}$, red long-short-dashed line. *Right*: evolution of $\Omega/\Omega_{\text{crit}}$ during the MS for the same models (same colour code).

starts its evolution with a v_{eq} that is accelerating, but this is due to the fact that, at the beginning of the MS, it is powered only through pp -chains, so it is still contracting. At the moment when the CNO cycle ignites, v_{eq} starts decelerating. The higher masses benefit from a stronger meridional-

circulation amplitude, so the surface accelerates during the evolution. The $25 M_{\odot}$ model marks the limit between the two behaviours, with a weak acceleration in the second half of the MS phase.

As shown in Fig. 3.17 on page 46, since v_{crit} decreases during the MS evolution, even a spinning-down model can reach the critical limit. This is exactly what happens to the $15 M_{\odot}$ model, which reaches the critical limit at the end of the MS ($X_c \approx 0.02$) although it has been decreasing during all its evolution. The last column of Table 4.1 on page 61 gives the central H mass fraction when the models reach the critical limit. The higher the mass, the sooner the limit is reached, although the higher masses start with lower initial $\Omega/\Omega_{\text{crit}}$ values (Fig. 4.3, *right*).

Once at the critical limit, all the models remain at the limit. In Fig. 4.3 (*right*), the 85 and especially the $200 M_{\odot}$ models seem to depart from $\Omega/\Omega_{\text{crit}} = 1$. Actually this is because the limit shown here is the Ω -limit, when only the centrifugal force is taken into account to counterbalance gravity. In the two most massive models however, the radiative acceleration is strong and they reach the so-called $\Omega\Gamma$ -limit: when both the centrifugal force and the radiative acceleration are taken into account, the total acceleration term at the surface of a star is:

$$\vec{g}_{\text{tot}} = \vec{g}_{\text{eff}} + \vec{g}_{\text{rad}} = \vec{g} + \vec{a}_{\text{cen}} + \vec{g}_{\text{rad}} \quad (4.1)$$

Maeder & Meynet (2000) show that the equation giving the critical velocity becomes in that case $\vec{g}_{\text{eff}} [1 - \Gamma_{\Omega}(\theta)] = \vec{0}$ where:

$$\Gamma_{\Omega}(\theta) = \frac{\kappa(\theta) L}{4\pi c G M \left(1 - \frac{\Omega^2}{2\pi G \bar{\rho}}\right)} \quad (4.2)$$

is the Eddington factor corrected for rotation (see their Eq. (2.9)). $\vec{g}_{\text{tot}}$ has two roots:

1. $v_{\text{crit},1}$ when $\vec{g}_{\text{eff}} = \vec{0}$, which is our usual definition of v_{crit} for the Ω -limit
2. $v_{\text{crit},2}$ when $\Gamma_{\Omega}(\theta) = 1$, which defines the $\Omega\Gamma$ -limit

For a large Eddington factor, the second root $v_{\text{crit},2}$ becomes lower than $v_{\text{crit},1}$ and the $\Omega\Gamma$ -limit is met first. Since the maximal value for $\frac{\Omega^2}{2\pi G \bar{\rho}}$ is 0.361, this occurs only for Eddington factors larger than 0.639.

The most massive of our models have Eddington factors that become larger than 0.639 in the second part of the MS evolution, so they meet the $\Omega\Gamma$ -limit. The true critical velocity is lowered by the radiative acceleration. Although the $\Omega/\Omega_{\text{crit}}$ ratio plotted in Fig. 4.3 (*right*) becomes lower than 1, these models are actually at the critical limit and remain so until the end of the MS phase.

At central H exhaustion, the moderate inflation of the radius brings the surface of all the models back to sub-critical velocities.

4.2.5 Mass loss and surface abundances

An interesting question arises now: can rotation provide the conditions for a heavy mass loss in stellar models that are supposed to undergo very weak line-driven winds? We recall that Meynet et al. (2006) show that at very low (but non-zero) metallicity, substantial amounts of mass can be lost by two processes:

1. low- Z stars are prone to reach the critical limit during the MS evolution and lose mass by a mechanical process;
2. the rotational mixing is very efficient in low- Z stars, so the surface is enriched, the effective metallicity rises and the line-driven winds are enhanced.

A $60 M_{\odot}$ model with $v_{\text{ini}} = 800 \text{ km s}^{-1}$ at $Z = 10^{-8}$ loses $36 M_{\odot}$ at the end of its evolution, among which $9 M_{\odot}$ are lost by the centrifugal force action during the MS and in a blue loop during CHeB,

Table 4.3: Total mass lost and surface abundances of C, N and O at the end of MS phase and at the end of the evolution for the models that undergo mass loss and surface enrichment. Masses are in $M_{\odot}$, velocities in km s^{-1} and abundances in mass fraction. The C abundance is the sum of ^{12}C and ^{13}C , the N abundance is the abundance in ^{14}N and the O abundance is the sum of ^{16}O , ^{17}O and ^{18}O .

M_{ini}	v_{ini}	end of MS					end of evolution				
		M_{lost}	He	C	N	O	M_{lost}	He	C	N	O
9	500	–	0.2426	4.60e-23	5.91e-21	2.35e-22	–	0.4279	5.50e-17	5.61e-15	1.85e-16
15	800	0.0306	0.2503	1.74e-14	1.81e-12	5.90e-14	0.0418	0.2646	6.36e-09	1.83e-07	3.38e-09
25	800	0.2252	0.2517	4.34e-13	4.02e-11	1.20e-12	0.2513	0.2615	3.53e-10	9.37e-09	1.86e-10
40	800	1.0070	0.2654	2.24e-12	1.90e-10	5.18e-12	2.0074	0.3045	5.17e-07	1.48e-05	6.16e-07
60	0	–	0.2400	0.00e+00	0.00e+00	0.00e+00	0.0001	0.2400	0.00e+00	0.00e+00	0.00e+00
60	800	2.3200	0.2906	6.84e-12	5.44e-10	1.39e-11	2.4061	0.5328	1.23e-05	3.54e-04	3.34e-05
85	0	0.0046	0.2400	0.00e+00	0.00e+00	0.00e+00	0.0681	0.4032	6.40e-04	3.88e-03	1.33e-04
85	800	4.4363	0.3362	1.74e-11	1.31e-09	3.19e-11	10.4239	0.6141	7.29e-06	1.81e-04	4.14e-05
200	0	0.0350	0.2400	0.00e+00	0.00e+00	0.00e+00	0.0509	0.2423	7.93e-13	6.12e-11	1.59e-12
200	800	16.4426	0.5183	8.82e-11	5.94e-09	1.23e-10	20.7609	0.6685	7.53e-05	2.09e-04	1.02e-04

and $27 M_{\odot}$ are lost by the enhancement of the line-driven winds during CHeB. So what happens when the metallicity is as low as strictly zero?

Table 4.3 shows the mass lost at the end of the MS (column 3). The mass lost during this phase is mainly due to the reaching of the critical limit, since the line-driven winds are extremely low or even absent. The total mass lost at the end of the MS phase remains modest in the present models: between $0.03 M_{\odot}$ for the $15 M_{\odot}$ model (which reaches the critical limit at the very end of the MS, when $X_c = 0.02$), and $16.4 M_{\odot}$ for the $200 M_{\odot}$ model (which spends more than half of the MS at the critical limit): the mechanical mass loss removes only the outermost layers, which have a low density. Moreover, the meridional circulation is low (compared to higher- Z stars), so the removal of a small amount of mass easily brings the surface to sub-critical velocity, and some time is needed before it accelerates again. Note that at the end of the MS, the $60 M_{\odot}$ at $Z = 10^{-8}$ from Meynet et al. (2006) has lost $2.376 M_{\odot}$ and ours 2.320 , so the order of magnitude is the same.

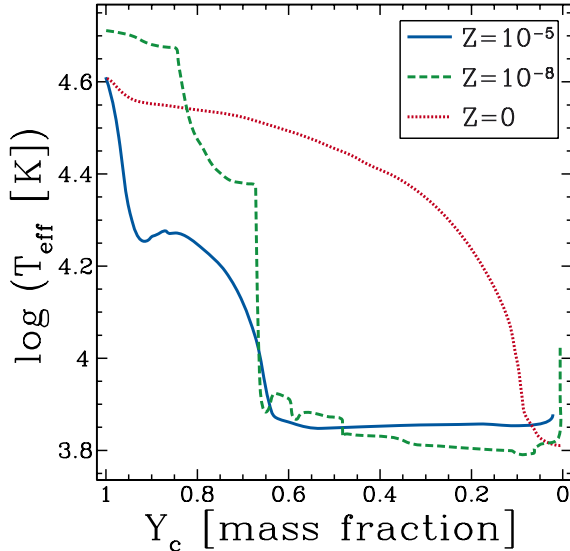


Figure 4.4: Evolution of the T_{eff} during core He burning for models of $60 M_{\odot}$ at various low Z . The $Z \neq 0$ models are taken from Meynet et al. (2006).

The difference between the evolution at extremely low- but non-zero metallicity and that at $Z = 0$ really starts after the MS (Fig. 4.4). We saw (Sect. 4.2.2 on page 62) that the metal-free models remain in the blue side of the HRD after H exhaustion. Most of the CHeB phase is spent with $\log T_{\text{eff}} > 4.0$, so the outer radius remains compact. This has two important consequences:

1. the gradient of Ω between core and surface is not stretched by a quickly inflating radius, so it keeps about the same order of magnitude as during the MS phase: the shear mixing does

not abruptly increase as is the case at higher Z ;

2. the outer convective zone appears late and remains thin, so the dredge-up does not dig deeply enough: most of the post-MS evolution occurs with a surface effective metallicity much lower than 10^{-6} .

As a result, the mass lost during CHeB remains very low: it amounts to at most $6 M_{\odot}$ in the case of the $85 M_{\odot}$ model, which is the model that loses the most mass at this stage. During that phase our $60 M_{\odot}$ model loses less than $0.1 M_{\odot}$ while the $60 M_{\odot}$ model at $Z = 10^{-8}$ from Meynet et al. (2006) loses during that same stage about $33 M_{\odot}$.

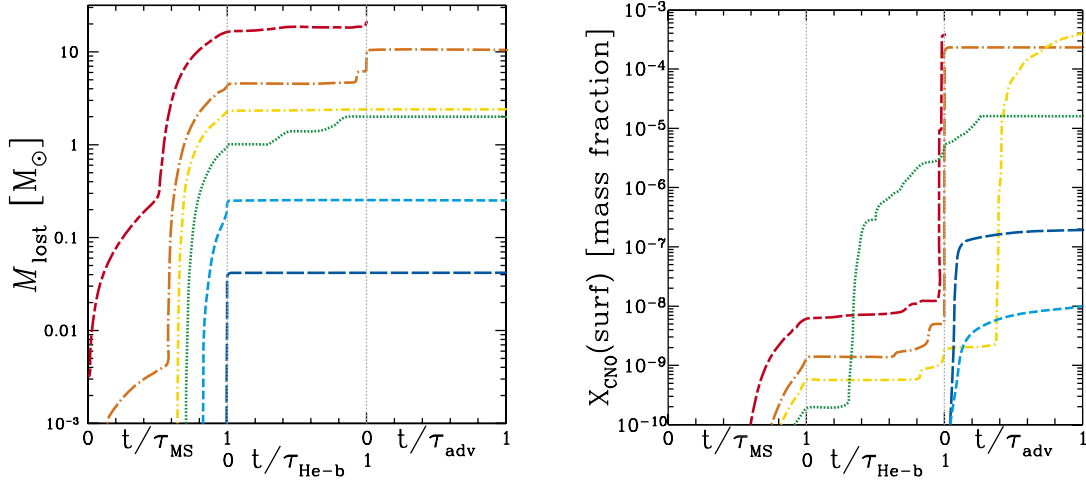


Figure 4.5: *Left*: evolution of the mass loss for all our rotating models at $Z = 0$ (same colour code as Fig. 4.3). The x -axis is a time axis divided in three parts: the fraction of the MS-phase, of the CHeB phase, and then of the advanced phases. *Right*: evolution of the surface C+N+O abundances for the same models (same colour code).

Figure 4.5 shows the evolution of the mass lost by the models (*left*) and their surface C+N+O abundances (*right*) during their whole life. The total mass lost is low for all the models. The advanced phases occur with an effective surface metallicity higher than 10^{-5} for the most massive models, but the duration is too short to play a role in the evolution of the mass. The $200 M_{\odot}$ model loses $20.7 M_{\odot}$, the $85 M_{\odot}$ model $10.4 M_{\odot}$. The models have to be as massive as $40 M_{\odot}$ or above to lose more than $1 M_{\odot}$.

In literature, the only study that does not compute the evolution at constant mass is Marigo et al. (2003). Their masses do not match ours exactly, so we can only make a qualitative comparison. Our non-rotating $200 M_{\odot}$ loses $0.05 M_{\odot}$, which is somewhere between their $120 M_{\odot}$ model ($0.02 M_{\odot}$ lost) and their $250 M_{\odot}$ model ($0.19 M_{\odot}$ lost). Our rotating $200 M_{\odot}$ model experiences a much stronger mass loss ($20.7 M_{\odot}$) than their rotating $250 M_{\odot}$ model ($3.35 M_{\odot}$). This is a factor of 6 times more than their more massive model. Their models have been started with a lower initial velocity, and despite the maximal coupling brought by the rigid-body rotation, they reach the critical limit at the very end of the MS evolution. The mass loss they adopt then ($\dot{M}_{\text{crit}} = 10^{-3} M_{\odot} \text{ yr}^{-1}$) is about two orders of magnitude larger than that resulting of our treatment of the critical mass loss, but the period of time during which they apply it is very short.

4.2.6 Kippenhahn diagrams

Figure 4.6 on page 70 shows the evolution of the structure for the models between 15 and $85 M_{\odot}$ from ZAMS to pre-supernova (pre-SN) stage. The x -axis is $\log(t - \tau_{\text{collapse}})$: the logarithm of the time remaining before core collapse.

Let us first look at the features of the non-rotating models (left column of Fig. 4.6). During the MS (i.e. between $\log(t - \tau_{\text{collapse}}) = 7-6$ in the $15 M_{\odot}$ panel), there is a zone of convective instability outside the H-burning core which is found in all the models. During CHeB, the CNO shell boost is clearly visible in the diagrams of the $25 M_{\odot}$, at the age $\log(t - \tau_{\text{collapse}}) = 5.5$: the H-burning shell suddenly becomes convective while the core is abruptly reduced. The only non-rotating model that develops an outer convective zone (OCZ) during CHeB is the $85 M_{\odot}$ model at an age $\log(t - \tau_{\text{collapse}}) = 4.7$. This OCZ remains thin most of the CHeB, but it suddenly deepens at the end. Although it quickly disappears thereafter, the dredge-up occurs after a strong CNO shell boost which has brought heavy elements very near the surface. Therefore it is the model that shows the largest enrichment level of all our set of models (see the right panel of Fig. 4.5 on previous page).

At standard metallicity, Limongi et al. (2000) find the transition from convective to radiative core C burning between 15 and $20 M_{\odot}$. At $Z = 0.001$ and $Z = 10^{-5}$, Hirschi (2007, Fig. 2) shows that the non-rotating $20 M_{\odot}$ model experiences convective core C burning while the rotating models of the same mass burn their carbon in a radiative core, which puts the limit at $20 M_{\odot}$. In the same work, the $Z = 10^{-8}$ models, both rotating and non-rotating, have convective core C burning, which means that at the lowest metallicities, the limit must be above $20 M_{\odot}$. From Limongi et al. (2001, Fig. 1), we deduce that at $Z = 0$, the transition occurs between 20 and $25 M_{\odot}$. Our models are consistent with this picture: Figure 4.6 shows that the $15 M_{\odot}$ model has a convective core C burning (starting at an age $\log(t - \tau_{\text{collapse}}) = 3$ in the figure) while the $25 M_{\odot}$ model has a radiative core at that phase. The shift of the transition between convective and radiative core C burning to higher mass at lower metallicity can be understood in the following way (see Hirschi et al. 2004): the C-burning core remains radiative if the energy production by nuclear burning is not high enough to compensate the energy lost by the neutrinos. In this case, the central entropy does not increase enough to create a convective zone. Typically, a bigger M_{α} implies a higher T_c during CHeB and thus a lower ^{12}C abundance. The C-burning phase is shorter, because less fuel is available, and does not develop a convective core. In Hirschi (2007), we find that at low Z , the M_{α} are smaller: for the $20 M_{\odot}$ non-rotating² models, $M_{\alpha} = 6.58$ at $Z = 0.001$, 6.24 at $Z = 10^{-5}$, and 4.43 at $Z = 10^{-8}$. The transition between convective and radiative C burning is thus expected to shift to higher mass at low Z .

In the models with $M \gtrsim 40 M_{\odot}$, the C-burning shell is convective until the end of the hydrostatic evolution. The convective C-burning shell even fuses with the convective He-burning shell. Note that the nuclear reaction network of our program is adapted to manage such a mixing.

What are the effects of rotation on the various features described above? Generally, it suppresses the convective instability outside the H-burning core during the MS by smoothing the abundance gradient of helium: more He means a reduction of the opacity, so the zone remains radiative. The CNO shell boost occurs in all the masses. It is well visible in the diagrams of the rotating 15 , 25 , and $40 M_{\odot}$ models, and we discover a similar boost in the 60 and $85 M_{\odot}$ models at the beginning of shell He burning (at $\log(t - \tau_{\text{collapse}}) = 3.5$ and 3.7 respectively). These latter boosts left no signature in the HRD. In the $\log T_c - \log \rho_c$ diagram, only the $85 M_{\odot}$ model shows the characteristic hook, the boost of the $60 M_{\odot}$ model being much weaker. We will see below (Sect. 4.3 on page 72) the impact of the boosts in the chemical abundances profiles.

We see that for the rotating models, below $40 M_{\odot}$ no outer convective zone (OCZ) develops after the MS. The $40 M_{\odot}$ model presents a very thin one, and the $85 M_{\odot}$ models a very early and short-lived one. On the contrary, the $60 M_{\odot}$ model develops a deep OCZ at the end of CHeB and keeps it until the onset of core Si burning. Without surprise, this model shows the largest surface enrichments of all the rotating models (see the right panel of Fig. 4.5).

In the rotating models, the C-burning shell experiences a few flashes, but otherwise it remains

²Since a similar equatorial velocity implies different Ω_{crit} ratio when Z vary, we compare here only the M_{α} of the non-rotating models.

4.2. Evolution at $Z = 0$

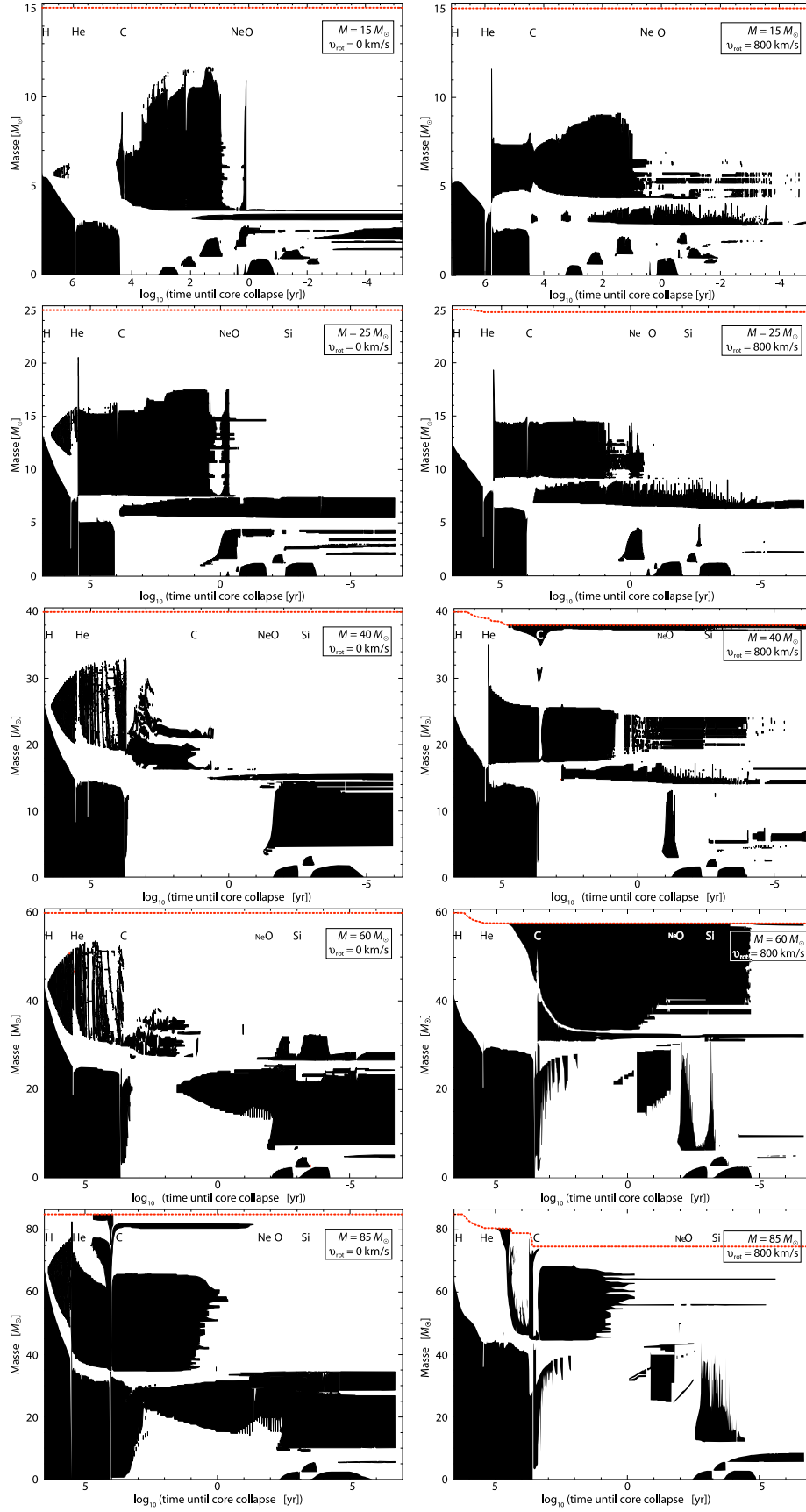


Figure 4.6: Kippenhahn diagrams (evolution of the structure) for the models between 15 and 85 $M_{\odot}$. Left column: without rotation; right column: with rotation. The black areas show the convective zones. The red dotted line at the top of the figures is the total mass of the model. The on-set of the different burning phases is indicated at the top of each window.

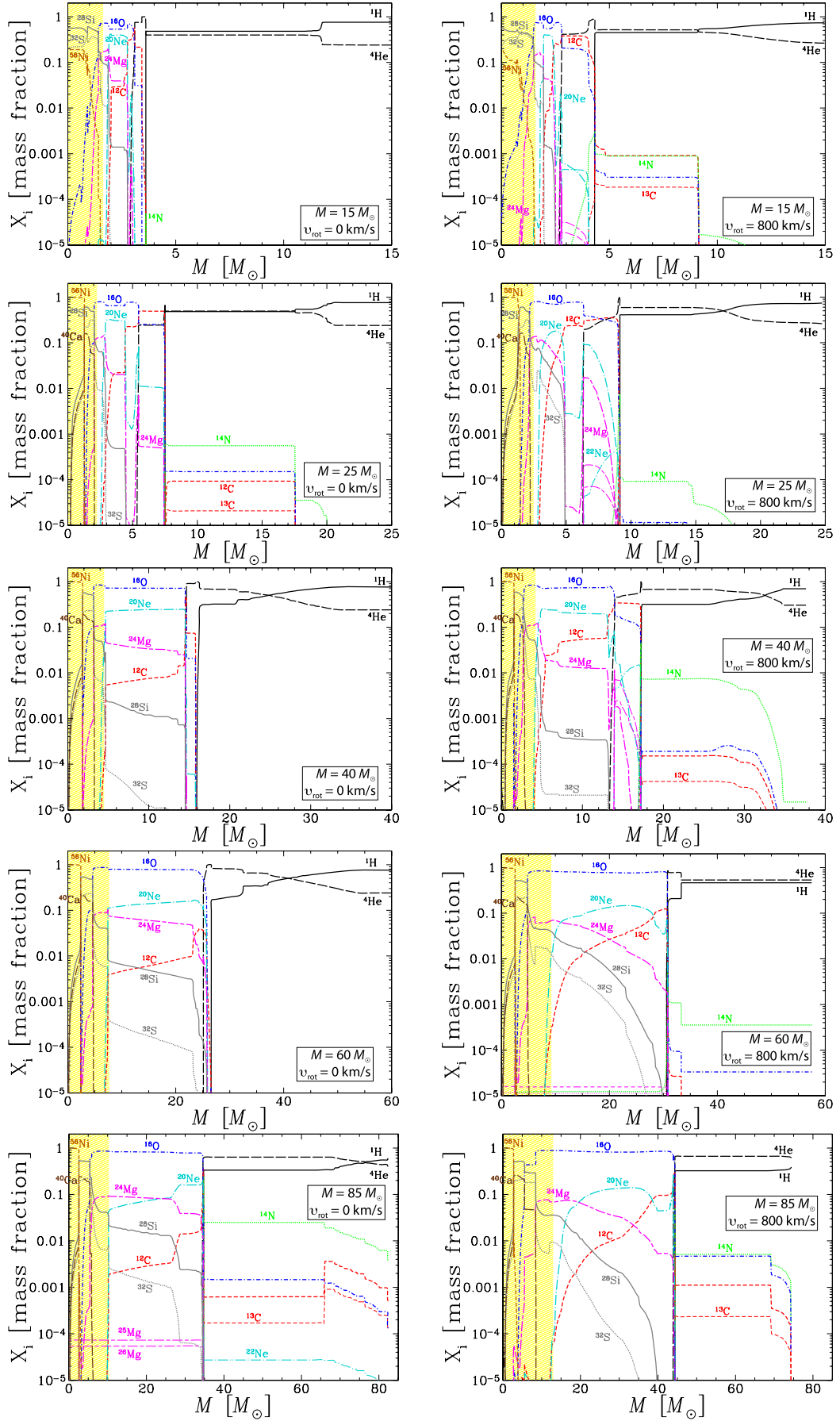


Figure 4.7: Abundances profiles at the pre-SN phase for the 15 to 85 $M_{\odot}$ models. Left column: without rotation; right column: with rotation. The yellow shaded area indicates the part of the star that will be trapped in the remnant.

radiative.

4.3 Final structure and yields

Figure 4.7 on previous page presents the final abundances profiles for the 15 to 85 $M_{\odot}$ models. In the case of the non-rotating models, there is an enormous difference whether the model experienced a H-shell boost or not. This boost determines whether the model will produce some primary nitrogen or not. In the rotating case, all the models experience this boost, although not at the same evolutionary stage: the lowest mass models do so during CHeB, while the most massive ones experience such a boost during shell He burning. As a result, all the rotating models produce ^{14}N . There is no clear difference whether this production occurs during core or shell He burning.

4.3.1 Core mass and yields

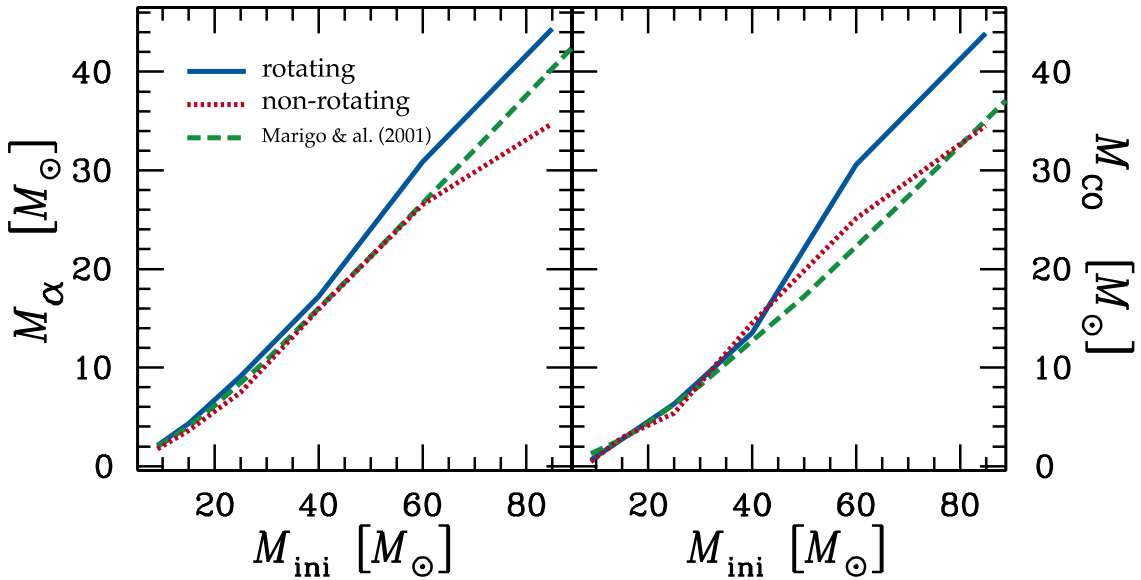


Figure 4.8: Mass of the cores at the end of the evolution (*left*: He-core mass; *right*: CO-core mass) for the rotating (blue solid line) and non-rotating (red dotted line) models. We show also the results obtained by Marigo et al. (2001) at the carbon ignition (green dashed line).

The computation of the yields strongly depends on the CO core mass. Table 4.2 on page 64 gives the values found for our models. We used the same definition of M_{CO} as in Heger et al. (2000a): the mass coordinate at which the abundance of helium drops below 10^{-3} . Figure 4.8 (*right*) shows the values of M_{CO} as a function of the initial mass. Rotation leads to larger CO cores (from 17% for the 25 $M_{\odot}$ models to 37% for the 9 $M_{\odot}$ models), except for the 15 and 40 $M_{\odot}$ models. However, if we take the definition of $M_{\text{CO}}^{\text{int}}$ from Hirschi et al. (2004) which includes the C and O mass produced in the He-burning shell, all the rotating models have larger M_{CO} than the non-rotating ones. A comparison with the value found by Marigo et al. (2001) shows that our non-rotating 40 and 60 $M_{\odot}$ models have slightly larger CO cores, but otherwise the differences between our two works are very small. The M_{CO} have not been determined at the same evolutionary stage, since our models are followed until the advanced phases while those of Marigo et al. (2001) are stopped at C ignition. As we will see in Sect. 4.3.2 on page 76, M_{CO} increases during C burning, so we would expect all our M_{CO} to be larger. However, the overshoot parameter used by Marigo et al. (2001) is larger than the one we use, which compensates for the difference.

Table 4.4: Total stellar yields at the pre-supernova stage, in $M_\odot$. In parenthesis, the “wind-only” contribution is given for the models for which no SN contribution is expected (see Sect. 4.3.4). The masses of the models are in $M_\odot$ and the velocities in km s^{-1}

M_{ini}	v_{ini}	$\frac{{}^3\text{He}}{10^{-4}}$	${}^4\text{He}$	${}^{12}\text{C}$	${}^{13}\text{C}$	${}^{14}\text{N}$	${}^{16}\text{O}$	${}^{17}\text{O}$	${}^{18}\text{O}$	${}^{22}\text{Ne}$	Z	$\Delta Y/\Delta Z$
9	0	-0.53	1.17	0.19	5.54e-11	2.68e-8	0.24	2.74e-11	4.45e-9	1.58e-7	0.43	2.72
9	500	-0.65	1.59	0.24	3.27e-5	1.81e-3	0.17	2.20e-7	8.20e-6	8.98e-6	0.41	3.88
15	0	-1.02	1.24	0.22	5.94e-8	5.13e-6	0.86	1.93e-10	2.68e-9	8.73e-7	2.53	0.49
15	800	-1.23	2.10	0.61	9.21e-4	4.99e-3	1.05	8.88e-7	7.17e-6	5.44e-4	2.10	1.00
25	0	-1.97	2.32	1.26	2.09e-4	5.69e-3	2.78	1.86e-6	1.79e-9	3.27e-6	4.93	0.47
25	800	-2.13	2.57	1.32	3.21e-5	1.08e-3	3.76	2.75e-7	7.78e-6	5.54e-4	5.75	0.45
		(-0.02)	(0.003)	(5e-14)	(1.59e-14)	(6.86e-12)	(2e-13)	(1.51e-15)	(6.51e-19)	(1.01e-26)	(7e-12)	
40	0	-3.21	2.94	0.23	4.85e-8	3.09e-7	7.25	2.29e-9	1.55e-8	3.63e-6	10.2	0.29
		(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	
40	800	-3.32	5.32	1.63	8.41e-4	1.00e-1	7.57	4.04e-6	6.74e-5	3.30e-2	11.4	0.47
		(-0.17)	(0.05)	(3e-8)	(7.96e-9)	(1.07e-6)	(4e-08)	(5.89e-11)	(5.43e-14)	(2.64e-11)	(1e-6)	
60	0	-4.87	4.58	0.18	9.07e-7	2.58e-6	13.6	1.69e-8	2.26e-7	1.89e-6	17.7	0.26
		(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	
60	800	-4.97	3.37	0.79	2.39e-4	1.20e-2	16.9	1.89e-5	1.33e-4	5.24e-5	21.7	0.16
		(-0.21)	(0.04)	(2e-8)	(7.10e-9)	(9.00e-7)	(9e-8)	(1.94e-10)	(8.18e-14)	(4.12e-11)	(1e-5)	
85	0	-7.22	11.70	0.28	1.46e-2	9.87e-1	19.7	6.63e-5	1.17e-7	1.18e-3	25.5	0.46
		(-0.004)	(0.01)	(3e-5)	(8.20e-6)	(2.42e-4)	(8e-6)	(7.26e-9)	(1.50e-11)	(3.99e-7)	(3e-4)	
85	800	-7.11	7.09	1.00	6.44e-3	1.40e-1	26.2	1.24e-4	2.32e-7	6.47e-6	31.6	0.22

From M_{CO} , we deduce the remnant mass M_{rem} by the relation between M_{CO} and M_{rem} established by Maeder (1992) on the basis of the residual baryons masses given by Wilson et al. (1985), Woosley (1986) and Thielemann et al. (1992).

The total stellar yields are the sum of the winds yields (mp_{im}^{winds}) and the pre-SN yields (mp_{im}^{preSN}). The pre-SN contribution from a star of initial mass m to the stellar yields of the element i is:

$$mp_{im}^{\text{preSN}} = \int_{M_{\text{rem}}}^{M_{\text{fin}}} [X_i(m_r) - X_i^0] dm_r \quad (4.3)$$

with X_i^0 the initial abundance (in mass fraction) of element i ($X_i^0 = 0$ for all elements except H and He), and $X_i(m_r)$ its final abundance at the Lagrangian coordinate m_r . Eq. (4.3) should be used for the elements that are only produced in the star, and whose initial abundance is not significant compared to the production. Helium is both produced and destroyed, and its initial abundance is not negligible compared to its production, so for this element we should use a more generic formula, proposed by Maeder (1981):

$$mp_{im}^{\text{preSN}} = \int_{M_{\text{rem}}}^{M_{\text{fin}}} X_i(m_r) \frac{X_{i,\text{max}}(m_r) - X_i^0}{X_{i,\text{max}}(m_r)} dm_r \quad (4.4)$$

where $X_{i,\text{max}}(m_r)$ is the maximal abundance reached for element i in the coordinate m_r . The latter formula gives the true stellar production of element i : it accounts for the fact that if element i is partly destroyed during the evolution, the destruction engulfs both the initial abundance and the new production. In this work, we have computed the He yields with Eq. (4.3), so in this case, the value we obtain do not represent the true production of the star, but the net change of He mass between the initial gas composition and the ejecta.

The wind contribution from the same star to the same element yields is:

$$mp_{im}^{\text{winds}} = \int_0^{\tau(m)} \dot{M}(m, t) [X_i^S(m, t) - X_i^0] dt \quad (4.5)$$

where $\tau(m)$ is the total lifetime of the star, $\dot{M}(m, t)$ the mass loss rate when the age of the star is t , and $X_i^S(m, t)$ the surface abundance (in mass fraction) of element i at age t . Most of the time, the stellar yields are dominated by the pre-SN contribution.

Table 4.4 presents the total stellar yields for the light elements which will not be significantly modified by the explosion (see Limongi & Chieffi 2002). We did not include the $200 M_{\odot}$ models because they did not undergo shell He burning, which noticeably modifies the results. The $9 M_{\odot}$ models were also stopped before central C burning, because of a strongly degenerate core at C ignition, which could lead to the total destruction of the star without further processing. However, the C ignition occurs at the edge of the core, so the star may survive the C burning. In that case, the star is supposed to collapse in a Type II supernova by electrons capture on a degenerate O-Ne-Mg core, because the mass included below the He-burning shell is lower than the limiting mass for Ne burning ($1.37 M_{\odot}$, see Nomoto 1984). In the case of the $15 M_{\odot}$ models, the evolution was stopped at the end of central O burning. According to Nomoto (1984), the star is expected to undergo an off-centre Si flash and to end with an Fe core because the mass inside the C-burning shell is higher than the Chandrasekhar mass. We will see in Sect. 4.3.2 on page 76 how the fact we did not follow the evolution sequence until the very end may affect the stellar yields computed for these two models.

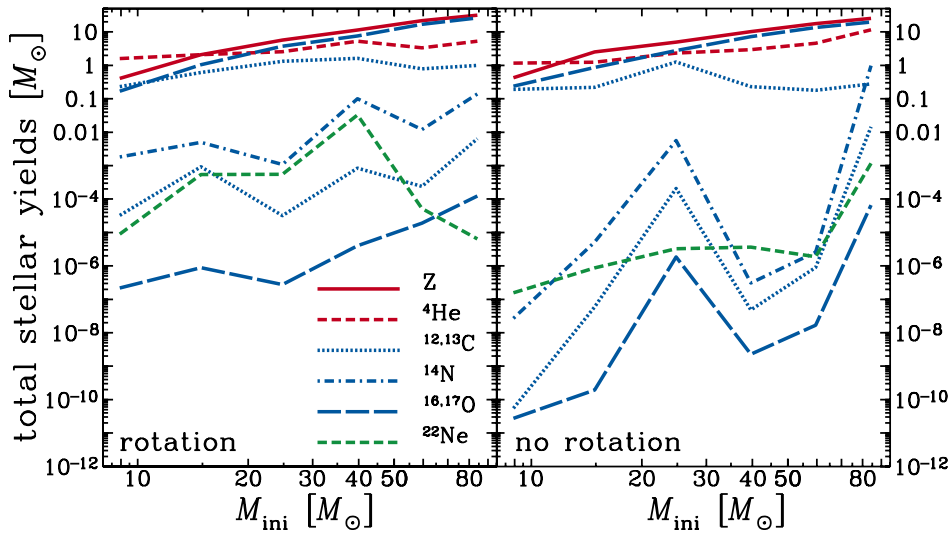


Figure 4.9: Yields of the light elements for the rotating (*left*) and non-rotating (*right*) models at $Z = 0$.

Rotation leads to larger yields for almost all the isotopes and for almost all the masses (Fig. 4.9). The exceptions are the non-rotating models that undergo a strong CNO shell boost (see page 63): the 25 and $85 M_{\odot}$ models produce larger amounts of ^{13}C and ^{14}N than their rotating counterparts. The $25 M_{\odot}$ model produces also more ^{17}O than the rotating one, and the $85 M_{\odot}$ model overtakes its rotating counterpart in the production of ^4He and ^{22}Ne . Otherwise, the enhancement of the yields brought by the rotational mixing can reach orders of magnitude, especially so for the production of primary ^{14}N : up to a factor of 10^6 in the case of the $40 M_{\odot}$ models. An important point is that rotation makes the production of primary nitrogen robust through the whole mass range, while there are enormous differences from mass to mass in the case without rotation. We recall also that, as shown by Meynet & Maeder (2002), rotation allows for primary nitrogen production in a wider range of metallicity.

Figure 4.10 shows a comparison of our CNO yields with those obtained for non-rotating models by Chieffi & Limongi 2004 (hereafter CL04) and Tominaga et al. 2007b (hereafter TUN07). Those are explosive yields³, but the elements we consider here are not supposed to be strongly affected

³CL04 compute them with the radiation-dominated shock approximation (see Chieffi & Limongi 2002, for the description of this explosion mechanism), and with an explosion energy $E_{\text{expl}} = 1.2 E_{51}$. TUN07 use the thermal bomb mechanism: the models with $M < 20 M_{\odot}$ have $E_{\text{expl}} = 1 E_{51}$, while the models with $M \geq 20 M_{\odot}$ are exploded as hypernovae (i.e. with $E_{\text{expl}} \geq 10 E_{51}$).

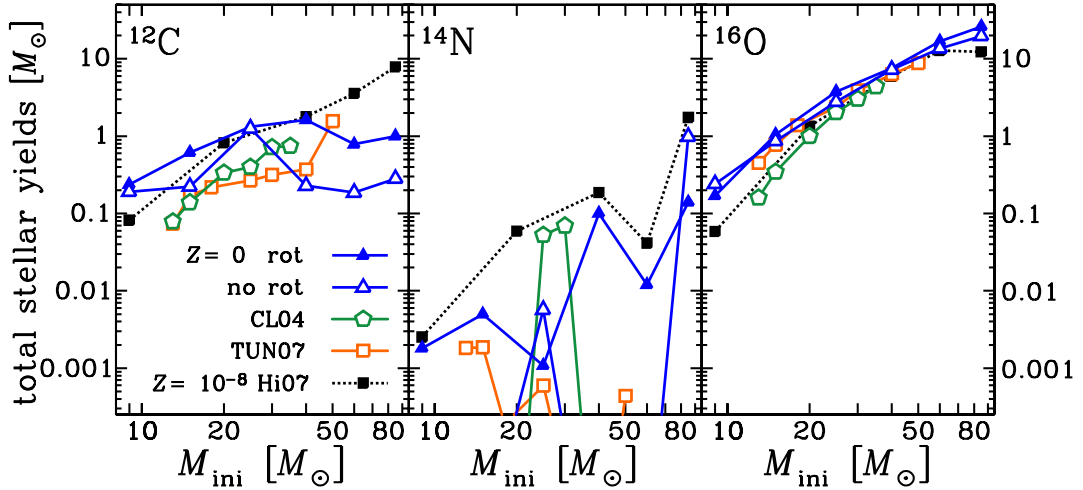


Figure 4.10: Yields comparison of our $Z = 0$ rotating (blue filled triangles) and non-rotating (blue empty triangles) models, with the non-rotating $Z = 0$ from Chieffi & Limongi (2004, CL04, green empty pentagons), from Tominaga et al. (2007b, TUN07, orange empty squares) and the rotating $Z = 10^{-8}$ models from Hirschi (2007, Hi07, black filled squares).

by the explosion. Our ^{12}C yields (*left*) compare within a factor of 3 with those from CL04 for the masses we have in common (15 and $25 M_{\odot}$), and within a factor of 4 with those from TUN07 (15, 25 and $40 M_{\odot}$ models). The $50 M_{\odot}$ model from TUN07 has a ^{12}C production that is a factor of 8 higher than our non-rotating $60 M_{\odot}$ model. The central panel of Fig. 4.10 shows the yields in ^{14}N , and well illustrates the mass-to-mass variations of the ^{14}N production in the non-rotating primordial models. CL04 find a non-negligible ^{14}N production for only two models, the $25 M_{\odot}$ and the $30 M_{\odot}$ models, which can be compared with the production of our $25 M_{\odot}$ model, but with a factor of almost 10 higher. This tends to confirm the CNO shell boost as a robust feature in this mass domain. The differences between the two sets of explosive yields are large. On the contrary, there is a good agreement in the ^{16}O yields of the three studies: they fall within a factor of 2.5 from each other. The ^{16}O yields are directly linked to M_{CO} , so if the models use similar physics and convection criterion, they should have similar yields. We can take the present result as a consistency check for our models.

On the same figure, we have also plotted the pre-SN yields obtained by Hirschi 2007 (hereafter Hi07) for $Z = 10^{-8}$ rotating models. Compared to the yields at $Z = 10^{-8}$, our 60 and $85 M_{\odot}$ models underproduce ^{12}C , while the lower masses overproduce it. The difference can be linked with the CNO shell boost: the lowest masses models undergo this boost during CHeB, which reduces their He-burning core and leads to a higher ^{12}C production. The ^{16}O yields for our $Z = 0$ models are also slightly larger than the one found at $Z = 10^{-8}$. This could be linked to our higher overshooting parameter, which leads to larger CO cores and thus a larger ^{16}O production.

An interesting point is the primary nitrogen production at extremely low metallicity. Meynet & Maeder (2002) show that this production increases when the metallicity decreases, and Hi07 confirms this trend. The primary nitrogen production depends strongly on the efficiency of the mixing, and we saw in Sect. 3.4.3 on page 49 how mixing changes with Z . We should thus expect a still higher production at $Z = 0$, however Fig. 4.10 (*centre*) shows that this is not the case. ^{14}N is systematically underproduced at $Z = 0$ compared to $Z = 10^{-8}$. In fact, this can be understood in view of the behaviour we described in Sect. 4.2.2 on page 62: the MS evolution at $Z = 0$ occurs at temperatures which allow some He burning, so the transition between H and He burning is smooth, the star remains in the blue part of the HRD, or if it crosses the HRD, it does so very slowly. Without a rapid core contraction and envelope expansion, the gradient of angular velocity at the border of the He-burning core remains shallow, so it triggers only little mixing. On the

contrary, when even only a small amount of metals is present, the transition between H and He burning is accompanied by a contraction of the core and a quick redwards evolution. Therefore, a steep angular velocity gradient is built at the border of the core, which triggers strong mixing, and enhances the ^{14}N production. On Fig. 4.10, we see that there is even two orders of magnitude between the yields of the $25 M_{\odot}$ model at $Z = 0$ and those of the $20 M_{\odot}$ model at $Z = 10^{-8}$. This can be explained as follows: the $Z = 10^{-8}$ model experiences a second CNO shell boost during shell He burning, which enhances its ^{14}N production by about two orders of magnitude.

4.3.2 Cores and yields at different burning stages

Often, stellar evolution calculations are stopped at the end of central He or C burning. If yields were computed at those stages, would they be close to the pre-SN yields determined at the end of the hydrostatic evolution or should we correct them by a large factor? How do the core masses and the yields vary from one burning stage to the other? To answer these questions, we have determined M_{α} and M_{CO} , and then the ejected masses of some elements at the end of each subsequent burning stage for the 15 to $85 M_{\odot}$ models. The ejected masses are calculated as mp_{im}^{preSN} (Eq. (4.3) on page 73), except that we do not subtract the initial abundance of the elements. The results for the rotating models are given in Table 4.5 on next page.

Core mass

While M_{α} changes by at most 5% after He exhaustion and gets its final value after core C burning, M_{CO} changes until the very end of the evolution and can gain up to 22% of its mass.

H, He

During the advanced phases, the evolution accelerates, so H and He have very little time to be processed further. After core C burning, the final value is reached for the two elements.

Carbon

^{12}C is both produced and destroyed in the He-burning zones, and mainly destroyed in the later reaction chains: its values are thus reduced between He exhaustion and pre-SN phase up to a factor of 3.

Nitrogen

^{14}N has a complicated history: at $Z = 0$, it depends strongly on the efficiency of the ^{12}C mixing between H- and He-burning zones for its production, and it is then destroyed in the He-combustion reactions. During central C burning, the amount of ^{14}N can change in an unpredictable way (increase or decrease), and with a great variation of amplitude: from 30% up to a factor of 22 in the case of the $60 M_{\odot}$ model. This latter value is the signature of the late shell boost we discovered in Fig. 4.6 on page 70. After C exhaustion, the remaining lifetime is so short in the models with $M \geq 40 M_{\odot}$ that the values remain the same until the end of the hydrostatic evolution. In the 15 and $25 M_{\odot}$ models, it is slightly decreased by the shell He burning, but by no more than 7%.

Oxygen

^{16}O is strongly produced during He burning, slightly consumed during C burning, and produced again during Ne burning (before being then of course destroyed during O burning). Once again, in the case of the 15 and $25 M_{\odot}$ models, the lifetime of the final burning stages is long enough to allow a net ^{16}O production thanks to the He-burning shell, so the value is increased by 6-10% between He exhaustion and pre-SN stage. In the most massive models, the remnant traps the inner parts up to most of the Ne-burning shell, and the lifetime of the final stages is too short to allow a

Table 4.5: Cores size and ejected masses of various elements computed at different burning stages for the rotating 15 to 85 $M_{\odot}$ models. The masses are in $M_{\odot}$.

stage	M_{α}	M_{CO}	H	${}^4\text{He}$	${}^3\text{He}/10^{-5}$	${}^{12}\text{C}$	${}^{13}\text{C}$	${}^{14}\text{N}$	ejected masses					${}^{16}\text{O}$	${}^{17}\text{O}$	${}^{18}\text{O}$	${}^{20}\text{Ne}$	${}^{22}\text{Ne}$	Z
									${}^{15}\text{N}$	${}^{15}\text{N}$	${}^{15}\text{N}$	${}^{15}\text{N}$							
									15	$M_{\odot}$									
end He-b	4.551	2.214	6.0970	5.5755	1.1075	0.88093	6.4906E-05	2.0347E-03	5.4202E-08	0.9520	2.500E-07	2.761E-06	3.88E-04	1.043E-05	1.8355				
end C-b	4.322	2.675	5.9657	5.3821	1.0883	0.71082	0.1967E-04	5.1366E-03	1.6915E-07	0.9808	9.160E-07	1.447E-05	0.19800	4.090E-04	2.0384				
end Ne-b	4.322	2.675	5.9657	5.3675	1.0882	0.70981	0.2091E-04	5.1183E-03	1.6924E-07	0.9909	9.128E-07	1.256E-05	0.20306	4.334E-04	2.0529				
end O-b	4.312	2.698	5.9653	5.3114	1.0882	0.61298	0.2151E-04	4.9882E-03	1.6834E-07	1.0484	8.878E-07	7.176E-06	0.18782	5.443E-04	2.1002				
									25	$M_{\odot}$									
end He-b	9.171	6.000	8.6001	8.3887	0.9602	1.8106	8.7965E-05	1.5480E-03	4.0073E-08	3.5272	4.119E-07	1.352E-08	0.0104	9.166E-06	5.3424				
end C-b	9.141	6.160	8.5904	8.0961	0.9602	1.5767	4.1472E-05	1.1647E-03	5.0456E-08	3.6133	3.100E-07	6.300E-06	0.3978	1.038E-03	5.6043				
end Ne-b	9.141	6.160	8.5904	8.0716	0.0960	1.4151	3.8973E-05	1.1291E-03	4.9641E-08	3.5435	2.932E-07	6.943E-07	0.6339	1.046E-03	5.6288				
end O-b	9.141	6.277	8.5903	8.0195	0.9602	1.3852	3.5485E-05	1.0888E-03	4.8653E-08	3.5470	2.773E-07	2.739E-06	0.6569	7.647E-04	5.6517				
end Si-b	9.141	6.302	8.5903	7.9162	0.9602	1.3178	3.2072E-05	1.0819E-03	4.6941E-08	3.7584	2.752E-07	7.778E-06	0.3671	5.544E-04	5.7492				
									40	$M_{\odot}$									
end He-b	17.210	12.775	8.9457	13.628	0.5226	2.8553	4.3196E-03	1.3333E-01	3.9991E-06	8.3186	7.779E-06	4.732E-08	0.0906	4.528E-05	11.380				
end C-b	17.220	13.112	8.9380	13.515	0.5229	2.8651	8.5001E-04	1.0053E-01	4.1354E-06	8.3113	4.094E-06	6.686E-05	0.1172	4.619E-02	11.421				
end Ne-b	17.220	13.266	8.9380	13.493	0.5229	2.7472	8.5003E-04	1.0053E-01	4.1255E-06	8.2213	4.043E-06	6.532E-05	0.3032	4.389E-02	11.407				
end O-b	17.220	13.381	8.9380	13.454	0.5229	1.6857	8.4137E-04	1.0053E-01	4.1145E-06	7.5799	4.043E-06	6.735E-05	1.8995	3.842E-02	11.419				
end Si-b	17.220	13.491	8.9380	13.423	0.5229	1.6308	8.4129E-04	1.0053E-01	4.1031E-06	7.5727	4.043E-06	6.737E-05	1.8715	3.306E-02	11.415				
									60	$M_{\odot}$									
end He-b	31.326	28.451	11.978	15.824	0.7321	2.8075	1.6737E-04	5.4202E-04	2.1193E-08	17.714	3.829E-07	5.004E-08	0.5745	1.506E-05	21.143				
end C-b	30.870	30.358	11.731	14.940	0.7202	2.7023	2.3901E-04	1.2045E-02	3.5635E-07	17.535	1.867E-05	1.338E-04	0.5777	5.262E-05	21.728				
end Ne-b	30.870	30.358	11.731	14.940	0.7202	2.6932	2.3916E-04	1.2045E-02	3.5507E-07	17.529	1.870E-05	1.338E-04	0.5925	5.262E-05	21.728				
end O-b	30.862	30.708	11.730	14.938	0.7202	0.9261	2.3908E-04	1.2054E-02	3.4084E-07	16.455	1.876E-05	1.332E-04	3.0658	5.238E-05	21.633				
end Si-b	30.862	30.695	11.729	14.933	0.7202	0.7885	2.3939E-04	1.2059E-02	3.4137E-07	16.897	1.890E-05	1.334E-04	2.2056	5.247E-05	21.675				
									85	$M_{\odot}$									
end He-b	44.602	40.232	9.8894	20.169	0.0063	3.2866	2.9808E-03	9.2941E-02	2.8675E-06	27.624	6.921E-05	2.729E-07	1.3807	6.390E-06	32.537				
end C-b	44.377	43.101	9.8553	20.105	0.0068	3.1342	6.4361E-03	1.3996E-01	4.6453E-06	26.937	1.242E-04	2.326E-07	1.4097	6.479E-06	31.838				
end Ne-b	44.377	43.202	9.8553	20.104	0.0068	3.1273	6.4366E-03	1.3997E-01	4.6442E-06	26.910	1.242E-04	2.326E-07	1.4154	6.478E-06	31.812				
end O-b	44.377	43.922	9.8553	20.090	0.0068	1.4874	6.4372E-03	1.3997E-01	4.6414E-06	25.772	1.242E-04	2.325E-07	3.7658	6.469E-06	31.626				
end Si-b	44.377	43.922	9.8553	20.090	0.0068	0.9996	6.4374E-03	1.3997E-01	4.6409E-06	26.236	1.242E-04	2.324E-07	2.7772	6.469E-06	31.626				

significant ^{16}O production by the He-burning shell, so the ejected mass in ^{16}O is reduced by 5-10%.

Heavy CNO isotopes

The heavy isotopes of the CNO elements (^{13}C , ^{15}N , $^{17,18}\text{O}$) have abundances that are mainly determined by the conditions at which the CNO cycle operates in the H-burning shell. The equilibrium abundances change with the temperature, and so does the time needed for the CNO cycle to be at the equilibrium. The variations for these isotopes are very unpredictable. The amplitude of the variations is large between He and C exhaustion, but after C exhaustion the values of the ejected masses vary by no more than about 30%.

Neon

There is a small production of ^{20}Ne during central He burning, but it is mainly produced during core C burning. The ejected mass is thus significantly increased after He exhaustion. It is then steadily increasing through all the advanced phases, except during the Si burning. It could be surprising at first that after central Ne burning the ejected Ne mass is slightly higher than before, but the Ne-exhausted core is trapped inside the remnant, so only the production by the C-burning shell is ejected. In the $M \geq 40 M_{\odot}$, the variation of the amount of ^{20}Ne ejected can vary by a factor of 2 to 15 between the end of core C burning and the pre-SN stage. Note that this element is supposed to be affected by the explosive processing, so the preSN yields are anyway subject to modifications.

Heavy Ne isotope

The heavy isotope ^{22}Ne is not followed in the reaction chains of the advanced burning phases: our network for these reactions includes only α -elements⁴. The ejected mass for ^{22}Ne presented in Table 4.5 shows thus the effects of the shell He burning on this isotope. In this case, it is mainly produced by $^{14}\text{N}(\alpha, \gamma)^{18}\text{F}(e^+ + \nu_e)^{18}\text{O}(\alpha, \gamma)^{22}\text{Ne}$, but at $T \approx 10^9$ K, the reaction $^{22}\text{Ne}(\alpha, n)^{25}\text{Mg}$ starts to be very efficient and destroys it. In all the models, the amount of ^{22}Ne ejected increases during core C burning, by a factor that can be very large (between 40 to 1000) in the models with $M \leq 40 M_{\odot}$. After Ne burning, the variations are much smaller, less than 40% except in the case of the $25 M_{\odot}$ models which decreases its ^{22}Ne production by 85% between core O burning and central Si exhaustion.

Total metals Z

The total metals amount is modified by the variation of the He amount and by the variation in the M_{CO} (which determines the remnant mass). The Z variation is higher for the lowest masses (14% for the $9 M_{\odot}$ model, 7% for the $15 M_{\odot}$ model) than for the highest masses (at most 3%).

What can we conclude about this study? Table 4.6 gives the burning stages that are required to estimate the ejected mass with an accuracy better than 20% for each element and for each mass. Generally, the shell He-burning phase is really necessary to get values that are significantly close to the ones which would be obtained in a complete evolution sequence. Note that a significant part of shell He burning occurs during the contraction before C ignition, so taking the yields at the beginning of core C burning would give a much better estimate than at the end of CHeB.

- The mass ejected in H and He would be overestimated if calculated only at He exhaustion, however by no more than 2% and 6% respectively.

⁴The elements followed in the network for the advanced stages are: α , ^{12}C , ^{16}O , ^{20}Ne , ^{24}Mg , ^{28}Si , ^{32}S , ^{36}Ar , ^{40}Ca , ^{44}Ti , ^{48}Cr , ^{52}Fe , and ^{56}Ni . The α -elements are the only ones which are really required in the late stage of the evolution to accurately follow the energy generation, and thus to correctly define the structure of our models. Our code uses a smaller network in the C, Ne, O, and Si burning zones, so as to reduce the time cost of the calculation (see Hirschi 2004, for more details).

Table 4.6: Burning stage required to get a value of the ejected mass close to the preSN value with an accuracy better than 20% for a given element (rotating models). Note that for the $15 M_{\odot}$ model, the values are compared to the one obtained at the end of O-b. If only the atom is written, it means that all the isotopes give the same results.

Burning stage	Mass [$M_{\odot}$]				
	15	25	40	60	85
end He-b	H	H	H	H	H
	He	He	He	He	He
		^{15}N	^{15}N		
	^{16}O	^{16}O	^{16}O	^{16}O	$^{16,18}\text{O}$ ^{22}Ne
end C-b	C	^{12}C			
			^{13}C	^{13}C	^{13}C
	N	^{14}N	^{14}N	N	N
	^{17}O	^{17}O	$^{17,18}\text{O}$	^{17}O	^{17}O
	^{20}Ne			^{22}Ne	
end O-b		^{13}C	^{12}C	^{12}C	
	^{18}O				
	^{22}Ne		Ne		
end Si-b					^{12}C
		^{18}O			
		Ne		^{20}Ne	^{20}Ne

- The mass ejected in ^{12}C calculated at He exhaustion would be overestimated by at most a factor of 3.
- The ^{14}N production is unpredictable before central C exhaustion, because of the possibility of going through a shell boost. After that phase, it would be overestimated compared to the pre-SN stage, but by no more than 7%.
- The mass ejected in ^{16}O calculated at He exhaustion would be either underestimated or overestimated (depending on whether the model has $M \leq 25 M_{\odot}$ or $M \geq 40 M_{\odot}$ respectively) by 5-10%.
- The heavy CNO isotopes production is unpredictable before the end of C burning. After that, it is quite stable in the models with $M \geq 40 M_{\odot}$ while the variations of the lower mass models are still higher (but no more than 30%).
- During core C burning, the ejected mass of ^{22}Ne can increase by orders of magnitude but after C exhaustion it would be overestimated by at most 85%.
- The variations in the ejected mass of ^{20}Ne are large, however those isotopes are supposed to be modified by the explosion, so even the pre-SN results should be corrected.
- In the case of the 9 and $15 M_{\odot}$ models, the total mass ejected in metals would be underestimated by 14% and 7% respectively if it were calculated at He exhaustion. After central C burning, it would be underestimated by less than 3%. For the higher mass models, the variations amount at most to 3%. The $85 M_{\odot}$ model is the only one for which the Z amount would be overestimated.

4.3.3 $\Delta Y / \Delta Z$

An interesting quantity is the helium-to-metal production ratio $\Delta Y / \Delta Z$. The last column of Table 4.4 gives this result for all the models. Figure 4.9 on page 74 shows that the curve of the He

4.3. Final structure and yields

Table 4.7: $\Delta Y/\Delta Z$ obtained by Eq. (4.6) for three different IMF: 1) the standard IMF from Salpeter 1955 (hereafter S55); 2) the IMF from Miller & Scalo 1979 (hereafter MS79), where the transition between the two power-laws is set at $10 M_{\odot}$; 3) the bi-modal IMF from Nakamura & Umemura 2001 (hereafter NU01), where we have set the first peak mass at $1 M_{\odot}$, and the second peak mass at $50 M_{\odot}$ or $100 M_{\odot}$. M_{down} and M_{up} are the bounding masses for the convolution, expressed in $M_{\odot}$. We also computed the $\Delta Y/\Delta Z$ obtained with the rotating models at $Z = 10^{-8}$ from Hirschi (2007).

Models	M_{down}	M_{up}	S55	MS79	IMF	
					NU01 (1 - 50 $M_{\odot}$)	NU01 (1 - 100 $M_{\odot}$)
$Z = 0$, no rotation	9	85	0.646	0.941	0.328	0.646
	9	120	0.643	0.929	0.393	0.649
	9	250	0.661	0.935	0.546	0.943
$Z = 0$, rotation	9	85	0.910	1.330	0.227	0.910
	9	120	0.880	1.286	0.237	0.336
	9	250	0.857	1.246	0.281	0.399
$Z = 10^{-8}$, rotation	9	85	1.633	2.376		
	9	120	1.586	2.296		

production as a function of the initial mass is flatter than that of the total metals production. Only the lowest mass model ($9 M_{\odot}$) produces a greater amount of He, whereas the more massive models produce more metals. In the case of the rotating models, the $15 M_{\odot}$ model is at the crossing of the two curves exactly, while the crossing is located at slightly lower mass in the non-rotating case. This means that for most of the models, $\Delta Y/\Delta Z$ will be smaller than 1. In comparison, the results for the fast rotating models at $Z = 10^{-8}$ from Hirschi (2007) show that the crossing is located at higher mass, somewhere between 20 and $40 M_{\odot}$. We can understand this difference as the result of the efficiency of mixing at “higher” metallicity: the boost of the H-burning shell is more efficient, so the He core is reduced and so are the metal yields.

We have computed the total $\Delta Y/\Delta Z$ resulting from a generation of Pop III stars. To do so, we have convolved the individual ΔY and ΔZ with an IMF, as follows:

$$\frac{\Delta Y}{\Delta Z} = \frac{\int_{M_{\text{down}}}^{M_{\text{up}}} \Delta Y \Phi(M) dM}{\int_{M_{\text{down}}}^{M_{\text{up}}} \Delta Z \Phi(M) dM} \quad (4.6)$$

where M_{down} and M_{up} limit the mass range of the stars having contributed to $\Delta Y/\Delta Z$, and $\Phi(M)$ is the IMF normalised so that $\int_{0.1}^{120} M \Phi(M) dM = 1$. The result is given in Table 4.7 (see the caption for the details on the IMF used). Compared to the S55 IMF, the MS79 IMF gives a bigger weight to the low masses, so the results obtained are higher. On the contrary, the NU01 IMF, with the second peak mass set to $50 M_{\odot}$ gives a bigger weight to the $60 M_{\odot}$ models results, which are the lowest ones of all masses, so the $\Delta Y/\Delta Z$ value is much lower. Since the $85 M_{\odot}$ models present higher individual $\Delta Y/\Delta Z$ values than the $60 M_{\odot}$ models, when the second peak mass is set to $100 M_{\odot}$, the convolved value is also higher.

Rotation gives higher $\Delta Y/\Delta Z$ values compared to no rotation in the case of the S55 or MS79 IMF, while it gives lower values in the case of the NU01 IMF.

The values we obtain for the Pop III stars are lower than the one determined from observations of HII regions in low- Z blue compact galaxies: $\Delta Y/\Delta Z = 2.3 \pm 1.0$ for Izotov & Thuan (1998) (revised to 2.8 ± 0.5 in Izotov & Thuan 2004 and then to 2.38 ± 0.45 in Izotov et al. 2007) or

even $\Delta Y/\Delta Z = 8.8 \pm 4.6$ for Holovaty & Melekh (2005). By correcting the Izotov & Thuan (2004) data for stellar absorption, Fukugita & Kawasaki (2006) find a flatter slope $\Delta Y/\Delta Z = 1.1 \pm 1.4$, which is compatible with the results we obtain for the rotating models with the MS79 IMF. As we can see in Table 4.7, the models at $Z = 10^{-8}$ give results which are much closer from the $\Delta Y/\Delta Z$ values determined from the Izotov & Thuan data. This is not a surprise, because the lowest metallicity reached by these data is $Z_{\odot}/55$, and we expect the Pop III chemical imprint to be quickly dominated by the subsequent stellar generations.

4.3.4 Fate of the models

The death of massive stars is usually associated with a supernova (SN) explosion: high speed ejecta and a compact remnant, either neutron star or black hole (BH). However this is true at “high” metallicity, when the star experiences a significant mass loss during its evolution, which reduces its final mass. Maeder (1992) show that the final mass of the most massive stars is even lower than that of stars with an initial mass $M_{\text{ini}} \leq 25 M_{\odot}$ (see Fig. 4.11).

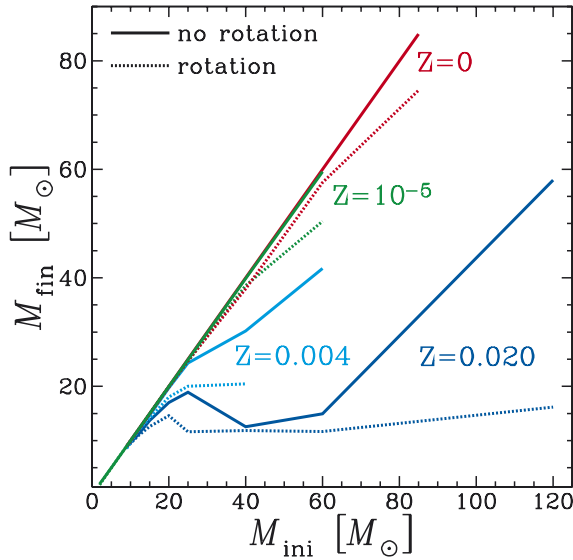


Figure 4.11: Relation $M_{\text{ini}}-M_{\text{fin}}$ for various Z . The models at $Z = 0.020$ are taken from Meynet & Maeder (2000), those at $Z = 0.004$ from Maeder & Meynet (2001), and those at $Z = 10^{-5}$ from Meynet & Maeder (2002).

At very low metallicity, this is no longer true. The star is still very massive at the end of its evolution, and this may drastically weaken the explosion or even inhibit it. In some other cases, instabilities can entirely disrupt the star. There is thus a whole variety of possible fates for low- Z massive stars. Heger & Woosley (2002) and Heger et al. (2003) determine the limiting M_{α} for which the fate will be:

- a Type II SN: $M_{\alpha} < 9 M_{\odot}$
- a BH by fallback: $9 M_{\odot} \leq M_{\alpha} < 15 M_{\odot}$
- a direct BH: $15 M_{\odot} \leq M_{\alpha} < 40 M_{\odot}$
- a pulsational pair-instability followed by a SN with BH formation: $40 M_{\odot} \leq M_{\alpha} < 64 M_{\odot}$
- a pair-instability SN (PISN): $64 M_{\odot} \leq M_{\alpha} \leq 133 M_{\odot}$
- or a direct BH collapse: $M_{\alpha} > 133 M_{\odot}$

The limits above are obtained by non-rotating models, and they may not be appropriate for rotating models, as shown by Fryer et al. (2001). Moreover, they are determined from bare He cores, which means that the increase of M_{α} by the H-burning shell is not taken into account. Note that

for the models with $M \geq 40 M_\odot$, this increase represents less than 10% of the M_α computed at the beginning of CHeB. Anyway, since we are lacking more accurate determinations, we applied them to all our models.

If we follow these limits, the fate we expect for our models is as follows (see also Fig. 4.12):

- Type II SN: rotating and non-rotating 9 and 15 $M_\odot$ models, non-rotating 25 $M_\odot$
- BH by fallback: rotating 25 $M_\odot$ model
- direct BH: rotating and non-rotating 40 and 60 $M_\odot$ models, non-rotating 85 $M_\odot$ model
- SN with BH formation after pulsational pair instability: rotating 85 $M_\odot$ model
- PISN: rotating and non-rotating 200 $M_\odot$ models

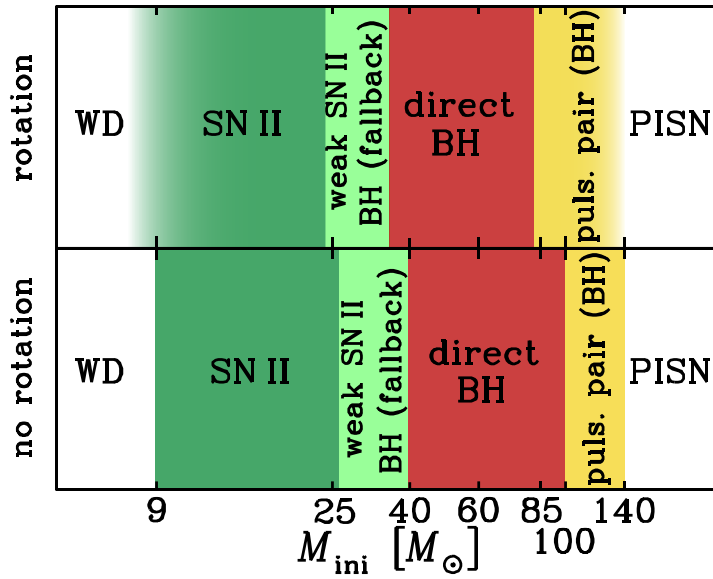


Figure 4.12: Expected fate of the non-rotating and rotating models at $Z = 0$. The limits for the non-rotating models are taken from Heger et al. (2003) since our models conform to them. In the rotating case, the limits between WD and SNII or between pulsational pair instability and PISN are not set by the models, so we have blurred them.

Rotation changes the mass of the He core, usually increasing it thanks to rotational mixing. This has consequences on the way the star ends its life. While the non-rotating 25 $M_\odot$ model ends as a Type II SN, the rotating one is expected to produce a weak explosion and a BH by fallback. The non-rotating 85 $M_\odot$ model remains in the direct BH domain, while the rotating one reaches a M_α that implies pulsational pair instability, followed by a Fe-core collapse into BH.

A star that ends its life as a BH is supposed to contribute to the chemical enrichment of its surroundings only through its winds. We give in parenthesis in Table 4.4 this “wind-only” contribution for the models that, according to the limits determined above, end their life as BH by fallback or by direct collapse. The winds yields are very small, since the mass loss is very modest (see Sect. 4.2.5 on page 66). In the case of the non-rotating models, no metals are shed in the ISM below 85 $M_\odot$, and the 85 $M_\odot$ models releases less than a thousandth of solar mass in metals. With rotation, the picture is not drastically changed, even the 60 $M_\odot$ model does not release more than $10^{-5} M_\odot$ in metals. As we mentioned above, the rotating 85 $M_\odot$ model undergoes pulsational pair instability, in which the outer layers are supposed to be ejected without the explosive processing. Its true yields are thus probably very close to the pre-SN yields.

Table 4.8: Some rotational properties of the iron core of our models: the mass M_{Fe} where the abundance of the sum $^{48}\text{Cr} + ^{52}\text{Fe} + ^{56}\text{Ni}$ amounts to 50% in mass fraction; the angular velocity Ω taken in the middle of the iron core (the profile of $\Omega(m)$ is almost flat in the core, so this value represents the mean value of Ω in the iron core); the maximum specific angular momentum j inside the core, corresponding to the first peak seen in Fig. 4.13 (left); the total angular momentum $\mathcal{L}$ and the parameter $\beta = E_{\text{rot}}/|E_{\text{pot}}|$, both taken at the border of the iron core.

Mass [$M_{\odot}$]	M_{Fe} [$M_{\odot}$]	Ω [s^{-1}]	j [$\text{cm}^2 \text{s}^{-1}$]	$\mathcal{L}$ [$\text{g cm}^2 \text{s}^{-1}$]	β %
25	1.327	0.4232	4.073e+16	5.837e+49	1.5833
40	1.555	0.3547	3.667e+16	5.963e+49	1.0046
60	2.516	0.1578	4.338e+16	1.152e+50	0.5449
85	2.779	0.1062	3.913e+16	1.182e+50	0.3624

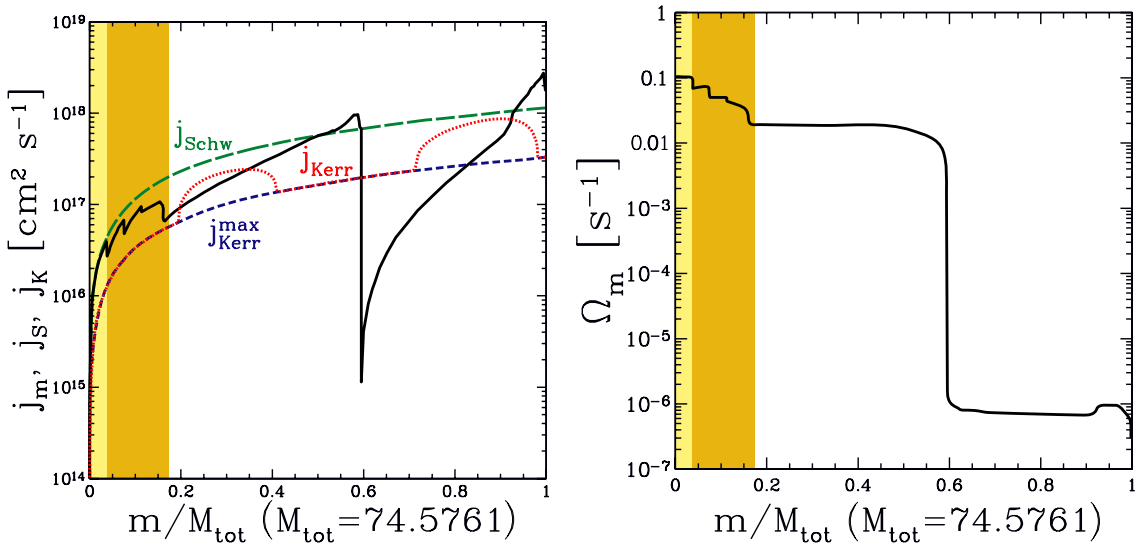


Figure 4.13: *Left*: profile of the specific angular momentum j_m inside our $85 M_{\odot}$ at the end of hydrostatic core Si-burning (solid line). The dotted line is $j_K = r_{\text{LSO}} c$ (Shapiro & Teukolsky 1983, p. 428), where the radius of the last stable orbit, r_{LSO} , is given by r_{ms} in formula (12.7.24) from Shapiro & Teukolsky (1983, p. 362) for circular orbit in the Kerr metric. j_K is the minimum specific angular momentum necessary to form an accretion disc around a rotating black hole. $j_{\text{Schw}} = \sqrt{12}Gm/c$ (long-dashed line) and $j_{\text{Kerr}}^{\text{max}} = Gm/c$ (short-dashed line) are the minimum specific angular momentum necessary for a non-rotating and a maximally-rotating black hole, respectively. The yellow area shows the extend of M_{Fe} , the orange one the part of the star which is supposed to be trapped in the remnant. *Right*: profile of Ω_m inside the same model.

Powerful explosions?

There is however a possibility that our rotating models may produce a powerful explosion at the moment of their death. Since they experience extremely low mass loss, they keep a high angular momentum content throughout their whole evolution. Table 4.8 gives the rotational characteristics of the iron core of 25 to $85 M_{\odot}$ models: they present all a high $\beta = E_{\text{rot}}/|E_{\text{pot}}|$ (or at least considered as high in Ott et al. 2004, for example). Figure 4.13 (left) on the current page shows that in the innermost parts of the $85 M_{\odot}$ model, the specific angular momentum is higher than that needed to create an accretion disc around a rotating BH. On the right panel of the same figure, we see that the angular velocity of the iron core is more than 5 orders of magnitude higher than the surface velocity.

According to Nomoto et al. (2003), there are indications that the most energetic supernovae, the hypernovae (HN), could be due to stellar rotation. Rotation is supposed to boost the explosion energy of a collapsing star by various mechanisms:

- it drives an anisotropy of the neutrinos sphere (Shimizu et al. 2001; Kotake et al. 2003)
- it lowers the critical neutrinos luminosity (the luminosity at which the stalled shock revives) (Yamasaki & Yamada 2005)
- it modifies the gravity (Burrows et al. 2005)
- it changes the geometry of the mass accretion, making it aspherical (Burrows et al. 2005)
- it generates vortices, which dredge up heat and increase the neutrinos luminosity (Burrows et al. 2005)

Our models are massive at the time of their death, they have heavy cores and they kept almost their entire envelope, but they have fast rotating iron cores, and therefore their final collapse could present a highly aspherical geometry. It would be extremely interesting to perform numerical simulations of the collapse of our models so we could check whether the energy released in the collapse is sufficient to overcome the high gravitation of the star and drive a successful explosion. For the time being, we can only say that we expect our models to undergo a stronger explosion than what is commonly admitted for Pop III stars of this mass range.

Gravitational waves emission?

The collapse of a star into a BH with even only a slight deviation from sphericity is expected to emit gravitational waves (GW). In the quadrupole approximation, the wave emission can be divided into three phases (Ott et al. 2004), represented in Fig. 4.14:

- I the *infall phase*: it correspond to the hydrodynamic collapse of the star. The absolute value of the quadrupole moment increases. Just a little before the core bounce, the increase decelerates;
- II the *bounce phase*: at core bounce, the wave signal decreases, reaching an absolute minimum a fraction of a millisecond after the bounce. It then increases again and reaches a maximum;
- III the *ring-down phase*: the signal consists then in small amplitude oscillations, following the core ring-down.

Rotation brings the asphericity required, so the collapse of a rotating star is considered as a possible source of GW emission. However, Mönchmeyer et al. (1991) point out that rotation also sustains the collapse with the help of the centrifugal force: the core bounce may occur at subnuclear densities, which modifies the GW emission (see also Ott et al. 2004). However, Ott et al. (2007), recently confirmed by Dimmelmeier et al. (2008), show that with a multi-D general relativistic (GR) treatment, a microphysical nuclear EOS and a simple treatment of deleptonisation, the effects of rotation are reduced: the gravitation is much stronger with the GR treatment and the bounce occurs at nuclear densities, even for the fastest rotators. They found that the amplitude of the GW emission is higher for faster rotation and higher degree of differential rotation in the core. Suwa et al. (2007) show that very massive collapsing Pop III stars may contribute to the GW background and be detectable by future facilities as DECIGO, the successor of LIGO. Without any true simulation, we cannot conclude anything, but with the results of Ott et al. (2007) in mind, we can say that there is a possibility that the explosion of fast rotating Pop III stars could contribute to the GW background.

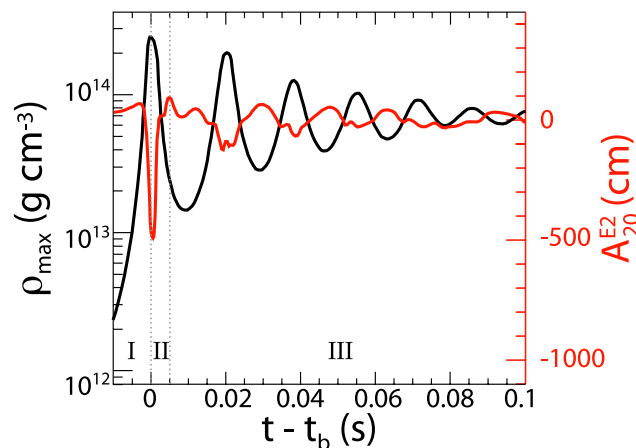


Figure 4.14: Gravitational waves (GW) emission during a core-collapse SN. t_b is the time of core bounce. Black: maximum density of the core; red: GW amplitude. The phases labelled I, II, and III are explained in the text. Adapted from Ott et al. (2004).

4.3.5 Chemical evolution of the Galaxy

Yields of extremely low-metallicity models are constrained by the most metal-poor stars observed in the halo of our Galaxy. They are supposed to trace the local chemical evolution of their surrounding medium at the time of their birth. The notion of time is important to keep in mind because stars with different masses have different lifetimes: the elements they produce are not released at the same moment. The oldest stars observed nowadays may have formed when only massive stars had time to contribute to the enrichment of the medium. If we want to understand the abundances observed at the surface of these old stars, we cannot just integrate the yields obtained by a set of models with an IMF, because we would lose track of the time. What we need is a complete chemical evolution model (CEM).

Such a CEM is computed by Chiappini et al. (2006b) for example⁵. They compare their predictions for the evolution of N/O and C/O upon the adoption of different sets of stellar yields with the surface ratios determined for the metal-poor stars observed by Israelian et al. (2004) and Spite et al. (2005). They show that the N/O ratio is much higher than what was predicted by a CEM using the yields of the slow-rotating $Z = 10^{-5}$ models from Meynet & Maeder (2002) down to $Z = 0$. When adding the yields of the fast-rotating $Z = 10^{-8}$ models from Hirschi (2007), the fit is much improved. The same improvement is found for the C/O ratio, which presents an upturn at low metallicity. Note that the high N/O and the C/O upturn of the low-metallicity stars is also observed in low-metallicity DLAs (Pettini et al. 2008b).

Their CEM is computed with the assumption that yields from $Z = 10^{-8}$ stellar models are valid down to $Z = 0$. But actually, there is a physical transition at $Z_{\text{trans}} \simeq 10^{-10}$: the stars below Z_{trans} present a PopIII-like behaviour (with some He burning already during the MS), and the stars above Z_{trans} have well separated H- and He-burning phases.

We computed a similar CEM, but this time including our $Z = 0$ pre-SN yields up to Z_{trans} , the yields of Hirschi (2007) up to $Z = 10^{-5}$, and then the yields from Meynet & Maeder (2002) and Hirschi et al. (2004). For the PopIII stellar models, we used an IMF that is truncated at $1 M_{\odot}$ but otherwise standard. Compared to the chemical evolution computed using the $Z = 10^{-8}$ yields down to $Z = 0$ (Fig. 4.15, *left panel*), the new computation predicts a reduction of the N/O and C/O ratios at extremely low metallicity ($Z \leq 6 \cdot 10^{-7}$). This trend is expected given the higher O and the lower C and N production that we obtain with the PopIII models (see Sect. 4.3.1 on page 72

⁵The details of the models can be found in Chiappini et al. (2006a), where they show that such a model reproduces nicely the metallicity distribution of the Galactic halo stars. This means that the timescale for the chemical enrichment in the halo is well constrained.

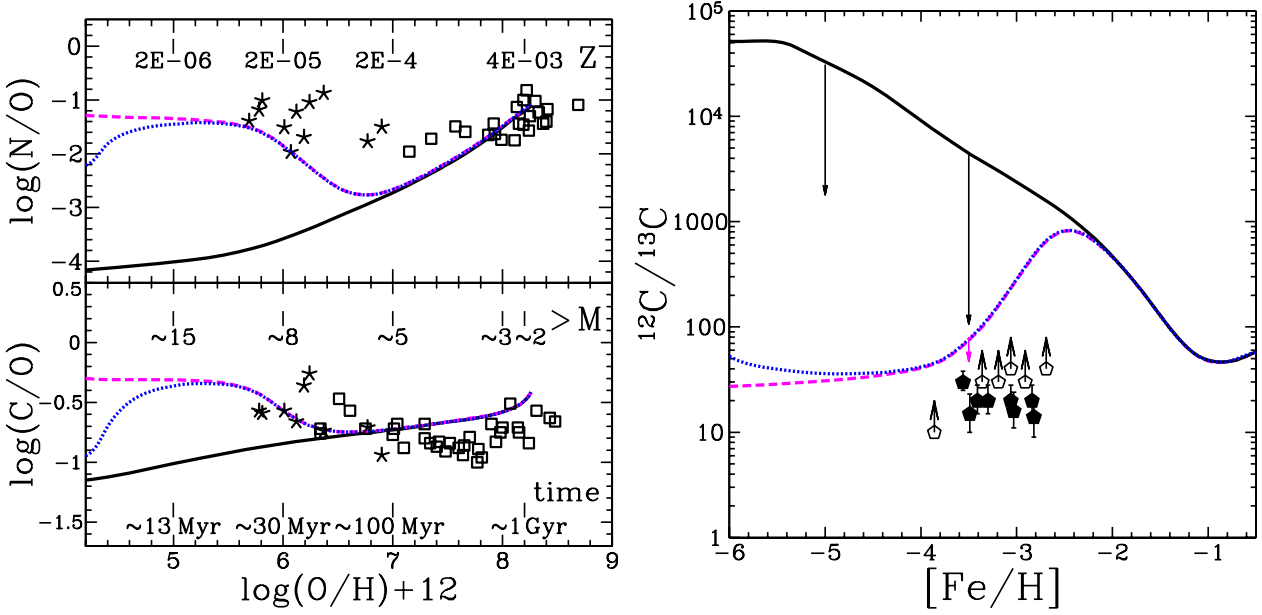


Figure 4.15: The black solid curve is the CEM obtained with the stellar yields of slowly rotating ($v_{\text{ini}} = 300 \text{ km s}^{-1}$) $Z = 10^{-5}$ models from Meynet & Maeder (2002) and Hirschi et al. (2004). The magenta dashed line includes the yields of fast rotating ($v_{\text{ini}} = 800 \text{ km s}^{-1}$) $Z = 10^{-8}$ models from Hirschi (2007) at very low metallicity. The blue dotted curve is obtained using the yields of the $Z = 0$ models presented in this paper up to $Z = 10^{-10}$. *Left*: evolution of the N/O and C/O ratios. Data points are from Israelian et al. (2004, open squares) and Spite et al. (2005, stars). *Right*: evolution of the $^{12}\text{C}/^{13}\text{C}$ ratio. Data points are *unmixed* stars from Spite et al. (2006): open pentagons are lower limits. The arrows going down from the theoretical curves indicate the final $^{12}\text{C}/^{13}\text{C}$ observed in giants (after the dredge-up), starting from the initial composition values given by the stellar models (see Chiappini et al. 2008).

and Fig. 4.10 on page 75). Note that a similar CEM computed with the “wind-only” contribution for our 25 to $60 M_{\odot}$ models leads to such a slow O enrichment that it would deviate from the CEM with $Z = 10^{-8}$ yields curve only at an O/H domain much lower than that shown in Fig. 4.15.

Another interesting quantity is the ratio $^{12}\text{C}/^{13}\text{C}$. A large N production comes with a high ^{13}C value since they are both the products of ^{12}C conversion by the CN-cycle in H-burning zones. We thus expect a low $^{12}\text{C}/^{13}\text{C}$ ratio from the yields of fast-rotating models. Chiappini et al. (2008) use the same set of yields as in Chiappini et al. (2006b), but this time to study the $^{12}\text{C}/^{13}\text{C}$ ratio. They show that this ratio is indeed much closer to the recent observations provided by Spite et al. (2006) upon the inclusion of fast-rotating $Z = 10^{-8}$ models. With the inclusion of our pre-SN PopIII yields (Fig. 4.15, *right panel*), we predict that the $^{12}\text{C}/^{13}\text{C}$ ratio should rise slightly again with decreasing $[Fe/H]$ at extremely low metallicity.

4.3.6 Discussion

With these results, we see that in PopIII stars, although the effects of rotation are important, they are less strong than in stars with even a tiny amount of metals ($Z = 10^{-8}$). There are two main causes for this:

1. the meridional circulation is extremely weak at $Z = 0$. The dependance of the velocity of the meridional currents in the outer cell in $1/\rho$ (see Sect. 3.16) makes it a factor of 100 lower compared to the value at standard metallicity, and still a factor of 4 lower compared to $Z = 10^{-5}$. The evolution of the angular momentum is close to local conservation, so the

critical limit is reached late. Moreover, only a very small amount of mass needs to be lost in order to bring the surface back to sub-critical rotation;

2. there is little structural readjustment at the end of the MS, because the core is already hot enough to burn some He during the main sequence, so the nuclear burnings are continuous. The transition between H and He burning is smooth and the models remain on the blue side of the Hertzsprung-Russell diagram for a long part of the core He burning. When they eventually reach the red side, their outer convective zone remains thin, so the dredge-up is not efficient and the surface enrichment stays low.

There is actually some mass loss where none was expected (because of the wind dependance on Z), but it remains modest even for the most massive of our models.

Could this picture be modified by changing the physical inputs in the models? The situation would be qualitatively the same had we taken a faster initial rotation rate. The main problem is the weakness of the meridional circulation, which does not efficiently feed the outer layers with angular momentum taken from the fast spinning core. So, even if the critical limit is reached a little earlier, the models still do not lose a lot of mass. For this work, we did not take into account the anisotropy of the winds in our models. We do not think it would change the picture presented here, because the loss of angular momentum by radiative mass loss is negligible. There is still an interesting possibility: the implementation of magnetic fields. They are expected to bring the strong core-envelope coupling that is lacking in $Z = 0$ models, as will be shown in the next section. However, both the existing prescription for the inclusion of magnetic fields in stellar models and the existence of magnetic fields in the early Universe are still debated (see the discussion in Sect. 4.4.5 on page 90).

The consequence of the low mass loss is very interesting for the supposed fate of the models. They end their hydrostatic evolution with a very fast spinning core and this could completely modify the final explosion. Without numerical simulations, we cannot of course be affirmative, but there are strong indications that the energy of the explosion could be boosted and drive a true SN where none (or only a weak one) was expected.

While without rotation the production of primary nitrogen is restricted to limited mass ranges, we showed that rotation makes it a robust feature at all masses, even though the ^{14}N yields for Pop III stars are lower than the production at $Z = 10^{-8}$. We included the yields of our models in a chemical evolution model of the Galactic halo. Our results predict that the N/O and the C/O ratios should drop at extremely low metallicity ($Z \leq 10^{-6}$). For the time being though, such low metallicities are far beyond our observational possibilities, so we will have to wait before we can put observational constraints on our results.

4.4 A case study: Can very massive Population III stars avoid pair-instability supernovae?

The study presented here has been the subject of an oral contribution in the following conferences:

First Stars III

held in Santa Fe, New Mexico (USA), 16-20 July 2007
(proceeding version on page 237, arXiv:0709.0202v1)

IAU Symposium 250

held in Kauai, Hawai'i (USA), 10-14 December 2007
(proceeding version on page 231, arXiv:0801.3397)

4.4.1 Motivations

Our $200 M_{\odot}$ model was not followed to the advanced stages, but most probably it would have fell into the pair-instability supernovae (PISN) regime. This fate is expected for the stars that end their life with a He core between 64 and $133 M_{\odot}$ (see Sect. 4.3.4 on page 81). If a great fraction of the Pop III stars were VMOs, we expect that a large number of them should have met such a fate. PISN are supposed to leave a very specific nucleosynthetic signature (Heger & Woosley 2002), with a strong odd-even effect: the absence of stable post-He burning stages deprives the star of the neutron excess needed to produce significant amounts of odd- Z nuclei. Also the lack of r- and s-process stops the nucleosynthesis around zinc.

However, Tumlinson et al. (2004) show that the PISN yields are a poor fit to the surface abundances observed in the most metal-poor stars of the halo of our Galaxy. Maybe the heavy initial mass range was not formed to begin with? This seems to be however extremely unlikely, given the Pop III formation simulations (Tan & McKee 2008; O’Shea & Norman 2007). Maybe the particular signature was quickly erased by the following generations of SNe? There are evidences that a second generation of metal-free stars can form in the relic HII regions of VMOs (Yoshida et al. 2006; O’Shea & Norman 2007), so probably the EMPs are born from a medium enriched by these *Pop III.2* stars and the chemical signature of VMOs was quickly lost. Or perhaps these stars avoided the PISN regime? The simplest way to avoid it would be to find a process which drives a heavy mass loss. We have seen that pure rotation does not suffice to drive a heavy mass loss at $Z = 0$. Would there be an additional mechanism which could help in this situation?

4.4.2 Added physics

In the previous models, we have neglected two natural consequences of the differential rotation: the anisotropy of the winds and the magnetic fields.

- The winds anisotropy arises from the variation of the gravitation with the colatitude (Maeder 1999). The effective gravity is lower at the equator, where the centrifugal force acts in the exactly inverse direction than the gravitation. At lower colatitude, the two vectors are no longer parallels (see Fig. 1.2 on page 3), and the norm of the centrifugal acceleration decreases with the colatitude, so the effective gravity results higher. Maeder (1999) shows that in case of rotation, the radiative flux on an isobar becomes:

$$\vec{F} = -\frac{L(P)}{4\pi G M_{\star}} \vec{g}_{\text{eff}} \quad (4.7)$$

where $M_{\star} = M \left(1 - \frac{\Omega^2}{2\pi G \rho_m}\right)$ is the reduced mass (with ρ_m the average density inside the considered surface level). The radiative flux will thus be stronger at the poles, driving a stronger mass loss in that direction. When matter is not expelled at the equator but at a lower colatitude, the angular momentum it takes with it is lower. The star loses mass but keeps more angular momentum than in the usual spherical mass loss.

- The magnetic field is produced in the radiative layers of a differentially-rotating star (Spruit 1999, 2002). An initial toroidal field at the beginning of the evolution is subject to the so-called Tayler-instability (Tayler 1973, 1980) which creates a vertical component. If the star is differentially rotating, the fields lines are wound up, which strengthens the horizontal field. The magnetic fields imposes a nearly rigid-body rotation to the star by maximally coupling the core and the envelope. We saw that our Pop III models were precisely lacking a strong coupling, and that was the reason why they behaved so differently from low- but non zero-metallicity stars. It is therefore interesting to study how the inclusion of magnetic fields would modify the evolution.

4.4.3 Input physics

We computed a $150 M_{\odot}$ model with $v_{\text{ini}}/v_{\text{crit}} = 0.56$ ($\Omega_{\text{ini}}/\Omega_{\text{crit}} = 0.75$), which corresponds to $v_{\text{eq}} = 1040 \text{ km s}^{-1}$ on the ZAMS. The program used for the calculation is `origin2006` (see Appendix A.3 on page 132), with the treatment of the Tayler-Spruit dynamo as in Maeder & Meynet (2005), and the correction for anisotropical winds as in Maeder (2002). As before, we used Kudritzki (2002) prescription for the radiative mass loss, with the same adaptations as described in Sect. 4.2.1 on page 61. For the WR mass loss, we adopted the Eldridge & Vink (2006) scaling with metallicity superimposed on the Nugis & Lamers (2000) rates.

For the sake of comparison, we computed also a model of the same mass without rotation.

4.4.4 Evolution and mass loss

Both models were evolved until the end of the CHeB phase. Of course, stellar evolution with magnetic fields is very interesting to study *per se* and would be worth a detailed description, but it is not the scope of the study we present here. We will thus let the interested reader turn to the literature dedicated to stellar evolution with magnetic fields (Maeder & Meynet 2004; Heger et al. 2005; Yoon & Langer 2005) and here we will only comment the HRD track and the mass loss history.

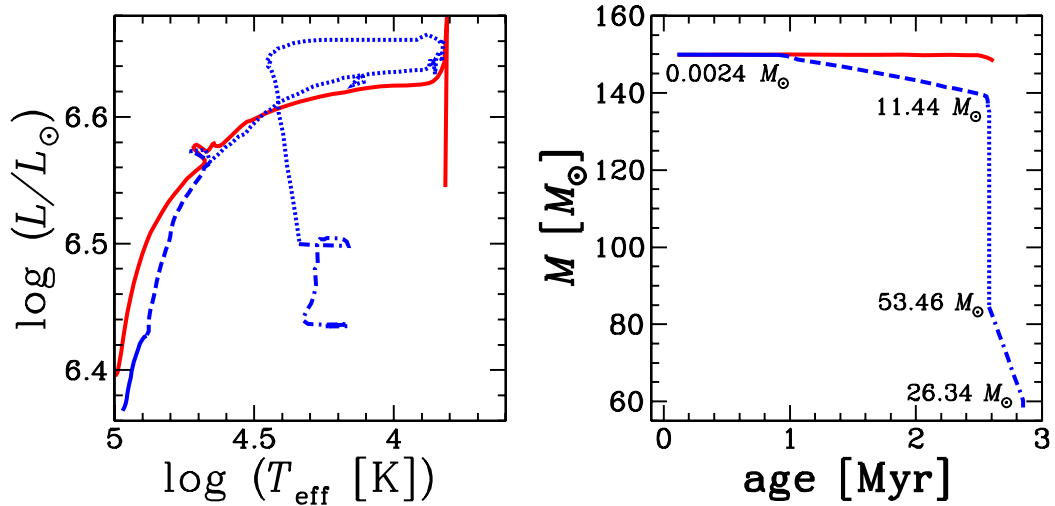


Figure 4.16: Evolution of a metal-free $150 M_{\odot}$ model with magnetic fields (red: non-rotating model; blue: rotating model). *Left*: tracks in the HRD. *Right*: Mass loss history.

Figure 4.16 present the evolution of the two models. During its whole evolution up to the end of core He-burning, the non-rotating model (red line) loses only $1.37 M_{\odot}$. This illustrates the weakness of radiative winds at $Z = 0$. The evolution of the rotating model (blue line) can be described by four distinct stages:

1. (*solid part, lower left corner*) The model starts its evolution on the MS with only radiative winds, losing only a little more than $0.002 M_{\odot}$. During this stage, it quickly accelerates, mainly because of the strong coupling exerted by the magnetic fields.
2. (*dashed part*) When its central content of hydrogen is still 0.58 in mass fraction, it reaches the critical velocity and starts losing mass by mechanical winds. It remains at the critical limit through the whole MS, but the mechanical winds remove only the most superficial layers that have become unbound, and less than 10% of the initial mass is lost at that stage ($11.44 M_{\odot}$). The model becomes also extremely luminous, and reaches the Eddington limit when

10% of hydrogen remains in the core. Precisely, it is the so-called $\Omega\Gamma$ -limit that is reached here, see Eq. (4.2) on page 66.

3. (*dotted part*) The combustion of helium begins as soon as the hydrogen is exhausted in the core, then the radiative H-burning shell undergoes a CNO flash, setting the model on its redward journey. The model remains at the $\Omega\Gamma$ -limit and loses a huge amount of mass. The strong magnetic coupling keeps bringing angular momentum to the surface and even the heavy mass loss is not able to let the model evolve away from the critical limit. The mass lost during that stage amounts to $53.46 M_{\odot}$. When the model starts a blue hook in the HR diagram, its surface conditions become those of a WR star ($X_{\text{surf}} < 0.4$ and $T_{\text{eff}} > 10'000$ K). The luminosity drops and takes the model away from the Γ -limit, marking the end of that stage.
4. (*solid part*) The rest of the core He-burning is spent in the WR conditions. The mass loss is strong but less than in the previous stage: another $26.34 M_{\odot}$ are lost.

At the end of core He-burning, the final mass of the model is only $M_{\text{fin}} = 58 M_{\odot}$, already below the minimum M_{α} needed for PISN ($M_{\alpha} \geq 64 M_{\odot}$). Note that the contraction of the core after helium exhaustion brings the model back to critical velocity, so this value for M_{fin} must be considered as an upper limit.

4.4.5 Discussion

From the results presented here it seems we can answer “yes” to the title’s question: under given conditions, very massive Pop III stars can lose large amounts of mass during their evolution, and so avoid the PISN explosion. We must however remain cautious: this is just an exploratory study and we cannot draw general conclusion based on it. Moreover, higher mass stars should experience higher mass loss rates, but would it be sufficient to help them also avoid the PISN domain?

Our results are based on assumptions that can be debated. First, we have assumed that at very low metallicity, the CNO lines can drive a WR wind, as suggested by Vink & de Koter (2005), but this point needs to be verified. In a more general perspective, we still lack a good mass loss prescription for the case $Z = 0$ strictly. Our treatment of the mass loss rate at the $\Omega\Gamma$ -limit is in the process of being refined: for the moment, mass loss is supposed to remove the angular momentum in a spherical configuration, while when the star is at the critical limit, most of the mass is expected to be lost in an equatorial disk. A more accurate treatment may lead to less mass loss for the same amount of angular momentum loss, so this could also change the picture.

Second, the magnetic fields implementation is still the subject of a controversy. Zahn et al. (2007) show that whereas the poloidal field, wound up by the differential rotation, is indeed able to produce a toroidal field, the latter does not necessarily loop back into amplifying the poloidal field. In their 3D simulations, based on the accurate formalism of Mathis & Zahn (2005), they observe no regeneration of the mean poloidal field, which is not affected by the Tayler instability and undergoes Ohmic diffusion. Another point to keep in mind is that the Tayler-Spruit dynamo needs a pre-existing seed field to be activated, and it is not yet clear whether magnetic fields were present in the early Universe (see Giovannini 2004, for a detailed discussion on this subject). In Spruit’s prescription, the initial amplitude of the field does not enter into play, while in the Mathis & Zahn (2005) prescription, the amplitude and the configuration of the magnetic field is a needed initial condition. Recently, Kahniashvili et al. (2008) set upper limits for a primordial magnetic field in the early Universe by analysing the polarisation of the Cosmic Microwave Background. They find that the comoving amplitude of the stochastic primordial magnetic field is ranging from 0.4 to 30 nG on a comoving length scale of 1 Mpc. Some work would yet be necessary to verify if these values allow a pertinent inclusion of magnetic fields in the evolution of primordial stars.

However, we think our results are interesting, because the computation has been accomplished with reasonable assumptions: the initial rotation rate used here was fast but not extreme, and the mechanisms called upon (anisotropy of the winds and magnetic fields) are not exotic ones, but two natural effects which arise when one properly treats the case of rotation.

4.5 A collaboration project: constraints on the possible variations of the fine structure constant by stellar models

The project I present in this section is led by Alain Coc (Nuclear Astrophysics group of the Centre de Spectrométrie Nucléaire et de Spectrométrie de Masse, Orsay, France) and Elisabeth Vangioni (Institut d'Astrophysique de Paris (IAP), France), in collaboration with Pierre Descouvemont (Université Libre de Bruxelles, Belgium), Jean-Philippe Uzan (IAP), and Keith Olive (William I. Fine Theoretical Physics Institute, University of Minnesota, USA). It has been the subject of an oral contribution in the following conference:

XXIVth IAP Colloquium “*Far Away: Light in the young Universe*”

held in Paris (France), 7 - 11 July, 2008

(no proceedings available)

4.5.1 Motivations

As we have seen in Sect. 2.1.2 on page 11, whereas the BBN prediction for D and ^4He are in close agreement with the observed values, the spectroscopically observed ^7Li abundance presents a discrepancy with the theoretical values. As shown recently by Bonifacio et al. (2007), the latest determinations of the Spite plateau remain a factor of 3 lower than the BBN prediction. Several solutions have been proposed, as shown in Coc et al. (2007) and Christlieb (2008):

- The first is of course to critically look at the BBN calculation itself, and search for uncertainties in the cross sections used. If some of the reactions leading to the production of the ^7Li were to be revised, the predicted production could be modified. However, this possibility has been ruled out by Cyburt et al. (2004), and also Angulo et al. (2005) who show that the $^7\text{Be}(\text{D},\text{p})^4\text{He}$ cross section could be even lower, and that would increase the discrepancy.
- We then turn to the observations: are the abundances analysis stained by systematic errors? The determination of the Li abundance directly depends on the assumed T_{eff} , so a modification of the T_{eff} scale could reduce the discrepancy (Meléndez & Ramírez 2004). However, Bonifacio et al. (2007) show that a change in the T_{eff} scale consequently affects the inferred surface gravity, leading to unlikely results. Another possibility of systematic errors is the use of 1D model atmospheres and the LTE approximation. This issue has been addressed by Asplund et al. (1999, 2003), who show that the 3D modelling reduces the ^7Li abundance by 0.2-0.3 dex while the non-LTE approximation increases it by also 0.2-0.3 dex, so the net result is a *status quo*.
- Is it possible that the low ^7Li surface abundance observed in the metal-poor halo stars is due to a depletion mechanism (rotational mixing, diffusion)? If the culprit is the rotational mixing, we would expect a large star-to-star scatter (Ryan et al. 1999), but this has not been observed: the scatter is 0.03 dex at most, which rules out this solution. An interesting possibility exists however, as shown by Richard et al. (2005): the atomic diffusion with turbulent mixing below the convective envelope. Models that include these mechanisms seem to reproduce the observed ^7Li abundances, but at the cost of adding a free parameter for the

turbulent mixing efficiency. Another problem with this solution is that it aggravates the discrepancy found for the abundance of the ${}^6\text{Li}$ (see e.g. Asplund et al. 2006, for a discussion on the ${}^6\text{Li}$ problem).

- When conventional explanations fail, it is always an occasion to turn to non-standard physics. How would the BBN prediction be modified if we allow the fundamental constant to be varying with time? This is the possibility we will explore here.

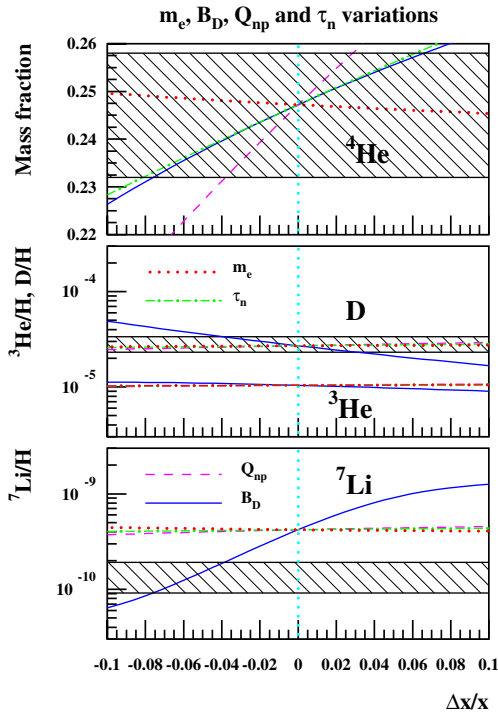


Figure 4.17: (Fig. 3 de Coc et al. 2007): primordial abundances as predicted by allowing for the variation of the electron mass m_e , the neutron lifetime τ_n , the mass difference between proton and neutron $Q_{np} = m_n - m_p$ and the deuterium binding energy B_D . The hatched zones represent the observed spectroscopic values (2σ).

between the positive detections (e.g. Webb et al. 2001; Murphy et al. 2003, 2008; Levshakov et al. 2007) and the negative ones (e.g. Quast et al. 2004; Srianand et al. 2004; Chand et al. 2006), and they are the subjects of a very lively debate. A promising technique is the measurements of the 18 cm OH molecular lines (Darling 2004; Kanekar et al. 2005). In principle, radio data are simpler to analyse, with more stable instruments, and a more direct reduction. However, this technique is yet restricted to low redshift objects ($z = 0.7 - 0.8$). In summary, with the actual precision of the measures, nothing can be excluded, nothing can be concluded.

Figure 4.17 presents the D, ${}^3,4\text{He}$ and ${}^7\text{Li}$ abundances predictions obtained by Coc et al. (2007) when letting some constants vary. We will focus on the variation of the binding energy of the deuterium, $\Delta B_D/B_D$. The link between the variation of the fine structure constant $\Delta\alpha/\alpha$ and that of $\Delta B_D/B_D$ is not direct. According to A. Coc (priv. comm., but see also Coc et al. 2007), a variation in α will have repercussions on the Yukawa coupling, the Higgs vacuum expectation value, and the

Many cosmological theories allow or need the variation of the fundamental constants, like the gravitation G , the ratio between the electron and proton mass $\mu = m_e/m_p$ or the coupling constants like the fine structure constant (or electromagnetic coupling) $\alpha = e^2/(\hbar c)$, the gravitation coupling $\alpha_G = Gm_p^2/(\hbar c)$, or the weak coupling $\alpha_W = G_F m_p^2 c/\hbar^3$. Among those theories, we can mention the Jordan-Brans-Dicke theory (Jordan 1949; Brans & Dicke 1961), the Kaluza-Klein theory (Kaluza 1921; Klein 1926), all the string-based theories and some quintessence theories. In these theories, the fundamental constants are neither fundamental nor constant, but just happen to have their particular value at our particular time, and should be considered as variable parameters characterising the state of the Universe. Usually, the time dependence of the fundamental constants is inferred through their dependence in a dynamical scalar field ϕ .

Are any variations observed to date? This point is still much debated. One of the main astronomical method to test the “constancy of the constants” is the analysis of high redshift quasar absorption systems: the fine splitting of resonance lines shows an amplitude proportional to α^2 . The ions that can be observed in high z QSO absorption systems and can be used to test the value of α are usually the SiIV doublet, OIII nebular lines or FeII lines. The redshift range that has been accessed so far is $0.5 < z < 3.6$. These observations are extremely demanding in term of spectral resolution, signal-to-noise, instrument calibration, and data reduction. With the available instruments, they are limited yet to the brightest quasars. The results of the measurements are divided

QCD energy scale. These terms in turn will affect the mass of the quarks, and thus the nucleons masses m_p and m_n and the omega and sigma mesons masses (m_ω , m_σ). Flambaum & Shuryak (2003) calculate the D binding energy (B_D) with a model where the nucleons interactions occur through the exchange of ω and σ mesons. Therefore, B_D is shown to be sensitive to a variation of the masses m_ω and m_σ , and thus to a variation of α . The relation between $\Delta B_D/B_D$ and $\Delta\alpha/\alpha$ depends on the model adopted for the scalar field. In the frame of the dilaton field (Campbell & Olive 1995), the relation is roughly $\Delta B_D/B_D \approx -(300 \pm 100)\Delta\alpha/\alpha$ (Eq. (28) and (34) in Coc et al. 2007), so a variation of a few percents in $\Delta B_D/B_D$ corresponds to an opposite sign variation of a few 10^{-5} in $\Delta\alpha/\alpha$. There is an interesting zone ($\Delta B_D/B_D \approx -0.04$) where the BBN predicted abundance of the ${}^7\text{Li}$ is reconciled with the spectroscopic data, and which is still marginally compatible with the observed D and ${}^4\text{He}$ values. Note that the BBN occurs at a redshift of 10^8 .

4.5.2 Description of the project

The 3α reaction would be particularly sensitive to a variation of the fine structure constant because it is a resonant reaction, and the variation would modify the Hoyle level and affect the ${}^8\text{Be}$. We define the following energies (see Fig. 4.18 below):

- $E_R({}^8\text{Be})$ as the energy of the ${}^8\text{Be}$ ground state with respect to the $\alpha+\alpha$ threshold;
- $E_R({}^{12}\text{C})$ as the energy of the Hoyle level with respect to the ${}^8\text{Be}+\alpha$ threshold, i.e. $E_R({}^{12}\text{C}) \equiv {}^{12}\text{C}(0_2^+) + Q_\alpha({}^{12}\text{C})$ where ${}^{12}\text{C}(0_2^+)$ is the excitation energy and $Q_\alpha({}^{12}\text{C})$ is the α particle separation energy;
- $Q_{\alpha\alpha\alpha}$ as the energy of the Hoyle level with respect to the 3α threshold so that

$$Q_{\alpha\alpha\alpha} = E_R({}^8\text{Be}) + E_R({}^{12}\text{C}). \quad (4.8)$$

- $\Gamma_\alpha({}^8\text{Be})$ as the partial width of the beryllium decay ($\alpha\alpha \leftrightarrow {}^8\text{Be} + \gamma$);
- $\Gamma_{\gamma,\alpha}({}^{12}\text{C})$ as the partial widths of ${}^8\text{Be} + \alpha \leftrightarrow {}^{12}\text{C}^* \rightarrow {}^{12}\text{C} + \gamma$.

Their standard values are given in Table 4.9. A variation in the strength of the nucleon-nucleon interaction, δ_{NN} , modifies the partial widths $\Gamma_\alpha({}^8\text{Be})$ and $\Gamma_{\gamma,\alpha}({}^{12}\text{C})$. The binding energy of the deuteron is related to δ_{NN} and it scales as

$$\Delta B_D/B_D = 5.716 \times \delta_{NN}. \quad (4.9)$$

We can also extract the sensitivity of $E_R({}^8\text{Be})$ and $E_R({}^{12}\text{C})$. They scale as

$$\begin{aligned} E_R({}^8\text{Be}) &= (0.09184 - 12.208 \times \delta_{NN}) \text{ MeV} \\ E_R({}^{12}\text{C}) &= (0.2876 - 20.412 \times \delta_{NN}) \text{ MeV}. \end{aligned} \quad (4.10)$$

Note that for $\delta_{NN} \gtrsim 0.007$, $E_R({}^8\text{Be})$ is negative and ${}^8\text{Be}$ would become stable. Using the bijective relation (4.9) between B_D and δ_{NN} we can also express our results as

$$\begin{aligned} E_R({}^8\text{Be}) &= (0.09184 - 2.136 \times \Delta B_D/B_D) \text{ MeV} \\ E_R({}^{12}\text{C}) &= (0.2876 - 3.570 \times \Delta B_D/B_D) \text{ MeV}. \end{aligned} \quad (4.11)$$

It follows that the energy of the Hoyle level with respect to the 3α threshold (and not with respect to ${}^8\text{Be}+\alpha$ threshold) is given by (see Eq. (4.8))

$$\begin{aligned} Q_{\alpha\alpha\alpha} &= (0.37945 - 5.706 \times \Delta B_D/B_D) \text{ MeV} \\ &= (0.37945 - 32.620 \times \delta_{NN}) \text{ MeV}. \end{aligned} \quad (4.12)$$

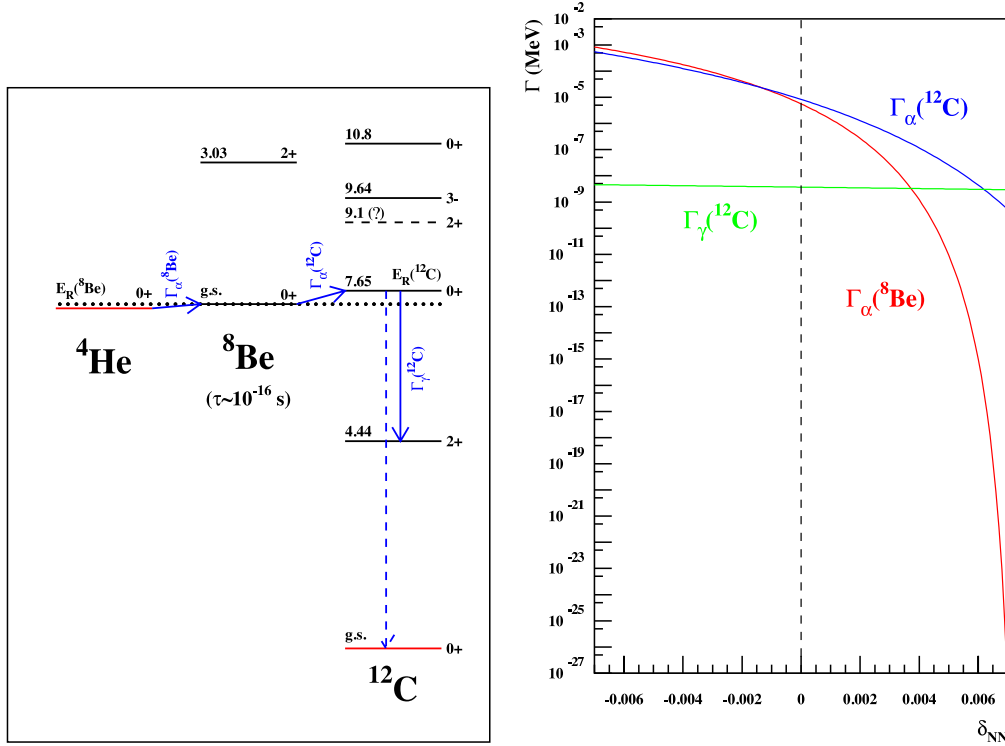


Figure 4.18: *Left*: Energy levels scheme for the 3α reaction. *Right*: Variations of the partial widths of ^8Be and ^{12}C as a function of δ_{NN} (see text).

Table 4.9: Nuclear data for the two steps of the 3α -reaction.

nucleus	J^π	E_R (keV)	Γ_α (eV)	Γ_γ (meV)
^8Be	0^+	91.84 ± 0.04	5.57 ± 0.25	—
^{12}C	0_2^+	287.6 ± 0.2	8.3 ± 1.0	3.7 ± 0.5

Note: See text for the definitions of the quantities.

References: (Tilley et al. 2004; Ajzenberg-Selove 1990; Audi et al. 2003)

Pierre Descouvemont established tabulated 3α rates as a function of the variation δ_{NN} . A comparison between the 3α rates obtained and those of the NACRe database is presented in Fig. 4.19 on next page. Note that on this figure, it is the log of the ratio between the rates that is represented, so the changes brought by a small variation of a few percents on δ_{NN} are enormous.

The idea of the project is to use these reaction rates in Pop III stellar models and see if we can constrain the possible values of δ_{NN} at a redshift of 20-15.

4.5.3 First models and results

With the program `origin2006` (Appendix A.3 on page 132), we computed models of $15 M_\odot$ without rotation, with initial composition $X = 0.75325$, $Y = 0.24675$ and $Z = 0$. For each model, we used a different 3α rate, corresponding to $-0.009 \leq \delta_{\text{NN}} \leq +0.006$. The models were followed until the end of central He burning.

Figure 4.20 on next page shows the HR diagram (*left*) for the models with δ_{NN} between -0.009 et +0.006 and the values of $\log T_c$ at the ignition of CNO cycle and core He burning (CHeB) (*right*). Table 4.10 on page 97 presents their characteristics at the end of CHeB.

On the ZAMS, the model with $\delta_{\text{NN}} = 0$ has not yet produced enough ^{12}C to be able to rely on

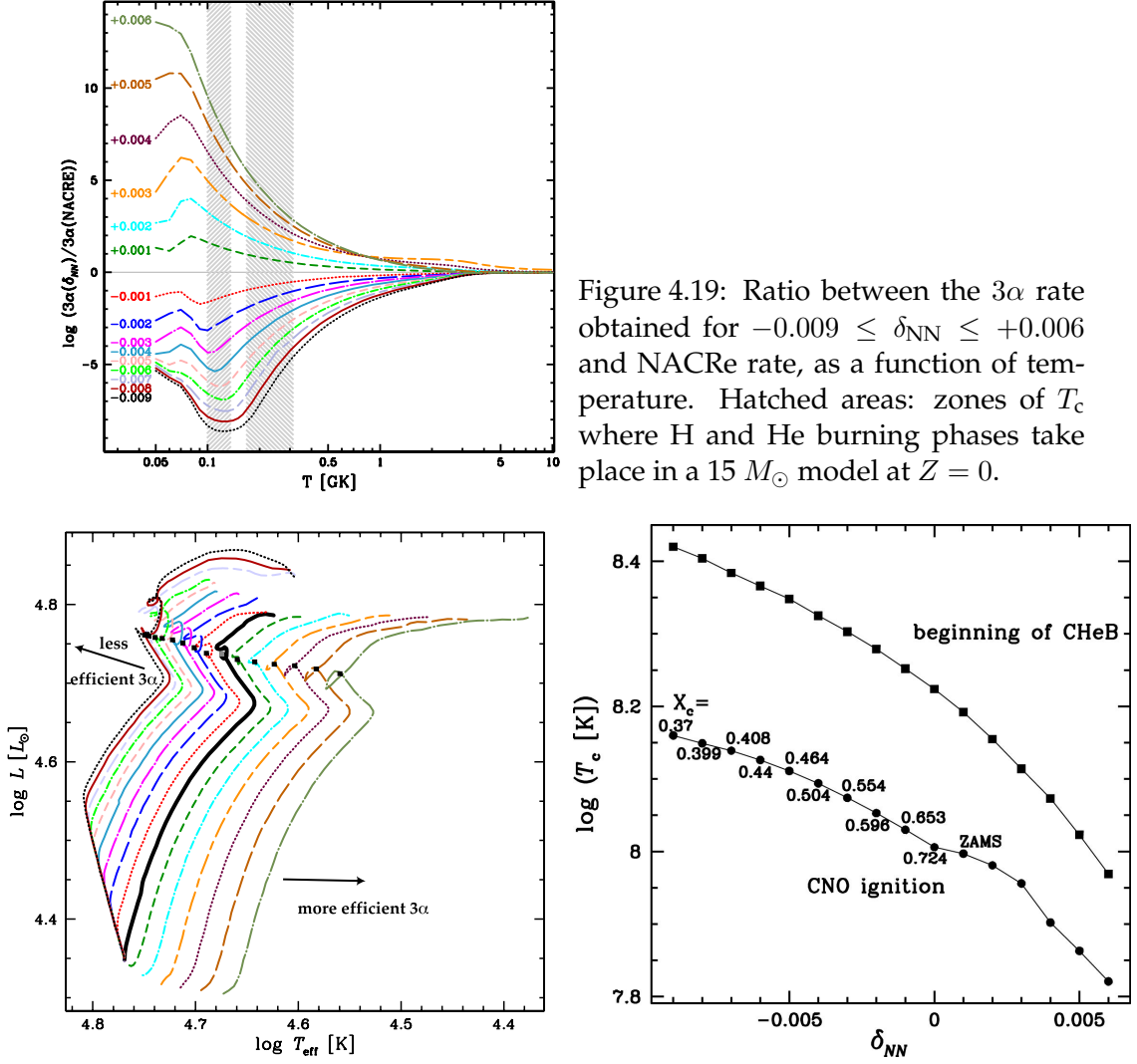


Figure 4.20: $15 M_\odot$ models with varying δ_{NN} . Left: HR diagram (same color code than Fig. 4.19; thick black: $\delta_{NN} = 0$). Right: Value of $\log T_c$ at the CNO cycle ignition (circles) and at the beginning of core He burning (squares).

the CNO cycle⁶, so it starts by continuing its initial contraction until the CNO cycle ignites. This event marks a little hook on Fig. 4.20 (right). Models with $\delta_{NN} < 0$ (i.e. a lower 3α rate) present a contraction phase which is longer for lower δ_{NN} : in these models, the less efficient 3α rates need a higher T_c to produce enough ^{12}C for triggering the CNO cycle. Models with $\delta_{NN} > 0$ (i.e. a higher 3α rate) are directly sustained by the CNO cycle on the ZAMS: the star can more easily counteract its own gravity. The initial contraction is stopped earlier, so H burning occurs at lower T_c and ρ_c (Fig. 4.20, right), i.e. at a slower pace. The MS lifetime τ_{MS} increases with increasing δ_{NN} . The difference between $\tau_{\text{MS}}(\delta_{NN}=0)$ and $\tau_{\text{MS}}(\delta_{NN}=-0.009)$ amounts to 17%, and to 19% between $\tau_{\text{MS}}(\delta_{NN}=0)$ and $\tau_{\text{MS}}(\delta_{NN}=+0.006)$.

While the differences in the 3α rates do not lead to strong effects in the evolution characteristics on the MS, the CHeB phase amplifies the differences between the models. The upper curve of Fig. 4.20 (right) shows the central temperature at the beginning of CHeB. There is a factor of 2.8 in temperature between the models with $\delta_{NN} = -0.009$ and $+0.006$. To get an idea of what this difference represents, we can relate these temperatures to the grid of Pop III models computed by

⁶We saw in Sect. 4.2.2 on page 62 that the lower mass allowing to be sustained by CNO cycle already on the ZAMS is $20 M_\odot$.

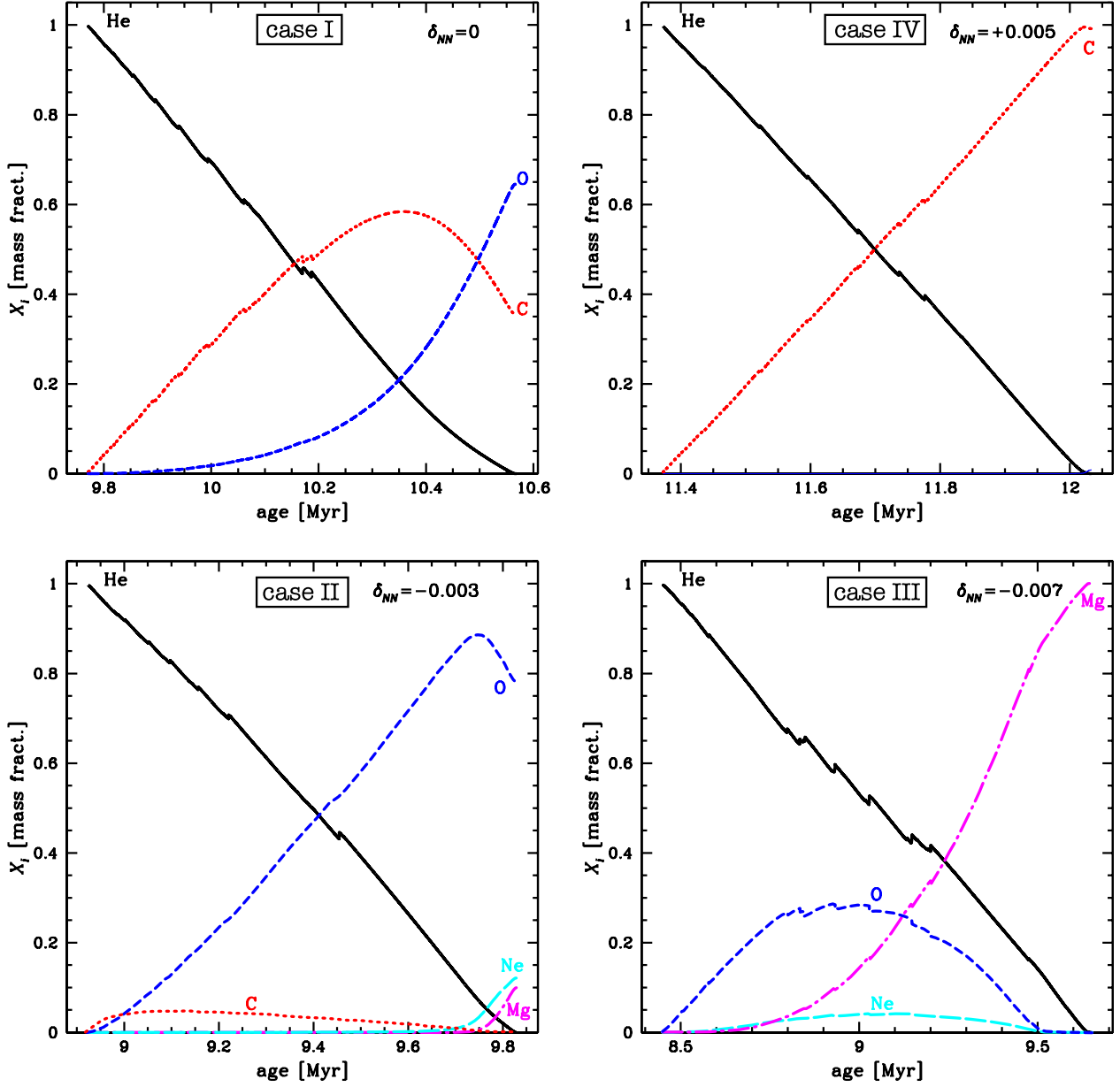


Figure 4.21: Evolution of the ^4He (black, solid line), ^{12}C (red, dotted), ^{16}O (blue, short-dashed), ^{20}Ne (cyan, long-dashed) and ^{24}Mg (magenta, dot-dashed) central mass fraction for the $15 M_{\odot}$ during core He burning. *Top left:* normal case ($\delta_{NN} = 0$); *top right:* strong 3α reaction ($\delta_{NN} = +0.005$); *bottom left:* weak 3α reaction ($\delta_{NN} = -0.003$); *bottom right:* very weak 3α reaction ($\delta_{NN} = -0.007$).

Table 4.10: Characteristics of the 15 $M_{\odot}$ models with δ_{NN} ranging from -0.009 to +0.006 at the end of core He burning. τ_{MS} and τ_{CHeB} are expressed in Myr, M_{CO} is in $M_{\odot}$, $X(\text{C})$ is in mass fraction (central values).

δ_{NN}	τ_{MS}	τ_{CHeB}	M_{CO}^a	$X(\text{C})^b$	C/O	case ^c
-0.009	8.224	1.344	3.84	$4.4e-10$	—	III
-0.008	8.285	1.276	3.83	$2.9e-10$	—	
-0.007	8.308	1.200	3.38	$8.5e-10$	—	
-0.006	8.401	1.168	3.61	$4.2e-07$	—	
-0.005	8.480	1.130	3.59	$5.9e-06$	$3.0e-05$	II
-0.004	8.672	0.933	3.60	$3.2e-05$	$5.2e-05$	
-0.003	8.790	0.905	3.60	$1.3e-04$	$1.7e-04$	
-0.002	9.046	0.892	3.61	$5.6e-04$	$6.4e-04$	
-0.001	9.196	0.888	3.70	0.013	0.014	I
0	9.640	0.802	3.65	0.355	0.550	
+0.001	9.937	0.720	3.61	0.695	2.278	
+0.002	10.312	0.684	3.62	0.877	7.112	
+0.003	10.677	0.664	3.62	0.958	22.57	IV
+0.004	10.981	0.659	3.62	0.981	52.43	
+0.005	11.241	0.660	3.61	0.992	123.9	
+0.006	11.447	0.661	3.55	0.996	270.2	

^a mass coordinate where the abundance of ^4He drops below 10^{-3}

^b Here, $X(\text{C}) = X(^{12}\text{C})$

^c see text for the description of the different cases.

Marigo et al. (2001). The 15 $M_{\odot}$ model with $\delta_{\text{NN}} = -0.009$ starts its CHeB at a higher temperature than a standard 100 $M_{\odot}$ of the same stage. In contrast, the model with $\delta_{\text{NN}} = +0.006$ starts its CHeB phase with a lower temperature than a standard 12 $M_{\odot}$ star at CNO ignition. Table 4.10 presents the characteristics of the models for each value of δ_{NN} at the end of CHeB. From these characteristics, we distinguish four different cases (see the last column of Table 4.10 and Fig. 4.21):

- I In the standard model and when δ_{NN} is very close to 0, ^{12}C is produced during He burning until the central temperature is high enough for the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction to become efficient: during the last part of the CHeB phase, the ^{12}C is processed into ^{16}O . The star ends its CHeB phase with a core composed of a mixture of $\sim 35\%$ of ^{12}C and $\sim 65\%$ ^{16}O (Fig. 4.21, *top-left*, on previous page).
- II If the 3α rate is weakened ($-0.005 \leq \delta_{\text{NN}} \leq -0.002$), ^{12}C is produced at a slower pace, and T_c is high from the beginning of the CHeB phase, so the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction becomes efficient very early: as soon as some ^{12}C is produced, it is immediately transformed into ^{16}O . The star ends its CHeB phase with a core composed mainly of ^{16}O , without any ^{12}C and with an increasing fraction of ^{24}Mg for decreasing δ_{NN} (Fig. 4.21, *bottom-left*).
- III For still weaker 3α rates ($\delta_{\text{NN}} \leq -0.006$), the central temperature during CHeB is such that the $^{16}\text{O}(\alpha, \gamma)^{20}\text{Ne}(\alpha, \gamma)^{24}\text{Mg}$ chain becomes efficient, reducing the final ^{16}O abundance. The star ends its CHeB phase with a core composed of nearly pure ^{24}Mg (Fig. 4.21, *bottom-right*). Because the abundances of both carbon and oxygen are completely negligible, we do not list the irrelevant value of C/O for these cases.
- IV If the 3α rate is strong ($\delta_{\text{NN}} \geq +0.003$), ^{12}C is very rapidly produced, but T_c is so low that the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction can hardly enter into play: ^{12}C is not transformed into ^{16}O . The star ends its CHeB phase with a core almost purely composed of ^{12}C (Fig. 4.21, *top-right*).

These results are summarised in Fig. 4.22 which shows the composition of the core at the end of the CHeB phase. One can clearly see the dramatic change in the core composition as a function of δ_{NN} showing a nearly pure Mg core at large and negative δ_{NN} , a dominantly O core at low but negative δ_{NN} , and a nearly pure C core at large and positive δ_{NN} . These results are qualitatively consistent with those found by Schlattl et al. (2004) for Population I type stars. Note that their cases with $\Delta E_{\text{R}} = \pm 100$ keV correspond roughly to our $\delta_{\text{NN}} \approx \pm 0.005$.

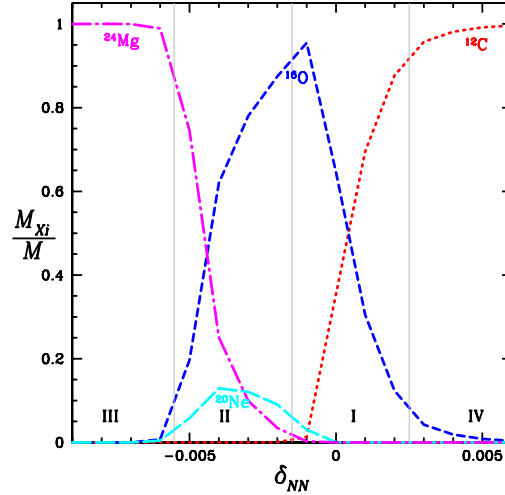


Figure 4.22: Chemical composition of the core at the end of CHeB in the 15 $M_{\odot}$ as a function of δ_{NN} .

4.5.4 Determination of the limits

All the models were followed without any numerical or evolutionary problem through the MS. The differences of lifetimes and tracks during this phase are not constraining enough to allow the exclusion of some values of the δ_{NN} variation between -0.009 and +0.006.

The CHeB phase amplifies the differences. At the end of CHeB, the models with $\delta_{\text{NN}} < -0.005$ have no more ^{12}C in the core, which means that the “standard” succession of the stellar evolution burning phases will not be respected (see the left panel of Fig. 4.22). Those models have no more ^{16}O or ^{20}Ne neither: everything has been transformed into ^{24}Mg . Note that at that phase, the central temperature is close to that which would allow the $^{24}\text{Mg}(\gamma, \alpha)^{20}\text{Ne}$ or $^{24}\text{Mg}(\alpha, \gamma)^{28}\text{Si}$ reactions to take place. Therefore, there is a possibility that the nucleosynthetic chain could go on despite the strange evolution. However, the Geneva code is conceived to follow the standard phases of stellar evolution, so it should be modified before being able to follow further the evolution of those odd objects.

The models with δ_{NN} between -0.002 et -0.005 end the CHeB phase with a central abundance of ^{12}C between 10^{-4} and 10^{-7} , which means that this fusion phase will be extremely short. The ^{20}Ne abundance at that stage is comprised between 0.04 and 0.10, so there will be a short neon photodisintegration phase. Moreover the ^{16}O abundance ranges between 0.94 and 0.44 so the oxygen fusion phase will be almost normal. While the succession of the burning phases seems preserved, one can however suppose that these models will present very different yields than the standard case with $\delta_{\text{NN}} = 0$. This point could be the subject of a future study. It is interesting to note here that since the carbon burning phase is very short (because of the very low ^{12}C abundance at the end of CHeB), the model will not have much time to lose entropy by neutrinos losses. We can suppose that the iron core will be hotter and bigger, so the remnant could be a black hole instead of a neutron star.

The models with $\delta_{\text{NN}} > 0$ end the CHeB with larger and larger ^{12}C abundances for increasing δ_{NN} . The carbon burning phase will thus be much longer for these models which will lose a lot of energy. A more suspicious feature is that the ^{16}O production becomes negligible or even null for $\delta_{\text{NN}} \geq +0.002$ (see case IV of Fig. 4.22). Yet the bulk of the ^{16}O production occurs during

CHeB: during C burning, the ^{16}O abundance is reduced by $^{16}\text{O}(\alpha, \gamma)^{20}\text{Ne}$, and during Ne burning, only a small fraction is produced by the disintegration reaction $^{20}\text{Ne}(\gamma, \alpha)^{16}\text{O}$. This would pose difficulties for explaining the high O overabundances observed in extremely iron-poor stars found in the Galactic halo (see Frebel et al. 2008).

From what precedes, we can conclude that with the $15 M_{\odot}$ models, the possible values for the δ_{NN} variation range between -0.005 and +0.002.

4.5.5 More massive models

Would the possible values of the δ_{NN} variation “allowed” by stellar models be different if determined in another mass range? Most probably, because CHeB occurs at higher T_c in more massive stars. The 3α reaction for very negative values of δ_{NN} becomes stronger with increasing T_c , and we have to check whether this could compensate the increased efficiency of the $^{16}\text{O}(\alpha, \gamma)^{20}\text{Ne}(\alpha, \gamma)^{24}\text{Mg}$ reaction chain, and thus allow a “normal” evolution for a larger range of δ_{NN} values. In the case of very positive δ_{NN} , the 3α rates become less extreme at higher T_c , which could allow values that had been excluded with $15 M_{\odot}$ models.

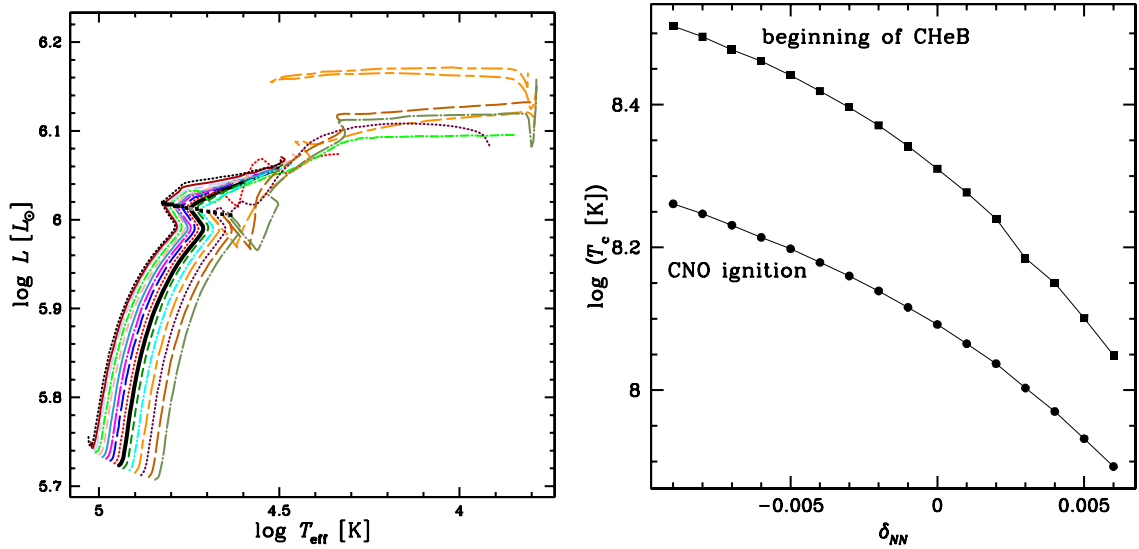


Figure 4.23: $60 M_{\odot}$ models with varying δ_{NN} . *Left*: HR diagram (same color code than Fig. 4.19 on page 95; thick black: $\delta_{\text{NN}} = 0$). *Right*: Value of $\log T_c$ at the CNO cycle ignition (circles) and at the beginning of core He burning (squares).

To verify the robustness of the limits established above, we have computed $60 M_{\odot}$ models (at $Z = 0$). Table 4.11 on page 101 presents the results. Figure 4.23 shows the HRD for those models and the central temperature at CNO and CHeB ignition. Figure 4.24 shows the chemical composition of the core at the end of the CHeB.

The behaviour we described for the $15 M_{\odot}$ models with $\delta_{\text{NN}} < 0$ is in fact more pronounced in the case of the $60 M_{\odot}$ models: ^{12}C and ^{16}O are exhausted at the end of CHeB for $\delta_{\text{NN}} = -0.005$ already. We can understand that by looking at Fig. 4.25: it shows the ratio between the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$, $^{16}\text{O}(\alpha, \gamma)^{20}\text{Ne}$ and $^{20}\text{Ne}(\alpha, \gamma)^{24}\text{Mg}$ reaction rates, and the 3α . We see that the former are a factor of 10 to 100 higher than the latter when $\log T_c \approx 8.48$, i.e. when there is still about 5% of helium in the core. Instead of a CO core, the models are left with an almost pure ^{24}Mg core.

For the $60 M_{\odot}$ models with $\delta_{\text{NN}} > 0$, the upper limit of the δ_{NN} variation is slightly higher, since $\delta_{\text{NN}} = +0.003$ seems to be still allowed. For numerical reasons, the model with $\delta_{\text{NN}} = +0.006$ has proven to be very difficult to follow at the end of CHeB and was stopped before complete He exhaustion.

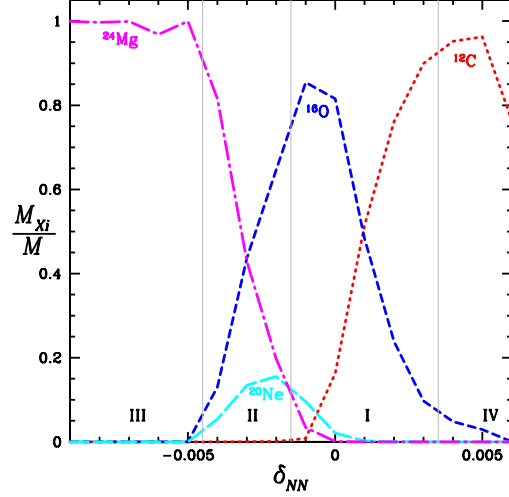


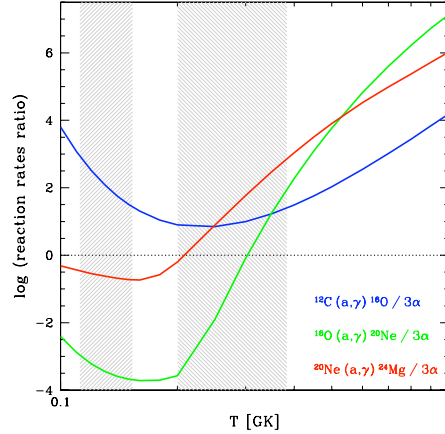
Figure 4.24: Chemical composition of the core at the end of CHeB in the $60 M_{\odot}$ as a function of δ_{NN} .

4.5.6 Discussion

Below $\delta_{\text{NN}} = -0.005$, the stars end the CHeB phase with a core that is completely deprived of carbon, oxygen and neon. This is due to the ^{12}C production by the 3α reaction becoming extremely weak compared to the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction (for which we have used the rates of Kunz et al. 2002). As soon as a little amount of ^{12}C is produced, it is transformed into ^{16}O , which in turn is transformed into ^{20}Ne and then ^{24}Mg due to the high temperature and density at which He burning occurs in these models.

Above $\delta_{\text{NN}} = +0.002$, the models end the CHeB with a pure ^{12}C core, which means that they do not produce any ^{16}O .

Figure 4.25: $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$, $^{16}\text{O}(\alpha, \gamma)^{20}\text{Ne}$ and $^{20}\text{Ne}(\alpha, \gamma)^{24}\text{Mg}$ reaction rate compared to NACRE 3α rate, as a function of temperature. Hatched areas: zones of T_{c} where the H and He burning occur in $60 M_{\odot}$ models.



From the preceding discussion, if we exclude a core composed exclusively of ^{24}Mg (case III), we must reject $\delta_{\text{NN}} < -0.005$ for the $15 M_{\odot}$ model. If we consider that a core only composed of ^{12}C is not acceptable either (case IV), we must reject $\delta_{\text{NN}} > +0.002$. If we consider that a reasonable value of C/O must lay close to unity, we must also reject case II and the allowed range for δ_{NN} is further restricted to -0.00 to $+0.002$. Similarly for the $60 M_{\odot}$ model, excluding cases III and IV leads to a limit $-0.004 \leq \delta_{\text{NN}} \leq +0.003$. The more stringent condition on $\text{C/O} \sim 1$ leads to $-0.001 \leq \delta_{\text{NN}} \leq +0.003$.

We have checked the limiting values for the δ_{NN} variation with stellar models of two different mass domains. For the $15 M_{\odot}$ models, the lower limit is slightly larger than for the $60 M_{\odot}$ models. This is the result of the fusion phases occurring at higher T_{c} in the more massive stars, at conditions where the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$, $^{16}\text{O}(\alpha, \gamma)^{20}\text{Ne}$ and $^{20}\text{Ne}(\alpha, \gamma)^{24}\text{Mg}$ reaction rates are largely dominant over the 3α rate. A weak 3α reaction is a bigger handicap in the high mass domain. In contrast, the

Table 4.11: Characteristics of the $60 M_{\odot}$ models with δ_{NN} ranging from -0.009 to +0.006. τ_{MS} and τ_{CHeB} are expressed in Myr, M_{CO} is in $M_{\odot}$, $X(\text{C})$ is in mass fraction. The mark “75” after M_{CO} indicates that it has been calculated as the mass coordinate where the C+O abundance rises above 0.75. The mark “75Mg” after M_{CO} indicates that it is the ^{24}Mg abundance which has become higher than 0.75.

δ_{NN}	τ_{MS}	τ_{CHeB}	M_{CO}	$X(\text{C})$	C/O	case
-0.009	3.061	0.374	22.31 ^{75Mg}	$2.3e-08$	--	III
-0.008	3.098	0.373	22.16 ^{75Mg}	$2.1e-08$	--	
-0.007	3.133	0.387	22.43 ^{75Mg}	$2.4e-08$	--	
-0.006	3.171	0.418	19.71 ^{75Mg}	$4.3e-05$	--	
-0.005	3.211	0.389	22.66 ^{75Mg}	$1.4e-08$	--	
-0.004	3.252	0.382	21.90	$3.5e-05$	$2.7e-04$	II
-0.003	3.294	0.347	21.76	$2.1e-04$	$4.8e-04$	
-0.002	3.338	0.328	22.27	$9.3e-04$	$1.4e-03$	
-0.001	3.379	0.347	18.40 ⁷⁵	0.008	0.009	I
0	3.418	0.299	21.37	0.163	0.200	
+0.001	3.458	0.267	21.59	0.513	1.062	
+0.002	3.495	0.244	21.14	0.761	3.193	
+0.003	3.534	0.259	21.14	0.899	9.233	
+0.004	3.571	0.265	19.86 ⁷⁵	0.952	19.89	IV
+0.005	3.607	0.808	27.22	0.963	32.85	
+0.006	3.644	--	--	--	--	

upper limit is larger for the $60 M_{\odot}$ models, because the 3α reaction rate is a little less extreme at higher T_{c} .

With Eq. 4.9 on page 93, a variation in δ_{NN} will result in a variation in the deuterium binding energy:

$$-0.023 < \Delta B_{\text{D}}/B_{\text{D}} < +0.011. \quad (4.13)$$

In principle, one would like to next convert the limit on δ_{NN} or B_{D} into a limit on the fundamental constants. As discussed in Sect. 4.5.1 on page 91 and in more detail in Coc et al. (2007), we can use gauge coupling unification to relate a variation in B_{D} to a variation of α_{em} through Λ . Ignoring any variation in the Yukawa couplings and Higgs vev, thus using $\Delta B_{\text{D}}/B_{\text{D}} = 18R\Delta\alpha_{\text{em}}/\alpha_{\text{em}}$, with $R = 36$ as is expected in the simplest grand unified theories, we obtain

$$-3.5 \times 10^{-5} < \frac{\Delta\alpha_{\text{em}}}{\alpha_{\text{em}}} < +1.8 \times 10^{-5}. \quad (4.14)$$

The limits on the variation of the fine structure constant derived above corresponds to a variation of α_{em} between the present time and a period around a redshift $z \sim 15 - 20$ where the Population III stars would have been present. These values are compatible with the similar limits (also assuming gauge coupling unification) on the variation of α_{em} at a redshift of 10^{10} from BBN predictions. They are larger by a factor of 10 than the values found in the claimed detections (Webb et al. 2001; Murphy et al. 2003, 2007) or a factor of 10 weaker than the limits from the non-detection (Chand et al. 2004; Srianand et al. 2004; Quast et al. 2004; Srianand et al. 2007) of a variation of α_{em} from quasar absorption systems at redshifts $z \leq 3.5$.

Actually, the major weakness of these results is the fact that the δ_{NN} variation is only taken into account for the 3α reaction. If the other rates are also touched, perhaps the limits found here would have to be revised, because they have been determined by anomalies in the evolution that are due to a competition between the efficiency of the various rates. However, being a resonant reaction, the 3α reaction is expected to be the most sensitive. Following Oberhummer et al. (2000), the

$^{16}\text{O}(\alpha, \gamma)^{20}\text{Ne}$ reaction is not expected to be sensitive to variations of α_{em} , while the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ could be more affected by such variations because of subthresholds in the ^{16}O nucleus. According to the same authors, this last reaction is expected to be strengthened by a weakening of the nucleon-nucleon interaction. In this case, the effects described for cases II and III would be more dramatic than the ones presented here, and so the limits might be tighter.

Is it really possible to exclude δ_{NN} values with stellar models? The criterion we applied was the possibility of a “normal” succession of burning phases ($\text{H} \rightarrow \text{He} \rightarrow \text{C} \rightarrow \text{Ne} \rightarrow \text{O} \rightarrow \text{Si}$). Note that with a modified code allowing to compute “non standard” fusion phases, we could probably follow these models further. The resulting yields would probably present large anomalies. Given the current state of abundance determination in extremely metal poor stars, it is highly improbable that the first stars would not produce fair amounts of ^{12}C and ^{16}O . The conservative case seem thus to offer a reasonable limit on the variations of the fundamental constants. However, the cosmological models do not give any clue concerning the temporal evolution of a possible variation of the constants. If the actual value was reached early in the past, the effect studied here would concern only the very first generation of stars, and for the time being, there are no direct observations of Pop III chemical imprints. Thus a weird nucleosynthetic result could be quickly erased by the “normal” one we can observe in the most metal-poor halo stars. On the contrary, if the variation was slow, following generations of stars should also be affected and observations should help us constrain the possible variations.

A new grid of stellar models

The results I'm presenting here arise from a collective work which is still in progress, involving the whole Geneva group. They will be completed within the next months.

5.1 Description of the project

One of the big achievements of the Geneva group between 1992 and 1999 was the computation of an extensive and homogenous database of stellar evolution models for masses between 0.8 and 120 solar masses and metallicities from $Z=0.001$ to 0.1 (referred hereafter as *the '92 grids*¹). The models are available through the CDS data centre in Strasbourg². Since these publications, many improvements have been brought to the code, the most important one being the treatment of rotation. Grids of stellar models are an important tool for the astronomical community, which makes a wide use of the '92 grids. Whenever rotating models are needed, the astronomers use the partial grids published in the series of papers *Stellar evolution with rotation*³. However, as I mentioned in Chapter 3 on page 27, they do not form an homogeneous set.

The group has thus undertaken the calculations of new grids of stellar models, both with and without rotation. The code has been improved and updated for this project, as described in Appendix A.3 on page 132.

5.1.1 Solar abundances

The '92 grids were computed with $Z = 0.020$ as the initial solar metallicity (corresponding to $Z = 0.0188$ at the age of the Sun), but recently, Asplund et al. (2005) have reanalysed the solar photospheric chemical composition. Their 3D study yields a lower metal abundance: $X = 0.7392$, $Y = 0.2486$, and $Z = 0.0122$. While these values bring a better agreement of the solar composition with that of the surrounding interstellar medium, they affect significantly the helioseismology results (Basu & Antia 2004; Turck-Chièze et al. 2004; Antia & Basu 2005; Bahcall et al. 2005a,c).

To reconcile the helioseismology with these new abundances, Antia & Basu (2005) and Bahcall et al. (2005b) suggest that if the Ne abundance is revised upwards, it may solve the problem. Precisely, Drake & Testa (2005) and Cunha et al. (2006) find a high Ne/O ratio in the solar neigh-

¹Schaller et al. (1992), Schaerer et al. (1993b,a), Charbonnel et al. (1993), Meynet et al. (1994), Charbonnel et al. (1996), Mowlavi et al. (1998), Charbonnel et al. (1999).

²See also <http://obswww.unige.ch/Recherche/evol/Geneva-grids-of-stellar-evolution>

³Meynet & Maeder (1997), Maeder (1997), Maeder & Zahn (1998), Maeder (1999), Meynet & Maeder (2000), Maeder & Meynet (2000, 2001), Meynet & Maeder (2002), Maeder (2002), Meynet & Maeder (2003, 2005), Hirschi et al. (2004, 2005).

bourhood. Landi et al. (2007) analyse the solar abundance in a coronal flare and find $\epsilon(\text{Ne}) = 8.11 \pm 0.12$, in excellent agreement with the value found by Cunha et al. (2006).

Although the question is still not settled (see the discussion in Basu & Antia 2008), we have decided to use the new Asplund et al. (2005) abundances with the high Ne from Cunha et al. (2006). The X , Y , and Z values obtained by Asplund et al. (2005) correspond to the solar surface values at an age of ~ 4.7 Gyr, while our models start on the ZAMS. During the evolution of the Sun, the surface values have been modified by microscopic diffusion and by mixing in the outer convective zone. Patrick Eggenberger has thus computed a series of solar models to find the initial abundance and calibrate the mixing length needed to obtain the adequate values at the age of the Sun. This yields to $X = 0.720$, $Y = 0.266$, and $Z = 0.014$ for the mixing length $\alpha = l/H_P = 1.6$.

5.1.2 Tests and calibrations

There are not many free parameters in the code, but they had to be calibrated before the computation of the grid could be started.

Main sequence width

The overshoot parameter affects the MS width. Figure 5.1 compares the observed width with a few non-rotating models (3, 2, and $1.7 M_\odot$) computed with an overshoot parameter $d_{\text{over}} = 0.1 H_P$. A second $1.7 M_\odot$ model was computed with $d_{\text{over}} = 0.05 H_P$. From this comparison, we adopted the following values: $d_{\text{over}} = 0.1 H_P$ for $M > 1.7 M_\odot$, $d_{\text{over}} = 0.05 H_P$ for $1.25 \leq M \leq 1.7 M_\odot$, and 0 below $1.25 M_\odot$. Note that we limited the tests to this narrow mass range because the rotational velocities of the corresponding rotating models are low on average: the MS width depends mostly on the overshoot parameter, and barely on the rotational effect, so the value determined here is valid for the grids both with and without rotation.

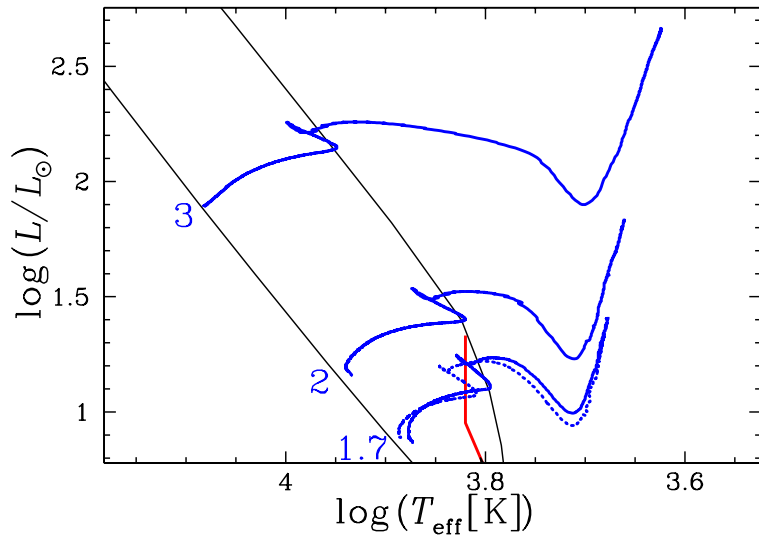


Figure 5.1: Calibration of the overshoot parameter by the MS width. Solid blue line: models computed with $d_{\text{over}} = 0.1 H_P$; dotted blue line: model computed with $d_{\text{over}} = 0.05 H_P$. The solid black line is the ZAMS-TAMS location obtained with the '92 grids. The red line is the observed TAMS.

Red giant branch location

As seen above (Sect. 5.1.1 on previous page), the mixing length for the outer convective zone was calibrated to fit the solar surface abundance values at the age of the Sun, but we had to test whether the red giant branch (RGB) location we obtain with this value ($\alpha = l/H_P = 1.6$) was correct or not. Figure 5.2 on next page shows the comparison of three models (1.5, 1.25, and $1.1 M_\odot$) with the red giants observed in four open clusters from the WEBDA database⁴:

⁴<http://www.univie.ac.at/webda/>

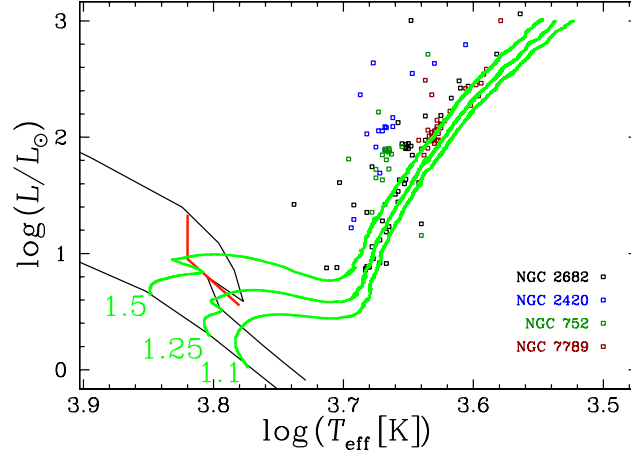


Figure 5.2: Test of the RGB location. Observations points are from the WEBDA database (see text).

NGC 7789:	1.7 Gyr	[Fe/H] = -0.08
NGC 2420:	1.1 Gyr	[Fe/H] = -0.26
NGC 752:	1.1 Gyr	[Fe/H] = -0.08
NGC 2682:	2.6 Gyr	[Fe/H] = +0.03.

The transformation from the colour-magnitude to the $\log L - \log T_{\text{eff}}$ system was done with the $B-V/T_{\text{eff}}$ relation from Flower (1996), corrected according to Sekiguchi & Fukugita (2000). The 1.1 and 1.25 $M_{\odot}$ models have turn-off (TO) ages older than the ages of the clusters (8.7 and 4.6 Gyr respectively), while the 1.5 $M_{\odot}$ model has a TO age comparable to the oldest cluster NGC 2682. The RGB of our models is located at the cool border of the clusters stars, which is satisfactory.

Surface enrichments

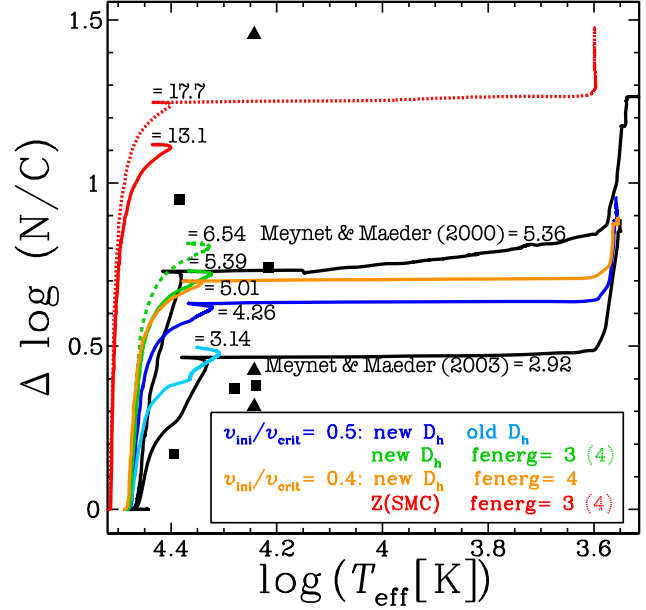
In the Geneva code, the coefficients for the rotational mixing are calculated on the basis of the internal parameters of the star. However, in the calculation of the shear diffusion coefficient D_{shear} there is one free parameter: the parameter α , which is the fraction of the excess energy in the shear that will contribute to the mixing (see Eq. (A.19) on page 128). In order to choose the value for this parameter, as well as the value of the initial v/v_{crit} , we have tested the enrichment level obtained at the end of the MS for 15 $M_{\odot}$ models. Figure 5.3 on next page shows the N/C ratio evolution as a function of the T_{eff} . It is interesting to note that the difference between the model taken from Meynet & Maeder (2000, with a surface enrichment of a factor of 5.36) and that taken from Meynet & Maeder (2003, with a surface enrichment of a factor of 2.92) differ only by the D_{shear} prescription from Talon & Zahn (1997) in the latter.

With this test, we decided to compute our rotating models with an initial $v/v_{\text{crit}} = 0.4$, and to set the α parameter to 4, which corresponds to a Richardson number $Ri_c = 1$. Note that in all these computations, the second free parameter A in Eq. (A.17) for the calculation of the horizontal turbulence coefficient D_h has been set to $A = 0.002$, which gives the best fit for the blue to red supergiants ratio.

Codes junction

The Geneva code uses the equation of state (EOS) of the ideal gas, which is suitable for massive and intermediate-mass stars. This EOS considers the stellar plasma as a mixture of perfect gases composed of atoms, ions, nuclei, and electrons. It takes into account the partial ionisation and the degeneracy of the gas. It is the resolution of the Saha equation that provides the degree of

Figure 5.3: Test of the enrichment levels, represented by the N/C ratio. The observational points are from Gies & Lambert (1992, squares), and Lennon (1993, triangles). The label “new D_h ” refers to the D_h prescription from Maeder (2003, see Eq. (A.17) on page 128). The parameter fenerg is the α parameter of Eq. (A.19) on page 128.



ionisation. For low-mass stars, however, this simple treatment is not valid anymore, because non-ideal effects become important, as the Coulomb interactions for example. Also the microscopic diffusion has time to develop and significantly change the surface abundances.

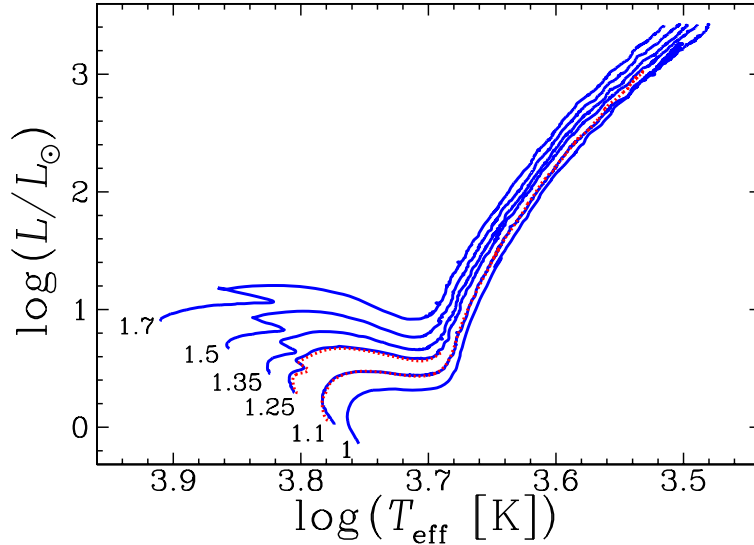


Figure 5.4: Test of the junction between `origin2006` (solid blue line) and the code used by Patrick Eggenberger for the smallest masses (dotted red line).

The lowest masses of our grids are thus calculated by Patrick Eggenberger whose code includes the MHD EOS (Hummer & Mihalas 1988; Mihalas et al. 1988; Daeppen et al. 1988), and the effects of microscopic diffusion (see Eggenberger et al. 2007, for a description). Figure 5.4 shows the junction between the two codes at masses 1.1 and 1.25 $M_\odot$ for the non-rotating models. In the case of the 1.1 $M_\odot$ models, the ZAMS location varies by 10% in luminosity and 1% in T_{eff} . In the case of the 1.25 $M_\odot$ models, the variation amounts to less than 0.5% in both L and T_{eff} , so we decided to set the transition between the two codes at that mass.

5.1.3 Physical ingredients

Models

The grids will cover the mass range between 0.4 and 120 $M_{\odot}$. In a first step, the massive stars will be evolved until the end of central C burning, the intermediate-mass stars until the early asymptotic giant branche (E-AGB), and the low-mass stars will be stopped just before the He flash. In a second step, the massive stars will be followed through the advanced stages to the pre-supernova, the intermediate-mass stars through the thermally-pulsing AGB (TP-AGB) phase, and the low-mass stars through the He flash and beyond.

Beside the solar metallicity $Z = 0.014$, there will be grids at $Z = 0.006$ ($\approx Z_{\text{LMC}}$), 0.002 ($\approx Z_{\text{SMC}}$), and also an over-solar one at $Z = 0.030$. An extension to lower metallicities is envisaged in the future ($Z = 10^{-5}$, $Z = 10^{-8}$). For the metallicities down to Z_{SMC} , we use a scaled solar mixture for the heavy elements. The initial He abundances for the various Z are determined by:

$$Y = Y_{\text{p}} + \frac{\Delta Y}{\Delta Z} \cdot Z \quad \text{with} \quad Y_{\text{p}} = 0.248 \quad \text{and} \quad \frac{\Delta Y}{\Delta Z} = 1.2857.$$

This value for Y_{p} is in agreement with the observations of Peimbert et al. (2007), within the error bars of the WMAP 5th year value (Komatsu et al. 2009). The value for $\Delta Y/\Delta Z$ is given by:

$$Y_{\odot} = 0.266 \stackrel{!}{=} 0.248 + 0.014 \frac{\Delta Y}{\Delta Z}.$$

Rotation

For the rotating models, the choice is set to fix the $v_{\text{ini}}/v_{\text{crit}}$ ratio rather than the initial equatorial velocity as was done until then. With such a large extent in mass, it would not have been possible to keep the same velocity for all the models, because whatever the choice it would have been over-critical for some models. Also it keeps a similar amplitude of the rotational effects throughout the whole mass and metallicity range.

The rotating models are computed with $v_{\text{ini}}/v_{\text{crit}} = 0.4$. For the massive stars at $Z_{\odot}$, this corresponds to initial velocities $v_{\text{ini}} \approx 250 - 400 \text{ km s}^{-1}$. We saw in Sect. 3.4.2 on page 43 that the initial velocity decreases quickly at the beginning of the main sequence (MS), so the probability to observe a star close to this value is very low. A more reasonable value to compare with observations is the mean MS velocity $\bar{v}_{\text{MS}}$. The same models have $\bar{v}_{\text{MS}} \approx 200 - 240 \text{ km s}^{-1}$, which is in good agreement with Huang & Gies (2006a) and Dufton et al. (2006), who find the peak of the observed velocity distribution probability at 200 and 250 km s^{-1} respectively for Galactic OB stars.

The description of the treatment of rotation in the Geneva code can be found in Appendix A.1 on page 125. In this new version of the code, we use Eq. (A.17) instead of Eq. (A.16) for the horizontal turbulence.

Mass loss

A variety of mass loss prescriptions exists in the literature, concerning stars of different types or at different evolutionary stages. Depending on the mass domain, we apply the following mass loss prescriptions:

- $M < 7 M_{\odot}$: the models are computed at constant mass during the MS, and then with the mass loss rates of Reimers 1975 (hereafter R75): $\dot{M} = 4 \cdot 10^{-13} \eta LR/M$ (with $\eta = 0.5$) on the red giant branch (RGB) or the AGB;
- $M = 7$: we use the de Jager et al. 1988 (hereafter dJ88) prescription during the MS, and then R75 as above;

- $7 < M < 15$: we use dJ88 throughout the whole evolution;
- $M \geq 15$: we use Vink et al. 2001 (hereafter VdKL01) during the MS, and then dJ88.

For the models that become a WR star, we apply the following prescription:

- during the WNL phase ($X_{\text{surf}} > 0$), we use the mass loss rates of Gräfenner & Hamann 2008 (hereafter GH08), and then we switch to the rates of Eldridge & Vink 2006 (hereafter EV06) for the WC and WO phase;
- above $40 M_{\odot}$, if the model enters the WNL phase during the MS, after H exhaustion we apply a fixed LBV mass loss rate $\dot{M} = 10^{-3} M_{\odot} \text{ yr}^{-1}$, and when $X_{\text{surf}} = 0$ we switch to the EV06 rates.

5.2 First grids: $Z = 0.014$ and $Z = 0.002$

To date, grids at two metallicities have been started: the solar one with $Z = 0.014$, and the SMC one with $Z = 0.002$. Tables of the global properties of the models are presented in Table 5.1 on page 111. A few models have not yet been calculated, others are still running. However, for most of the models, the MS evolution has been achieved, so we can already present some results.

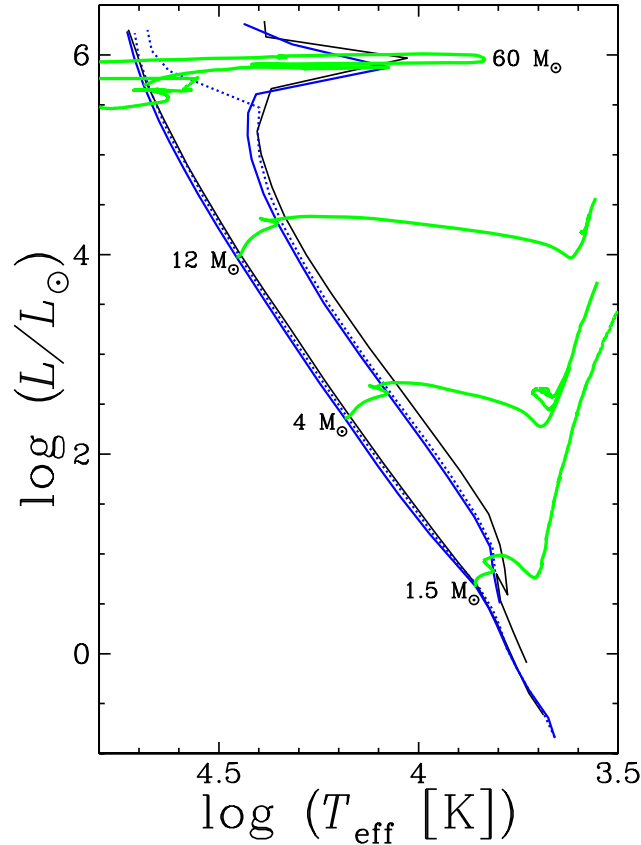


Figure 5.5: ZAMS location and MS width for all the models. Solid blue line: non-rotating $Z = 0.014$ models; dotted blue line: rotating models; solid thin black line: values from the $Z = 0.020$ '92 grids (Schaller et al. 1992).

Figure 5.5 shows the ZAMS location and the MS width given by the models. The rotating ZAMS is slightly shifted towards lower T_{eff} , as expected (see Sect. 3.4.1 on page 39). For the low- and intermediate-mass stars, the MS width is similar with and without rotation. For the massive

stars, the MS width increases slightly with rotation between 12 and 32 $M_{\odot}$. For the highest masses, rotation favours the entrance into the WR phase, so the MS width is reduced. Compared to the ZAMS location obtained with the '92 grids, the non-rotating ZAMS is slightly shifted to higher T_{eff} , as expected from the choice of a lower Z for the present grids (see Sect. 3.4.1 on page 38). The MS is wider in the old grids because of a larger overshoot parameter.

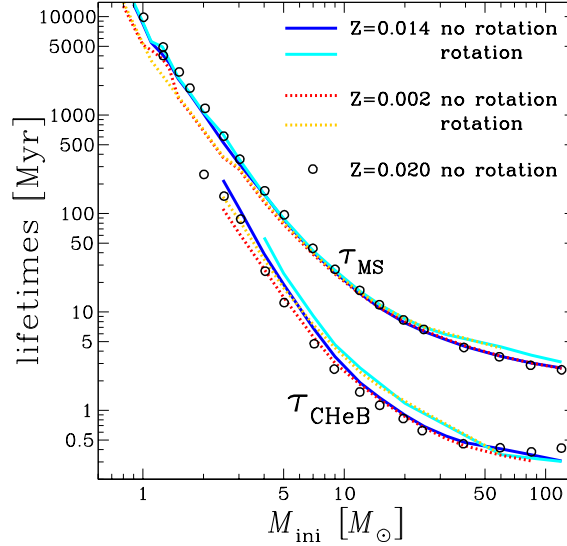
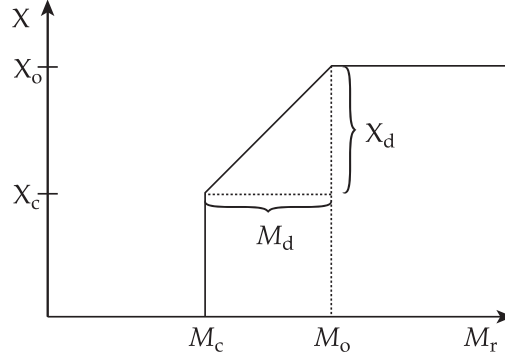


Figure 5.6: Lifetimes for all the models, on the MS (τ_{MS}) and during the core He burning (CHeB) phase (τ_{CHeB}). The black empty circles are the values for the solar metallicity of the '92 grids (Schaller et al. 1992).

Figure 5.6 presents the lifetimes on the MS (τ_{MS}) and on the CHeB phase (τ_{CHeB}) for all the models, rotating and non-rotating, at $Z = 0.014$ and $Z = 0.002$. In the domain of the massive stars, the difference in τ_{MS} due to metallicity is much smaller than the difference between rotating and non-rotating models at a given Z . On the contrary, the low-mass and the lightest intermediate-mass stars are more affected by metallicity than by rotation. In stars with $M > 9 M_{\odot}$, the opacity is dominated by that of electron scattering $\kappa_{\text{es}} \approx 0.2(1 + X)$, which is not much dependent in the heavy elements content. In the lower end of the mass spectrum, the cool atmospheres are very sensitive to the metal content: a high opacity reduces the luminosity and thus increases the lifetime. The solar non-rotating values for τ_{MS} compare well with the results of the '92 grids.

The HR diagrams (pages 112 to 115) reveal the sensitivity of the blue loop in the Cepheids mass domain. In the case of the non-rotating models at $Z = 0.002$, the loops are much wider and cover a larger mass range than at $Z = 0.014$ (3 to 9 $M_{\odot}$ vs 5 to 9 $M_{\odot}$). Rotation decreases the extent and the occurrence of the loops: at $Z = 0.014$ they are even completely suppressed, while at $Z = 0.002$, they are much shorter and occur only between 5 and 7 $M_{\odot}$. The reason why no Cepheids occur with the rotating models at $Z = 0.014$ has not yet been elucidated. Lauterborn et al. (1971) explain that a blue loop occurs when the core potential $\phi_c = M_c/R_c$ becomes smaller than a critical potential ϕ_{crit} . Due to an H abundance gradient outside the He core, the core potential ϕ_c must in fact be replaced by an effective potential $\phi_{\text{eff}} = h \cdot M_c/R_c$ with h determined by the mass of the zone in which exists an H gradient (M_d in Fig. 5.7), and the difference of H abundance in this zone (X_d): $\log h = 8.5 X_d M_d/M$. Any diffusion mechanism outside the He core will increase h and therefore ϕ_{eff} . It might not be able to become smaller than ϕ_{crit} and thus the star might stay along the Hayashi line. As a matter of fact, the D_h determined with the prescription from Maeder (2003) leads to a more efficient mixing at the border of the core, so a reduction of the loops compared to the models computed with the old prescription for D_h is expected. Note however that this mechanism alone does not explain the complete disappearance of the loops. More work is still needed to

Figure 5.7: Idealised H profile during shell H burning. M_c is the mass at the border of the He core, X_0 the initial H abundance (from Lauterborn et al. 1971).



understand this result. One possibility exists: the Cepheids population at solar metallicity are formed only by non- or very slowly-rotating stars. To date, we could not find enough information in the observations to draw a clear picture.

At $Z = 0.014$, the lowest mass to become a WR star⁵ is $60 M_\odot$ in the case without rotation, while rotation already allows the $32 M_\odot$ model to enter the WR stage during CHeB. Meynet & Maeder (2005) explain this by two effects of the internal mixing:

1. the freshly synthesised elements produced in the core are diffused towards the envelope,
2. the mass of the core is bigger.

Less mass needs to be removed before the enriched layers are encountered, and the diffusion of He favours a blue track in the HR diagram.

At $Z = 0.002$, the lowest mass to become a WR star is shifted to higher mass: $85 M_\odot$ for the non-rotating models. Here again, rotation lowers the limit, since the $60 M_\odot$ model becomes a WR star. We did not yet compute the 40 and $32 M_\odot$ models, so this is not a firm limit yet.

Figures 5.12 to 5.15 (pages 116 to 119) show the $\log T_c - \log \rho_c$ diagrams. For the time being, they are presented without any comment.

Further analysis will follow when we have completed the computations.

⁵Here, models with $\log T_{\text{eff}} > 4.0$ and $X_{\text{surf}} < 0.3$ are considered a WR stars

Table 5.1: Properties of the models at $Z = 0.014$ and $Z = 0.002$. The masses are in $M_{\odot}$, the velocities in km s^{-1} , and the durations in Myr. Y_{surf} , N/C, and N/O are given in mass fraction.

$Z = 0.014$															$Z = 0.002$																
M_{ini}	v_{ini}	$\bar{v}_{\text{MS}}$	τ_{MS}	M	end H burning v	Y_{surf}	N/C	N/O	τ_{CHeB}	M	end He burning v	Y_{surf}	N/C	N/O	M_{fin}	v_{ini}	$\bar{v}_{\text{MS}}$	τ_{MS}	M	end H burning v	Y_s	N/C	N/O	τ_{CHeB}	M	end He burning v	Y_s	N/C	N/O		
0.7	50	1.72	36203.9	0.700	0.48	0.222	0.29	0.11		0.7					0.8	0	0	12798.6	0.800	0	0.154	0.30	0.11								
0.8	50	1.85	22582.7	0.800	0.76	0.237	0.30	0.12		0.8	50	1.92	14047.9	0.800	1.05	0.8	50	1.92	14047.9	0.800	1.05	0.154	0.30	0.11							
0.8	50	1.85	22582.7	0.800	0.76	0.237	0.30	0.12		0.8	50	1.92	14047.9	0.800	1.05	0.8	50	1.92	14047.9	0.800	1.05	0.154	0.30	0.11							
0.9	0	0	13416.5	0.900	0	0.205	0.29	0.11	0.900	0	0	7938.6	0.900	0	0	0.9	0	0	7938.6	0.900	0	0.104	0.30	0.11							
0.9	50	2.98	14024.2	0.900	1.22	0.247	0.31	0.12		0.9	50	4.41	8462.8	0.900	1.97	0.9	50	4.41	8462.8	0.900	1.97	0.239	0.33	0.12							
1	1	0	8499.0	1.000	0	0.211	0.29	0.11		1	0	0	5165.9	1.000	0	1	0	0	5165.9	1.000	0										
1	50	2.79	8781.1	1.000	1.87	0.253	0.31	0.12		1	50	4.23	5300.5	1.000	2.84	1	50	4.23	5300.5	1.000	2.84	0.242	0.33	0.12							
1.1	0	0	5482.8	1.100	0	0.203	0.29	0.11		1.1	0	0	4589.7	1.100	0	1.1	0	0	4589.7	1.100	0	0.251	0.29	0.12							
1.1	50	3.72	5582.7	1.100	2.67	0.256	0.31	0.12		1.1	50	6.00	3573.5	1.100	3.62	1.1	50	6.00	3573.5	1.100	3.62	0.243	0.32	0.12							
1.25	0	0	4139.3	1.250	0	0.266	0.29	0.12		1.25	0	0	3716.4	1.250	0	1.25	0	0	3716.4	1.250	0	0.251	0.29	0.12							
1.25	150	144	4935.5	1.250	159	0.266	0.30	0.12		1.25						1.25															
1.35	0	0	3393.4	1.350	0	0.266	0.29	0.12		1.35	0	0	2958.6	1.350	0	1.35	0	0	2958.6	1.350	0	0.251	0.29	0.12							
1.5	0	0	2333.7	1.500	0	0.266	0.29	0.12		1.5	0	0	1497.1	1.500	0	1.5	0	0	1497.1	1.500	0	0.251	0.29	0.12							
1.5	148	128	2400.1	1.500	134	0.266	0.30	0.12		1.5	150	136	1547.5	1.500	146	1.5	150	136	1547.5	1.500	146	0.256	0.94	0.25							
1.7	0	0	1671.8	1.700	0	0.266	0.29	0.12		1.7	0	0	1082.6	1.700	0	1.7	0	0	1082.6	1.700	0	0.251	0.29	0.12							
1.7	155	132	1721.7	1.700	128	0.266	0.30	0.12		1.7	151	127	1111.6	1.700	123	1.7	151	127	1111.6	1.700	123	0.252	0.47	0.17							
2	0	0	1039.1	2.000	0	0.266	0.29	0.12		2	0	0	689.0	2.000	0	2	0	0	689.0	2.000	0	0.251	0.29	0.12							
2	164	136	1072.8	2.000	128	0.266	0.30	0.12		2	185	151	708.7	2.000	143	2	185	151	708.7	2.000	143	0.252	0.76	0.23							
2.5	0	0	532.9	2.500	0	0.266	0.29	0.12	219.8	2.489	0	0.284	1.42	0.36	2.5	0	0	371.8	2.500	0	0.251	0.29	0.12		111.5	2.476	0	0.264	1.68	0.38	
2.5	173	145	637.7	2.500	133	0.268	1.14	0.28		2.5	199	157	380.9	2.500	141	2.5	199	157	380.9	2.500	141	0.251	0.78	0.23	148.0	2.469	5.19	0.279	5.93	0.60	
3	0	0	323.2	3.000	0	0.266	0.29	0.12	113.8	2.987	0	0.285	1.51	0.39	3	0	0	282.5	3.000	0	0.251	0.29	0.12		63.47	2.973	0	0.255	1.43	0.33	
3	181	144	360.0	3.000	125	0.267	0.88	0.24		3	202	166	290.9	3.000	149	3	202	166	290.9	3.000	149	0.253	5.15	0.47	86.53	2.960	4.31	0.276	11.88	0.67	
4	0	0	157.7	4.000	0	0.266	0.29	0.12	38.99	3.965	0	0.283	1.52	0.40	4	0	0	131.1	4.000	0	0.251	0.29	0.12		27.33	3.967	0	0.251	1.15	0.29	
4	194	154	157.0	4.000	134	0.266	0.92	0.25	57.189	3.966	3.17	0.291	2.08	0.50	4	218	175	135.9	4.000	158	0.252	5.70	0.49	34.33	3.936	3.25	0.264	11.00	0.63		
5	0	0	87.11	5.000	0	0.266	0.29	0.12	19.13	4.960	0	0.279	1.48	0.38	5	0	0	77.56	5.000	0	0.251	0.29	0.12		14.05	4.962	0	0.251	1.01	0.27	
5	204	162	89.32	5.000	145	0.266	1.03	0.26	24.638	4.950	2.48	0.289	2.76	0.51	5	228	182	80.78	5.000	164	0.252	6.85	0.52	17.62	4.914	2.75	0.261	11.78	0.64		
7	0	0	40.57	6.973	0	0.266	0.29	0.12	6.919	6.891	0	0.280	1.55	0.40	7	0	0	38.56	6.999	0	0.251	0.29	0.12		5.610	6.940	0	0.251	1.08	0.28	
7	220	173	41.96	6.976	163	0.267	1.82	0.35	9.190	6.830	1.62	0.287	3.89	0.56	7	245	194	40.03	6.999	187	0.252	11.58	0.57	7.475	6.836	2.04	0.263	17.62	0.68		
9	0	0	25.44	8.959	0	0.266	0.29	0.12	3.532	8.669	0	0.283	1.63	0.43	9	0	0	24.23	8.998	0	0.251	0.29	0.12		3.028	8.894	0	0.251	1.23	0.29	
9	233	184	26.60	8.945	177	0.267	1.92	0.36	4.646	8.562	1.26	0.292	3.88	0.58	9	257	205	24.98	8.996	201	0.253	11.52	0.58	4.328	8.337	1.38	0.265	17.11	0.70		
12	0	0	15.40	11.955	0	0.266	0.29	0.12	1.915	11.498	0	0.303	1.91	0.51	12	0	0	15.16	11.992	0	0.251	0.29	0.12		1.829	11.921	0	0.251	1.07	0.27	
12	249	200	16.16	11.932	201	0.267	1.95	0.36	2.696	10.391	0.65	0.296	3.40	0.55	12	273	217	15.82	11.968	218	0.254	10.53	0.59	2.448	11.495	1.39	0.302	19.12	0.88		
15	0	0	11.04	14.920	0	0.266	0.29	0.12	1.344	13.054	0	0.311	2.45	0.57	15	0	0	11.03	14.974	0	0.251	0.29	0.12		1.286	14.877	0	0.251	0.34	0.13	
15	261	212	11.92	14.919	234	0.269	2.03	0.38							15	285	228	11.73	14.913	236	0.257	13.42	0.64	1.734	14.342	1.14	0.323	23.22	1.00		
20	0	0	7.766	19.677	0	0.266	0.29	0.12	0.899	14.613	0.11	0.427	7.58	1.29	20	303	252	7.942	19.909	0	0.251	0.29	0.12		0.869	19.686	0	0.251	0.29	0.12	
20	277	225	8.514	19.652	254	0.272	1.54	0.35	1.183	11.658	0.11	0.427	7.58	1.29	20	303	252	8.631	19.902	278	0.260	7.72	0.63	1.242	18.277	0.76	0.374	17.95	1.41		
25	0	0	6.309	24.195	0	0.266	0.29	0.12	0.688	15.032	0	0.496	29.23	3.67	25	0	0	6.468	24.785	0	0.251	0.29	0.12		0.663	24.313	0	0.251	0.29	0.12	
25	285	233	6.978	24.107	288	0.267	0.58	0.19							25	315	266	7.059	24.760	301	0.260	4.19	0.56	0.957	15.433	0.19	0.492	27.41	3.84		
32	0	0	5.224	30.117	0	0.266	0.29	0.12	0.549	17.127	0	0.564	102.7	10.75	32	0	0	5.316	31.525	0	0.251	0.29	0.12		0.517	29.692	0	0.251	0.29	0.12	
32	299	240	5.921	29.694	311	0.283	1.09	0.36	0.586	9.054	123	0.272	0.00	0.00	32	0	0														
40	0	0	4.451	36.499	0	0.266	0.29	0.12	0.470	15.790	0	0.570	166.2	14.25	40	0	0	4.527	39.128	0	0.251	0.29	0.12		0.435	29.807	0	0.402	6.37	1.64	
60	0	0	3.539	36.147	0	0.497	145.9	61.42							60	0	0	3.967	57.857	0	0.251	0.29	0.12		0.350	37.252	0	0.662	75.10	98.95	
60	346	205	4.436	41.995	130	0.808	67.95	36.73	0.357	16.893	67	0.262	0.00	0.00	60	377	328	4.286	56.347	185	0.422	4.21	1.46	0.360	18.769	50	0.353	0.00	0.00		
85	0	0	3.030	49.164	0	0.628	112.8	70.03	0.354	18.090	0	0.256	0.00	0.00	85	0	0	3.030	81.033	0	0.251	0.29	0.12		0.307	49.040	0	0.726	71.88	100.71	
85	374	203	3.642	55.631	114	0.875	84.67	78.22	0.328	24.914	56	0.256	0.00	0.00	85																
120	0	0	2.692	67.247	0	0.725	97.23	75.27	0.306	31.012	0	0.249	0.00	0.00	120	0	0	2.661	112.067	0	0.251	0.29	0.12								
120	394	196	3.103	70.013	71	0.916	80.30	86.31	0.303	35.027	48	0.240	0.00	0.00	120																

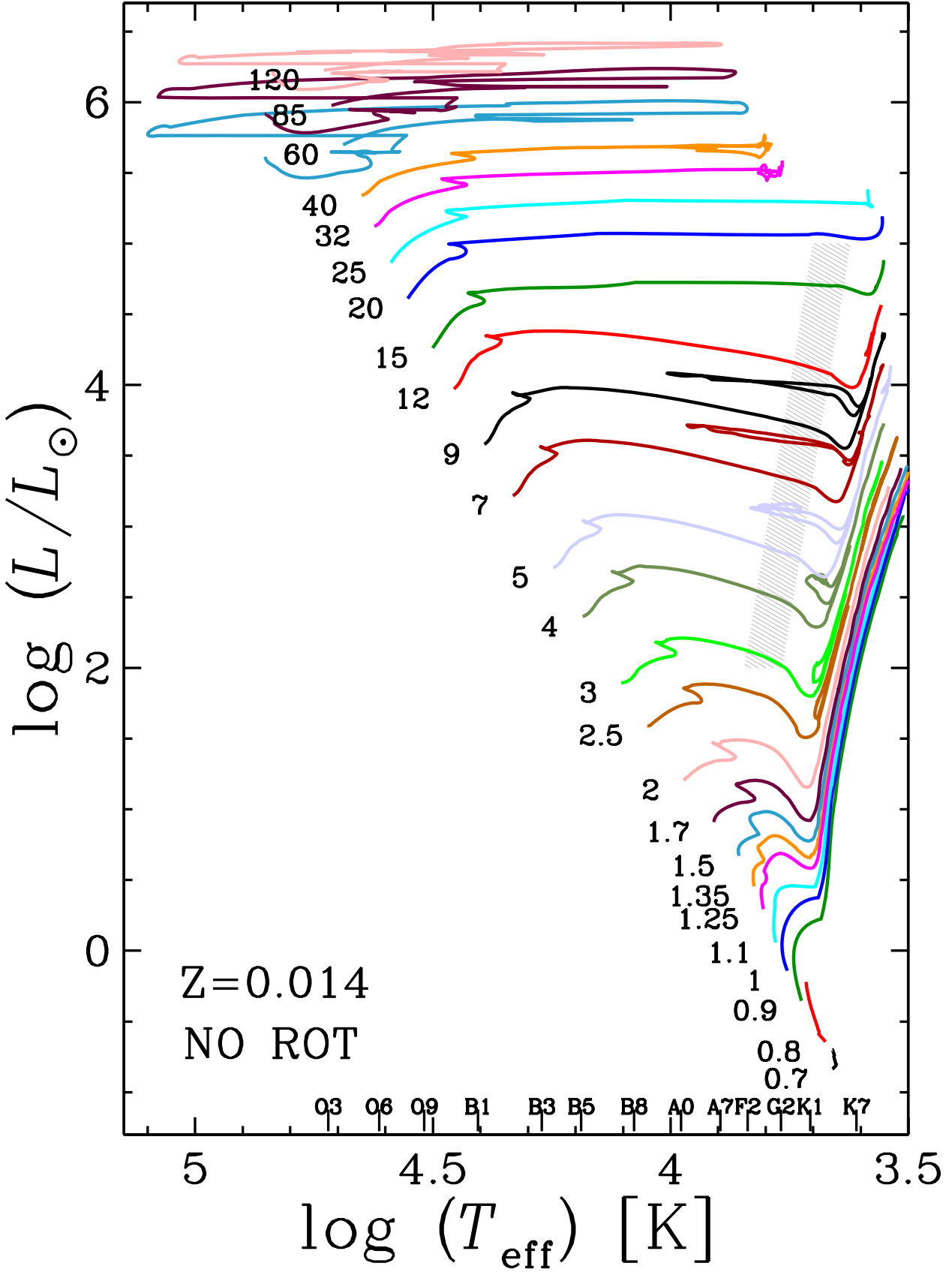
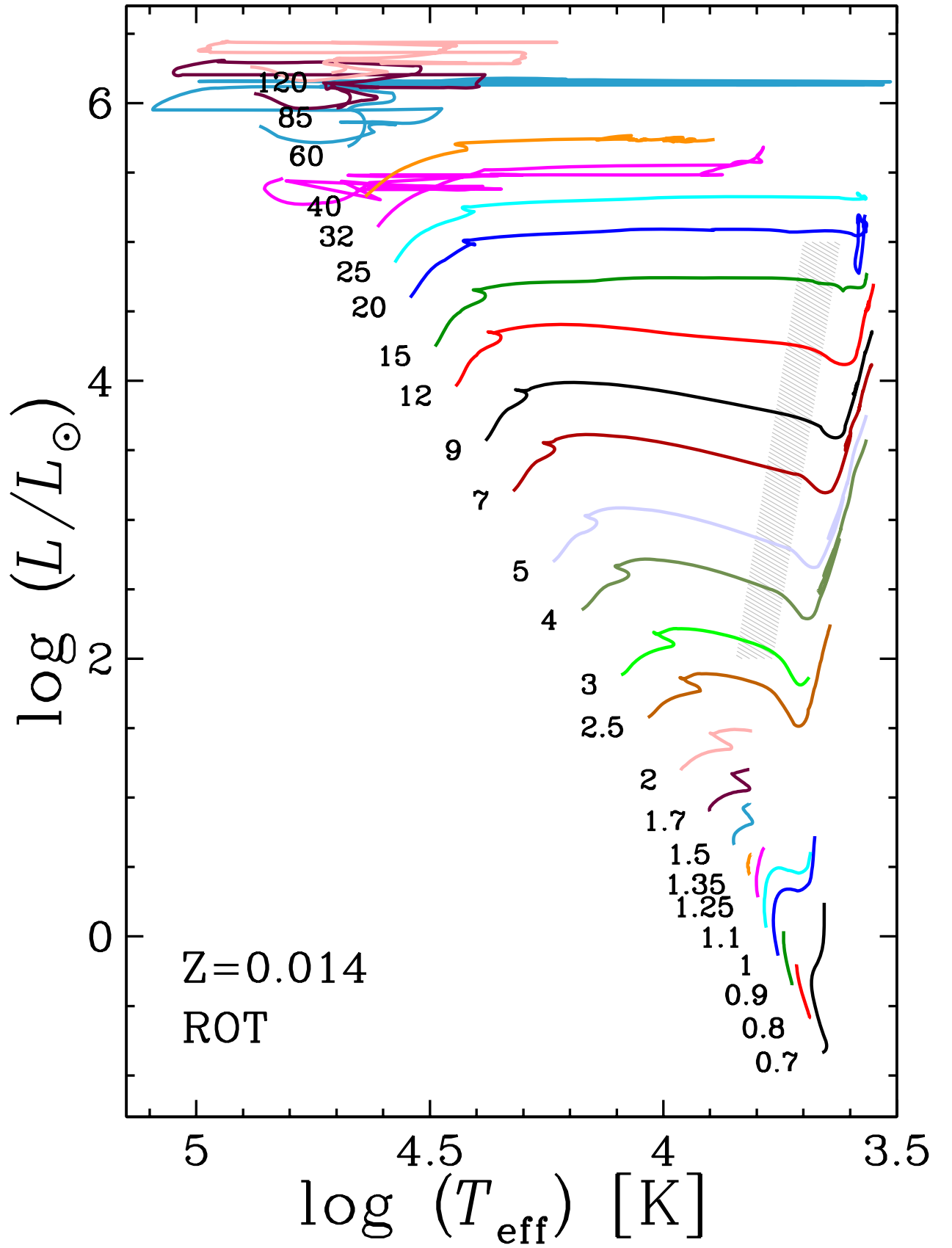


Figure 5.8: Theoretical HR diagram of all the non-rotating models at $Z = 0.014$. The models with $M \geq 1.7 M_{\odot}$ are calculated with an overshoot parameter $d_{\text{over}} = 0.1 H_P$, the models with $1.25 M_{\odot} \geq M \geq 1.5 M_{\odot}$ with an overshoot parameter $d_{\text{over}} = 0.05 H_P$, and 0 below. The Cepheids instability strip (grey shaded area) is plotted at the Galactic value given by Tammann et al. (2003). The spectral types are taken from Lang (1992).

Figure 5.9: Same as Fig. 5.8 for the rotating models, with $v_{\text{ini}}/v_{\text{crit}} = 0.4$.

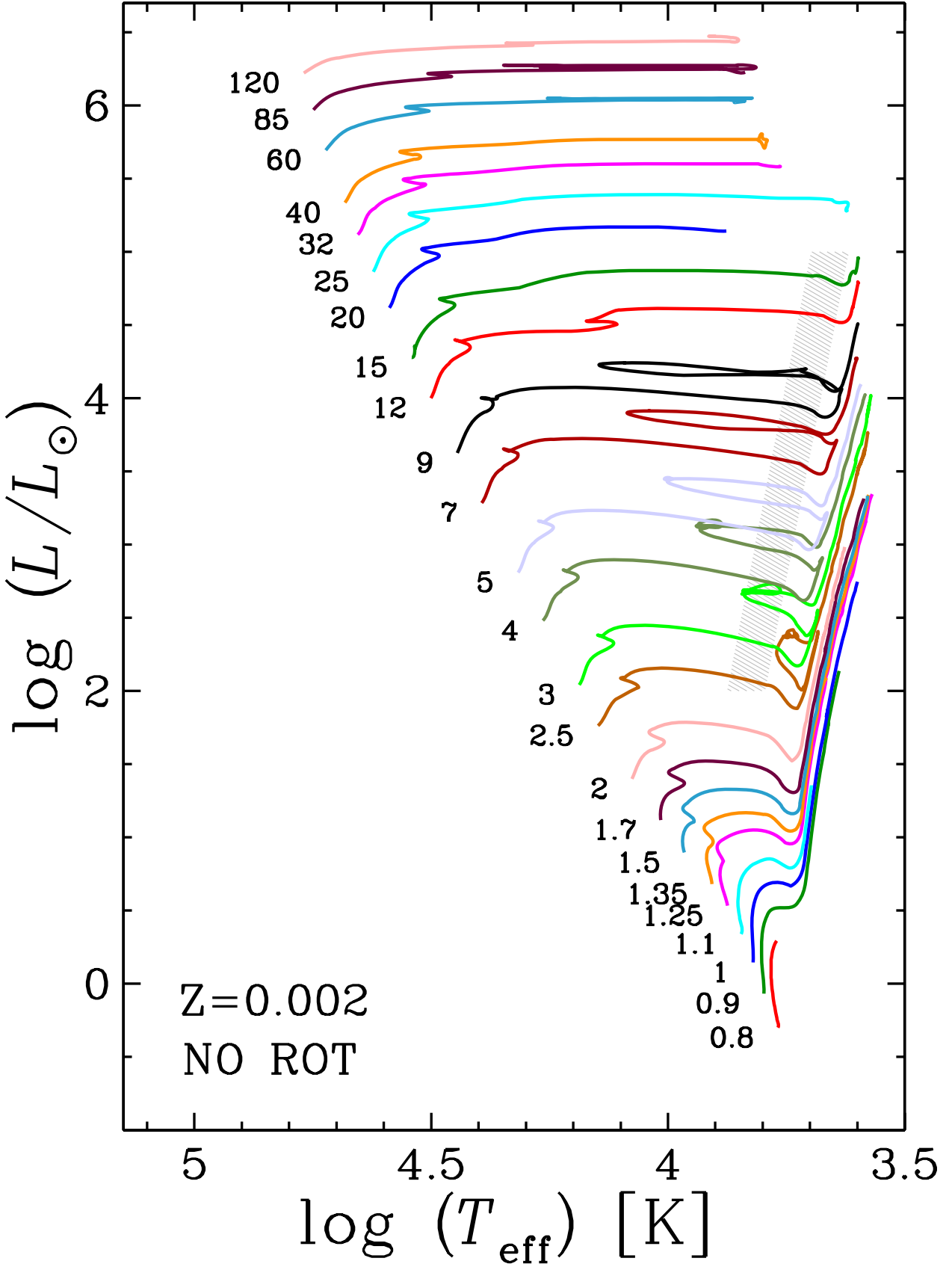


Figure 5.10: Theoretical HR diagram of all the non-rotating models at $Z = 0.002$. The models with $M \geq 1.7 M_{\odot}$ are calculated with an overshoot parameter $d_{\text{over}} = 0.1 H_P$, the models with $1.25 M_{\odot} \geq M \geq 1.5 M_{\odot}$ with an overshoot parameter $d_{\text{over}} = 0.05 H_P$, and 0 below. The Cepheids instability strip (grey shaded area) is plotted at the SMC value given by Tammann et al. (2003).

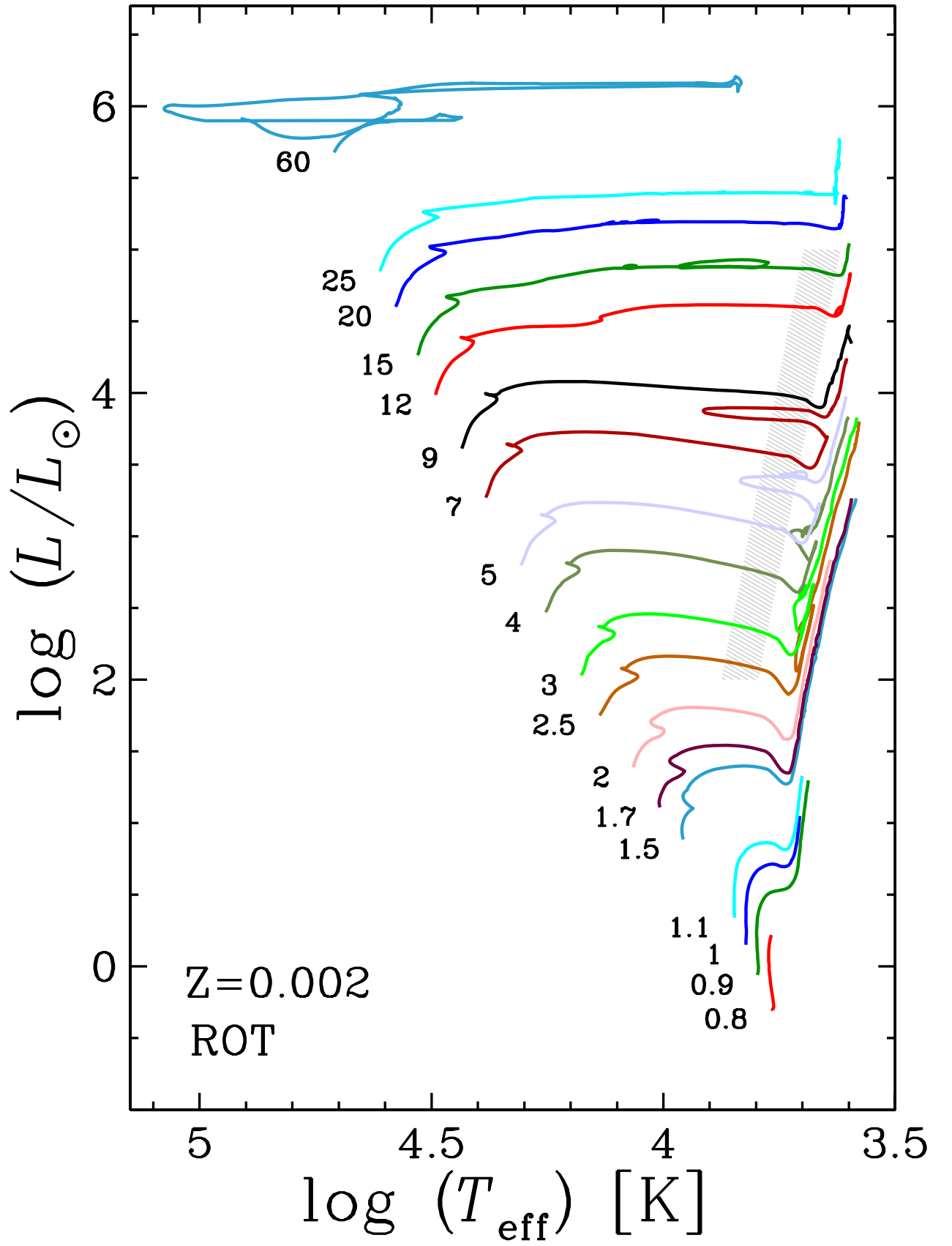


Figure 5.11: Same as Fig. 5.10 for the rotating models, with $v_{\text{ini}}/v_{\text{crit}} = 0.4$.

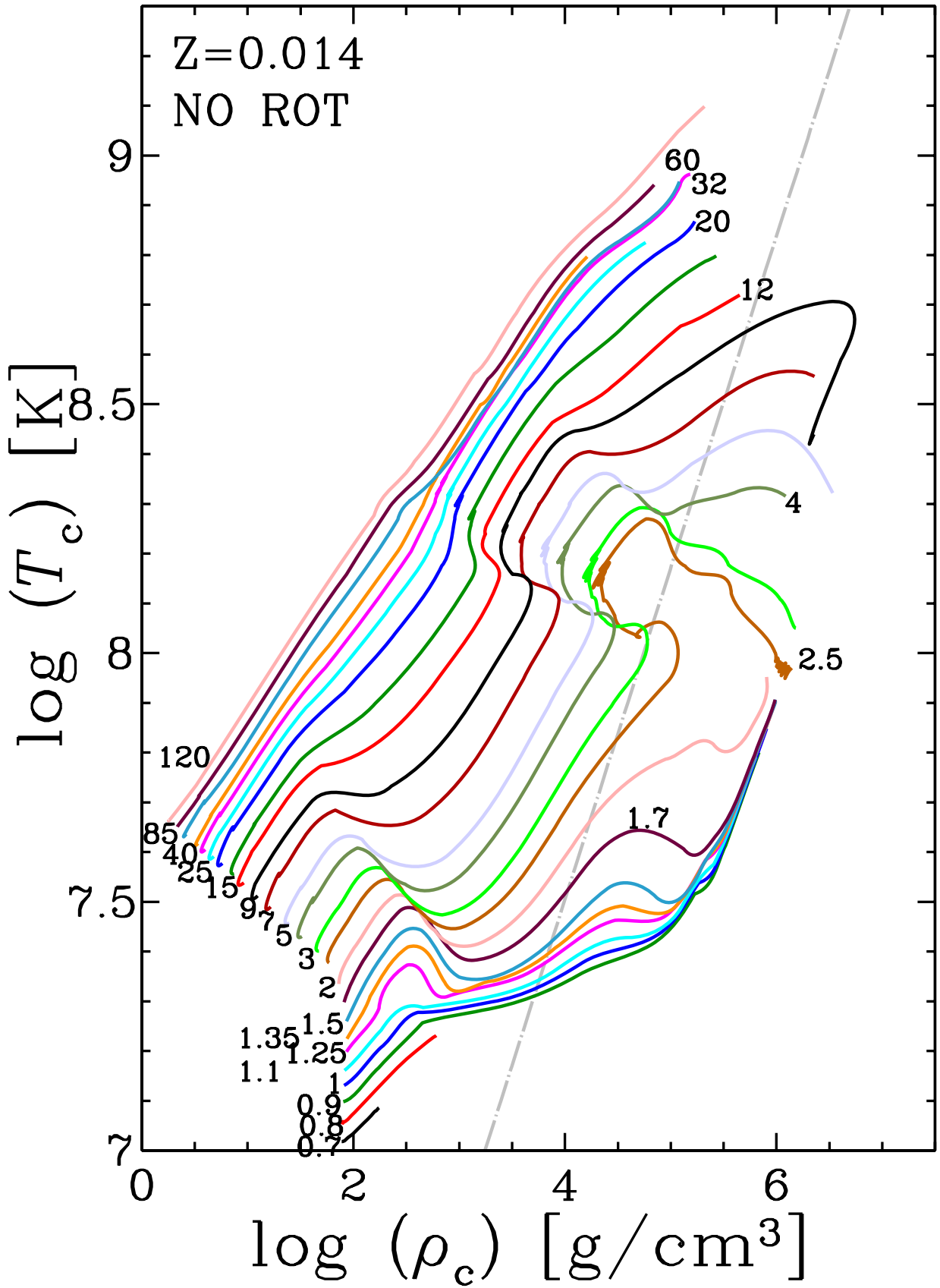


Figure 5.12: $\log T_c - \log \rho_c$ diagram of all the non-rotating models at $Z = 0.014$. The grey dot-dashed line is the limit between ideal and degenerate (non-relativistic) gas.

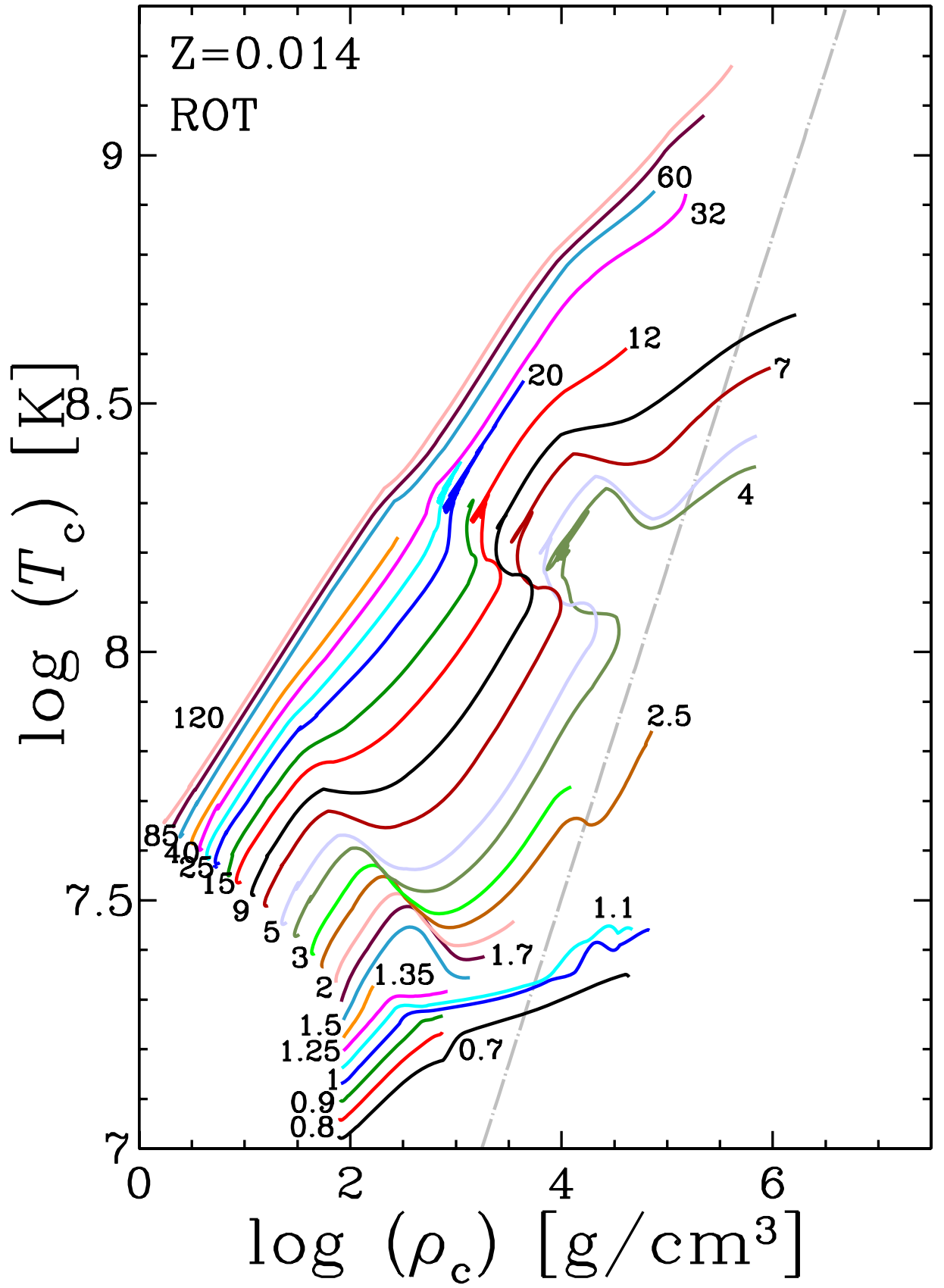


Figure 5.13: Same as Fig. 5.12 for the rotating models, with $v_{\text{ini}}/v_{\text{crit}} = 0.4$.

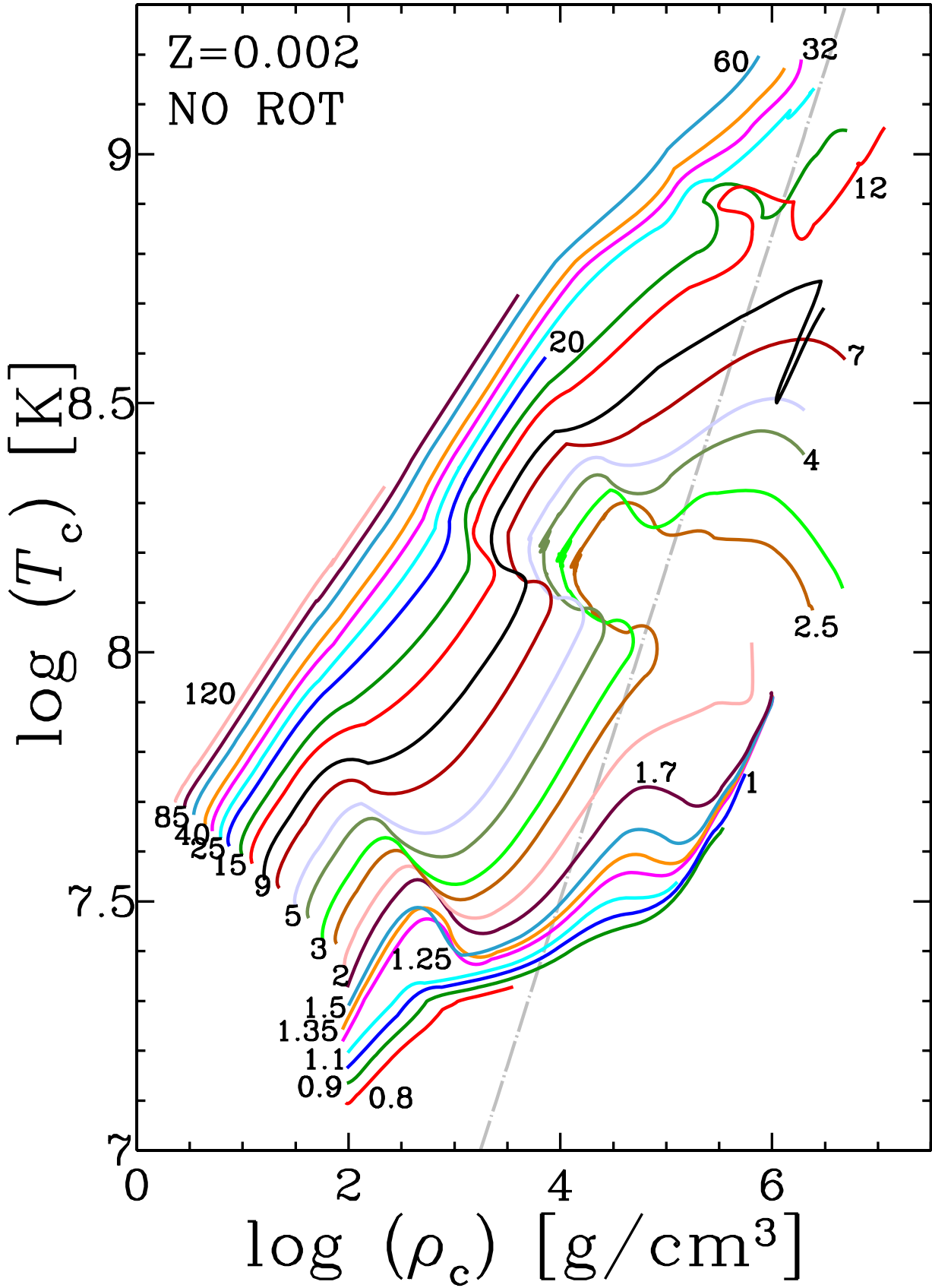


Figure 5.14: $\log T_c - \log \rho_c$ diagram of all the non-rotating models at $Z = 0.002$. The grey dot-dashed line is the limit between ideal and degenerate (non-relativistic) gas.

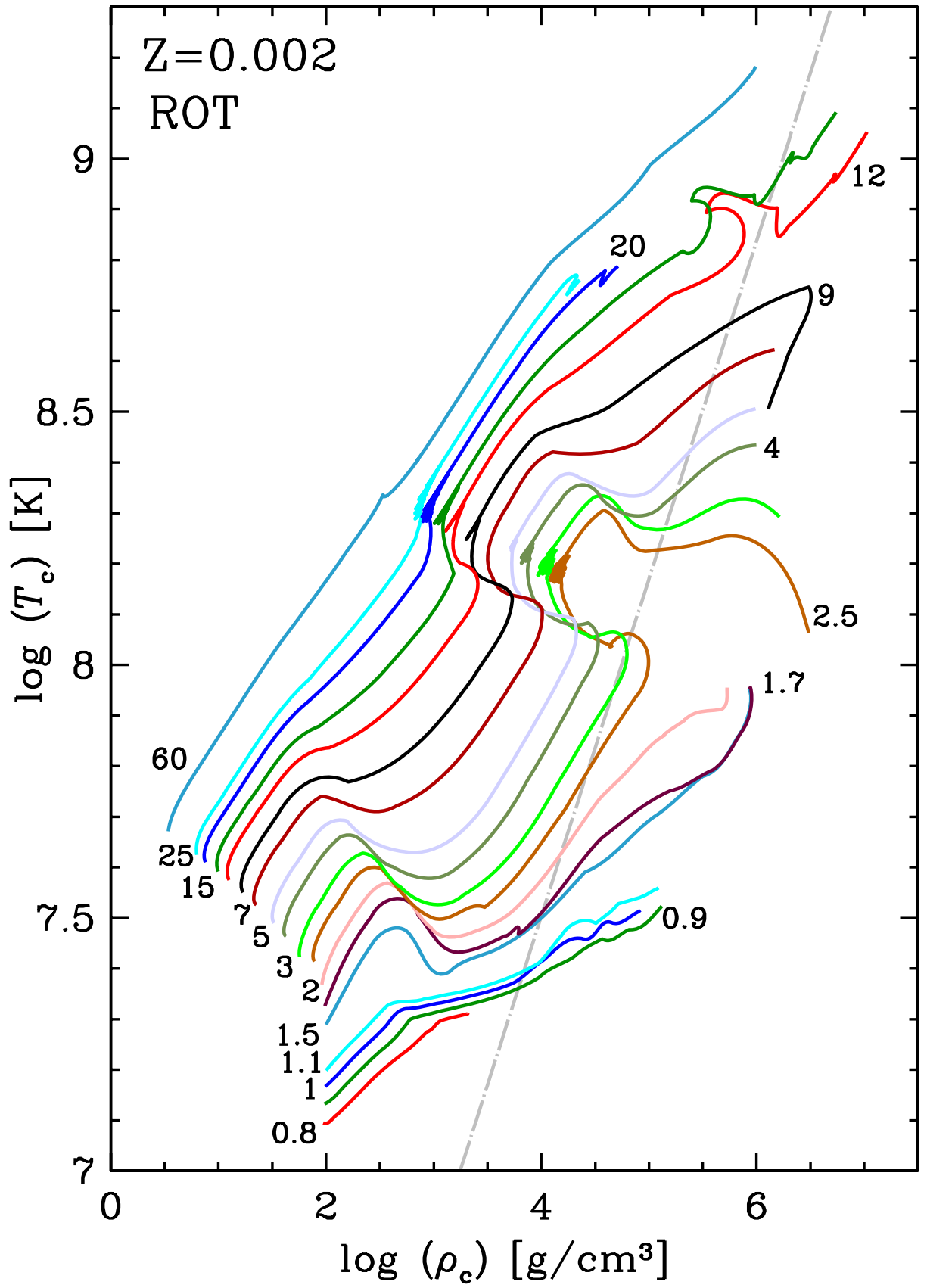


Figure 5.15: Same as Fig. 5.14 for the rotating models, with $v_{\text{ini}}/v_{\text{crit}} = 0.4$.

Conclusion and perspectives

6.1 Rotation at various metallicities

The effects of rotation on the evolution of stars at various metallicities Z have been studied with a coherent set of stellar models. The evolution of the equatorial velocity results from a delicate interplay between the mass loss, which removes angular momentum from the star, and the meridional circulation, which couples the core to the envelope and brings angular momentum towards the surface. Both mechanisms are sensitive to the metallicity, both showing a decrease with Z . It has been shown that as long as $Z \neq 0$, the effects of rotation are stronger with decreasing Z . The stars are more prone to reach the critical limit during the main sequence (MS), which leads to a mechanical mass loss, and the surface enrichments are stronger, which enhances the radiative winds.

The results of the models have been compared to the observed population of B stars in the Galaxy and the Magellanic Clouds. In the view that Be stars are B type stars rotating close to the critical limit, it has been shown that to account for the observed $\text{Be}/(\text{B} + \text{Be})$ ratio, one needs to suppose that the ejection disc occurs already when the star is rotating at 70% of the critical limit. A better agreement is also obtained if one supposes that the distribution of initial rotational rates are shifted to higher velocities at low metallicity.

After the MS, rotation favours a redwards evolution in the Hertzsprung-Russell (HR) diagram. The Ω -gradients are stretched and the shear mixing becomes very strong. The outer convective zone deepens in mass and dredges-up freshly synthesised heavy elements. The surface metallicity increases in a dramatic way, and a heavy mass loss ensues. The wind composition is rich in both H- and He-burning products and shows similarities with the abundance patterns observed at the surface of the metal-poor halo stars.

6.2 Evolution of Population III stars

The effects of rotation on the evolution of Population III (Pop III) stars have been studied. The Geneva code has been modified for this purpose. It has been shown that the trend encountered at very low metallicity concerning the effects of rotation is no longer valid at $Z = 0$. The stars are so compact that the meridional circulation becomes very weak, and the stars evolve almost in a regime of local angular momentum conservation. When the critical limit is reached, the removal of only very little mass is sufficient to bring the surface back to sub-critical values. A forthcoming study will address this particular point: what is the metallicity dependence of the wind that is due to the centrifugal force, linked to the strength of the meridional circulation?

After the MS, the models remain in the blue part of the HR diagram, or evolve very slowly

towards the red. This is due to the fact that the stars are already hot enough to burn some He during the MS: there is no structural readjustment at H exhaustion. The Ω -gradients are not stretched as it is the case at non-zero metallicity, and the surface enrichment remains modest. The outer convective zone does not deepens significantly in mass, so the dredge-up is not efficient: the evolution proceeds with a surface metallicity that is always much lower than 10^{-6} , and the mass loss remains low.

The rotational mixing at the border of the He core brings some C and O in the H-burning shell and boosts the CNO cycle, producing large amounts of primary nitrogen. In the case of the non-rotating models, this production occurs only in very specific mass domains in which the structure allows a contact of the growing He core with the H shell. Pre-SN yields have been calculated for the rotating and non-rotating models. The difference between rotating and non-rotating yields for ^{14}N and the heavy CNO isotopes can reach orders of magnitude. The rotating yields have been included in a chemical evolution model for the Galaxy. They predict a decrease of the N/O and C/O ratio at extremely low metallicity ($Z \leq 6 \cdot 10^{-7}$).

The stars end their life having lost less than 10% of their initial mass. This situation allows the core to keep a high amount of angular momentum until the pre-supernova stage. Strong deviations to the sphericity are expected, which have been related with powerful explosions. Although the rotating Pop III stars do not lose much more mass than their non-rotating counterparts, they could undergo a SN explosion where none was expected. The follow-up of our models beyond the pre-SN stage by a multi-D code is in project (see below).

The influence of the adjunction of a magnetic field on the mass loss of a very massive Pop III star has been studied. The field provides the strong core-envelope coupling that is missing in Pop III stars, and allows a strong mass loss. Would this be sufficient to explain the absence of the typical signature of pair-instability supernovae in the very metal-poor stars of the Galactic halo? The exploration of a wider mass domain is needed before we can give a firm answer. Anyway, this study shows the interest to further address the role of a magnetic field in the evolution of Pop III stars.

The modelling of Pop III stars has been used as a physics laboratory to explore two cosmological aspects: the role of the dark matter (DM) in the formation of Pop III stars, and the possible variation of the fine structure constant. Concerning the DM influence, it has been shown that DM densities above $2\text{-}5 \cdot 10^{10} \text{ GeV cm}^{-3}$ can lead to the complete freeze-out of the evolution on the ZAMS for times longer than the age of the Universe. With regard to the fine structure constant variation, it has been shown that stellar constraints can be put on the possible variation amplitude at a redshift $z \approx 15 - 20$. The link between the fine structure value and that of the deuterium binding energy leads to the determination of varying 3α rates, and their integration in a stellar evolution code drives very different evolution characteristics. For very weak 3α rates, the star ends the central He-burning phase with a core that is composed purely of ^{24}Mg , while for very strong 3α rates, the core is purely composed of ^{12}C . In both case, no ^{16}O is produced, which is highly doubtful and allows the elimination of such values. For the case where no such extreme results are obtained, it would be interesting to follow the evolution until the pre-SN phase to see how the abundance anomalies are transmitted through the advanced phases and affect the final yields.

6.3 New grids of stellar models

The Geneva group as a whole has undertaken the publication of new grids of stellar models, both with and without rotation. The code has been reformed and updated for this purpose. Many tests have been performed to control the accuracy of the results obtained with the new version. The calibration and determination of some free parameters of the code have been carried out. The calculation of the models have been started for two metallicities: the solar one at $Z = 0.014$, and

the small Magellanic Cloud one at $Z = 0.002$.

This work is still in progress, so only preliminary results could be given.

6.4 Perspectives

6.4.1 Work on the code

Numerical aspects

The Geneva code is our essential tool, and we must dedicate some of our time to its care. One of my priorities after the completion of this thesis is to work on the portability of the code. We need a tool that is stable on all platforms, independently of any compiler, so we can be free of the institute choice about hard- or software. It will be also an opportunity to clean some parts of the code that make the computations too heavy.

Another project on a longer term is to parallelise the code. The computation of a full evolutionary sequence with the inclusion of all the rotation effects is extremely time consuming. The speed of the processors are not really increasing anymore, so we need to spread the calculation on several processors to really gain some time.

Physical aspects

An important project regarding the code is for the group to increase the reaction network so we can follow the s -process. The network we have is adapted to follow accurately the evolution of the energy release and the structure adjustment. The s -process elements can be added as a parallel network by post-processing. The possibility to follow the s -process is of particular interest in the case of very low-metallicity models: the strong primary nitrogen production of the rotating models is accompanied by the production of ^{13}C and ^{22}Ne that are efficient neutrons providers. Some EMP halo stars present anomalies in the s -process element abundances, so it would be tempting to study what our models would predict.

6.4.2 Pop III stars

We saw that an important aspect in the formation of Pop III stars is the accretion onto the protostar. If the protostar is rotating close to the critical limit, we expect that its radiative flux will be anisotropic, so the feedback on the in-falling matter will be very different than in the spherical case. This aspect will be the subject of a study already undertaken in collaboration with Jonathan Tan (University of Florida, USA), Christopher McKee (University of California, Berkeley) and Cyril Georgy (Geneva Observatory).

In our study of Pop III stars, we addressed the question of their structure and evolution, as well as their nucleosynthetic signature, but we left entirely untouched the aspect of their ionisation power. Rotation changes the lifetime, the luminosity and the colour of the stars, so it must affect the number of ionising photons that are emitted during their life. In the case of the Pop III stars, this aspect is of particular interest, since they are responsible for the reionisation of the early Universe. I plan to address this point in a future study.

The model we have calculated with the inclusion of a magnetic field has shown that the evolution is strongly affected by the magnetic coupling. It allows a strong mass loss, and may affect deeply the yields and the outcome at the final stage. Although it is not yet clear whether magnetic fields were present in the early Universe, it may be an interesting ingredient to include in a simulation. I plan to explore this aspect in a near future.

At the pre-supernova stage, our rotating models have kept a large amount of angular momentum in the core. This should most likely change the dynamics of the explosion. Only multi-

dimensional studies could provide a firm answer to this supposition. In collaboration with Rolf Walder (Université de Lyon, France) we plan to integrate the pre-SN structures obtained for the Pop III models in a 3D hydrodynamical code and study this aspect.

To date, no direct observations of Pop III stars have been possible, so we cannot put observational constraints on our models. With the forthcoming LAMOST and SSS surveys, we expect to multiply by a factor of ~ 100 the number of EMPs, and thus to get statistically significant trends of the abundances patterns at extremely low metallicity. There is also the hope to observe stars with $[\text{Fe}/\text{H}] < -6.0$ or even -7.0 (Christlieb et al. 2008). They will not yet be true Pop III stars, but they might be their direct descendants and allow to put more accurate nucleosynthetic constraints on the stellar models. In another domain of astronomy, gravitational lensing has allowed to detect extremely high-redshift galaxies. A few dedicated instruments are in the process of being installed in the purpose of detecting $\text{Ly}\alpha$ emitters or luminous drop-outs at $z > 7$. Among them, we can mention the Dark Age z Lyman-alpha Explorer (DAZLE¹), that is being installed on the VLT, the Gemini Genesis Survey (GGS²) on Gemini South, or WFC3³ on Hubble. We can expect that the emission of the highest- z galaxies is dominated by the flux of Pop III stars, and that through population synthesis models we can have insights on the characteristics of the primordial stars.

6.4.3 Grids

The calculations for the two first grids will be completed within the next months. They will be followed by the computations of grids at other metallicities ($Z = 0.030, 0.006, 10^{-5}$). After the first step of calculations up to the end of carbon burning, the models will be followed further, to the pre-supernova stage for the massive stars, the thermally-pulsing AGB phase for the intermediate-mass models, and beyond the He flash for the low-mass stars.

The publication of the stellar grids are important, but nowadays it is also essential to provide a web interface where the astronomical community can get the data they need. The interface must provide HR locations not only in the theoretical units of $\log T_{\text{eff}} - \log L$, but also in the observers units of colour - magnitude, where several filters must be available. The interface must also provide isochrones. An important project besides the completion of grids themselves is to build this interface.

¹<http://www.ast.cam.ac.uk/optics/dazle>

²http://geminigenesis.org/?page_id=5

³<http://www.stsci.edu/hst/wcf3>

Numerical aspects

A.1 The treatment of rotation in the Geneva code

I will here recall the main developments of the effects of rotation for their inclusion in the Geneva code. The details of the derivations can be found in the series of papers published by the Geneva group under the generic name: “Stellar evolution with rotation”.

A.1.1 The Roche model

In the code, rotation is treated within the frame of the Roche model; *i.e.* the gravitational potential is approximated by GM_r/r where M_r is the mass inside the equipotential with a mean radius r . The radius r that labels each equipotential is defined by $r = (V_r/(4/3\pi))^{1/3}$ where V_r is the volume (deformed by rotation) inside the equipotential considered.

In the Roche model and for a conservative rotation law (*i.e.* such that the centrifugal force can be derived from a potential)¹, the shape of a meridian at the surface of an equipotential is given by couples of r and θ values satisfying the following equation:

$$\frac{GM_r}{r} + \frac{1}{2}\Omega_r^2 r^2 \sin^2 \theta = \frac{GM_r}{r_{\text{pol}}}, \quad (\text{A.1})$$

where r is the radius at colatitude θ , Ω_r the angular velocity at that radius, M_r the mass inside the considered equipotential surface, and r_{pol} the radius at $\theta = 0$. Thus the shape of a given equipotential is determined by three parameters M_r , Ω_r , and r_{pol} . The first two, M_r , and Ω_r are independent variables. The third one is derived from the first two and the equations of stellar structure. Setting $x = \left(\frac{GM_r}{\Omega_r^2}\right)^{-1/3} r$, one can write Eq. (A.1) (see Kippenhahn & Thomas 1970)

$$\frac{1}{x} + \frac{1}{2}x^2 \sin^2 \theta = \frac{1}{x_{\text{pol}}}. \quad (\text{A.2})$$

With this change of variable, the shape of an equipotential is uniquely determined by only one parameter x_{pol} .

With M the total mass of the star and Ω the angular velocity at the surface, Eqs. (A.1) or (A.2) give the shape of a meridian line at the stellar surface. Setting

$$f = \frac{R_{\text{eq}}}{R_{\text{pol}}}, \quad (\text{A.3})$$

¹In the case of solid body rotation, the centrifugal force is conservative, and it is possible to define equipotential surfaces. This is no longer the case for shellular rotation. In that case, however, the shape of the isobaric surfaces are given by the same expression as the one giving the shape of the equipotentials for solid body rotation provided some changes in variables are performed (see Meynet & Maeder 1997).

where R_{eq} is the equatorial radius, one easily obtains from Eq. (A.1) that

$$R_{\text{pol}} = \left(\frac{GM}{\Omega^2} \right)^{1/3} \left(\frac{2(f-1)}{f^3} \right)^{1/3} = \left(\frac{GM}{\Omega^2} \right)^{1/3} x_{\text{pol}}. \quad (\text{A.4})$$

The above equation relates the inverse of the oblateness f to R_{pol} .

As recalled above, the critical angular velocity, Ω_{crit} , corresponds to the angular velocity at the equator of the star such that the centrifugal force balances the gravity exactly. The classical critical angular velocity (or the Ω -limit) in the frame of the Roche model is given by

$$\Omega_{\text{crit}} = \left(\frac{2}{3} \right)^{3/2} \sqrt{\frac{GM}{R_{\text{pol,crit}}^3}}, \quad (\text{A.5})$$

where $R_{\text{pol,crit}}$ is the polar radius when the surface rotates with the critical velocity. The critical velocity is given by

$$v_{\text{crit}} = \sqrt{\frac{2}{3} \frac{GM}{R_{\text{pol,crit}}}}. \quad (\text{A.6})$$

By replacing Ω in Eq. (A.4) by $(\Omega/\Omega_{\text{crit}}) \cdot \Omega_{\text{crit}}$ and using Eq. (A.5), one obtains a relation between R_{pol} , f , and $R_{\text{pol,crit}}$,

$$R_{\text{pol}} = \frac{3}{2} R_{\text{p,crit}} \left(\frac{\Omega_{\text{crit}}}{\Omega} \right)^{2/3} \left(\frac{2(f-1)}{f^3} \right)^{1/3}. \quad (\text{A.7})$$

Thus one has

$$\frac{\Omega}{\Omega_{\text{crit}}} = \left(\frac{3}{2} \right)^{3/2} \left(\frac{R_{\text{pol,crit}}}{R_{\text{pol}}} \right)^{3/2} \sqrt{\frac{2(f-1)}{f^3}}. \quad (\text{A.8})$$

With a good approximation (see below), one finds that $R_{\text{pol,crit}}/R_{\text{pol}} \simeq 1$ and therefore

$$\frac{\Omega}{\Omega_{\text{crit}}} \simeq \left(\frac{3}{2} \right)^{3/2} \sqrt{\frac{2(f-1)}{f^3}}. \quad (\text{A.9})$$

This equation allows the determination of Ω_{crit} from Ω and f obtained by any model with $\Omega < \Omega_{\text{crit}}$; however, the closer to the critical value, the better. We will see in Sect. 3.3.1 the extent of the validity of the assumption $R_{\text{pol,crit}}/R_{\text{pol}} = 1$.

Setting v the velocity at the equator, one finds that

$$\frac{v}{v_{\text{crit}}} = \frac{\Omega R_{\text{eq}}}{\Omega_{\text{crit}} R_{\text{e,crit}}} = \frac{\Omega}{\Omega_{\text{crit}}} \frac{R_{\text{eq}}}{R_{\text{pol}}} \frac{R_{\text{pol}}}{R_{\text{pol,crit}}} \frac{R_{\text{pol,crit}}}{R_{\text{e,crit}}}, \quad (\text{A.10})$$

where $R_{\text{e,crit}}$ is the equatorial radius when the surface rotates with the critical velocity. Using Eq. (A.7) above, and the fact that in the Roche model $R_{\text{pol,crit}}/R_{\text{e,crit}} = 2/3$, one obtains

$$\frac{v}{v_{\text{crit}}} = \left(\frac{\Omega}{\Omega_{\text{crit}}} 2(f-1) \right)^{1/3} \quad (\text{A.11})$$

which is illustrated in Fig. A.1 on next page

An interesting quantity is the ratio of the centrifugal acceleration a_{cen} to the gravity g_e at the equator (not corrected for the rotation effect or the radiation pressure):

$$\frac{a_{\text{cen}}}{g_e} = \frac{\Omega^2 R_{\text{eq}}^3}{GM} = \left(\frac{\Omega}{\Omega_{\text{crit}}} \right)^2 \left(\frac{2}{3} \right)^3 f^3 \left(\frac{R_{\text{p}}}{R_{\text{pol,crit}}} \right)^3, \quad (\text{A.12})$$

where we use Eq. (A.5) and divide/multiply by $R_{\text{pol,crit}}^3$. Replacing $R_{\text{pol,crit}}/R_{\text{pol}}$ by its expression deduced from Eq. (A.7), we obtain

$$\frac{a_{\text{cen}}}{g_e} = 2(f-1). \quad (\text{A.13})$$

We can check that at the critical limit, when $f = 3/2$, then $a_{\text{cen}} = g_e$.

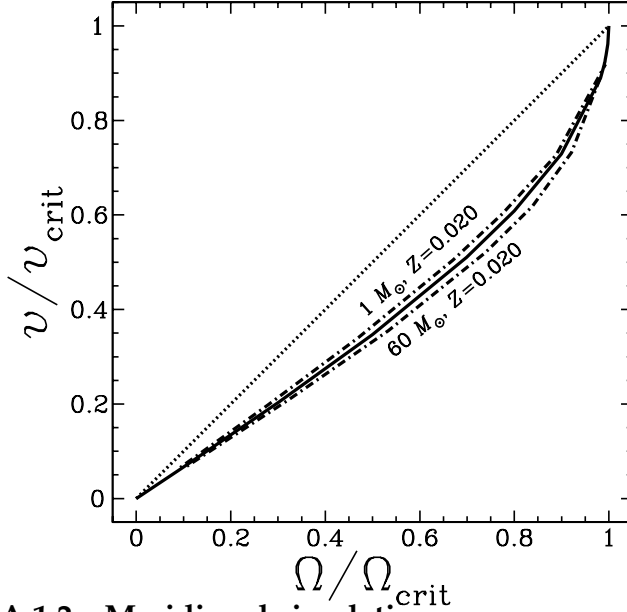


Figure A.1: Relation between v/v_{crit} and $\Omega/\Omega_{\text{crit}}$ in the frame of the Roche model. The continuous line is obtained with Eq. (A.11); the dashed lines show the result obtained with stellar models of 1 and 60 $M_{\odot}$ at $Z = 0.020$ using Eq. (A.11), see Sect. 3.3.1.

A.1.2 Meridional circulation

As mentioned in Chapter 1, the meridional circulation arises from von Zeipel's theorem. The vertical component of the meridional circulation velocity is $u(r, \theta) = U(r) P_2(\cos \theta)$ with $P_2(x)$ the second Legendre polynomial². The formulation of the radial amplitude $U(r)$ has been determined by Zahn (1992) and Maeder & Zahn (1998) as:

$$U(r) = \frac{P}{\rho g C_P T [\nabla_{\text{ad}} - \nabla_{\text{rad}} + (\varphi/\delta) \nabla_{\mu}]} \left(\frac{L}{M_{\star}} [E_{\Omega} + E_{\mu}] + \frac{C_P}{\delta} \frac{\partial \Theta}{\partial t} \right) \quad (\text{A.14})$$

where C_P is the specific heat, $\nabla_{\text{ad}} = \frac{P\delta}{\rho T C_P}$ the adiabatic gradient, $M_{\star} = M \left(1 - \frac{\Omega^2}{2\pi g \bar{\rho}}\right)$, and $\Theta = \bar{\rho}/\rho$ is the ratio of the variation of the density to the average density on an equipotential. φ and δ arise from the generic equation of state: $\frac{d\rho}{\rho} = \alpha \frac{dP}{P} + \varphi \frac{d\mu}{\mu} - \delta \frac{dT}{T}$. E_{Ω} and E_{μ} are terms which depend on the Ω - and μ -distributions respectively (see Maeder & Zahn 1998, for details on these expressions). E_{Ω} is the dominant term in the big parenthesis, and it can be reasonably approximated by:

$$E_{\Omega} \simeq \frac{8}{3} \left[1 - \frac{\Omega^2}{2\pi g \bar{\rho}} \right] \left(\frac{\Omega^2 r^3}{GM} \right) \quad (\text{A.15})$$

The term in brackets is known as the *Gratton-Öpik term*. It may become largely negative in the outermost layers of the stars, where $\bar{\rho}$ becomes very small, which leads to $U(r) < 0$. In this case, the circulation goes down along the polar axis from the surface inwards, and then rises in the equatorial plane outwards to the surface.

A.1.3 Horizontal turbulence

The shellular-rotation hypothesis is based upon the postulate that in differentially rotating stars, the horizontal turbulence, *i.e.* the turbulence along an isobar, is very strong. This is due to the fact that there is no restoring force in that direction, as the buoyancy force (the restoring force of the density gradient) in the vertical direction. Zahn (1992) relates the diffusion coefficient to the viscosity due to horizontal turbulence:

$$D_{\text{h}} \approx \nu_{\text{h}} = \frac{1}{c_{\text{h}}} r |2V(r) - \alpha U(r)| \quad (\text{A.16})$$

²The Legendre polynomials are a set of orthogonal functions that are used to solve Laplace's equation $\nabla^2 \Phi = 0$ in spherical coordinates when there is an azimuthal symmetry $\partial \varphi = 0$. The second-order Legendre polynomial is $P_2(x) = \frac{1}{2} (3x^2 - 1)$

where c_h is a constant of order 1, $V(r)$ is the horizontal component of the meridional circulation velocity, $U(r)$ its vertical component, and in this expression $\alpha = \frac{1}{2} \frac{d \ln(r^2 \bar{\Omega})}{d \ln r}$.

More recently, Maeder (2003) after revisiting the assumptions made by Zahn (1992) proposes a more general expression for D_h :

$$D_h = A r (r \bar{\Omega}(r) V |2V - \alpha U|)^{1/3} \quad \text{with} \quad A = \left(\frac{3}{400 n \pi} \right)^{1/3}. \quad (\text{A.17})$$

Note that the form of this expression is the usual form for a viscosity: $\nu = \frac{1}{3} \ell v$ where the velocity v consists in the geometric mean of three relevant velocities:

- the velocity $|2V - \alpha U|$ as in Zahn (1992);
- the horizontal component V of the meridional circulation;
- the average local rotational velocity.

A.1.4 Shear

Differential rotation induces shear turbulence at the interface of layers having different rotational velocities. A layer remains stable if the excess kinetic energy due to the differential rotation does not overcome the buoyancy force. It is known as the *Richardson criterion* and is expressed as:

$$Ri = \frac{N^2}{(\partial u / \partial z)^2} > Ri_c$$

where u is the horizontal velocity, z the vertical coordinate, and $N^2 = \frac{g\delta}{H_P} (\nabla_{\text{ad}} - \nabla + (\varphi/\delta)\nabla_\mu)$ is the Brunt-Väisälä frequency. Ri_c is the critical value for stability. It is usually set $Ri_c = 1/4$, but Canuto (2002) shows that turbulence can occur with a larger Ri number.

The coefficient of diffusion by dynamical shear implemented in the code is described in Hirschi et al. (2004):

$$D_{\text{dyn}} = \frac{1}{3} r \Delta\Omega \Delta r \quad (\text{A.18})$$

where r is the mean radius of the zone constituted of consecutive shells with $Ri < Ri_c = 1/4$, $\Delta\Omega$ is the variation of Ω over this zone and Δr is the extend of the zone. The timescale for the dynamical shear is just as it sounds: the dynamical timescale $\tau_{\text{dyn}} = \sqrt{r^3/(Gm_r)}$. The prescription for D_{dyn} is valid when the thermal adjustment timescale $\tau_{\text{th}} = \frac{d^2}{12K}$ (for a sphere with diameter d and a thermal diffusivity K) is longer than the dynamical timescale.

Secular shear arises when the thermal dissipation reduces the buoyancy force. The coefficient of diffusion by secular shear turbulence is determined by Maeder (1997) as:

$$D_{\text{shear}} = \frac{4(K + D_h)}{\left[\frac{\varphi}{\delta} \nabla_\mu \left(1 + \frac{K}{D_h} \right) + (\nabla_{\text{ad}} - \nabla_{\text{rad}}) \right]} \cdot \frac{H_P}{g\delta} \left[\frac{\alpha}{4} \left(0.8836 \Omega \frac{d \ln \Omega}{d \ln r} \right)^2 - (\nabla' - \nabla) \right] \quad (\text{A.19})$$

with $K = \frac{4acT^3}{3\kappa\rho^2C_P}$ the thermal diffusivity, α the fraction of the excess energy in the shear that will contribute to mixing, and $(\nabla' - \nabla)$ the difference between the internal non-adiabatic gradient and the local gradient.

In the zones where $Ri < Ri_c$, the shear mixing coefficient is eventually set to:

$$D_{\text{shear,final}} = D_{\text{dyn}} + D_{\text{shear}}$$

A.1.5 Transport of angular momentum

The transport of angular momentum inside a star is implemented following the prescription of Zahn (1992). This prescription has been complemented by the works of Talon & Zahn (1997) and Maeder & Zahn (1998). In the radial direction, it obeys the following equation:

$$\rho \frac{d}{dt} (r^2 \bar{\Omega})_{Mr} = \frac{1}{5r^2} \frac{\partial}{\partial r} (\rho r^4 \bar{\Omega} U(r)) + \frac{1}{r^2} \frac{\partial}{\partial r} \left(\rho D r^4 \frac{\partial \bar{\Omega}}{\partial r} \right) \quad (\text{A.20})$$

The first term in the right hand side of this equation is the divergence of the *advected* flux of angular momentum, while the second term is the divergence of the *diffused* flux. The coefficient D is the total diffusion coefficient, taking into account the various instabilities which transport angular momentum (convection, shears).

A.1.6 Transport of chemical species

Chaboyer & Zahn (1992) show that the horizontal turbulence competes efficiently with the advective term of the meridional circulation. The horizontal flow tends to homogenise the layer in such a way that the transport of chemical species by both meridional circulation and horizontal turbulence can be computed as a diffusive process which coefficient is:

$$D_{\text{eff}} = \frac{1}{30} \frac{|r U(r)|^2}{D_h} \quad (\text{A.21})$$

The change of abundance for a given chemical element i in the shell with Lagrangian coordinate r is thus:

$$\rho \frac{dX_i}{dt} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(\rho r^2 [D + D_{\text{eff}}] \frac{\partial X_i}{\partial r} \right) + \left(\frac{dX_i}{dt} \right)_{\text{nucl}} \quad (\text{A.22})$$

where D is the same as in Eq. (A.20). The last term accounts for the change of abundance due to nuclear reactions.

A.1.7 Mass loss

Rotation increases the radiative mass-loss rate by a factor

$$F_{\Omega} = \frac{(1 - \Gamma)^{\frac{1}{\alpha} - 1}}{\left[1 - \frac{\Omega^2}{2\pi G \rho_m} - \Gamma \right]^{\frac{1}{\alpha} - 1}} \quad (\text{A.23})$$

(see Maeder & Meynet 2000, for the detailed developement) with the Eddington factor $\Gamma = L/L_{\text{Edd}} = \kappa L / (4\pi c G M)$ (where κ is the electron-scattering opacity), and α the force multiplier parameter depending on T_{eff} . According to Lamers et al. (1995), the empirical values of α are:

$\alpha = 0.52$	for $T_{\text{eff}} \geq 4.40$
0.24	$4.25 \leq T_{\text{eff}} < 4.40$
0.21	$4.10 \leq T_{\text{eff}} < 4.25$
0.17	$3.95 \leq T_{\text{eff}} < 4.10$
0.15	$T_{\text{eff}} < 3.95$

The effective mass-loss rate of a rotating model is thus $\dot{M}(\Omega) = F_{\Omega} \cdot \dot{M}(\Omega = 0) = F_{\Omega} \cdot \dot{M}_{\text{rad}}$.

A.2 Adaptations brought to the code for the modelling of Population III stars

We saw (Chapter 3 and 4) that the lack of metals modifies the evolution of Pop III stars. The Geneva code had to be adapted to such peculiarities. The modifications are added directly in the code and called upon by new input parameter (`ipop3`) which can take the value 0 (normal evolution) or 1 (Pop III evolution).

A.2.1 Nuclear-burning networks

The main aspect consists in joining the H- and He-burning networks, which are normally separated in the Geneva code. In this respect, the modifications I brought to the Geneva code (in its version Meynet & Maeder 2002) are based on the master's thesis of Feijóo (1999).

During the MS, after the calculation of the H-burning reactions rates (variable `eps`), the program continues to that of the He-burning reactions rates (`epsy`) if `ipop=1`. In the original program, there was a test on the temperature to avoid too weak He-burning reactions to be accounted for, so I removed this test to allow an earlier onset of the 3α reaction. The temperature test was kept only for the $^{22}\text{Ne}(\alpha, n)^{25}\text{Mg}$ and $^{22}\text{Ne}(\alpha, \gamma)^{26}\text{Mg}$ reactions. In case the extended network NeNa-MgAl is used, the temperature test is also maintained for the neutron capture reactions.

At the end of the calculation of the He-burning reactions rates, the program tests whether we are still in the H-burning phase ($X_c \neq 0$) or if we are already in the He-burning phase:

- if there is no H left, the program calculates normally
 - the pressure and temperature derivatives of the energy production rate:

$$\text{epsp} = d \log \epsilon / d \log P$$

$$\text{epst} = d \log \epsilon / d \log T,$$
 - the mean energy production between the shells j and $j + 1$:

$$\text{en} = \sqrt{|\epsilon(j+1) \cdot \epsilon(j)|},$$
 - the energy production in units of MeV per nucleon (variable `zensi`);
- if we are still in the MS, the program calculates all these quantities for each burning network, and then combines them as follows:
 - $\text{epsp} = \frac{d \log \epsilon}{d \log P} = \frac{d \log (\epsilon_H + \epsilon_{He})}{d \log P} = \frac{\epsilon_H}{\epsilon_H + \epsilon_{He}} \frac{d \log \epsilon_H}{d \log P} + \frac{\epsilon_{He}}{\epsilon_H + \epsilon_{He}} \frac{d \log \epsilon_{He}}{d \log P}$
 - $\text{en} = \sqrt{|\epsilon_H(j+1) + \epsilon_{He}(j+1)| \cdot |\epsilon_H(j) + \epsilon_{He}(j)|}$
 - $\text{zensi} = \max(\text{zensi}(\text{H}), \text{zensi}(\text{He}))$

With the joining of the two reaction networks, the number of elements that are followed during H burning is increased and more reactions are accounted for, so I had to add them to the matrix that computes the change of abundances.

In some places of the program, the elements with an abundance lower than 10^{-9} are set to 0. For Pop III calculation, this limit has to be enlarged, because otherwise the CNO cycle cannot be ignited, or is artificially delayed (we recall that at CNO ignition, the abundance of ^{12}C is around 10^{-12}), so I set it at 10^{-75} to let any trace element appear.

A.2.2 Mass loss

Kudritzki (2002) proposes a mass loss formula that is valid for very low-metallicity stars close to the Eddington limit $L_{\text{Edd}} = 4\pi cGM/\kappa$ (with κ the electron-scattering opacity):

$$\log \dot{M} = \frac{Q_0 - Q_{\min}}{(-[Z]_{\min})^{0.5}} \left[\log \left(\frac{Z}{Z_{\odot}} \right) - [Z]_{\min} \right]^{0.5} + Q_{\min} \quad (\text{A.24})$$

with $[Z]_{\min}$, $Q_{\min}$, and Q_0 depending on the luminosity through a polynomial formula:

$$y = a_0 + a_1 \bar{L} + a_2 \bar{L}^2 \quad \text{where} \quad \bar{L} = \log \left(\frac{L}{L_{\odot}} \right) - 6.0$$

with the coefficients a_0 , a_1 , and a_2 as follows:

$T_{\text{eff}} \text{ (K)}$	y	a_0	a_1	a_2
60'000	$[Z]_{\min}$	-3.40	-0.40	-0.65
	$Q_{\min}$	-8.00	-1.20	2.15
	Q_0	-5.99	1.00	1.50
50'000	$[Z]_{\min}$	-3.85	-0.05	-0.60
	$Q_{\min}$	-10.35	3.25	0.00
	Q_0	-4.85	0.50	1.00
40'000	$[Z]_{\min}$	-4.45	0.35	-0.80
	$Q_{\min}$	-11.75	3.65	0.00
	Q_0	-5.20	0.93	0.85

This prescription is not exactly aimed at the Pop III stars, but it is used by Marigo et al. (2003) with some adaptations (see Sect. 4.2.1 on page 61) and to date, it is the best prescription for extremely low-metallicity very massive stars. I have thus implemented it in the code, where it is called by `imloss=9`.

A.3 Rejuvenation of the Geneva code: *origin2006*

A.3.1 Evolution of the program

Since ~2002, the members of the Geneva group have developed the code each in a different direction: André Maeder and Georges Meynet have implemented the treatment of magnetic fields according to the prescription of Spruit (1999, 2002), Raphael Hirschi has added the computation of the advanced phases until the preSN stage, Patrick Eggenberger has modified the code for the computation of low-mass stars and asteroseismology, and I have added the adaptations for the computation of Pop III stars as presented in the previous section.

In the idea of maintaining a coherent entity for the whole group, and also to have an extended tool for the stellar grids (Chapter 5 on page 103), we have decided to fuse the different versions of the code into one, and I have been tasked with this work. Since Patrick Eggenberger's modifications have affected the very structure of the code, we have not included them but only the three other versions.

A.3.2 Updates

Such a deep revision was a good opportunity to update the atomic data used in the code, as the reaction rates and the opacities tables.

Opacities

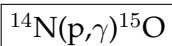
The opacities depend on the chemical abundances, and thus they must be computed for the choice we made with the solar abundance. Several groups of research in atomic physics provide the computation of the opacities for a given mixture with a web interface, among which the OPAL group (<http://physci.llnl.gov/Research/OPAL/opal.html>).

With the OPAL code, I have generated opacity tables for the new mixture. For the low-temperature regime, we got the suitable mixture of the Ferguson et al. (2005) updated molecular opacity tables provided at <http://webs.wichita.edu/physics/opacity/>.

Reaction rates

Since the NACRE compilation of Angulo et al. (1999), some of the reaction rates have been updated or revised by various nuclear physics research groups. I have described the various updates, the comparison with the NACRE values, and the effects on stellar evolution found in the literature in a small document. I join it below, it is dated from May 2006.

Update of nuclear reaction rates



New rates

- Angulo & Descouvemont (2001): re-analysis of Schröder et al. (1987) data, determination of the astrophysical factor with the *R*-matrix formalism. A table of reaction rates is given in the temperature range $0.005 \leq T_9 \leq 2$.

Astrophysical factor: [NACRE]	$S(0) = 3.2 \pm 0.8 \text{ keV b}$
[AD01]	$S(0) = 1.77 \pm 0.20 \text{ keV b}$

Reaction rates: $[AD01]/[NACRE] \simeq 0.5$ at $T_9 \leq 0.1$
 $\simeq 2$ at $T_9 > 0.1$

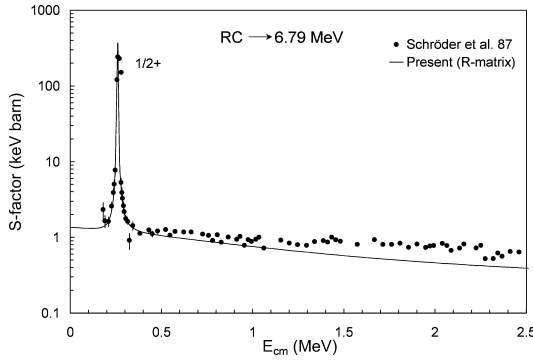


Fig. 4. The S -factor data of the $^{14}\text{N}(p,\gamma)^{15}\text{O}$ reaction for the capture process into the 6.79 MeV state of ^{15}O from Ref. [6] are shown together with the best fit (see text).

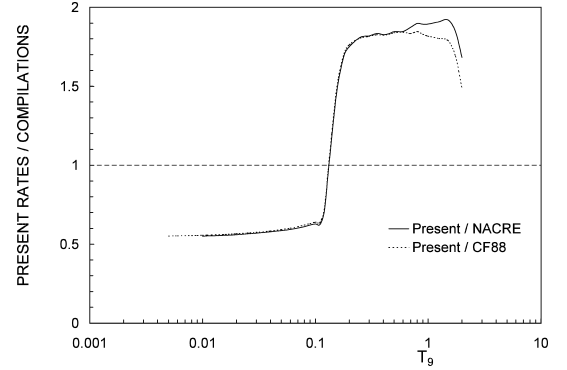


Fig. 6. Present rates compared to the NACRE and the CF88 rates for temperatures of $0.005 \leq T_9 \leq 2$ (see text).

- Mukhamedzhanov et al. (2003): new cross-section measurements at the U-120M cyclotron in Prague (Czech Republic), analysed by R -matrix formalism. A table of reaction rates is given in the temperature range $0.007 \leq T_9 \leq 0.1$.

Astrophysical factor: [NACRE] $S(0) = 3.2 \pm 0.8$ keV b
 [Mukh03] $S(0) = 1.70 \pm 0.22$ keV b

Reaction rates: [Mukh03]/[NACRE] $\simeq 0.54$ at $T_9 \leq 0.1$
 [Mukh03]/[AD01] $\simeq 1$

- Runkle et al. (2005): 155-524 keV measurements on the LENA facility of 1MV Van der Graaff accelerator (Durham, USA) analysed by R -matrix formalism.

Astrophysical factor: [NACRE] $S(0) = 3.2 \pm 0.8$ keV b
 [LENA05] $S(0) = 1.68 \pm 0.09$ keV b

- LUNA Collaboration et al. (2006): low energies measurements on the LUNA facility of Gran Sasso 400kV accelerator. Thermonuclear reaction rates are calculated for $0.09 \leq T_9 \leq 0.3$

Astrophysical factor: [NACRE] $S(0) = 3.2 \pm 0.8$ keV b
 [LUNA06] $S(0) \simeq 1.7$ keV b

Reaction rates: [LUNA06]/[NACRE] $\simeq 0.6$ at $T_9 \leq 0.1$
 $\simeq 0.9$ at $T_9 > 0.1$

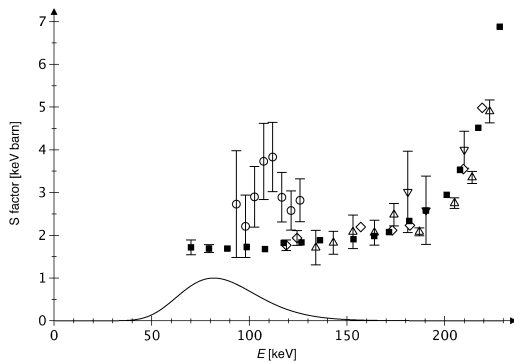


Fig. 2. Astrophysical S -factor for the $^{14}\text{N}(p,\gamma)^{15}\text{O}$ reaction from the present work (filled squares) and from previous studies: circles [8], inverted triangles [7], diamonds [16,17], triangles [18]. Error bars ($\pm 1\sigma$ statistical uncertainty) are only shown where they are larger than the symbols used. The Gamow peak for $T_6 = 80$ is also shown. The systematic uncertainties are given in the text and in Table 1.

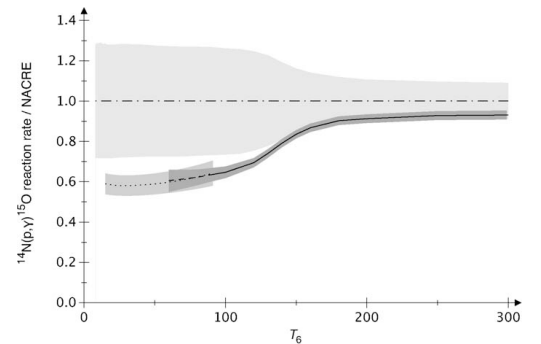


Fig. 3. Thermonuclear reaction rate relative to the NACRE [11] rate. Dot-dashed line: NACRE rate. Solid (dashed) line: present work and more than 90% (more than 50%) of the Gamow peak covered by experimental data. Dotted line: Extrapolation-based rate from Ref. [17]. The shaded areas indicate quoted upper and lower limit for the NACRE rate [11] and $\pm 1\sigma$ statistical uncertainty for the rate from Ref. [17] and the present work.

3α

New rates

- Fynbo et al. (2005): inverse reaction measurement at the IGISOL facility of Jyväskylä accelerator (Finland). A new resonance is found at 11 MeV, leading to higher reaction rates for $T_9 \lesssim 0.05$ but lower rates for $T_9 \gtrsim 1$.

Reaction rates:

$$\begin{aligned} [\text{ISOL05}]/[\text{NACRE}] &\simeq 2 && \text{at } T_9 < 0.025 \\ &\simeq 0.8 - 0.93 && \text{at } 0.025 < T_9 < 0.1 \\ &\simeq 0.1 && \text{at } T_9 > 1 \end{aligned}$$

 $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$

New rates

- Kunz et al. (2002) new measurements at the Stuttgart Dynamitron accelerator analysed by *R*-matrix formalism. A table of reaction rates is given in the temperature range $0.04 \leq T_9 \leq 10$.

Astrophysical factor:

$$[\text{CF88}]^3 \quad S(300) = 100^{+100}_{-50} \text{ keV b}$$

$$[\text{Kunz02}] \quad S(300) \simeq 165 \pm 50 \text{ keV b}$$

Reaction rates:

$$\begin{aligned} [\text{Kunz02}]/[\text{NACRE}] &\simeq 0.6 - 0.8 && \text{at } T_9 < 0.6 \\ &\simeq 1.1 - 1.4 && \text{at } T_9 > 0.6 \end{aligned}$$

- Schürmann et al. (2005) first direct measurements of ^{16}O recoils in the energy range 1.9–4.2 MeV at the ERNA facility of 4MV Dynamitron tandem accelerator (Bochum, Germany). A new resonance was found at 4.888 MeV. A table of $S(E)$ is given in the range $1.903 \leq E(\text{MeV}) \leq 4.917$.

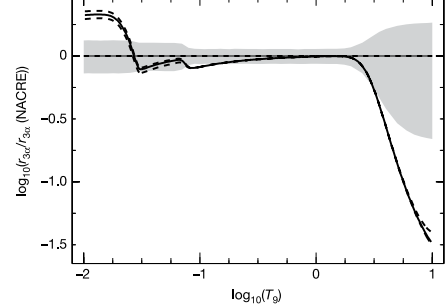


Figure 3 The triple- α reaction rate from this work, $r_{3\alpha}$, relative to the value from the current NACRE compilation³, $r_{3\alpha}(\text{NACRE})$. T_9 is the temperature in 10^9 K. Solid line, our rate including only the Hoyle resonance; dashed lines, our rate including the broad 0^+ resonance and its interference with the Hoyle resonance; and grey band, estimated error band from NACRE³ (the uncertainty in the position of their assumed 2^+ resonance is not included). We assume that the reduced- γ -decay width of the broad 0^+ resonance is equal to that of the Hoyle resonance, and the dashed lines differ only in the sign of the interference term.

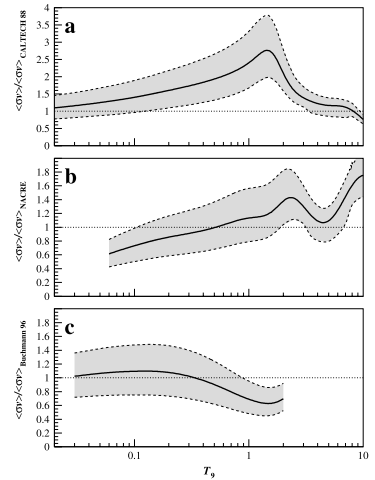


Fig. 3.—Comparison (ratio) of our new reaction rate with previous rates from the compilations of (a) CALTECH (CF88), (b) NACRE (Angulo et al. 1999), and (c) Buchmann (1996). The straight line refers to the adopted rate and the dashed lines refer to the high and low rates of this work, respectively. Take notice of the different temperature dependence of the reaction rate obtained by the three evaluations.

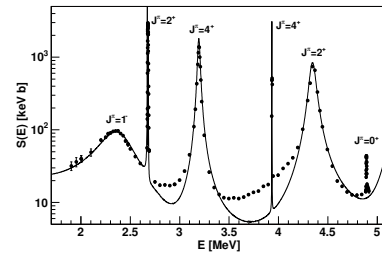


Fig. 3. Total $S(E)$ factor of the reaction $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$. Data near the narrow resonances at $E = 2.68, 3.9$, and 4.9 MeV are thick-target yields. The solid line represents the sum of the different amplitudes extracted from a recent *R*-matrix calculation [22]. Error bars shown are statistical only.

³Caughlan & Fowler (1988)

- Hammer et al. (2005) new measurements combining three different experiments at the EUROGAM and GANDI facilities of the Stuttgart Dynamitron accelerator, analysed by R -matrix formalism. An analytical expression valid for the temperature range $0.001 \leq T_9 \leq 10$ with maximum uncertainty of 8% is given.

Reaction rates:

$$[\text{Hammer05}]/[\text{NACRE}] \simeq 0.8 \quad \text{at } T_9 \sim 0.2$$

$$[\text{Hammer05}]/[\text{Kunz02}] \simeq 1$$

$$N_A \langle \sigma v \rangle = r_1 + r_2 + r_3 + r_4 \quad [\text{cm}^3 (\text{s} \cdot \text{mol})^{-1}]$$

$$r_1 = \frac{a_0}{T_9^2 (1 + a_1 T_9^{-2/3})^2} \exp\left(-\frac{a_2}{T_9^{1/3}} - \left(\frac{T_9}{a_3}\right)^2\right) \quad r_2 = \frac{a_4}{T_9^2 (1 + a_5 T_9^{-2/3})^2} \exp\left(-\frac{a_6}{T_9^{1/3}}\right)$$

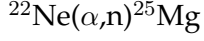
$$r_3 = \frac{a_7}{T_9^{3/2}} \exp\left(-\frac{a_8}{T_9}\right) \quad r_4 = \frac{a_9}{T_9^{2/3}} (1 + a_{10} T_9^{1/3}) \exp\left(-\frac{a_{11}}{T_9^{1/3}}\right).$$

The fit parameters for the analytical expression of the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction rate are:

$$a_0 = 1.51 \times 10^8; \quad a_1 = 0.0666; \quad a_2 = 32.12; \quad a_3 = 1.03; \quad a_4 = 1.11 \times 10^9; \quad a_5 = 0.735; \\ a_6 = 32.12; \quad a_7 = 0.0; \quad a_8 = 0.0; \quad a_9 = 16200; \quad a_{10} = 2.19 \times 10^6; \quad a_{11} = 38.814.$$

$^{22}\text{Ne} + \alpha\text{-capture}$

New rates



- Jaeger et al. (2001): new measurements at the RHINOCEROS facility of the Dynamitron accelerator (Stuttgart). This work reduces the uncertainties by a factor of 5 compared to Nacre's. A table of reaction rates is given in the temperature range $0.1 \leq T_9 \leq 1$, as well as an analytical expression in the range $0.1 \leq T_9 \leq 10$.

Reaction rates:

$$[\text{Jaeger01}]/[\text{NACRE}] \simeq 0.6 \text{ at } T_9 < 0.6$$

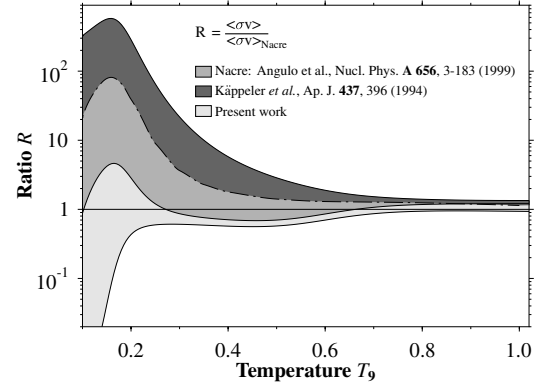
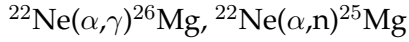


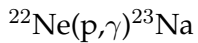
FIG. 2. Comparison of the bands of uncertainty, marked with different grey scales, obtained for the reaction rate of $^{22}\text{Ne}(\alpha, n)^{25}\text{Mg}$ from the determination of this work and the compilations of Käppeler *et al.* [12] and NACRE [11]. All rates have been normalized here to NACRE. At $T_9 = 0.2$ the reduction of the uncertainty obtained in the present work is about a factor of 100.



- Karakas et al. (2006): re-analysis of Giesen et al. (1994) data in the scope of reducing the uncertainties in the rates.

Reactions of the NeNa - MgAl cycle

New rates



- Hale et al. (2002): new measurements of $^{22}\text{Ne}(p, \gamma)^{23}\text{Na}$ at the FN tandem accelerator (Durham, USA). A table of reaction rates is given in the temperature range $0.02 \leq T_9 \leq 2$.

Reaction rates:

$$[\text{Hale02}]/[\text{NACRE}] \rightarrow 10^{-3} \text{ at } 0.03 < T_9 < 0.2$$

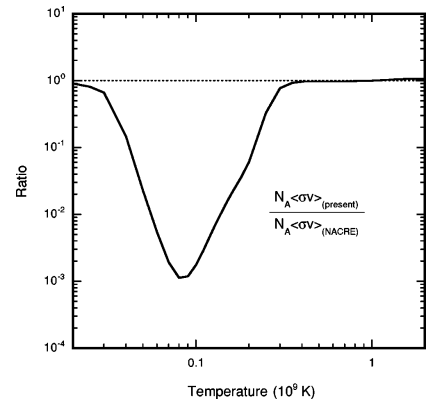


FIG. 10. Ratio of the present recommended rate to previous results [36].

$^{23}\text{Na}(p,\gamma)^{24}\text{Mg}$ and $^{23}\text{Na}(p,\alpha)^{20}\text{Ne}$

- Hale et al. (2004): new measurements of $^{23}\text{Na}(p,\gamma)^{24}\text{Mg}$ and $^{23}\text{Na}(p,\alpha)^{20}\text{Ne}$ at the FN tandem accelerator (Durham, USA). Analytical formulas and tables of reaction rates are given in the temperature range $0.01 \leq T_9 \leq 5$.

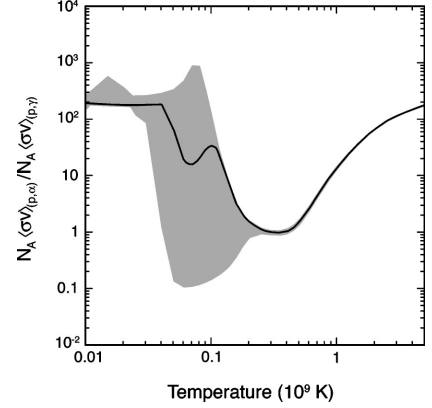


FIG. 10. Ratio of reaction rates for (p,α) and (p,γ) reactions. The solid line is the ratio calculated using the recommended rates and the shaded area denotes the uncertainty in the ratio. The latter includes the correlations in the uncertainties for the two reactions.

Effects on stellar evolution

$^{14}\text{N}(p,\gamma)^{15}\text{O}$ is the slowest reaction of CNO-cycle. For **low-mass stars** [1,2], it plays a role at the end of MS, when pp -chains are not sufficient to sustain the star and CNO-cycle starts in. A slower reaction rate means a slower H-burning process, and shallower temperature profile leads to a more extended and slightly hotter core. The turnoff point is modified towards higher luminosities. In **intermediate-mass stars** [2,3], the slower rate implies an evolution on the MS at higher luminosities. The blue-loop during core He-b, very sensitive to internal structure, gets significantly shorter with the slower rates. In the TP phase, the peak luminosity is higher and the interpulse duration is longer for slower rates.

In **low-** and **intermediate-mass stars**, 3α seems to play an important role in the He-shell flash [2,3], a larger rate inducing a higher peak luminosity and subsequently a deeper dredge-up. The C/O ratio in the core and the total ^{12}C yields are larger by about a factor 2 for higher 3α rate.

$^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ is extremely difficult to determine since its cross-section is very small and extrapolation at low energies are very complex due to a mixture of groundstate and cascade transitions with complicated energy dependences. Yet it is a key reaction for the evolution of **massive stars** towards late phases [4,5]. It governs the C/O ratio of the core at the end of He-b and subsequently the yields of ^{12}C and C-b elements. It plays also a role in the limit mass between the regime of convective or radiative core C-b (around $20 M_{\odot}$). In **intermediate-mass stars** [3,5], the effect of varying the $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ rate is well within the variations that may arise from the choice of the semi-convection treatment.

$^{22}\text{Ne} + \alpha$ -capture play a role in the ratio $^{24}\text{Mg} : ^{25}\text{Mg} : ^{26}\text{Mg}$ of the Mg yields [6].

Comparative studies in stellar evolution

[1] Imbriani et al. (2004)

Study: $^{14}\text{N}(p,\gamma)^{15}\text{O}$ in low-mass stars, link with age determination for GCs.

Code: FRANEC (Straniero et al. 1997).

Models: $\sim 0.8 M_{\odot}$ with $0.0001 \leq Z \leq 0.006$.

Rates: [CF88] vs [LUNA04].

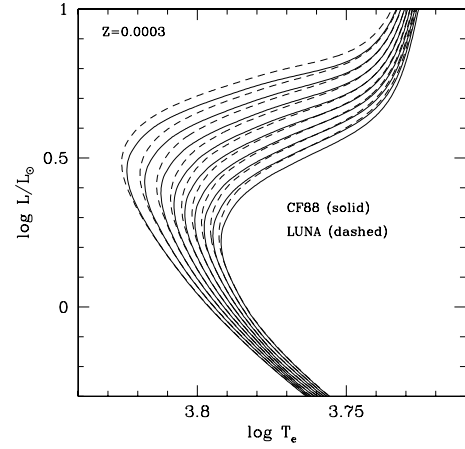


Fig. 2. Isochrones for Globular Clusters obtained with different rates of the $^{14}\text{N}(p,\gamma)^{15}\text{O}$ reaction: CF88 (solid) and LUNA (dashed). The brightest isochrone (of each set) is the youngest (10 Gyr), while the fainter is the oldest (18 Gyr). The age step between two adjacent isochrones is 1 Gyr. The metallicity is $Z = 0.0003$ (or $[M/H] = -1.82$).

[2] Weiss et al. (2005):

Study: $^{14}\text{N}(p,\gamma)^{15}\text{O}$ and 3α in metal-poor or -free low-mass stars, solar intermediate-mass star. Focus on He ignition and thermally unstable He-b.

Code: Garching code (Weiss & Schlattl 2000).

Models: 0.8 and $1.0 M_{\odot}$ with $Z = 0.001, 0.0001, 0.00001$, a PopIII $1 M_{\odot}$ and a solar $5 M_{\odot}$.

Rates: [Adelb98] vs [LUNA04] for $^{14}\text{N}(p,\gamma)^{15}\text{O}$ and [NACRE] analytical formula vs [ISOL05] for 3α .

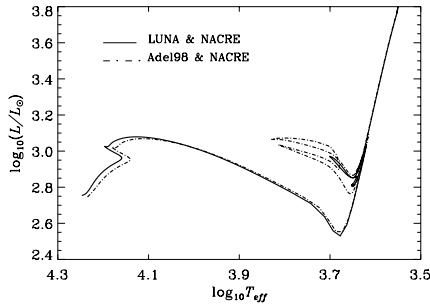


Fig. 4. Evolution in the HRD for the Pop. I model with $M = 5 M_{\odot}$, showing the influence of changing the $^{14}\text{N}(p,\gamma)^{15}\text{O}$ reaction rate. The two cases are indicated in the figure.

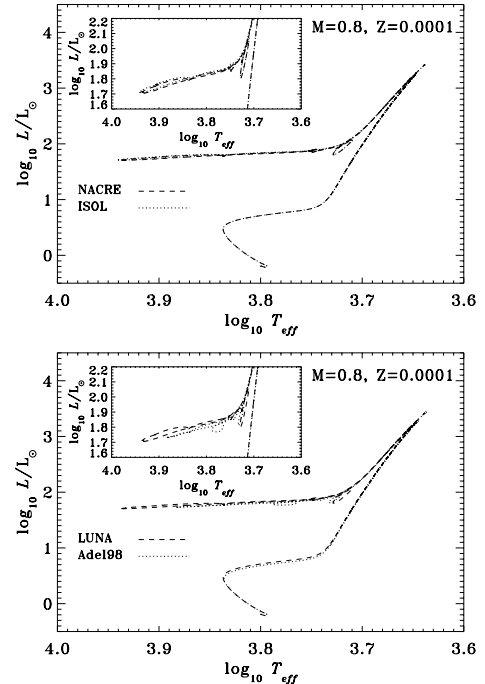


Fig. 2. Evolution of a stellar model of $0.8 M_{\odot}$ and $Z = 10^{-4}$ from ZAMS to HB. In the upper panel we varied the 3α - and in the lower one the $^{14}\text{N}(p,\gamma)^{15}\text{O}$ -rate. The insets show the approach to the HB in greater detail.

[3] Herwig et al. (2006):

Study: $^{14}\text{N}(\text{p},\gamma)^{15}\text{O}$, 3α , and $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ in intermediate-mass stars. Focus on AGB evolution.

Code: EVOL (Herwig 2004)

Models: $2 M_{\odot}$ with $Z = 0.01$.

Rates: [NACRE] analytical formulas, with a correcting factor adapted to match the rates of [LUNA04] and [ISOL05], and the uncertainties on $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$.

[4] Weaver & Woosley (1993):

Study: $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ in massive stars.

Code: KEPLER (Weaver et al. 1978)

Models: $12\text{--}40 M_{\odot}$.

Rates: [CF88] with a multiplying factor varying from 0.5 to 3.0.

[5] Imbriani et al. (2001):

Study: $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ in low-mass to massive stars.

Code: FRANEC (Straniero et al. 1997)

Models: $0.8\text{--}25 M_{\odot}$.

Rates: [CF88] vs [CFHZ85].

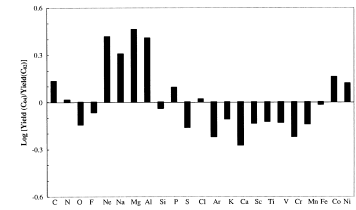


FIG. 13.—Logarithmic ratio between the yields produced by the $\text{C}_{0.4}$ run and those produced by the $\text{C}_{0.2}$ run

[6] Karakas et al. (2006):

Study: $^{22}\text{Ne} + \alpha$ -capture in intermediate-mass stars.

Code: Mount Stromlo Stellar Structure Program (Frost & Lattanzio 1996)

Models: $5 M_{\odot}$ at $Z = 0.020, 0.008, 0.004, 10^{-4}$.

Rates: different set of rates (cf. table below).

Table 6. In the first row we show the yields of ^{25}Mg (in roman font) and ^{26}Mg (in italics) in solar masses for different stellar models ($M(M_{\odot})$, Z) that use the new recommended rates for the $^{22}\text{Ne} + \alpha$ reactions. In other rows, we show the percentage difference between models computed using other estimates of the $^{22}\text{Ne} + \alpha$ reactions and the yields from row 1 (see definition in the text).

$^{22}\text{Ne}(\alpha,\gamma)^{26}\text{Mg}$	$^{22}\text{Ne}(\alpha,n)^{25}\text{Mg}$	(5.0.0001)	(5.0.004)	(5.0.008)	(5.0.02)
recommended	recommended	5.80(−4)	4.84(−4)	3.49(−4)	6.49(−5)
		2.28(−3)	1.11(−3)	6.84(−4)	1.17(−4)
standard ^a	standard ^a	−3.42	29.6	36.6	45.6
		9.05	7.50	3.51	−5.95
K94 ^b	K94 ^b	22.5	31.5	38.2	52.6
		9.80	8.64	5.68	−2.99
NACRE ^c	NACRE ^c	20.4	27.9	34.6	44.8
		15.3	16.0	15.9	8.82
upper limit	upper limit	3.34	8.94	11.5	24.7
		13.8	17.0	18.9	26.7
lower limit	lower limit	−8.93	−7.95	−9.98	−14.7
		−14.2	−16.2	−17.5	−16.6
lower limit	upper limit	5.10	11.5	13.5	20.7
		0.46	0.21	−1.46	−4.13
upper limit	lower limit	−9.95	−8.94	−11.2	−16.5
		2.47	3.82	4.96	7.82

^aStandard Case: Kippeler et al. (1994, K94) for the $^{22}\text{Ne} + \alpha$ reactions; reaction rates for the NeNa and MgAl chains described in the Appendix

^bK94: K94 for the $^{22}\text{Ne} + \alpha$ reactions; NACRE recommended rates for the NeNa and MgAl chains

^cNACRE: NACRE recommended rates for the $^{22}\text{Ne} + \alpha$ reactions and the NeNa and MgAl chains

We used the NetGen database⁴ to generate the reaction rates tables. A table summarising the reactions and their reference is joined below. Some reactions for which we used until then an analytical formula are now tabulated: they are marked by a star in the table.

⁴<http://www-astro.ulb.ac.be/Netgen/>

Fichiers de taux de réaction appelés dans *energ*

Main reactions

He-b			
Reaction	file	reference	He-b
$^1\text{H}(p, \nu e^+)^2\text{H}$	ppnu	[1] Angulo et al. (1999)	Reaction
$^3\text{He}(^3\text{He}, 2p)^4\text{He}$	he3h	[1]	$^4\text{He}(\alpha\alpha, \gamma)^{12}\text{C}$
$^3\text{He}(\alpha, \gamma)^7\text{Be}$	he3a	[1]	$^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$
$^7\text{Be}(p, \gamma)^8\text{B}$	be7g	[1]	$^{13}\text{C}(\alpha, n)^{16}\text{O}$
$^{12}\text{C}(p, \gamma)^{13}\text{N}$	c12g	[1]	$^{14}\text{N}(\alpha, \gamma)^{18}\text{F}$
$^{13}\text{C}(p, \gamma)^{14}\text{N}$	c13g	[1]	$^{16}\text{O}(\alpha, \gamma)^{20}\text{Ne}$
$^{14}\text{N}(p, \gamma)^{15}\text{O}$	n14g	$T_9 \leq 0.1$: [2] Mukhamedzhanov et al. (2003)	$^{17}\text{O}(\alpha, n)^{20}\text{Ne}$
		$T_9 > 0.1$: [1] (<i>low rates</i>)	$^{18}\text{O}(\alpha, \gamma)^{22}\text{Ne}$
$^{15}\text{N}(p, \gamma)^{16}\text{O}$	n15g	[1]	$^{20}\text{Ne}(\alpha, \gamma)^{24}\text{Mg}$
$^{15}\text{N}(p, \alpha)^{12}\text{C}$	n15a	[1]	$^{22}\text{Ne}(\alpha, n)^{25}\text{Mg}$
$^{16}\text{O}(p, \gamma)^{17}\text{F}$	o16g	[1]	
$^{17}\text{O}(p, \gamma)^{18}\text{F}$	o17g	[1]	$^{22}\text{Ne}(\alpha, \gamma)^{26}\text{Mg}$
$^{17}\text{O}(p, \alpha)^{14}\text{N}$	o17a	[1]	
$^{18}\text{O}(p, \gamma)^{19}\text{F}$	o18g	[1]	
$^{18}\text{O}(p, \alpha)^{15}\text{N}$	o18a	[1]	

Reactions of the NeNa-MgAl cycle

He-b			
Reaction	file	reference	He-b
$^{19}\text{F}(p, \gamma)^{20}\text{Ne}$	f19g	[1] Angulo et al. (1999)	Reaction
$^{19}\text{F}(p, \alpha)^{16}\text{O}$	f19a	[1]	$^{14}\text{C}(p, \gamma)^{15}\text{N}$
$^{20}\text{Ne}(p, \gamma)^{21}\text{Na}$	ne20	[1]	$^{15}\text{N}(\alpha, \gamma)^{19}\text{F}$
$^{21}\text{Ne}(p, \gamma)^{22}\text{Na}$	ne21	[6] Iliadis et al. (2001)	$^{17}\text{O}(\alpha, \gamma)^{21}\text{Ne}$
$^{22}\text{Ne}(p, \gamma)^{23}\text{Na}$	ne2g	[7] Hale et al. (2002)	$^{18}\text{O}(\alpha, n)^{21}\text{Ne}$
$^{23}\text{Na}(p, \alpha)^{20}\text{Ne}$	na3a	[6]	$^{19}\text{F}(\alpha, p)^{22}\text{Ne}$
$^{23}\text{Na}(p, \gamma)^{24}\text{Mg}$	na3g	[6]	$^{21}\text{Ne}(n, \alpha)^{18}\text{O}$
$^{24}\text{Mg}(p, \gamma)^{25}\text{Al}$	mg4g	[6]	$^{21}\text{Ne}(\alpha, \gamma)^{25}\text{Mg}$
$^{25}\text{Mg}(p, \gamma)^{26}\text{Al}^g$	mg5g	[6]	$^{24}\text{Mg}(\alpha, \gamma)^{28}\text{Si}$
$^{25}\text{Mg}(p, \gamma)^{26}\text{Al}^m$	mg5m	[6]	$^{25}\text{Mg}(\alpha, n)^{28}\text{Si}$
$^{26}\text{Mg}(p, \gamma)^{27}\text{Al}$	mg6g	[6]	
$^{26}\text{Al}^g(p, \gamma)^{27}\text{Si}$	al6g	[6]	
$^{27}\text{Al}(p, \alpha)^{24}\text{Mg}$	al7a	[6]	
$^{27}\text{Al}(p, \gamma)^{28}\text{Si}$	al7g	[6]	

★ = new files

A.3.3 Modifications

I have described all the modifications and the new usage of the code `origin2006` in a document aimed to the members of the group, and thus written in french. I join it below without translation, the content being extremely technical and most probably of internal use only.

Origin2006

1 Programme

`Origin2006`: version modifiée pour intégrer le calcul des phases avancées selon Hirschi 2004 et le calcul de Population III. Cette version est basée sur `originV2` de Meynet incluant le traitement du magnétisme et de la circulation méridienne selon formule simplifiée.

Les sous-routines se trouvent dans `/home/evol/ekstrom/origin2006/`. Les makefiles sont:

- `make_evosol2006`
- `make_evolver2006`

Il est recommandé de compiler sans l'option *fast* avec le fortran de la dernière version de Sun Studio. Pour se mettre dans cet environnement: `ugvt sun_studio`.

2 Fichier d'input

Ajout

Une ligne a été rajoutée:

```
INETBURN= 3 ISUGI= 1 IDIFCON= 0 IR2PA= 0 IPOP3= 1
```

`inetburn` =0: calcul sans les phases avancées

=3: calcul avec réseau complet

→ il faut le mettre =3 dès le début du calcul

le programme modifie `inetburn` au fil des phases avancées:

```
inetburn=3 et ipout=1 : H-b
```

```
3          2 : He-b
```

```
3          3 : C-b
```

```
4          3 : Ne-b
```

```
5          3 : O-b
```

```
6          3 : Si-b
```

`isugi` =0: discrétisation habituelle des équations de conservation et de transport d'énergie

=1: discrétisation selon Sugimoto (1970)

changement automatique seulement après épuisement de l'He dans le coeur

→ il faut le mettre =1 dès le début du calcul

=5: possibilité de maintenir la discrétisation habituelle un peu plus longtemps

`idifcon` =0: traitement habituel des zones convectives

=1: traitement de la convection comme diffusion dans les phases où le temps évolutif est plus court que le temps convectif
→ il faut le mettre =0 au début et
=1 lorsque le coeur convectif fluctue ++ pendant la fusion O

ir2pa =0: calcul habituel
=1: prise en compte de accélération $\ddot{r}$ dans les équations de structure
(attention: implémenté seulement dans *girsu.f*, pas utilisable en l'état)

ipop3 =0: calcul d'étoiles normales avec $Z \neq 0$
=1: calcul d'étoiles primordiales

Modification

Le format de `FAKTOR` a été modifié pour permettre de l'augmenter drastiquement dans les phases avancées: `f8.4` → `e8.2`

NB: on trouvera dans `/home/evol/ekstrom/origin2006/` un fichier texte nommé *formats* avec un exemple du nouveau format d'inputs, permettant de l'implémenter par simple copier-coller.

3 Petits fichiers

3.1 Nouveaux

nsugi.save

Ce nouveau fichier doit se trouver dans le répertoire de calcul. Il contient la valeur du paramètre `nsugi`. Pendant la fusion H et He, il est égal au `m` du dernier modèle calculé. A la fin de l'He, il va progressivement passer à `-1`, ce qui permet la transition en douceur de la discrétisation habituelle à celle de Sugimoto.

NB: Lorsqu'on lance un nouveau calcul, il faut simplement créer ce fichier avec un entier > 0 . Si on a commencé la fusion C et qu'on reprend le calcul en arrière à l'He, il ne faut pas oublier de remettre $nsugi \geq m$ du modèle qu'on reprend.

netb.inc, netburn.inc, r2points.inc

Ces fichiers contiennent les déclarations pour les variables nécessaires aux phases avancées. Ils doivent être présents dans le répertoire du programme (comme *evol.inc*)

netdef.in

Ce nouveau fichier contient les éléments suivis dans les phases avancées ainsi que leurs abondances initiales. Il doit se trouver dans le répertoire de calcul. Des fichiers d'exemples pour différentes métallicités se trouvent dans `/home/evol/ekstrom/inputs/netdef.inZXXX`.

netalu.dat

Ce nouveau fichier contient les abondances initiales des éléments du réseau NeNa-MgAl. Il doit se trouver dans le répertoire de calcul. Des fichiers d'exemples pour différentes métallicités et différents type de mélanges (solaire: `s`; alpha-enhanced: `a`) se trouvent dans `/home/evol/ekstrom/inputs/netaluZXXX.dat`.

netinit.inCNE, netinit.inCNEO

Ces nouveaux fichiers contiennent les éléments du réseau des phases avancées. Ils se trouvent dans le répertoire d'inputs.

vit.datCNE, vit.datCNEO

Ces nouveaux fichiers contiennent les réactions des phases avancées. Ils se trouvent dans le répertoire d'inputs.

liste_tables3

Ce fichier contient les liens vers les fichiers contenant les nouveaux taux de réaction tabulés avec NetGen (<http://www-astro.ulb.ac.be/Netgen/>). Il est également centralisé dans le répertoire d'inputs. Certains taux ont été mis à jour, et des fichiers ont été rajoutés (voir page 140).

3.2 Modifiés**evol.inc**

Ce fichier a été modifié pour contenir le chemin du repertoire centralisé d'inputs (variable `input_dir`).

3.3 Déplacés**ftfp.dat, kappa93.dat, alphaenhanced.dat, solar.dat**

Ces fichiers ont été placés dans un répertoire d'inputs centralisé, ce qui évite de devoir les copier dans chaque répertoire de calcul.

4 Fichiers de sortie

Le fichier `.l` a subi beaucoup de modifications puisque de nouvelles informations liées aux phases avancées y sont désormais écrites (trop longues à énumérer ici). On trouvera ci-dessous uniquement les modifications apportées aux fichiers `.a`, `.g` et `.v`. Des listes explicatives des variables sont jointes ci-dessous (voir pages 151 à 153). *NB*: Le fichier *formats* (voir page 142) contient également les nouveaux formats pour ces fichiers. Il permet d'implémenter ces formats dans les sous-routines d'analyse par simple copier-coller.

.a

Le fichier d'abondance s'écrit dorénavant sur une seule ligne afin de faciliter le traitement avec SuperMongo. Les éléments ont été classés par ordre croissant de A (en gras les modifications):

```

write(23,798) nm,u1,u2,u5,u37,u7,u8,u9,u10,u38,u12,u13,u14,u6,u11
$,u39,u40,u41,u25,u26,u30,u27,u28,u29,u15,u42,u17,u18,u19,u20,u43
$,u22,u23,u24,u16,u21,u44,u45,u46,u31,u32,u36,u33,u34,u35
798 format(1x,i5,1x,1pe14.7,0pf9.4,42(1x,e12.6))

```

.g

Le fichier d'évolution a subi quelques modifications, de format ou de variables. Son écriture se fait selon l'appel suivant:

```

      write(20,851) nm,u1,u2,xl,xtt,u5,u7,b33(1),u8,u9,u10,u12,u13,u14
      $,u6,u11,qmnc,xte,xmdot,rhoc,tc,u15,u17,b33(m),u18,u19,u20,u22,
      $ u23,u24,u16,u21,ybe7(m)*7,yb8(m)*8,flube7,flub8,snube7,snub8
      $,rapcrj,rot1,rotm,xobla,u27,u33,alpro6,lcnom,xmcno,scno
$,xjspe1,xjspe2
      $,vcrlm,vcri2m,vequam,rapomm,eddesm
      $,vcrit1,vcrit2,vequat,rapom2,eddesc
$,bmomit,btot,ekrote,epote,ekine,erade
      $,(uudrawc(ii),ii=1,40)
851 format(i6,1x,1pe20.13,0pf9.4,2(1x,f5.3),2(1x,e14.7),1p,9(1x,e14.7)
      $,1x,0pf6.4,3x,f5.3,1x,f7.3,2(1x,f5.3),2(1x,e14.7),1p,9(1x,e14.7),
      $2(1x,e10.3),2(1x,e10.3),2(1x,e10.3),0pf12.8,6(1x,1pe10.3),
      $1x,i4,1x,0pf9.4,1x,1pe9.2,0p,2(1x,f10.3),3x,3(1x,f8.2)
      $,2(1x,f8.6),3(1x,f8.2)
      $,2(1x,f8.6),6(1x,1pe14.7),0p,40f6.3)

```

.v

Dans le fichier de structure, certaines variables inutilisées ont été éliminées, de nouvelles sont apparues, des changements de format ont été effectués. Le fichier s'écrit sur une seule ligne même si ialflu=1. Si on traite le réseau des phases avancées, l'écriture est faite selon l'appel suivant:

```

      write(29,795) j,vm,x2,x3,x4,v1,x(j),y(j),xc(j),xo(j),eps(j)
      $,epsy(j),epsc(j),radm,x6,zensi(j),epsn ,x10,x11,x12,x13,x14,x15
      $,psi,epsyy(j),epsyc(j),epsyo(j),eg,adim,x8,beta,y3(j),xc13(j)
      $,xn14(j),xn15(j),xo17(j),xo18(j),x20(j),x22(j),x24(j),x25(j)
      $,x26(j),vmyhelio,omegi(j),xmdudp(j),xricha(j),difcon(j),difsch(j)
      $,difdef(j),vmasse,dlodlr(j),xnprim(j),xkther(j),ucicoe(j)
      $,vcicoe(j),dcirch(m-j+1),echpry(j),gravy(j),dhoriz(m-j+1)
      $,dhoriz(m-j+1),omegp(j),vr(j),vomegi(j),dmago(m-j+1),dmagx(m-j+1)
      $,etask(m-j+1),bvais2(m-j+1),bphi(m-j+1),alven(m-j+1),xqmin(m-j+1)
      $,x7,vmye,xf19(j),xne21(j),xna23(j),xal26g(j),xal27(j),xsi28(j)
      $,xc14(j),xf18(j),xneut(j),xprot(j),xbid(j)
$,(abelx(ii,j),ii=1,nbelx)
795 format(i4,3(f10.7,1x),f14.11,1x,e14.6,4(1x,e14.7),3x,1p,3(e11.4
      $,1x),2x,e11.4,1x,0pf11.6,1x,1pe12.5,1x,e11.4,3x,6(e12.5,1x),e9.2
      $,1x,e9.2,1x,e10.2,1x,e11.2,3x,4(e12.5,1x),5x,0p,4(e14.7,1x),2x
      $,4(e14.7,1x),2x,3(e14.7,3x),f9.6,2x,1p,6(3x,e12.5),1x,0p,f9.4
      $,20(1x,e15.8),2(1x,f9.6),1p,11(1x,e14.7),8(1x,e14.7))

```

Si on ne traite pas les phases avancées, le n° de format est 838, qui ne diffère de 795 que par l'absence des variables abelx(nbelx,ldi).

5 Nouvelles routines

5.1 Phases avancées

netrates.f

Ce fichier contient trois routines: *netinit*, *readnetZA* et *readreac*.

- *netinit*: est appelée au début d'une série de calcul. Elle initialise le réseau des éléments des phases avancées en lisant le fichier *netdef.in* dans le répertoire de calcul. Elle définit également la valeur de *nbelx* (nombre d'éléments suivis dans le vecteur *abelx*).
- *readnetZA*: est appelée par *netinit*. Elle vérifie les éléments suivis par le réseau.
- *readreac*: est appelée par *netinit*. Elle lit le fichier *vit.dat* dans le répertoire d'inputs et transfère les taux de réactions dans la matrice *vgrid*.

netburn.f

Ce fichier contient trois routines principales: *netburning*, *contribreac* et *calcrates*.

- *netburning*: c'est le *netnew* des phases avancées.
- *contribreac*: est appelée par *netburning*. Elle calcule les contributions de chaque réaction nucléaire aux abondances des éléments suivis (cf. matrice *A* de *netnew*).
- *calcrates*: est appelée par *energ*. Elle calcule μ_e et le facteur de screening. Elle interpole les taux de réactions stockés dans *vgrid*.

Il contient également la petite sous-routine *inversemat* d'inversion de matrice, appelée par *netburning*, et qui fait elle-même appel à *girl*.

girsu.f

La sous-routine *gisu* est appelée au début de *gi* si *dzeit* < temps de conduction thermique et *isugi* $\geq$ 1. Cette routine utilise la discrétisation de Sugimoto.

5.2 Population III

netnewp3.f, netwkip3.f, netalup3.f

Les sous-routines *netnewp3*, *netwkip3* et *netalup3* calculent les changements d'abondance des éléments par les réactions du double réseau H-b + He-b pendant la séquence principale. La fusion de l'He peut se faire au choix selon le réseau He-b seul ou les deux réseaux joints (par défaut, réseau He-b seul). Ces sous-routines sont appelées dans *main* et *henyey* à la place des sous-routines habituelles *netnew*, *netwki* et *netalu* sur la condition *ipop3*=1.

5.3 Autre

enint.f

La sous-routine *enint* est appelée dans *main* avant l'écriture dans l'unité 9. Elle calcule diverses grandeurs qui seront écrites dans le fichier *.g*:

epote:	énergie gravitationnelle	$-G \sum_j (m_j/r_j) dm_j$
ekrote:	énergie rotationnelle	$(1/2) \sum_j I_j \Omega_j^2$
ekine:	énergie cinétique du gaz	$(3/2) k N_A \sum_j (T_j/\mu_j) dm_j$
erade:	énergie de radiation	$(4\pi/3) (4\sigma/c) \sum_j T_j^4 dr_j^3$

momspe.f

La sous-routine *momspe* est appelée dans *main* juste avant l'écriture dans l'unité 9. Elle calcule le moment spécifique (ΩR^2) à la masse 3 et 5 $M_\odot$, valeurs qui seront écrites dans le fichier .g.

6 Modifications apportées aux routines

main

Fichier: *mainrgva2006.f*

- ADV · rajout des éléments des phases avancées `abelx(i, ldi)`
- si `ialflu=1`: mise à zéro de l'abondance initiale du 28Si. La variable `xsi28(ldi)` ne contient que les variations dues au cycle NeNa-MgAl et la variable `abelx(1, ldi)` contient l'abondance initiale et ses variations par les phases avancées
- appel de `grapmui` aussi si `idifcon=1`
- POPIII · appel différencié de *netnewp3* ou *netwkip3* au lieu de *netnew* ou *netwki*
- AUTRE · lecture des abondances initiales du réseau NeNa-MgAl dans le fichier *netalu.dat*
- appel à la nouvelle sous-routine *enint*
- appel à la nouvelle sous-routine *momspe*
- modification du format d'écriture du fichier .g. Il inclut les variables calculées dans *enint* et *momspe* (voir page 152).
- modification du format d'écriture du fichier .a. Il est écrit sur une seule ligne et les éléments sont classés par ordre croissant de A (voir page 151).
- diminution automatique de `fitm` lors du départ vers le rouge
- création d'un fichier *runfile* si la série s'est terminée sans accroc. Cela permet de ne relancer le calcul suivant de manière automatique que s'il existe (cf. /home/evol/ekstrom/inputs/go_evol)

henyey

Fichier: *henyeygva2006.f*

- ADV · appel de *omconv* conditionné à `idifcon=0`
- appel de *grapmui* aussi si `idifcon=1`
- modif du nombre limite d'itération
- modif du calcul de `vmyhelio` pour inclure les éléments des phases avancées (ainsi que ceux du cycle NeNa-MgAl)
- rajout des éléments des phases avancées dans le fichier .v
- POPIII · appel différencié de *netnewp3* ou *netwkip3* au lieu de *netnew* ou *netwki*
- AUTRE · modif du format d'écriture du fichier .v pour qu'il s'écrive sur une seule ligne même si `ialflu=1` (voir page 153)
- transmission de `xxmu(j)=vmyhelio` par le **common**/ekinet/

gi

Fichier: *gir.f*

ADV · appel à la nouvelle sous-routine *gisu*

energ

Fichier: *energva2006.f*

- ADV · appel de *calcrates* pour le calcul de *epsc*, *epsp1*, *epst1* et en lors du C-b
 · rajout des éléments des phases avancées et *z* considéré comme ^{56}Ni
- POPIII · jonction des réseaux H-b et He-b. Les variables *epsp1*, *epst1*, *en* et *zensi* sont calculées différemment dans ce cas.
- AUTRE · taux de réactions nucléaires actualisés avec NetGen. De nouveaux fichiers de réactions tabulées ont été introduits (voir page 140)
 · rajout des réactions du cycle NeNa-MgAl dans le calcul de *dy* lors du He-b (si *ialflu*=1)

interpol

Fichier: *interpol6gva2006.f*

- AUTRE · va lire le fichier *liste_tables3* qui contient les chemins vers les fichiers contenant les nouveaux taux de réactions (/home/evol/ekstrom/taux/NetGen2006/).

interx

Fichier: *interxgva2006.f*

- ADV · rajout des éléments des phases avancées

netnew

Fichier: *netnewr2006.f*

- ADV · rajout des éléments des phases avancées
 · modif des mises à zéro
 · appel à *netburning* lors du C-b

netwki

Fichier: *netwkigva2006.f*

- ADV · appel à *netburning* lors du C-b
- AUTRE · élagage de la sous-routine puisqu'on n'y entre que si *ialflu*=1

chemie

Fichier: *chemiervgva2006.f*

- ADV · on ne moyenne les abondances que si *idifcon*=0
 · pas de mise à zéro de l'abondance d'hélium (besoin des α pour les phases avancées)
 · pas de mise à zéro de l'abondance du carbone si on est dans le Ne-b. De même, on saute les changements d'abondances du ^{20}Ne , ^{24}Mg , ^{16}O et ^{12}C
 · rajout des éléments des phases avancées pour moyenner les abondances

- on ne moyenne l'He que si $epsc(j) = 0$
- mise à zéro des abondances $< 1.e-75$
- appel à *coedif* et *diffbr* aussi si $idifcon=1$
- POPIII · lors de l'He-b, moyenne de l'abondance de l'H seulement si on utilise le réseau He seul

chemit

Fichier: *chemit2006.f*

- ADV · mêmes changements que dans *chemie*
- AUTRE · rajout des éléments du cycle NeNa-MgAl

chemold

Fichier: *chemoldgva2006.f*

- ADV · si $idifcon=1$, on quitte
- rajout des éléments des phases avancées pour moyenner les abondances
- on ne moyenne l'He que si $epsc(j) = 0$

dichte

Fichier: *dichte2006.f*

- ADV · modif du calcul de $vmy1$, $vmy0$, $vmye$ pour inclure les éléments des phases avancées (ainsi que ceux du cycle NeNa-MgAl)

coedif

Fichier: *coedif_2006.f*

- ADV · ajout du calcul du shear dynamique
- mise à zéro de $dshear(n)$ et $deffl(n)$ si $irot=0$

coedif2

Fichier: *coedif2_2006.f*

- ADV · mise à zéro de $dshear(n)$ et $deffl(n)$ si $irot=0$

diffbr

Fichier: *diffbrgva_impli2006.f*

- ADV · ajout des éléments des phases avancées
- effet de la diffusion appliqué à $y(j)$ seulement si $epsc(j) = 0.0$
- mise à zéro des abondances $< 1.e-25$ (au lieu de $1.e-15$ avant), pas de mise à zéro de $y(j)$
- POPIII · pas de transformation $y(j) = y(j) + x(j)$ et $x(j) = 0.0$ dans les couches où $epsy(j) \neq 0.0$ et $x(j) \neq 0.0$
- mise à zéro des abondances $< 1.e-75$

diffusion

Fichier: *diffusion.f*

POPIII · on saute les mises à zéro

difcou

Fichier: *difcou.f*

POPIII · rajout des éléments de He-b même si $x > 0$

grapmui, grapmuib

Fichier: *grapmuigva2006.f*, *grapmuib2006.f*

ADV · nouvelle variable amu^3 pour calcul de μ avec éléments des phases avancées

momevo

Fichier: *momevo2006.f*

AUTRE · transmission des variables bmomit (moment d'inertie total), btot (moment angulaire total), ainsi que $\text{bmomin}(j)$ (moment d'inertie de la coquille j) par le **common**/ekinet/

couran, courom

Fichier: *couran_impli2006.f*, *courom_impli2006.f*

ADV · valeur maximum de $\text{dzeit}/\text{tdiff} \rightarrow 30$ (au lieu de 20)

POPIII · rajout des éléments de He-b même si $x > 0$ (dans *couran*)

interx

Fichier: *interxgva2006.f*

ADV · ajout des éléments des phases avancées

kappa

Fichier: *kappa96_afalpenh2006.f*, *kappa96_solar2006.f*

ADV · ajout des éléments des phases avancées pour le contrôle des abondances (ainsi que ceux du cycle NeNa-MgAl)

schrit

Fichier: *schritgva2006.f*

ADV · ajout des éléments des phases avancées
· différence relative pour ω

tpsp, write4, write8

Fichier: *tpspgva2006.f*, *write4rgva2006.f*, *write8rgva2006.f*

ADV · ajout des éléments des phases avancées

vomcon

Fichier: *vomconmagn2006.f*

ADV · homogénéisation dans les zones convectives seulement si `idifcon=0`

xloss

Fichier: *xlossr2006.f*

- POPIII · `imloss=9`: nouvelle loi de perte de masse pour très basses métallicité (Kudritzki 2002)
- AUTRE · mise à jour de `imloss=6`: on passe de Vink et al. (2000) à Vink et al. (2001), avec dépendance en Z de la densité du vent
- mise à jour de `imloss=7` (Nugis & Lamers 2000): adaptation de la dépendance en Z selon Eldridge & Vink (2006).
- `imloss=10`: nouvelle loi pour RSG et AGB (van Loon et al. 2005)

zeit

Fichier: *zeit.f*

- ADV · pas temporels différenciés selon le stade de combustion
- possibilité de version plus lente (par défaut en commentaires)

.a file (origin2006)

n°	variable	format	grandeur	description
1	nm	i6		numéro du modèle
2	u1	1pe20.13		âge
3	u2	9.4	$M/M_{\odot}$	
4	u5	e12.6	$X(1)$	)
5	u37	e12.6	$^3\text{He}(1)$	)
6	u7	e12.6	$Y(1)$	)
7	u8	e12.6	$^{12}\text{C}(1)$	)
8	u9	e12.6	$^{13}\text{C}(1)$	)
9	u10	e12.6	$^{14}\text{N}(1)$	)
10	u38	e12.6	$^{15}\text{N}(1)$	)
11	u12	e12.6	$^{16}\text{O}(1)$	)
12	u13	e12.6	$^{17}\text{O}(1)$	)
13	u14	e12.6	$^{18}\text{O}(1)$	)
14	u6	e12.6	$^{20}\text{Ne}(1)$	)
15	u11	e12.6	$^{22}\text{Ne}(1)$	)
16	u39	e12.6	$^{24}\text{Mg}(1)$	)
17	u40	e12.6	$^{25}\text{Mg}(1)$	)
18	u41	e12.6	$^{26}\text{Mg}(1)$	)
19	u25	e12.6	$^{19}\text{F}(1)$	)
20	u26	e12.6	$^{21}\text{Ne}(1)$	)
21	u30	e12.6	$^{23}\text{Na}(1)$	)
22	u27	e12.6	$^{26}\text{Al}(1)$	)
23	u28	e12.6	$^{27}\text{Al}(1)$	)
24	u29	e12.6	$^{28}\text{Si}(1)$	)
25	u15	e12.6	$X(m)$	)
26	u42	e12.6	$^3\text{He}(m)$	)
27	u17	e12.6	$Y(m)$	)
28	u18	e12.6	$^{12}\text{C}(m)$	)
29	u19	e12.6	$^{13}\text{C}(m)$	)
30	u20	e12.6	$^{14}\text{N}(m)$	)
31	u43	e12.6	$^{15}\text{N}(m)$	)
32	u22	e12.6	$^{16}\text{O}(m)$	)
33	u23	e12.6	$^{17}\text{O}(m)$	)
34	u24	e12.6	$^{18}\text{O}(m)$	)
35	u16	e12.6	$^{20}\text{Ne}(m)$	)
36	u21	e12.6	$^{22}\text{Ne}(m)$	)
37	u44	e12.6	$^{24}\text{Mg}(m)$	)
38	u45	e12.6	$^{25}\text{Mg}(m)$	)
39	u46	e12.6	$^{26}\text{Mg}(m)$	)
40	u31	e12.6	$^{19}\text{F}(m)$	)
41	u32	e12.6	$^{21}\text{Ne}(m)$	)
42	u36	e12.6	$^{23}\text{Na}(m)$	)
43	u33	e12.6	$^{26}\text{Al}(m)$	)
44	u34	e12.6	$^{27}\text{Al}(m)$	)
45	u35	e12.6	$^{28}\text{Si}(m)$	)

.g file (*origin2006*)

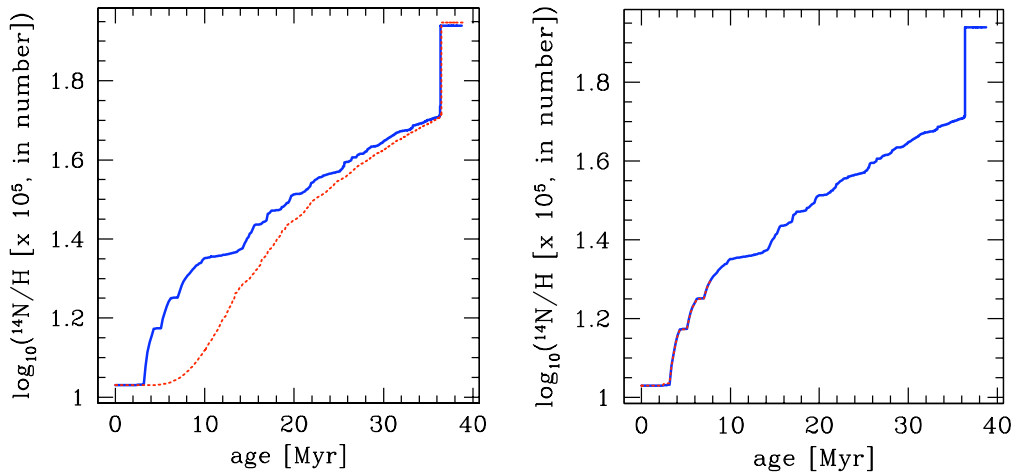
n°	variable	format	grandeur	description
1	nm	i6		numéro du modèle
2	u1	1pe20.13		âge
3	u2	f9.4	$M/M_{\odot}$	
4	xl	f5.3	$\log(L/L_{\odot})$	
5	xth	f5.3	$\log(T_{\text{eff}})$	
6	u5	e14.7	$X(1)$	)
7	u7	e14.7	$Y(1)$	)
8	b33	1pe14.7	${}^3\text{He}(1)$	)
9	u8	1pe14.7	${}^{12}\text{C}(1)$	)
10	u9	1pe14.7	${}^{13}\text{C}(1)$	)
11	u10	1pe14.7	${}^{14}\text{N}(1)$	)
12	u12	1pe14.7	${}^{16}\text{O}(1)$	)
13	u13	1pe14.7	${}^{17}\text{O}(1)$	)
14	u14	1pe14.7	${}^{18}\text{O}(1)$	)
15	u6	1pe14.7	${}^{20}\text{Ne}(1)$	)
16	u11	1pe14.7	${}^{22}\text{Ne}(1)$	)
17	qmnc	f6.4	M_{cc}	fraction de masse du coeur convectif
18	xte	f5.3	$\log(T_{\text{eff}})_{\text{non corr.}}$	T_{eff} non corrigée (<i>corrvind</i>)
19	xmdot	f7.3	$\log(\dot{M})$	
20	rhoc	f5.3	$\log(\rho_c)$	
21	tc	f5.3	$\log(T_c)$	
22	u15	e14.7	$X(m)$	)
23	u17	e14.7	$Y(m)$	)
24	b33m	1pe14.7	${}^3\text{He}(m)$	)
25	u18	1pe14.7	${}^{12}\text{C}(m)$	)
26	u19	1pe14.7	${}^{13}\text{C}(m)$	)
27	u20	1pe14.7	${}^{14}\text{N}(m)$	)
28	u22	1pe14.7	${}^{16}\text{O}(m)$	)
29	u23	1pe14.7	${}^{17}\text{O}(m)$	)
30	u24	1pe14.7	${}^{18}\text{O}(m)$	)
31	u16	1pe14.7	${}^{20}\text{Ne}(m)$	)
32	u21	1pe14.7	${}^{22}\text{Ne}(m)$	)
33	ybe7m	1pe10.3	${}^7\text{Be}(m)$	)
34	yb8m	1pe10.3	${}^8\text{B}(m)$	)
35	flube7	1pe10.3		flux de ν dû à ${}^7\text{Be}(e^-, \nu){}^7\text{Li}$
36	flub8	1pe10.3		flux de ν dû à ${}^8\text{B}(e^+ \nu){}^8\text{Be}$
37	snu7	1pe10.3		cf. 35 mais en SNU
38	snu8	1pe10.3		cf. 36 mais en SNU
39	rapcrj	f12.8	$\Omega/\Omega_{\text{crit}}$	
40	rot1	1pe10.3	$\Omega(1)$	Ω en surface
41	rotrm	1pe10.3	$\Omega(m)$	Ω au centre
42	xobla	1pe10.3	$R_{\text{pol}}/R_{\text{eq}}$	
43	u27	1pe10.3	${}^{26}\text{Al}(1)$	abondance en surface
44	u33	1pe10.3	${}^{26}\text{Al}(m)$	abondance au centre
45	alpro6	1pe10.3	$\dot{M}(\Omega)/\dot{M}(0)$	correction de la perte de masse due à la rotation
46	lcnom	i4		couche avec max de CNO
47	xmcno	f9.4		masse de la couche lcnom
48	scno	1pe9.2		somme des éléments CNO dans couche lcnom
49	xjspe1	f10.3	$j(3 M_{\odot})$	moment spécifique à $3 M_{\odot}$ [unités de collapsar]
50	xjspe2	f10.3	$j(5 M_{\odot})$	moment spécifique à $5 M_{\odot}$ [unités de collapsar]
51	vcrit1m	f8.2	$v_{\text{crit},1}$	1 ^{re} racine
52	vcrit2m	f8.2	$v_{\text{crit},2}$	2 ^e racine
53	vequam	f8.2	v_{eq}	(avant convergence)
54	rapomm	f8.6	$\Omega/\Omega_{\text{crit}}$	limite $\Gamma\Omega$
55	eddesm	f8.6	Γ_{Edd}	
56	vcrit1	f8.2	$v_{\text{crit},1}$	1 ^{re} racine
57	vcrit2	f8.2	$v_{\text{crit},2}$	2 ^e racine
58	vequa	f8.2	v_{eq}	(après convergence)
59	rapom2	f8.6	$\Omega/\Omega_{\text{crit}}$	limite $\Gamma\Omega$
60	eddesc	f8.6	Γ_{Edd}	
61	bmomit	1pe14.7	I	moment d'inertie total
62	btot	1pe14.7	$\mathcal{L}_{\text{tot}}$	moment cinétique total
63	ekrote	1pe14.7	E_{rot}	énergie cinétique de la rotation
64	epote	1pe14.7	E_{pot}	énergie potentielle
65	ekine	1pe14.7	E_{gaz}	énergie thermique du gaz
66	erade	1pe14.7	E_{rad}	énergie de la radiation
67 - 106	uudrawc	f6.3		limites des zones convectives.

n°			description		n°		variable		format		grandeur		description	
1	j	i4	n° de couche (l : surface, m : centre)		47	difcon	1pe12.5	D_{conv}					coefficient de diffusion convective	
2	vm	f10.7			48	difsch	1pe12.5	D_{sh}					coeff. de diffusion due au cisaillement	
3	x2	f10.7			49	difdef	1pe12.5	D_{eff}					coeff. de diffusion de la circ. mérid. et de la turb.	
4	x3	f10.7			50	vmasse	f9.4	M_r					masse incluse dans la coquille $[M_{\odot}]$	
5	x4	f14.11			51	vmolnr	e15.8	$\partial \ln \Omega / \partial \ln r$					gradient de vitesse angulaire	
6	vl	e14.6			52	xnprim	e15.8						(pas utilisé)	
7	x	e14.7			53	xkther	e15.8	K					coeff. de diffusion thermique	
8	y	e14.7			54	uicoe	e15.8	$U(r)$					composante verticale de la circ. mérid.	
9	xc	e14.7			55	vcoie	e15.8	$V(r)$					composante tangentielle de la circ. mérid.	
10	xo	e14.7			56	dcirch	e15.8						coeff. de diffusion de circ. mérid. (seulement si $\text{imagn}=1$)	
11	eps	1pe11.4	énergie produite par H-b		57	echprry	e15.8	H_P					échelle de pression	
12	epsy	1pe11.4	énergie produite par He-b		58	gravy	e15.8	g					gravité	
13	epsz	1pe11.4	énergie produite par C-b		59	dhoriz	e15.8	D_h					coeff. de diffusion de la turb. horiz.	
14	radm	1pe11.4	(moyenne entre j et j+1)		60	dhoriz	e15.8	$D_{h,\text{Zahn}}$					nouveau D_h de Zahn	
15	x6	f11.6			61	omegp	e15.8	$\Omega(r)$					calculé seulement avec conserv. du mom. cin.	
16	zensi	1pe12.5	facteur de la convection ($< 0 : \text{rad}, > 0 : \text{conv.}$)		62	vr	e15.8	$r_{\text{prev.}}$					rayon du modèle précédent	
17	epsn	1pe11.4			63	vomegi	e15.8	$\Omega(r)_{\text{prev.}}$					vitesse ang. du modèle préc.	
18	x10	1pe12.5			64	dmago	e15.8	$D_{\text{magn},\Omega}$					coeff. de diffusion magn. pour Ω	
19	x11	1pe12.5			65	dmagx	e15.8	D_{magn}					coeff. de diff. magn. pour les esp. chimiques	
20	x12	1pe12.5			66	etask	e15.8	η					utilisé dans le traitement du magnétisme	
21	x13	1pe12.5			67	bvais2	e15.8	N^2					carré de la fréquence de Brunt-Väisälä	
22	x14	1pe12.5			68	bphi	e15.8	B_{ϕ}					champ magnétique toroidal	
23	x15	1pe12.5			69	alven	e15.8	ω_A					fréquence d'Alfvén	
24	psi	1pe9.2	degré de dégénérescence		70	xqmin	e15.8						grad. de rotation min. pour amplification du champ magn.	
25	psxy	1pe9.2			71	x7	f9.6	μ					couche j+1, calculé dans <i>dichte</i>	
26	epsyc	1pe10.2			72	vmeye	f9.6	μ_e					poids moléculaire des e^-	
27	epsyo	1pe11.2			73	xf19	1pe14.7	^{19}F						
28	eg	1pe12.5			74	xne21	1pe14.7	^{21}Ne						
29	adim	1pe12.5	(moyenne entre j et j+1)		75	xna23	1pe14.7	^{23}Na						
30	x8	1pe12.5	opacité		76	xal26g	1pe14.7	^{26}Al						
31	beta	1pe12.5			77	xsal27	1pe14.7	^{27}Al						
32	y3	e14.7			78	xxi28	1pe14.7	^{28}Si						
33	xc13	e14.7			79	xcl4	1pe14.7	^{14}C						
34	xn14	e14.7			80	xf18	1pe14.7	^{18}F						
35	xn15	e14.7			81	xneut	1pe14.7	n						
36	xo17	e14.7			82	xprot	1pe14.7	p						
37	xo18	e14.7			83	xbid	1pe14.7							
38	x20	e14.7			84	abelx(1,8)	1pe14.7	^{28}Si						
39	x22	e14.7			85	abelx(2,8)	1pe14.7	^{32}S						
40	x24	e14.7			86	abelx(3,8)	1pe14.7	^{36}Ar						
41	x25	e14.7			87	abelx(4,8)	1pe14.7	^{40}Ca					seulement si	
42	x26	e14.7			88	abelx(5,8)	1pe14.7	^{44}Ti					in et burn ≥ 1	
43	vmhelio	f9.6	poids moléculaire moyen de la couche j		89	abelx(6,8)	1pe14.7	^{48}Cr						
44	omegi	1pe12.5	vitesse angulaire		90	abelx(7,8)	1pe14.7	^{52}Fe						
45	xdmudp	1pe12.5	gradient de composition chimique		91	abelx(8,8)	1pe14.7	^{56}Ni						
46	xricha	1pe12.5	nombre de Richardson											

A.3.4 Tests

The first step was to control that we get the same results with the new code as with the original one if we use the same configuration.

Figure A.3 on page 156 shows an example of such a test between the code used by Georges Meynet and André Maeder (*originV2*) and *origin2006* in its basic configuration (no Pop III stars, no advanced phases). The model is a $9 M_{\odot}$ at $Z = 0.020$ with $v_{\text{ini}} = 200 \text{ km s}^{-1}$. The agreement between the two codes is satisfactory, except for the surface enrichment (next to last line). I checked the cause of the difference, and found that it was linked to the timesteps used. The code adapts it automatically, but while the original code determines the timesteps only from the innermost shell, the new code determines it from the most active shell, which leads to smaller timesteps. A computation with *origin2006* but with the timestep adapted only from the central evolution gives the same results (Fig. A.2). I then tested the code in the Pop III + advanced phases



Same model as Fig. A.3 on page 156. *Left:* *origin2006* (red dotted line) with the timesteps determined from the most active shell. *Right:* *origin2006* with the timesteps determined from the centre only.

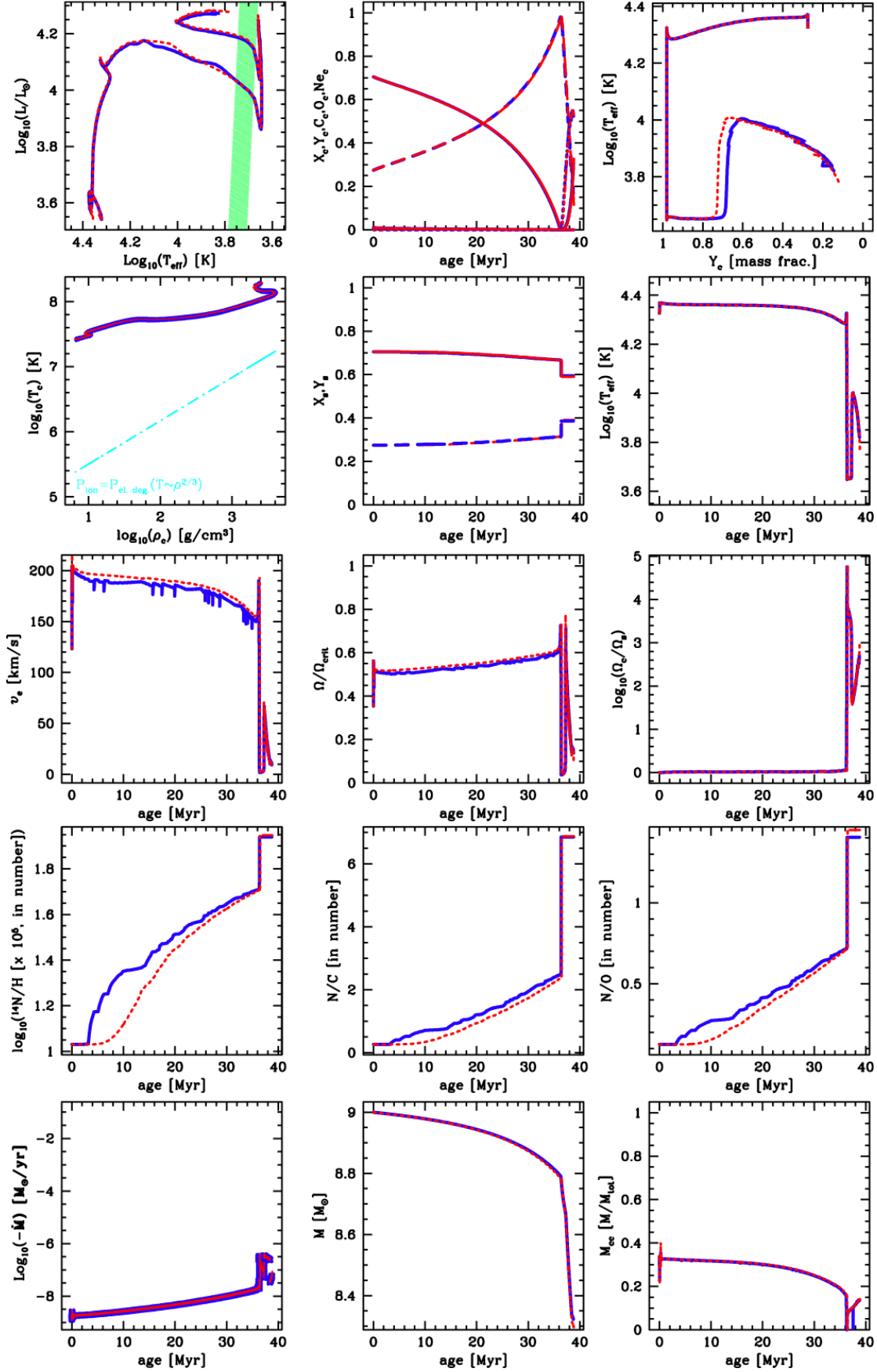
configuration. Since these two extensions were already fused in the program I used for the models of Chapter 4, they were tested together. Figure A.4 on page 157 shows an example of the results on a $9 M_{\odot}$ model with $v_{\text{ini}} = 500 \text{ km s}^{-1}$ at $Z = 0$. The two models have an almost identical behaviour, with a small deviation in the way the surface enrichment occurs. However the level of enrichment is fairly similar, and the difference is mostly that *origin2006* shows smoother curves, so we considered this test as satisfactory.

The second step was to check the compatibility of the different implementations. For example, the parameter *inetburn* has to be switch on from the beginning of the evolution, but it should not interfere with the normal evolution during the first phases. Also the treatment of the advanced phases has never been associated to the treatment of the NeNa-MgAl cycle.

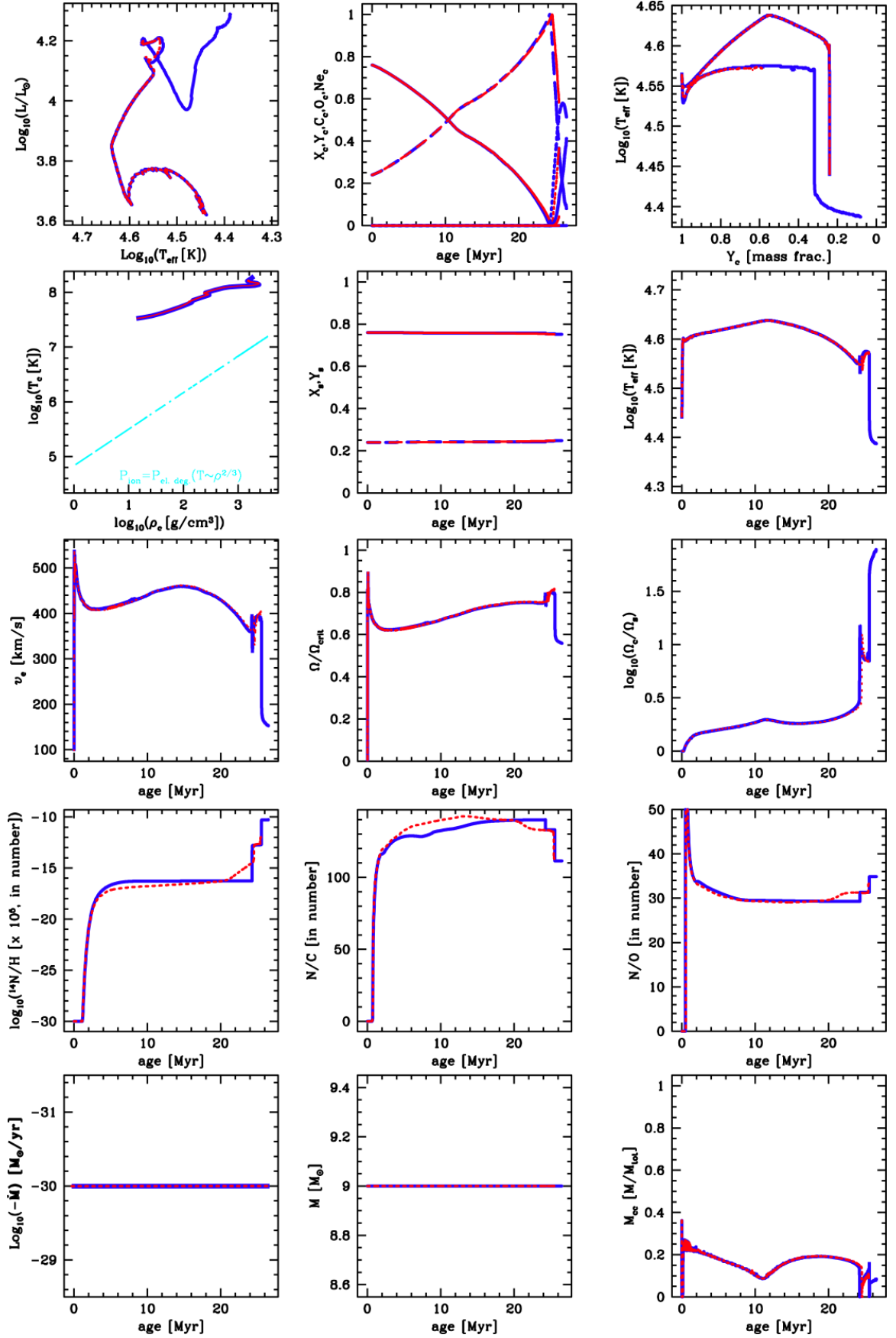
Figure A.5 on page 158 shows the result of the evolution of a $15 M_{\odot}$ model with $v_{\text{ini}} = 300 \text{ km s}^{-1}$ at $Z = 0.020$, with the parameter *inetburn*=0 or 3. In this case, the agreement is almost perfect. Before testing the association *inetburn*=3 and *ialflu*=1, I had to check the differences brought by the treatment of the NeNa-MgAl cycle. Besides the aspect of the new energy source from an extended network, one of the main source of difference resides in a different treatment of the mixing and burning phases. Normally, in a convective zone, the chemical species are homogenised, then the change of abundance through reaction rates is calculated, and then the convective zone is mixed again. When the NeNa-MgAl cycle is treated, the neutrons are followed and since they interact only locally, the species are mixed only after the burning. Figure A.6 on page 159 shows a comparison of the results obtained with and without the extended network. The

main difference is a shorter lifetime (by about 5%). When `inetburn=3` (Fig. A.7 on page 160), we get the same kind of difference, but with a slightly lower amplitude (about 3%).

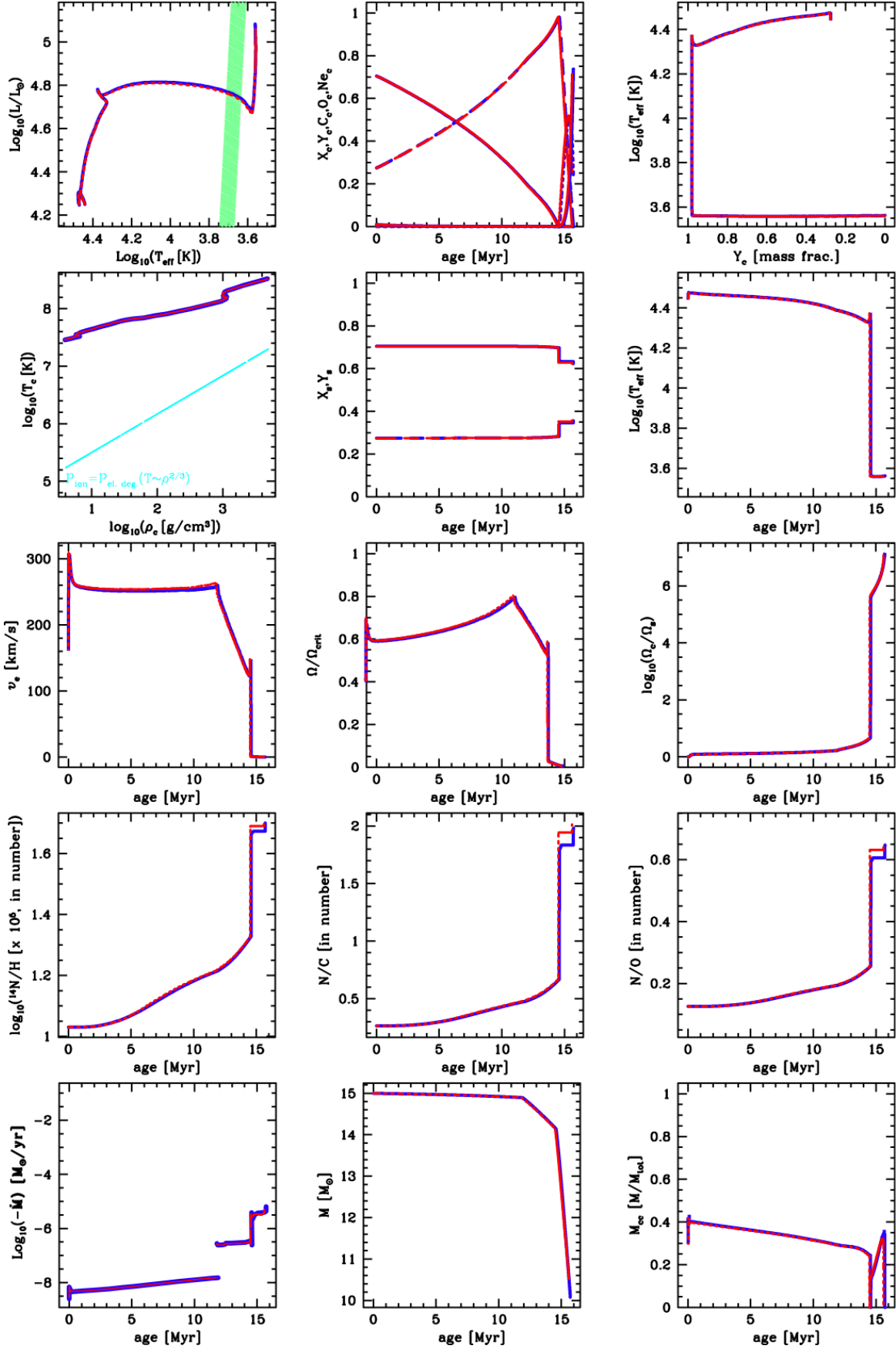
Given these good results, we considered that `origin2006` had passed the exam and could be used with confidence.



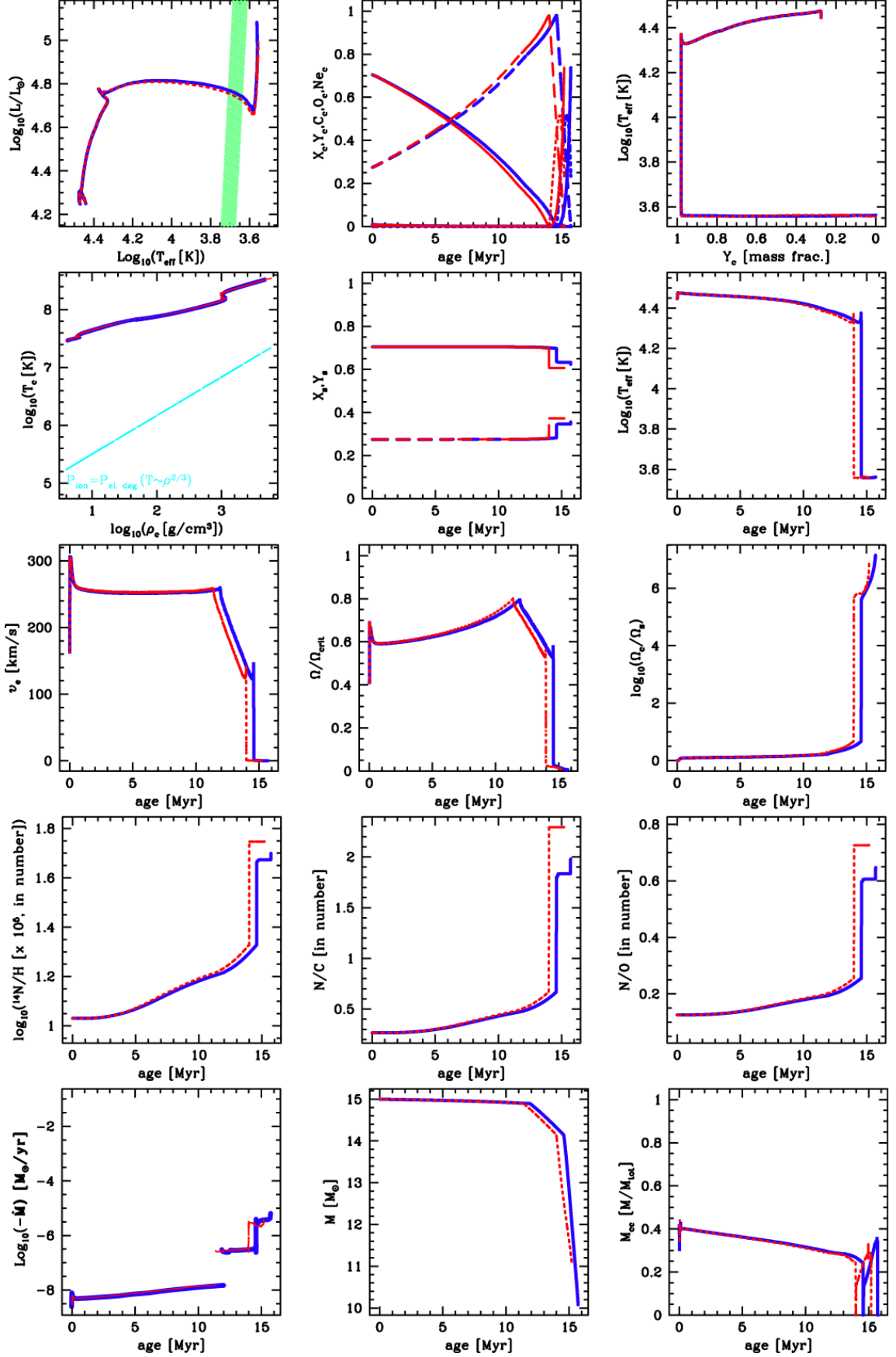
Evolution of a 9 $M_{\odot}$ model with $v_{\text{ini}} = 200 \text{ km s}^{-1}$ at $Z = 0.020$. Comparison between a computation with Georges Meynet's code (blue line), and a computation with *origin2006* when the parameters are set to *inetburn*=0 and *ipop3*=0 (red line).



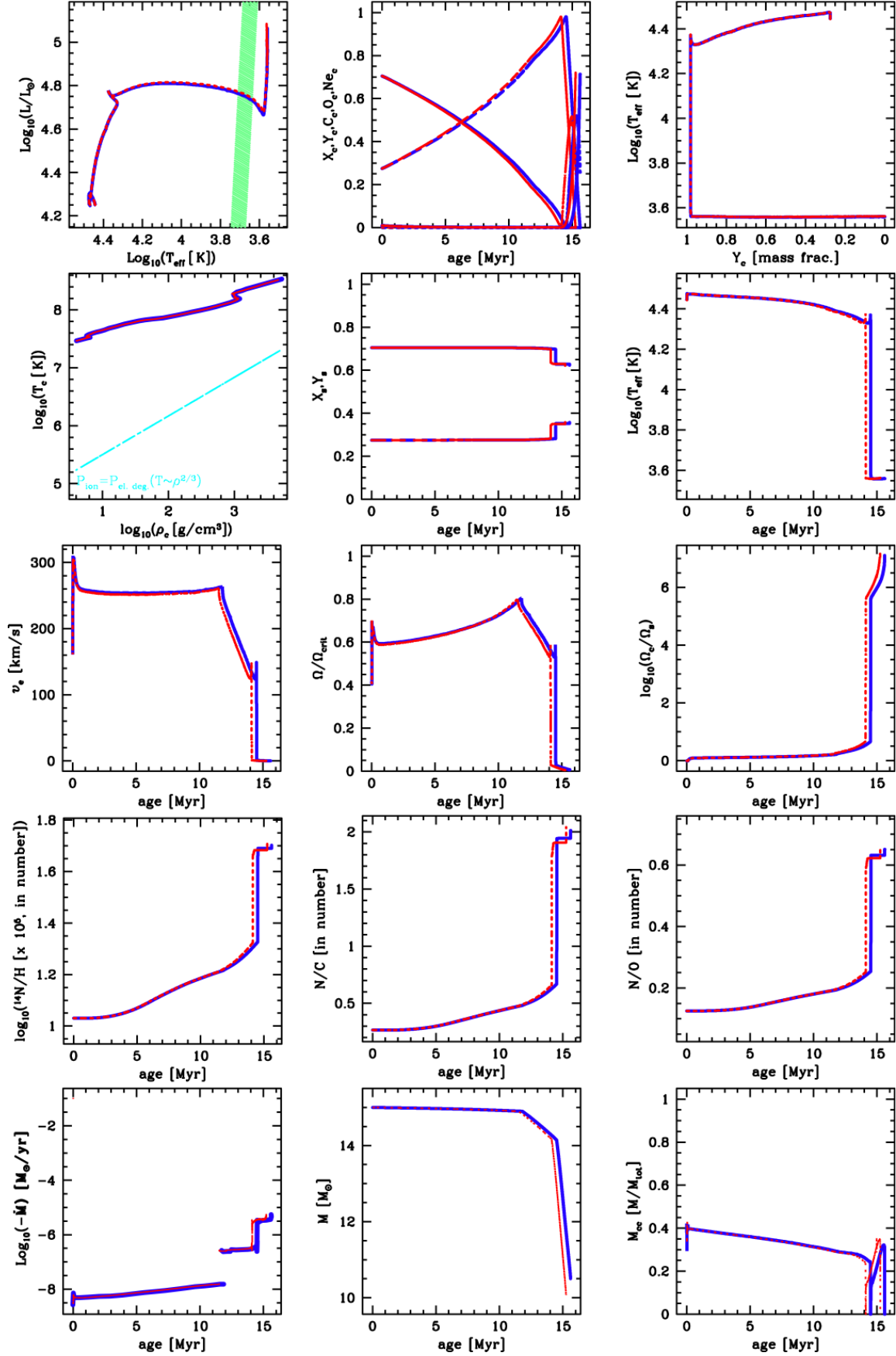
Evolution of a 9 $M_{\odot}$ model with $v_{\text{ini}} = 500 \text{ km s}^{-1}$ at $Z = 0$. Comparison between a computation done with the program advanced used for the models of Chapter 4 (blue line), and a computation with *origin2006* when the parameters are set to *inetburn*=3 and *ipop3*=1 (red line).



Evolution of a $15 M_{\odot}$ model with $v_{\text{ini}} = 300 \text{ km s}^{-1}$ at $Z = 0.020$. Comparison between a computation done with the parameter `inetburn=0` (blue line) and `inetburn=3` (red line).



Evolution of a $15 M_{\odot}$ model with $v_{\text{ini}} = 300 \text{ km s}^{-1}$ at $Z = 0.020$. Comparison between a computation done with the parameter $ialflu=0$ (blue line) and $ialflu=1$ (red line).



Evolution of a $15 M_{\odot}$ model with $v_{\text{ini}} = 300 \text{ km s}^{-1}$ at $Z = 0.020$. Comparison between a computation done with the parameter $ialflu=0$ (blue line) and $ialflu=1$ (red line) when $inethburn=3$.

Publications related to this thesis

B.1 List of papers

Refereed articles

- DARK MATTER ANNIHILATIONS IN POP III STARS
M. Taoso, G. Bertone, G. Meynet & **S. Ekström**
2008 Phys. Rev. D, **78**, id. 123510
- EFFECTS OF ROTATION ON THE EVOLUTION OF PRIMORDIAL STARS
S. Ekström, G. Meynet, C. Chiappini, R. Hirschi & A. Maeder
2008 A&A **489**, 685
- A NEW IMPRINT OF FAST ROTATORS: LOW $^{12}\text{C}/^{13}\text{C}$ RATIOS IN EXTREMELY METAL-POOR HALO STARS
C. Chiappini, **S. Ekström**, G. Meynet, R. Hirschi, A. Maeder & C. Charbonnel
2008 A&A **479**, L9
- EVOLUTION TOWARDS THE CRITICAL LIMIT AND THE ORIGIN OF BE STARS
S. Ekström, G. Meynet, A. Maeder & F. Barblan
2008 A&A **478**, 467
- A STRONG CASE FOR FAST STELLAR ROTATION AT VERY LOW METALLICITIES
C. Chiappini, R. Hirschi, G. Meynet, **S. Ekström**, A. Maeder & F. Matteucci
2006 A&A **449**, L27
- THE EARLY STAR GENERATIONS: THE DOMINANT EFFECT OF ROTATION ON THE CNO YIELDS
G. Meynet, **S. Ekström** & A. Maeder
2006 A&A **447**, 623

Publications in conference proceedings

- POWERFUL EXPLOSIONS AT $Z = 0$?
S. Ekström, G. Meynet, R. Hirschi, A. Maeder
in IAU Symposium 255: *Low-Metallicity Star Formation: From the First Stars to Dwarf Galaxies*
Eds L. K. Hunt, S. Madden & R. Schneider
2008 IAU Symp. 255, p. 194 (arXiv:0807.5050)

B.1. List of papers

- CAN VERY MASSIVE STARS AVOID PAIR-INSTABILITY SUPERNOVAE?

S. Ekström, G. Meynet, A. Maeder

in IAU Symposium 250: *Massive Stars as Cosmic Engines*

Eds F. Bresolin, P.A. Crowther & J. Puls

2008 IAU Symp. 250, p. 209 (arXiv:0801.3397)

and

in *First Stars III*, Eds T. Abel, A. Heger & B. O'Shea

2007 AIP Conf. Ser. Vol. 990, p. 220 (arXiv:0709.0202v1)

- EFFECTS OF ROTATION ON MASS LOSS FOR POPULATION III STARS

S. Ekström, G. Meynet, A. Maeder

In *Stellar Evolution at Low Metallicity: Mass Loss, Explosions, Cosmology*

Eds. H.J.G.L.M Lamers, N. Langer, T. Nugis & K. Annuk

2006 ASP Conf. Ser. Vol. 353, p. 141 (astro-ph/0511080)

Poster proceedings

- CNO PRODUCTION IN THE FIRST GENERATION STARS

S. Ekström, G. Meynet, A. Maeder

Proceedings of *Nuclei in the cosmos IX*, online publication at PoS website (<http://pos.sissa.it>)

2006 PoS(NIC-IX)091 (astro-ph/0610310)

- FAST ROTATING FIRST STARS

S. Ekström, G. Meynet, A. Maeder

In IAU Symposium 228: *From Lithium to Uranium: Elemental tracers of early cosmic evolution*,

Cambridge University Press, Eds. V. Hill, P. François & F. Primas

2005 IAU Symp. 228, p. 163

B.2 Refereed articles

PHYSICAL REVIEW D **78**, 123510 (2008)

Dark matter annihilations in Population III stars

Marco Taoso,^{1,2} Gianfranco Bertone,² Georges Meynet,³ and Silvia Ekström³

¹*INFN, Sezione di Padova, via Marzolo 8, Padova, 35131, Italy*

²*Institut d'Astrophysique de Paris, UMR 7095-CNRS, Université Pierre et Marie Curie, 98 bis Boulevard Arago 75014, Paris, France*

³*Geneva Observatory, University of Geneva, Maillettes 51, 1290 Sauverny, Switzerland*

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We study the impact of the capture and annihilation of weakly interacting massive particles (WIMPs) on the evolution of Population III stars. With a suitable modification of the Geneva stellar evolution code, we study the evolution of 20 and 200 $M_{\odot}$ stars in dark matter halos with densities between 10^8 and 10^{11} GeV/cm^3 during the core H-burning phase, and, for selected cases, until the end of the core He-burning phase. We find that for WIMP densities higher than $5.3 \cdot 10^{10} (\sigma_p^{\text{SD}}/10^{-38} \text{ cm}^2)^{-1} \text{ GeV cm}^{-3}$ the core H-burning lifetime of 20 $M_{\odot}$ and 200 $M_{\odot}$ stars exceeds the age of the Universe, and stars are sustained only by WIMP annihilations. We determine the observational properties of these “frozen” objects and show that they can be searched for in the local Universe thanks to their anomalous mass-radius relation, which should allow unambiguous discrimination from normal stars.

DOI: 10.1103/PhysRevD.78.123510

PACS numbers: 95.35.+d, 97.10.Bt, 97.10.Cv

In the standard cosmological model, the matter density of the Universe is dominated by an unknown component, approximately 5 times more abundant than baryons, dubbed dark matter (DM). Among the many DM candidates proposed in the literature, weakly interacting massive particles (WIMPs), i.e. particles with mass $\mathcal{O}(100)$ GeV and weak interactions, appear particularly promising, also in view of their possible connection with well motivated extensions of the standard model of particle physics (see Ref. [1] for recent reviews on particle DM, including a discussion of ongoing direct, indirect, and accelerator searches). Despite their weak interactions, WIMPs can lead to macroscopic effects in astrophysical objects, provided that they have a sizeable scattering cross section off baryons. In this case, in fact, DM particles traveling through stars can be captured and sink at the center of the stars. Direct searches and astrophysical arguments, however, severely constrain the strength of DM-baryons interactions (see e.g. Ref. [2] and references therein). Since the capture rate is proportional to the product of the scattering cross section times the local DM density, large effects are thus expected in regions where the DM density is extremely high (this was already noticed in the context of the so-called “cosmions” [3]). Recent progress in our understanding of the formation and structure of DM halos has prompted a renewed interest in the consequences of DM capture in stars, in particular, in the case of white dwarfs [4], compact objects [5], and main sequence stars [6] at the Galactic center, where the DM density could be extremely high [7].

Alternatively, one may focus on the first stars, which are thought to form from gas collapsing at the center of 10^6 – $10^8 M_{\odot}$ DM halos at redshift $z \lesssim 10$ – 30 . The consequences of DM annihilations in Pop III stars were first investigated by Spolyar, Freese, and Gondolo [8], who have shown that the energy released by WIMP annihilations in these mini-halos, during the formation of a proto-

star, may exceed any cooling mechanism, thus leading to a new phase of stellar evolution (see also Ref. [9]). The formation of proto-stars with masses between $6M_{\odot}$ and $600M_{\odot}$ in DM halos of $10^6 M_{\odot}$ at $z = 20$ can actually be delayed by $\sim 10^3$ – 10^4 yrs [10]. It was subsequently shown in Refs. [11,12], that the annihilation of DM particles captured at the center of the star, due to scattering off the stellar nuclei, can lead to an energy injection that overwhelms nuclear reactions. Under these circumstances, the core H-burning phase of Pop III stars, in DM halos of density of $10^{11} \text{ GeV cm}^{-3}$, is substantially prolonged, especially for small mass stars ($M_* < 40M_{\odot}$) [10].

In this paper, we perform a detailed study of the impact of DM capture and annihilation on the evolution of Pop III stars implementing in the Geneva stellar evolution code [13] a WIMP luminosity term as described below. With respect to previous analyses, this already allows us to properly take into account the stellar structure in the calculation of the capture rate that we compute, following Ref. [14], as

$$C = 4\pi \int_0^{R_*} dr r^2 \frac{dC(r)}{dV} \quad (1)$$

with

$$\begin{aligned} \frac{dC(r)}{dV} &= \left(\frac{6}{\pi}\right)^{1/2} \sigma_{\chi N} \frac{\rho_i(r)}{M_i} \frac{\rho_{\chi}}{m_{\chi}} \frac{v^2(r)}{\bar{v}^2} \frac{\bar{v}}{2\eta A^2} \\ &\times \left\{ \left(A_+ A_- - \frac{1}{2} \right) [\chi(-\eta, \eta) - \chi(A_-, A_+)] \right. \\ &\quad \left. + \frac{1}{2} A_+ e^{-A_-^2} - \frac{1}{2} A_- e^{-A_+^2} - \frac{1}{2} \eta e^{-\eta^2} \right\}, \\ A^2 &= \frac{3v^2(r)\mu}{2\bar{v}^2\mu_-^2}, \quad A_{\pm} = A \pm \eta, \quad \eta^2 = \frac{3v_*^2}{2\bar{v}^2} \\ \chi(a, b) &= \frac{\sqrt{\pi}}{2} [\text{Erf}(b) - \text{Erf}(a)] = \int_a^b dy e^{-y^2}, \\ \mu_- &= (\mu_i - 1)/2, \quad \mu_i = m_{\chi}/M_i, \end{aligned} \quad (2)$$

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where $\rho_i(r)$ is the mass density profile of a given chemical element in the interior of the star and M_i refers to its atomic mass, while ρ_χ , m_χ , and $\bar{v}$ are, respectively, the WIMP mass and the WIMP density and velocity dispersion at the star position. The velocity of the star with respect to an observer, labeled as v_* , is assumed to be equal to $\bar{v}$, giving therefore $\eta = \sqrt{3}/2$. The radial escape velocity profile depends on $M(r)$, i.e. the mass enclosed within a radius r , $v^2(r) = 2 \int_r^\infty GM(r')/r'^2 dr'$. The WIMP scattering cross section off nuclei, $\sigma_{\chi,N}$, is constrained by direct detection experiments and for a WIMP mass of 100 GeV the current upper limits are $\sigma_{\text{SI}} = 10^{-43} \text{ cm}^2$ [15] and $\sigma_{\text{SD}} = 10^{-38} \text{ cm}^2$ [16], respectively, for spin-independent and spin-dependent WIMP interactions off a proton. We will adopt these reference values throughout the paper, but the capture rate can be easily rescaled for other scattering cross sections by using Eq. (3). The spin-independent interactions with nucleons inside nuclei add up coherently giving an enhancement factor A^4 with respect to the interaction with a single nucleon: $\sigma_{\chi,N}^{\text{SI}} = A^4 \sigma_{\chi,p}$, where A is the mass number. There is no such enhancement for the spin-dependent interactions. We consider the contribution to the capture rate from WIMP-hydrogen spin-dependent interactions and WIMP-helium ^4He spin-independent interactions, neglecting the presence of other elements because of their very low abundance. The contribution of helium, for the WIMPs parameters above, is found to be negligible with respect to that from hydrogen.

Once captured, WIMPs get redistributed in the interior of the star reaching, in a characteristic time τ_{th} , a thermal distribution [17]:

$$n_\chi(r) = n_0 e^{(-r^2)/(r_\chi^2)} \quad \text{with} \quad r_\chi = \sqrt{\frac{3kT_c}{2\pi G \rho_c m_\chi}} \quad (3)$$

with T_c and ρ_c referring to the core temperature and density. The distribution results are quite concentrated toward the center of the star: e.g. for a $20M_\odot$ star immersed in a WIMP density of $\rho_\chi = 10^9 \text{ GeV cm}^{-3}$ at the beginning of the core H-burning phase we obtain $r_\chi = 2 \times 10^9 \text{ cm}$, a value much lower than the radius of the star, $R_* = 10^{11} \text{ cm}$. This consideration underlines the importance of an accurate spatial resolution in the core to properly treat the luminosity produced from WIMPs annihilations. We have also checked that regardless of the extremely high concentrations of WIMPs obtained at the center of the stars, the gravity due to WIMPs is completely negligible. The number of scattering events needed for DM particles to thermalize with the nuclei in the star is of order $\approx m_\chi/M_H$, thus an upper limit on the thermalization time can be obtained as $\tau_{\text{th}} = (m_\chi/M_H)/(\sigma_{\text{SD}} \bar{n}_H \bar{v})$ where $\bar{n}_H$ is the average density on the star.

The WIMPs emissivity is $\epsilon_\chi(r) = 4\pi(\sigma v) m_\chi c^2 n_\chi^2(r)$ and the total WIMPs luminosity, L_χ , is simply the integral over the volume of the star. For the annihilation cross

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section times relative velocity (σv), we assume the value $3 \times 10^{-26} \text{ cm}^2$, as appropriate for a thermal WIMP, but note that the total WIMP luminosity at equilibrium does not depend on this quantity. After a time $\tau_\chi = \sqrt{C(\sigma v) \pi^{-3/2} r_\chi^{-3}}$ an equilibrium between capture and annihilation is established, and this incidentally allows to determine the normalization constant n_0 above.

We have checked that the two transients τ_χ and τ_{th} remain much smaller, during the evolution of the star, than the Kelvin-Helmoltz time scale τ_{KH} and the time scale needed for the nuclear reactions to burn a small hydrogen fraction, e.g. $\Delta X_c = 0.002$, of the convective core, τ_{nuc} :

$$\tau_{\text{KH}} = \frac{GM_*^2}{R_* L_*}, \quad \tau_{\text{nuc}} = \frac{q_c \Delta X_c M_* 0.007 c^2}{L_*},$$

where the * labels quantities relative to the star and q_c is the core convective mass fraction. This argument justifies the assumption of equilibrium between capture and annihilation and the use of the radial distribution in Eq. (3). We assume here an average WIMP velocity $\bar{v} = 10 \text{ km s}^{-1}$, the virial velocity in a halo of 10^5 – $10^6 M_\odot$ at $z = 20$ (see [11] and references therein). As for the DM density, semi-analytic computations of the adiabatic contraction of DM halos [8], in agreement with the results extrapolated from simulations of first star formation [18], suggest DM densities of order $10^{12} \text{ GeV cm}^{-3}$ or even higher.

We have implemented the effects of WIMPs annihilation in the Geneva stellar evolution code (see Ref. [13] for details) and followed the evolution of $20M_\odot$ and $200M_\odot$ stars for different DM densities. We show in Fig. 1 the evolutionary tracks for the $20M_\odot$ model, and show for comparison (black line) the case of a standard Pop III star without WIMPs. For DM densities smaller than 10^9 GeV cm^{-3} the evolutionary tracks closely follow that of a normal star and they are not shown for simplicity. The position of the star at the beginning of the core H-burning phase (zero-age main sequence, or ZAMS) is obtained when, after a short transient, the luminosity produced at the center of the star equals the total luminosity and the star settles down in a stationary regime. For increasing DM densities the WIMPs luminosity produced at the center overwhelms the luminosity from nuclear reactions and makes the star inflate, producing therefore a substantial decrease of the effective temperature and a moderate decrease of the star luminosity at the ZAMS position, with respect to the standard scenario. For $\rho_\chi = 10^{10} \text{ GeV cm}^{-3}$, the energy produced by WIMPs present in the star at a given time, estimated as $E_\chi \approx L_\chi \tau_{\text{KH}}$, is, at the ZAMS, ~ 0.8 times the gravitational potential energy of the star, and the star therefore starts to contract. In this phase, the core temperature, and consequently also the nuclear reactions, increase. When the latter become comparable with the WIMPs luminosity, the standard situation is recovered and the evolutionary track joins the classical

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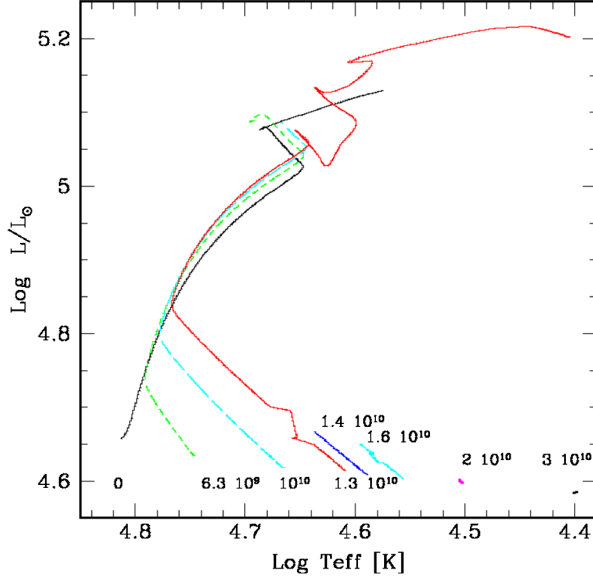


FIG. 1 (color online). Evolutionary tracks of a Pop III $20M_{\odot}$ star for different WIMP densities (labels in units of GeV cm^{-3}). We have adopted a WIMP model with $m_{\chi} = 100 \text{ GeV}$ and $\sigma_p^{\text{SD}} = 10^{-38} \text{ cm}^2$.

tracks of a star without WIMPs. An important difference from standard evolution is that in the first phase, the nuclear reactions are slowed down and therefore the core H-burning lifetime is prolonged. For dark matter densities $\rho_{\chi} \leq 1.6 \times 10^{10} \text{ GeV cm}^{-3}$, the picture is qualitatively the same, and for these models we only show in Fig. 1 the first phases of the evolution. In Fig. 2, we show the core temperature as a function of the DM density, at different stages of the core H-burning phase. At high DM densities hydrogen burns at much lower core temperatures than in the usual scenario, until a certain mass fraction is reached, e.g. $X_c = 0.3$ for $\rho_{\chi} = 10^{10} \text{ GeV cm}^{-3}$ and the standard evolutionary track is joined. For increasing DM densities the nuclear reaction rate is more and more delayed until the contraction of the star is inhibited, due to the high DM energy accumulated, and the evolution is frozen. In Fig. 1 for $\rho_{\chi} = 2 \times 10^{10} \text{ GeV cm}^{-3}$ and $\rho_{\chi} = 3 \times 10^{10} \text{ GeV cm}^{-3}$ the stars seems to remain indefinitely at the ZAMS position. In Fig. 3 we show the core H-burning lifetime as a function of the DM density. In the case of a $20M_{\odot}$ model, for $\rho \leq 10^9 \text{ GeV cm}^{-3}$ the core H-burning phase is prolonged by less than 10% but the delay increases rapidly for higher DM densities. Extrapolating the curve we determine a critical density, $\rho_c = 2.5 \times 10^{10} \text{ GeV cm}^{-3}$, beyond which the core H-burning lifetime is longer than the age of the Universe. All the calculations have been repeated for the $200M_{\odot}$ model and we find that both the $20M_{\odot}$ and $200M_{\odot}$ stars evolutions are stopped for DM densities higher than $5.3 \times 10^{10} \left(\frac{\sigma_p^{\text{SD}}}{10^{-38} \text{ cm}^2} \right)^{-1} \text{ GeV cm}^{-3}$. We have

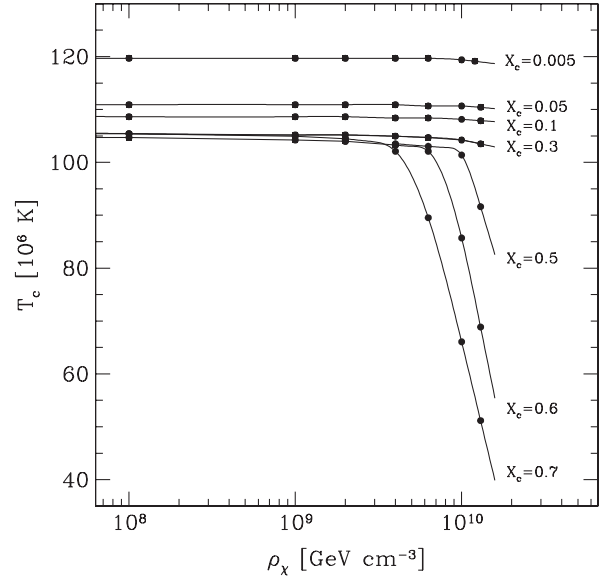
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FIG. 2. Temperature of the core as a function of the DM density for the $20M_{\odot}$ model, at different stages of the core H-burning phase. X_c denotes the mass fraction of hydrogen at the center ($X_c = 0.76$ at the beginning of the core H-burning phase). WIMP parameters as in Fig. 1.

also verified that the results weakly depend on the WIMP mass, e.g. the core H-lifetime is modified by a factor 0.2% and 5%, respectively, for $m_{\chi} = 10 \text{ GeV}$ and $m_{\chi} = 100 \text{ GeV}$, if $\rho_{\chi} = 10^{10} \text{ GeV cm}^{-3}$.

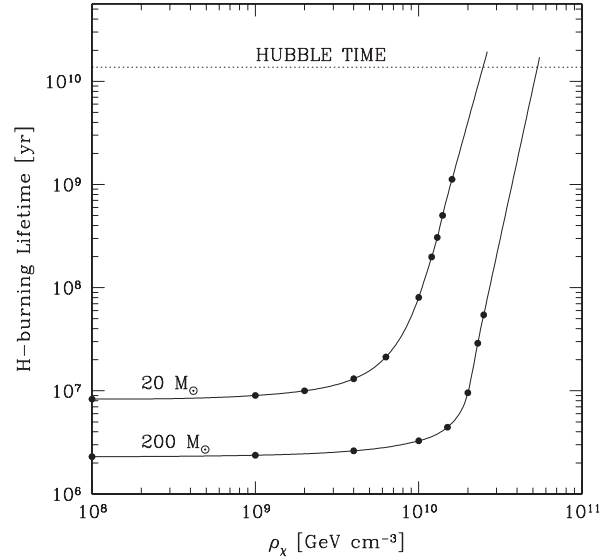


FIG. 3. Variation of the core H-burning lifetime as a function of the WIMP densities for the Pop III 20 and $200M_{\odot}$ models. WIMP parameters as in Fig. 1.

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It is remarkable that under these circumstances, frozen Pop III stars can survive until the present epoch and can be searched for as an anomalous stellar population. In Fig. 4 we show the effective temperature and gravity acceleration at the surface of these frozen Pop III stars, kept in the H-burning phase, for different DM densities. Frozen stars would thus appear much bigger and with much lower surface temperatures with respect to normal stars with the same mass and metallicity. Our results are qualitatively consistent with the preliminary estimates in [11, 12] and the analysis in [10]. However, for a given DM density, we obtain a somewhat longer core H-burning lifetime with respect to [10], possibly due to their use of an approximated expression for the capture rate. We have also followed, for selected models, the evolution during the core He-burning phase. During this evolutionary stage, the dark matter luminosity is lower than the nuclear reaction luminosity, therefore the impact of DM annihilations is found to be rather weak. For the $20M_{\odot}$ model and for $\rho_{\chi} = 1.6 \times 10^{10} \text{ GeV cm}^{-3}$ the He-lifetime is prolonged by a factor 1.2, rather than a factor 37 found for the H-burning phase for the same DM density.

In conclusion, we have adapted a stellar evolution code to study the evolution of Pop III stars in the presence of WIMPs. We have shown that provided the DM density remains above a critical value, the annihilation of WIMPs captured by Pop III stars can dramatically alter the evolution of these objects and prolong their lifetime beyond the age of the Universe. We have determined the properties of these “frozen” stars and determined the observational properties that may allow to discriminate these objects from ordinary stars.

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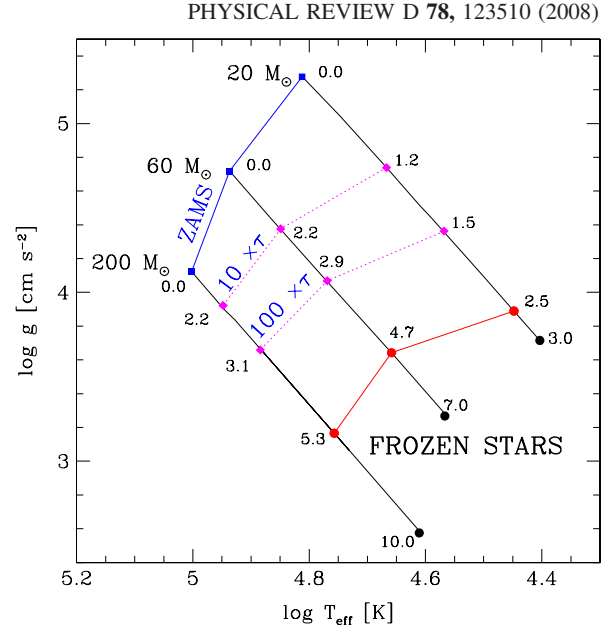


FIG. 4 (color online). ZAMS positions of 20, 60, and $200M_{\odot}$ Pop III stars in the g vs T_{eff} plane for different DM densities (labels in units of GeV cm^{-3}). Big red (gray) circles correspond to the critical WIMP density (see text). The lines labeled as 10τ and 100τ correspond to models with a lifetime prolonged by 10 and 100 times with respect to the case without WIMPs. WIMP parameters as in Fig. 1.

for useful discussions. During the completion of this work we became aware of a related work done independently by Yoon, Iocco, and Akiyama [19]. Their results, obtained with an independent stellar evolution code, appear to be in good agreement with our own.

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Effects of rotation on the evolution of primordial stars

S. Ekström¹, G. Meynet¹, C. Chiappini^{1,2}, R. Hirschi³, and A. Maeder¹¹ Geneva Observatory, University of Geneva, Maillettes 51, 1290 Sauverny, Switzerland
e-mail: Sylvia.Ekstrom@obs.unige.ch² Osservatorio Astronomico di Trieste, via G.B. Tiepolo 11, 34131 Trieste, Italy³ University of Keele, Keele ST5 5BG, UK

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ABSTRACT

Context. Although still beyond our observational abilities, Population III stars are interesting objects from many perspectives. They are responsible for the re-ionisation of the inter-galactic medium. They also left their chemical imprint in the early Universe, which can be deciphered in the most metal-poor stars in the halo of our Galaxy.

Aims. Rotation has been shown to play a determinant role at very low metallicity, bringing heavy mass loss where almost none was expected. Is this still true when the metallicity strictly equals zero? The aim of our study is to answer this question, and to determine how rotation changes the evolution and the chemical signature of the primordial stars.

Methods. We have calculated seven differentially-rotating stellar models at zero metallicity, with masses between 9 and 200 $M_{\odot}$. For each mass, we also calculated a corresponding model without rotation. The evolution is followed up to the pre-supernova stage.

Results. We find that $Z = 0$ models rotate with an internal profile $\Omega(r)$ close to local angular momentum conservation, because of a very weak core-envelope coupling. Rotational mixing drives an H-shell boost due to a sudden onset of the CNO cycle in the shell. This boost leads to a high ^{14}N production, which can be as much as 10^6 times higher than the production of the non-rotating models. Generally, the rotating models produce much more metal than their non-rotating counterparts. The mass loss is very low, even for the models that reach critical velocity during the main sequence. It may however have an impact on the chemical enrichment of the Universe, because some of the stars are supposed to collapse directly into black holes. They would contribute to the enrichment only through their winds. While in that case non-rotating stars would not contribute at all, rotating stars may leave an imprint on their surrounding. Due to the low mass loss and the weak coupling, the core retains a high angular momentum at the end of the evolution. The high rotation rate at death probably leads to a much stronger explosion than previously expected, changing the fate of the models. The inclusion of our yields in a chemical evolution model of the Galactic halo predicts log values of N/O, C/O and $^{12}\text{C}/^{13}\text{C}$ ratios of -2.2 , -0.95 and 50 respectively at log O/H +12 = 4.2.

Key words. stars: evolution – stars: rotation – stars: chemically peculiar – stars: supernovae: general

1. Introduction

Population III (PopIII) stars occupy a key position in the evolution of the Universe. They are the first sources of light that re-ionised the Universe. Also, they are the first producers of the heavy elements in the early Universe¹.

Because of the peculiar conditions of the early Universe, especially the lack of coolants provided by the metals, the very first metal-free stars that formed are supposed to be very massive, with a typical mass $M \approx 10^2\text{--}10^3 M_{\odot}$ (Abel et al. 2002; Bromm et al. 2002). Nakamura & Umemura (2001) propose a bimodal IMF, with one peak at $M \approx 10^2 M_{\odot}$ and the other at $M \approx 1 M_{\odot}$. The bimodality is due to a threshold in the initial density of the cloud. Since this first suggestion that PopIII may not all be very massive objects, several studies have shown that the radiation field of a pre-existing star has a strong impact on subsequent star formation (Omukai & Yoshii 2003; O’Shea et al. 2005; Greif & Bromm 2006). This would allow lower-mass metal-free stars to appear. Moreover Silk & Langer (2006) showed that even if the early Universe was free of magnetic fields, magnetic seeds can be generated in the disc surrounding

an accreting stellar progenitor. These seeds can be amplified through a dynamo process and become strong enough to play an important role in the accretion rate. This again would reduce substantially the final mass range of the first stars. Though it was once suggested that the most metal-poor stars known to this day may be low-mass PopIII that have undergone surface pollution (Shigeyama et al. 2003), it is now widely accepted that metal-free stars were too massive to still be present nowadays ($M > 0.8 M_{\odot}$). The observation facilities are not yet able to reach far enough for a direct detection (except maybe for the detection of pair-instability supernovae at intermediate redshifts, see Scannapieco et al. 2005). The only way to constrain their physical properties is indirect observation of their chemical or radiative signature. Although it is not firmly established whether the most metal-deficient stars of the halo of our Galaxy are truly second generation stars (Salvadori et al. 2007), most authors consider that they are born from a cloud enriched by only a few primordial progenitors (Christlieb et al. 2002; Umeda & Nomoto 2003). Recently, accurate measurements of their surface abundances have started to allow some constraints to be placed on the theoretical results of PopIII evolution (Cayrel et al. 2004; Barklem et al. 2005; Cohen et al. 2007; Aoki et al. 2007).

In view of their importance, PopIII stars have been the subject of many evolutionary calculations. Amongst the latest ones, most present the evolution until the AGB phase or central C

¹ According to the standard Big Bang Nucleosynthesis model, only elements with nucleon number $A \leq 7$ were formed at a significant level during the first minutes of the Universe (e.g. Iocco et al. 2007).

ignition (Chieffi et al. 2001; Marigo et al. 2001, 2003; Siess et al. 2002; Gil-Pons et al. 2007; Lau et al. 2008). Others are focused on the final stages (Heger & Woosley 2002; Umeda & Nomoto 2005). To our knowledge, there are only two studies that include rotating models: the one by Marigo et al. (2003), and the one by Heger et al. (2000b). Marigo et al. (2003) study very massive objects with masses between 120 and 1000 $M_{\odot}$. In their work, rotating stars are treated as rigid bodies and the effects of rotation intervene mainly through a correction factor applied on the radiative mass loss (see Sect. 2.2). In these models, no rotational mixing is accounted for. Heger et al. (2000b) follow the evolution of differentially-rotating stellar models of 15–250 $M_{\odot}$ from the zero-age main sequence (ZAMS) to core collapse, but the detailed results of these models have not been published.

The present paper explores the effects of differential rotation on primordial massive stars. We think it is worth doing such a study for two main reasons. First, for metallicities between 0.004 and 0.040, the inclusion of the effects of differential rotation in stellar evolution codes has allowed us to improve the agreement between the models and observations in many aspects: it has allowed us to better reproduce the observed surface abundances (Heger & Langer 2000; Meynet & Maeder 2000), the ratio of blue-to-red supergiants at low metallicity (Maeder & Meynet 2001), the variation with metallicity of the WR/O ratios and of the numbers of type Ibc to type II supernovae (Meynet & Maeder 2003, 2005). It is interesting to see what a code that has been successfully compared with observations can predict in a metallicity domain where no direct observations are available. Second, rotation has been shown to have important effects on low metallicity stars (Meynet et al. 2006). During the Main Sequence (MS), the radiative winds are very low because of the lack of metals, so the star cannot lose its angular momentum. If it starts its evolution with a sufficiently high equatorial velocity, it may reach the critical limit. In that case, the outer layers are no longer bound and some mass is lost, probably through a *decretion* disc (Owocki 2005) that may be quickly erased by the radiation. Further in the evolution, the rotational mixing brings freshly synthesised metals to the surface. The opacity of the outer layers are enhanced and the radiative mass loss is increased. While low metallicity stars were supposed to lose almost no mass at all, it has been shown that fast rotation can dramatically enhance the mass loss and thus modify drastically the evolution of these stars. How does a total absence of metals alter the above picture? It is interesting to extend the study to strictly $Z = 0$ models.

The paper is organized as follow: in Sect. 2, we describe the physical ingredients of the models; Sect. 3 is dedicated to the presentation of the evolution of the models; in Sect. 4, we present the pre-SN structure and the yields obtained; we study the effect of their inclusion in a galactic chemical evolution code in Sect. 5, and then a discussion follows in Sect. 6.

Electronic tables of the evolutionary sequences and the final structure files are available at <http://obswww.unige.ch/Recherche/evol/Rotating-primordial-stars>

2. Physical ingredients of the models

The stellar models presented here have masses ranging from 9 to 200 $M_{\odot}$. The initial composition is $X = 0.76$, $Y = 0.24$ and $Z = 0$. The models with masses between 25 and 85 $M_{\odot}$ were computed until the end of the hydrostatic core Si burning. The 9 $M_{\odot}$ model developed a strongly degenerated core before carbon ignition and was thus stopped then. The 15 $M_{\odot}$ model was stopped at the end of oxygen burning also because of a too

Table 1. Rotational properties of the models. The masses are in solar mass units, and the velocities in km s^{-1} . The last column gives the central hydrogen mass fraction when the model reaches the critical limit.

Mass	v_{ini}	v/v_{crit}	$\bar{v}$ (MS)	$X_{\text{c,b}}$
9	500	0.54	430	–
15	800	0.72	626	0.018
25	800	0.63	657	0.253
40	800	0.56	700	0.326
60	800	0.52	740	0.360
85	800	0.48	754	0.399
200	800	0.40	733	0.428

degenerate core at that time. Because of numerical difficulties, the 200 $M_{\odot}$ was stopped shortly after central He exhaustion.

The computation was carried out with the Geneva stellar evolution code. The version of the code is the one used in Hirschi et al. (2004) that includes the necessary implementations to continue through advanced stages up to the pre-SN. It has been slightly modified to join the reaction networks of H and He burning during the MS, as required for zero-metallicity models.

The reaction rates are taken from the NACRE database (http://pntpm.ulb.ac.be/Nacre/barre_database.htm). The opacities come from Iglesias & Rogers (1996), complemented at low temperature by the molecular opacities of Alexander & Ferguson (1994). The treatment of convective instability is done according to the Schwarzschild criterion. As in Hirschi (2007), convection is treated as a diffusive process from oxygen burning onwards. An overshooting parameter $\alpha_{\text{over}} = 0.2 H_P$ is used for H- and He-burning cores only.

2.1. Rotation

The effects of rotation are computed in the framework of the Roche model (see Ekström et al. 2008b) and according to the shellular-rotation hypothesis (see Zahn 1992). The code couples the centrifugal force to the internal structure equations (Maeder & Meynet 2000b). This modifies the hydrostatic equilibrium and the surface conditions. The rotation-induced instabilities taken into account are the meridional circulation, the secular and dynamical shears.

- The meridional circulation arises from the impossibility for a rotating star to be in hydrostatic and radiative equilibrium at the same time (the von Zeipel theorem). The vertical component of the meridional circulation velocity is $u(r, \theta) = U(r) P_2(\cos \theta)$ with $P_2(x)$ the second Legendre polynomial. The formulation of the radial amplitude $U(r)$ has been determined by Zahn (1992) and Maeder & Zahn (1998) as:

$$U(r) = \frac{P}{\rho g C_P T [\nabla_{\text{ad}} - \nabla_{\text{rad}} + (\varphi/\delta) \nabla_{\mu}]} \cdot \left(\frac{L}{M_{\star}} [E_{\Omega} + E_{\mu}] + \frac{C_P}{\delta} \frac{\partial \Theta}{\partial t} \right)$$

where C_P is the specific heat, $\nabla_{\text{ad}} = \frac{P \delta}{\rho T C_P}$ the adiabatic gradient, $M_{\star} = M \left(1 - \frac{\Omega^2}{2\pi g \rho_m} \right)$, and $\Theta = \bar{\rho}/\rho$ is the ratio of the variation of the density to the average density on an equipotential. φ and δ arise from the generic equation of state: $\frac{d\rho}{\rho} = \alpha \frac{dP}{P} + \varphi \frac{d\mu}{\mu} - \delta \frac{dT}{T}$. E_{Ω} and E_{μ} are terms that depend on the Ω - and μ -distributions respectively (see the details of

these expressions in Maeder & Zahn 1998). E_Ω is the dominant term in the round brackets, and it can be reasonably approximated by: $E_\Omega \approx \frac{8}{3} \left[1 - \frac{\Omega^2}{2\pi g \bar{\rho}} \right] \left(\frac{\Omega^2 r^3}{GM} \right)$. The term in square brackets is known as the *Gratton-Öpik term*. It may become largely negative in the outermost layers of the stars, where $\bar{\rho}$ becomes very small, which leads to $U(r) < 0$. In this case, the circulation descends along the polar axis from the surface inwards, and then rises in the equatorial plane outwards to the surface, bringing angular momentum to the surface.

- Differential rotation induces shear turbulence at the interface of layers having different rotational velocities. A layer remains stable if the excess kinetic energy due to the differential rotation does not overcome the buoyancy force. This is known as the *Richardson criterion* and is expressed as $Ri = \frac{N^2}{(\partial u / \partial z)^2} > Ri_c$, where u is the horizontal velocity, z the vertical coordinate, and $N^2 = \frac{g\delta}{H_p} (\nabla_{ad} - \nabla + (\varphi/\delta)\nabla_\mu)$ is the Brunt-Väisälä frequency. $Ri_c = 1/4$ is the critical value for stability. The coefficient of diffusion by dynamical shear implemented in the code is described in Hirschi et al. (2004):

$$D_{\text{dyn}} = \frac{1}{3} r \Delta\Omega \Delta r$$

where r is the mean radius of the zone constituted of consecutive shells with $Ri < Ri_c = 1/4$, $\Delta\Omega$ is the variation of Ω over this zone and Δr is the extend of the zone. The timescale for the dynamical shear is $\tau_{\text{dyn}} = \sqrt{r^3/(Gm_r)}$. The prescription for D_{dyn} is valid when the thermal adjustment timescale $\tau_{\text{th}} = \frac{d^2}{12K}$ (for a sphere with diameter d and a thermal diffusivity K) is longer than the dynamical timescale.

- Secular shear arises when the thermal dissipation reduces the buoyancy force. The coefficient of diffusion by secular shear turbulence was determined by Maeder (1997) as:

$$D_{\text{shear}} = \frac{4(K + D_h)}{\left[\frac{\varphi}{\delta} \nabla_\mu \left(1 + \frac{K}{D_h} \right) + (\nabla_{ad} - \nabla_{rad}) \right]} \cdot \frac{H_p}{g\delta} \left[\frac{\alpha}{4} \left(0.8836 \Omega \frac{d \ln \Omega}{d \ln r} \right)^2 - (\nabla' - \nabla) \right]$$

with $K = \frac{4acT^3}{3\kappa\rho^2C_p}$ the thermal diffusivity, α the fraction of the excess energy in the shear that will contribute to mixing, and $(\nabla' - \nabla)$ the difference between the internal non-adiabatic gradient and the local gradient. The latter term is extremely small and is neglected in the calculation. D_h is the coefficient for the horizontal turbulence. It is determined as in Zahn (1992): $D_h = \frac{1}{c_h} r |2V(r) - \alpha U(r)|$, where c_h is a constant of order 1, $V(r)$ is the horizontal component of the meridional circulation velocity, $U(r)$ its vertical component, and in this expression $\alpha = \frac{1}{2} \frac{d \ln(r^2 \bar{\Omega})}{d \ln r}$.

In the zones where $Ri < Ri_c$, the shear mixing coefficient is eventually set to:

$$D_{\text{shear}} = D_{\text{dyn}} + D_{\text{shear}}$$

The transport of angular momentum inside a star is implemented following the prescription of Zahn (1992), complemented by the works of Talon & Zahn (1997) and Maeder & Zahn (1998). In the radial direction, it obeys the following equation:

$$\rho \frac{d}{dt} (r^2 \bar{\Omega})_{M_r} = \frac{1}{5r^2} \frac{\partial}{\partial r} (\rho r^4 \bar{\Omega} U(r)) + \frac{1}{r^2} \frac{\partial}{\partial r} \left(\rho D r^4 \frac{\partial \bar{\Omega}}{\partial r} \right).$$

The first term in the right hand side of this equation is the divergence of the *advected* flux of angular momentum, while the second term is the divergence of the *diffused* flux. The coefficient D is the total diffusion coefficient, taking into account the various instabilities which transport angular momentum (convection, shear). After the end of the MS, the timescale of the evolution shortens, so only the diffusive term is kept in the formulation of the transport of the angular momentum.

The transport of chemical elements by the meridional circulation has been shown to be satisfactorily approximated by a diffusion process (see Chaboyer & Zahn 1992) throughout the whole evolution.

Since very low- or zero-metallicity stars are more compact than their metal-rich counterparts, a similar ratio of the initial velocity to the critical velocity² v/v_{crit} leads to a higher equatorial velocity. In order to explore the effects of rotation on metal-free models, we chose a high initial velocity, 800 km s⁻¹. Note that for the 60 $M_\odot$ model, this initial velocity corresponds to a total angular momentum content that is similar to a solar metallicity model of the same mass rotating at 300 km s⁻¹. The 9 $M_\odot$ model was computed with a lower initial velocity (500 km s⁻¹) because 800 km s⁻¹ is overcritical on the ZAMS for this model. Table 1 presents the initial equatorial velocities of the models, the corresponding v/v_{crit} ratio and the mean velocity during MS.

In order to be able to compare our results with other works, we computed, for each mass, a corresponding model without rotation.

2.2. Mass loss

Zero-metallicity stars are supposed to evolve at constant mass due to a scaling of the wind with metallicity of the form $\dot{M} \propto (Z/Z_\odot)^{0.5}$. However, according to Kudritzki (2002), H and He lines are able to remove some mass if the luminosity is sufficiently high. We thus adopted the Kudritzki (2002) prescription for the models presenting a luminosity $\log(L/L_\odot) > 6$. The 85 and 200 $M_\odot$ models were computed with it from the start, while we turned it on during the MS evolution of the 60 $M_\odot$. Since Kudritzki's prescription does not correspond to the strictly $Z = 0$ case, we used the same adaptations as in Marigo et al. (2003): 1) the surface metallicity is set $Z = \max(Z_{\text{min}}, Z_{\text{eff}})$, where $Z_{\text{min}} = 10^{-4} Z_\odot$ is the lowest value in the validity range of Kudritzki's prescription, and $Z_{\text{eff}} = 1 - X - Y$ is the effective surface metallicity of the model (allowing for self-enrichment); and 2) the effective temperature is set $T_{\text{eff}} = \min(T_{\text{eff}}, 60\,000 \text{ K})$, i.e. we did not extrapolate Kudritzki's formula beyond its validity domain but constrained $T_{\text{eff}} \leq 60\,000 \text{ K}$ for the calculation of the mass loss. The models with a mass lower than 60 $M_\odot$ never reach $\log(L/L_\odot) > 6$, so they were computed without radiative mass loss.

Rotation, by lowering the effective gravity at the surface, helps the radiation remove mass from the star. As shown in Maeder & Meynet (2000a), the line-driven mass loss is enhanced by a factor which depends on the angular velocity such that the mass loss in the case of rotation becomes:

$$\dot{M}(\Omega) = F_\Omega \cdot \dot{M}(\Omega = 0) \quad \text{where} \quad F_\Omega = \frac{(1 - \Gamma)^{\frac{1}{\alpha} - 1}}{\left[1 - \frac{\Omega^2}{2\pi G \rho_m} - \Gamma \right]^{\frac{1}{\alpha} - 1}}.$$

² The critical velocity v_{crit} is reached when the gravitational acceleration is exactly counterbalanced by the centrifugal force. In the framework of the Roche model, one has: $v_{\text{crit}} = \sqrt{\frac{2}{3} \frac{GM}{R_{\text{pb}}}}$, where R_{pb} is the polar radius at break-up limit.

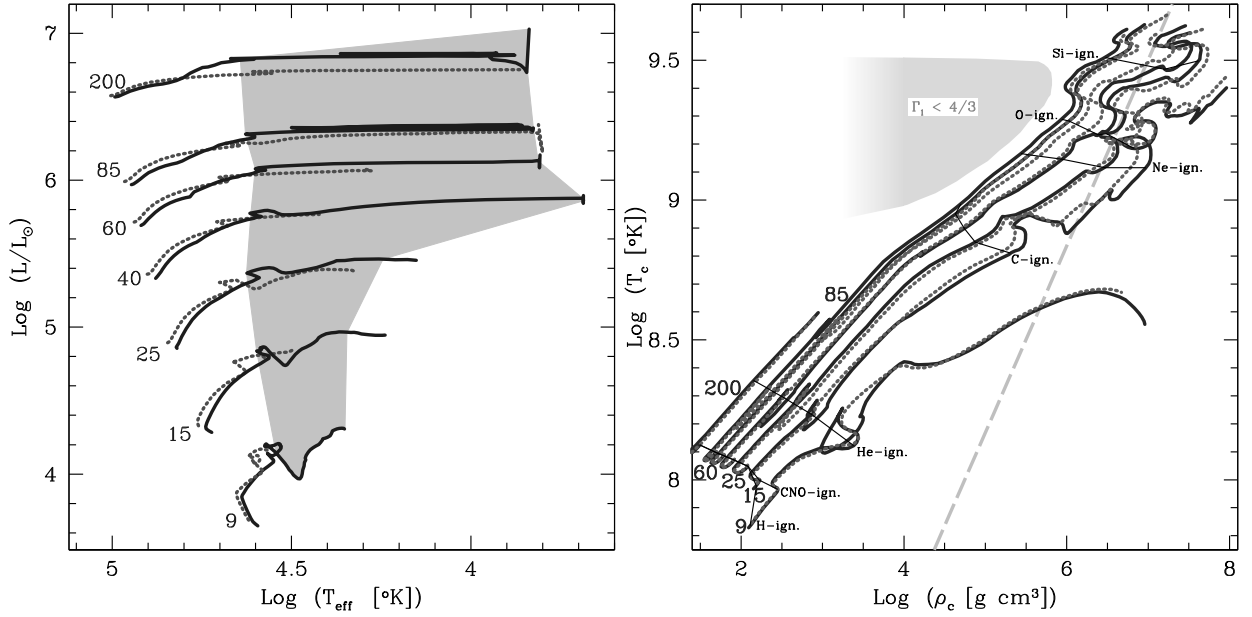


Fig. 1. Evolution of $Z = 0$ models (with rotation: solid lines; without rotation: dotted lines). *Left:* Hertzsprung Russell diagram. The grey area shows the zone of the diagram where He burns in the core of the rotating models. *Right:* central temperature vs. central density diagram. The ignition of the different burning stages are given for the rotating models. The grey area indicates the zone of the diagram where the pair-creation instability occurs. The dashed grey line shows the locus of the points where the perfect gas pressure is equal to the completely-degenerate non-relativistic pressure.

We see that this expression depends also on the Eddington factor $\Gamma = L/L_{\text{Edd}} = \kappa L/(4\pi cGM)$ where κ is the electron-scattering opacity.

Moreover, there is another situation in which the models can lose mass: it is when the external layers of the star reach the critical velocity. The over-critical layers are no longer bound to the star, although it is not yet clear whether they are expelled in the interstellar medium or remain in the surroundings, perhaps in the form of a disc. However, the removal of these layers brings the surface back to sub-critical velocity until the evolution of the star accelerates it again. In the present calculation, as in [Meynet et al. \(2006\)](#), once the model reaches the break-up limit, we adapted the mass loss rate to keep the surface just under the critical velocity. This treatment was applied to all the models except the $9 M_{\odot}$, which never reached the critical limit.

3. Evolution

3.1. Hertzsprung Russell diagram and central conditions

In the left panel of Fig. 1, we present the Hertzsprung Russell diagram (HRD) of our rotating and non-rotating models, and in the right panel, their tracks in the $\log T_c - \log \rho_c$ diagram. We notice the expected characteristics of rotating models: the ZAMS is shifted towards lower effective temperature and luminosity with respect to the non-rotating case³. Then, when the evolution proceeds, the tracks become more luminous, and the main-sequence turn-off is shifted to cooler temperature: the core of the rotating models is refuelled by fresh H brought by the mixing. It thus grows, leading to an enhancement of the luminosity.

The 9 and $15 M_{\odot}$ models are powered only by *pp*-chains when they arrive on the ZAMS and thus they continue their

initial contraction. It is only when they have produced a sufficient amount of carbon through the 3α reaction (about 10^{-12} in mass fraction) that the CNO cycle can start, drawing a hook in the $\log T_c - \log \rho_c$ diagram. The onset of the CNO cycle in these models can also be seen in the HRD: their tracks evolve towards the blue side of the diagram, until the energy provided by the CNO cycle stops the contraction and bends the tracks back into the usual MS feature. In the rotating $9 M_{\odot}$ model, this happens at an age of 12.2 Myr (when the central H mass fraction is $X_c = 0.439$) while in the non-rotating one it happens a little earlier, at an age of 10.9 Myr (but at a similar burning stage: $X_c = 0.439$). In the case of the non-rotating $15 M_{\odot}$ model, it happens after merely 1.5 Myr ($X_c = 0.695$), while it takes 2 Myr ($X_c = 0.677$) in the case of the rotating one. [Marigo et al. \(2001\)](#) find that the mass limit for CNO ignition to occur on the ZAMS is $20 M_{\odot}$. Our results are consistent with that limit.

After central H exhaustion, the core He-burning phase (CHeB) starts right away: the core was already hot enough to burn a little He during the MS so it does not need to contract much further. The transition between core H burning and He burning is smooth because the model is continuously sustained by central nuclear burning. This prevents the models from starting a redward evolution, so they remain in the blue part of the HRD at the beginning of CHeB. The H-burning shell is powered only by *pp*-chains and remains radiative. The main part of the luminosity is provided by the core (see in Fig. 2 *left*, the case of the rotating $15 M_{\odot}$ model). Something particular happens to the rotating models during the CHeB phase: because of rotational mixing, some carbon produced in the core is diffused towards the H-burning shell, allowing a sudden ignition of the CNO cycle in the shell. This boost of the shell leads to a retraction of the convective core and a decrease of the luminosity. At the same time, it transforms the quiet radiative H-burning shell into an active convective one (see Fig. 2 *centre*). Later, the model takes the structure of a higher metallicity one: a convective He-burning core,

³ This shift is due to the sustaining effect of the rotation: the gravity is counter-balanced both by the gas pressure and the centrifugal force in such a way that the star behaves like a lower mass one.

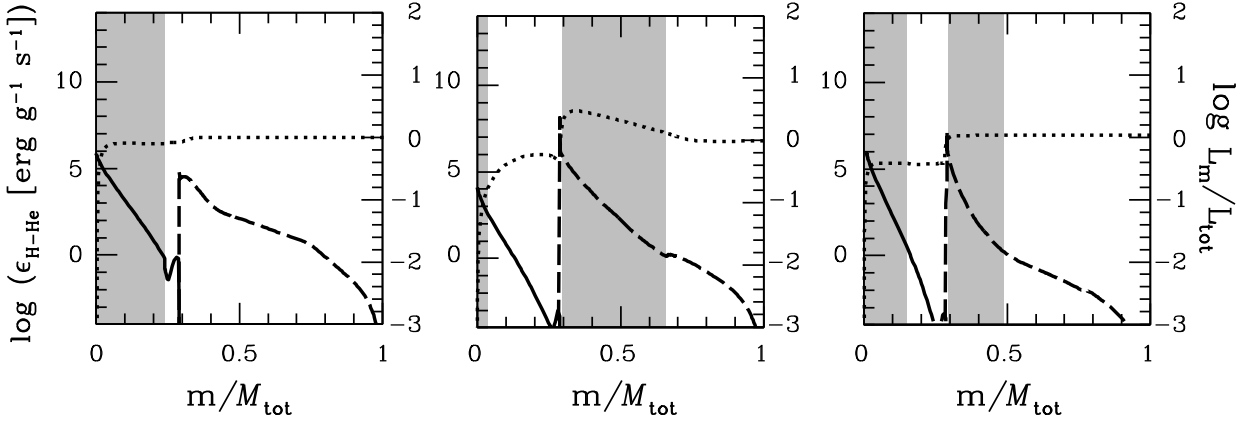


Fig. 2. Internal structure of the $15 M_{\odot}$ rotating model at different stages of the core He burning. The solid line is the He-burning energy production and the dashed line the H-burning energy production. The dotted line is the fraction of the total luminosity (scale on the right). The grey areas show the convective zones. *Left:* before the CNO shell boost (age: 13.739 Myr; $Y_c = 0.4632$) – *centre:* at the moment of the boost (age: 13.744 Myr; $Y_c = 0.4919$) – *right:* after the boost (age: 13.757 Myr; $Y_c = 0.4840$).

convective H-burning shell and expanding envelope (see Fig. 2 *right*). This boost appears in Fig. 1 (*left*) as a deep V-shaped feature in the rotating tracks of the 9 and $15 M_{\odot}$ models, and as a slight depression in the 25 and $40 M_{\odot}$ tracks: the luminosity decreases at the same time as the T_{eff} cools down. The only non-rotating models that present the same feature are the 25 and $85 M_{\odot}$. In this case, the structure is such that the growing convective core reaches the bottom of the quiet radiative H-burning shell, triggering the same event as in the rotating models. Such a CNO boost leaves an imprint in the $\log T_c - \log \rho_c$ diagram: a second hook can be seen in the tracks, which takes the shape of a complex loop in the case of the $9 M_{\odot}$ track.

The crossing of the HRD is slow, unlike higher metallicity models. As we mentioned above, the nuclear burning is continuous inside the core, therefore the $Z = 0$ models do not go through a phase of rapid core contraction and its mirror effect of fast envelope expansion. On the HRD, we note that the crossing of the diagram takes place during the whole CHeB (grey area).

In the non-rotating set, all the models with mass between 9 and $60 M_{\odot}$ end their life as blue supergiants (BSG), while the 85 and $200 M_{\odot}$ become red supergiants (RSG). This compares well with the tracks obtained by Marigo et al. (2001). As in their study, all the models with masses between 9 and $60 M_{\odot}$ start and finish core He burning in the blue part of the HRD. Only the highest masses models ($M \geq 70 M_{\odot}$) end their central He combustion on their Hayashi line.

Generally, the rotating tracks end their evolution in a cooler part of the HRD than their non-rotating counterparts. The only exception is the $85 M_{\odot}$, which ends its life as a BSG because of a blue loop after CHeB. The rotating 15 and $25 M_{\odot}$ end their evolution in the blue part of the HRD, without any redward incursion. The rest of the rotating models become RSG. None of our models meet the conditions to be Wolf-Rayet stars. The $85 M_{\odot}$ star ends its life with a $\log T_{\text{eff}}$ higher than 4.3, but its surface abundance of hydrogen remains too high.

3.2. Lifetimes

Table 2 shows the lifetimes for the MS and CHeB phases. During the MS, rotation induces an increase in the lifetime from 12.8% for the $200 M_{\odot}$ to 24.5% for the $15 M_{\odot}$ model. The models with higher initial v/v_{crit} show a greater increase. For the models that

Table 2. Final masses, total lifetime, MS and core He-burning durations, masses of the He- and CO-core for our models at the pre-SN stage. Masses are in $M_{\odot}$, velocities in km s^{-1} and times in Myr.

Mass	v_{ini}	M_{fin}	τ_{life}	τ_{H}	τ_{He}	M_{He}	M_{CO}
9	0	9.00	22.0	19.50	1.99	1.73	0.46
9	500	9.00	27.1	23.90	2.58	2.09	0.63
15	0	15.00	11.6	10.60	0.83	3.62	2.92
15	800	14.96	14.4	13.20	0.98	4.31	2.71
25	0	25.00	7.29	6.64	0.56	7.51	5.39
25	800	24.75	8.47	7.88	0.50	9.14	6.30
40	0	40.00	5.03	4.60	0.36	15.95	14.52
40	800	37.99	5.88	5.41	0.41	17.22	13.53
60	0	59.99	3.96	3.60	0.30	26.52	25.11
60	800	57.59	4.57	4.20	0.31	30.86	30.58
85	0	84.93	3.42	3.05	0.31	34.77	34.50
85	800	74.57	3.84	3.52	0.28	44.38	43.92
200	0	199.95	2.53	2.27	0.23	101.63	95.69
200	800	183.56	2.83	2.56	0.24	109.65	95.97

reach the critical limit, we find an almost linear relation between the relative enhancement of the MS lifetime $\Delta\tau_{\text{H}}$ and the initial ratio v/v_{crit} of the form

$$\Delta\tau_{\text{H}} = \frac{\tau_{\text{H}}(v/v_{\text{crit}}) - \tau_{\text{H}}(0)}{\tau_{\text{H}}(0)} = 0.362 v/v_{\text{crit}}$$

(when τ_{H} is expressed in Myr) which reproduces the values of $\Delta\tau_{\text{H}}$ obtained in our models with an accuracy better than 7%.

For our non-rotating $9 M_{\odot}$, the MS lifetime is shorter by 12% than that found by Marigo et al. (2001), which is consistent with our lower overshoot parameter: the convective core of our model is about 10% smaller than theirs. They also have a higher initial hydrogen abundance ($X = 0.77$). Gil-Pons et al. (2007) provide two models of $9 M_{\odot}$, one computed without overshoot and the other with the overshoot theory presented by Eldridge & Tout (2004). Their calculation of the convective boundaries could be approximated by $\alpha_{\text{over}} = 0.3 H_{\text{P}}$ (Gil-Pons, priv. comm.). The lifetime of our model is about 21% shorter than the model they calculated with this higher overshoot. But it is also 11% shorter than their model calculated without any overshoot. In a previous paper (Gil-Pons et al. 2005), they show the MS evolution of the convective core and of the H and He abundances for this model. The transition from pp -chain burning to CNO cycle occurs

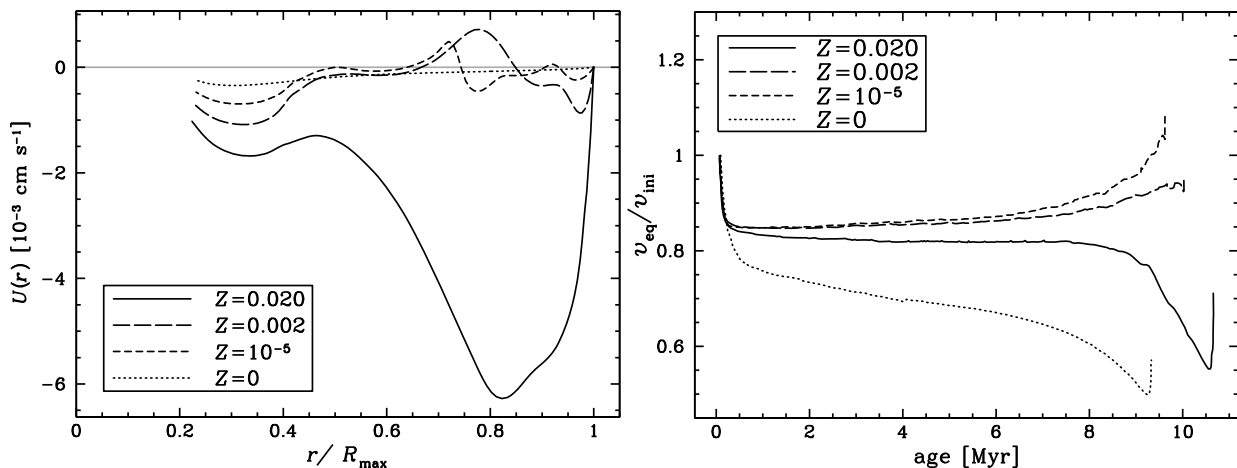


Fig. 3. Models of $20 M_{\odot}$ at various metallicities, with $\Omega_{\text{ini}}/\Omega_{\text{crit}} = 0.5$. *Left:* internal profile of $U(r)$, where $u(r, \theta)$ the vertical component of the velocity of the meridional circulation is $u(r, \theta) = U(r) P_2(\cos \theta)$ with P_2 the second Legendre polynomial. The radius is normalized to the outer one. All the models are at the same evolutionary stage, when the central H mass fraction is about 0.40. *Right:* evolution of the equatorial velocity, normalized to the initial velocity.

earlier in their model (4.4 Myr) than in ours (10 Myr), and in a much more abrupt way (see their Fig. 4). The convective core expands suddenly, the H abundance rises in the core and thus allows a longer lifetime. The difference in the time at which the CNO cycle ignites arises most probably from differences in the reaction rates and in the equation of state used.

In the case of our $15 M_{\odot}$ non-rotating model, we again find a difference of about 10% to the work of Marigo et al. (2001), consistent with the difference found in the $9 M_{\odot}$ models. Our lifetime is also 12% shorter than that calculated by Siess et al. (2002) for their (non-rotating) $15 M_{\odot}$ model, although the overshooting parameter they use is the same as ours. The difference arises from the treatment of the light elements (^7Be , ^7Li , and ^8B): they are followed explicitly by their code, while ours considers them to be in equilibrium.

In order to compare our results with those of Marigo et al. (2003) at higher masses, we computed the evolution of a non-rotating $120 M_{\odot}$ star up to the end of the MS. Once more the MS lifetime of our model is found to be 10% shorter than the one computed by the above authors.

3.3. Evolution of the equatorial velocity

Our models are started on the ZAMS with a flat rotation profile. As described in Meynet & Maeder (2000), the meridional circulation then establishes very quickly (1–2% of the MS time) an equilibrium profile of $\Omega(r)$ inside the star with the core spinning faster than the surface.

After this first adjustment phase, the evolution of the surface (equatorial) velocity results from a delicate interplay between the mass loss (which removes angular momentum at the surface) and the meridional circulation (which brings angular momentum from the core to the surface). How does this interplay behave at $Z = 0$, and what are the differences with the situation at higher metallicities? To find an answer, we compared four models of $20 M_{\odot}$ with $\Omega_{\text{ini}}/\Omega_{\text{crit}} = 0.50$ at various metallicities, computed with the same code as our set of $Z = 0$ models⁴. In the left panel of Fig. 3, we show the internal profile of the radial component of the meridional circulation. Let us focus on the outer cell, which

transports the angular momentum outward, when $U(r)$ is negative. The amplitude of the meridional circulation is a factor of 6 higher in the standard metallicity model ($Z = 0.020$) than in the $Z = 0.002$ one. This factor amounts to 25 when we compare with the $Z = 10^{-5}$ model, and reaches 100 with the $Z = 0$ one. This illustrates the effect of the Gratton-Öpik term (see Sect. 2.1) in the expression of the meridional circulation velocity $U(r)$: when the metallicity decreases, stars are more compact, so the density increases, and thus $U(r)$ decreases. In the right panel of Fig. 3, we see the resulting evolution of the equatorial velocity. At standard metallicity, although the amplitude of the meridional circulation is large, the loss of angular momentum through the radiative winds has the strongest effect, and the equatorial velocity slows down. At low or very low Z , the meridional circulation is weak, but the mass loss is so diminished that the models are spinning up. In the case of $Z = 0$, there is no mass loss to remove mass and angular momentum, but the meridional circulation is so weak that the evolution of $\Omega(r)$ is very close to local angular momentum conservation, $\Omega r^2 = \text{const.}$; because of the natural inflation of the external radius, the surface of the model has to slow down.

The behaviour described above is valid for all the models with $M_{\text{ini}} < 40 M_{\odot}$: they are slowing down during the MS evolution, except at the very end of it when they contract briefly before central H exhaustion. Above $40 M_{\odot}$, the models are spinning up on the MS. This is not surprising, since the meridional circulation is higher in more massive stars (which have a less dense envelope), allowing the transport of angular momentum that is lacking in lower mass models.

During evolution, because of the inflation of the radius, the value of the critical velocity decreases. So even a decelerating model may reach the critical velocity during its evolution. This is indeed what happens with our models. In Fig. 4 we present the evolution of the $\Omega/\Omega_{\text{crit}}$ ratio during the MS. As mentioned in Sect. 2.1, the $9 M_{\odot}$ model is the only one that does not reach the critical limit. For the other models, the higher the mass the sooner the critical limit is attained. The last column of Table 1 gives the central mass fraction of hydrogen when each model reaches the critical velocity.

Once at the critical limit, all the models remain at the limit. In Fig. 4, the 85 and especially the $200 M_{\odot}$ models seem to

⁴ These models are taken from Ekström et al. (2008b).

Table 3. Total mass lost and surface abundances of C, N and O at the end of MS phase and at the end of the evolution for the models that undergo mass loss and surface enrichment. Masses are in $M_{\odot}$, velocities in km s^{-1} and abundances in mass fraction. The C abundance is the sum of ^{12}C and ^{13}C , the N abundance is the abundance in ^{14}N and the O abundance is the sum of ^{16}O , ^{17}O and ^{18}O .

M_{ini}	v_{ini}	End of MS					End of evolution				
		M_{lost}	He	C	N	O	M_{lost}	He	C	N	O
9	500	—	0.2426	4.60e-23	5.91e-21	2.35e-22	—	0.4279	5.50e-17	5.61e-15	1.85e-16
15	800	0.0306	0.2503	1.74e-14	1.81e-12	5.90e-14	0.0418	0.2646	6.36e-09	1.83e-07	3.38e-09
25	800	0.2252	0.2517	4.34e-13	4.02e-11	1.20e-12	0.2513	0.2615	3.53e-10	9.37e-09	1.86e-10
40	800	1.0070	0.2654	2.24e-12	1.90e-10	5.18e-12	2.0074	0.3045	5.17e-07	1.48e-05	6.16e-07
60	0	—	0.2400	0.00e+00	0.00e+00	0.00e+00	0.0001	0.2400	0.00e+00	0.00e+00	0.00e+00
60	800	2.3200	0.2906	6.84e-12	5.44e-10	1.39e-11	2.4061	0.5328	1.23e-05	3.54e-04	3.34e-05
85	0	0.0046	0.2400	0.00e+00	0.00e+00	0.00e+00	0.0681	0.4032	6.40e-04	3.88e-03	1.33e-04
85	800	4.4363	0.3362	1.74e-11	1.31e-09	3.19e-11	10.4239	0.6141	7.29e-06	1.81e-04	4.14e-05
200	0	0.0350	0.2400	0.00e+00	0.00e+00	0.00e+00	0.0509	0.2423	7.93e-13	6.12e-11	1.59e-12
200	800	16.4426	0.5183	8.82e-11	5.94e-09	1.23e-10	20.7609	0.6685	7.53e-05	2.09e-04	1.02e-04

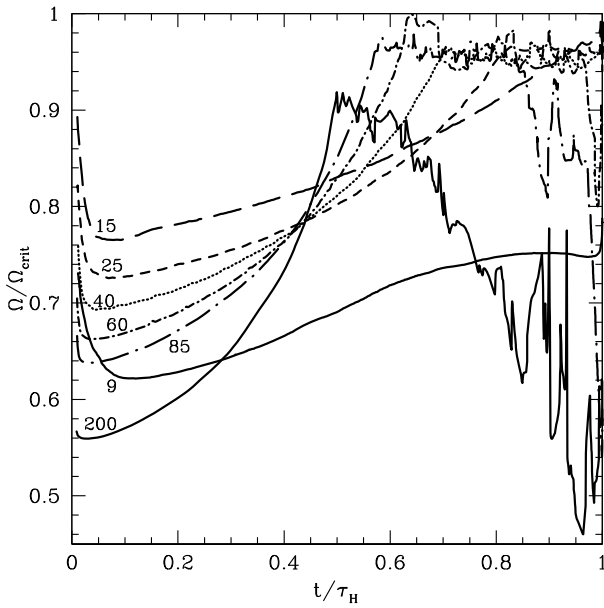


Fig. 4. Evolution of the $\Omega/\Omega_{\text{crit}}$ ratio during the MS. All the models start the MS with $v_{\text{eq}} = 800 \text{ km s}^{-1}$, except the $9 M_{\odot}$ which starts the MS at 500 km s^{-1} .

depart from $\Omega/\Omega_{\text{crit}} = 1$, but this is because the limit shown here is only the Ω -limit, where the centrifugal force alone is taken into account to counterbalance the gravity. In the two above models however, the radiative acceleration is strong and they reach the so-called $\Omega\Gamma$ -limit, that is the second root of the equation giving the critical velocity: $g_{\text{eff}} [1 - \Gamma] = 0$ (see [Maeder & Meynet 2000a](#), for the detailed derivation). The true critical velocity is lowered by the radiative acceleration, and though the plotted $\Omega/\Omega_{\text{crit}}$ ratio becomes lower than 1, these models are actually at the critical limit and remain at this limit until the end of the MS. At central H exhaustion, the moderate inflation of the radius brings the surface back to sub-critical velocities.

3.4. Mass loss and surface abundances

In [Meynet et al. \(2006\)](#), we show how rotating low-metallicity stars may lose more mass than expected mainly through two processes: first the reaching of the break-up velocity, which leads to a mechanical mass loss of moderate amplitude through a decretion disc; second the enrichment of the surface because of

rotational mixing, which leads to a very large enhancement of the radiative mass loss. Now what happens if the metallicity is as low as zero? The total mass lost at the end of MS remains modest in the present models (Table 3): between $0.03 M_{\odot}$ for the $15 M_{\odot}$ model (which reaches the critical limit only at the very end of the MS, when $X_{\text{c}} = 0.02$), and $16.4 M_{\odot}$ for the $200 M_{\odot}$ model (which reaches the break-up limit at $X_{\text{c}} = 0.43$). The mechanical mass loss removes only the outermost layers, which have a very low density, so only a little mass is removed during this phase.

After the MS, as mentioned in Sect. 3.1, the models remain in the blue part of the HRD most of the CHeB phase and then their redward incursion is moderate. The outer convective zone that develops remains very thin, so the dredge-up does not dig deeply enough (see the Kippenhahn diagrams in Fig. 5). Most of the post-MS evolution occurs with a surface metallicity much lower than 10^{-6} . As a result, the mass lost during CHeB remains very low: it amounts to at most $6 M_{\odot}$ in the case of the $85 M_{\odot}$ model, which is the model that loses the most mass at this stage.

The total mass lost by the models during the whole evolution is small. The $200 M_{\odot}$ model loses $20.7 M_{\odot}$, the $85 M_{\odot}$ model $10.4 M_{\odot}$. The models have to be as massive as $40 M_{\odot}$ or above to lose more than $1 M_{\odot}$. This picture is very different to the evolution with just a small fraction of metals: the $60 M_{\odot}$ at $Z = 10^{-8}$ presented in [Meynet et al. \(2006\)](#) loses $36 M_{\odot}$, among which $27 M_{\odot}$ are due to the enhancement of the radiative winds by the surface self-enrichment. At $Z = 0$, a similar model loses $2.4 M_{\odot}$ only. To check if the choice of a higher initial equatorial velocity could significantly enhance the mass loss in $Z = 0$ models, we have computed a test model of $40 M_{\odot}$ with $v_{\text{ini}} = 900 \text{ km s}^{-1}$ (i.e. $\Omega/\Omega_{\text{crit}} = 0.83$). We evolved it until the end of CHeB. During the MS evolution, it reaches the critical limit when its central H mass fraction is $X_{\text{c}} = 0.43$ and loses $1.702 M_{\odot}$ until the end of the MS. The total mass lost at the end of CHeB amounts to $1.734 M_{\odot}$ which compares well with the $1.571 M_{\odot}$ lost by the model with a slower v_{ini} at the same stage. We can thus conclude that the choice of a higher v_{ini} is not determinant in altering mass loss.

The non-rotating models do not lose mass at all: even the $200 M_{\odot}$ model loses only a few hundredths of a solar mass. We will see in Sect. 4.3 that some of our models may be able to contribute to the chemical enrichment of the medium only through their wind ejecta. In that context, while the non-rotating models do not contribute at all, the rotating ones will leave a modest imprint on their surroundings.

We cannot directly compare our results with the literature: the only study that does not compute the evolution at constant mass is the one by [Marigo et al. \(2003\)](#) and their masses do

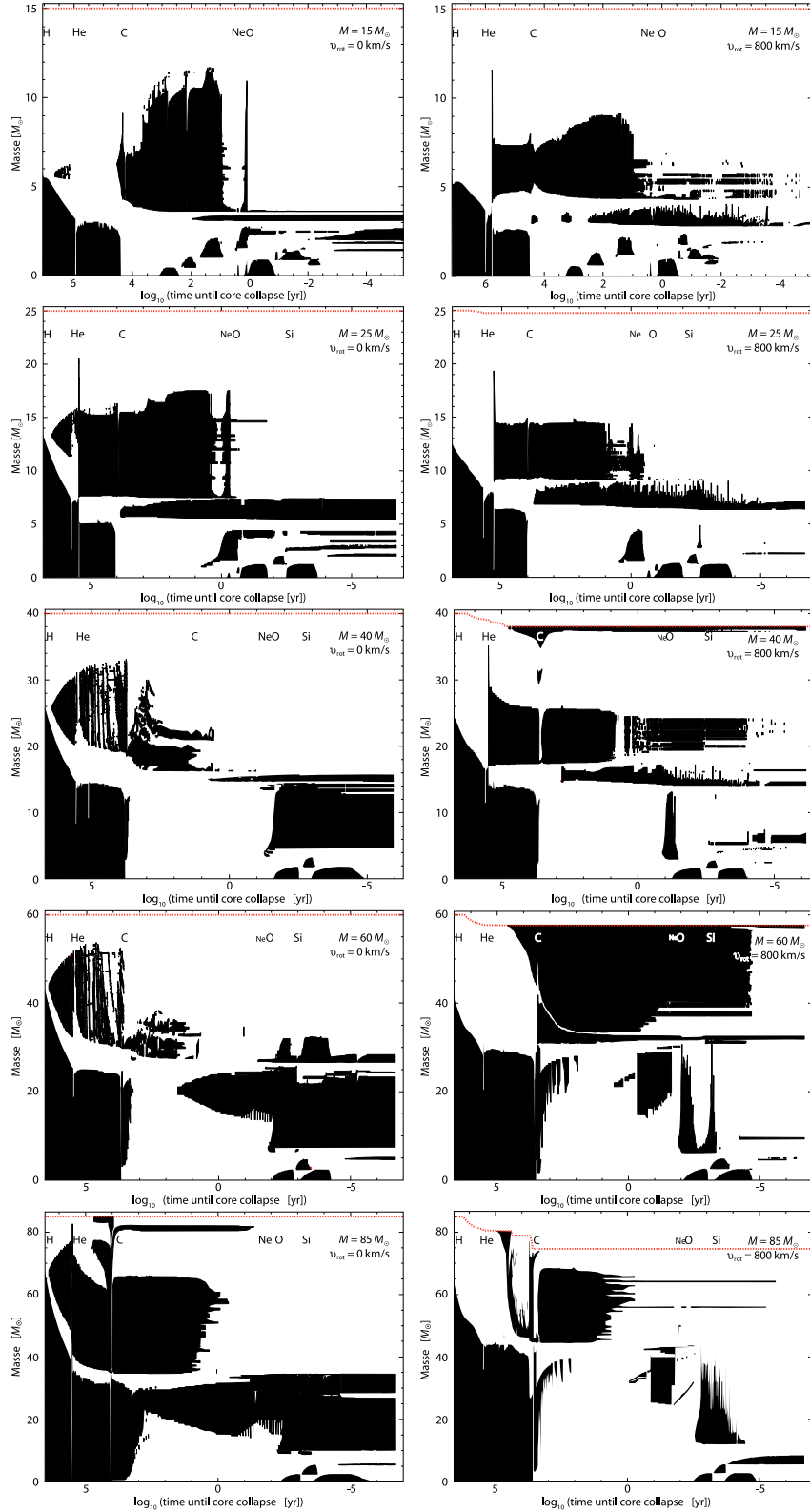


Fig. 5. Kippenhahn diagrams (evolution of the structure) for the models between 15 and 85 $M_{\odot}$. *Left column:* without rotation; *right column:* with rotation. The black areas show the convective zones. The (red) dotted line at the top of the figures is the total mass of the model. The on-set of the different burning phases is indicated at the top of each window.

not match ours exactly. Nevertheless, we can make a qualitative comparison. Our rotating $200 M_{\odot}$ model experiences a much stronger mass loss than what they found for their rotating $250 M_{\odot}$ model, which loses $3.35 M_{\odot}$ only, that is 6 times less than our slightly less heavy model. Their models have been computed with a lower initial velocity, and despite the maximal coupling brought by the rigid body rotation, they attain the critical limit only at the very end of the MS phase. The mass loss they adopt then ($\dot{M}_{\text{crit}} = 10^{-3} M_{\odot} \text{ yr}^{-1}$) is about 2 orders of magnitude higher than that resulting from our treatment of the critical mass loss, but the period during which they apply it is short.

4. Final structure and light element yields

4.1. Kippenhahn diagrams

In Fig. 5 we present the Kippenhahn diagrams (evolution of the structure) of the models between 15 and $85 M_{\odot}$. Generally, rotation suppresses the convective instability outside the H-burning core by smoothing the abundance gradient of helium above the core. The CNO shell boost is clearly visible in the diagrams of the non-rotating $25 M_{\odot}$ and the rotating 15, 25 and $40 M_{\odot}$ models: the H-burning shell suddenly becomes convective and the core is abruptly reduced. We can discover in the rotating $60 M_{\odot}$ diagram a similar boost, but only at the beginning of shell He burning. This boost left no signature in the HRD.

As mentioned above, the rotating 15 and $25 M_{\odot}$ models remain in the blue part of the HRD and do not develop any outer convective zone (OCZ). The rotating $40 M_{\odot}$ presents a very thin one and the $85 M_{\odot}$ a very early and short-lived one. On the contrary, the $60 M_{\odot}$ develops a deeper OCZ at the end of CHeB and for almost the rest of its evolution until the pre-SN stage. Its surface enrichment is thus larger than that presented by the $85 M_{\odot}$ model. The only non-rotating model that develops an OCZ is the $85 M_{\odot}$. Although this convective zone disappears quickly, it suffices to dredge up heavy elements brought near the surface by the CNO shell boost, and it is the model that shows the highest level of enrichment at the end of its evolution (see Table 3).

Limongi et al. (2000) show that at standard metallicity, the transition from convective to radiative core C burning occurs between 15 and $20 M_{\odot}$. From Fig. 1 of Limongi et al. (2001), we deduce that at $Z = 0$, this transition occurs at slightly higher masses, between 20 and $25 M_{\odot}$. We note that our $15 M_{\odot}$ models have convective C-burning cores, while cores are radiative in the case of our $25 M_{\odot}$ models, in agreement with Limongi et al. (2001). In the non-rotating models with $M \geq 40 M_{\odot}$, the shell C burning is convective up to the end of the hydrostatic evolution, while in the rotating models, the shell experiences a few flashes but otherwise remains radiative.

4.2. Stellar yields

The yields calculated from stellar models are strongly correlated to the value of the CO-core mass, M_{CO} . We give the values of M_{CO} of our models in Table 2. They were determined as in Heger et al. (2000a) as the mass coordinate at which the abundance of He drops below 10^{-3} (in mass fraction). From these values, we determined the remnant masses of our models, M_{rem} , with the relation between M_{rem} and M_{CO} described in Maeder (1992)

In Table 4 we present the total stellar yields for the light elements that are not significantly modified by the explosion (Limongi & Chieffi 2002). The total yields are the sum of the pre-SN yields ($mp_{\text{im}}^{\text{preSN}}$) and the wind yields ($mp_{\text{im}}^{\text{winds}}$). The

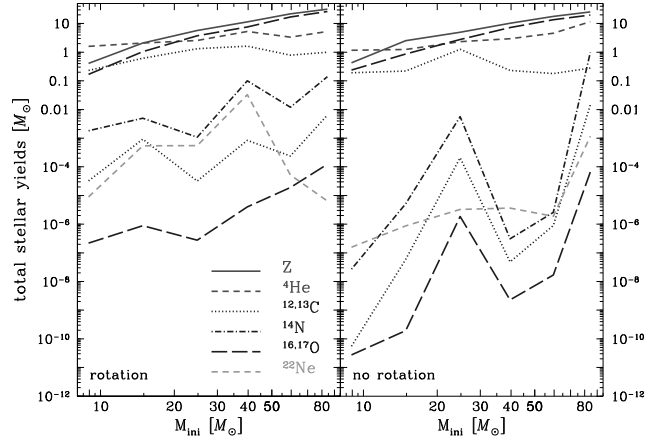


Fig. 6. Total stellar yields as a function of initial mass for our $Z = 0$ models. *Left:* models with rotation; *right:* non-rotating models.

pre-SN contribution from a star of initial mass m to the stellar yield of an element i is:

$$mp_{\text{im}}^{\text{preSN}} = \int_{M_{\text{rem}}}^{M_{\text{fin}}} [X_i(m_r) - X_i^0] dm_r$$

with X_i^0 the initial abundance (in mass fraction) of element i ($X_i^0 = 0$ for all elements except He), and $X_i(m_r)$ its final abundance at the Lagrangian coordinate m_r . The winds contribution is:

$$mp_{\text{im}}^{\text{winds}} = \int_0^{\tau(m)} \dot{M}(m, t) [X_i^S(m, t) - X_i^0] dt$$

where $\tau(m)$ is the final age of the star with initial mass m , $\dot{M}(m, t)$ the mass loss rate when the age of the star is t , and $X_i^S(m, t)$ the surface abundance (in mass fraction) of element i at age t . Most of the time, the total stellar yields are dominated by the pre-SN contribution.

We did not include the $200 M_{\odot}$ models in Table 4 because they have not undergone the shell He-burning phase, which noticeably modifies the results. The $9 M_{\odot}$ models were also stopped before central C burning, but as we mentioned, the core was strongly degenerate at carbon ignition, which could lead to the complete destruction of the star without further processing. Note that the carbon ignition occurred at the edge of the core, so there is a possibility that the star survived C burning. In that case, since the mass included below the He-burning shell is less than the limiting mass for Ne burning ($1.37 M_{\odot}$), the star is expected to collapse in a type II SN by electron capture on a degenerate O-Ne-Mg core (Nomoto 1984). This would modify the pre-SN yields presented here since the C burning would have processed the matter of the core. We would thus expect a reduction of ^{12}C and an enhancement of ^{13}C , ^{14}N , ^{16}O and ^{22}Ne . In the case of the $15 M_{\odot}$ models, the evolution was stopped at the end of central O burning. At that time, the mass interior to the C-burning shell is higher than the Chandrasekhar mass, so the star is expected to undergo an off-centre Si flash and to end with an Fe core (Nomoto 1984). We then expect the yields presented here to be modified slightly (reduction of C, N and Ne, enhancement of O). We have checked the amplitude of the variation when the yields are calculated at the end of O burning or at the end of Si burning in the models that have been followed until the pre-SN phase, so we do not expect the yields of the $15 M_{\odot}$ models given in Table 4 to vary by more than 15%.

Table 4. Total stellar yields at the pre-supernova stage, in $M_{\odot}$. In parenthesis, the “wind-only” contribution is given for the models for which no SN contribution is expected (see Sect. 4.3). The masses of the models are in $M_{\odot}$ and the velocities in km s^{-1} .

M_{ini}	v_{ini}	$\frac{{}^3\text{He}}{10^{-4}}$	${}^4\text{He}$	${}^{12}\text{C}$	${}^{13}\text{C}$	${}^{14}\text{N}$	${}^{16}\text{O}$	${}^{17}\text{O}$	${}^{18}\text{O}$	${}^{22}\text{Ne}$	Z
9	0	-0.53	1.17	0.19	5.54e-11	2.68e-8	0.24	2.74e-11	4.45e-9	1.58e-7	0.43
9	500	-0.65	1.59	0.24	3.27e-5	1.81e-3	0.17	2.20e-7	8.20e-6	8.98e-6	0.41
15	0	-1.02	1.24	0.22	5.94e-8	5.13e-6	0.86	1.93e-10	2.68e-9	8.73e-7	2.53
15	800	-1.23	2.10	0.61	9.21e-4	4.99e-3	1.05	8.88e-7	7.17e-6	5.44e-4	2.10
25	0	-1.97	2.32	1.26	2.09e-4	5.69e-3	2.78	1.86e-6	1.79e-9	3.27e-6	4.93
25	800	-2.13	2.57	1.32	3.21e-5	1.08e-3	3.76	2.75e-7	7.78e-6	5.54e-4	5.75
		(-0.02)	(0.003)	(5e-14)	(1.59e-14)	(6.86e-12)	(2e-13)	(1.51e-15)	(6.51e-19)	(1.01e-26)	(7e-12)
40	0	-3.21	2.94	0.23	4.85e-8	3.09e-7	7.25	2.29e-9	1.55e-8	3.63e-6	10.2
		(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
40	800	-3.32	5.32	1.63	8.41e-4	1.00e-1	7.57	4.04e-6	6.74e-5	3.30e-2	11.4
		(-0.17)	(0.05)	(3e-8)	(7.96e-9)	(1.07e-6)	(4e-08)	(5.89e-11)	(5.43e-14)	(2.64e-11)	(1e-6)
60	0	-4.87	4.58	0.18	9.07e-7	2.58e-6	13.6	1.69e-8	2.26e-7	1.89e-6	17.7
		(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
60	800	-4.76	3.33	0.79	2.39e-4	1.20e-2	16.9	1.89e-5	1.33e-4	5.24e-5	21.7
		(-0.21)	(0.04)	(2e-8)	(7.10e-9)	(9.00e-7)	(9e-8)	(1.94e-10)	(8.18e-14)	(4.12e-11)	(1e-6)
85	0	-7.22	11.70	0.28	1.46e-2	9.87e-1	19.7	6.63e-5	1.17e-7	1.18e-3	25.5
		(-0.004)	(0.01)	(3e-5)	(8.20e-6)	(2.42e-4)	(8e-6)	(7.26e-9)	(1.50e-11)	(3.99e-7)	(3e-4)
85	800	-6.15	5.31	1.00	6.44e-3	1.40e-1	26.2	1.24e-4	2.32e-7	6.47e-6	31.6

Rotation leads to larger yields for almost all the isotopes and for almost all the masses (see Fig. 6). The exceptions are the non-rotating models that undergo a strong CNO shell boost: the 25 and 85 $M_{\odot}$ models, which produce larger amounts of ${}^{13}\text{C}$ and ${}^{14}\text{N}$. The 25 $M_{\odot}$ model overtakes its rotating counterpart also in the production of ${}^{17}\text{O}$, and the 85 $M_{\odot}$ model produces more ${}^4\text{He}$ and ${}^{22}\text{Ne}$ than the rotating one. Otherwise, the enhancement of the yields brought by the rotational mixing may reach orders of magnitude, especially so for the production of primary ${}^{14}\text{N}$: a factor of 10^5 between the rotating and non-rotating 9 $M_{\odot}$ models and even a factor of 10^6 in the case of the 40 $M_{\odot}$ models. We note that rotation makes the production of ${}^{14}\text{N}$ robust through the whole mass range, while there are enormous differences from mass to mass in the non-rotating models.

Figure 7 shows a comparison of the CNO yields of all our $Z = 0$ models with the yields of the non-rotating models at $Z = 0$ from Chieffi & Limongi (2004) and those of the rotating models at $Z = 10^{-8}$ from Hirschi (2007). For ${}^{12}\text{C}$ (left) and ${}^{16}\text{O}$ (right), our non-rotating 15 and 25 $M_{\odot}$ yields compare within a factor of 2 or 3 with those of Chieffi & Limongi (2004). Their 25 $M_{\odot}$ produces a huge amount of ${}^{14}\text{N}$ (centre), even a factor ~ 10 higher than ours, so the CNO boost of the H-burning shell seems to be a robust feature in this mass domain. Compared to the ${}^{12}\text{C}$ produced at $Z = 10^{-8}$, the production of our rotating 60 and 85 $M_{\odot}$ models is smaller, while it is almost equal or larger for the 9 to 40 $M_{\odot}$ models. This can be understood because our high mass models do not undergo a shell boost, so their He-burning core is larger, which leads to a lower ${}^{12}\text{C}$ yield. The ${}^{16}\text{O}$ production is about the same at $Z = 10^{-8}$ and $Z = 0$. Our PopIII models have slightly larger ${}^{16}\text{O}$ yields than the models of Hirschi (2007), but we use a higher overshooting parameter which leads to larger CO cores and thus greater ${}^{16}\text{O}$ production.

An interesting point is the production of primary nitrogen at very low metallicity. Meynet & Maeder (2002) show that this production increases when the metallicity decreases, and Hirschi (2007) confirms the trend. We should thus expect a still higher production at $Z = 0$, but this is not the case. ${}^{14}\text{N}$ is produced systematically less at $Z = 0$ than at $Z = 10^{-8}$. We relate this change of behaviour to the fact that the $Z = 0$ stars burn their

hydrogen at temperatures hot enough to burn some He as well and do not evolve into the red part of the HR diagram (or do so very slowly during CHeB) as can be seen in Fig. 1 (left). Due to this fact, the stars undergo much less mixing during the CHeB phase. Indeed, no redwards evolution means that the gradient of the angular velocity at the border of the He-burning core remains much shallower and triggers much less mixing. On the contrary, when a small amount of metal is present, stars begin to burn their hydrogen through the CNO cycle from the start. Thus the central temperatures during the MS phases are well below those required for He-burning. At the end of the MS phase, the core contracts and the stars evolve quickly to the red. A steep angular velocity gradient appears at the core border and triggers strong mixing. Figure 7 shows that there is even a difference of almost two orders of magnitude between the production of the 20 $M_{\odot}$ model at $Z = 10^{-8}$ and the 25 $M_{\odot}$ model at $Z = 0$. This can be explained as follows: the $Z = 10^{-8}$ model experiences a second CNO shell boost during shell He burning, which enhances its ${}^{14}\text{N}$ production by about two orders of magnitude.

4.3. Fate of the models

At high metallicity, massive stars experience very strong mass loss. Their final mass is therefore very low (even smaller than the final mass of stars with an initial mass $M_{\text{ini}} \leq 25 M_{\odot}$, see Maeder 1992). Thus, the fate of all massive stars is a SN explosion, whatever their initial mass.

At very low metallicity, this is no longer true. Heger & Woosley (2002) and Heger et al. (2003) determine the limiting M_{He} for which the fate will be:

- a type II SN: $M_{\text{He}} < 9 M_{\odot}$;
- a black hole (BH) by fallback: $9 M_{\odot} \leq M_{\text{He}} < 15 M_{\odot}$;
- a direct BH: $15 M_{\odot} \leq M_{\text{He}} < 40 M_{\odot}$;
- a pulsational pair-instability followed by a SN with BH formation: $40 M_{\odot} \leq M_{\text{He}} < 64 M_{\odot}$;
- a pair-instability SN (PISN): $64 M_{\odot} \leq M_{\text{He}} \leq 133 M_{\odot}$;
- or a direct BH collapse: $M_{\text{He}} > 133 M_{\odot}$.

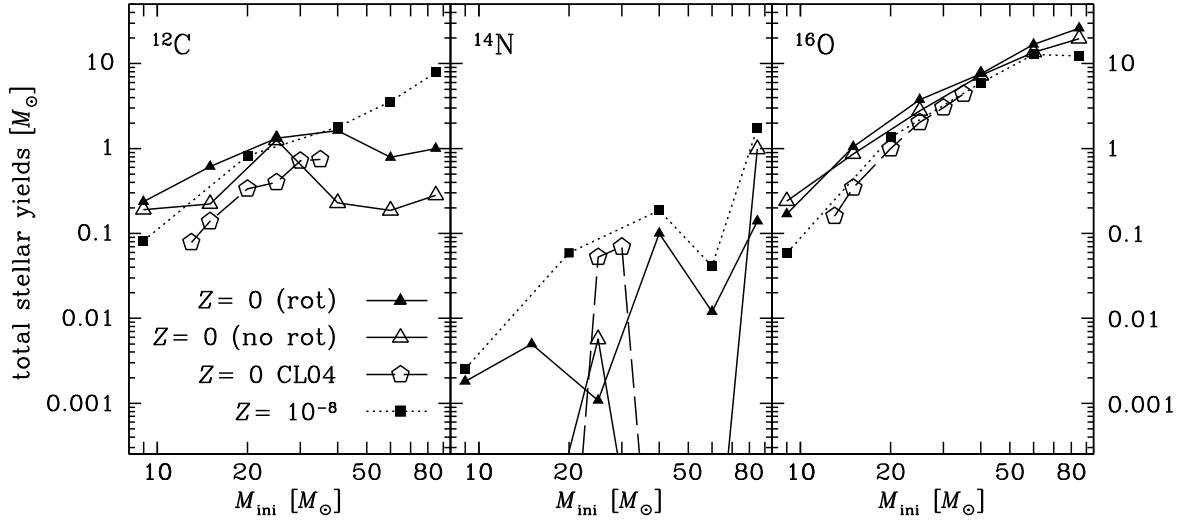


Fig. 7. Yield comparison between the non-rotating $Z = 0$ models from Chieffi & Limongi (2004, CL04, open pentagons), the rotating $Z = 10^{-8}$ models from Hirschi (2007, filled squares) and our rotating (filled triangles) and non-rotating (open triangles) $Z = 0$ models. *Left:* ^{12}C ; *centre:* ^{14}N ; *right:* ^{16}O .

Table 5. Some rotational properties of the iron core of our models: the mass M_{Fe} where the abundance of the sum $^{48}\text{Cr} + ^{52}\text{Fe} + ^{56}\text{Ni}$ amounts to 50% in mass fraction; the angular velocity Ω taken in the middle of the iron core (the profile of $\Omega(m)$ is almost flat in the core, so this value represents the mean value of Ω in the iron core); the maximum specific angular momentum j inside the core, corresponding to the first peak seen in Fig. 8 (*left*); the total angular momentum $\mathcal{L}$ and the parameter $\beta = E_{\text{rot}}/|E_{\text{pot}}|$, both taken at the border of the iron core.

Mass [$M_{\odot}$]	M_{Fe} [$M_{\odot}$]	Ω [s^{-1}]	j [$\text{cm}^2 \text{s}^{-1}$]	$\mathcal{L}$ [$\text{g cm}^2 \text{s}^{-1}$]	β %
25	1.327	0.4232	4.073e+16	5.837e+49	1.5833
40	1.555	0.3547	3.667e+16	5.963e+49	1.0046
60	2.516	0.1578	4.338e+16	1.152e+50	0.5449
85	2.779	0.1062	3.913e+16	1.182e+50	0.3624

Note that the limits above are obtained by non-rotating models, and they may not be appropriate for rotating models, as shown by Fryer et al. (2001). But since we are lacking more accurate determinations, we applied them to all our models. A star that ends its life as a BH is supposed to contribute to the chemical enrichment of its surrounding only through its winds. We give in parenthesis in Table 4 this “wind-only” contribution for the models that, according to the limits determined above, end their life as BH by fallback or by direct collapse.

The wind yields are very small, since we have seen that the winds are very weak (Sect. 3.4). The 25 $M_{\odot}$ model for instance, releases a total metal amount of less than $10^{-7} M_{\odot}$ through its winds while it produces a little more than 5 $M_{\odot}$ in the SN scenario. The 60 $M_{\odot}$ model contributes to the enrichment of the early Universe with a mere $10^{-5} M_{\odot}$ of metals. Note that the net enrichment depends of course on the amount of metal ejected, but also on the volume in which it is diluted. The wind contribution is expected to be a slow one, so it will be diluted in a smaller volume. The net enrichment may not be much smaller than a SN contribution diluted in a larger volume.

Rotation changes the mass of the He-cores at the end of the evolution. They are usually more massive because of the rotationally-induced mixing, and this may have consequences for the way the star ends its life. While the 25 $M_{\odot}$ model

without rotation ends as a type II SN, the rotating one is supposed to produce a weak explosion and a BH by fallback. The non-rotating 85 $M_{\odot}$ model remains in the direct BH domain while the rotating one reaches a He-core mass that implies pulsational pair-instability, followed by an Fe-core collapse into a BH. Its true yields must thus be very close to the pre-SN value.

As mentioned in Sect. 3.1, none of our models meet the conditions for becoming WR stars. They end their evolution having kept their envelope, and thus seem to fail to become a GRB progenitor as defined in the collapsar model of Woosley (1993). However, we have seen that the rotating 85 $M_{\odot}$ is likely to undergo pulsational pair instability⁵ and thus lose some mass in this process. If the mass lost is large, it may become a GRB progenitor since the very low core-envelope coupling has maintained a high angular momentum in the core. Fig. 8 (*left*) shows that in the innermost parts of this model, the specific angular momentum is higher than that needed to create an accretion disc around a BH.

More generally, we expect that rotation will change the way a star collapses. Shimizu et al. (2001) show that an anisotropy of only 10% in the neutrino flux can boost the energy of the explosion. Walder et al. (2005) and Yamasaki & Yamada (2005) show that the rotation-induced asphericity of the matter accretion rate is an even more important factor that helps produce a robust explosion. The geometry of the system leads most probably to a jet-like explosion. Table 5 gives the rotational characteristics of the iron core of our models and Fig. 8 (*right*) the distribution of angular velocity inside the 85 $M_{\odot}$ model. All models have kept a high total angular momentum, and meet the conditions to produce a maximally rotating BH. Though detailed calculations would be needed to ascertain this point, the true fate of our models could be much more explosive than previously supposed. In the chemical evolution model discussed below (see Sect. 5), we shall use the total yields, implicitly assuming that the material above M_{rem} will be ejected.

The centrifugal force has a stabilising effect on the collapsing core, so the core bounce may occur at sub-nuclear densities. This alters the emission of gravitational waves (GW).

⁵ This process has not been followed in the present models.

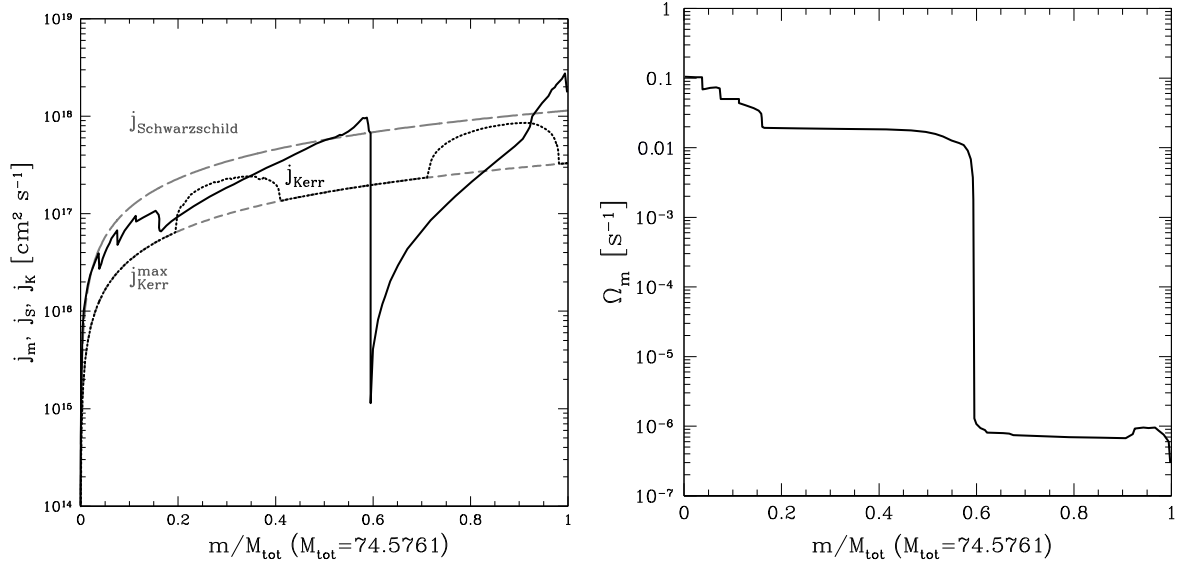


Fig. 8. *Left:* profile of the specific angular momentum j_m inside our $85 M_{\odot}$ model at the end of hydrostatic core Si-burning (solid line). The dotted line is $j_K = r_{\text{LSO}} c$ (Shapiro & Teukolsky 1983, p. 428), where the radius of the last stable orbit, r_{LSO} , is given by r_{ms} in formula (12.7.24) from Shapiro & Teukolsky (1983, p. 362) for a circular orbit in the Kerr metric. j_K is the minimum specific angular momentum necessary to form an accretion disc around a rotating black hole. $j_{\text{Schwarzschild}} = \sqrt{12} Gm/c$ (long-dashed line) and $j_{\text{Kerr}}^{\text{max}} = Gm/c$ (short-dashed line) is the minimum specific angular momentum necessary for a non-rotating and a maximally-rotating black hole, respectively. *Right:* profile of Ω_m inside the same model.

Ott et al. (2004) show that rotation increases the amplitude of GW because of the induced anisotropy. But if rotation is really fast, for $\beta = E_{\text{rot}}/|E_{\text{pot}}| > 0.3\%$, the amplitude of GW becomes smaller, and the characteristic frequency is shifted to lower frequencies. Except for the $85 M_{\odot}$ star, our models have a high β so their GW emission is expected to be damped by their high rotation rate. Suwa et al. (2007) show that very massive collapsing PopIII stars may contribute to the GW background and be detectable by future facilities such as DECIGO, the successor of LIGO. In that framework, our $85 M_{\odot}$ model, with $\beta = 0.3\%$, is expected to contribute to this background.

5. Chemical evolution of the Galaxy with $Z=0$ yields

Yields of extremely low-metallicity models are constrained by the most metal-poor stars observed in the halo of our Galaxy. They are supposed to trace the local chemical evolution of their surrounding medium at the time of their birth. The notion of time is important because stars with different masses have different lifetimes; the elements they produce are not released at the same moment. The oldest stars observed nowadays may have formed when only massive stars had time to contribute to the enrichment of the medium. If we want to understand the abundances observed at the surface of these old stars, we cannot just integrate the yields obtained by a set of models with an IMF, because we would lose track of the time. What we need is a complete chemical evolution model (CEM).

Such a CEM is computed by Chiappini et al. (2006b) for example⁶. They compare their predictions for the evolution of N/O and C/O upon the adoption of different sets of stellar yields

⁶ The details of the models can be found in Chiappini et al. (2006a), where they show that such a model reproduces the metallicity distribution of the Galactic halo stars. This means that the timescale for the chemical enrichment in the halo is well constrained.

with the surface ratios determined for the metal-poor stars observed by Israelian et al. (2004) and Spite et al. (2005). They show that the N/O ratio is much higher than what was predicted by a CEM using the yields of the slow-rotating $Z = 10^{-5}$ models from Meynet & Maeder (2002) down to $Z = 0$. When adding the yields of the fast-rotating $Z = 10^{-8}$ models from Hirschi (2007), the fit is much improved. The same improvement is found for the C/O ratio, which presents an upturn at low metallicity. Note that the high N/O and the C/O upturn of the low-metallicity stars is also observed in low-metallicity DLAs (Pettini et al. 2008).

Their CEM is computed with the assumption that yields from $Z = 10^{-8}$ stellar models are valid down to $Z = 0$. But actually, there is a physical transition at $Z_{\text{trans}} \approx 10^{-10}$: the stars below Z_{trans} present a PopIII-like behaviour (with some He burning already during the MS), and the stars above Z_{trans} have well separated H- and He-burning phases.

We computed a similar CEM, but this time including our $Z = 0$ pre-SN yields up to Z_{trans} , the yields of Hirschi (2007) up to $Z = 10^{-5}$, and then the yields from Meynet & Maeder (2002) and Hirschi et al. (2004). For the PopIII stellar models, we used an IMF that is truncated at $1 M_{\odot}$ but is otherwise standard. Compared to the chemical evolution computed using the $Z = 10^{-8}$ yields down to $Z = 0$ (Fig. 9, left panel), the new computation predicts a reduction of the N/O and C/O ratios at extremely low metallicity ($Z \leq 6 \times 10^{-7}$). This trend is expected given the higher O and the lower C and N production of the PopIII models (Sect. 4.2 and Fig. 7). Note that a similar CEM computed with the “wind-only” contribution for our 25 to $60 M_{\odot}$ models leads to such a slow O enrichment that it would deviate from the CEM with $Z = 10^{-8}$ yield curve only at an O/H domain much lower than that shown in Fig. 9.

Another interesting quantity is the ratio $^{12}\text{C}/^{13}\text{C}$. A large N production comes with a high ^{13}C value since they are both the products of ^{12}C conversion by the CN-cycle in H-burning zones. We thus expect a low $^{12}\text{C}/^{13}\text{C}$ ratio from the yields of

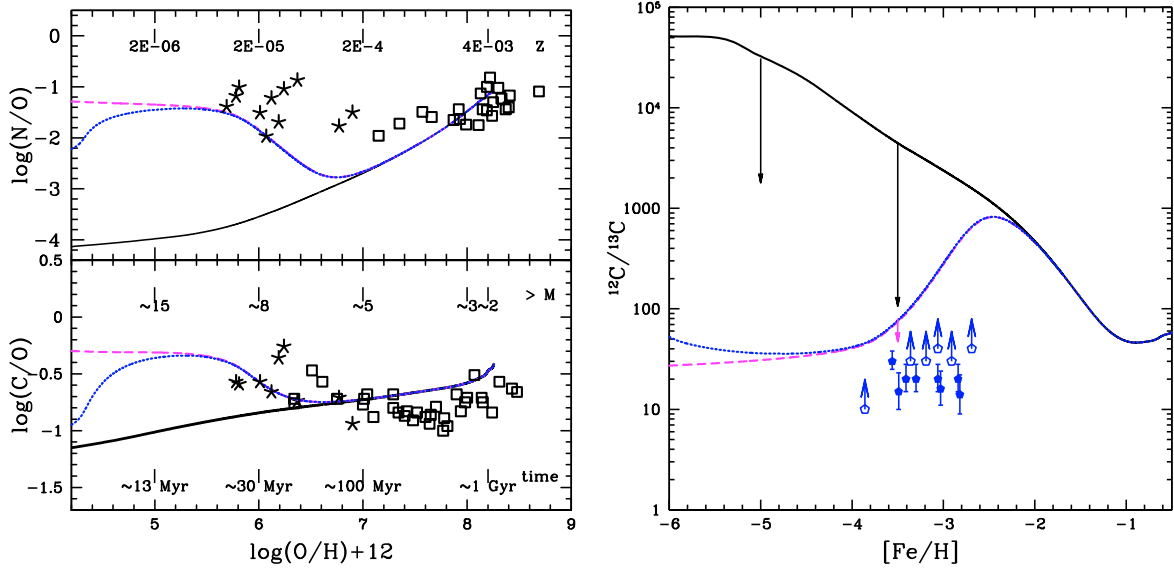


Fig. 9. Solid (black) curve: the CEM obtained with the stellar yields of slow rotating $Z = 10^{-5}$ models from Meynet & Maeder (2002) and Hirschi et al. (2004). Dashed (magenta) line includes the yields of fast rotating $Z = 10^{-8}$ models from Hirschi (2007) at very low metallicity. Dotted (blue) curve is obtained using the yields of the $Z = 0$ models presented in this paper up to $Z = 10^{-10}$. *Left:* evolution of the N/O and C/O ratios. Data points are from Israelian et al. (2004, open squares) and Spite et al. (2005, stars). *Right:* evolution of the $^{12}C/^{13}C$ ratio. Data points are unmixed stars from Spite et al. (2006): open pentagons are lower limits. The arrows going down from the theoretical curves indicate the final $^{12}C/^{13}C$ observed in giants (after the dredge-up), starting from the initial composition values given by the stellar models (see Chiappini et al. 2008).

fast-rotating models. Chiappini et al. (2008) use the same set of yields as in Chiappini et al. (2006b), but this time to study the $^{12}C/^{13}C$ ratio. They show that this ratio is indeed much closer to the recent observations provided by Spite et al. (2006) upon the inclusion of fast-rotating $Z = 10^{-8}$ models. With the inclusion of our pre-SN Pop III yields (Fig. 9, right panel), we predict that the $^{12}C/^{13}C$ ratio should rise slightly again with decreasing $[Fe/H]$ at extremely low metallicity.

6. Discussion and conclusion

We computed a set of metal-free stellar models with masses between 9 and 200 $M_{\odot}$, following their evolution until the pre-supernova stage. The set was composed of seven differentially rotating models with an initial velocity of 800 km s^{-1} (except for the 9 $M_{\odot}$ model which started the main sequence with 500 km s^{-1}), and a comparison set of non-rotating models. We showed that in Pop III stars, although the effects of rotation are important, they are less strong than in stars with even a tiny amount of metals ($Z = 10^{-8}$). There are two main causes for this:

1. The meridional circulation is extremely weak at $Z = 0$. The dependance of the outer cell in $1/\rho$ (see Sect. 3.3) makes it a factor of 100 lower compared to the value at standard metallicity, and still a factor of 4 lower compared to $Z = 10^{-5}$. The evolution of the angular momentum is close to local conservation, so the critical limit is reached late. Moreover, only a very small amount of mass needs to be lost in order to bring the surface back to sub-critical rotation.
2. There is little structural readjustment at the end of the MS, because the core is already hot enough to burn some He during the main sequence, so the nuclear burning is continuous. The transition between H and He burning is smooth and the models remain on the blue side of the Hertzsprung-Russell diagram for a long part of the core He burning. When they

eventually reach the red side, their outer convective zone remains thin, so the dredge-up is not efficient and the surface enrichment stays low. The envelope remains transparent to radiation and the mass loss is negligible.

There is actually a mass loss where none was expected (because of the wind dependance on Z), but it remains modest even for the most massive of our models.

Could this picture be modified by changing the physical inputs in the models? We have shown that the situation is qualitatively the same when we take a faster initial rotation rate. The main problem is the weakness of the meridional circulation, which does not efficiently feed the outer layers with angular momentum taken from the fast spinning core. So, even if the critical limit is reached a little earlier, the models still do not lose a lot of mass. For this work, we did not take into account the anisotropy of the winds in our models. We do not think it would change the picture presented here, because the loss of angular momentum by radiative mass loss is negligible. There is still an interesting possibility: the implementation of magnetic fields. They have been shown to change many features in stellar evolution (Heger et al. 2005; Maeder & Meynet 2005), and to provide the strong core-envelope coupling that is lacking in the $Z = 0$ models. With such a coupling, the mass loss becomes much larger and the evolution may be strongly altered (Ekström et al. 2008a).

With the initial velocities considered here, our models do not follow a chemically homogeneous evolution (Maeder 1987; Heger & Langer 2000). In the frame of the physics we used, homogeneous evolution for Pop III stars would need faster rotation rates. When the magnetic dynamo mechanism proposed by Spruit (1999, 2002) is taken into account, chemical mixing becomes more efficient and homogeneous evolution can be achieved with smaller initial rotation rates (Yoon & Langer 2005; Yoon et al. 2006; Woosley & Heger 2006). Note however that at extremely low metallicities, the weakening of the

meridional circulation becomes a real handicap for the models to evolve chemically homogeneously. [Hirschi et al. \(2008\)](#) find that at $Z = 10^{-8}$, a $40 M_{\odot}$ model (computed with magnetic fields) starting its evolution with $v/v_{\text{crit}} = 0.55$ does not rotate fast enough to follow such an homogeneous evolution. We expect that at $Z = 0$ the problem would be still more acute. The consequences of such extreme effects of rotation is left for a future work.

The only prescription for a stellar dynamo existing to date, the Tayler-Spruit formalism ([Spruit 1999, 2002](#)), is still debated: recent works have cast some doubts on the efficiency of the Tayler-Spruit dynamo mechanism. For instance, [Zahn et al. \(2007\)](#) have shown that although the Pitts and Tayler instability indeed develops, it does not succeed in driving a dynamo. Moreover, this dynamo needs a pre-existing magnetic-field seed, and it is not yet clear whether magnetic fields were present in the early Universe or not (see [Giovannini 2004](#), for a detailed discussion on this subject).

We presented pre-supernova yields for our stellar models. We included them in a chemical evolution model of the Galactic halo. Our results predict that the N/O and the C/O ratios should drop at extremely low metallicity ($Z \leq 10^{-6}$). For the time being though, the observational discrimination between the two curves is far beyond our observational possibilities. We also gave the rotational characteristics of the iron core of our models and showed that since they do not lose much angular momentum during their evolution, they keep high rotation rates in the core, so their final explosion may be much more energetic than sometimes assumed. We showed that the most massive of our models may contribute to the emission of the gravitational wave background.

Many interesting questions and issues were not treated in this study. Our mass coverage was too coarse to accurately constrain the limiting masses for undergoing a CNO shell boost, for example, or for the different fates at the end of the evolution. Even without knowing if magnetic fields were indeed present in the early Universe, it would be interesting to study the role they could play in primordial stellar evolution. As mentioned above, they could drastically change the picture presented here. We have also left for a forthcoming paper the question of re-ionisation. Rotation changes the luminosity, the T_{eff} and the lifetimes of the models, so it should change the ionising power of the first stars.

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LETTER TO THE EDITOR

A new imprint of fast rotators: low $^{12}\text{C}/^{13}\text{C}$ ratios in extremely metal-poor halo stars

C. Chiappini^{1,2}, S. Ekström¹, G. Meynet¹, R. Hirschi³, A. Maeder¹, and C. Charbonnel^{1,4}

¹ Observatoire Astronomique de l'Université de Genève, 1290 Sauverny, Switzerland
 e-mail: Christina.chiappini@obs.unige.ch

² Osservatorio Astronomico di Trieste, via G. B. Tiepolo 11, 34131 Trieste, Italia

³ Keele University, Lennard-Jones Lab., Keele ST5 5BG, UK

⁴ LATT, CNRS UMR 5572, OMP, Université Paul Sabatier Toulouse 3, 14 Av. E. Belin, 31400 Toulouse, France

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ABSTRACT

Context. Fast stellar rotation is currently the most promising mechanism for producing primary nitrogen in metal-poor massive stars. Chemical evolution models computed with the inclusion of the yields of fast rotating models at a metallicity $Z = 10^{-8}$ can account for the high N/O abundances observed in normal metal-poor halo stars. If, as believed, intermediate mass stars did not have enough time to contribute to the interstellar medium enrichment at such low metallicities, the above result constitutes a strong case for the existence of fast rotators in the primordial Universe.

Aims. An important result of stellar models of fast rotators is that large quantities of primary ^{13}C are produced. Hence, our goal is to investigate the consequence of fast rotation on the evolution of the $^{12}\text{C}/^{13}\text{C}$ ratio in the interstellar medium at low metallicity.

Methods. We compute the evolution of the $^{12}\text{C}/^{13}\text{C}$ ratio for the first time at very low metallicities upon the inclusion of fast rotators at $Z = 10^{-8}$.

Results. We predict that, if fast rotating massive stars were common phenomena in the early Universe, the primordial interstellar medium of galaxies with a star formation history similar to the one inferred for our galactic halo should have $^{12}\text{C}/^{13}\text{C}$ ratios between 30–300. Without fast rotators, the predicted $^{12}\text{C}/^{13}\text{C}$ ratios would be ~ 4500 at $[\text{Fe}/\text{H}] = -3.5$, increasing to $\sim 31\,000$ at around $[\text{Fe}/\text{H}] = -5.0$. Current data on very metal-poor giant normal stars in the galactic halo agree better with chemical evolution models including fast rotators. The expected difference in the $^{12}\text{C}/^{13}\text{C}$ ratios, after accounting for the effects of the first dredge-up, between our predictions with/without fast rotators is of the order of a factor of 2–3. However, larger differences (a factor of ~ 60 – 90) are expected for giants at $[\text{Fe}/\text{H}] = -5$ or turnoff stars already at $[\text{Fe}/\text{H}] = -3.5$. To test our predictions, challenging measurements of the $^{12}\text{C}/^{13}\text{C}$ in more extremely metal-poor giants and turnoff stars are required.

Key words. stars: abundances – stars: rotation – Galaxy: evolution – Galaxy: abundances – Galaxy: halo – early Universe

1. Introduction

In the last few years, a burst of new information on cosmic abundances, provided by high-resolution spectra of very metal-poor stars in the halo has had a considerable impact on our understanding of both stellar nucleosynthesis and the chemical enrichment of the primordial interstellar medium (ISM). In particular, it is in the halo of the Milky Way that the oldest and most metal-poor stars in the Universe are observable, born at times (or equivalent redshifts) still out of reach of the deepest surveys of primordial galaxies.

Several teams (e.g. Cayrel et al. 2004; Cohen et al. 2008; Barklem et al. 2005; Aoki et al. 2007) have studied the chemical composition of halo metal-poor stars (stars with less than 1/300 down to 1/10 000 the solar iron abundance). These low-mass stars have lifetimes comparable to the age of the Universe, and they retain in their atmospheres the elemental abundances of the gas at the time of their birth. Hence, these stars contain a memory of the unique nucleosynthesis contribution of the first stellar generations to the enrichment of the ISM, offering a local benchmark for cosmology. These massive stars are long dead but there are hopes of observing them directly in very high-redshift galaxies. The current way to constrain the properties of the first generations of stars and check whether the very different

primordial environment has produced noticeable effects concerning their properties is to search for their imprints on the oldest extremely metal-poor stars (EMPs) in our galactic halo.

Recently a sample of EMP ($[\text{Fe}/\text{H}] \leq -3$) halo stars has become available. Using UVES on the VLT, Cayrel et al. (2004) provided abundance measurements of unprecedented accuracy for several elements in a fairly large sample of metal-poor *normal* stars (to distinguish from the C-rich EMP stars, see Beers & Christlieb 2005). Their data have revealed a striking homogeneity in the chemical properties of halo stars. In particular, a very low scatter for the $[\alpha/\text{Fe}]$ ratios was found. In a subsequent paper, the authors (Spite et al. 2005, hereafter S05) again report some unexpected results, namely: these same metal-poor stars show a high N/O ratio suggesting high levels of production of primary nitrogen in massive stars (see below). Furthermore, a large scatter in their N/O ratios (much larger than their quoted error bars) was found. S05 also report a slight increase in the C/O of the same stars with decreasing metallicity.

As discussed in Chiappini et al. (2005, 2006a,b), the data of S05 happen to be in a very interesting metallicity range: at such low metallicities, their observed stars are probably made of only massive star ejecta diluted by the primordial ISM. Indeed, chemical evolution models (CEM) of the halo (Chiappini et al. 2006a; Prantzos 2003) show that an $8 M_{\odot}$ star dies at $[\text{Fe}/\text{H}] \sim -3.5$.

This means that, below this metallicity, the ISM is enriched exclusively by massive stars. Moreover, according to these models the contribution of AGB stars is negligible below $[\text{Fe}/\text{H}] \sim -2.5$. More important, these theoretical predictions seem to be confirmed by very recent observations. Meléndez & Cohen (2007) have shown that the $^{25,26}\text{Mg}/^{24}\text{Mg}$ ratios in halo dwarfs are low and that AGB stars would have played a minor role below $[\text{Fe}/\text{H}] \sim -2.0$.

If AGB stars¹ indeed had not had enough time to contribute to the ISM enrichment at such low metallicities and massive stars are not producers of primary nitrogen (as predicted by standard stellar models, e.g., Woosley et al. 2002), one would expect to observe a decline in the N/O or N/Fe ratios towards low Z , contrary to what has been found by S05. Hence, the high levels of N/O observed in halo stars have to be a result of the nucleosynthesis taking place in the metal-poor massive stars, suggesting a revision of standard models for the nucleosynthesis in massive stars.

The effects of stellar axial rotation are numerous and at low metallicity may lead to a drastic revision of current wisdom (see Meynet et al. 2006; Hirschi 2007). The stellar models of the Geneva group, including rotation (Maeder & Meynet 2000, 2001), have proved to be successful in explaining some observations that could not be explained by non-rotating models, namely, the observed number ratio of Wolf Rayet stars to O-type stars for different metallicities, the observed ratio of WN to WC for metallicities lower than solar, and the observed ratio of type Ib/Ic to type II supernovae at different metallicities. One of the consequences of rotation is that carbon and oxygen, produced in the He-burning core, are transported by rotational mixing into the H-burning shell, where they are transformed into primary ^{13}C and ^{14}N . Interestingly, the efficiency of this process increases when the initial mass and rotational velocity increase (see Maeder & Meynet 2000; Meynet & Maeder 2002b), producing more N for fast rotators. Another fundamental prediction from models including rotation is that the rotational mixing also increases with decreasing metallicity.

Chiappini et al. (2006b) computed CEMs adopting the new calculations of Hirschi (2007) for the evolution of massive stars at very low metallicities under the assumption of an almost constant initial ratio $v_{\text{rot}}/v_{\text{crit}}$ as a function of metallicity² (i.e. where the v_{rot} increases towards lower metallicities). It was shown that, in such a framework, massive stars can produce large amounts of nitrogen, and this explains the data of S05. Moreover, these same models naturally predict a C/O upturn at low metallicities. It should be noticed that this is currently the only way to explain the CNO halo abundances of *normal* stars giving strong support to the idea that stars rotate faster at low metallicities. One important caveat is that the abundances of C, N, and O in low metallicity red giant branch stars, could still suffer from systematic non negligible corrections due to 3D and NLTE effects (see Asplund 2005). Are there other imprints of fast rotators that are less affected by the above uncertainties? The $^{12}\text{C}/^{13}\text{C}$ ratio is largely unaffected by uncertainties in the adopted atmosphere parameters (Spite et al. 2006, hereafter S06), although in this case the stellar evolution effects taking place in these old, low-mass stars have to be correctly accounted for in order to recover their pristine values.

In this Letter we suggest that fast-rotating massive stars could also have left an imprint on the evolution of the $^{12}\text{C}/^{13}\text{C}$ ratio in the earliest phases of the ISM enrichment. In fact, the same rotational mixing responsible for the nitrogen production would also produce non negligible amounts of ^{13}C . We predict, for the first time, that the impact of an early population of fast rotators on the evolution of the $^{12}\text{C}/^{13}\text{C}$ abundance ratio in the very metal-poor ISM is huge. *If fast rotators existed, low (~ 30 – 300) $^{12}\text{C}/^{13}\text{C}$ ratios should be observed in the very metal-poor ISM of galaxies polluted essentially only by massive stars.*

This Letter is organized as follows. The adopted stellar yields are briefly presented in Sect. 2. Our results are shown in Sect. 3. Section 4 is devoted to our conclusion and to a discussion of this finding's impact on different research areas.

2. ^{13}C yields for very metal-poor stars

As this Letter focuses on the very metal-poor range (below $[\text{Fe}/\text{H}] \simeq -3$ or $\log(\text{O}/\text{H}) + 12 \simeq 7$) where no/few intermediate mass star would have had time to contribute to the ISM enrichment (see Meléndez & Cohen 2007)³, we concentrate on the effects of rotation on the stellar yields of metal-poor massive stars.

The ^{13}C is produced in H-burning regions, where the CN cycle converts ^{12}C into ^{13}C . As discussed in the previous section, rotation triggers the production of primary ^{13}C , by allowing the diffusion of ^{12}C produced in He-burning zones into H-burning ones. As a numerical illustration, a $40 M_{\odot}$ at $Z = 10^{-8}$ ejects a quantity of newly synthesized ^{13}C equal to $6.3 \times 10^{-2} M_{\odot}$ when $v_{\text{rot}}^{\text{ini}} = 700 \text{ km s}^{-1}$ (Hirschi 2007). For comparison, models without rotation predict a ^{13}C yield of $3.3 \times 10^{-5} M_{\odot}$ for $Z = 10^{-6}$ and $M = 35 M_{\odot}$ (Chieffi & Limongi 2004), or a ^{13}C yield of $2.2 \times 10^{-4} M_{\odot}$ for $Z = 0.001$ and $M = 40 M_{\odot}$ (Tominaga et al. 2007).

Here we adopt the ^{12}C and ^{13}C yields of Hirschi (2007) computed for $Z = 10^{-8}$ and initial rotational velocities, $v_{\text{rot}}^{\text{ini}} = 600$ to 800 km s^{-1} , depending on the stellar mass. For higher metallicities the adopted yields are from Meynet & Maeder (2002a, hereafter MM02), for $v_{\text{rot}}^{\text{ini}} = 300 \text{ km s}^{-1}$. The justification for the higher velocities at $Z = 10^{-8}$ relies on the following assumption: stars begin their evolution on the ZAMS with a rotational velocity such that the ratio $v_{\text{rot}}/v_{\text{crit}}$ remains almost constant (around 0.5–0.6) with mass and metallicity, implying higher rotational velocities at lower metallicities. As a consequence the stellar yields of ^{13}C of Hirschi (2007) are greater by 2 to 3 orders of magnitude than the ones of MM02 for $Z = 10^{-5}$ and $v_{\text{rot}}^{\text{ini}} = 300 \text{ km s}^{-1}$.

3. Results

Here we show the impact of the stellar yields discussed above on the predictions of the $^{12}\text{C}/^{13}\text{C}$ evolution in the galactic halo. We adopt the same CEM of Chiappini et al. (2006a), briefly described below.

Our model assumes that the galactic halo was formed by a combination of infall plus outflow. Our main assumptions are: a) a Gaussian infall ($f(t) \propto e^{-(t-t_0)^2/2\sigma^2}$, with $t_0 = 0.1 \text{ Gyr}$ and

¹ Classically the best candidates for the production of primary N through hot-bottom burning during the asymptotic giant branch (AGB) phase (Siess 2007).

² v_{crit} represents the critical rotational velocity, i.e., the equatorial velocity such that the centrifugal acceleration compensates for the gravity.

³ The effect of rotation on the evolution of low- and intermediate-mass stars (LIMS) and their impact on the galactic ISM will be discussed in a forthcoming paper. In the present work the low- and intermediate-mass stars are included with the same yield prescriptions as adopted in our previous work (Chiappini et al. 2003), with $v_{\text{rot}}^{\text{ini}} = 300 \text{ km s}^{-1}$.

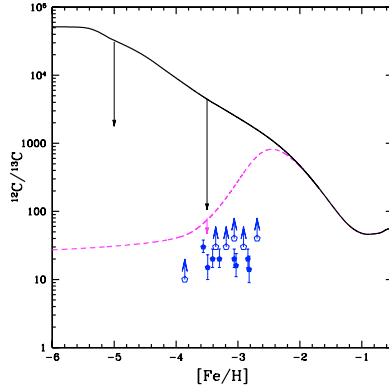


Fig. 1. Predicted evolution of the $^{12}\text{C}/^{13}\text{C}$ ratio according to a CEM computed with different stellar yield sets for metallicities below $Z = 10^{-5}$: **a)** solid line – a model computed under the assumption that the lowest metallicity yield table of MM02 ($Z = 10^{-5}$) is valid down to $Z = 0$ (without fast rotators) and **b)** dashed line (magenta in the electronic version) a model computed with the contribution of $Z = 10^{-8}$ fast rotators. The data are the *unmixed* stars of S06. The open symbols represent lower limits. The arrows indicate the final $^{12}\text{C}/^{13}\text{C}$ observed in giants, starting from the initial values given by models with (light/magenta arrow) and without fast rotators (dark arrows).

$\sigma = 0.05$); b) an outflow rate of 8 times the star formation rate; c) a Schmidt law for the star formation rate (see also Prantzos 2003). This model is able to reproduce the observed halo metallicity distribution (see Fig. 4 of Chiappini et al. 2006a). While in our previous model, without outflow (Chiappini et al. 2006b) a $8 M_{\odot}$ star would die at a metallicity $[\text{Fe}/\text{H}] \sim -2.2$, in the new model with outflow the same star would die when the metallicity in the ISM was $[\text{Fe}/\text{H}] \sim -3.8$. The latter model is favoured by the observed halo metallicity distribution.

In Fig. 1 we show our model predictions. The different curves show the same CEM computed with different sets of stellar yields at low metallicities. A model computed without fast rotators predicts very high $^{12}\text{C}/^{13}\text{C}$ ratios at low metallicities, namely: ~ 2400 at $[\text{Fe}/\text{H}] = -3.0$, ~ 4500 at $[\text{Fe}/\text{H}] = -3.5$, increasing to $\sim 31\,000$ around $[\text{Fe}/\text{H}] = -5.0$. When the contribution of the fast rotators is taken into account, much lower $^{12}\text{C}/^{13}\text{C}$ ratios are predicted: ~ 295 at $[\text{Fe}/\text{H}] = -3.0$, ~ 80 at $[\text{Fe}/\text{H}] = -3.5$ and ~ 30 at $[\text{Fe}/\text{H}] = -5.0$. In other words, we predict smaller ISM $^{12}\text{C}/^{13}\text{C}$ ratios (by a factor of ~ 1000 at $[\text{Fe}/\text{H}] = -5.0$ and of ~ 60 at $[\text{Fe}/\text{H}] = -3.5$) upon the inclusion of fast rotators. We notice that the impact of fast rotators could be even greater than what has been computed here. This would be the case if, for instance, fast rotators would still play some role at slightly higher metallicities. However, at present, stellar yields computed with high rotational velocities for higher metallicities ($Z \sim 10^{-5}$) are not available.

Currently, the only data available in the literature for the $^{12}\text{C}/^{13}\text{C}$ ratio at such low metallicities is from S06. They obtained new $^{12}\text{C}/^{13}\text{C}$ isotopic ratios for the same stars as studied in S05. Here we only plot their so-called *unmixed* stars, which are giants that have undergone the first dredge-up but are still below the luminosity function bump and hence have not experienced the thermohaline-mixing (e.g., Charbonnel & Zahn 2007). To recover their initial $^{12}\text{C}/^{13}\text{C}$ ratio relevant to the comparison with our CEM predictions, we have computed the evolution of the carbon isotopic ratio in stellar models of $0.85 M_{\odot}$, $[\text{Fe}/\text{H}] = -3.5$ (typical of the Spite sample) from the pre-main sequence up to

the end of the first dredge-up on the red giant branch, assuming different initial values for the $^{12}\text{C}/^{13}\text{C}$ ratio.

Starting from an initial $^{12}\text{C}/^{13}\text{C}$ ratio corresponding to the prediction of a CEM without fast rotators at $[\text{Fe}/\text{H}] = -3.5$, we obtain a post dredge-up value of 106. This value is clearly higher than the observed ratio in the Spite *unmixed* sample (~ 20 – 30). In the case of an initial value of 80 as predicted by a model that takes the contribution of the fast rotators at this same metallicity into account, a value of 47 is obtained, i.e., slightly higher than, but marginally consistent with, the observational data. Starting from an initial value of 50 (see previous discussion), the observational values can be reached (~ 30). Thus, even though our predictions for the ISM $^{12}\text{C}/^{13}\text{C}$ ratio with and without fast rotators differ by a factor of 60 at $[\text{Fe}/\text{H}] = -3.5$, in the case of very metal-poor giants, the expected effect would be smaller (a factor of 2–3), but still significant. Larger differences are obtained at lower metallicities. At $[\text{Fe}/\text{H}] = -5$, starting from an initial value of 31 000 (predicted by CEM without fast rotators), we obtain a post dredge-up value of 1790 (Fig. 1). On the other hand, essentially no change is obtained when starting from the initial values of our CEM which include fast rotators (in the latter case the initial ^{13}C is already so high that the effect of the 1st dredge-up turns out to be negligible). Thus, at such low metallicities the expected effect is greater (a factor of 60!).

In summary, we offer a robust theoretical prediction both for the ISM of primordial galaxies and for the EMPs in our galactic halo. Our predictions could in principle be checked, once C-isotopic ratios had been measured in normal EMP turnoff stars at $[\text{Fe}/\text{H}] \sim -3.5$ or giants at lower metallicities. An additional test of our predictions will probably come from the fast evolving field of abundance measurements in the ISM of high-redshift galaxies (Pettini 2006). The effect of fast rotators should appear in systems where the main contributors to the gas enrichment are the massive stars. From an observational point of view, the most suitable systems for measuring the $^{12}\text{C}/^{13}\text{C}$ are the damped Ly α systems (DLAs) because the absence of strong galactic winds would not allow the measurement of the isotopic shifts. Levshakov et al. (2006) reported a lower limit $^{12}\text{C}/^{13}\text{C} > 80$ in a DLA at $z = 1.15$. However in this case, the metallicity of the system is $[\text{Fe}/\text{H}] \sim -1$ and the role of fast rotators could be minor. The value found by Levshakov et al. cannot be compared to our models in Fig. 1 because the chemical evolution of DLAs most probably proceeded in a radically different way than for the galactic halo (see Chiappini et al. 2003; Dessauges-Zavadsky et al. 2007). The impact of fast rotators in DLAs will be the subject of a forthcoming paper.

It is worth noticing that $^{12}\text{C}/^{13}\text{C}$ ratios have recently been measured (Carretta et al. 2005) in globular cluster unevolved stars, in the metallicity range $[\text{Fe}/\text{H}] \sim -2.0$ to ~ -0.8 . A low value for this ratio was found (~ 8), suggesting the pollution of the studied stars by previously evolved AGB stars. More recently, Decressin et al. (2007) have suggested that the low $^{12}\text{C}/^{13}\text{C}$ values (together with many other observed properties of globular clusters) could be explained better by instead assuming that the stars we observe today in globular clusters were polluted by a previously evolved generation of fast-rotating massive stars.

A final remark concerns the contribution of super-AGB stars (SAGB), which in principle could enrich the ISM in ^{13}C and ^{14}N (Siess 2007; Gil-Pons et al. 2007), but were not included in the present calculations as the stellar yields for these objects are currently not available. As SAGBs have masses between 8 – $12 M_{\odot}$ (depending on the metallicity) one could wonder if these objects could enrich the ISM early on both in ^{13}C and ^{14}N , thus being good candidates to explain the very metal-poor data on CNO,

without the need of fast rotators. However, this does not seem to be the case as the very metal-poor data of Spite et al. falls in a metallicity range where the main contributors to the ISM enrichment are stars with masses between 10 and 30 $M_{\odot}$ provided our enrichment timescale, constrained by the halo metallicity distribution, is correct (see Chiappini et al. 2006a). We should however keep this caveat in mind and this point certainly deserves further study once stellar yields for SAGBs become available.

4. Discussion and conclusions

In this Letter we have studied the impact of fast rotators on the chemical evolution of the $^{12}\text{C}/^{13}\text{C}$ ratio in the halo of the Milky Way. We predict that, if fast rotating massive stars were common phenomena in the early Universe, the ISM at an epoch where the chemical enrichment was essentially due to massive stars ($[\text{Fe}/\text{H}] < -3.0$) should have had $^{12}\text{C}/^{13}\text{C}$ ratios between 30–300. Without fast rotators, the predicted $^{12}\text{C}/^{13}\text{C}$ ratios would be ~ 4500 at $[\text{Fe}/\text{H}] = -3.5$, increasing to $\sim 31\,000$ around $[\text{Fe}/\text{H}] = -5.0$.

We also computed the expected surface abundance modifications during the evolution of a typical, very metal-poor, low mass star, and concluded that if fast rotators did play a role (i.e. the ISM in the earliest phases had low $^{12}\text{C}/^{13}\text{C}$ ratios), the EMPs observed today ($[\text{Fe}/\text{H}] \lesssim -3.0$) should have similar $^{12}\text{C}/^{13}\text{C}$ ratios to the ones reported by S06, whereas values a factor of ~ 2 – 3 higher would be obtained without fast rotators. A stronger effect is predicted at lower metallicities where the two different scenarios would lead to differences of about 60.

If the first stars were confirmed to be fast rotators, achieving initial rotational velocities of about 600 – 800 km s^{-1} , the impact on several other research areas will be enormous. The high CNO content on the surface of very metal-poor fast rotators could be a key to produce the cooling needed to form low-mass stars, even at the very first phases of the Universe. A profound impact is to be expected on the progenitors of long-duration gamma-ray bursts (Yoon et al. 2006; Hirschi 2007). Stars born with such high velocities may have their supernovae explosion affected by the fast rotation of the core.

Our results on the CNO nucleosynthesis are also crucial in interpreting the emission line spectra of star-forming galaxies at high redshifts. For instance Pettini et al. (2002) measured abundances of several elements in the ambient interstellar medium of the Lyman break galaxy MS1512-cB58 and showed that, while O and other α -elements have already reached $\sim 2/5$ of their solar values, the production and mixing within the ISM of Fe-peak and N apparently lagged behind, indicating an age of a few tenths of Myrs (Matteucci & Pipino 2002). Although this galaxy has a much higher metallicity (~ -0.8) than the ones considered in the present work, it is interesting to notice that if the timescale

for the release of primary N changes (as it would be the case if fast rotators play a significant role), the conclusions drawn from high- z emission line observations can be affected.

In addition to nitrogen and carbon, fast rotation also affects other elements, such as ^{22}Ne , ^{25}Mg , ^{26}Mg , or the weak s-process elements, to cite only a few of them. The discussion of these other elements will appear in a future paper.

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Evolution towards the critical limit and the origin of Be stars

S. Ekström, G. Meynet, A. Maeder, and F. Barblan

Geneva Observatory, University of Geneva, Maillettes 51, 1290 Sauverny, Switzerland
e-mail: Sylvia.Ekstrom@obs.unige.ch

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ABSTRACT

Context. More and more evidence leads to considering classical Be stars as rotating close to the critical velocity. If so, then the question that arises is the origin of this high surface velocity.

Aims. We determine which mechanisms accelerate the surface of single stars during the main sequence evolution. We study their dependence on the metallicity and derive the frequency of stars with different surface velocities in clusters of various ages and metallicities.

Methods. We have computed 112 stellar models of four different initial masses between 3 and 60 $M_{\odot}$, at four different metallicities between 0 and 0.020, and with seven different values of the ratio $\Omega/\Omega_{\text{crit}}$ between 0.1 and 0.99. For all the models, computations were performed until either the end of the main sequence evolution or until the critical limit was reached.

Results. The evolution of surface velocities during the main sequence lifetime results from an interplay between meridional circulation (bringing angular momentum to the surface) and mass loss by stellar winds (removing it). The dependence on metallicity of these two mechanisms plays a key role in determining, for each metallicity, a limiting range of initial masses (spectral types) for stars able to reach or at least approach the critical limit. Present models predict a higher frequency of fast rotating stars in clusters with ages between 10 and 25 Myr. This is the range of ages where most of Be stars are observed. To reproduce the observed frequencies of Be stars, it is necessary to first assume that the Be star phenomenon already occurs for stars with $v/v_{\text{crit}} \geq 0.7$ and, second, that the fraction of fast rotators on the zero-age main sequence is higher at lower metallicities. Depending on the stage at which the star becomes a Be star, it may present either larger or less enrichments in nitrogen at the surface.

Key words. stars: evolution – stars: rotation – stars: emission-line, Be

1. Introduction

The aim of the present work is to study how the surface velocity varies as a function of time during the main sequence (MS) evolution for stars of various initial masses, metallicities, and rotation. Although predictions of the evolution of the surface velocity can already be found in the literature (see Heger & Langer 2000; Meynet & Maeder 2000, 2005), the published results lack homogeneity in their physical ingredients and cover ranges of initial velocities that are too narrow to provide valuable answers to the following questions:

- When is very fast rotation encountered in the course of the evolution of massive single stars in the MS phase?
- What is the percentage of stars rotating faster than a given value of $v_{\text{eq}}/v_{\text{crit}}$ in stellar clusters of various ages and metallicities?
- What are the expected chemical enrichments at the surface of these very fast rotating stars?
- What are the initial conditions required for single stars to reach the critical velocity limit (defined as the rotational velocity such that the centrifugal acceleration balances the gravity at the equator exactly)?
- Do these conditions depend on the mass and metallicity, and if so, how?

To know which stars can encounter the critical limit during the MS evolution is interesting in at least two astrophysical contexts: first, the study of Be stars; second, the very metal-poor stars representative of the stellar generations in the early Universe. Let us briefly describe the link with these two research areas.

1.1. Be stars and the critical limit

It is now widely accepted that the “Be phenomenon” is linked to fast rotation. Indeed, classical Be stars are B-type stars that exhibit line emission over the photospheric spectrum. This emission probably arises from an outwardly moving equatorial disk (Porter & Rivinius 2003) whose formation is (at least in part) due to the fast rotation of the star. If the star is rotating with $v_{\text{eq}}/v_{\text{crit}} \sim 0.95$, the additional velocity needed to launch material into orbit is close to the speed of sound in the outer layers (Townsend et al. 2004). Therefore many instabilities, such as non-radial pulsations, begin to be effective for orbital ejection (Owocki 2004). Very interesting is that recent interferometric observations strongly support the idea that these stars are very fast rotators. The oblateness of the Be star Achernar (Domiciano de Souza et al. 2003) indeed suggests that the star is rotating near the critical limit, and Meilland et al. (2007) have for the first time obtained direct evidence that the disk around the Be star α Arae is in Keplerian rotation. The wind geometry is compatible with a thin disk wind and a polar-enhanced wind. These authors also find that α Arae is rotating very close to its critical rotation. Even the mystery behind the β Ceph case seems to be elucidated: β Ceph is a known Be star but also a known slow rotator, thus challenging the fast rotation scenario. However, β Ceph is a multiple system. Schnerr et al. (2006) have shown that β Ceph, the slow rotator, produces most of the light but is not responsible for the emission lines. The emission lines emanate from a much fainter companion, the Be star, which they consider as probably fast rotating.

If the above observations do support the view that Be stars are at the critical limit, there are still some doubts that all Be stars are indeed at this limit. For instance, on the basis of the observed distribution of the $v \sin i$ for Be stars, Cranmer (2005) concludes that, while late type Be stars could be at the critical limit, early Be type stars might rotate at only 40% of their critical velocity. Let us note that the interpretation of the observations is stained with some theoretical uncertainties. A good example is the way the deduced velocities are affected because most of the radiative flux emanates from the polar regions for a fast rotating star (von Zeipel 1924). The low-velocity regions near the pole will thus have a stronger impact on the spectrum than the dimmer faster regions near the equator. This implies that the rotational velocity of very fast rotators has probably been systematically underestimated (Collins 2004; Townsend et al. 2004). Cranmer (2005) accounts for this effect in his work, but the problem is complex enough that the results are uncertain. Whether at the critical limit or not, Be stars represent wonderful laboratories for studying the effects of extreme rotation.

1.2. Very metal poor stars and the critical limit

The link between metal-free or very metal-poor stars and the question of the evolution of the surface velocity certainly deserves a lot of attention, since it might deeply affect the evolution of these stars. Let us recall that the radiation-driven stellar winds at low metallicity are much weaker than at high metallicity, thus it is commonly accepted that mass loss for very metal-poor stars is very inefficient. This might not be true, however, when rotation is taken into account in the models for at least two reasons (see Meynet et al. 2006b, to get a more detailed discussion of these points). First and somewhat paradoxically, since the stars are losing a very little amount of mass by radiatively driven stellar winds, they have more chance losing a large amount of mass by rotational ejection during the MS evolution. Indeed, because of the low line-driven winds, they will not be able to lose their angular momentum. They are thus more prone to reach the critical limit during their evolution (Maeder & Meynet 2001; Meynet et al. 2006a). At the critical limit, as in the Be case, stellar matter is launched into an equatorial disk that will eventually dissipate by radiative effects. In that case, the material will be lost by the star. If correct, such an evolution would thus promote mass loss through a mechanical wind triggered by rotation. This “mechanical wind”, as stressed above, appears to be more significant in metal-poor regions where the radiation-driven stellar winds are expected to be very low. Second, rotational mixing enhances the surface CNO content, which will lead to some increase in the outer layers opacities. This may revive the line-driven winds and lead to strong mass loss.

Such mass loss, either triggered by rotational ejection or through the enhancement of the CNO content of the outer layers, has (if realised) significant consequences on the evolution of the first stellar generations, as well as on their nucleosynthesis (Ekström et al. 2006; Hirschi 2007; Decressin et al. 2007). Also such mass loss may be significant in the process leading to the formation of a collapsar, which is the most favoured model for the progenitors of the long gamma ray bursts (Woosley 1993; Yoon & Langer 2005; Woosley & Heger 2006; Meynet & Maeder 2007).

In the present work, we use rotating stellar models to address the questions of

- The possible origin of Be stars from a purely theoretical point of view: can models reproduce the way the number of

Be stars varies with the metallicity or with the age? Is their number high enough to account for the observed number of Be stars (supposing that Be stars are star rotating with values of $v_{\text{eq}}/v_{\text{crit}}$ higher than a given value)?

- The behaviour of rotating stars at very low or zero metallicity: how does the lack of metals influence the evolution of the equatorial velocity?

Of course, the answers given to these questions will strongly depend on the stellar models used. However, we think that this is worth to be done, because the stellar models we used were computed with physical ingredients that made predictions consistent with many observational features of massive stars (see Meynet & Maeder 2005, and the references therein on the series of papers by the Geneva group on rotating models), and thus it is interesting to explore what these models say about fast-rotating stars in more detail. For very fast rotation, the present models probably still need to be improved by incorporating additional physics (e.g. pulsation), and we hope that the results obtained here will guide us towards improvements in the future.

The paper is organised as follow. In Sect. 2 the physical ingredients of the models are presented. We also recall some basic relations from the Roche model useful for discussing some properties of our models on the ZAMS. In Sect. 3, we present various characteristics of our models on the ZAMS as a function of the initial mass, velocity, and metallicity. The evolution of the models on the MS is presented in Sect. 4. The question of the initial conditions required to reach a given value of $v_{\text{eq}}/v_{\text{crit}}$ during the MS phase is the subject of Sect. 5. The frequency of stars with velocities exceeding a given value of $v_{\text{eq}}/v_{\text{crit}}$ is presented in Sect. 6. The sensitivity of the predicted frequencies to various hypotheses made in this work are discussed in Sect. 7. Possible links with observational features of Be stars will be presented in Sect. 8. Conclusions and future perspectives are the subject of Sect. 9.

Let us mention that electronic tables are available for all our ZAMS and evolutionary models at <http://obswww.unige.ch/Recherche/evol/Critical-limit-and-Be-stars>

2. Physics of the models

2.1. Input physics

Since the detailed description of the physics of the models has already been presented in detail in previous papers (see references below), we shall keep the discussion very short here. The physics of rotation (transport of the angular momentum and of the chemical species) are treated as in Meynet & Maeder (2005). These models are based on the theory of the transport mechanisms induced by rotation proposed by Zahn (1992) and further complemented by the works of Talon & Zahn (1997) and Maeder & Zahn (1998).

We computed the evolution of five different masses, $M = 1, 3, 9, 20, 60 M_{\odot}$, at four different metallicities, $Z = 0.020, 0.002, 0.00001, 0$. For each combination of $M-Z$, we computed models with seven different initial rotational rates, $\omega = \Omega/\Omega_{\text{crit}} = 0.10, 0.30, 0.50, 0.70, 0.80, 0.90, 0.99$. The 3 to $60 M_{\odot}$ models were evolved until either the beginning of core helium burning or reaching the break-up limit. For the $1 M_{\odot}$ models, only the ZAMS models are presented.

The initial composition of the models for the metallicities $Z = 0.020, 0.002, 0.00001$, and 0 are respectively $X = 0.7050, 0.7545, 0.7599$, and 0.76 and $Y = 0.275, 0.2435, 0.24002$, and 0.24. For the heavy elements, we used the same mixture

as the one used in the OPAL opacity tables: the solar mixture of Grevesse & Noels (1993) for $Z = 0.020$ and 0.002 and the α -enhanced mixture of Weiss (1995) for $Z = 0.00001$ ¹. The nuclear reaction rates are from the NACRE database².

The convection was treated according to the Schwarzschild criterion. The size of the convective core is increased by a moderate overshooting of $0.1 H_p$.

The 3 $M_\odot$ models were computed without mass loss, and the 9 $M_\odot$ models were computed with the de Jager et al. (1988) prescription. For the higher mass models we used the prescription proposed by Kudritzki & Puls (2000, hereafter KP00) and de Jager et al. (1988) in the temperature domain not covered by the first prescription. The effects of rotation on the mass loss rates are taken into account as explained in Maeder & Meynet (2000a).

2.2. The Roche model

In all the derivations, we use the Roche model; i.e. we approximate the gravitational potential by GM_r/r where M_r is the mass inside the equipotential with a mean radius r . The radius r that labels each equipotential is defined by $r = (V_r/(4/3\pi))^{1/3}$ where V_r is the volume (deformed by rotation) inside the equipotential considered³.

In the frame of the Roche model and for a conservative rotation law (i.e. such that the centrifugal force can be derived from a potential), the shape of a meridian at the surface of an equipotential is given by couples of r and θ values satisfying the following equation:

$$\frac{GM_r}{r} + \frac{1}{2}\Omega_r^2 r^2 \sin^2 \theta = \frac{GM_r}{r_p}, \quad (1)$$

where r is the radius at colatitude θ , Ω_r the angular velocity at that radius, M_r the mass inside the considered equipotential surface, and r_p the radius at $\theta = 0$. Thus the shape of a given equipotential is determined by three parameters M_r , Ω_r , and r_p . The first two, M_r , and Ω_r are independent variables. The third one is derived from the first two and the equations of stellar structure. Setting $x = \left(\frac{GM_r}{\Omega_r^2}\right)^{-1/3} r$, one can write Eq. (1) (see Kippenhahn & Thomas 1970)

$$\frac{1}{x} + \frac{1}{2}x^2 \sin^2 \theta = \frac{1}{x_p}. \quad (2)$$

With this change of variable, the shape of an equipotential is uniquely determined by only one parameter x_p .

With M the total mass of the star and Ω the angular velocity at the surface, Eqs. (1) or (2) give the shape of a meridian line at the stellar surface. Setting

$$f = \frac{R_e}{R_p}, \quad (3)$$

where R_e is the equatorial radius, one easily obtains from Eq. (1) that

$$R_p = \left(\frac{GM}{\Omega^2}\right)^{1/3} \left(\frac{2(f-1)}{f^3}\right)^{1/3} = \left(\frac{GM}{\Omega^2}\right)^{1/3} x_p. \quad (4)$$

The above equation relates the inverse of the oblateness f to R_p .

As recalled above, the critical angular velocity, Ω_{crit} , corresponds to the angular velocity at the equator of the star such that the centrifugal force balances the gravity exactly. The classical critical angular velocity or the Ω -limit (to distinguish it from the $\Omega\Gamma$ -limit as defined in Maeder & Meynet 2000a) in the frame of the Roche model is given by

$$\Omega_{\text{crit}} = \left(\frac{2}{3}\right)^{3/2} \left(\frac{GM}{R_{p,\text{crit}}^3}\right)^{1/2}, \quad (5)$$

where $R_{p,\text{crit}}$ is the polar radius when the surface rotates with the critical velocity. The critical velocity is given by

$$v_{\text{crit}} = \left(\frac{2}{3}\frac{GM}{R_{p,\text{crit}}}\right)^{1/2}. \quad (6)$$

By replacing Ω in Eq. (4) by $(\Omega/\Omega_{\text{crit}}) \cdot \Omega_{\text{crit}}$ and using Eq. (5), one obtains a relation between R_p , f , and $R_{p,\text{crit}}$,

$$R_p = \frac{3}{2}R_{p,\text{crit}} \left(\frac{\Omega_{\text{crit}}}{\Omega}\right)^{2/3} \left(\frac{2(f-1)}{f^3}\right)^{1/3}. \quad (7)$$

Thus one has

$$\frac{\Omega}{\Omega_{\text{crit}}} = \left(\frac{3}{2}\right)^{3/2} \left(\frac{R_{p,\text{crit}}}{R_p}\right)^{3/2} \left(\frac{2(f-1)}{f^3}\right)^{1/2}. \quad (8)$$

With a good approximation (see below), one finds that $R_{p,\text{crit}}/R_p \approx 1$ and therefore

$$\frac{\Omega}{\Omega_{\text{crit}}} \approx \left(\frac{3}{2}\right)^{3/2} \left(\frac{2(f-1)}{f^3}\right)^{1/2}. \quad (9)$$

This equation is quite useful since it allows determination of Ω_{crit} from quantities obtained with a model computed for any $\Omega < \Omega_{\text{crit}}$; however, the closer to the critical value, the better. This is not the case of Eq. (7) which involves $R_{p,\text{crit}}$ whose knowledge can only be obtained by computing models at the critical limit. Below, we see to what extent the use of expression (9) provides a good approximation of Ω_{crit} .

Setting v the velocity at the equator, one finds that

$$\frac{v}{v_{\text{crit}}} = \frac{\Omega R_e}{\Omega_{\text{crit}} R_{e,\text{crit}}} = \frac{\Omega}{\Omega_{\text{crit}}} \frac{R_e}{R_p} \frac{R_p}{R_{p,\text{crit}}} \frac{R_{p,\text{crit}}}{R_{e,\text{crit}}}, \quad (10)$$

where $R_{e,\text{crit}}$ is the equatorial radius when the surface rotates with the critical velocity. Using Eq. (7) above, and the fact that in the Roche model $R_{p,\text{crit}}/R_{e,\text{crit}} = 2/3$, one obtains

$$\frac{v}{v_{\text{crit}}} = \left(\frac{\Omega}{\Omega_{\text{crit}}}\right)^2 \frac{2(f-1)}{f^3}. \quad (11)$$

An interesting quantity is the ratio of the centrifugal acceleration a_{cen} to the gravity g_e at the equator

$$\frac{a_{\text{cen}}}{g_e} = \frac{\Omega^2 R_e^3}{GM} = \left(\frac{\Omega}{\Omega_{\text{crit}}}\right)^2 \left(\frac{2}{3}\right)^3 f^3 \left(\frac{R_p}{R_{p,\text{crit}}}\right)^3, \quad (12)$$

¹ See <http://www-phys.llnl.gov/Research/OPAL/opal.html> for exact references.

² See http://pntpm.ulb.ac.be/Nacre/barre_database.htm

³ ZAMS models are supposed to rotate like solid bodies, the centrifugal force is conservative, and it is possible to define equipotential surfaces. This is no longer the case for shellular rotation. In that case, however, the shape of the isobaric surfaces are given by the same expression as the one giving the shape of the equipotentials for solid body rotation provided some changes in variables are performed (see Meynet & Maeder 1997).

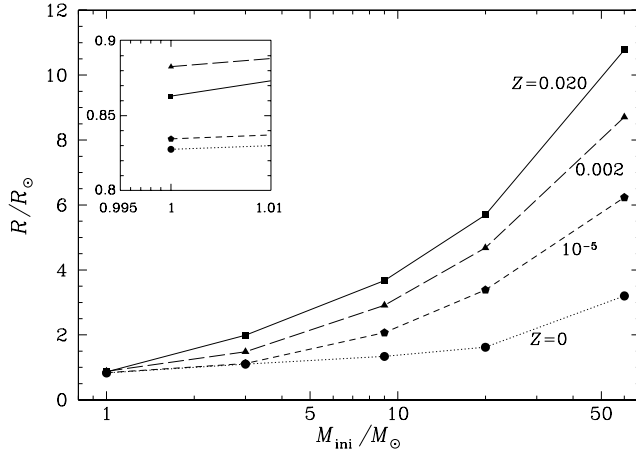


Fig. 1. Variations in the radius as a function of initial mass, for various metallicities for non-rotating stars. The inset zoom shows the small difference in radii in the case of the $1 M_{\odot}$ models.

where we use Eq. (5) and divide/multiply by $R_{p,\text{crit}}^3$. Replacing $R_{p,\text{crit}}/R_p$ by its expression deduced from Eq. (7), we obtain

$$\frac{a_{\text{cen}}}{g_e} = 2(f - 1). \quad (13)$$

We can check that at the critical limit, when $f = 3/2$, then $a_{\text{cen}} = g_e$.

3. Basic parameters on the zero age main sequence

We consider that a star has arrived on the ZAMS, when a mass fraction of 0.003 of hydrogen has been transformed into helium at the centre. At this stage, the star is supposed to have a solid body rotation. We present here the stellar radii, the total luminosity, the variation with the colatitude of the effective temperature, and the moment of inertia of the models. We then show the relations between the initial rotation and the angular momentum content, $\nu_{\text{ini}}/\nu_{\text{crit}}$ and $\Omega_{\text{ini}}/\Omega_{\text{crit}}$.

3.1. Radii, luminosities, and effective temperatures

Considering first the case without rotation, we can see in Fig. 1 that there is a relation between radius and mass of the form $R \sim M^{0.6}$ at standard metallicity. This relation becomes shallower with decreasing Z , down to $R \sim M^{0.3}$ at $Z = 0$. This comes mainly from two causes: 1) first, at lower metallicity, the opacities are lower; 2) the CNO elements are less abundant, or even absent in metal-free stars. To compensate for the energy radiated by the surface when smaller amounts of CNO elements are present, the central regions must extract energy from nuclear reactions at a higher temperature regime. To reach this regime, the star contracts more during the pre-MS phase and is thus more compact on the ZAMS. For example, in the case of the $60 M_{\odot}$ models, the radius at $Z = 0$ is reduced by a factor 3.36 in comparison with the radius at standard metallicity. The $1 M_{\odot}$ models are running only on pp -chains and show very small differences in their radii: only about 5% between $Z = 0.020$ and $Z = 0$ (see the inset zoom in Fig. 1).

In Fig. 2, we present the variation in the polar radius R_p as a function of $\omega = \Omega/\Omega_{\text{crit}}$, normalized to its non-rotating value, for the standard metallicity models. We see that increasing ω

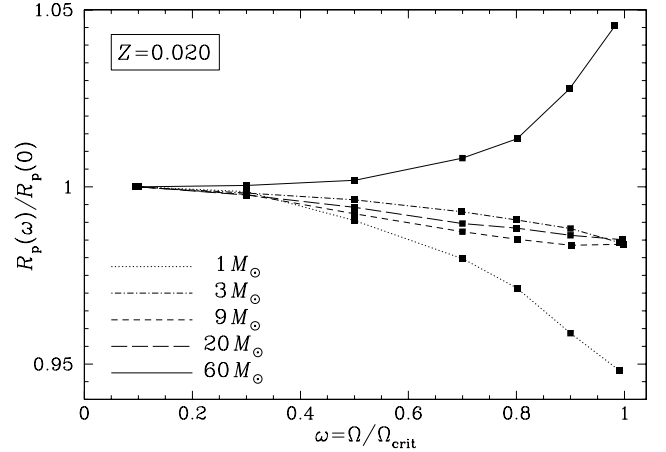


Fig. 2. Variations in the polar radius as a function of the ratio $\omega = \Omega/\Omega_{\text{crit}}$, normalized to the non-rotating value, for various masses at standard metallicity.

tends to decrease the radius in the case of the 1 to $20 M_{\odot}$, a trend that can be explained as follows: rotation plays the role of a sustaining force in the star. Thus, the central temperature and density in rotating stars are lower, subsequently decreasing the luminosity and the radius. The change of the polar radius for the $1 M_{\odot}$ model amounts to about 5%. In the case of the 3 , 9 , and $20 M_{\odot}$ models, the assumption of $R_{p,\text{crit}}/R_p = 1$ (Sect. 2.2) is absolutely valid, since the decrease only amounts to 1.5%. Why are the nearly completely radiative stars such as the $1 M_{\odot}$ stellar models more deformed for a given value of ω than stars with a significant convective core? It comes from the fact that the density gradient in convective zones is shallower than in radiative ones, allowing more mass to be contained in a given volume. This strengthens gravity and the resistance to deformation.

For the $60 M_{\odot}$ models, the change in the polar radius has the same amplitude as for the $1 M_{\odot}$ model but in the opposite direction, showing an increase of about 4.5%. How such a difference of behaviour between the lowest and highest mass models might be explained? In the high-mass star range, radiation pressure contributes more to the total pressure. It is the highest in the polar regions where the effective gravity (gravity decreased by the centrifugal acceleration) is the highest (von Zeipel theorem). A look at Fig. 6 shows that $T_{\text{eff}}(\text{pole})$ is higher when ω is higher (10% between $\omega = 0.1$ and $\omega = 0.99$), and therefore $P_{\text{rad}} \sim T^4$ is stronger. This tends to inflate the polar radius in massive stars.

Rotation deforms the star, the ratio R_e/R_p increasing with ω as shown in Fig. 3. We have plotted the simplified relation obtained by Eq. (9), assuming $R_{p,\text{crit}}/R_p = 1$ (continuous line), as well as the relation obtained using Eq. (8) applied to two different masses (1 and $60 M_{\odot}$) at $Z = 0.02$. In this case, we observe the direct effect of the behaviour seen in Fig. 2. For the $1 M_{\odot}$, the decrease in R_p with increasing ω makes f higher than the continuous line, joining it only at the critical limit since $R_{e,\text{crit}}/R_{p,\text{crit}}$ is always equal to 1.5 in the Roche model frame, while the inverse behaviour of the $60 M_{\odot}$ leads to the inverse result. Let us note also that in all cases, the equatorial radius becomes longer than 1.1 times the polar one only for $\omega \geq 0.75$.

In Fig. 4, we show the behaviour of the luminosity $L(\omega)$ of our $60 M_{\odot}$ models as a function of ω for the different metallicities considered. The value of the luminosity is normalized to its non-rotating value. One sees that the luminosity decreases by at most 5 to 10% when rotation increases up to the critical

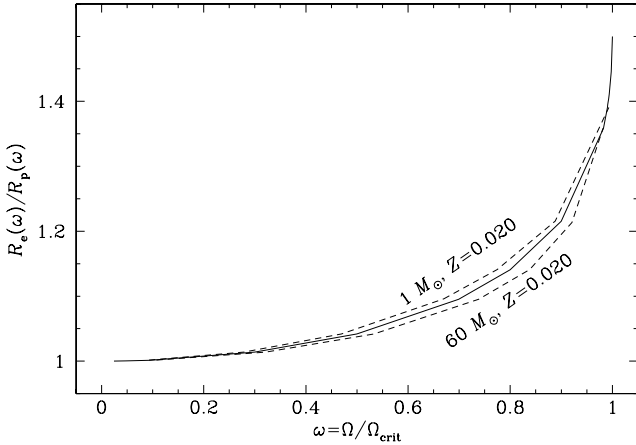


Fig. 3. Variation in the ratio $R_e(\omega)/R_p(\omega)$ (equatorial over polar radius) as a function of $\omega = \Omega/\Omega_c$. The continuous line shows the relation given by Eq. (9), assuming $R_{p,crit}/R_p = 1$. The dashed lines show the relations for the $Z = 0.020$ models with $1 M_\odot$ and $60 M_\odot$ using Eq. (8).

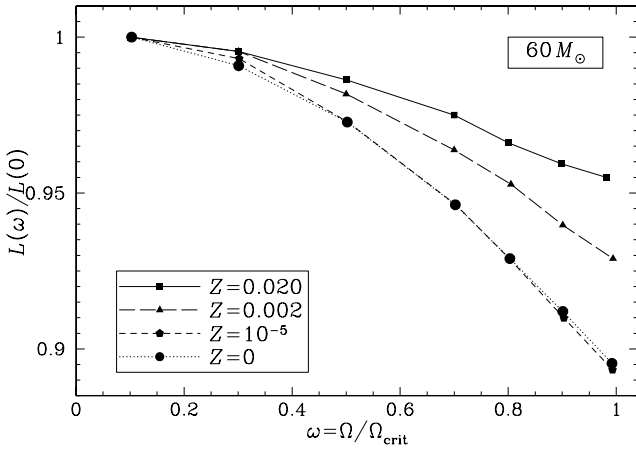


Fig. 4. Variations in the total luminosity as a function of ω , normalized to the non-rotating value, for $60 M_\odot$ at various metallicities.

limit. As mentioned above, this is because the centrifugal acceleration helps in sustaining the star against the gravity and allows the luminosity to take a value corresponding to a non-rotating lower initial mass star. We can see that the effects of rotation are stronger at lower metallicity. This is related to the fact that low metallicity stars are more compact. The regions having a temperature high enough to undergo nuclear burning are more extended in the star and thus overlap regions that are affected more by rotation.

Figure 5 is the same as Fig. 4, but this time by fixing the metallicity ($Z = 0.020$) and varying the mass. The $1 M_\odot$ is the most affected by rotation, with a decrease of 22.5% in its luminosity at break-up. The higher masses only show a decrease of 4.5 to 7%. This again is related to the energy production through pp -chains in the $1 M_\odot$ model being more extended in the interior of the star. As above, a non-negligible part of the luminosity is produced in regions that may suffer from the effects of rotation.

Figure 6 shows the variation with colatitude in the effective temperature at the surface of a $20 M_\odot$ star for various rotation rates at standard metallicity. The ratio $(T_{\text{eff}}(\text{pole}) - T_{\text{eff}}(\text{equator}))/T_{\text{eff}}(\text{equator})$ becomes superior to 10% only for

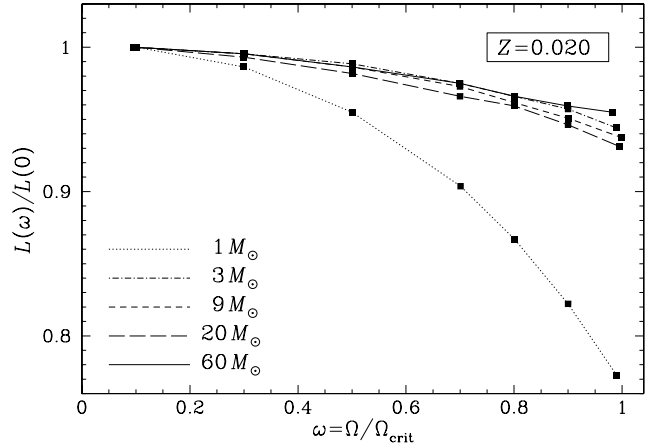


Fig. 5. Variations in the total luminosity as a function of ω , normalized to the non-rotating value, for various masses at standard metallicity.

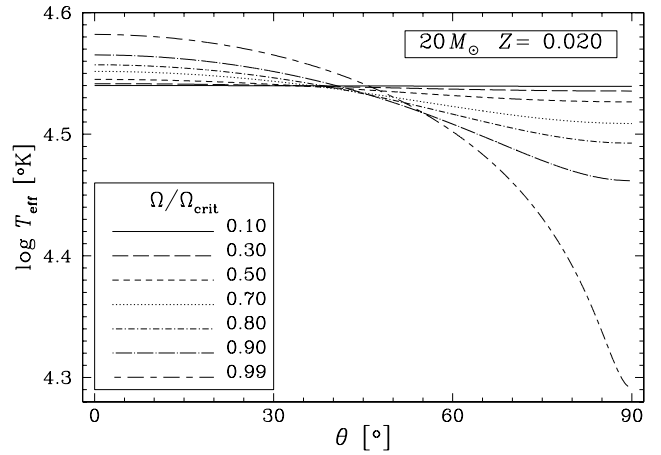


Fig. 6. Variations in the effective temperature T_{eff} as a function of colatitudinal angle θ , for the various values of rotational rates, in the $20 M_\odot$ model at standard metallicity.

$\omega > 0.7$. Near break-up, the effective temperature of the polar region is about a factor two higher than that of the equatorial one. We may compare this result with the recent work of Espinosa Lara & Rieutord (2007) who have studied the structure and dynamics of rapidly rotating stellar models with a two-dimension code. They computed a model with $\omega = 0.82$ for which the ratio $(T_{\text{eff}}(\text{pole}) - T_{\text{eff}}(\text{equator}))/T_{\text{eff}}(\text{equator})$ is 14%. Our closer model ($\omega = 0.80$) shows a slightly higher ratio of 16%, though we should expect a lower value due to the lower ω . Let us note that the 2D simulation has been carried out within a spherical box, which according to the authors, may soften the latitudinal variations. It would be interesting to check how much of the discrepancy remains were the 2D computation done without such a container.

3.2. Inertia, angular momenta, and rotational energies

Figure 7 presents the variations in the moment of inertia with the initial mass of the models. These variations show almost no dependence on ω , so only the case of $\omega = 0.1$ is plotted. The moments of inertia have been obtained by summing the contributions of each shell i of the model: $I = \sum_i (2/3) r_i^2 dm_i$, where r_i

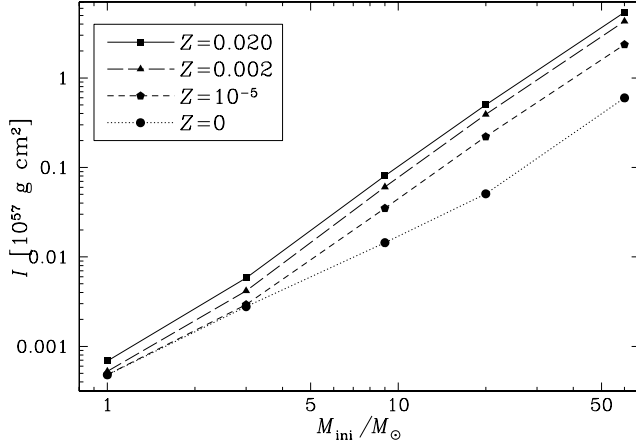


Fig. 7. Variations in the moment of inertia as a function of initial mass for various metallicities (models with $\Omega/\Omega_{\text{crit}} = 0.1$).

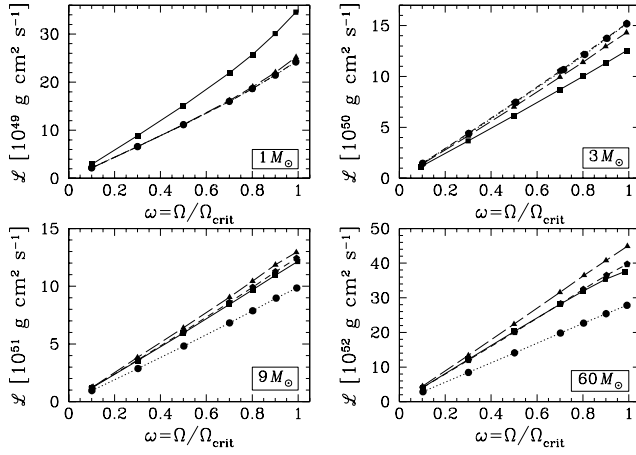


Fig. 8. Variations in the total angular momentum content of the models on the ZAMS as a function of initial rotation rate $\omega = \Omega/\Omega_{\text{crit}}$ for various metallicities, plotted according to the initial mass of the models. The coding for Z is the same as in Fig. 7.

is the mean radius of the shell, i.e. the radius in $P_2(\cos \theta) = 0$. Thus, r_i accounts for the rotational distortion up to the second-order Legendre polynomial, and the above way of calculating the moment of inertia is correct at this level of approximation. Let us mention that rotation deforms only the outermost layers of the model, which contain very little mass. This is why the moment of inertia does not change much with or without rotation.

We note that, at a given metallicity, the moment of inertia I increases with the initial mass. The increase amounts to about 4 orders of magnitudes between 1 and 60 $M_\odot$. As can be seen in Fig. 7, the relation between I and M is almost linear on a logarithmic scale, with a slope of 2.2 for the non-zero metallicities and of 1.7 for $Z = 0$. The difference in the slopes can be linked to the difference in the way the radii increase with increasing mass. From the mass-radius relations obtained in Sect. 3.1 ($R \sim M^{0.6}$ at standard metallicity and $R \sim M^{0.3}$ at $Z = 0$), we indeed expect a relation $I \sim M^{2.2}$ and $I \sim M^{1.6}$ at $Z = 0.02$ and 0, respectively. For a given initial mass, the moments of inertia are smaller at lower metallicities, as expected due to smaller radii at lower Z .

The variation in the total angular momentum content ($\mathcal{L} = \sum_i I_i \Omega_i$) as a function of ω is shown in Fig. 8. Note that the highest point on the right of the figures gives an estimate of the

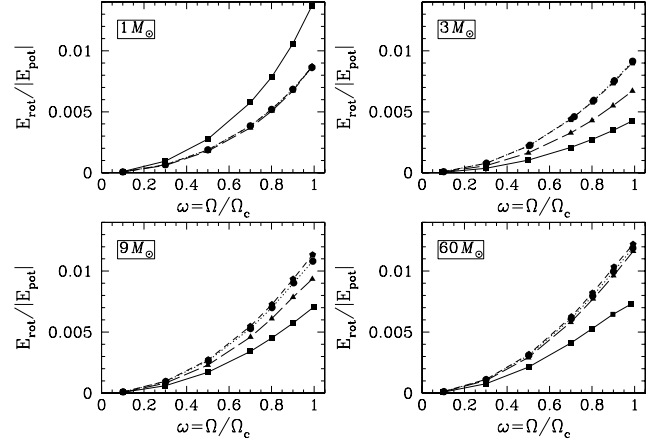


Fig. 9. Variations in the ratio between rotational and potential energy as a function of the ratio $\omega = \Omega/\Omega_{\text{crit}}$ for various metallicities, plotted according to the initial mass of the model. The coding for Z is the same as in Fig. 7.

maximum angular momentum content on the ZAMS (for solid body rotation). Since the moment of inertia of the star does not depend much on the rotation velocity, and since we suppose solid body rotation on the ZAMS, one obtains quasi linear relations between $\mathcal{L}$ and ω . We note that the metallicity dependence remains modest at least in the range of metallicities between 0.00001 and 0.020. In the 9 to 60 $M_\odot$ range, the value of ω being kept constant, there is at first a slight increase in the angular momentum content when decreasing the metallicity (compare the curve with the black squares for the $Z = 0.020$ models with the curve with the triangles corresponding to 0.002 models). Then for still lower metallicities, the angular momentum decreases with the metallicity (the curve for the $Z = 0.00001$ models overlaps the $Z = 0.020$ models) reaching its lower values for the metal-free stellar models.

In the mass range between 9 and 20 $M_\odot$, stars having the same angular momentum content on the ZAMS lead to a very similar value for ω , whatever their metallicity between 0.00001 and 0.02. Only if the metallicity is zero would the value of ω corresponding to the same angular momentum content be much higher. As a numerical example, a 20 $M_\odot$ on the ZAMS with $\mathcal{L} = 0.3 \times 10^{53} \text{ cm}^2 \text{ g s}^{-1}$ has a ratio $\omega = 0.5$ when Z ranges between 0.00001 and 0.020. This angular momentum content corresponds to $\omega = 0.8$ when $Z = 0$.

The rotational kinetic energy ($E_{\text{rot}} = \sum_i (1/2) I_i \Omega_i^2$) expressed as a fraction of the binding energy ($E_{\text{pot}} = -G \sum_i (M_i/r_i) dm_i$) is shown in Fig. 9. The rotational energy amounts to at most one percent of the binding energy. This is consistent with the well known fact that the effects of the centrifugal acceleration remains quite modest on the hydrostatic structure of the stellar interior, even when the surface rotates with a velocity near the critical one. For a 60 $M_\odot$ stellar model at $Z = 0.020$, the binding energy is $1.8 \times 10^{51} \text{ erg}$, while its $Z = 0$ counterpart has a binding energy equal to $5.3 \times 10^{51} \text{ erg}$, i.e. about a factor 3 higher. Let us recall for comparison that the binding energy of a neutron star is of the order of 10^{53} erg , about two orders of magnitude greater.

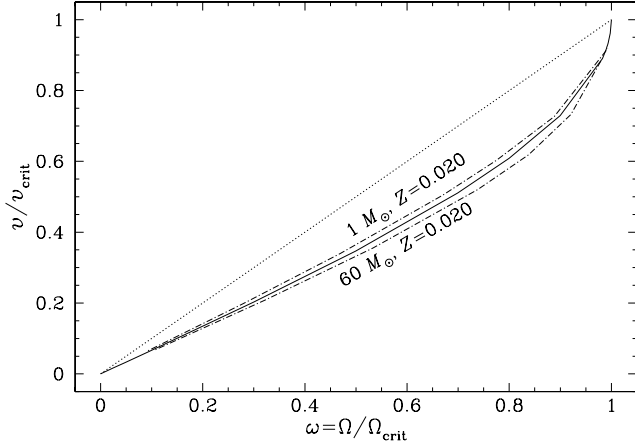


Fig. 10. Relation between v/v_{crit} and $\omega = \Omega/\Omega_{\text{crit}}$ obtained in the frame of the Roche model. The continuous line is obtained assuming $R_{p,\text{crit}}/R_p = 1$ (see text and Eqs. (9) and (11)). The dot-dashed lines show the relations for the $Z = 0.020$ models with 1 and $60 M_{\odot}$ using Eq. (8). The dotted line is the line of slope 1.

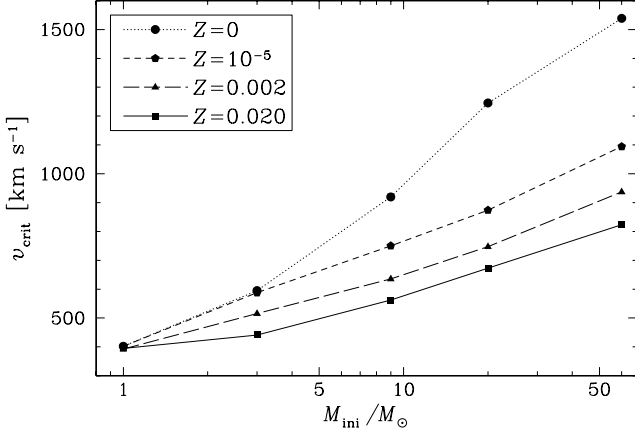


Fig. 11. Variations in the critical equatorial velocity on the ZAMS as a function of the initial mass, for various metallicities.

3.3. Relations between $\Omega/\Omega_{\text{crit}}$ and v/v_{crit}

The relations between v/v_{crit} and $\Omega/\Omega_{\text{crit}}$ obtained in the frame of the Roche model (see Eq. (11)) for the 1 and $60 M_{\odot}$ stellar models at $Z = 0.02$ are shown in Fig. 10. In case we suppose that the polar radius remains constant ($R_{p,\text{crit}}/R_p = 1$), then Eq. (9) can be used and one obtains a unique relation between $\Omega/\Omega_{\text{crit}}$ and v/v_{crit} , independent of the mass, metallicity, and evolutionary stage considered (continuous line). One sees that the values of v/v_{crit} are lower than that of $\Omega/\Omega_{\text{crit}}$ by at most $\sim 25\%$. At the two extremes, the ratios are of course equal.

3.4. Velocities on the ZAMS: v_{crit} , v

Table 1 gives the angular velocity, the period, the polar radius, and the equatorial velocity at the critical limit on the ZAMS for the various models. Let us note that, for obtaining the value of Ω_{crit} or of whatever quantities at the critical limit, it is necessary to compute models at the critical limit. Here we have computed models very near, but not exactly at, the critical limit, since at that point numerical singularities are encountered. As explained in Sect. 2.2, using Eq. (9) we could have estimated

Table 1. Angular velocity, rotational period, polar radius, and equatorial velocity at the critical limit on the ZAMS (solid body rotation).

M $M_{\odot}$	Z	Ω_{crit} s^{-1}	P_{crit} h	$R_{p,\text{crit}}$ $R_{\odot}$	v_{crit} km s^{-1}
1	0	4.9E-04	3.6	0.8	400
1	10^{-5}	4.9E-04	3.6	0.8	400
1	0.002	4.6E-04	3.8	0.8	390
1	0.020	4.6E-04	3.8	0.8	395
3	0	5.3E-04	3.3	1.1	595
3	10^{-5}	5.1E-04	3.4	1.1	585
3	0.002	3.4E-04	5.1	1.4	515
3	0.020	2.2E-04	8.1	2.0	440
9	0	6.5E-04	2.7	1.4	920
9	10^{-5}	3.5E-04	4.9	2.0	750
9	0.002	2.2E-04	8.1	2.8	635
9	0.020	1.5E-04	11.7	3.6	560
20	0	7.3E-04	2.4	1.6	1245
20	10^{-5}	2.5E-04	7.0	3.3	870
20	0.002	1.6E-04	11.1	4.6	745
20	0.020	1.2E-04	15.2	5.6	670
60	0	4.6E-04	3.8	3.2	1540
60	10^{-5}	1.7E-04	10.6	6.4	1095
60	0.002	1.0E-04	16.9	8.7	935
60	0.020	7.0E-05	24.9	11.3	820

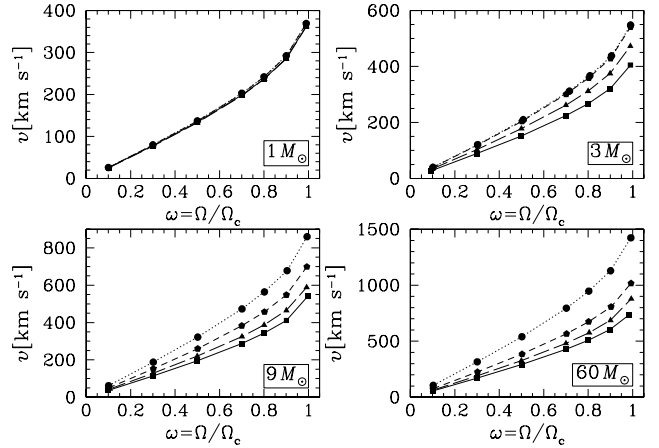


Fig. 12. Variations in the surface equatorial velocity on the ZAMS as a function of initial rotation rate $\omega = \Omega/\Omega_{\text{crit}}$ for various metallicities and masses. The coding for Z is the same as in Fig. 11.

Ω_{crit} from whatever model was computed with a lower initial velocity. Doing this for the whole range of initial masses, metallicities and velocities, we would have obtained values within 10% of those shown in Table 1. For the models between 3 and $20 M_{\odot}$, the error would always be inferior to 3.5% because in this mass range, $R_p(\omega)/R_p(0)$ remains very near 1 (see Fig. 2) and Eq. (9) is a good approximation of Eq. (8). For the 1 and $60 M_{\odot}$, the errors would be larger, amounting to a maximum of 9.3 and 6.7% respectively, showing the need to use instead Eq. (8) if $\omega \ll 1$. In general the error decreases drastically when ω increases, so to obtain the values given in Table 1, we have used our faster rotating model, deriving the value of Ω_{crit} (using Eq. (9)) with an error less than 0.001%. The critical period, P_{crit} is then simply given by $P_{\text{crit}} = 2\pi/\Omega_{\text{crit}}$. The polar radius is obtained

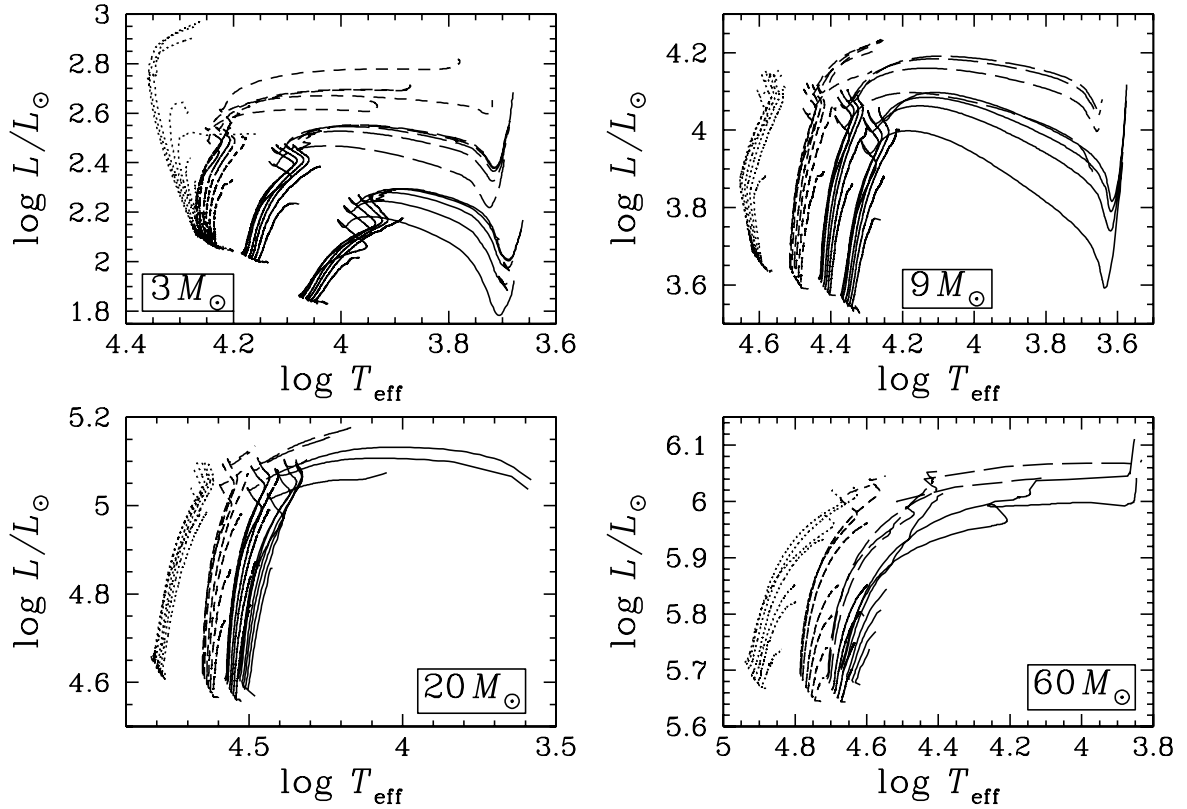


Fig. 13. Evolutionary tracks in the theoretical Hertzsprung-Russell diagrams for all the models computed beyond the ZAMS. The effective temperature corresponds to a surface-averaged effective temperature. The computations are stopped when the star reaches the critical limit or before the beginning of the core He-burning phase.

from Eq. (5) and the critical equatorial velocity is deduced from $v_{\text{crit}} = 1.5 R_{\text{p,crit}} \Omega_{\text{crit}}$.

The variation in the critical velocity as a function of the initial mass and metallicity is shown in Fig. 11. For a given initial mass, the critical velocity is higher at lower Z : e.g. for the $60 M_{\odot}$ model, the critical velocity goes from slightly more than 800 km s^{-1} at $Z = 0.020$ to more than 1500 km s^{-1} at $Z = 0$. It is a consequence of the smaller radii of stars at low metallicity (see Eq. (6)) as can be seen from Fig. 1.

At a given metallicity, the critical velocity becomes higher for higher initial masses. This reflects the fact that the ratio of the mass to the radius increases with the star mass (cf. Sect. 3.1). The critical rotation periods are between 2.4 and 25 h for the whole domain of masses and metallicities explored in the present work. Let us just mention here that, as will be described in Sect. 5, the critical velocity evolves with time, so the values given here are values at the beginning of the star's evolution and should not be used throughout the whole MS phase.

Figure 12 shows the relations between v and ω on the ZAMS for different initial mass models at various metallicities. The corresponding values are also given in Table 2. We see that, for a given value of ω , higher surface velocities are obtained at lower metallicities and for higher initial masses.

4. Evolution on the main sequence

4.1. Hertzsprung-Russell diagram and lifetimes

Evolutionary tracks in the Hertzsprung-Russell diagram for the present models are shown in Fig. 13. The models have evolved

until either reaching Ω_{crit} or just before the onset of central helium burning. For each mass, the effect of reducing the metal content shifts the tracks towards the blue part of the diagram and, for 3 and $9 M_{\odot}$, towards higher luminosities. In each metallicity subgroup, the effect of raising the rotational rate shifts the tracks towards lower luminosities and cooler temperatures on the ZAMS. When the star evolves, rotational mixing continuously refuels the convective core in fresh hydrogen and brings newly produced helium into the radiative zone. The enrichment of the radiative envelope in helium makes the star brighter and bluer by decreasing the opacity. If we look at what happens to the convective core (Fig. 14), we see that at first, a higher rotational rate leads to a smaller convective core, due to the lower central temperature. Then, we see that the decrease in the core is slower because of the refuelling in hydrogen. At the end of the MS, the core is much larger in the fast rotating models. In that respect rotation acts as a core overshoot.

In Table 3 the lifetimes of our stellar models are given, as well as the time at which the critical limit (if reached) is first encountered during the MS phase. For the models that were not pursued until the end of the MS phase, we give an estimate of the MS lifetime (indicated by an asterisk) obtained by inspections of the variation in the central hydrogen mass fraction with time. Note that we suppose that the star, once it reaches the critical limit, remains in the vicinity of this limit. This behaviour has been confirmed by previous computations in which models were pursued beyond the point where they first reached the critical limit (see Ekström et al. 2006). Most of the $60 M_{\odot}$ stellar models encounter the critical limit very early. In that case only

Table 2. The first line indicates the equatorial velocity in km s^{-1} on the ZAMS for each initial mass and metallicity. For the 3, 20, and 60 $M_{\odot}$ stellar mass models, the second line gives the time-averaged value of the velocity over the adjustment period from solid to non-solid rotation at the beginning of the MS phase (see Sect. 4.2).

M	Z	$\Omega/\Omega_{\text{crit}}$							
		0.1	0.3	0.5	0.7	0.8	0.9	0.99	
1	0	26	80	137	203	242	292	369	
1	10^{-5}	26	80	137	202	242	291	369	
1	0.002	25	77	133	197	237	285	362	
1	0.020	26	78	134	199	237	286	362	
3	0	40	121	210	312	367	438	548	
		39	110	179	258	299	342	397	
3	10^{-5}	39	119	204	301	359	429	541	
		38	107	174	249	293	340	391	
3	0.002	34	103	178	262	312	375	473	
		32	92	151	218	255	300	348	
3	0.020	28	89	152	224	267	321	405	
		29	83	137	199	231	269	312	
9	0	62	188	322	473	564	678	860	
		62	171	279	393	462	532	607	
9	10^{-5}	50	152	260	383	457	548	699	
		48	132	220	322	378	444	511	
9	0.002	44	128	220	325	387	465	589	
		40	111	186	269	316	371	421	
9	0.020	37	113	195	287	343	411	541	
		36	102	169	247	291	340	384	
20	0	85	256	434	644	766	918	1147	
		77	214	349	512	584	672	769	
20	10^{-5}	58	177	303	447	532	639	819	
		55	156	263	386	454	535	626	
20	0.002	49	150	258	380	454	545	686	
		45	129	221	323	379	444	515	
20	0.020	45	136	233	344	409	490	639	
		41	118	200	291	342	397	459	
60	0	106	316	540	795	947	1129	1423	
		97	276	476	688	827	952	1140	
60	10^{-5}	75	225	385	566	674	807	1017	
		71	213	364	533	632	751	880	
60	0.002	63	191	326	480	577	687	879	
		57	175	300	440	517	609	708	
60	0.020	56	171	292	428	509	603	732	
		50	157	268	389	458	534	600	

the time of reaching the limit is given. We see that for $\omega = 0.7$ the MS lifetime is increased by 15–23% (depending on the initial mass and metallicity) in comparison with the models with $\omega = 0.1$.

4.2. Evolution of the equatorial velocity

The evolution of the rotation velocities at the stellar surface directly depends, of course, on the expansion/contraction movements of the star. But besides this, it also strongly depends on two factors: the internal mechanism of angular momentum transport (internal coupling) and the mass loss.

An extreme case of internal angular momentum transport is the one that imposes solid body rotation at each moment in the course of the evolution of the star. A strong coupling is then realised between the contracting core and the expanding envelope. The spinning up of the central regions is transmitted to the

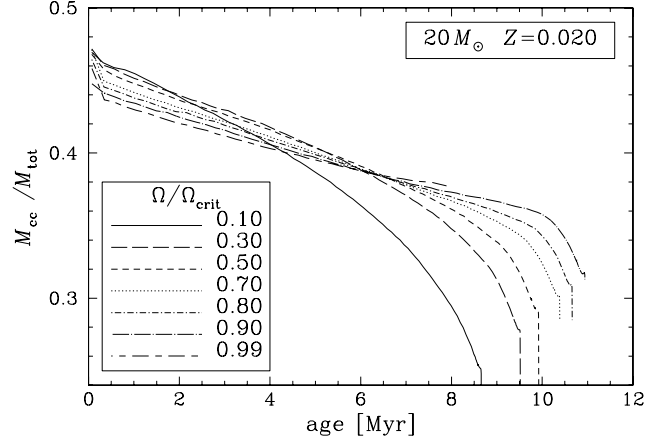


Fig. 14. Evolution in the size of the convective core during MS, for the various values of rotational rate, in the 20 $M_{\odot}$ at standard metallicity.

stellar surface. Thus, during the MS phase, the surface velocity approaches the critical limit and may even reach it, depending on the initial conditions (see Sackmann & Anand 1970; Langer 1997; Meynet et al. 2007). Another extreme case is the one of no transport of angular momentum. Each stellar layer keeps its own angular momentum. The variation in Ω is then simply governed by the local conservation of the angular momentum and, at the surface, $\Omega \sim R^{-2}$. When the radius increases, the surface angular velocity decreases more rapidly than the classical critical angular velocity (see Eq. (5)), thus the star evolves away from the critical limit. In the present models, without magnetic fields, the situation lies between these two extreme cases. A moderate coupling is mainly exerted by meridional circulation, which is more efficient than shear transport, as far as transport of angular momentum is concerned (see e.g. Meynet & Maeder 2000).

The second significant effect governing the evolution of angular momentum is mass loss. The strong mass losses during the MS phase, expected for high-mass stars at high metallicities, decrease the angular momentum of the stars. The strength of this decrease in the angular momentum depends on how matter is ejected. In the extreme case where all the matter would be ejected along narrow jets centred on the rotational axis, nearly no angular momentum would be lost, while if all the matter is ejected near the equator plane, this maximizes the amount of angular momentum lost. Depending on the geometry of the wind, the surface velocity may thus decrease or increase. In the present work, we did not account for the effects of wind anisotropies (which would be strong only for a relatively narrow range of masses and velocities, see Meynet & Maeder 2003), thus, in the present case, strong mass loss always leads to a decrease in the surface velocity.

Figure 15 shows the evolution of the surface velocity expressed as a fraction of the critical velocity during the MS phase for 20 $M_{\odot}$ stellar models. Quite generally the curves can be decomposed into three parts, whose importance depends on the initial rotation and metallicity:

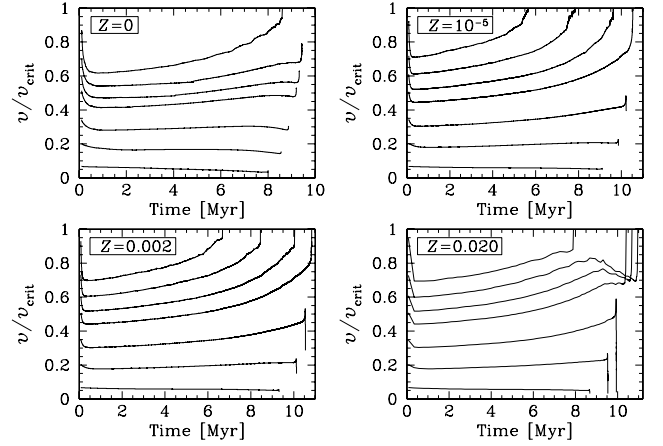
1. At the very beginning, there is a short adjustment period, which lasts for a few percent of the MS lifetime, during which meridional circulation transports angular momentum from the outer parts of the star to the inner ones. This slows down the surface of the star. Then, in the interior, shear turbulence becomes active and erodes the gradients built by the meridional circulation. Under the influence of these

Table 3. Lifetimes (in Myr) during the MS phase. An asterisk means that the value was estimated (see text).

Z	Ω/Ω_c						
	0.1	0.3	0.5	0.7	0.8	0.9	0.99
$3 M_\odot$							
0	215.6	227.8	237.8	249*	255*	264*	275*
				(223.7)	(206.2)	(189.0)	(170.6)
10^{-5}	224.9	243.6		261.5	265.6	271*	271*
						(270.6)	(213.4)
0.002	274.2	301.3	312.3	320.9	325.4	331*	331*
						(316.4)	(237.9)
0.020	357.0	399.6	424.4	439.2	444.4	454*	484*
						(446.7)	(302.9)
$9 M_\odot$							
0	20.31	22.11	23.24	24.31	24.74	25*	25*
						(24.77)	(16.12)
10^{-5}	26.62	28.70	29.98	30.85	31.29	31.8*	32.4*
						(31.30)	(25.03)
0.002	27.77	30.11	31.32	32.26	32.87	33.5*	34.0*
						(31.84)	(26.00)
0.020	27.59	30.33	31.62	32.62	33.0*	34.0*	34.0*
					(32.91)	(28.89)	(23.05)
$20 M_\odot$							
0	8.003	8.554	8.868	9.199	9.324	9.434	9.44*
							(8.616)
10^{-5}	9.099	9.853	10.213	10.54*	10.68*	10.73*	10.73*
				(10.533)	(9.626)	(7.843)	(5.651)
0.002	9.303	10.117	10.527	10.85*	11.00*	11.06*	11.15*
				(10.840)	(10.033)	(8.444)	(6.712)
0.020	8.663	9.521	9.928	10.39*	10.66*	10.96*	10.96*
				(10.389)	(10.662)	(10.937)	(7.892)
$60 M_\odot$							
0	3.633	3.802	3.924				
				(3.386)	(2.538)	(2.259)	(1.173)
10^{-5}	3.874	4.112					
			(3.696)	(2.746)	(2.200)	(1.525)	(0.783)
0.002	3.994	4.251					
			(4.432)	(2.818)	(2.336)	(1.717)	(1.087)
0.020	3.714	3.978					
			(3.960)	(2.426)	(2.031)	(1.595)	(1.179)

two counteracting effects, the Ω -profile converges toward an equilibrium configuration (Zahn 1992; Denissenkov et al. 1999; Meynet & Maeder 2000). From Fig. 15, one sees that for lower initial values of v/v_{crit} the decrease is much weaker than for higher values. It may even completely disappear. It illustrates the dependence of the meridional velocity on Ω . This feature also shows that, in the present theoretical context, we cannot have stars that would remain near the critical limit starting from the ZAMS. Indeed, our most extreme case, the model with $\Omega/\Omega_{\text{crit}} = 0.99$ evolves away from the critical limit just after the ZAMS.

Of course we can wonder to what extent this behaviour is dependent on the chosen initial conditions. For instance would this behaviour be the same, if one had computed realistic pre-MS evolution? Probably in that case the model would not have solid body rotation on the ZAMS and thus the above feature is in part artificial. It is difficult to say more, however, due to the complexity of the processes affecting the angular momentum of stars during their formation process. The point to keep in mind is that *we cannot have non-evolved critically rotating stars in the present models*. Let us stress that these stars should be very rare and their absence should not have any impact on the frequency of stars predicted to rotate at or near the critical velocity. The above effect also implies

**Fig. 15.** Evolution of the ratio v/v_{crit} during the core hydrogen-burning phase for the $20 M_\odot$ models at various metallicities. The computations were stopped as soon as the star reaches the critical limit or just before the beginning of the core He-burning phase.

that the velocities on the ZAMS given in Table 2 are probably quantities not directly comparable to observed ones. Indeed, in the most optimistic case, where ZAMS models indeed rotate as solid body, the chance of seeing stars at the very beginning of their relaxation period is very low. More realistic values of the initial velocities would correspond to time-averaged values computed over the period during which about ten percent of the initial content in hydrogen has been burnt in the core. These values are also given in Table 2. It is important to note that models treating advection as a diffusion would miss this first relaxation period. Indeed during this phase, meridional currents build the gradient of Ω , a process that diffusion cannot do, since it always erodes any gradient.

2. As explained for instance in Maeder & Meynet (2000b), after the first adjustment period, a large outer cell of meridional circulation sets in, transporting angular momentum from the inner parts of the star to the outer ones⁴. If this transport is rapid enough and the stellar winds not too intense, then v/v_{crit} increases. (An essential distinction has to be made here: it is the ratio v/v_{crit} that increases, not necessarily the equatorial velocity itself, see Fig. 18 and Sect. 5 for discussion on this point.) This is the case for all the models with $\omega \geq 0.3$ at $Z \geq 0.00001$. For metal-free stars, the increase in v/v_{crit} only occurs for the models with $\omega \geq 0.7$. This is a consequence of the Gratton-Öpik effect (see the term in brackets in Eq. (14)) making the meridional circulation velocity lower at lower Z .
3. At the end of the MS phase, the increase of v/v_{crit} accelerates when the star contracts after the blue hook in the HR diagram. The rapid change of the velocity at that stage is due to the rapid change in the radius, not to any transport mechanisms.

Looking at the right bottom panel of Fig. 15 (models at $Z = 0.020$), another interesting effect can be discussed. We see for

⁴ In our model, we have two cells of meridional circulation during the MS phase, an inner one where matter moves clockwise and an outer one where matter moves counter-clockwise. The velocities in the outer cell are much higher than the velocities of the inner cell. As a consequence, the outer cell clearly dominates the transport of the angular momentum in the star, transporting it from inner regions towards outer ones.

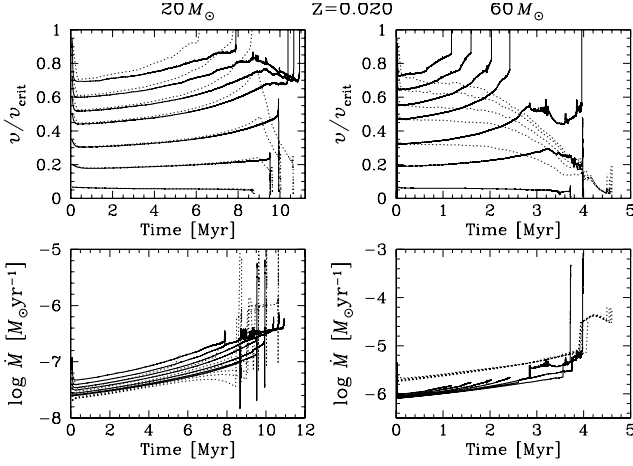


Fig. 16. Comparison of the evolution of v/v_{crit} (top) and $\log \dot{M}$ (bottom) for the $20 M_{\odot}$ models (left) and the $60 M_{\odot}$ models (right) at standard metallicity, when using Kudritzki & Puls (2000) (continuous lines) or Vink et al. (2000) (dotted lines).

instance that the $20 M_{\odot}$ stellar models with an initial value of v/v_{crit} between 0.5 and 0.7 (ω between 0.7 and 0.9) show a bump around the age of 9 Myr. This occurs because, at that time for these models, mass loss increases enough to efficiently remove the angular momentum brought to the surface by the meridional currents. The surface velocity thus evolves away from the critical limit, reaching it again only at the very end of the MS phase. This feature is not seen at lower metallicity because mass loss rates are lower. This trend was already shown by Maeder & Meynet (2001). For a given degree of coupling, the mass loss rates play a very critical role in the evolution of the surface rotation.

In order to check the validity of our results, we computed another set of 20 and $60 M_{\odot}$ models at $Z = 0.020$ with the mass loss prescription of Vink et al. (2000, hereafter VdKL00). In the case of the $20 M_{\odot}$ models, the mass loss rates obtained in the new set are slightly lower than the ones obtained with Kudritzki & Puls (2000, KP00). The $\omega \geq 0.8$ models thus meet the break-up limit earlier in their evolution. The $\omega = 0.7$ model also shows a bump in the velocity, as described above, but then it stays far from the break-up limit, even at the end of the MS, unlike the case with the KP00 rates. This leads to the minimum ω required to reach the break-up limit during evolution being slightly higher with VdKL00 rates, while the fast rotators spend a little more time at the break-up limit (Fig. 16, left panels). But the qualitative result is generally similar to the one obtained with KP00 rates. In the $60 M_{\odot}$ case, the opposite happens: VdKL00's rates are much stronger (by a factor 2 during the MS), and thus no model reaches the break-up limit anymore, even the one starting its evolution with $\omega = 0.99$ (Fig. 16, right panels). The most massive stars' evolution is dramatically influenced by the choice of the mass loss rates, and it can be concluded that, in that case, the comparison between theoretical models and observations instead constitutes a check on the mass loss rates more than anything else.

Coming back to our initial set of models, we present in Table 4 the time-averaged velocities during the MS phase or during the period before the reaching of the critical limit. This table is useful for connecting the values of ω on the ZAMS with observable rotational properties during the MS phase (see Sect. 5).

Table 4. Time-averaged equatorial velocities in km s^{-1} corresponding to various ZAMS values of $\omega = \Omega/\Omega_{\text{crit}}$. The average is taken either on the whole MS phase or on the part of the evolution before the critical limit is reached. For these latter models, the mass fraction of hydrogen at the centre when the surface velocity reaches the critical limit is indicated in parenthesis.

M	Z	$\omega = \Omega/\Omega_{\text{crit}}$						
		0.1	0.3	0.5	0.7	0.8	0.9	0.99
3	0	38	108	198	294 (5e-2)	333 (0.1)	373 (0.17)	423 (0.26)
3	10^{-5}	33	95	167	247	297	353 (7e-3)	416 (0.30)
3	0.002	27	80	139	207	246	298 (0.1)	352 (0.38)
3	0.020	23	69	120	181	215	258 (3e-2)	311 (0.38)
9	0	56	151	265	394	477	569 (1e-5)	666 (0.35)
9	10^{-5}	39	117	204	305	363	435 (3e-2)	515 (0.29)
9	0.002	33	99	173	256	305 (3e-4)	369 (9e-2)	425 (0.31)
9	0.020	30	90	157	236	284 (2e-2)	341 (0.21)	387 (0.36)
20	0	54	169	295	446	507	588	721 (0.13)
20	10^{-5}	45	144	254	389 (3e-3)	469 (0.15)	557 (0.33)	651 (0.48)
20	0.002	37	120	213	323 (2e-3)	387 (0.14)	456 (0.30)	528 (0.43)
20	0.020	32	107	189	279 (2e-4)	325 (1e-4)	369 (6e-4)	447 (0.30)
60	0	63	248	456	645 (0.20)	830 (0.38)	930 (0.45)	1167 (0.61)
60	10^{-5}	63	221	408 (0.16)	587 (0.37)	687 (0.47)	797 (0.57)	890 (0.67)
60	0.002	48	179	323 (5e-5)	474 (0.40)	552 (0.45)	641 (0.54)	727 (0.63)
60	0.020	37	140	258 (8e-2)	400 (0.38)	472 (0.44)	547 (0.51)	609 (0.56)

4.3. Surface enrichment

In Fig. 17 the tracks are plotted in the plane $(N/C)/(N/C)_{\text{ini}}$ versus P , where P is the rotational period in hours. During the evolution the surface is progressively enriched in CNO-burning products, i.e. enriched in nitrogen and depleted in carbon. At the same time, the rotational period increases. The hatched regions correspond to zones where stars would rotate at or faster than the critical limit. We note the following features:

- As already mentioned in previous works (see e.g. Maeder & Meynet 2001), for a given value of the initial mass, of ω , and for a given evolutionary stage, the surface enrichments are stronger in metal-poor stars. This is related mainly to the fact that the gradients of Ω are steeper in metal poor stars due the slower meridional currents in more compact stars. We note that surface enrichments for $3 M_{\odot}$ stellar models at $Z = 0.02$ remain in the range of a few percent, even for high rotation rates. This means that for these low mass stars, observations

will probably not be able to see such small enrichments. The same initial mass models at $Z = 0.002$ present enhancements of up to 45%.

- The surface enrichments are also higher in more massive stars, a feature that is due to the higher efficiency of the shear mixing when the initial mass increases.
- Stars with higher initial velocities reach higher N/C ratios. Let us recall here that the computations were stopped when the critical limit was reached, explaining why, for instance, the maximum surface enrichment indicated for the $9 M_{\odot}$ stellar model at $Z = 0.02$ is lower for $\Omega/\Omega_{\text{crit}} = 0.99$ than for $\Omega/\Omega_{\text{crit}} = 0.8$ and 0.9 . If the model with $\Omega/\Omega_{\text{crit}} = 0.99$ were continued, it would reach higher values than those shown in Fig. 17.
- In general, stars near the critical limit show nitrogen enhancements. There are, however, cases where no enhancements or very small ones would be expected. These cases are the stars starting their evolution with very high rotation rates and still being nearly non-evolved. For instance, in the case of the $9 M_{\odot}$ stellar model at $Z = 0.02$ with $\Omega/\Omega_{\text{ini}} = 0.99$, v/v_{crit} becomes superior to 0.7, while still no significant surface enrichment in nitrogen has occurred. Another case where no observable surface enrichment is expected is for stars at $Z = 0.02$ and with initial masses around $3 M_{\odot}$ (see lower left panel).
- No stars with masses between 3 and $20 M_{\odot}$ and with rotation periods longer than about 5 days are expected to be at the critical limit.

5. Initial conditions for stars reaching nearly critical velocities

Let us first recall that a key ingredient of the ratio v/v_{crit} is the behaviour of v_{crit} itself. Figure 18 shows how it evolves in time in the case of the $20 M_{\odot}$ model at standard metallicity with $\omega = 0.7$ initially ($v/v_{\text{crit}} = 0.5$). During evolution, v_{crit} gets lower, and in the present case the final value amounts to only half the initial one. It can be explained by the modifications of the stellar parameters: some mass is lost and the radius is slowly inflating, so the surface gravity decreases and can be more easily counterbalanced by the centrifugal force. In the example shown in Fig. 18, we see that, even though the equatorial velocity is slowly decreasing during the MS, the ratio v/v_{crit} increases steadily. It is essential to take this evolutionary effect into account when one wants to determine the v/v_{crit} ratio at any given time. Keeping the initial value would lead to a completely wrong result. Let us mention that, in that perspective, our results are not contradictory with the observations of Huang & Gies (2006b) or Abt (2003), who find a clear spin down with evolution for the stars in their sample. Except the most extreme ones with $\omega = 0.99$ – 0.90 , all our standard metallicity models in the mass range 3 – $9 M_{\odot}$ show a steady or decreasing equatorial velocity, though their v/v_{crit} ratios increase.

From our computations, we can determine, for each model specified by its initial mass and metallicity, the values of $\Omega/\Omega_{\text{crit}}$ on the ZAMS required to reach given limits of v/v_{crit} during the MS phase. These values are plotted in Fig. 19. Using Table 2, it is possible to translate these limits on $\Omega/\Omega_{\text{crit}}$ into limits on the equatorial velocities on the ZAMS or at the beginning of the MS phase. Using Table 4, a similar correspondence can be obtained with values of the averaged velocity during the MS phase or during the phase before reaching the critical velocity.

At the standard metallicity, we note that in general the values of $\Omega/\Omega_{\text{crit}}$ needed on the ZAMS to reach given values of v/v_{crit}

decrease when the mass increases. This comes from the fact that meridional currents are more rapid in the less dense envelope of more massive stars. Let us recall that, in the outer layer, the dominant term in the expression for the radial component of the meridional circulation $U(r)$ is mainly (Gratton-Öpik term)

$$U(r) \sim \left[1 - \frac{\Omega^2}{2\pi G \bar{\rho}} \right] \left(\frac{\Omega^2 r^3}{GM} \right) \quad (14)$$

where $\bar{\rho}$ means the average over the considered equipotential. Thus the outer cell of the meridional circulation will be more active in less dense envelopes. In the upper left panel of Fig. 19, the bump around the $20 M_{\odot}$ stellar model comes from this model suffering significant mass loss at the end of its MS evolution. This discourages reaching nearly critical values (except at the very end of the MS evolution, see Fig. 15). In the case of the $60 M_{\odot}$ model, when the mass loss rates of KP00 are used, the nearly critical values occur well before the star undergoes similar enhancements of the mass loss rate, and thus, the minimum values of $\Omega/\Omega_{\text{crit}}$ are lower than for the $20 M_{\odot}$. The situation is quite different if the mass loss rates from VdKL00 are used (see Fig. 16).

For metallicities $Z = 0.002$ and 0.00001 , we find that the minimum values of $\Omega/\Omega_{\text{crit}}$ required to reach near critical velocities also decrease when the initial mass increases. More massive stars thus reach the critical limit more easily. We also note that for the $60 M_{\odot}$ at $Z = 0.002$, the minimum value of $\Omega/\Omega_{\text{crit}}$ to reach the critical velocity is around 0.7, while at $Z = 0.00001$ it is lowered to values equal to 0.5. This is mainly due to the weak radiative winds of the metal-poor models. Since radiative winds are triggered by metal lines, a low metal content implies low mass loss rates. We have already seen in Sect. 4.2 that there is a competition between the efficiency of the mass loss (removing some angular momentum and thus decelerating the surface) and the efficiency of the meridional circulation (bringing the angular momentum from the core to the surface and thus accelerating the surface). From our results, we can say that, even though the efficiency of the meridional circulation is lower at low Z , the effect of lowering the mass loss rates plays the major role and the total budget favours a significant acceleration of the surface, at least down to $Z = 0.00001$. Let us note that at some points, the curves are superposed: for instance for the $9 M_{\odot}$ at $Z = 0.002$, the curve for reaching the critical limit and those for reaching values of $v/v_{\text{crit}} \geq 0.8$ and 0.9 go through the same point. This simply reflects the fact that, when such a star reaches the values $v/v_{\text{crit}} \geq 0.8$, it then also reaches the critical limit.

For metal-free stars, one notes that, for $M > 9 M_{\odot}$, the minimum values of $\Omega/\Omega_{\text{crit}}$ to approach critical velocities during the MS evolution are in general higher than at higher metallicities. This reflects the greater compactness of these stars. The metal-free $3 M_{\odot}$ models, in contrast reach the critical limit much more easily than the models at higher metallicities. We can see that their tracks in the HR diagram (see Fig. 13) evolve quite blueward. This evolution is due to the fact that metal-free stars in this mass domain must contract over a large part of their MS evolution to compensate for the lack of CNO elements. In such a situation, the evolution of Ω is mainly governed by the local conservation of the angular momentum (nearly no coupling), which favours the approach of the critical limit when the star contracts. Typically for a $3 M_{\odot}$ stellar model, the minimum value of $\Omega/\Omega_{\text{crit}}$ is around 0.6 at $Z = 0$ for reaching the critical limit during the MS phase, while it is higher than 0.8 at standard metallicity.

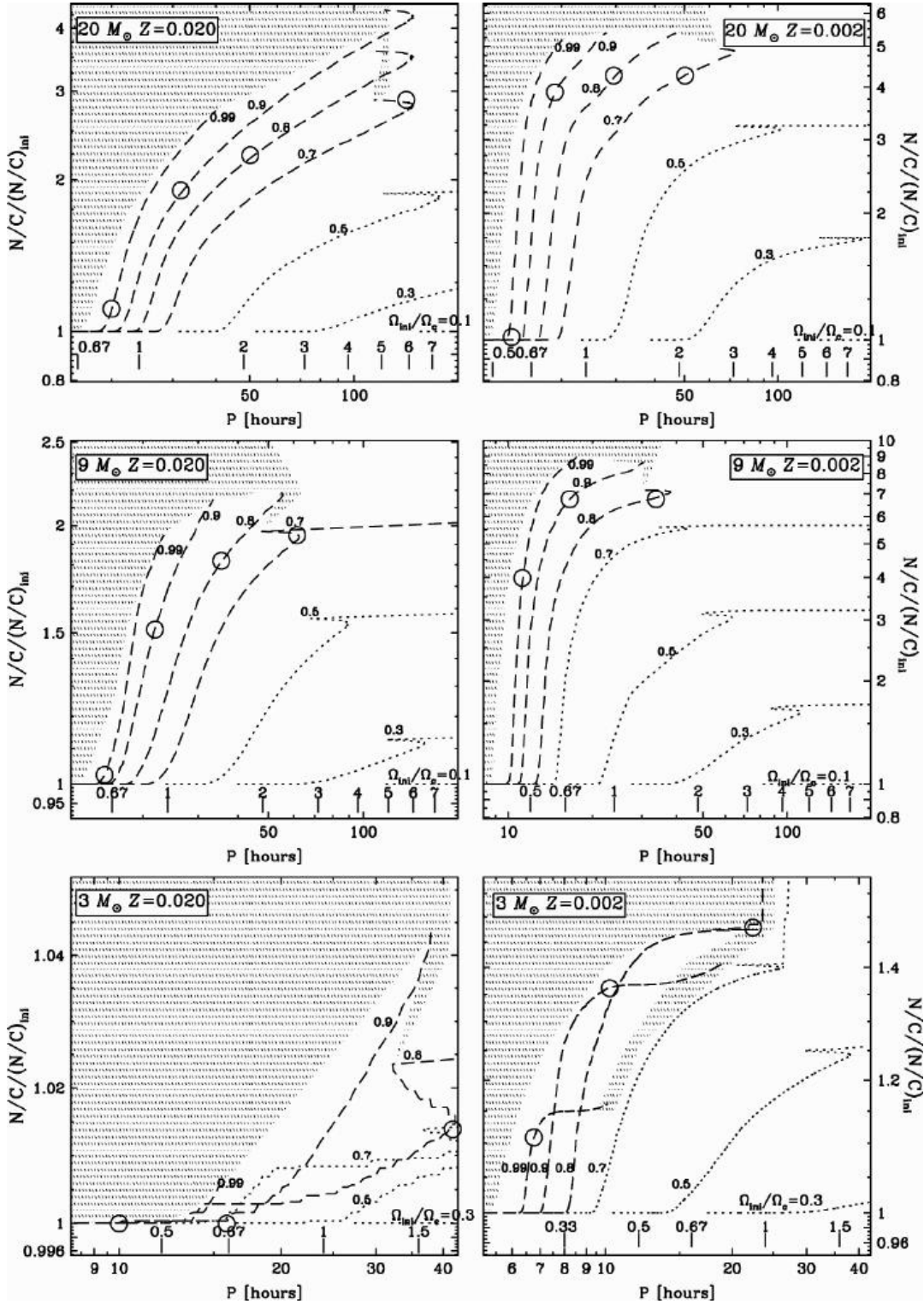


Fig. 17. Evolutionary tracks in the plane surface N/C ratio, normalized to its initial value, versus the rotational period in hours for different initial mass stars, various initial velocities, and the metallicities $Z = 0.02$ and 0.002 . Positions of some periods in days are indicated at the bottom of the figure. The dotted tracks never reach the critical limit during the MS phase. The short dashed tracks reach the critical limit during the MS phase. The dividing line between the shaded and non-shaded areas corresponds to the entrance into the stage when the star is at the critical limit during the MS evolution. Big circles along some tracks indicate the stage when v/v_{crit} becomes superior to 0.7 . If Be stars are rotating at velocities superior to 70% of the critical velocity, the present models would predict that they would lie in the region between the big circles and the dividing line. Beware the different vertical scales used when comparing similar masses at different metallicities.

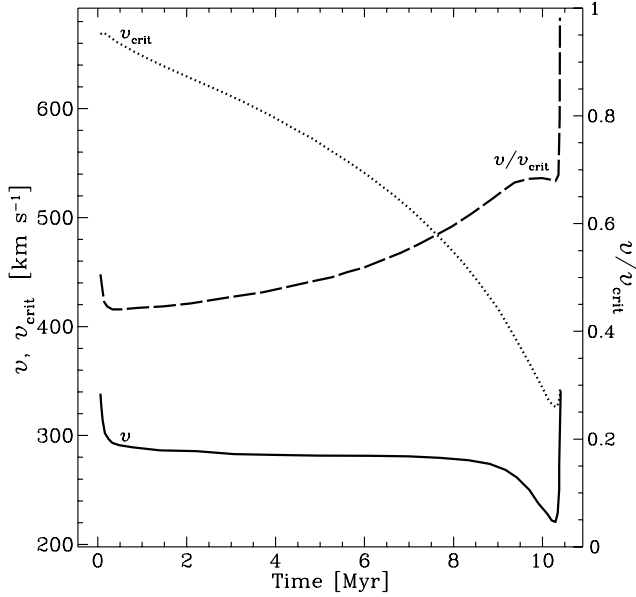


Fig. 18. Evolution of the equatorial velocity v (continuous line), the critical velocity v_{crit} (dotted line), and their ratio (dashed line) for the $20 M_{\odot}$ at standard metallicity with $\omega = 0.70$ initially ($v/v_{\text{crit}} = 0.50$).

6. Rotation of stars in clusters of various ages and metallicities

Let us call $n(t, w)$ the number of MS stars in a cluster of age t having surface rotational velocities, such that $v/v_{\text{crit}} \geq w$. We only consider stars brighter than about two magnitudes below the turn off and call $N(t)$ the total number of MS stars in the same cluster, in the same interval of magnitudes. The ratio $n(t, w)/N(t)$ can be expressed as

$$\frac{n(t, w)}{N(t)} = \frac{\int_{M_{\min}}^{M_{\max}} \bar{n}(M) p(M, t, w) dM}{\int_{M_{\min}}^{M_{\max}} \bar{n}(M) f(M, t) dM}$$

where $M_{\min}$ is the mass of stars that are at two magnitudes below the turn off, $M_{\max}$ the mass of stars at the turn off, $\bar{n}(M)$ the number of stars born with an initial mass M , and $p(M, t, w)$ the fraction of these stars that, at the age t , are on the MS and have surface velocities such that $v/v_{\text{crit}} \geq w$. $f(M, t)$ is the fraction of stars of initial mass M that are on the MS at the time t . In the following, we explain how these different values are obtained from our rotating stellar models.

- $M_{\max}$: In the case of non-rotating stellar models, at a given metallicity we have a one-to-one relation between the initial mass and the MS lifetime. It suffices then to take the initial mass value of the star that has a MS lifetime equal to the considered age of the cluster to obtain the mass of stars at the turn off, i.e. $M_{\max}$. When rotation is accounted for, the situation is more complicated, since the MS lifetimes increase with the initial velocities. Thus the maximum mass of a star that can still be on the MS at a given age corresponds to the mass of the rotating star with the maximum initial rotation velocity (here $\Omega/\Omega_{\text{crit}} = 0.99$ on the ZAMS) and with MS lifetime that corresponds to the age considered. Of course in a given cluster there is little chance of having at turn off such a star due to the very small number of stars with such a high velocity. This effect will be taken into account through the initial velocity distribution function in $f(M, t)$ (see below).

- $M_{\min}$: To estimate the lower bound $M_{\min}$, we have taken about 70% of the value of $M_{\max}$. We indeed checked, using theoretical isochrones (Meynet et al. 1993), that for ages between 8 and 316 Myr, the typical mass of stars two visual magnitudes below the turn off is in that range.
- $\bar{n}(M)$: The number of stars born in the mass interval between $M_{\min}$ and $M_{\max}$ is proportional to $\int_{M_{\min}}^{M_{\max}} M^{-2.35} dM$, when a Salpeter's slope of the initial mass function is considered.
- $f(M, t)$: as explained above, stars with a mass $M_{\max}$ will still be in their MS phase, provided they started their evolution with $\Omega/\Omega_{\text{crit}} \geq 0.99$. Stars of this mass but with slower initial rotation will have already evolved away from the MS. Thus the fraction of stars of mass $M_{\max}$ that are still on the MS at time t is equal to

$$f(M_{\max}, t) = \int_{\omega_{\min}(M, t)=0.99}^{\omega=1.00} d_p(\omega) d\omega$$

where $\omega_{\min}(M, t)$ is the minimum value of ω for stars of initial mass M to still be on the MS at time t , and d_p the probability density for a star to start its evolution with a given value ω (See Fig. 20). For each mass and age, one can determine $\omega_{\min}(M, t)$, and similar integrals as the one shown above can be performed.

- $p(M, t, w)$: From figures similar to Fig. 15, we can extract, for each initial mass, metallicity, and rotation, the time at which v/v_{crit} becomes superior to w (note that once $v/v_{\text{crit}} \geq w$, it remains greater for the rest of the MS phase⁵, the only exception being the $20 M_{\odot}$ at $Z = 0.020$). From such data, one can deduce that for a star of $9 M_{\odot}$ with $Z = 0.020$ and an age equal to 25 Myr, the minimum value of $\Omega/\Omega_{\text{crit}}$ required on the ZAMS to reach values of v/v_{crit} superior to 0.7 during the MS phase is 0.83. Using an initial distribution of $\Omega/\Omega_{\text{crit}}$ (see Fig. 20), one can deduce the value of $p(M, t, w)$.

We now need to specify the function we use for the initial distribution of $\Omega/\Omega_{\text{crit}}$. Huang & Gies (2006a) provide a distribution of equatorial velocities corrected for the $\sin i$ effect from their observations of 496 OB-type stars. This distribution is shown in Fig. 20 (left). Of course this distribution does not correspond to a distribution on the ZAMS, since stars are picked up at different phases of their evolution during the MS phase. We can reasonably consider, however, that these equatorial velocities are representative of the values for the average velocities of stars in the MS phase. We can then associate a value of ω on the ZAMS to each of these values using the relation obtained from our $9 M_{\odot}$ at standard metallicity between the averaged equatorial velocity during the MS phase and ω on the ZAMS. Proper normalization has been applied to transform the probability densities for velocities into probability densities for ω , i.e. the normalization has been chosen so that $\int_0^{420} D_p(v) dv = \int_0^1 d_p(\omega) d\omega = 1$, where D_p and d_p are the probability densities for v (see Fig. 20, left panel) and ω , respectively. The cumulative distribution of ω on the ZAMS that we obtained in this way is shown in the right panel of Fig. 20.

In Fig. 21 the variation in the number fraction of stars at the critical limit (continuous thick lines) in clusters of different ages is shown. We also plotted the fraction of stars having v/v_{crit} values superior to given limits (see caption). Due to the coarse grid of mass we used here, we shall not attach too much importance

⁵ Note that we used the extrapolated lifetimes (marked with an asterisk in Table 3) for the models whose computation was stopped before completing the MS phase.

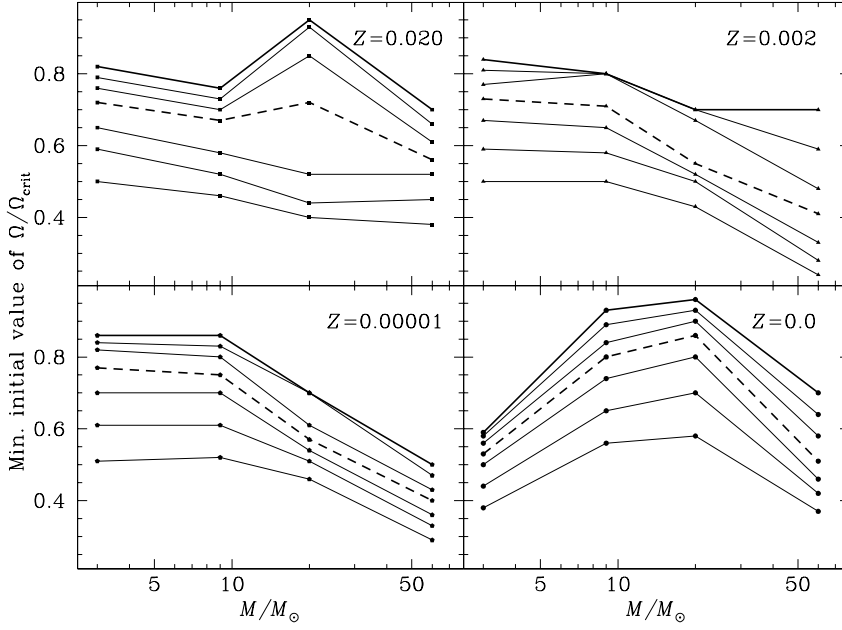


Fig. 19. Minimum values of $\omega = \Omega/\Omega_{\text{crit}}$ on the ZAMS required for reaching values of v/v_{crit} superior to 0.4 (bottom line), 0.5, 0.6, 0.7 (dashed), 0.8, 0.9 and 0.99 (top heavy line), respectively, during the MS phase. The initial metallicity considered is indicated in each panel.

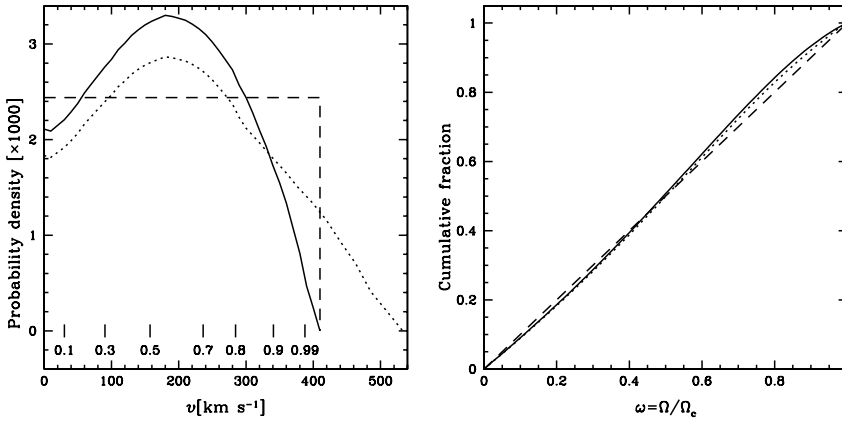


Fig. 20. *Left panel:* distribution of the equatorial velocities as given by Huang & Gies (2006a) (continuous line). The dotted line shows the case when it is assumed that, above 300 km s⁻¹, the velocities are underestimated (see text). The dashed line shows a flat distribution. At the bottom of the figure, the correspondence between ZAMS values of $\Omega/\Omega_{\text{crit}}$ and average velocities during the MS phase as deduced from our 9 $M_{\odot}$ at standard metallicity (Table 4) is indicated. *Right panel:* cumulative distribution function of $\omega = \Omega/\Omega_{\text{crit}}$ deduced from the data by Huang & Gies (2006a) with the same assertions as in the left panel.

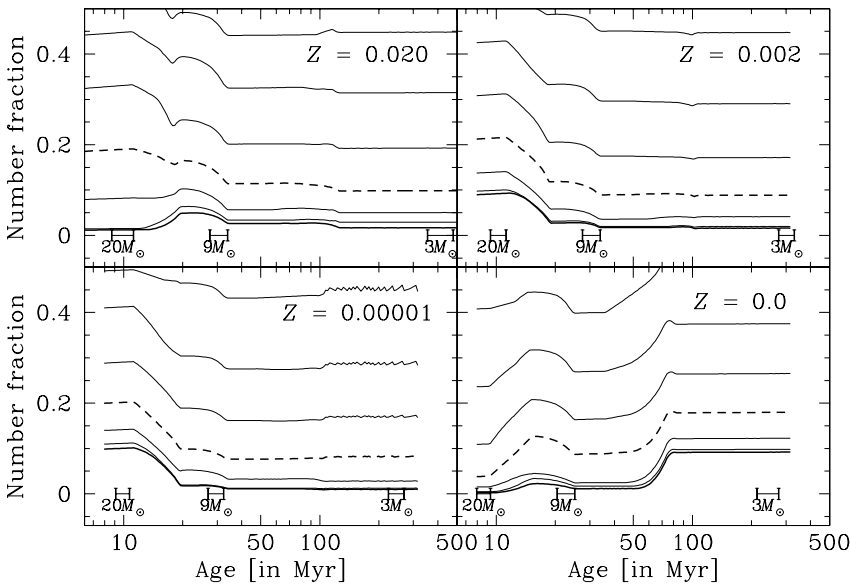


Fig. 21. Variation in the fraction of stars with v/v_{crit} superior or equal to 0.4, 0.5, 0.6 (three thin continuous lines from the top), 0.7 (thick dashed line), 0.8, 0.9 (two thin continuous lines), and at the critical limit (bottom thick solid line) in clusters of various ages and metallicities. At the bottom of each panel, the range of MS lifetimes are indicated for the 3, 9 and 20 $M_{\odot}$ stellar models.

to the details of these curves but will comment on their general behaviour:

- At standard metallicity, we note a bump around ages 20–25 Myr for the curves corresponding to the critical limit and for $v/v_{\text{crit}} \geq 0.9$ and 0.8. This kind of behaviour can be explained in the following way: at younger ages than the bump, mass loss discourages reaching the critical limit, at older ages than the bump, meridional currents are not efficient enough for transporting the angular momentum outwards (denser envelopes in stars of lower initial masses).
- For metallicities $Z = 0.002$ and 0.00001 , we note that the bump shifts to younger ages and reaches higher values. This is because of the lower mass loss rates experienced by stars at low metallicity. This allows more massive stars, which have less dense envelopes and thus stronger meridional currents, to undergo a more rapid acceleration of their surface and to reach the critical limit more easily.
- We note that the curves are very similar at the metallicities $Z = 0.002$ and 0.00001 . Thus, from the point of view of the surface velocities, we would not expect any significant changes over this metallicity range.
- For metal-free stars, the curves are very different than at $Z = 0.00001$ and 0.020 : at ages younger than about 20–32 Myr, stars have more difficulty reaching the critical limit. This comes from the absence of metals making these stars very compact, with dense outer layers and very slow meridional circulation currents (note that if the mass range were extended towards higher masses, this bump might appear at very young ages). At ages older than about 80 Myr, on the other hand, stars at the critical limit are much more numerous than at higher metallicities. This comes from the fact that stars with initial masses equal or below $3\text{--}5 M_{\odot}$ contract for a large part of their MS phases (see the metal-free $3 M_{\odot}$ stellar models) and thus can reach the break-up limit quite early.
- At $Z = 0.020$, the fraction of stars at the critical limit is at most 5% and can reach values of slightly more than 10% at lower metallicities. The number fraction of stars with $v/v_{\text{crit}} \geq 0.9$ tightly follows the curve of stars at the critical limit, indicating, that when this value is reached, the surface velocity rapidly evolves towards the critical limit. The fraction of stars with $v/v_{\text{crit}} \geq 0.7$ reaches values up to 20–25% and does so at an earlier age: around 10 Myr and below.

7. Discussion of the results

7.1. Sensitivity on the ZAMS model

We recalled in Sect. 4.2 that our way of building our ZAMS models makes it impossible to maintain a high value of v/v_{crit} from the ZAMS during the beginning of the MS phase. Activation of the meridional circulation imposes a rapid slowdown of the surface, so the star needs to evolve somewhat in the MS phase in order to again approach the critical limit. At this point, one can ask two questions: 1) would it be possible to build ZAMS models with higher angular momentum content, which would remain near the critical limit throughout the MS phase? 2) How would such behaviour, if possible, change the above results concerning the frequency of stars near the critical limit?

The answer to the first question is yes. It would suffice to distribute the angular momentum or the angular velocity in the ZAMS model differently, for instance, by imposing a non-uniform distribution of Ω on the ZAMS, allowing more angular momentum to be contained in the inner parts of the star. But this

would not change the present results concerning the frequencies of stars near the critical limit. Indeed, these frequencies are computed on the basis of the observed frequencies of stars with given averaged rotational velocities during the MS phase. Stars with higher angular momentum than the most rapid rotators computed here will present higher values of the averaged rotational velocities during the MS and are thus already implicitly accounted for.

7.2. Sensitivity on the velocity distribution

Let us first note that the measure of the rotational velocity through the Doppler effect is not free of spurious effects. First this technique only gives access to $v \sin i$ where i is the angle between the line of sight and the rotational axis. This quantity cannot be known except if some other observations as e.g. a rotational period obtained from variability induced by the presence of dark spot at the surface become available. But even in that case, the knowledge of v requires knowing of the radius (more precisely of the equatorial radius). Moreover, if the surface rotates differentially like the solar surface, the period thus obtained might only be partially representative of the surface rotation. Second, due to the von Zeipel theorem, the fast rotating regions of the star, i.e. the zones near the equator, are less luminous than the polar regions, which have low linear velocities. This effect produces an underestimate of the rotational velocities. Townsend et al. (2004) obtain the following behaviour between the FWHM of the HeI 4471 line for a B2-type star and the equatorial velocity. The relation is nearly perfectly linear for $\sin i = 1$ and $v/v_{\text{crit}} < 0.7$ (for a standard metallicity $9 M_{\odot}$ star, which corresponds to velocities inferior to 300 km s^{-1} on the ZAMS). In that velocity regime, the FWHM measurements, due to the above effects, underestimate the velocities by at most 3–4%.

For $v/v_{\text{crit}} > 0.7$, the relation is no longer linear, the FWHM increases much more slowly with the velocity, and it even goes through a maximum, decreasing a little for velocities near the critical limit. In that case, the FWHM becomes a poor indicator of the surface velocity, a star rotating at the critical limit presenting the same line broadening as a star rotating at 80% of the critical velocity. In other words, this technique may underestimate the real velocity of the star by 20–25% for very fast rotators (typically for $v/v_{\text{crit}} > 0.75$, i.e. for velocities superior to about 340 km s^{-1} , for a $9 M_{\odot}$ at standard metallicity). To study this effect, we did the following experiment. We modified the velocity distribution given by Huang & Gies (2006a) by assuming that between 300 and 420 km s^{-1} all the velocities v should be enhanced by a factor equal to $1 + (v - 300)/120 \times 0.3$ (i.e. a linear enhancement amounting at most to a factor 1.3 at 420 km s^{-1}). Using such a modified velocity distribution, we find that the fraction of stars at the critical limit for an age equal to 25 Myr at $Z = 0.020$ would be equal to 0.055 instead of 0.049 with the original velocity distribution of Huang & Gies (2006a, Fig. 20). Thus this effect would not significantly change the results.

To obtain a fraction of stars at the critical limit of about 10% at $Z = 0.020$, it is necessary to considerably increase the fraction of stars born with $\Omega/\Omega_{\text{crit}}$ greater than 0.6 on the ZAMS. Typically passing from the original velocity distribution to a uniform distribution, one obtains that the fraction of stars with values of $\Omega/\Omega_{\text{crit}}$ on the ZAMS superior to 0.6, 0.7, 0.8, and 0.9, passes from 37.5, 26.5, 15.5, and 6.5% to 40, 30, 20, and 10% respectively. Using such a uniform velocity distribution, one finds that the fraction of stars at the critical limit equal 12% at $Z = 0.020$ and age equal to 25 Myr (instead of 5% with the original velocity distribution).

7.3. Sensitivity on mass loss and magnetic fields

Mass loss, as we have already discussed above, plays a key role in preventing the high mass stars to reach the critical limit. We showed that, by using two different laws for the mass loss rate, very different behaviours are obtained for the $60 M_{\odot}$ models at standard metallicity. It underlines the importance of this ingredient in shaping the evolution of the surface velocity in this mass range. For lower initial mass stars, typically for $20 M_{\odot}$, mass loss becomes lower and lower (at least in the absence of any surface magnetic field).

We can wonder whether mass loss anisotropies would have any impact on the results obtained in this work (which assumed spherical winds). Let us recall that, when a star is rotating at $v/v_{\text{crit}} \geq 0.9$, winds of hot stars will become much more intense at the pole than at the equator. This effect would accelerate the evolution towards the critical limit. However, accounting for this effect would hardly change the evolution we computed here. Indeed, v/v_{crit} becomes greater than about 0.8–0.9 only when the surface velocity increases in an already very rapid way (see Fig. 15).

A magnetic field, following the Tayler-Spruit dynamo (Spruit 2002), will tend to impose solid body rotation during the MS phase (Maeder & Meynet 2005), thus making the approach of the critical limit easier (stronger coupling between the core and the envelope). We can expect that the inclusion of this effect would have an strong impact on the present results, making the fraction of stars at the critical limit greater than in the present work. It has, however, to be noted that at present the Tayler-Spruit dynamo is still very much debated, and some more work is needed before assessing whether this mechanism can be active in stars or not.

8. Possible links with the Be stars

There are good indications that the Be star phenomenon is related to the fast rotation of the star. For instance, Martayan et al. (2006a) show that the initial velocities of the Be stars are significantly higher than the initial velocities of the normal B stars, giving some support to the view that only stars with a sufficiently high initial velocity can go through a Be episode. Let us note that, when material is ejected into a Keplerian disk, the star loses angular momentum, slows down, and evolves away from the limit. Secular evolution will then bring the star back towards this limit. Probably the phases in between the loss of material are much longer than the phases during which the star loses material into the Keplerian disk. Thus most of the Be stars might be observed during these quiet phases when they are evolving back towards the limit. It means that the rotation of observed Be stars might be somewhat lower than the rotation at which outbursts occur. If the link between Be stars and fast rotation is now well-accepted, it is not, however, clear what rotation rate is needed for a star to become a Be star or why this process is mainly restricted to early B type stars. Is rotation the only key parameter governing the transformation of a star into a Be star? When does such a transformation occur? In the present section we would like to provide some answers to these questions.

Comparisons between number fractions of fast rotating stars as predicted by the present models and as observed in clusters (see caption) is shown in Fig. 22. We see first that if we consider only those stars that are strictly at the critical limit (hatched zone I), the prediction falls well below the observed points, all the more so as there are also rapidly rotating stars that are not Be stars, or Be stars that are observed between outbursts. Thus,

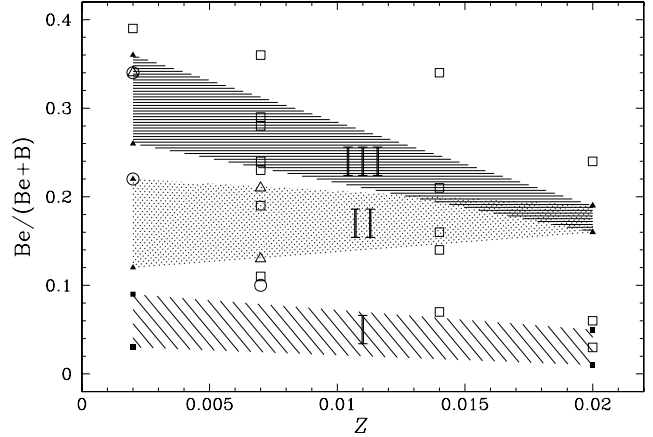


Fig. 22. The empty symbols correspond to the observed fraction of Be stars among B0-B3 type objects in clusters with ages ranging from 10 to 25 Myr. Data were taken from Wisniewski & Bjorkman (2006, circles), Keller et al. (1999, triangles) and Maeder et al. (1999, squares). The filled symbols correspond to predictions of the given models for $Z = 0.002$ and $Z = 0.020$ and for ages equal to 10 and 25 Myr, respectively (we estimated the fraction of Be in a two-magnitude interval below the turn off, as did Maeder et al. 1999). The hatched zone labelled by I corresponds to stars predicted at the critical limit. The hatched zone labelled by II shows the fractions of stars with $v/v_{\text{crit}} \geq 0.7$. These two cases were obtained assuming a velocity distribution from Huang & Gies (2006a). The third hatched region, labelled III, corresponds to stars with $v/v_{\text{crit}} \geq 0.7$ and assumes that the initial distribution of rotation contains faster rotators at low metallicity (see text).

according to the present stellar models, the number fraction of stars at the critical limit is not sufficient to account for the observed fraction of Be stars.

Let us mention that there are two proposed channels for the formation of Be stars: 1) the single star channel, where the critical limit is reached because of the natural evolution of a single rotating star; and 2) the binarity channel, with the spin-up of the member of a binary system through mass transfer from its companion. According to McSwain & Gies (2005a,b), about 75% of the detected Be stars may have been spun-up by binary mass transfer, while most of the remaining Be stars were very likely rapid rotators at birth. There are indeed some known binaries among the observed Be stars, but in most cases, no binarity has yet been clearly assessed. It may be very difficult to detect, however, because of the long orbital period and the very small orbital motions. In some cases, decades of careful measurements have been necessary before detecting a companion (see Gies et al. 1998 for ϕ Per, Maintz et al. 2005 for 59 Cyg, Rivinius et al. 2004 for FY CMa). According to Negueruela (2007), the binarity channel may be significant, but it is unlikely that most Be stars can form through this channel. From a theoretical point of view, Pols et al. (1991) deduce that probably 40% (at most 60%) of the Be stars may form through mass transfer spin-up. Consequently, the binary channel alone could account for a $(\text{Be}/(\text{B}+\text{Be}))$ ratio of the order of 8–10% if we take the $(\text{Be}/(\text{B}+\text{Be}))$ value found by Martayan et al. (2006b). In that last case, our prediction thus remains too low even when discounting the binarity channel contribution.

In the frame of the single star scenario, if we assume that the Be phenomenon may occur already at $v/v_{\text{crit}} \geq 0.7$ (see hatched zone II), then the situation improves somewhat; however, the theoretical results do not show any increase in the fraction of Be stars with decreasing metallicity, as some observations indicate

(Maeder et al. 1999; Wisniewski & Bjorkman 2006). It might be that part of this trend is due to an observational bias. For instance McSwain & Gies (2005b) find that Be stars are generally found among the brighter B-stars in their survey of southern sky Galactic clusters. A similar trend was found by Keller et al. (2001) in η and χ Per. Since metal-poor clusters are more distant, observations will mainly pick up the brighter end of the B stars sample where the fraction of Be stars is higher. However, it is not clear that such a bias will have a strong impact when clusters of similar ages are compared; provided the observed samples are complete and the count is performed in the same absolute magnitude interval, then the observed number ratios at different metallicities can be compared. To reproduce the observed trend with the metallicity obtained by Maeder et al. (1999) and more recently by Wisniewski & Bjorkman (2006) with the present models, we have to assume that the fraction of fast rotators at birth is higher in metal-poor regions than in metal-rich one. As a numerical example, supposing that the initial distribution of rotation at $Z = 0.002$ is given by a uniform distribution (Fig. 20), we then obtain the hatched zone III, which agrees reasonably well with the observations. From the discussion above, we deduce that to reproduce the observational number fraction of Be stars, we first have to relax the hypothesis that Be stars are strictly at the critical limit: the Be phenomenon should already occur for $v/v_{\text{crit}} \gtrsim 0.7$; second, the distribution of initial values of $\Omega/\Omega_{\text{crit}}$ on the ZAMS at low metallicity should contain a higher fraction of fast rotating stars.

Let us discuss these two conclusions. The first conclusion would tell us that, although fast rotation plays a significant role in transforming the star into a Be star, it is not sufficient alone. Above some value of v/v_{crit} , some other processes, such as pulsations, must help in launching material into a disk.

Now, what about the second conclusion of a higher fraction of fast rotators at low Z ? Keller (2004) measured the rotational velocity of 100 MS early B-type stars in clusters of the LMC and compared the results with observations of early B-type stars in clusters of the solar neighbourhood. He finds that the LMC stars are faster rotators than the galactic ones: the mean values of $v \sin i$ is 116 km s^{-1} for the galactic stars and 146 km s^{-1} for the LMC stars. Part of the difference can be due to the difference in mass loss between galactic and LMC stars, but part might also be due to differences in the initial distribution of the velocities. Penny et al. (2004), on the other hand, find no difference between the velocities of O-type stars in the Magellanic Clouds and in the Galaxy. Thus, the picture is at present not clear and probably new surveys, such as the VLT-Flames survey, will provide new insights on that question very soon. At the moment, we still consider the question as open. Present results would, however, clearly favour a higher fraction of stars born with high values of $\Omega/\Omega_{\text{crit}}$ at low metallicities.

A point that the models have to explain is when the Be phenomenon appears during the MS phase. Unfortunately no clear picture emerges from the observations. Some authors find the highest fraction of Be stars among the spectral types B0-B2, appearing most often at ages between 13–25 Myr (Fabregat & Torrejón 2000). Recently, Frémat et al. (2006), determined the fundamental parameters of the Be stars that would be observable by COROT, found that the main part of their stellar sample falls in the second half of the MS lifetime. This would favour a view according to which the Be star phenomenon tends to appear at the end of the MS phase. Other authors (as e.g. Zorec & Briot 1997) conclude that the Be phenomenon is not associated with a particular evolutionary phase and can occur during the whole MS phase, at least for some range of initial masses (see

Zorec et al. 2005). With the adopted distribution of initial velocities, present models predict that most near-critical velocity stars will be found at evolved stages during the MS phase and would thus provide an explanation for the observation of Fabregat & Torrejón (2000). From a physical point of view, we think that the Be star phenomenon may be limited to a given spectral type range for the following reasons. For the earlier spectral types than the favoured one, mass loss prevents the star from approaching the critical limit, and for the later spectral types, the meridional currents are not strong enough to accelerate the envelope. Models would predict a shift towards earlier spectral type when the metallicity decreases due to the decrease in mass loss with the metallicity (extending the range towards earlier spectral type) and the decrease in the meridional velocity in denser layers (increasing the minimum initial mass for meridional currents being efficient enough to accelerate the surface). Of course, beyond these physical reasons are the effects of a possible variation in the distribution of initial rotation as a function of metallicity, which would somewhat blur the above picture. And now, if we relax the condition for becoming a Be star to $v/v_{\text{crit}} \gtrsim 0.7$, we allow this phenomenon to appear earlier in the life of a star.

The present models predict the approach of high values of v/v_{crit} (0.8–0.99) only when the star is already evolved into the MS phase. As mentioned above, the use of a different internal profile of Ω in our ZAMS models would allow us to explore ever more extreme cases of rotation, where the star may reach the critical limit, or at least remain near to it, from the beginning of the MS phase. However, these stars should be very rare. The need for the stars to be somewhat evolved to reach high surface velocities implies that these stars should present at their surface N-enrichments. These surface enrichments might, however, be very small for stars in the lower part of the mass range, possibly evolving into a Be star phase (see the cases of our $3 M_{\odot}$ stellar model at $Z = 0.020$ in Fig. 17). Observations of N surface enrichments for Be stars are still scarce, probably because of the difficulty of determining abundances from greatly widened absorption lines. Frémat et al. (2004) obtain N/C ratios for four Be stars that are clearly higher than what is observed at the surface of slowly rotating B-type stars. They obtain $(\text{N/C})/(\text{N/C})_{\text{ini}}$ of the order of a factor 2, which would correspond closely to what is expected for near critically rotating $9 M_{\odot}$ stars at $Z = 0.020$ (see middle panel of Fig. 17).

9. Conclusions

From the above computations we can deduce that

1. As long as the mass loss rates are not too high, meridional currents bring the v/v_{crit} ratio close to 1 during the MS phase, by advecting angular momentum from the inner regions to the outer layers.
2. Since the velocity of the meridional currents in the outer layers scales with the inverse of the density, the process becomes more efficient for stars of higher initial mass and/or higher initial metallicity.
3. At high metallicity, however, mass loss becomes greater and greater and can prevent the stars from reaching the break-up limit.
4. In stellar clusters, one expects that the fraction of stars having $v/v_{\text{crit}} \gtrsim 0.8$ reaches its maximum for ages between 20–32 Myr at $Z = 0.020$. This range of ages shifts to older ages at higher metallicities and to younger ages at lower metallicities. Such a shift results from the dependence on the metallicity and the mass of the intensity of the mass loss

rates and of the velocities of the meridional currents. If we relax the condition and look at stars with $v/v_{\text{crit}} \gtrsim 0.7$, the maximum number will be found in clusters of ages around 10 Myr and below and do so at all non-zero metallicities.

5. Be stars might be the natural outcome of stars with an initial rotational velocity in the upper tail of the initial velocity distribution. To explain their frequency from the rotating stellar models presented here, minimum values of $v/v_{\text{crit}} \gtrsim 0.7$ have to be reached during the MS for the Be star phenomenon to occur.
6. Depending on when this limit is reached, one expects more or less high surface enrichments. If the limit is reached very early during the MS phase, no enrichment is expected, while if this limit is reached at the end of the MS phase, high N/C and N/O ratios are expected. For Be stars developing out of small initial mass stars (typically from $3 M_{\odot}$ at standard metallicity) one expects small surface enrichments or none at all.
7. To reproduce the higher fractions of Be stars at low metallicity, we must assume that, in metal-poor regions, a higher number of stars are born with high values of $\Omega/\Omega_{\text{crit}}$.

In a future work, it will be interesting to explore the effects of a magnetic field. As already mentioned above, this will probably change the results presented here significantly, allowing the stars to approach the critical limit more easily. It will be interesting to see if, in that case, it would still be necessary to invoke a higher proportion of fast rotators at low Z .

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A strong case for fast stellar rotation at very low metallicities

C. Chiappini^{1,2}, R. Hirschi³, G. Meynet², S. Ekström², A. Maeder², and F. Matteucci⁴

¹ Osservatorio Astronomico di Trieste, via G. B. Tiepolo 11, 34131 Trieste, Italia
 e-mail: Christina.Chiappini@obs.unige.ch

² Observatoire Astronomique de l'Université de Genève, 1290 Sauverny, Switzerland

³ Dept. of Physics and Astronomy, University of Basel, 4056 Basel, Switzerland

⁴ Dipartimento di Astronomia, Università degli Studi di Trieste, via G. B. Tiepolo 11, 34131 Trieste, Italia

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ABSTRACT

We investigate the effect of new stellar models taking rotation into account and computing for a metallicity $Z = 10^{-8}$ on the chemical evolution of the earliest phases of the Milky Way. These models were computed under the assumption that the ratio of the initial rotation velocity to the critical velocity of stars is roughly constant with metallicity. This naturally leads to faster rotation at lower metallicity, as metal-poor stars are more compact than metal-rich ones. We find that the new $Z = 10^{-8}$ stellar yields have a tremendous impact on the nitrogen enrichment of the interstellar medium for $\log(\text{O}/\text{H}) + 12 < 7$ (or $[\text{Fe}/\text{H}] < -3$). We show that including $Z = 10^{-8}$ stellar yields in chemical evolution models, both high N/O and C/O ratios are obtained in the very-metal poor metallicity range, in agreement with observations. Our results give further support to the idea that stars at very low metallicities could have rotational velocities of the order of 600–800 km s⁻¹.

Key words. stars: rotation – Galaxy: evolution

1. The still unexplored very low metallicity range

The extremely metal-poor halo stars ($[\text{Fe}/\text{H}] < -3$, $\log(\text{O}/\text{H}) + 12 < 7$, Cayrel et al. 2004; Beers & Christlieb 2005; Spite et al. 2005, hereafter S05) are not only relics of the earliest phases of the formation of the Milky Way (MW), but also represent the lowest metallicities measured in the universe. So far, high- z observations were not able to find objects in this very low metallicity range. Therefore, it is in the halo of the MW that we have the unique possibility of studying the evolution of abundance ratios at metallicities never explored before.

In particular, until very recently the nitrogen enrichment in low metallicity environments could be studied only through HII region abundances in the outer parts of spiral galaxies or in blue compact galaxies (both showing $\log(\text{O}/\text{H}) + 12 \geq 7$, e.g. Izotov et al. 2006) or Damped Lyman Alpha systems (DLAs, $[\text{Fe}/\text{H}] > -2.5$, Wolfe et al. 2005). The very metal-poor halo stars of S05 are unique since they provide the opportunity to study the nitrogen enrichment of the interstellar medium (ISM) at metallicities never explored before in chemical evolution models¹. New effects are observed at such low metallicities. Recent measurements of nitrogen abundances in

metal-poor stars (S05) show a high N/O ratio suggesting high levels of production of primary nitrogen in massive stars. Moreover, the N/O abundance ratios in metal-poor stars exhibit a large scatter (roughly 1 dex, much larger than their quoted error bars), although none of the stars measured so far has N/O ratios as low as the ones observed in DLAs.

In a recent paper Chiappini et al. (2005, hereafter CMB05) studied the implications of this new data set on our understanding of nitrogen enrichment in the MW. By the time the latter paper was published there was no set of stellar yields able to explain the very metal-poor data of S05. Using the so-called population III stellar yields available in the literature did not solve the problem either (see also Ballero et al. 2006 for chemical evolution models computed with Pop. III stellar yields provided by different authors). In CMB05 it was concluded that the only way to account for the new data was to assume that stars at low metallicity rotate sufficiently fast to enable massive stars to contribute much larger amounts of nitrogen. It was predicted that massive stars born with metallicities below $Z = 10^{-5}$ should produce a factor of 10 up to a few times 10² more nitrogen (depending on the stellar mass) than the values given by Meynet & Maeder (2002a, hereafter MM02) for $Z = 10^{-5}$ and $v_{\text{rot}}^{\text{ini}} = 300 \text{ km s}^{-1}$. The physical motivation for this would be an increase in the rotational velocity in very metal-poor stars (Maeder et al. 1999; Meynet et al. 2006) and hence an increase in the nitrogen yields (MM02 – see below). In this framework it is possible to understand the apparently contradictory finding

¹ In this Letter we concentrate on the so-called *normal* very metal-poor stars, since our goal is to explain the mean ISM enrichment at this low metallicity range. Thus, we will not address the carbon-rich-ultra-metal-poor stars – see Beers & Christlieb (2005) for a recent review on this subject.

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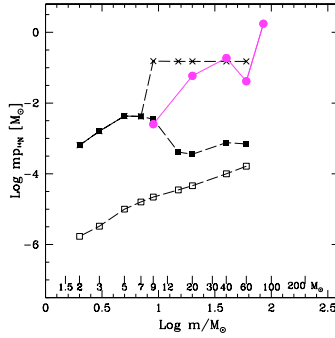


Fig. 1. Stellar yields for ^{14}N at low metallicities for the whole stellar mass range. The yields of MM02 for stellar models with and without rotation for $Z = 10^{-5}$ are shown by filled and open squares, respectively. The asterisks connected by the long-dashed line show the ad hoc stellar yields adopted for $Z < 10^{-5}$ in the heuristic model of CMB05 (see text). The dots show the new stellar yields computed for massive stars born at $Z = 10^{-8}$. The new computations of Hirschi (2006) for $Z = 10^{-8}$ lead to a large increase in N yields for masses above $\sim 20 M_{\odot}$, similar to the predictions of CMB05. This agreement is striking considering that they were obtained from completely different approaches.

by S05 of a large scatter in N/O and the almost complete lack of scatter in $[\alpha/\text{Fe}]$ ratios of the same very metal-poor halo stars (Cayrel et al. 2004). In fact, the scatter in the N/O abundance ratios could be related to the distribution of the stellar rotational velocities as a function of metallicity.

Whether the above suggestions were physically plausible remained to be assessed by stellar evolution models computed at lower metallicities, which took rotation and mass loss into account. New stellar evolution models have been computed for metallicities $Z = 10^{-8}$ (Meynet et al. 2006; Hirschi 2006) for massive stars. The new calculations show that, if the stars at all Z start their lives on the zero age main sequence (ZAMS) with on average a fraction of ~ 0.5 the critical velocity, the low Z stars easily reach break-up velocity during MS evolution. Fast rotation also contributes producing a more efficient mixing at lower Z , thereby leading to a large production of N in massive stars. In this Letter we show the impact of these new stellar yields on the chemical enrichment of C, N, and O of the earliest phases of the MW. The new stellar yields are presented in Sect. 2. Our results are shown in Sect. 3 and our conclusions drawn in Sect. 4.

2. New yields for very metal-poor massive stars

This Letter focuses in the very metal-poor range (below $[\text{Fe}/\text{H}] \simeq -3$ or $\log(\text{O}/\text{H}) + 12 \simeq 7$) where no intermediate mass star would have had time to contribute to the ISM enrichment (according to our non-stochastic approach). The effect of rotation on the evolution of low and intermediate mass stars (LIMS) and their impact on the galactic ISM at higher metallicities will be discussed elsewhere. Here we adopt the stellar yields for LIMS given in MM02 but aware of the limitations already discussed in Chiappini et al. (2003).

For massive stars we adopt the new yields shown in Fig. 1. Filled squares show stellar yields of MM02 for their lowest

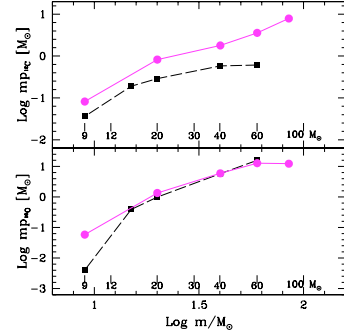


Fig. 2. Stellar yields for ^{12}C (upper panel) and for ^{16}O (lower panel) for massive stars. Curves labeled as in Fig. 1.

metallicity case ($Z = 10^{-5}$) resulting from models with rotation ($v_{\text{rot}}^{\text{ini}} = 300 \text{ km s}^{-1}$), while open symbols stand for models computed with $v_{\text{rot}}^{\text{ini}} = 0 \text{ km s}^{-1}$. The asterisks connected by the long-dashed line show the ad hoc stellar yields for metallicities $Z < 10^{-5}$ adopted in the heuristic model of CMB05. The dots show the new results obtained by Hirschi (2006; see also Meynet et al. 2006) for $Z = 10^{-8}$. The stellar yields for the $Z = 10^{-8}$ case were computed according to the following assumption: stars begin their evolution on the ZAMS with approximately the same angular momentum content, regardless of their metallicity. At solar metallicity, observations indicate a mean rotational velocity of a $60 M_{\odot}$ star on the MS of the order of 200 km s^{-1} , which corresponds to an initial angular momentum of the order of $2 \times 10^{53} \text{ g cm}^2 \text{ s}^{-1}$ (see Meynet et al. 2006 for details). At a metallicity of $Z = 10^{-8}$, this corresponds to $v_{\text{rot}}^{\text{ini}} = 800 \text{ km s}^{-1}$. In other words, we adopt a rotational velocity such that the ratio between $v_{\text{rot}}^{\text{ini}}/v_{\text{breakup}}$ remains almost constant (around 0.5) with mass and metallicity (see Hirschi 2006 for details). The very interesting result is that the new computations by Hirschi (2006) for $Z = 10^{-8}$ predict a large increase in the N yields for stars above $20 M_{\odot}$, similar to the ad hoc yields of CMB05 (see Fig. 1). Figure 2 shows a comparison of the stellar yields from stellar evolution models with rotation for $Z = 10^{-8}$ ($v_{\text{rot}}^{\text{ini}} = 800 \text{ km s}^{-1}$) and 10^{-5} ($v_{\text{rot}}^{\text{ini}} = 300 \text{ km s}^{-1}$) for ^{12}C (upper panel) and ^{16}O (lower panel). The carbon yields for the $Z = 10^{-8}$ case are systematically higher than the ones at $Z = 10^{-5}$ (although by significantly lower factors than in the case of ^{14}N – see Sect. 3). The differences between the $Z = 10^{-8}$ and 10^{-5} yields shown in Figs. 1 and 2 are to be ascribed mainly to the different $v_{\text{rot}}^{\text{ini}}$ adopted in each case, the physics involved being essentially the same.

3. Chemical evolution at very low metallicities

The adopted chemical evolution model for the MW is the so-called “two-infall model”, which is the same model as was used in CMB05. Figure 3 shows our model predictions for the evolution of nitrogen and carbon for different assumptions of stellar yields at low metallicity. The solid curves show a model computed with MM02 stellar yields (as in Chiappini et al. 2003²). This model was computed under the assumption that the $Z = 10^{-5}$ yield table ($v_{\text{rot}}^{\text{ini}} = 300 \text{ km s}^{-1}$) would be valid

² For the solar metallicity we are adopting the new calculations presented in Hirschi et al. (2004).

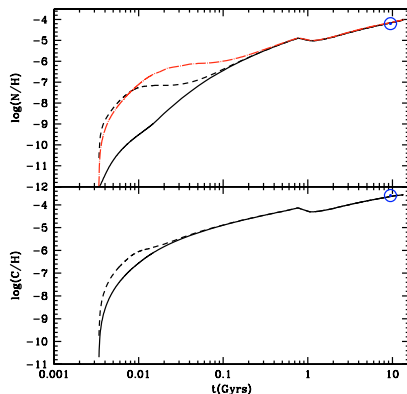


Fig. 3. Predicted evolution of nitrogen (*upper panel*) and carbon (*lower panel*) according to a chemical evolution model computed with different stellar yield sets for metallicities below $Z = 10^{-5}$: **a)** solid line – a model computed under the assumption that the lowest metallicity yield table of MM02 ($Z = 10^{-5}$) is valid down to $Z = 0$; **b)** dot-dashed line (red in the online version) – the model of CMB05 where an ad hoc higher yield of nitrogen is assumed for metallicities below 10^{-5} ; **c)** dashed line – the new model presented in this Letter adopting new stellar yields computed by new stellar evolution models with faster rotation for $Z = 10^{-8}$ massive stars.

down to $Z = 0$. As can be seen in Fig. 4, this model cannot explain the high levels of N/O or the C/O upturn observed in the very metal-poor halo stars of S05³. The dot-dashed line shows the *heuristic model* of CMB05. This model is the same as the one represented by the solid line except that for metallicities $Z < 10^{-5}$ the yields of nitrogen were strongly increased in comparison to the ones given in MM02 for massive stars (the adopted yields in the case of this model are shown in Fig. 1 by the asterisks connected by a long-dashed line). As a consequence, this model produces more nitrogen at the beginning of galaxy evolution (as shown in Fig. 3, upper panel, dot-dashed curve), leading to large N/O ratios at low metallicities (Fig. 4, upper panel, dot-dashed curve). However, it was unclear whether stellar evolution models at such low metallicities could predict such a large enhancement of nitrogen and what would be the impact for C and O.

The dashed curves in Figs. 3 and 4 show our most recent model computed with the new stellar yields at $Z = 10^{-8}$ of Hirschi (2006) for massive stars, assuming them to be valid down to $Z = 0$ ⁴. The evolution of nitrogen predicted by this model (dashed line in Fig. 3, upper panel) is similar to the one predicted by the CMB05 model (dot-dashed line), except for the later times (as expected since the ad hoc yields of CMB05 are higher than the ones of Hirschi for masses below $20 M_{\odot}$).

³ One of the main assumptions when comparing chemical evolution predictions with abundance data is that they represent the pristine abundances from the ISM, from which the stars formed. Therefore, objects that could have undergone mixing processes should be avoided. The data shown here are in principle *unmixed stars* (S05).

⁴ The physics adopted in the $Z = 10^{-8}$ models should be valid down to $Z \sim 10^{-10}$, which represents the metallicity limit below which massive stars first enter the phase of H-burning via the pp chain, followed by the 3α reaction, which then allows the CNO cycle to proceed, as in $Z = 0$ (Pop. III) stars.

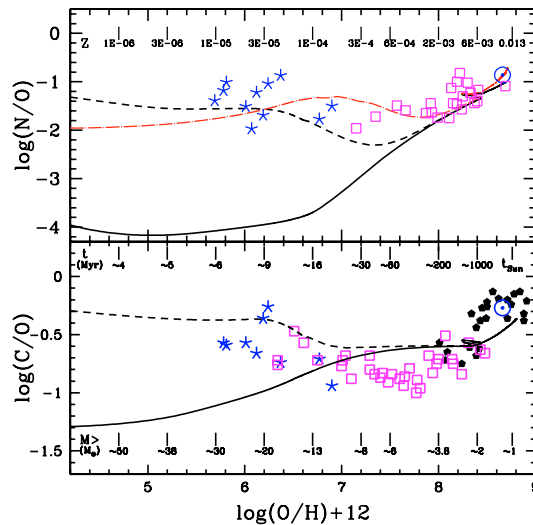


Fig. 4. *Upper panel:* solar vicinity diagram $\log(N/O)$ vs. $\log(O/H) + 12$. The data points are from Israelian et al. (2004 – large squares), S05 (asterisks). Models are labeled as in Fig. 3. *Lower panel:* solar vicinity diagram $\log(C/O)$ vs. $\log(O/H) + 12$. The data are from Spite et al. (S05 – asterisks), Israelian et al. (2004 – squares), Nissen (2004 – filled pentagons). Solar abundances (Asplund 2005, and references therein) are also shown.

The similarity of both curves is striking as they were obtained following completely independent approaches. This result implies that faster rotation is able to account for the S05 observations. Some differences are also seen for the C evolution (see Fig. 3, lower panel). Our new model predicts a C/O upturn at low metallicities (Fig. 4, lower panel, dashed curve). This upturn results from the strong production of primary nitrogen. Indeed, high production of primary nitrogen implies a very active H-burning shell, which contributes a large part of the total luminosity of the star. As a consequence, part of the total luminosity compensated by the energy produced in the helium core is reduced, making the average core temperature and thus the efficiency of the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction lower. More efficient mixing also leads to greater mass loss, decreasing the He-core size. Higher C/O ratios are thus obtained at the end of the He-burning phase.

We note that the effects obtained in this Letter, namely, the high N/O and the C/O upturn at very low metallicities can be explained without invoking Pop. III stars (i.e. without changing the IMF or including zero-metallicity stellar yields) and hence do not necessarily imply the signature of Pop. III stars as previously claimed in the literature (e.g. Akerman et al. 2004). The only shortcoming of the present model is that it predicts slightly higher [C/Fe] with respect to the observations of S05 (around 0.3 dex larger at $[\text{Fe}/\text{H}] \sim -3$ up to 0.9 dex at $[\text{Fe}/\text{H}] \sim -4$). However, in this case there are two large uncertainties, namely a) the stellar yields for Fe in massive stars (strongly dependent on the adopted mass cut) and b) the [C/Fe] ratios of S05 could still be affected by uncertainties due to the first dredge-up dilution of C on the giant branch (Iben 1965) and NLTE/3D corrections.

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Intermediate mass stars of this same low metallicity could also produce large amounts of nitrogen, if this production is linked to high rotational velocities. In this case, a flatter curve in the N/O diagram would be obtained. In fact, as can be seen in Fig. 4, the agreement between model predictions for N/O and C/O observations worsens at $\log(\text{O}/\text{H}) + 12 > 7$, where the AGB contribution starts to be effective. Super AGB stars, not included in the present Letter, could also contribute to the N enrichment of the ISM (Siess, priv. comm.). Stellar evolution models with rotation for LIMS computed up to the final evolutionary phases are being developed and will soon be available to be incorporated in chemical evolution models.

4. Discussion and conclusions

In this Letter, we computed chemical evolution models adopting the very recent calculations of Hirschi (2006) for the evolution of massive stars at very low metallicities under the assumption of an almost constant ratio $v_{\text{rot}}^{\text{ini}}/v_{\text{breakup}}$ as a function of metallicity (i.e. where the $v_{\text{rot}}^{\text{ini}}$ increases towards lower metallicities). In such a framework, massive stars can produce large amounts of nitrogen (of the order of the ones suggested in CMB05).

At present, this is the only way to explain the high nitrogen abundances measured recently in *normal* halo stars (S05). This gives further support to the idea that stars rotate faster at very low metallicities. The new stellar evolution models also produce some extra carbon at $Z \leq 10^{-5}$. As a consequence, an upturn is produced in the C/O at $\log(\text{O}/\text{H}) + 12 < 7$ metallicities. This result is obtained without the need of introducing Pop. III stars. Finally, our results show that the C and N yields strongly depend on metallicity for $Z \leq 10^{-5}$, once rotation is taken into account. This has a strong impact on the chemical enrichment of the very metal-poor ISM. It is thus important to avoid extrapolating the stellar yields in this metallicity range when computing chemical evolution models, even if the timespan to reach $Z = 10^{-5}$ is just a few Myrs.

The only alternative explanation of the observations considered in this Letter would be if AGB stars contributed to the ISM enrichment before $[\text{Fe}/\text{H}] \sim -3$. This would mean that the timescales for the chemical evolution of the halo would be very different from the ones adopted here. Our model assumes that the halo was formed by an exponential infall with

an e-folding time of ~ 1 Gyr. The enrichment timescales do not change if the infall timescale is increased by a factor of 2 (nor if the star formation efficiency is divided by 2). Models assuming outflows could delay the ISM enrichment so that AGBs would have time to contribute. However, in this case it should be seen if such models could still fit the halo abundance ratios well. The chemical evolution model presented here (without outflows) also agrees well with several other abundance ratios (François et al. 2004).

Here we show the results for the mean ISM enrichment history, assuming that at a given metallicity all stars of a given mass rotate at the same velocity. Further computations are in progress, and we envision being able to compute chemical evolution models in which a distribution of rotational velocities is assumed for each metallicity. These models would predict a scatter in N/O to be compared to the observed one (forthcoming paper).

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The early star generations: the dominant effect of rotation on the CNO yields

G. Meynet, S. Ekström, and A. Maeder

Geneva Observatory, 1290 Sauverny, Switzerland
 e-mail: georges.meynet@obs.unige.ch

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ABSTRACT

Aims. We examine the role of rotation on the evolution and chemical yields of very metal-poor stars.

Methods. The models include the same physics, which was applied successfully at the solar Z and for the SMC, in particular, shear diffusion, meridional circulation, horizontal turbulence, and rotationally enhanced mass loss.

Results. Models of very low Z experience a much stronger internal mixing in all phases than at solar Z . Also, rotating models at very low Z , contrary to the usual considerations, show a large mass loss, which mainly results from the efficient mixing of the products of the 3α reaction into the H-burning shell. This mixing allows convective dredge-up to enrich the stellar surface in heavy elements during the red supergiant phase, which in turn favours a large loss of mass by stellar winds, especially as rotation also increases the duration of this phase. On the whole, the low Z stars may lose about half of their mass. Massive stars initially rotating at half of their critical velocity are likely to avoid the pair-instability supernova. The chemical composition of the rotationally enhanced winds of very low Z stars show large CNO enhancements by factors of 10^3 to 10^7 , together with large excesses of ^{13}C and ^{17}O and moderate amounts of Na and Al. The excesses of primary N are particularly striking. When these ejecta from the rotationally enhanced winds are diluted with the supernova ejecta from the corresponding CO cores, we find $[\text{C}/\text{Fe}]$, $[\text{N}/\text{Fe}]$, $[\text{O}/\text{Fe}]$ abundance ratios that are very similar to those observed in the C-rich, extremely metal-poor stars (CEMP). We show that rotating AGB stars and rotating massive stars have about the same effects on the CNO enhancements. Abundances of s-process elements and the $^{12}\text{C}/^{13}\text{C}$ ratio could help us to distinguish between contributions from AGB and massive stars.

Key words. stars: evolution – stars: rotation – stars: abundances – stars: mass loss – nuclear reactions, nucleosynthesis, abundances

1. Introduction

Interest in the evolution of extremely metal-poor stars has been stimulated recently by at least two types of observing programmes. First, the detection of very far galaxies at redshifts well beyond 6 (see e.g. Pelló et al. 2005) opens the way to detection of galaxies whose colours will be dominated by extremely metal-poor stars (Schaerer 2002, 2003). Second, as a complement to the observation of the deep Universe, the detection of nearby, very metal-poor halo stars provides very interesting clues to the early chemical evolution of our Galaxy (Beers et al. 1992; Beers 1999; Christlieb et al. 2004; Bessell et al. 2004; Cayrel et al. 2004; Spite et al. 2005). These works have shown the following very interesting results:

- *The measured abundances of many elements at very low metallicity present a very small scatter (Cayrel et al. 2004).* At first sight this appears difficult to understand. Indeed, in the early Universe, stars are believed to form from the ejecta of a small number of supernovae (may be only one). For instance the Argast et al. models (2000, 2002) predict that for $[\text{Fe}/\text{H}] < -3$, type II supernovae enrich the

interstellar medium only locally. Since the chemical composition of supernova ejecta may differ a lot from case to case, large scatter of the abundances is expected at very low metallicity. For most of the elements, however, this strong scatter is not observed, at least down to a metallicity of $[\text{Fe}/\text{H}] \sim -4.0$ (Cayrel et al. 2004). This might indicate that, already at this low metallicity, stars are formed from a well-mixed reservoir composed of ejecta from stars of different initial masses.

- *These observations also show that there is no sign of enrichments by pair-instability supernovae, at least down to a metallicity of $[\text{Fe}/\text{H}]$ equal to -4 .* Let us recall that these supernovae have very massive stars as progenitors, with initial masses between approximately 140 and 260 $M_{\odot}$ (Barkat et al. 1967; Bond et al. 1984; Heger & Woosley 2002). Such massive stars are believed to form only in very metal-poor environments. At the end of their lifetime, they are completely destroyed when they explode as a pair-instability supernova. In this way they may strongly contribute to the enrichment of the primordial interstellar medium in heavy elements. The composition of the ejecta of pair-instability

supernovae is characterised by a well marked odd-even effect and a strong zinc deficiency. These two features are not observed in the abundance pattern of very metal-poor halo stars. Does this mean that at $[\text{Fe}/\text{H}]$ equal to -4 , pair-instability supernovae no longer dominate the chemical evolution of galaxies or that such stars are not formed? If formed, could these stars skip the pair instability or have different nucleosynthetic outputs from those currently predicted by theoretical models?

- *The N/O ratios observed at the surface of halo stars by Israelian et al. (2004) and Spite et al. (2005) indicate that important amounts of primary nitrogen should be produced by very metal-poor massive stars (Chiappini et al. 2005).* The physical conditions for such important productions of primary nitrogen by very metal-poor massive stars remain to be found.
- *If most stars at a given $[\text{Fe}/\text{H}]$ present a great homogeneity in composition, a small group, comprising about 20–25% of the stars with $[\text{Fe}/\text{H}]$ below -2.5 , show very large enrichments in carbon.* These stars are known as C-rich extremely metal-poor (CEMP) stars. The observed $[\text{C}/\text{Fe}]$ ratios are between ~ 2 and 4 , showing a large scatter. Other elements, such as nitrogen and oxygen (at least in the few cases where the abundance of this element could be measured), are also highly enhanced. Interestingly, the two most metal-poor stars known up to now, the Christlieb star or HE 0107-5240, a halo giant with $[\text{Fe}/\text{H}] = -5.3$, and the subgiant or main-sequence star HE 1327-2326 with $[\text{Fe}/\text{H}] = -5.4$ (Frebel et al. 2005) belong in this category. To explain such high and scattered CNO abundances, obviously a special process has to be invoked (see Sect. 6 below).

The results outlined above clearly indicate that new scenarios for the formation and evolution of massive stars at very low Z need to be explored.

Among the physical ingredients that could open new evolutionary paths in very metal-poor environments, rotation certainly appears a very interesting possibility. First, for metallicities Z between 0.004 and 0.040 , the inclusion of rotation improves the agreement between the models and observations in many respects by allowing us to reproduce the observed surface abundances (Heger & Langer 2000; Meynet & Maeder 2000), the ratio of blue-to-red supergiants at low metallicity (Maeder & Meynet 2001), the variation with the metallicity of the WR/O ratios and of the numbers of type Ibc to type II supernovae (Meynet & Maeder 2003, 2005). Most likely, stars are also rotating at very low metallicity, and one can hope that the same physical model assumptions that improve the physical description of stars at $Z \geq 0.004$ would also apply to the very low metallicity domain.

Second, if the effects of rotation are already quite significant at high metallicity, one expects that they are even more important at lower metallicity. For instance, it was shown in previous works that the chemical mixing becomes more efficient for lower metallicity for a given initial mass and velocity (Maeder & Meynet 2001; Meynet & Maeder 2002). This comes from the fact that the gradients of Ω are much steeper in the lower metallicity models, so they trigger more efficient

shear mixing. The gradients are steeper because less angular momentum is transported outwards by the meridional currents, whose velocity scales as the inverse of the density in the outer layers (see the Gratton-Öpik term in the expression for the meridional velocity in Maeder & Zahn 1998).

Third, rotation can induce mass loss in two ways. The first way, paradoxically, is linked to the fact that very metal-poor stars are believed to lose little mass by radiatively driven stellar winds. Indeed, in the radiatively driven wind theory, the mass loss scales with the metallicity of the outer stellar layers as $(Z/Z_\odot)^\alpha$ with α between 0.5 and 0.8 (Kudritzki et al. 1987; Vink et al. 2001). Thus lowering the metallicity by a factor $200\,000$ (as would be the case for obtaining the metallicity of the Christlieb star) would thus lower the mass loss rates by a factor 450 , or even by a greater factor if the metallicity dependence becomes stronger at lower Z , as suggested by Kudritzki (2002). Now since metal-poor stars lose little mass, they also lose little angular momentum (if rotating), so they have a greater chance of reaching the break-up limit during the Main Sequence phase (see for instance Fig. 9 in Meynet & Maeder 2002). At break-up, the outer stellar layers become unbound and are ejected whatever their metallicity. The break-up is reached more easily when we take into account that massive rotating stars have polar winds as shown by Owocki et al. (1996) and Maeder (1999). When most of the mass is lost along the rotational axis, little angular momentum is lost.

Another way for rotation to trigger enhancements of the mass loss comes from the mixing induced by rotation. In general, rotational mixing favours the evolution into the red supergiant stage (see Maeder & Meynet 2001), where mass loss is higher. It also enhances the metallicity of the surface of the star and, in this way, boosts the radiatively driven stellar winds (see below).

Thus there are very good reasons for exploring the effects of rotation at very low metallicity, which we have attempted in this work. This was also the aim of the recent work by Marigo et al. (2003), who compute Pop III massive stellar models with rotation, assuming solid-body rotation. In this very interesting piece of work, they mainly study the effects of reaching the break-up limit. However, since they did not include the rotational mixing of the chemical elements, they could not explore the effects of rotation on the internal chemical composition of the stars. Also, solid body rotation is just the extreme case of coupling the internal rotation, which ignores the physics and timescales of the internal transport. In the present models, the transport of both the angular momentum and the chemical species are treated in a consistent way, and, as we shall see, rotational mixing has very important consequences on both the stellar yields and the mass loss rates.

In Sect. 2, we briefly recall the main physical ingredients of the models. The evolutions of fast-rotating massive star models at very low Z are described in Sect. 3. The evolution of the internal chemical composition is the subject of Sect. 4, while the ejected masses of various isotopes are presented in Sect. 5. The case of the CEMP stars is discussed in Sect. 6. Section 7 summarises the main results and raises a few questions to be explored in future works.

2. Physical ingredients

The computation of our models was done with the Geneva evolution code. The opacities were taken from Iglesias & Roger (1996) and complemented at low temperatures by the molecular opacities of Alexander (<http://web.physics.twsu.edu/alex/wwwdra.htm>). The nuclear reaction rates were based on the NACRE data basis (Angulo et al. 1999). The treatment of rotation included the hydrostatic effects described in Meynet & Maeder (1997) and the effects of rotation on mass loss rates according to Maeder & Meynet (2000). In particular, we accounted for the wind anisotropies induced by rotation as in Maeder (1999). Meridional circulation was implemented according to Maeder & Zahn (1998), but including the new D_h coefficient as described in Maeder (2003). Roughly compared to the old D_h , the new one tends to reduce the size of the convective core and to allow larger enrichment of the surface in CNO-processed elements. The reader is referred to these papers for a detailed description of the effects. The convective instability was treated according to the Schwarzschild criterion without overshooting. The radiative mass loss rates are from Kudritzki & Puls (2000) when $\log T_{\text{eff}} > 3.95$ and from de Jager et al. (1988) otherwise. The mass loss rates depend on metallicity as $\dot{M} \sim (Z/Z_\odot)^{0.5}$, where Z is the mass fraction of heavy elements at the surface of the star. As we shall see, this quantity may change during the evolution of the star.

A specific treatment for mass loss was applied at break-up. At break-up, the mass loss rate adjusts itself in such a way that an equilibrium is reached between the two following opposite effects. 1) The radial inflation due to evolution, combined with the growth of the surface velocity due to the internal coupling by meridional circulation, brings the star to break-up, and thus some amount of mass at the surface is no longer bound to the star. 2) By removing the most external layers, mass loss brings the stellar surface down to a level in the star that is no longer critical. Thus, at break-up, we should adapt the mass loss rates, in order to maintain the surface layers at the break-up limit. In practice, however, since the critical limit contains mathematical singularities, we considered that during the break-up phase, the mass loss rates should be such that the model stays near a constant fraction (for example, 0.98) of the limit. At the end of the MS phase, the stellar radius inflates so rapidly that meridional circulation is unable to continue to ensure the internal coupling, and the break-up phase ceases naturally.

In this first exploratory work, we focused our attention on stars with initial masses of $60 M_\odot$ and $7 M_\odot$ in order to gain insight into the properties of both massive and AGB stars at low Z . The evolution was computed until the end of the core helium burning phase (core carbon burning phase in the case of the $60 M_\odot$ rotating model at $Z = 10^{-8}$). Two metallicities were considered for the $60 M_\odot$ models: $Z = 10^{-8}$ and $Z = 10^{-5}$. Only this last metallicity was considered for the $7 M_\odot$ model. Of course we do not know if stars with $Z = 10^{-8}$ have ever formed; however, it is not possible at the present time to exclude such a possibility. Indeed it might be that the first star generations produce very little amounts of heavy elements, due to the strong fallback of ejected material onto black holes at the

Table 1. Initial abundances in mass fraction for models at $Z = 10^{-8}$.

Element	Initial abundance
H	0.75999996
^3He	0.00002554
^4He	0.23997448
^{12}C	7.5500e-10
^{13}C	0.1000e-10
^{14}N	2.3358e-10
^{15}N	0.0092e-10
^{16}O	67.100e-10
^{17}O	0.0300e-10
^{18}O	0.1500e-10
^{19}F	0.0020e-10
^{20}Ne	7.8368e-10
^{21}Ne	0.0200e-10
^{22}Ne	0.6306e-10
^{23}Na	0.0882e-10
^{24}Mg	3.2474e-10
^{25}Mg	0.4268e-10
^{26}Mg	0.4897e-10
^{27}Al	0.1400e-10
^{28}Si	3.2800e-10
^{56}Fe	3.1675e-10

end of their lifetimes. Moreover, as we shall see, the behaviour of the $Z = 10^{-5}$ and 10^{-8} massive star models are qualitatively similar, indicating that the evolutionary scenarios explored here might apply to a broad range of initial metallicities.

The initial mixture of heavy elements was taken as equal to the one used to compute the opacity tables (Iglesias & Roger 1996, Weiss alpha-enhanced elements mixture). The initial composition for models at $Z = 10^{-8}$ is given in Table 1. The models at $Z = 10^{-5}$ have the same initial mixture of heavy elements. More precisely, the mass fractions for all the isotopes heavier than ^4He were multiplied by 10^3 ($=10^{-5}/10^{-8}$).

Nothing is known on the rotational velocities of such stars. However, there are some indirect indications that stars at lower Z could have higher initial rotational velocities: 1) realistic simulations of the formation of the first stars in the Universe show that the problem of the dissipation of the angular momentum is more severe at very low Z than at the solar Z . Thus these stars might begin their evolution with a higher amount of angular momentum (Abel et al. 2002). 2) There are some observational hints that the distribution of initial rotation might contain more fast rotators at lower Z (Maeder et al. 1999). 3) Even if stars begin their life on the ZAMS with the same total amount of angular momentum for all metallicities, then the stars at lower metallicity rotate faster as a consequence of their smaller radii.

The three arguments listed above would favour the choice of a higher value for the initial rotational velocity than those adopted for solar models. To choose this value we proceeded in the following way. First we supposed that the stars begin their evolution on the ZAMS with approximately the same

Table 2. Properties of the stellar models at the end of the H- and He-burning phases. M_{ini} is the initial mass, Z the initial metallicity, v_{ini} the initial velocity, $\bar{v}$ the mean equatorial rotational velocity during the MS phase defined as in Meynet & Maeder (2000), t_{H} the H-burning lifetimes, M the actual mass of the star, v the actual rotational velocity at the stage considered, Y_{s} the helium surface abundance in mass fraction. N/C and N/O are the ratios of nitrogen to carbon, respectively, of nitrogen to oxygen at the surface of stars in mass fraction; C, N, O are the abundances of carbon, nitrogen, and oxygen at the surface in mass fractions. The numbers in parentheses indicate the power of ten, i.e., $7.54(-10) = 7.54 \times 10^{-10}$.

M_{ini} $M_{\odot}$	Z	v_{ini} km s^{-1}	$\bar{v}$ km s^{-1}	End of H-burning						End of He-burning						
				t_{H} Myr	M $M_{\odot}$	v km s^{-1}	Y_{s}	N/C	N/O	t_{He} Myr	M $M_{\odot}$	v km s^{-1}	Y_{s}	C	N	O
60	10^{-8}	0	0	3.605	59.817	0	0.24	0.31	0.03	0.292	59.726	0	0.24	$7.54(-10)$	$2.34(-10)$	$6.71(-9)$
60	10^{-8}	800	719	4.004	57.624	591	0.27	103	9.77	0.522	23.988	0.02	0.76	$1.97(-4)$	$1.02(-2)$	$2.85(-4)$
60	10^{-5}	800	636	4.441	53.846	567	0.34	40	0.82	0.544	37.280	0.57	0.80	$5.06(-5)$	$2.07(-3)$	$4.24(-5)$

angular momentum content, whatever the metallicity. At solar metallicity, observation provides values for the mean observed rotational velocity on the MS phase (around 200 km s^{-1} for OB stars). Stellar models allowed us to estimate the initial angular momentum required to achieve such values (around $2.2\text{--}2.5 \times 10^{53} \text{ g cm}^2 \text{ s}^{-1}$). Adopting such an initial value of the angular momentum, we found that a $60 M_{\odot}$ stellar model at $Z = 10^{-8}$ has a velocity on the ZAMS of 800 km s^{-1} . This is the value of the initial velocity we adopt in the present work.

3. Evolution of a massive rotating star at very low metallicity

3.1. Rotation and mass loss during the main sequence phase

Figure 1 shows the evolutionary tracks during the main sequence (MS) phase for the $60 M_{\odot}$ stellar models at $Z = 10^{-8}$. Table 2 presents some properties of the models at the end of the core H- and He-burning phases. From Fig. 1, we see that rotation produces a small shift of the tracks toward lower luminosities and T_{eff} . This effect is due to both atmospheric distortions (note that surface-averaged effective temperatures are plotted in Fig. 1 as explained in Meynet & Maeder 2000) and to the lowering of the effective gravity (see e.g. Kippenhahn & Thomas 1970; Maeder & Peytremann 1970; Collins & Sonneborn 1977). The MS lifetime of the rotating model is enhanced by 11%.

These results show that rotation does not affect the UV outputs of very metal-poor massive stars much (the UV outputs of the first massive star generations might contribute a lot to the reionization of the early Universe, see e.g. Madau 2003). Only if a significant fraction of primordial stars would rotate so fast that they follow the path of homogeneous evolution (Maeder 1987), could rotation increase the ionizing power. In that case, the star would remain in the blue part of the HR diagram and would have a much longer lifetime.

Figure 2 shows the evolution of the ratio $\Omega/\Omega_{\text{crit}}$ at the surface during the MS phase. At $Z = 10^{-8}$, the model with $v_{\text{ini}} = 800 \text{ km s}^{-1}$ reaches the break-up limit when the mass fraction of hydrogen at the centre $X_{\text{c}} \approx 0.40$. The star stays at break-up for the remaining of its MS life with an enhanced

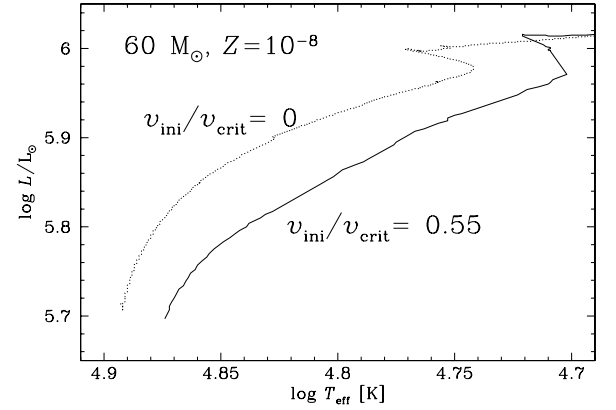


Fig. 1. Evolutionary tracks in the HR diagram for $60 M_{\odot}$ stellar models at $Z = 10^{-8}$.

mass loss rate. As a consequence, the model ends its MS life with $57.6 M_{\odot}$, having lost 4% of its initial mass. Despite the star staying in the vicinity of the break-up limit during an important part of its MS lifetime, it does not lose very large amounts of mass, due to the fact that only the outermost layers of the stars are above the break-up limit and are ejected. These layers have low density and thus contain little mass.

A model with the same initial velocity, but with a metallicity three orders of magnitude higher, reaches the break-up limit very early in the MS phase, when $X_{\text{c}} \approx 0.56$. This comes from the fact that when the metallicity increases, a given value of the initial velocity corresponds to a higher initial value of the $v_{\text{ini}}/v_{\text{crit}}$ ratio. The model at $Z = 10^{-5}$ ends its MS life with $53.8 M_{\odot}$, having lost 10% of its initial mass.

In order to discuss the effects of a change of rotation, it is interesting to compare this last result with the one obtained in Meynet & Maeder (2002) for a $60 M_{\odot}$ model at $Z = 10^{-5}$ with $v_{\text{ini}} = 300 \text{ km s}^{-1}$. This last model reaches the break-up velocity much later, only when $X_{\text{c}} \approx 0.01$. At $Z = 10^{-5}$, the velocity 300 km s^{-1} thus appears as the lower limit for the initial rotation, allowing a $60 M_{\odot}$ star to reach the break-up limit during its MS phase. The $60 M_{\odot}$ star ends its MS life with $59.7 M_{\odot}$, having lost only 0.5% of its initial mass. Note that the 2002 models were computed with slightly different

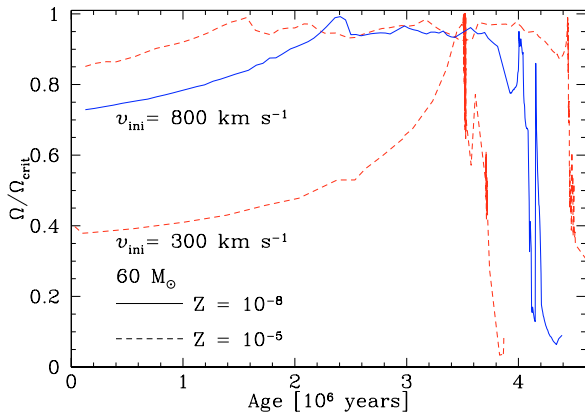


Fig. 2. Evolution of $\Omega/\Omega_{\text{crit}}$ at the surface of $60 M_{\odot}$ models at $Z = 10^{-8}$ (continuous line) and $Z = 10^{-5}$ (upper dashed line) with $v_{\text{ini}} = 800 \text{ km s}^{-1}$. The case of the $60 M_{\odot}$ model at $Z = 10^{-5}$ with $v_{\text{ini}} = 300 \text{ km s}^{-1}$ from Meynet & Maeder (2002) is also shown (lower dashed line).

physical ingredients than those used to compute the models discussed in this paper (different expression for D_h and for the mass loss rates). However, at these very low metallicities, radiatively driven winds remain quite modest, and the transport of the angular momentum, mainly driven by the meridional circulation, does not depend much on the expression of D_h .

During the MS phase, the surface of the rotating stars is enriched in nitrogen and depleted in carbon as a result of rotational mixing. Figure 3 shows that the N/C ratios are enhanced by more than two orders of magnitude at the end of the H-burning phase. More precisely, at the surface of the $Z = 10^{-8}$ model, nitrogen is enhanced by a factor 27 and carbon decreased by a factor 12. All other physical ingredients being the same, one also sees that the model with the lowest metallicity is also the one with the greatest surface enrichments. This well agrees with the trend already found in our previous works (Maeder & Meynet 2001; Meynet & Maeder 2002), which results from the steep gradients of angular velocity that build up in very metal-poor stars that favours shear mixing. Let us emphasise, however, that during the MS phase, the arrival of CNO-processed material at the surface does not change the metallicity of the outer layers. Indeed, rotational mixing brings nitrogen to the surface but depletes carbon and oxygen, keeping the sum of CNO elements constant, and the metallicity as well. Thus during the MS phase, the enhanced mass loss rates undergone by the rotating models are entirely due to the mechanical effect of the centrifugal force. As we shall see, this is no longer the case during the core He-burning phase.

3.2. Rotation and mass loss during the post main-sequence phases

Rotational mixing that occurs during the MS phase and still continues to be active during the core He-burning phase deeply modifies the internal chemical composition of the star. This has important consequences during the post-MS phases and deeply changes the evolution of the rotating models with respect to

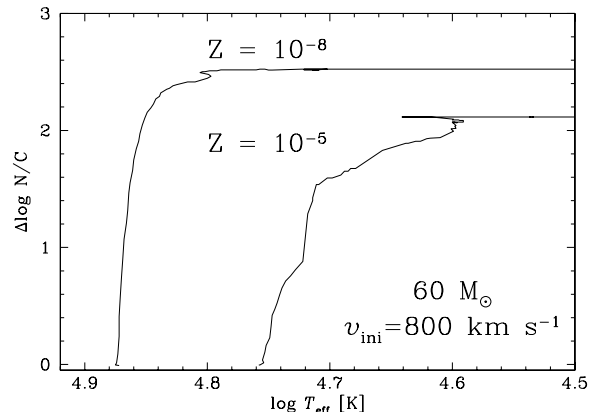


Fig. 3. Evolution as a function of $\log T_{\text{eff}}$ of the excess at the surface for the ratio N/C (expressed in dex) compared to the initial ratio. N and C are the abundances of nitrogen and carbon at the surface of the star.

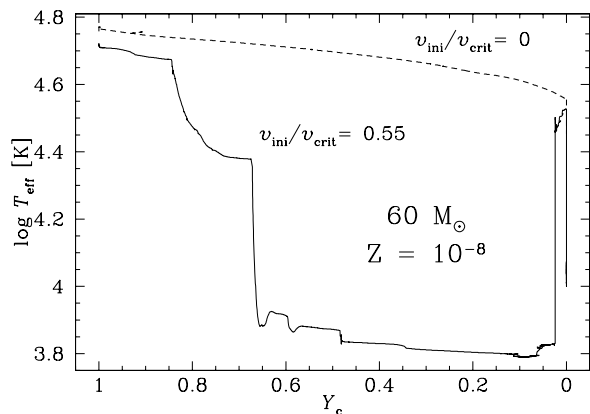


Fig. 4. Evolution of $\log T_{\text{eff}}$ as a function of Y_c , the mass fraction of ^4He at the centre, for a non-rotating (dashed line) and rotating (continuous line) $60 M_{\odot}$ model at $Z = 10^{-8}$.

their non-rotating counterparts. Among the most striking differences, one notes the following:

- 1) Rotation favours redwards evolution in the HR diagram as was already shown by Maeder & Meynet (2001), and as illustrated in Fig. 4. One sees that the non-rotating model remains on the blue side during the whole core He-burning phase, while the 800 km s^{-1} model at $Z = 10^{-8}$ starts its journey toward the red side of the HR diagram early in the core helium burning stage, when $Y_c \approx 0.67$ (Y_c is the mass fraction of helium at the centre of the star model). The same is true for the corresponding model at $Z = 10^{-5}$. Let us recall that this behaviour is linked to the rapid disappearance of the intermediate convective zone associated to the H-burning shell (see Fig. 5 and Maeder & Meynet 2001).
- 2) Redwards evolution enhances the mass loss. In the cases of our $60 M_{\odot}$ stellar models, it brings the stars near the Humphreys-Davidson limit, i.e., near $\log L/L_{\odot} = 6$ and T_{eff} in a broad range around 10^4 K . Near this limit, the mass loss rates (here from de Jager et al. 1988) become very important. For instance, the model represented in the left

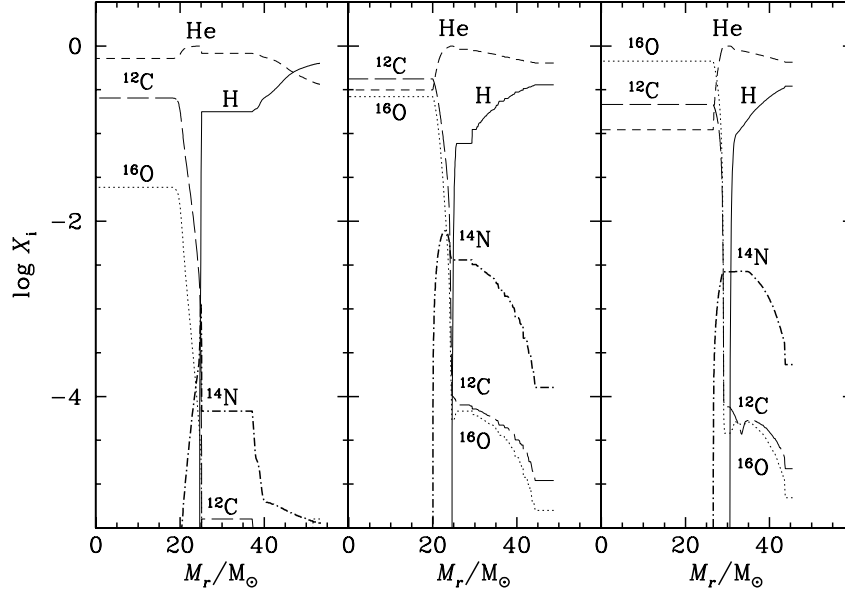


Fig. 5. Chemical composition of a $60 M_{\odot}$ stellar model at $Z = 10^{-5}$ with $v_{\text{ini}} = 800 \text{ km s}^{-1}$ when it evolves from the blue to the red part of the HR diagram. The model shown in the left panel has $\log L/L_{\odot} = 6.129$ and $\log T_{\text{eff}} = 4.243$; in the middle panel, it has $\log L/L_{\odot} = 6.130$ and $\log T_{\text{eff}} = 4.047$; in the right panel, it has $\log L/L_{\odot} = 6.145$ and $\log T_{\text{eff}} = 3.853$.

- panel of Fig. 5 ($\log L/L_{\odot} = 6.129$, $\log T_{\text{eff}} = 4.243$) is still far to the left hand side of the Humphreys-Davidson limit. Its mass loss rate is $\log(-\dot{M}) = -5.467$, where $\dot{M}$ is expressed in $M_{\odot}$ per year. The model in the right panel ($\log L/L_{\odot} = 6.145$, $\log T_{\text{eff}} = 3.853$) is in the vicinity of the Humphreys-Davidson limit. Its mass loss rate is equal to -4.616 , i.e., more than seven times higher. During this transition, the overall metallicity at the surface does not change and remains equal to the initial one (here $Z_{\text{ini}} = 0.00001$). We observe a similar transition in the case of the $Z = 10^{-8}$ stellar model.
- 3) During the core He-burning phase, primary nitrogen is synthesized in the H-burning shell, due to the rotational diffusion of carbon and oxygen produced in the helium core into the H-burning shell (Meynet & Maeder 2002). This is illustrated well in Fig. 5 for the $Z = 10^{-5}$ rotating model and in Fig. 6 for the model at $Z = 10^{-8}$.
 - 4) In contrast to what happens during the MS phase, rotational mixing during the core He-burning phase induces large changes in the surface metallicity. These changes occur only at the end of the core He-burning phase, although the conditions for their apparition result from the mixing that occurs during the whole core He-burning phase. Indeed, rotational mixing progressively enriches the outer radiative zone in CNO elements, thus enhancing its opacity slowly. When, in the $Z = 10^{-8}$ stellar model, the abundance of nitrogen in the outer layers becomes approximately 10^{-8} in mass fraction (i.e., has increased by two orders of magnitude with respect to the initial value), these outer layers become convective. The outer convective zone then rapidly deepens in mass and dredges up newly synthesized elements to the surface. From this stage onwards, the surface metallicity increases in a spectacular way, as can be seen in Fig. 6. For instance, the

rotating $60 M_{\odot}$ at $Z = 10^{-8}$ has a surface metallicity of 10^{-2} at the end of its lifetime, i.e., similar to that of the Large Magellanic Cloud!

- 5) The consequence of such large surface enrichments on the mass loss rates remains to be studied in detail using models of stellar winds with the appropriate physical characteristics (position in the HR diagram and chemical composition). In the absence of such sophisticated models, we applied the usual rule here, namely $\dot{M}(Z) = (Z/Z_{\odot})^{1/2} \dot{M}(Z_{\odot})$, where Z is the metallicity of the outer layers. With this prescription, the surface enhancement of the metallicity is responsible for the large decrease in the stellar mass that can be seen in Fig. 6.
- 6) During the late stages of the core helium-burning phase, as a result of mass loss and mixing, the star may evolve along a blue loop in the HR diagram (see Fig. 4). When the star evolves bluewards, the global stellar contraction brings the outer convective zone, which evolves like a solid body rotating shell to break-up (Heger & Langer 1998). At this stage of the evolution, the luminosity is not far from the Eddington limit and the star may reach the $\Omega\Gamma$ -limit (Maeder & Meynet 2000). This multiplies the mass loss rates by very large factors.
- 7) During the last 24 000 years of its lifetime, the model presents abundance patterns characteristic of WNL stars at its surface.
- 8) As a result of mixing and mass loss rates, the duration of the core He-burning phase is much longer in the rotating model. The present $60 M_{\odot}$ model with $v_{\text{ini}} = 800 \text{ km s}^{-1}$ at $Z = 10^{-8}$ has a helium-burning lifetime that is $\sim 80\%$ longer than the corresponding lifetime of the non-rotating model (see Sect. 4 below for more explanations).

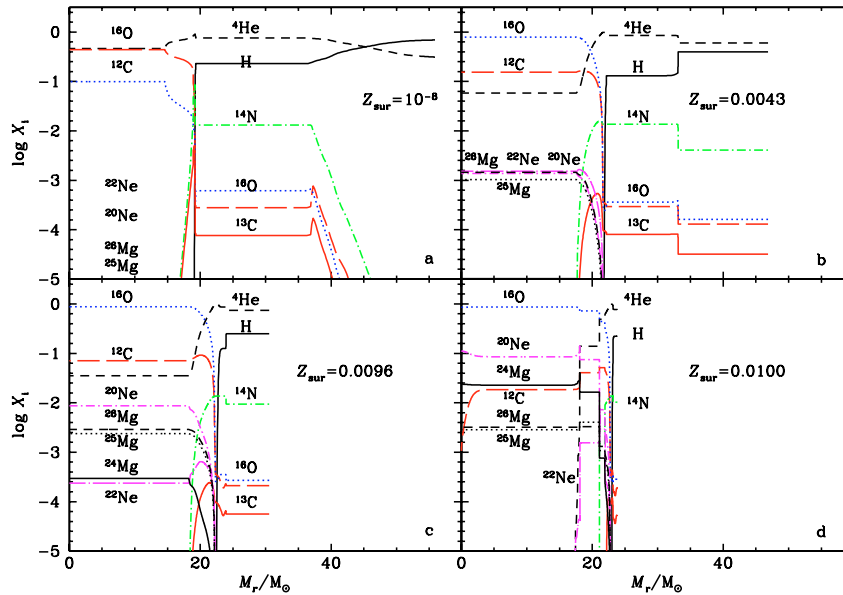


Fig. 6. Variations of the abundances (in mass fraction) as a function of the Lagrangian mass within a $60 M_{\odot}$ star with $v_{\text{ini}} = 800 \text{ km s}^{-1}$ and $Z = 10^{-8}$. The four panels show the chemical composition at four different stages at the end of the core He-burning phase: in panel **a**) the model has a mass fraction of helium at the centre, $Y_c = 0.11$ and an actual mass $M = 54.8 M_{\odot}$. **b**) $Y_c = 0.06$, $M = 48.3 M_{\odot}$. **c**) $Y_c = 0.04$, $M = 31.5 M_{\odot}$. **d**) End of the core C-burning phase, $M = 23.8 M_{\odot}$. The actual surface metallicity Z_{surf} is indicated in each panel.

The different effects described above are all due to the mixing induced by rotation and they all tend to enhance the quantity of mass lost by stellar winds. The rotating $60 M_{\odot}$ at $Z = 10^{-8}$ loses about $36 M_{\odot}$ during its lifetime. About $2 M_{\odot}$ are lost due to break-up during the MS phase, $\sim 3 M_{\odot}$ are lost when the star is in the red part of the HR diagram (with surface metallicity equal to the initial one), $27 M_{\odot}$ are lost due to the effect of the enhancement of the surface metallicity, the remaining $4 M_{\odot}$ are lost when the star evolves along the blue loop and reaches the $\Omega\Gamma$ -limit. One sees that, by far, the most important effect is due to the increase in the surface metallicity.

Paradoxically the corresponding model at higher metallicity ($Z = 10^{-5}$) loses less mass (a little less than 40% of the total mass). This can be understood from the following facts: first less primary nitrogen is synthesized due to slightly less efficient chemical mixing when the metallicity increases, thus the surface does not become as metal rich as in the model at $Z = 10^{-8}$. Second and for the same reason as above, the outer convective zone does not deepen as far as in the more metal-poor model. These two factors imply that the maximum surface metallicity reached in this model, which is equal to 0.0025, is about a factor 4 below the one reached by the $Z = 10^{-8}$ model. Finally, the blue loop does not extend that far into the blue side, and the surface velocity always remains well below the break-up limit during the whole blueward excursion.

In order to investigate to what extent the behaviour described above depends on the physical ingredients of the model, we compare the present results with those of a rotating model ($v_{\text{ini}} = 800 \text{ km s}^{-1}$) of a $60 M_{\odot}$ star at $Z = 10^{-5}$ with a different prescription for the mass loss rates (Vink et al. 2000, 2001 instead of Kudritzki & Puls 2000), with the Ledoux criterion instead of the Schwarzschild one for determining the size of

the convective core, with a core overshoot of $\alpha = 0.2 H_p$ and the old prescription for the horizontal diffusion coefficient D_h . This model is described in Meynet et al. (2005). In this case, the outer convective zone deepens farther into the stellar interior and thus produces a greater enhancement of the surface metallicity (the same order as the one we obtained in the present $Z = 10^{-8}$ $60 M_{\odot}$ model). Higher enhancements of the surface metallicity then induces greater mass loss by stellar winds. More important than these differences, however, we shall retain here that the results are qualitatively similar to those obtained in our previous models. In particular, the mechanism of surface metallicity enhancement occurs in both models and appears to be a robust process.

3.3. Do very metal-poor, very massive stars end their lives as pair-instability supernovae?

Might the important mass loss undergone by the rotating models prevent the most massive stars from going through pair instability? According to Heger & Woosley (2002), progenitors of pair-instability supernovae have helium core masses between ~ 64 and $133 M_{\odot}$. This corresponds to initial masses between about 140 and $260 M_{\odot}$. Thus the question is whether stars with initial masses above $140 M_{\odot}$ can lose a sufficient amount of mass to have a helium core that is less than about $64 M_{\odot}$ at the end of the core He-burning phase. From the values quoted above, it would imply the loss of more than $(140 - 64) = 76 M_{\odot}$, which represents about 54% of the initial stellar mass. From the computation performed here, one can expect that such a scenario is possible, where a $60 M_{\odot}$ loses more than 60% of its initial mass. However, more extensive computations are needed to check whether the rotational mass loss could indeed

prevent the most massive stars from going through this pair instability. Were this the case, it would explain why the nucleosynthetic signature of pair-instability supernovae is not observed in the abundance pattern of the most metal-poor halo stars observed up to now. At least this mechanism could restrain the mass range for the progenitors of pair-instability supernovae, pushing the minimum initial mass needed for such a scenario to occur to higher values. Let us also note that the luminosity of the star comes nearer to the Eddington limit when the initial mass increases. When rotating, such stars will then encounter the $\Omega\Gamma$ -limit (Maeder & Meynet 2000) and very likely undergo strong mass losses.

4. Evolution of the interior chemical composition

As discussed above, rotational mixing changes the chemical composition of stellar interiors in an important way. This is illustrated well by Figs. 6 and 7, which show the internal chemical composition of our rotating and non-rotating $60 M_{\odot}$ stellar models at four different stages at the end of the core He-burning phase (models at $Z = 10^{-8}$).

Comparing panels a of Figs. 6 and 7, one sees that a large convective shell is associated to the H-burning shell in the rotating model, while such a shell is absent in the non-rotating model. This contrasts with what happens at the beginning of the core He-burning phase, where the intermediate convective zone associated to the H-burning shell was absent in the rotating model (or at least much smaller), while in the non-rotating model, the intermediate convective zone was well-developed (see above). Why is there this difference between the beginning and the end of the core He-burning phase? At the beginning of the core He-burning phase, the disappearance of the intermediate convective shell was a consequence of the rotational mixing that operated during the core H-burning phase and that brought some freshly synthesized helium into this region. More helium in this region means less hydrogen and also some decrease in the opacity, both of which inhibit the development of convection (cf. Maeder & Meynet 2001). Now, at the end of the core He-burning phase, we have He-burning products that are brought into the H-burning shell. These products, mainly carbon and oxygen, act as catalysts for the CNO cycle and make the H-burning shell more active, thus favouring convection. This mechanism enriches this zone not only in primary ^{14}N , but also in primary ^{13}C . Looking at the H-rich envelope in the non-rotating model at the same stage, one sees that all these elements have much lower abundances. Actually they fall well below the minimum ordinate of the figure.

If one now compares the chemical composition of the CO cores when $Y_c \sim 0.11$ (see panels a of Figs. 6 and 7), one notes the following points. First, the abundances in ^{12}C , ^{16}O and ^{20}Ne are approximately equal in both the rotating and non-rotating models. This comes from the fact that the CO core masses are approximately the same in both models. On the other hand in the rotating model, the abundance of ^{22}Ne is greatly enhanced, as the abundances of ^{25}Mg and ^{26}Mg . The abundance of ^{22}Ne results from the conversion of primary ^{14}N , which has diffused into the He-burning core. Thus the resulting high abundance of ^{22}Ne is also of primary origin. The isotopes

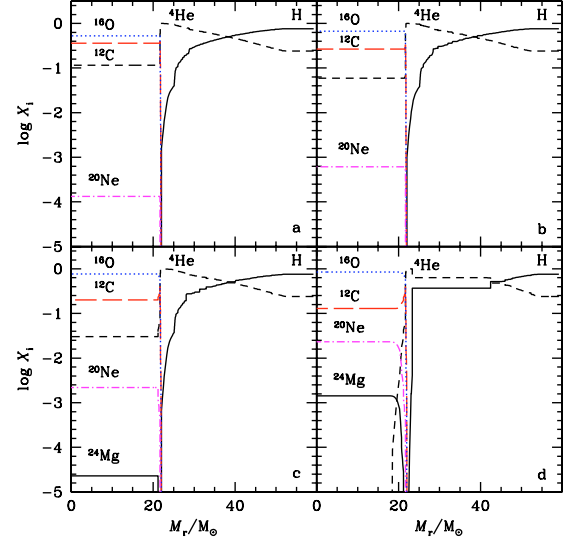


Fig. 7. Same as Fig. 6 for a $60 M_{\odot}$ star with $v_{\text{ini}} = 0 \text{ km s}^{-1}$ and $Z = 10^{-8}$. The four panels show the chemical composition at four different stages at the end of the core He-burning phase: Panel a) $Y_c = 0.12$, $M = 59.74 M_{\odot}$. b) $Y_c = 0.06$, $M = 59.74 M_{\odot}$. c) $Y_c = 0.03$, $M = 59.73 M_{\odot}$. d) $Y_c = 0.0$, $M = 59.73 M_{\odot}$. The surface metallicity is equal to 10^{-8} at the four evolutionary stages.

of magnesium are produced by the reactions $^{22}\text{Ne}(\alpha, \gamma)^{26}\text{Mg}$ and $^{22}\text{Ne}(\alpha, n)^{25}\text{Mg}$. Their high abundances also result from primary nitrogen diffusion into the He-core.

The CO-core mass (cf. Table 3) in the rotating model is slightly smaller than in the non-rotating one. This contrasts to what happens at higher metallicity, where rotation tends to increase the CO-core mass (see Hirschi et al. 2004). Again, this results from the mechanism of primary nitrogen production, which induces a large convective zone associated to the H-burning shell, which then prevents this shell from migrating outwards and, thus, the CO core from growing in mass. Let us recall that in rotating models at solar metallicity, there is no primary nitrogen production due to the less efficient mixing at higher metallicity (see Meynet & Maeder 2002); thus there is no increase in the H-burning shell activity.

In panel b of Fig. 6, as explained in the previous section, one sees the outer convective zone extending inwards and bringing CNO elements to the surface. In panels c and d, mass-loss efficiently removes these outer layers. At the corresponding stages in the non-rotating model, the outer envelope is not enriched in heavy elements and keeps its mass.

At the end of the He-burning phase (see panels d), the abundance of ^{12}C is significantly smaller in the rotating model than in the non-rotating one. At the same time, the abundances of ^{20}Ne and ^{24}Mg are significantly greater. This is a consequence of helium diffusion into the He-core at the end of the He-burning phase. Let us recall that ^{12}C is destroyed by alpha capture (to produce ^{16}O), while ^{20}Ne and ^{24}Mg are produced by alpha captures on, respectively, ^{16}O and ^{20}Ne . For what concerns the other isotopes of neon and magnesium, one sees that in the rotating models, much higher abundances of ^{25}Mg and ^{26}Mg are reached due to the transformation of the ^{22}Ne at

Table 3. Helium-, CO-core mass and mass of the remnants (respectively m_α , m_{CO} , and m_{rem}) of 60 $M_\odot$ stellar models with and without rotation at $Z = 10^{-8}$. The total mass ejected (m_{ej}) and the mass ejected of various chemical species ($m(X_i)$) are given in solar masses. The values of some isotope ratios (in mass fractions) are also indicated. The case of matter ejected by stellar winds only is distinguished from the case of matter ejected by both the stellar winds and the supernova explosion.

	$M_{\text{ini}}/M_\odot$	Z	v_{ini} [km s ⁻¹]	$M_{\text{ini}}/M_\odot$	Z	v_{ini} [km s ⁻¹]
	60	10^{-8}	0	60	10^{-8}	800
m_α	23.08			23.83		
m_{CO}	21.61			18.04		
m_{rem}	6.65			5.56		
	Mass ejected			Mass ejected		
	WIND	SN+WIND		WIND	SN+WIND	
m_{ej}	0.28	53.35		36.17	54.44	
$m(^4\text{He})$	6.62e-02	19.58		21.46	23.85	
$m(^{12}\text{C})$	2.08e-10	2.066		4.78e-03	4.26e-01	
$m(^{13}\text{C})$	2.84e-12	5.84e-09		1.25e-03	1.37e-03	
$m(^{14}\text{N})$	6.44e-11	1.90e-07		1.97e-01	2.20e-01	
$m(^{16}\text{O})$	1.85e-09	12.61		6.08e-03	13.54	
$m(^{17}\text{O})$	8.27e-13	6.39e-10		7.50e-06	8.60e-06	
$m(^{18}\text{O})$	4.14e-12	6.53e-10		1.58e-08	4.44e-03	
	Isotopic ratios			Isotopic ratios		
$^{12}\text{C}/^{13}\text{C}$	73.24	3.54e+08		3.82	311	
N/C	0.31	9.20e-08		41.2	0.52	
N/O	0.03	1.51e-08		32.4	0.02	

the end of the core helium-burning phase. The neutrons liberated by the $^{22}\text{Ne}(\alpha, n)^{25}\text{Mg}$ reaction can be captured by iron peak elements, producing some amount of s-process elements (see e.g. Baraffe et al. 1992). In view of the important changes to the interior chemical composition due to rotation, there is good chance that the s-process in the present rotating massive star models is quite different from the one obtained in non-rotating models. This will be examined in later papers.

5. Chemical composition of the winds and of the supernova ejecta

5.1. Wind composition

Let us first discuss the chemical composition of the winds. The total mass lost, as well as the quantities of various chemical elements ejected by stellar winds, are given in Tables 3 and 4. The models at $Z = 10^{-5}$ were computed with an extended nuclear reaction network including the Ne-Na and Mg-Al chains, which is why in Table 4 the wind-ejected masses of these elements can be indicated. The stellar yields – i.e., the mass of an isotope newly synthesized and ejected by the star – can be obtained by subtracting the mass of the isotope initially

Table 4. Same as Table 3 for rotating stellar models at $Z = 10^{-5}$. The two 60 $M_\odot$ models were computed with different physical ingredients, see text.

	$M_{\text{ini}}/M_\odot$	Z	v_{ini} [km s ⁻¹]	$M_{\text{ini}}/M_\odot$	Z	v_{ini} [km s ⁻¹]
	60	10^{-5}	800	60	10^{-5}	800
m_α	36.90			30.69		
m_{CO}	28.60			27.95		
m_{rem}	8.69			8.50		
	Mass ejected			Mass ejected		
	WIND	SN+WIND		WIND	SN+WIND	
m_{ej}	23.10	51.31		29.31	51.50	
$m(^4\text{He})$	12.20	18.99		12.60	14.58	
$m(^{12}\text{C})$	3.34e-04	5.84e-01		1.45e-02	2.46	
$m(^{13}\text{C})$	6.81e-05	1.47e-04		3.81e-03	2.58e-02	
$m(^{14}\text{N})$	9.78e-03	2.51e-02		4.29e-02	1.87e-01	
$m(^{15}\text{N})$	3.21e-07	2.15e-06		1.55e-06	1.68e-05	
$m(^{16}\text{O})$	2.72e-04	18.12		3.29e-02	17.32	
$m(^{17}\text{O})$	4.59e-07	2.40e-06		2.78e-05	1.32e-04	
$m(^{18}\text{O})$	2.82e-08	1.34e-03		1.63e-08	2.07e-04	
$m(^{19}\text{F})$	1.95e-09			1.10e-08		
$m(^{20}\text{Ne})$	7.64e-06			1.29e-05		
$m(^{21}\text{Ne})$	1.84e-08			6.99e-08		
$m(^{22}\text{Ne})$	4.80e-07			3.35e-05		
$m(^{23}\text{Na})$	1.22e-06			5.61e-06		
$m(^{24}\text{Mg})$	4.41e-06			6.21e-06		
$m(^{25}\text{Mg})$	3.95e-07			6.96e-07		
$m(^{26}\text{Mg})$	5.68e-07			3.07e-06		
$m(^{27}\text{Al})$	5.49e-06			7.75e-06		
	Isotopic ratios			Isotopic ratios		
$^{12}\text{C}/^{13}\text{C}$	4.90	3970		3.81	95.3	
N/C	29.3	0.04		2.96	0.08	
N/O	36.0	0.001		1.30	0.01	

present in that part of the star¹ from the ejected masses given in Tables 3 and 4.

According to Table 3, the non-rotating model ejects only half a percent of its total mass through stellar winds, which is completely negligible. Moreover, this material has exactly the same chemical composition as does the protostellar cloud from which the star formed. If, at the end of its lifetime, all the stellar material is swallowed by the black-hole resulting from the star collapse, the nucleosynthetic contribution of such stars would be zero. In contrast, the rotating models lose more than 60% of their initial mass through stellar winds. This material is strongly enriched in CNO elements. Even if all the final stellar mass is engulfed into a black-hole at the end of

¹ This quantity may be obtained by multiplying the initial abundance of the isotope considered (given in Table 1) by m_{ej} .

the evolution, the nucleosynthetic contribution of such stars remains quite significant. As already noted above, the corresponding model at $Z = 10^{-5}$ loses less mass by stellar winds (see Table 4). However, the amounts of mass lost remain large and they present strong enrichments in CNO elements, as in the case of the $Z = 10^{-8}$ rotating model. Also ^{23}Na and ^{27}Al are somewhat enhanced in the wind material.

Other striking differences between the rotating and non-rotating models concern the $^{12}\text{C}/^{13}\text{C}$, N/C, and N/O ratios (see Tables 3 and 4). The wind of the non-rotating model shows solar ratios ($^{12}\text{C}/^{13}\text{C} = 73$, N/C = 0.31, and N/O = 0.03 in mass fractions). The wind of rotating models is characterised by very low $^{12}\text{C}/^{13}\text{C}$ ratios, around 4–5 (close to the equilibrium value of the CN cycle) and by very high N/C (between about 3 and 40) and N/O ratios (between 1 and 36). Thus wind material presents the signature of heavily CNO-processed material.

5.2. Total ejecta composition (wind and supernova ejecta)

In order to estimate the quantity of mass lost at the time of the supernova explosion (if any), it is necessary to know the mass of the remnant. This quantity is estimated with the relation of Arnett (1991) between the mass of the remnant and the mass of the carbon-oxygen core. The masses of the different elements ejected are then simply obtained by integrating their abundance in the final model between m_{rem} (see Tables 3 and 4) and the surface. Since the evolution of the present models was stopped before the presupernova stage was reached, the masses of ^{12}C and ^{16}O obtained here might still be somewhat modified by the more advanced nuclear phases.

How does the contribution of the two models (rotating and non-rotating) at $Z = 10^{-8}$ compare when both the wind and the supernova contribute to the ejection of the stellar material? First, one sees that the total mass ejected (through winds and supernova explosion) is very similar (on the order of $54\text{--}55 M_{\odot}$), due to the fact that the two models have similar CO core masses. Second, one sees that the amount of ^4He ejected by the rotating model is slightly higher, whereas the amount of ^{12}C is lower due to the effect discussed above (α -captures at the end of the core He-burning phase). The quantity of ^{16}O ejected is similar in both models. Third, the most important differences between the rotating and non-rotating models occur for ^{13}C , ^{14}N , ^{17}O , and ^{18}O . The abundances of these isotopes are increased by factors between $10^4\text{--}10^7$ in the ejecta of the rotating model. The first three isotopes are produced in the H-burning shell (CNO cycle) and are mainly ejected by the winds, while the last one, produced at the interface between the CO-core and the He-burning shell, is ejected at the time of the supernova explosion. Fourth, rotation also deeply affects the ratios of light elements in the ejected material (see Tables 3 and 4). The effects of rotation are qualitatively similar to those obtained when comparing the composition of the wind material of rotating and non-rotating stellar models. Rotation decreases the $^{12}\text{C}/^{13}\text{C}$ ratios from 3.5×10^8 in the non-rotating case to 311 in the rotating case, while it increases the N/C and N/O ratios,

which have values of $\sim 10^{-7}$ and 10^{-8} respectively, when $v_{\text{ini}} = 0 \text{ km s}^{-1}$, and of 0.5 and 0.02 when $v_{\text{ini}} = 800 \text{ km s}^{-1}$.

In the ejecta of rotating models, composed of both wind and supernova material, the $^{12}\text{C}/^{13}\text{C}$ ratio is higher than in pure wind material, and the N/C and N/O ratios are smaller. This comes from the fact that the supernova ejecta are rich in helium-burning products characterized by a very high $^{12}\text{C}/^{13}\text{C}$ ratio and by very low N/C and N/O ratios.

At this point we can ask if the rotating star had lost no mass through stellar winds, and if all the stellar material were ejected at the time of the supernova explosion, would the composition of the ejecta be different with respect to the case discussed above, where part of the material is ejected by the winds and part by the supernova explosion. Let us recall that stellar winds remove layers from the stars at an earlier evolutionary stage than do supernova explosions. If some of these layers, instead of being ejected by the winds, had remained locked inside the star, they would have been processed further by the nuclear reactions. Thus their composition at the end of the stellar evolution would be different from the one obtained if they had been ejected at an earlier time by the winds. Obviously for such differences to be important, mass loss must remove the layers at a sufficiently early time. If it does so only at the very end of the evolution, there would be no chance for the layers to be processed much by the nuclear reactions, and there would be no significant difference whether the mass were ejected by the winds or by the supernova explosion. Actually, this is what happens in our rotating models. As indicated above, the mass is removed at the very end of the He-burning phase, and only material from the H-rich envelope is ejected. Thus, if this material were ejected only at the time of the supernova explosion, it would have kept the same chemical composition as the one in Table 3.

As a result, the chemical composition of the ejecta (wind and supernova) does not depend much on the mass loss, but is deeply affected by rotation. However, the stellar winds may of course be of primary importance if the whole final mass of the star is swallowed in a black-hole at the end of the evolution. In that case, the star will contribute to the interstellar enrichment only by its winds.

Comparing the data given in the right part of Table 3 with the left part of Table 4, one can see the effect produced by an increase in the initial metallicity by three orders of magnitude (all other things being equal). Interestingly, we see that the differences between the two models are in general much smaller than those between the rotating and non-rotating model at a given metallicity. The total ejected mass, and the masses of ^4He , ^{12}C , ^{16}O are similar within factors between 0.8 and 1.4. The quantities of ^{14}N and ^{13}C are within an order of magnitude, and the masses of ^{17}O and ^{18}O differ by a factor 3. We are thus far from the factors $10^4\text{--}10^7$ between the results of the rotating and non-rotating models at $Z = 10^{-8}$! The effects of rotation at extremely low metallicity are much larger than the effects of a change in the initial Z content.

The results given on the right side of Table 4 corresponds to the model described in Meynet et al. (2005). It differs from the present models by the mass loss and mixing prescription (see Sect. 3.2). As already emphasized above, the results are

qualitatively very similar. However, quantitatively, they present some differences. For instance, the quantity of ^{12}C in the model presented on the right side of Table 4 is larger by a factor 4 compared to the value given on the left side of the same table. The right model presents a smaller helium core, an effect mainly due to higher mass loss rates. This favours larger ejections of carbon by the winds and also by the supernova, since smaller helium cores lead to higher C/O ratios at the end of the helium-burning phase. In the right model, the quantity of ^{16}O is decreased by about 4%. The ejected masses of ^{13}C and ^{17}O are increased by factors of 176 and 55, respectively. The masses of the other isotopes differ by less than an order of magnitude.

6. Link with the extremely metal-poor C-rich stars

6.1. Observations and existing interpretations

Spectroscopic surveys of very metal-poor stars (Beers et al. 1992; Beers 1999; Christlieb 2003) have shown that CEMP stars account for up to about 25% of stars with metallicities lower than $[\text{Fe}/\text{H}] \sim -2.5$ (see e.g. Lucatello et al. 2004). A star is said to be C-rich if $[\text{C}/\text{Fe}] > 1$. A large proportion of these CEMP stars also present enhancements in their neutron capture elements (mainly *s*-process elements). A few of them also appear to exhibit large enhancements in N and O. The most iron-deficient stars observed so far are CEMP stars. These stars are HE 0107-5240, a giant halo star, and HE 1327-2326, a dwarf or subgiant halo star. The star HE 0107-5240 ($[\text{Fe}/\text{H}] = -5.3$) presents the following CNO surface abundances: $[\text{C}/\text{Fe}] = 4.0$, $[\text{N}/\text{Fe}] = 2.3$, and $[\text{O}/\text{Fe}] = 2.4$ (Christlieb et al. 2004; Bessell et al. 2004). The ratio $^{12}\text{C}/^{13}\text{C}$ has also been tentatively estimated by Christlieb et al. (2004), who suggest a value of about 60, but with a great uncertainty. They can, however, rule out a value inferior to 50 (let us recall that the solar ratio is ~ 73). The star HE 1327-2326 has $[\text{Fe}/\text{H}] = -5.4$ and CNO surface abundances: $[\text{C}/\text{Fe}] = 4.1$, $[\text{N}/\text{Fe}] = 4.5$, $[\text{O}/\text{Fe}] < 4.0$ (Frebel et al. 2005).

The origin of the high carbon abundance is still an open question and various scenarios have been proposed:

1. **The primordial scenarios:** in this case the abundances observed at the surface of CEMP stars are the abundances of the cloud from which the star formed. The protostellar cloud was enriched in carbon by one or a few stars from a previous generation. For instance, Umeda & Nomoto (2003) propose that the cloud from which HE 0107-5240 formed was enriched by the ejecta of one Pop III $25 M_{\odot}$ star, which had exploded with low explosion energy (on the order of 3×10^{50} erg) and experienced strong mixing and fallback at the time of the supernova explosion. The mixing is necessary to create the observed high-level enrichments in light elements, and the fallback is necessary to retain a large part of the iron peak elements. Limongi et al. (2003) suggest that the cloud was enriched by the ejecta of two supernovae from progenitors with masses of about 15 and $35 M_{\odot}$.
2. **The accretion/mass transfer scenarios:** some authors have proposed that this particular abundance pattern results from accretion of interstellar material and from a companion (for instance an AGB star, as proposed by Suda et al. 2004). As far as the nucleosynthetic origin is concerned, this scenario is not fundamentally different from the first one.
3. **The in situ scenarios:** finally, some authors have explored the possibility that the star itself could have produced the particular abundance pattern seen at its surface (Picardi et al. 2004). The overabundance of nitrogen might easily be explained in the frame of this scenario, if the star had begun its evolution with the high carbon and oxygen overabundance. In fact we did perform a test calculation of a non-rotating $0.8 M_{\odot}$ stellar model at $[\text{Fe}/\text{H}] = -5.3$ with an initial value of $[\text{C}/\text{Fe}]$ and $[\text{O}/\text{Fe}]$ equal to 4.0 and 2.3, respectively, i.e. equal to the abundances observed at the surface of HE 0107-5240. We found that, when the star reaches the value of the effective temperature ($T_{\text{eff}} = 5100 \pm 150$ K) and of gravity ($\log g = 2.2 \pm 0.3$) of HE 0107-5240 (Christlieb et al. 2002), the surface nitrogen enrichment is well within the range of the observed values. However, it appears difficult to invoke similar processes to explain the high carbon and oxygen enhancements (see Picardi et al. 2004).

6.2. No “in situ” CN production

An abundance pattern typical of CEMP stars has been observed at the surface of non-evolved stars (Norris et al. 1997; Plez & Cohen 2005; Frebel et al. 2005). Among the most recent observations, let us mention the subgiant or dwarf star HE 1327-2326 (Frebel et al. 2005, see above) and the dwarf star G77-61 (Plez & Cohen 2005). The initial mass of G77-61 is estimated to be between 0.3 and $0.5 M_{\odot}$, and it has an $[\text{Fe}/\text{H}] = -4.03$, $[\text{C}/\text{Fe}] = 2.6$, $[\text{N}/\text{Fe}] = 2.6$, and a $^{12}\text{C}/^{13}\text{C}$ ratio of 5 ± 1 . In this case, there is no way for the star, which burns its hydrogen through the pp chains, to produce nitrogen. There is even less possibility of producing surface enhancements of carbon and oxygen. Therefore, the “in situ” scenario can be excluded, at least for this star. In that case, only the first and second scenarios are possible. The same is true for explaining the very high overabundances of carbon and nitrogen at the surface of HE 1327-2326.

The values observed at the surface of non-evolved and evolved stars are shown in Fig. 8. In the second case, the surface may have been depleted in carbon and oxygen and enriched in nitrogen due to the dredge-up that occurs along the red giant branch. On the other hand, in the case of the non-evolved stars, as explained above, this mechanism cannot be invoked, and the measured abundances reflect the abundances of the cloud that gave birth to the star. On the whole, the distributions of elements is similar for evolved and non-evolved stars, which favours the primordial scenario.

In the following, we explore the first two scenarios using our fast-rotating models. The abundance pattern observed at the surface of CEMP stars seems to be a mixture of hydrogen and helium burning products. Since rotation allows these products to coexist in the outer layers of stars (both in massive and

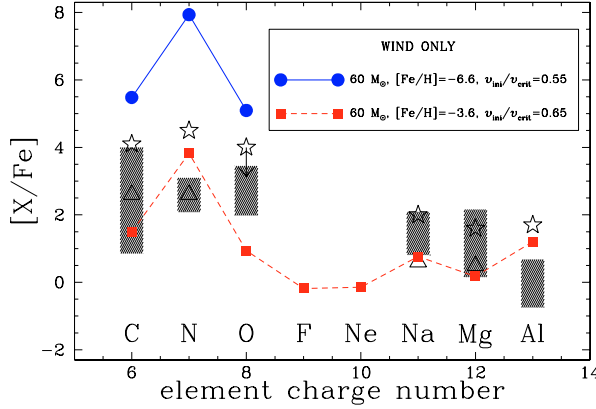


Fig. 8. Chemical composition of the wind of rotating $60 M_{\odot}$ models (solid circles and squares). The hatched areas correspond to the range of values measured at the surface of giant CEMP stars: HE 0107-5240, $[\text{Fe}/\text{H}] \approx -5.3$ (Christlieb et al. 2004); CS 22949-037, $[\text{Fe}/\text{H}] \approx -4.0$ (Norris et al. 2001; Depagne et al. 2002); CS 29498-043, $[\text{Fe}/\text{H}] \approx -3.5$ (Aoki et al. 2004). The empty triangles (Plez & Cohen 2005, $[\text{Fe}/\text{H}] \approx -4.0$) and stars (Frebel et al. 2005, $[\text{Fe}/\text{H}] \approx -5.4$, only an upper limit is given for $[\text{O}/\text{Fe}]$) correspond to non-evolved CEMP stars (see text).

intermediate mass stars), this seems a useful direction for our research.

6.3. Comparison with wind composition

Let us first see if the CEMP stars could be formed from material made up of massive star winds, or at least heavily enriched by winds of massive stars. At first sight, such a model might appear quite unrealistic, since the period of strong stellar winds is rapidly followed by the supernova ejection, which would add the ejecta of the supernova itself to the winds. However, for massive stars at the end of their nuclear lifetime, a black hole, that swallows the whole final mass, might be produced. In that case, the massive star would contribute to the local chemical enrichment of the interstellar medium only through its winds. Let us suppose that such a situation has occurred and that the small halo star we observe today was formed from the shock induced by the stellar winds with the interstellar material. What would its chemical composition be? Its iron content would be the same as the iron abundance of the massive star. Indeed, the iron abundance of the interstellar medium would have no time to change much in the brief massive star lifetime, and the massive star wind ejecta are neither depleted nor enriched in iron.

The abundances of the other elements in the stellar winds for our two rotating $60 M_{\odot}$ at $Z = 10^{-8}$ and 10^{-5} are shown in Fig. 8. The ordinate $[X/\text{Fe}]$ is given by the following expression:

$$[X/\text{Fe}] = \log \left(\frac{X}{X_{\odot}} \right) - \log \left(\frac{X(\text{Fe})}{X(\text{Fe})_{\odot}} \right),$$

where X is the mass fraction of element X in the wind ejecta, and $X_{\odot}$ in the Sun. Similarly, the symbols $X(\text{Fe})$ and $X(\text{Fe})_{\odot}$ refer to the mass fraction of ^{56}Fe in the wind material or in the Sun. Here we suppose that $\log(X(\text{Fe})/X(\text{Fe})_{\odot}) \sim [\text{Fe}/\text{H}]$, since

the mass fraction of hydrogen remains approximately constant, whatever the metallicity between $Z = 10^{-8}$ and 0.02. The values of $[\text{Fe}/\text{H}]$ are those corresponding to the initial metallicity of the models (for $Z = 10^{-8}$ one has $\log(X(\text{Fe})/X(\text{H})) = -9.38$, see Table 1). The solar abundances are those chosen by Christlieb et al. (2004) and Bessell et al. (2004) in their analysis of the star HE 0107-5240, and they correspond to the solar abundances obtained recently by Asplund et al. (2005). In particular, $\log(X(\text{Fe})/X(\text{H}))_{\odot} = -2.80$, thus $[\text{Fe}/\text{H}] = -6.6$ at $Z = 10^{-8}$, and $[\text{Fe}/\text{H}] = -3.6$ at $Z = 10^{-5}$.

From Fig. 8, we see that the winds are strongly enriched in CNO elements. The model at $Z = 10^{-5}$, computed with an extended nuclear reaction network, allows us to look at the abundances in the winds of heavier elements. The wind material is also somewhat enriched in Na and Al. Before comparing with the observations, let us first note:

- 1) The more metal-poor model is shifted toward higher values compared to the metal rich one. If, in both models, the mass fraction of element X were the same in the wind ejecta, then one would expect a shift by 3 dex when the iron content of the model goes from $[\text{Fe}/\text{H}] = -3.6$ to $[\text{Fe}/\text{H}] = -6.6$. The actual shift is approximately 3.6 dex, slightly more than the iron content difference of 3 dex between the two models. The additional 0.6 dex comes from the more efficient mixing in the metal-poorer models.
- 2) In the frame of the hypotheses made here, i.e. a halo star made from the wind ejecta that triggered its formation, we should compare the wind composition from a massive star model with the same initial iron content as in the halo star we considered. The range of iron contents in the models, $[\text{Fe}/\text{H}]$ equal from -6.6 to -3.6 , covers the range of iron content of the CEMP plotted in Fig. 8, whose $[\text{Fe}/\text{H}]$ are between -5.4 and -3.5 . However, the $[\text{Fe}/\text{H}] = -6.6$ model is well below the lower bound of the observed $[\text{Fe}/\text{H}]$, making this model less interesting for comparisons with the presently available observations. In that respect the $[\text{Fe}/\text{H}] = -3.6$ model, which has an iron content that is comparable to the iron-richest stars observed, is more interesting.
- 3) Any dilution with some amount of interstellar material would lower the abundance of the element X without changing the mass fraction of iron. In that case the values plotted in Fig. 8 are shifted to lower values, but the relative abundances of the elements will not change (as far as the main source of the elements considered are the wind ejecta).

Keeping in mind these three comments, it appears that what has to be compared with the observations are more the relative abundances between the CNO elements than the actual values of the $[X/\text{Fe}]$ ratios, which will depend on the initial metallicity of the model considered, as well as on the dilution factor.

From Fig. 8, and Tables 3 and 4, one sees that, for the two metallicities considered here, the wind material of rotating models is characterised by N/C and N/O ratios between ~ 1 and 40 and by $^{12}\text{C}/^{13}\text{C}$ ratios around 4–5. These values are compatible with the ratios observed at the surface of CS 22949-037 (Depagne et al. 2002): $\text{N}/\text{C} \sim 3$ and $^{12}\text{C}/^{13}\text{C} \sim 4$. The observed value for N/O (~ 0.2) is smaller than the range of

theoretical values, but greater than the solar ratio (~ 0.03). Thus the observed N/O ratio also bears the mark of some CNO processing, although slightly less pronounced than in our stellar wind models.

On the whole, a stellar wind origin for the material composing this star does not appear out of order in view of these comparisons, all the more so if one considers that, in the present comparison, there is no fine tuning of some parameters in order to obtain the best agreement possible. The theoretical results are directly compared to the observations. Moreover only a small subset of possible initial conditions has been explored.

Other CEMP stars present, however, lower values for the N/C and N/O ratios and higher values for the $^{12}\text{C}/^{13}\text{C}$ ratio. For these cases, it appears that the winds of our rotating $60 M_{\odot}$ models appear to be too strongly CNO-processed (too high N/C and N/O ratios and too low $^{12}\text{C}/^{13}\text{C}$ ratios). Better agreement would be obtained if the observed abundances also stem from material coming from the CO-core and ejected either by strong late stellar winds or in a supernova explosion.

6.4. Expression of abundance ratios in total ejecta (winds and supernova)

To find the initial chemical composition of stars that would form from a mixture of wind and supernova ejecta with interstellar medium material, let us define X_{ej} as the mass fraction of element X in the ejecta (wind and supernova). This quantity can be obtained from the stellar models and computed according to the expression below:

$$X_{\text{ej}} = \frac{X_{\text{wind}}m_{\text{wind}} + X_{\text{SN}}m_{\text{SN}}}{m_{\text{wind}} + m_{\text{SN}}},$$

where X_{wind} and X_{SN} are the mass fractions of element X in the wind, in the supernova ejecta. Here m_{wind} and m_{SN} are the masses ejected by the stellar winds and at the time of the supernova explosion. To obtain the mass of the remnants, we adopted the relation obtained by Arnett (1991) between the masses of the remnant and the CO core. This method is the same as the one adopted by Maeder (1992).

The total mass ejected by the star, $m_{\text{ej}} = m_{\text{wind}} + m_{\text{SN}}$, is mixed with some amount of interstellar material m_{ISM} . The mass fraction of element X in the material composed from the ejecta mixed with the interstellar medium will be

$$X = \frac{X_{\text{ej}}m_{\text{ej}} + X_{\text{ini}}m_{\text{ISM}}}{m_{\text{ej}} + m_{\text{ISM}}} = \frac{X_{\text{ej}} \frac{m_{\text{ej}}}{m_{\text{ISM}}} + X_{\text{ini}}}{\frac{m_{\text{ej}}}{m_{\text{ISM}}} + 1},$$

where X_{ini} is the mass fraction of element X in the interstellar medium. In our case the interstellar medium is very metal poor so that one can consider $X_{\text{ini}} \sim 0$ for the heavy elements synthesised in great quantities by the star (note that this cannot be done for nitrogen ejected by the non-rotating $60 M_{\odot}$ stellar model). We also suppose that $m_{\text{ej}} \ll m_{\text{ISM}}$ and thus $X = (X_{\text{ej}}m_{\text{ej}})/m_{\text{ISM}}$. Using these expressions, one can write

$$[\text{Fe}/\text{H}] = \log \left(\frac{X(\text{Fe})_{\text{ej}}}{X(\text{Fe})_{\odot}} \right) + \log \left(\frac{m_{\text{ej}}}{m_{\text{ISM}}} \right),$$

assuming, as we did above, that $X(\text{H})_{\odot}/X(\text{H})_{\text{ej}} \approx 1$. Values of $[\text{X}/\text{Fe}]$ are obtained using the expression

$$[\text{X}/\text{Fe}] = [\text{X}/\text{H}] - [\text{Fe}/\text{H}] = \log \left(\frac{X_{\text{ej}}}{X_{\odot}} \right) - \log \left(\frac{X(\text{Fe})_{\text{ej}}}{X(\text{Fe})_{\odot}} \right).$$

One needs to have an estimate for both $m_{\text{ej}}/m_{\text{ISM}}$ (the dilution factor) and for the mass fraction of iron in the ejecta $X(\text{Fe})_{\text{ej}}$. A precise quantitative determination of $X(\text{Fe})_{\text{ej}}$ and $m_{\text{ej}}/m_{\text{ISM}}$ from theory is quite difficult. For instance, for a given initial mass, the quantity of iron ejected by the supernova can vary by orders of magnitudes depending on the mass cut, the energetics of the supernova, and the geometry of the explosion (see e.g. Maeda & Nomoto 2003). On the other hand, the dilution factor will depend on the energetics of the supernova, among other parameters. In the absence of any very precise guidelines, we determined the two unknown quantities, the dilution factor, and the mass of ejected iron, requiring that the mixture will have $[\text{Fe}/\text{H}] = -5.4$ and $[\text{O}/\text{Fe}] = +3.5$. The first value corresponds to the values observed at the surface of the star HE 1327-2326 (Frebel et al. 2005), and the second value is below the upper limit of $[\text{O}/\text{Fe}] (<4.0)$ found for this star. Doing so, one can write,

$$[\text{X}/\text{Fe}] = [\text{X}/\text{O}] + [\text{O}/\text{Fe}] = \log \left(\frac{X_{\text{ej}}}{X_{\odot}} \right) - \log \left(\frac{X(\text{O})_{\text{ej}}}{X(\text{O})_{\odot}} \right) + 3.5,$$

where $X(\text{O})$ is the mass fraction of oxygen. The mass fraction of ejected iron can be estimated from

$$[\text{O}/\text{Fe}] = \log \left(\frac{X(\text{O})_{\text{ej}}}{X(\text{O})_{\odot}} \right) - \log \left(\frac{X(\text{Fe})_{\text{ej}}}{X(\text{Fe})_{\odot}} \right) = 3.5,$$

and the dilution factor can be obtained from

$$[\text{Fe}/\text{H}] = \log \left(\frac{X(\text{Fe})_{\text{ej}}}{X(\text{Fe})_{\odot}} \right) + \log \left(\frac{m_{\text{ej}}}{m_{\text{ISM}}} \right) = -5.4.$$

6.5. Results from the “wind plus supernova ejecta” model

Using the above formulae, let us now discuss what can be expected for the chemical composition of a very metal-poor star formed from such a mixture. This includes the CNO elements for which the present models can give consistent estimates and the case of our models at $Z = 10^{-8}$ ($[\text{Fe}/\text{H}] = -6.6$), which are the models that are compatible with the requirement that the mixture of ejecta and ISM material must have an $[\text{Fe}/\text{H}] = -5.4$. Obviously our second series of models at $Z = 10^{-5}$ ($[\text{Fe}/\text{H}] = -3.6$) does not fit this requirement. Imposing $[\text{O}/\text{Fe}] = 3.5$ and $[\text{Fe}/\text{H}] = -5.4$ implies masses of iron that are being ejected on the order of $1 \times 10^{-3} M_{\odot}$ and mixed with a mass of interstellar medium of about $2 \times 10^5 M_{\odot}$. The mass of ejected iron (actually in the form of ^{56}Ni) is very small compared to the classical values of $0.07\text{--}0.10 M_{\odot}$. On the other hand, this quantity can be very small if a large part of the mass falls back onto the remnant (Umeda & Nomoto 2003). The mass of interstellar gas collected by the shock wave can be related to the explosion energy E_{exp} through (see Shigeyama & Tsujimoto 1998)

$$M_{\text{ISM}} = 5.1 \times 10^4 M_{\odot} \left(\frac{E_{\text{exp}}}{10^{51} \text{ erg}} \right).$$

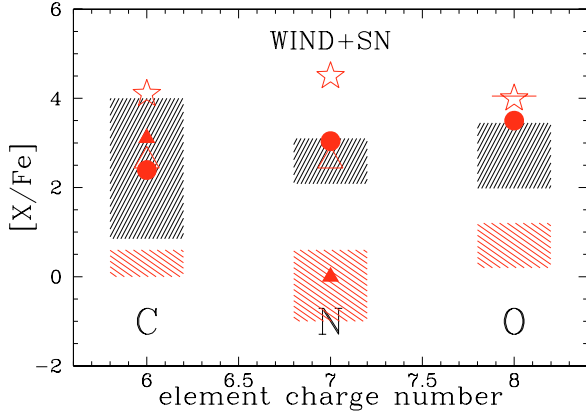


Fig. 9. Chemical composition of the ejecta (wind and supernova) of $60 M_{\odot}$ models: solid circles and triangles correspond to models at $Z = 10^{-8}$ ($[\text{Fe}/\text{H}] = -6.6$) with and without rotation. The $[\text{N}/\text{Fe}]$ ratio for the non-rotating model is equal to 0, i.e., no N-enrichment is expected. The hatched areas (moving from top right down to the left) correspond to the range of values measured at the surface of giant CEMP stars (same stars as in Fig. 8). The empty triangles (Plez & Cohen 2005) and stars (Frebel et al. 2005, only an upper limit is given for $[\text{O}/\text{Fe}]$) correspond to non-evolved CEMP stars (see text). The hatched areas (L to R from top) show the range of values measured at the surface of normal halo giant stars by Cayrel et al. (2004) and Spite et al. (2005, unmixed sample only, see text).

A mass of $2 \times 10^5 M_{\odot}$ would correspond to an energy equal to 4×10^{51} erg, i.e., a value well in the range of energies released by supernova explosion. Thus imposing $[\text{O}/\text{Fe}] = 3.5$ and $[\text{Fe}/\text{H}] = -5.4$ does not imply unrealistic values for the mass of iron that is ejected and for the mass of interstellar medium swept up by the shock wave of the supernova explosion.

The theoretical ratios for the CNO elements are shown in Fig. 9 and compared with the ratios observed at the surface of CEMP stars and of normal giant halo stars by Cayrel et al. (2004) and Spite et al. (2005). For oxygen, the value of 3.5 is obtained by construction, so it does not provide any constraint; however, see the previous paragraph. More interesting, of course, are the carbon and nitrogen abundances. One sees that both the non-rotating and rotating models might account for some level of C-enrichment that is compatible with the range of values observed at the surface of CEMP stars. However, only the rotating models produce N-rich material at this level. Figure 9 also shows that the predicted value for the N/C and N/O ratios from the rotating model appear to agree with the observed values of these ratios at the surface of CEMP stars. Thus as expected, the addition of material from the CO core (here ejected at the time of the supernova explosion) to the wind material (mainly enriched in CNO-processed material), reduces the N/C and N/O ratios.

The theoretical values in the wind-plus-supernova ejecta model for the ratio $^{12}\text{C}/^{13}\text{C}$ are between 100 and 4000 for the rotating models (see Tables 3 and 4). The value predicted by the non-rotating model is much higher, on the order of 10^8 . Compared to the observed values, which are between 4 and 60, the value of the non-rotating model is in excess by at least seven

orders of magnitude. The situation is much more favourable for the rotating models. In this last case, the predicted values are still somewhat too high, but by much lower factors.

Other proportions probably exist between wind and supernova mass ejecta than those considered here, which would provide a better fit to the observed surface abundances of CEMP stars. Also more models need to be calculated for exploring the set of initial parameters leading to a good correspondence between theory and observations. From the large range of results obtained by different initial conditions, there is little doubt, that such a given set of parameters exists. The $^{12}\text{C}/^{13}\text{C}$ ratio appears extremely sensitive to input parameters, so it may be a powerful tool for a closer identification of the exact nucleosynthetic site.

As can be seen from Fig. 9, the abundances observed at the surface of the normal giant stars by Cayrel et al. (2004) and Spite et al. (2005) are not far from solar ratios, and are well below the range of values observed at the surface of CEMP stars. Only the subset of stars qualified as unmixed by Spite et al. (2005), i.e., presenting no evidence of C to N conversion, has been plotted here. Probably these stars are formed from a reservoir of matter made up of the ejecta of different initial mass stars, convolved with a proper distribution of the initial rotation velocities, while the C-rich stars require some special circumstances involving a few or maybe only one nucleosynthetic event.

6.6. Chemical composition of the envelopes of E-AGB stars

One can wonder whether intermediate mass stars could also play a role in explaining the peculiar abundance pattern of CEMP stars. For instance, Suda et al. (2004) propose that the small halo star, observed today as a CEMP star, was the secondary in a close binary system. The secondary might have accreted matter from its evolved companion, an AGB star, and might have thus acquired at least part of its peculiar surface abundance pattern.

The physical conditions encountered in the advanced phases of an intermediate mass star are not so different from the one realised in massive stars. Thus the same nuclear reaction chains can occur and lead to similar nucleosynthetic products. Also the lifetimes of massive stars (on the order of a few million years) are not very different from the lifetimes of the most massive intermediate mass stars; typically a $7 M_{\odot}$ has a lifetime on the order of 40 Myr, only an order of magnitude higher than a $60 M_{\odot}$ star. Moreover, the observation of s-process element overabundances at the surface of some CEMP stars also point toward a possible Asymptotic Giant Branch (AGB) star origin² for the material composing the CEMP stars.

To explore this scenario, we computed a $7 M_{\odot}$ with $v_{\text{ini}} = 800 \text{ km s}^{-1}$ at $Z = 10^{-5}$, and with the same physical ingredients as the $60 M_{\odot}$ stellar models of the present paper.

² Note that massive stars also produce s-process elements. The massive star s-process elements have low atomic mass number (A inferior to about 90) and are known under the name of the weak component of the s-process.

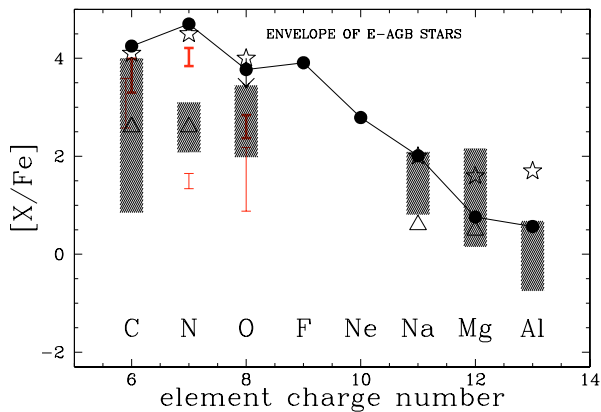


Fig. 10. Chemical composition of the envelopes of E-AGB stars compared to abundances observed at the surface of CEMP stars (hatched areas). The continuous line shows the case of a $7 M_{\odot}$ at $Z = 10^{-5}$ ($[\text{Fe}/\text{H}] = -3.6$) with $v_{\text{ini}} = 800 \text{ km s}^{-1}$. The vertical lines (shown as “error bars”) indicate the ranges of values for CNO elements in the stellar models of Meynet & Maeder (2002) (models with initial masses between 2 and $7 M_{\odot}$ at $Z = 10^{-5}$). The thick and thin lines correspond to rotating ($v_{\text{ini}} = 300 \text{ km s}^{-1}$) and non-rotating models. The empty triangles (Plez & Cohen 2005) and stars (Frebel et al. 2005, only an upper limit is given for $[\text{O}/\text{Fe}]$) correspond to non-evolved CEMP stars.

In contrast to the $60 M_{\odot}$ models, the $7 M_{\odot}$ stellar model loses little mass during the core H- and He-burning phase, so that the star has still nearly its whole original mass at the early asymptotic giant branch stage (the actual mass at this stage is $6.988 M_{\odot}$). This is because the star never reaches the break-up limit during the MS phase; and, due to rotational mixing and dredge-up, the metallicity enhancement at the surface only occurs very late, when the star evolves toward the red part of the HR diagram after the end of the core He-burning phase. At this stage, the outer envelope of the star is enriched in primary CNO elements, and the surface metallicity reaches about 1000 times the initial metallicity. If such a star is in a close binary system, there is good chance that mass transfer occurs during this phase of expansion of the outer layers. In that case, the secondary may accrete part of the envelope of the E-AGB star.

From the $7 M_{\odot}$ stellar model, we can estimate the chemical composition of the envelope at the beginning of the thermal pulse AGB phase. Here we call envelope all the material above the CO-core. The result is shown in Fig. 10 (continuous line with solid circles). We also plotted the values obtained from the models of Meynet & Maeder (2002) for initial masses between 2 and $7 M_{\odot}$ at $Z = 10^{-5}$ and with $v_{\text{ini}} = 0$ and 300 km s^{-1} .

Before discussing the comparisons with observations, let us make two remarks: 1) as was the case for the theoretical predictions of the massive star winds, the values given here have not been adjusted to fit the observed values but result from the basic physics of the models; 2) the initial metallicity of our AGB models ($[\text{Fe}/\text{H}] = -3.6$) is at the higher range of values of the metallicities observed for the CEMP stars. However, based on the results from our massive star models at $[\text{Fe}/\text{H}] = -6.6$ and -3.6 (see Fig. 8), we see that the overall pattern of the abundances will probably remain quite similar for a lower

initial metallicity. Only a shift toward higher values along the vertical axis is expected when the initial metallicity of the model is decreased.

Looking at Fig. 10, one can note the three following points:

1. The envelope of rotating intermediate mass stars presents a chemical composition in carbon, nitrogen, and oxygen that agrees well with the observed value at the surface of CEMP stars. In particular, compared to the wind material of massive stars (see Fig. 8), the N/C ratios and N/O ratios are in better agreement. The non-rotating models cannot account for the high overabundances in nitrogen and oxygen.
2. The $^{12}\text{C}/^{13}\text{C}$ ratios in our rotating models are between 19 and 2500, with the lowest values corresponding to the most massive intermediate-mass star models. The non-rotating models give values between 3×10^5 and 2×10^6 . Again here, rotating models agree much better with the observed values, although very low $^{12}\text{C}/^{13}\text{C}$ values (on the order of 4–5, as observed e.g. at the surface of the dwarf halo star G77 61, see Plez & Cohen 2005) seem to be reproduced only by massive star models (wind material only).
3. For sodium and aluminum, the ratios predicted by our $7 M_{\odot}$ model with $v_{\text{ini}} = 800 \text{ km s}^{-1}$ fit the observed values well. In the case of magnesium, good agreement is also obtained.

Thus we see that the envelopes of AGB stellar models with rotation show a very similar chemical composition to the one observed at the surface of CEMP stars. It is, however, still difficult to say that rotating intermediate mass star models are better than rotating massive star models in this respect. Probably, some CEMP stars are formed from massive star ejecta and others from AGB star envelopes. Interestingly at this stage, some possible ways to distinguish between massive star wind material and AGB envelopes do appear. Indeed, we just saw above that massive star wind material is characterised by a very low $^{12}\text{C}/^{13}\text{C}$ ratio, while intermediate mass stars seem to present higher values for this ratio. The AGB envelopes would also present very high overabundances of ^{17}O , ^{18}O , ^{19}F , and ^{22}Ne , while wind of massive rotating stars present a weaker overabundance of ^{17}O and depletion of ^{18}O , ^{19}F , and ^{22}Ne . As discussed in Frebel et al. (2005), the ratio of heavy elements, such as the strontium-to-barium ratio, can also give clues to the origin of the material from which the star formed. In the case of HE 1327-2326, Frebel et al. (2005) give a lower limit of $[\text{Sr}/\text{Ba}] > -0.4$, which suggests that strontium was not produced in the main s-process occurring in AGB stars, thus leaving the massive star hypothesis as the best option, in agreement with the result from $^{12}\text{C}/^{13}\text{C}$ in G77-61 (Plez & Cohen 2005) and CS 22949-037 (Depagne et al. 2002).

7. Conclusion

We have proposed a new scenario for the evolution of very metal-poor massive stars. This scenario requires no new physical processes, as it is based on models that have been extensively compared to observations of stars at solar composition and in the LMC and SMC. The changes with respect to classical scenarios are twofold and are both induced by fast rotation: first, rotational mixing deeply affects the chemical composition

of the material ejected by the massive stars; second, rotation significantly enhances the mass lost by stellar winds. The mass loss rates are increased mainly because the mixing process is so strong that the surface metallicity is enhanced by several orders of magnitude. This leads to strong radiative winds during the evolution in the red part of the HR diagram. The strongest mass loss occurs at the very end of the core He-burning phase. The proposed scenario may allow very massive stars to avoid the pair instability.

We show that material ejected by rotating models has chemical compositions that show close similarities to the peculiar abundance pattern observed at the surface of CEMP stars. We explored the three possibilities of CEMP stars made of: 1) massive star wind material; 2) total massive star ejecta (wind plus supernova ejecta); 3) and material from E-AGB star envelopes. Interestingly, from the models computed here, one can order these three possibilities according to the degree of richness in CNO processed material. From the richest to the poorest, one has the wind material, the E-AGB envelope, and the total ejecta of massive stars. The imprints on the abundance pattern of CEMP stars are thus not the same, depending on which source is involved. There is good hope that in the future, it will be possible to distinguish them.

Other interesting questions will be explored in the future with these rotating metal-poor models. Among them let us briefly mention:

- *What is the enrichment in new synthesized helium by the first stellar generations?* This is a fundamental question already asked long ago by Hoyle & Tayler (1964). A precise knowledge of the helium enrichment caused by the first massive stars (Carr et al. 1984; Marigo et al. 2003) is important in order to correctly deduce the value of the cosmological helium from the observed abundance of helium in low metallicity regions (see e.g. Salvaterra & Ferrara 2003). Production of helium by the first massive stars may also affect the initial helium content of stars in globular clusters. If the initial helium content of stars in globular clusters is increased by 0.02 in mass fraction, the stellar models will provide ages for the globular clusters that are lower by roughly 15%, i.e., 2 Gyr starting from an age of 13 Gyr (Shi 1995; see also the interesting discussion in Marigo et al. 2003). In the case of our rotating $60 M_{\odot}$ at $Z = 10^{-8}$, 22% of the initial mass is ejected in the form of new synthesised helium by stellar winds. Thus the models presented here will certainly lead to new views on the question of the helium enrichment at very low metallicity, provided of course, they are representative of the evolution of the majority of massive stars at very low Z .
- *What are the sources of primary nitrogen in very metal-poor halo stars?* Primary ^{14}N is produced in large quantities in our rotating models. In our previous work on this subject (Meynet & Maeder 2002), we discussed the yields from stellar models at $Z = 10^{-5}$ with $v_{\text{ini}} = 300 \text{ km s}^{-1}$. Such an initial velocity corresponds to a ratio $v_{\text{ini}}/v_{\text{crit}}$ of only 0.25. This value is lower than the value of ~ 0.35 reached by solar metallicity models with $v_{\text{ini}} = 300 \text{ km s}^{-1}$. With such a low initial ratio of $v_{\text{ini}}/v_{\text{crit}}$, we found that the

main sources of primary nitrogen were intermediate mass stars with initial masses around about $3 M_{\odot}$. However, as already shown in Meynet & Maeder (2002), the yield in ^{14}N increases rapidly when the initial velocity increases. As a numerical example, the yield in primary nitrogen for the $Z = 10^{-5}$, $60 M_{\odot}$ with $v_{\text{ini}}/v_{\text{crit}}$ equal to 0.25 was $7 \times 10^{-4} M_{\odot}$, while the corresponding model with $v_{\text{ini}}/v_{\text{crit}}$ equal to 0.65 produces a yield of $\sim 2 \times 10^{-1}$; i.e., it increased by a factor of nearly 300! Interestingly, these high yields of primary ^{14}N from short-lived massive stars seem to be required for explaining the high N/O ratio observed in metal-poor halo stars (Chiappini et al. 2005). Note that massive intermediate mass stars, whose lifetimes would be only an order of magnitude higher than those of the most massive stars, could also be invoked to explain the high N/O ratio observed in very metal-poor stars. The age-metallicity relation is not precise enough to allow us to distinguish between the two.

- *How does rotation affect the yields of extremely metal-poor stars?* Other elements such as ^{13}C , ^{17}O , ^{18}O , and ^{22}Ne are also produced in much greater quantities in the rotating models. Computations are now in progress for extending the range of initial parameters explored and to study the impact of such models on the production of these isotopes, as well as on s-process elements.

We think that the fact that stars rotate, and may even rotate fast especially at low metallicity, has to be taken into account for obtaining more realistic models of the extremely metal-poor stars that formed in the early life of the Universe.

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B.3 Conference proceedings

Low-Metallicity Star Formation: From the First Stars to Dwarf Galaxies

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Powerful explosions at $Z = 0$?

Sylvia Ekström¹, Georges Meynet¹, Raphael Hirschi² & André Maeder¹

¹Geneva Observatory, University of Geneva, Maillettes 51 - CH 1290 Sauverny, Switzerland

²University of Keele, Keele, ST5 5BG, UK

Abstract. Metal-free stars are assumed to evolve at constant mass because of the very low stellar winds. This leads to large CO-core mass at the end of the evolution, so primordial stars with an initial mass between 25 and 85 $M_{\odot}$ are expected to end as direct black holes, the explosion energy being too weak to remove the full envelope.

We show that when rotation enters into play, some mass is lost because the stars are prone to reach the critical velocity during the main sequence evolution. Contrarily to what happens in the case of very low- but non zero-metallicity stars, the enrichment of the envelope by rotational mixing is very small and the total mass lost remains modest. The compactness of the primordial stars lead to a very inefficient transport of the angular momentum inside the star, so the profile of $\Omega(r)$ is close to $\Omega r^2 = \text{const.}$ As the core contracts, the rotation rate increases, and the star ends its life with a fast spinning core. Such a configuration has been shown to modify substantially the dynamics of the explosion. Where one expected a weak explosion or none at all, rotation might boost the explosion energy and drive a robust supernova. This will have important consequences in the way primordial stars enriched the early Universe.

Keywords. stars: evolution, stars: rotation, stars: chemically peculiar, supernovae: general

1. The first stars

Population III (Pop III) stars occupy a key position in the history of the Universe, as a link between the pure H-He Universe at the beginning of times, and the metal-rich one we observe nowadays around us. Their evolution is important, because during their life they form the first heavy elements, but their death is also important, because this is the moment at which the newly synthesised elements are released in the surrounding medium, starting the chemical enrichment of the Universe.

While the first studies on the star formation at $Z = 0$ concluded that only very massive objects could form (Bromm et al. 2002; Abel et al. 2002), later findings show that the radiative feedback of the very first stars could lead to a second generation of metal-free stars with lower initial mass (Omukai and Yoshii 2003; O’Shea et al. 2005; Greif and Bromm 2006). Could these stars contribute to the chemical enrichment of the Universe? The consensual picture is that most of them are directly swallowed by a black hole.

2. Evolution at $Z = 0$

Since they are deprived of metals, the massive Pop III stars cannot rely on the CNO cycle to sustain their gravity. The pp -chains are not very sensitive in temperature and cannot completely halt the initial collapse. The stars continue their contraction until the central temperature is high enough to allow some carbon to be produced through the 3α reaction.

As a consequence, the main sequence (MS) occurs at much higher central temperature (T_c) and density (ρ_c), as shown in Fig. 1 (*left*). The great compactness implies that the

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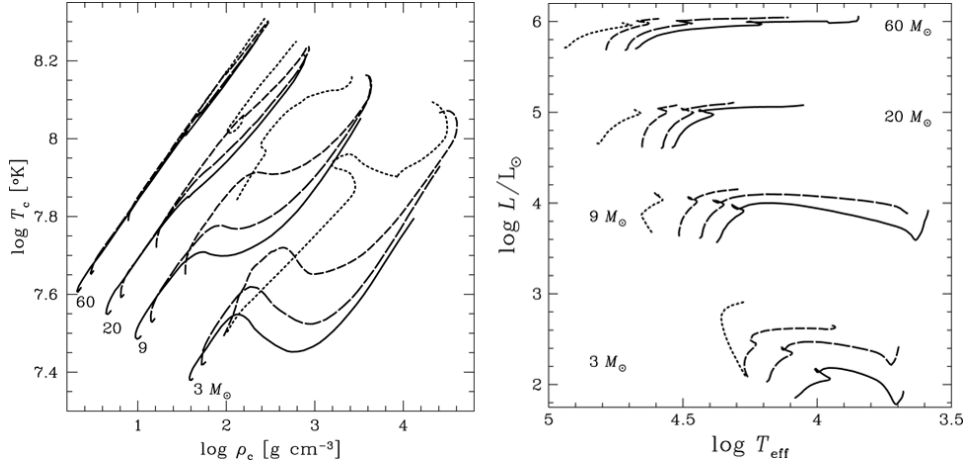


Figure 1. Evolution during the main sequence of massive stars at various metallicities: $Z = 0.020$ (solid line); $Z = 0.002$ (long-dashed line); $Z = 10^{-5}$ (short-dashed line); $Z = 0$ (dotted line). *Left:* $\log T_c - \log \rho_c$ diagram. *Right:* HR diagram.

zero-age main sequence (ZAMS) is shifted toward higher effective temperature (T_{eff}), as shown in Fig. 1 (*right*). Without any metal, the envelope is transparent, so the stars remain in the blue side of the HR diagram throughout the whole MS.

The lack of metals has another consequence: the radiative winds are supposed to scale with the metallicity as $\dot{M} \propto (Z/Z_\odot)^\alpha$ with $\alpha = 0.5$ (Kudritzki and Puls 2000; Nugis and Lamers 2000; Kudritzki 2002) or 0.7-0.8 (Vink et al. 2001; Vink and de Koter 2005; Mokiem et al. 2007). At $Z = 0$, we thus expect no radiative winds, and most Pop III models are computed with the hypothesis of constant mass. In fact, Kudritzki (2002) shows that very low-metallicity stars close to the Eddington limit $L_{\text{Edd}} = 4\pi cGM/\kappa$ (with κ the electron-scattering opacity and obvious meaning for the other quantities) are subject to a weak but non-null mass loss. Vink and de Koter (2005) show that there is a flattening in the metallicity dependence of the WR winds below $Z \approx 10^{-3}$. Also Smith and Owocki (2006) show that continuum-driven eruptive mass ejections can occur at any metallicity, even $Z = 0$, leading to strong mass loss episodes like those observed in luminous blue variables (LBVs).

Even if the mass loss is non null, it is weak anyway, so the Pop III stars end their life with a large helium core (M_α), as shown by Marigo et al. (2001). In such a case, the expected fate of the stars can deviate from the usual “SN explosion with neutron star or black hole remnant” seen at higher metallicity. Heger et al. (2003) determine the fate of massive stars at various metallicities as a function of their M_α at the pre-supernova (preSN) stage:

- a Type II SN: $M_\alpha < 9 M_\odot$
- a BH by fallback: $9 M_\odot \leq M_\alpha < 15 M_\odot$
- a direct BH: $15 M_\odot \leq M_\alpha < 40 M_\odot$
- a pulsational pair-instability followed by a SN with BH formation: $40 M_\odot \leq M_\alpha < 64 M_\odot$
- a pair-instability SN (PISN): $64 M_\odot \leq M_\alpha \leq 133 M_\odot$
- or a direct BH collapse: $M_\alpha > 133 M_\odot$

Relating M_α with the initial mass of the star in the case of $Z = 0$, this means that stars with a mass between 25 and 140 $M_\odot$ or above 260 $M_\odot$ will not contribute at all to the enrichment of the early Universe.

Could this picture be revised?

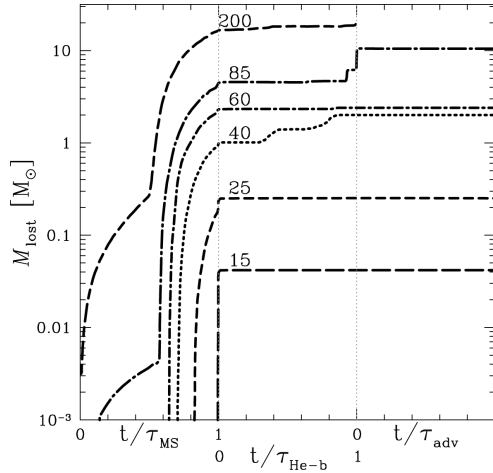


Figure 2. Mass lost during the evolution of Pop III models of various masses (Ekström et al. 2008a): $15 M_{\odot}$ (long-dash line); $25 M_{\odot}$ (short-dashed line); $40 M_{\odot}$ (dotted line); $60 M_{\odot}$ (dot-short dashed line); $85 M_{\odot}$ (dot-long dashed line); $200 M_{\odot}$ (short dash-long dashed line). The x -axis is a temporal axis, divided in three parts: the fraction of the MS evolution, the fraction of the core He burning phase, and the fraction of the rest of the advanced phases.

3. Rotation effects

Meynet et al. (2006) show that at low (but non-zero) metallicity, rotation drastically changes the evolution of massive stars. Two processes are involved:

(a) During the MS, because of the low radiative winds, the star loses very little mass and thus very little angular momentum. As the evolution proceeds, the stellar core contracts and spins up. If a coupling exists between the core and the envelope (*i.e.* meridional currents or magnetic fields), the surface may be accelerated up to the critical velocity and the star may experience a mechanical mass loss due to the centrifugal acceleration. The matter is launched into a decretion disc (Owocki 2005), which may be dissipated later by the radiation field of the star.

(b) Rotation induces an internal mixing that depends on the gradient of the rotational rate Ω . After H exhaustion, the core contracts and the envelope expands, stretching the Ω -gradient. The mixing becomes strong and enriches the surface in heavy elements. Rotation also favours a redward evolution after the MS, allowing the star to spend more time in the cooler part of the HR diagram, where mass loss is increased. The outer convective zone dives deep inside the star and dredges up freshly synthesised heavy elements. The surface metallicity is dramatically enhanced (by a factor of 10^6 for a $60 M_{\odot}$ at $Z_{\text{ini}} = 10^{-8}$). The opacity of the envelope is increased, and the radiative winds may thus be drastically enhanced.

Meynet et al. (2006) show that at $Z = 10^{-8}$, while a non-rotating $60 M_{\odot}$ model loses only $0.27 M_{\odot}$, a similar model computed with an initial velocity $v_{\text{ini}} = 800 \text{ km s}^{-1}$ loses $36 M_{\odot}$, *i.e.* 60 % of its initial mass.

Now what happens if the metallicity is $Z = 0$ strictly?

Ekström et al. (2008a) show that the first process (the mechanical mass loss during the MS) is slightly reduced because the meridional circulation is weak, so the star almost evolves in a regime of local angular momentum conservation $\Omega r^2 = \text{const}$, and reaches the critical velocity late in its MS evolution. The second process (the enrichment of the surface) barely occurs at $Z = 0$. This can be explained by the fact that some He is burnt already during the MS: at H exhaustion, the star is hot enough to move on to core He burning without any structural readjustment, and the He burning occurs mainly in the

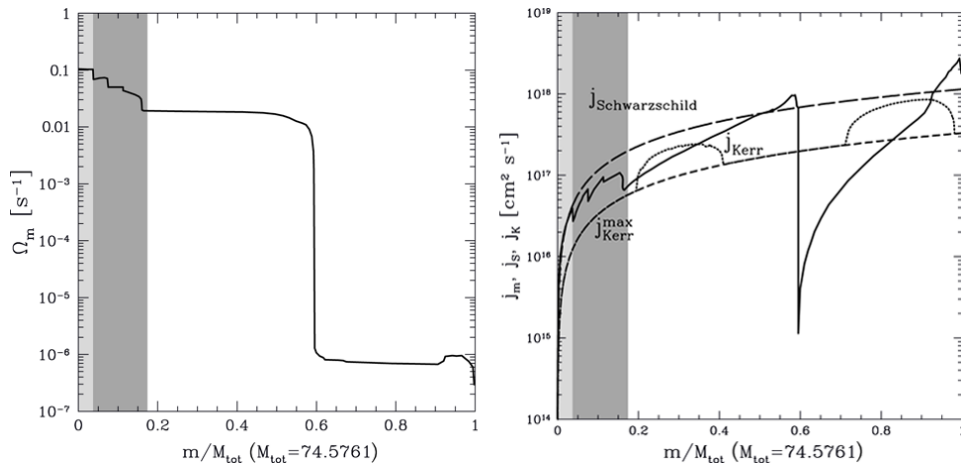


Figure 3. Pre-supernova structure of a Pop III $85 M_{\odot}$ model. The light grey shaded area marks the border of the remnant, and the dark grey that of the CO core. *Left:* Ω profile. *Right:* specific angular momentum profile. $j_{\text{Schwarzschild}} = \sqrt{12}Gm/c$ (long-dashed line) and $j_{\text{Kerr}}^{\text{max}} = Gm/c$ (short-dashed line) is the minimum specific angular momentum necessary for a non-rotating and a maximally-rotating black hole, respectively. j_{Kerr} (dotted line) is the minimum specific angular momentum necessary to form an accretion disc around a rotating black hole.

blue side of the HR diagram. The stretching of the Ω -gradient does not occur, and the outer convective zone, if any, remains thin. Most of the post-MS evolution occurs with a surface metallicity much lower than 10^{-6} .

As shown in Fig. 2, most of the mass is lost during the MS, and it amounts at most to 10% of the initial mass.

4. Explosion

A direct consequence of the evolution close to the local angular momentum conservation is that the models keep a high angular momentum content in the core throughout their whole evolution. Figure 3 (*left*) shows that the angular velocity of the iron core of a $85 M_{\odot}$ model at the pre-supernova stage is more than 5 orders of magnitude higher than the surface velocity. On the right panel of the same figure, we see that in the innermost parts, the specific angular momentum of the same model is higher than that needed to create an accretion disc around a rotating BH.

According to Nomoto et al. (2003), there are indications that the most energetic supernovae, the hypernovae (HN), could be due to stellar rotation. Rotation is supposed to boost the explosion energy of a collapsing star by various mechanisms:

- it drives an anisotropy of the neutrinos sphere (Shimizu et al. 2001; Kotake et al. 2003),
- it lowers the critical neutrinos luminosity (the luminosity at which the stalled shock revives) (Yamasaki and Yamada 2005),
- it modifies the gravity (Burrows et al. 2005),
- it changes the geometry of the mass accretion, making it aspherical (Burrows et al. 2005),
- it generates vortices, which dredge up heat and increase the neutrinos luminosity (Burrows et al. 2005).

The models presented here are massive at the time of their death, they have heavy cores and they almost kept their entire envelope, but they have fast rotating iron cores, and therefore their final collapse could present a highly aspherical geometry. It would be

extremely interesting to perform numerical simulations of the collapse of these rotating models so we could check whether the energy released in the collapse is sufficient to overcome the high gravitation of the star and drive a successful explosion. For the time being, we can only say that we expect rotating Pop III stars to undergo a stronger explosion than what is commonly admitted for non-rotating Pop III stars of this mass range.

5. Conclusions

The lack of metals affects strongly the stellar evolution at $Z = 0$. One of the main result is to reduce dramatically the radiative mass loss. This means that rotating stars will keep all their angular momentum until the end of their evolution. At the pre-supernova stage, they will present a fast-rotating iron core, so we expect the collapse to be highly aspherical. This may boost the explosion energy and allow a strong supernova explosion where only a weak one or none was expected.

Note that the inclusion of magnetic fields could alter these results by providing the core-envelope coupling which is lacking in Pop III stars. However, in that case, a strong mass loss would be expected (Ekström et al. 2008b), so the evolution of the star would anyway be strongly affected with regard to the consensual picture.

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Can Very Massive Stars Avoid Pair-Instability Supernovae?

Sylvia Ekström, Georges Meynet and André Maeder

Geneva Observatory, University of Geneva, Maillettes 51 - CH 1290 Sauverny, Switzerland
email: sylvia.ekstrom@obs.unige.ch, georges.meynet@obs.unige.ch,
andre.maeder@obs.unige.ch

Abstract. Very massive primordial stars ($140 M_{\odot} < M < 260 M_{\odot}$) are supposed to end their lives as PISN. Such an event can be traced by a typical chemical signature in low metallicity stars, but at the present time, this signature is lacking in the extremely metal-poor stars we are able to observe. Does it mean that those very massive objects were not formed, contrarily to the primordial star formation scenarios? Could it be possible that they avoided this tragic fate?

We explore the effects of rotation, anisotropical mass loss and magnetic field on the core size of very massive Population III models. We find that magnetic fields provide the strong coupling that is lacking in standard evolution metal-free models and our $150 M_{\odot}$ Population III model avoids indeed the pair-instability explosion.

Keywords. stars: evolution – stars: rotation – stars: chemically peculiar – stars: mass loss – stars: magnetic fields

1. Introduction

According to Heger *et al.* (2003), the fate of single stars depends on their He-core mass (M_{α}) at the end of the evolution. They have shown that at very low metallicity, the stars having $64 M_{\odot} < M_{\alpha} < 133 M_{\odot}$ will undergo pair-instability and be entirely disrupted by the subsequent supernova. This mass range in M_{α} has been related to the initial mass the star must have on the main sequence (MS) through standard evolution models: $140 M_{\odot} < M_{\text{ini}} < 260 M_{\odot}$. Since we will present here a non-standard evolution, we will rather keep in mind the M_{α} range.

The typical mass of Population III (Pop III) stars is explored by early structure formation studies and chemistry considerations about cooling. Different studies (see Abel *et al.* 2002; Bromm *et al.* 2002, among others) give the same conclusion: Pop III stars are supposed to be massive or very massive, even when a bimodal mass distribution allows the formation of lower mass components (see Nakamura & Umemura 2001). Therefore we expect that many among them should die as pair-instability supernovae (PISN).

1.1. A typical chemical signature which remains unobserved

These PISN events are supposed to leave a typical chemical signature. According to Heger & Woosley (2002), the complete disruption of the star leads to a very strong odd-even effect: the absence of stable post-He burning stages deprives the star of the neutron excess needed to produce significant amounts of odd- Z nuclei. Also the lack of r - and s -process stops the nucleosynthesis around zinc. Even if one mixes these yields with the yields of zero-metallicity $12 - 40 M_{\odot}$ models (which end up as standard Type II SNe), the PISN signature remains and should be observable.

However, using those yields, Tumlinson *et al.* (2004) have shown that it provides only a very poor fit to the abundances pattern observed in the metal-poor stars known today.

The odd-even effect is not observed, and the models significantly over-produce Cr and under-produce V, Co and Zn.

The most metal-deficient stars are supposed to be formed in a medium enriched by only one or a few SNe. The absence of the chemical signature of the PISN is a strong argument against their existence. But how could that be?

1.2. Simple solutions

The simplest solution to explain this absence is to suppose that the mass domain in question was not formed in the primordial clouds. Maybe the primordial IMF was not as top-heavy as we actually think, and the most massive stars formed then could very well be too small for such a fate. However, recent works on primordial stars formation seem to rule out this possibility (see Yoshida *et al.* 2006; O'Shea & Norman 2007).

Another possibility is that the signature was very quickly erased by the next generations of stars. Maybe the metal-poor stars we observe are enriched by more SNe than we actually think, and the later contributions are masking the primordial ones. Only the observation of more and more metal-deficient stars will provide an answer to that possibility.

One can also wonder whether there would be a way for those stars to avoid their fate. In this context, the simplest solution is to suppose that some mechanism could lead to such a high mass loss that the conditions for pair-instability would no more be met in the central regions. The aim of the present work is to explore this possibility.

2. Rotation at very low metallicity

Radiatively-driven winds are supposed to scale with metallicity as $Z^{(0.5-0.86)}$. Thus, very low or even no mass loss is expected when Z approaches to 0.

However, we have shown (Meynet *et al.* 2006) that rotation can change the mass loss history of low metallicity stars in a dramatic way. Two processes are involved:

(a) During MS, because of the low radiative winds, the star loses very little mass and thus very little angular momentum. As the evolution proceeds, the stellar core contracts and spins up. If a coupling exists between the core and the envelope (*i.e.* meridional currents or magnetic fields), the surface may be accelerated up to the critical velocity and the star may lose mass by a mechanical wind due to centrifugal acceleration. The matter is launched into a decretion disk, which may be dissipated later by the radiation field of the star.

(b) Rotation induces an internal mixing that enriches the surface in heavy elements. The effective surface metallicity is enhanced by a factor that has been shown to be very

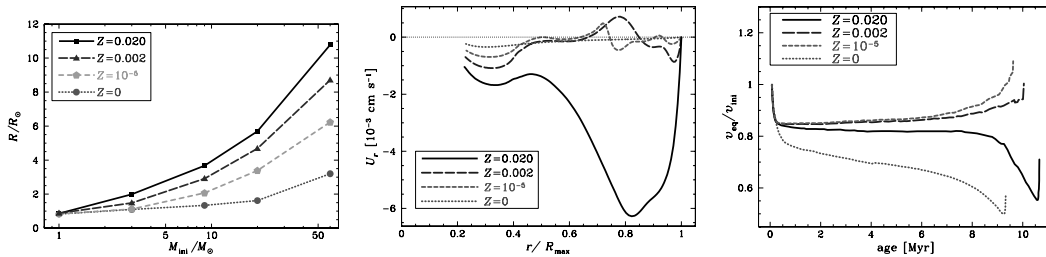


Figure 1. *Left:* Variations in the radius as a function of initial mass for various metallicities. *Centre:* Amplitude of the radial component of the meridional circulation inside $20 M_{\odot}$ models at the same evolutionary stage ($X_c = 0.4$) and having the same rotation rate ($v/v_{\text{crit}} = 0.5$) but various metallicities. *Right:* Evolution of the equatorial velocity during the MS. The models are taken from Ekström *et al.* (2008).

large (up to 10^6 for a $60 M_{\odot}$ at $Z_{\text{ini}} = 10^{-8}$). Rotation also favours a redward evolution after the MS, allowing the star to spend more time in the cooler part of the HR diagram. The opacity of the envelope is increased, and the radiative winds may thus be drastically enhanced.

Both effects add up and lead to strong mass loss at very low metallicity.

2.1. Rotation at $Z = 0$

But what happens when the metallicity drops down to $Z = 0$ strictly? The absence of carbon prevents the star to start burning hydrogen through CNO-cycle, but pp -chains are not energetic enough to sustain the star, so it contracts longer during its formation. At metallicities $Z = 0.020$, 0.002 , 10^{-5} , and 0 respectively, the radii on the ZAMS will be in a ratio $3.5 : 2.8 : 2 : 1$ for a $20 M_{\odot}$ model (see Fig. 1, *left*), that is the $Z = 0$ star will be twice as compact as the $Z = 10^{-5}$ one. This has a direct influence on the amplitude of the outer cell of the meridional circulation, which depends on the compactness of the star as $U_r \propto \left[1 - \frac{\Omega^2}{2\pi G \bar{\rho}}\right]$. For the same metallicities, the amplitude of the outer cell will be in a ratio $100 : 17 : 4 : 1$ (Fig. 1, *centre*). This means that at $Z = 0$, the core-envelope coupling is almost null. Figure 1 (*right*) shows the evolution of the equatorial velocity during the MS for our $20 M_{\odot}$ models. At standard metallicity, the meridional circulation is strong, but the mass loss is also strong, so the model spins down. At lower but non-zero metallicities, the mass loss drops, and though the core-envelope coupling is weak, the models spin up. At $Z = 0$, the mass loss is null, but the model evolves almost in a regime of local conservation of angular momentum. Thus, the natural inflation of the radius during the MS leads to a spin down of the model. The model may reach the critical limit, but later in its MS evolution (for a given initial mass and velocity), and the mechanical mass loss remains very small.

At central H exhaustion, the core of the $Z = 0$ model is already hot enough to burn some helium, so the transition between H-burning and He-burning is smooth, without any structural readjustment. The model remains in the blue part of the Hertzsprung-Russell diagram until very late in the central He-burning phase (Fig. 2). When it eventually evolves towards the red part, the outer convective zone is very thin, so the enrichment of the surface remains very low. Whereas at $Z = 10^{-8}$ we get a strong post-MS mass loss, at $Z = 0$ it is negligible.

Rotation alone seems thus to fail in providing a way to lose sufficient mass at $Z = 0$.

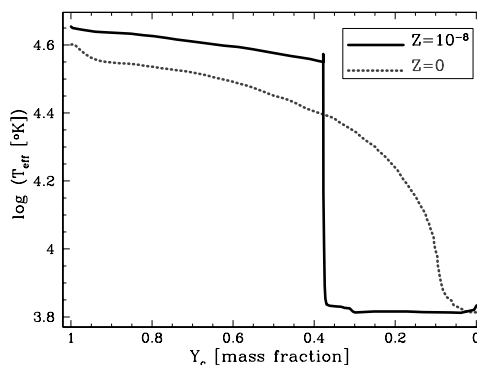


Figure 2. Evolution of T_{eff} during core He-burning for two models of $60 M_{\odot}$. The $Z = 10^{-8}$ model is taken from Hirschi (2007).

2.2. Two natural effects of rotation

In the previous calculations, we have neglected two mechanisms that arise naturally with rotation and could change this picture.

The first one is the wind anisotropy (see Maeder 1999): when a star rotates fast, it becomes oblate. As shown by von Zeipel theorem, the poles become hotter than the equator, and the radiative flux is no more spherically symmetric: it gets much stronger in the polar direction than in the equatorial plane. Since the mass is lost preferentially near the poles, it removes much less angular momentum than in the spherical configuration, and thus the star may reach the critical limit much earlier.

The second mechanism is the magnetic fields. According to Spruit (2002) the differential rotation may amplify an existing magnetic field through the Tayler-Spruit dynamo mechanism, and provide a strong core-envelope coupling. This coupling will be able to accelerate the surface very early in the evolution, and maintain the star at critical limit throughout its entire evolution.

3. Evolution with anisotropical winds and magnetic fields

3.1. Ingredients

With the help of the two effects mentioned above, we have computed an exploratory model. Its mass has been chosen to be $150 M_{\odot}$ and the initial ratio between the equatorial velocity and the critical one to be $v_{\text{ini}}/v_{\text{crit}} = 0.56$. Let us mention that this ratio is higher than the one needed to account for the observed average velocities at solar metallicity, but it is still a “reasonable” value, not an extreme one.

The computation has been accomplished using the Geneva code with up to date nuclear reaction rates obtained with NETGEN (<http://www-astro.ulb.ac.be/Netgen/>). The opacity tables come from OPAL (<http://www-phys.llnl.gov/Research/OPAL/opal.html>) with the extension at low temperature by Ferguson *et al.* (2005). The initial composition is $X = 0.753$, $Y = 0.247$ and of course $Z = 0$.

The radiative mass loss prescription is an important ingredient of the modelization of massive stars. Here we have used Kudritzki (2002). Since this prescription is not aimed at the case $Z = 0$ strictly, we have used the same adaptations as in Marigo *et al.* (2003). The Wolf-Rayet (WR) mass loss rate is taken from Nugis & Lamers (2000) with the metallicity scaling from Eldridge & Vink (2006). For the calculation, we have taken the effective surface metallicity $Z_{\text{eff}} = (1 - X - Y)_{\text{surf}}$ so that the enrichment of the surface is accounted for. We must stress that this Z_{eff} is mainly composed by CNO elements but no iron. It is usually considered that WR winds are triggered by Fe lines, whereas the CNO lines determine only v_{∞} , so we expect that no WR winds can take place at $Z = 0$. But Vink & de Koter (2005) have shown that when the metallicity gets really low, the CNO lines take over the role of Fe lines in the line driving, and this is what we have assumed here.

When the model reaches the critical limit, the mechanical mass loss has been treated as described in Meynet *et al.* (2006).

The treatment of anisotropic winds has been implemented as in Maeder (2002) and the effect of magnetic fields as in Maeder & Meynet (2005).

3.2. Result

In Fig. 3, we present the evolution in the HR diagram (left panel) and the evolution of mass with time (right panel). The grey line shows a non-rotating model computed with the same physics for comparison.

During its whole evolution up to the end of core He-burning, the non-rotating model loses only $1.37 M_{\odot}$. This illustrates the weakness of radiative winds at $Z = 0$.

The evolution of the rotating model (black line) can be described by four distinct stages:

(a) (*continuous part*, lower left corner) The model starts its evolution on the MS with only radiative winds, losing only a little more than $0.002 M_{\odot}$. During this stage, it accelerates quickly, mainly because of the strong coupling exerted by the magnetic fields.

(b) (*dashed part*) When its central content of hydrogen is still 0.58 in mass fraction, it reaches the critical velocity and starts losing mass by mechanical wind. It remains at the critical limit through the whole MS, but the mechanical wind removes only the most superficial layers that have become unbound, and less than 10% of the initial mass is lost at that stage ($11.44 M_{\odot}$). The model becomes also extremely luminous, and reaches the Eddington limit when 10% of hydrogen remains in the core. Precisely, it is the so-called $\Omega\Gamma$ -limit that is reached here. Due to the fast rotation, the maximum Eddington factor allowed is reduced; at the same time, because of the high luminosity, the critical velocity is reduced in comparison with the one derived from the classical Ω -limit (see Maeder & Meynet 2000, for details).

(c) (*dotted part*) The combustion of helium begins as soon as the hydrogen is exhausted in the core, then the radiative H-burning shell undergoes a CNO flash, setting the model on its redward journey. The model remains at the $\Omega\Gamma$ -limit and loses a huge amount of mass. The strong magnetic coupling keeps bringing angular momentum to the surface and even the heavy mass loss is not able to let the model evolve away from the critical limit. The mass lost during that stage amounts to $53.46 M_{\odot}$. When the model starts a blue hook in the HR diagram, its surface conditions become those of a WR star ($X_{surf} < 0.4$ and $T_{eff} > 10^4$ K). The luminosity drops and takes the model away from the Γ -limit, marking the end of that stage.

(d) (*continuous part*) The rest of the core He-burning is spent in the WR conditions. The mass loss is strong but less than in the previous stage: another $26.34 M_{\odot}$ are lost.

At the end of core He-burning, the final mass of the model is only $M_{fin} = 58 M_{\odot}$, already below the minimum M_{α} needed for PISN ($M_{\alpha} \geq 64 M_{\odot}$). Note that the contraction of the core after helium exhaustion brings the model back to critical velocity, so this value for M_{fin} must be considered as an upper limit.

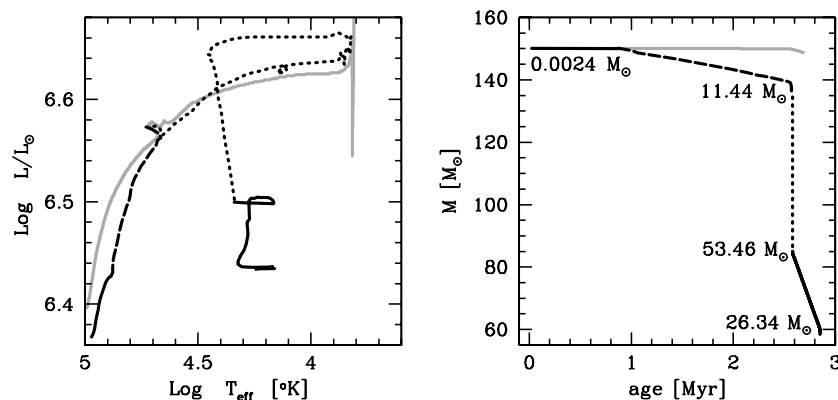


Figure 3. Black line: rotating model; *continuous part*: beginning of MS ($X_c = 0.753$ down to 0.58; *dashed part*: rest of the MS; *dotted part*: beginning of core He-burning phase ($Y_c = 1.00$ down to 0.96); *continuous part*: rest of the He-burning. Grey line: non-rotating model for comparison. **Left panel:** evolution in the Hertzsprung-Russell diagram; **Right panel:** evolution of the mass of the model. The mass indicated is the mass lost at each stage, not a summation.

4. Summary

The model we presented here is exploratory. We cannot draw general conclusions from it. But our model shows that heavy mass loss is possible even at $Z = 0$, and the answer to our title's question is: *yes, under certain conditions, very massive stars can indeed avoid PISN.*

Some aspects need yet to be clarified. Is the WR mass loss rate we have used really valid? Can the CNO lines alone really drive a WR wind? In a more general perspective, we still lack a good mass loss recipe for the strict $Z = 0$ case. A word of caution must also be cast on the inclusion of magnetic fields. The validity of the Tayler-Spruit dynamo is still under debate, and more work need to be done before we may confidently rely on results that have been obtained with the actual treatment. Moreover, there is not a clear consensus whether magnetic fields were present in the early Universe or not. In the Taylor-Spruit dynamo, the mechanism amplifies a pre-existing field, so if none were present at the start, we cannot use it.

Anyway, the physics used in the present model is today's "state of the art" and it is interesting to study what can be achieved with it. Our result is encouraging, because the computation has been accomplished with reasonable assumptions: the initial rotation rate used here was fast but not extreme, and the mechanisms called upon (anisotropy of the winds and magnetic fields) are not exotic ones, but two natural effects which arise when one treats properly the case of rotation.

After this first step, more work is needed. First, we have to refine the mass loss treatment at $\Omega\Gamma$ -limit and check if the mass loss stays as strong as here. If it does so, we have to check if our result are valid in the whole PISN mass domain. Higher mass models should experience higher mass loss, but it is necessary to check if it would still be sufficient to help them avoid pair-instabilities.

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Can Very Massive Stars Avoid Pair-Instability Supernovae?

S. Ekström, G. Meynet and A. Maeder

Geneva Observatory, University of Geneva, CH - 1290 Sauverny, Switzerland

Abstract. Very massive primordial stars ($140M_{\odot} < M < 260M_{\odot}$) are suggested to end their lives as pair-instability supernovae. Such an event can be traced by a typical chemical signature in low metallicity stars, but at the present time, this signature is lacking in the extremely metal-poor stars we are able to observe. Does it mean that those very massive objects did not form, contrarily to the primordial star formation scenarios? Could they avoid this tragical fate? We explore the effects of rotation, anisotropic mass loss and magnetic fields on the core size of a very massive Population III model, in order to check if its mass is sufficiently modified to prevent the pair instability. We obtain that a Population III model of $150M_{\odot}$ with $v/v_{\text{crit}} = 0.56$ computed with the inclusion of wind anisotropy and Tayler-Spruit dynamo avoids the pair instability explosion.

Keywords: stars: Population III; supernovae: pair instability; stars: rotation; stars: mass loss

PACS: 97.10.Kc; 97.10.Me; 97.20.Wt; 97.60.Bw

WHERE ARE THE PAIR-INSTABILITY SUPERNOVAE ?

According to [1], the fate of single stars depends on their He-core mass (M_{α}) at the end of the evolution. They have shown that at very low metallicity, the stars having $64M_{\odot} < M_{\alpha} < 133M_{\odot}$ will undergo pair-instability and be entirely disrupted by the subsequent supernova. This mass range in M_{α} has been related to the initial mass the star must have on the main sequence (MS) through standard evolution models: $140M_{\odot} < M_{\text{ini}} < 260M_{\odot}$. Since we will present here a non-standard evolution, we will rather keep in mind the M_{α} range.

The typical mass of Population III (Pop III) stars is explored by early structure formation studies and chemistry considerations about cooling. Different studies (e.g., [2, 3]) give the same conclusion: Pop III stars are massive or very massive, even when a bimodal mass distribution allows the formation of lower mass components [4]. Therefore we expect that many among them should die as pair-instability supernovae (PISN).

A typical chemical signature which remains unobserved

These PISN events are supposed to leave a typical chemical signature. According to [5], the complete disruption of the star leads to a very strong odd-even effect: the absence of stable post-He burning stages deprives the star of the neutron excess needed to produce significant amounts of odd- Z nuclei. Also the lack of r - and s -process stops the nucleosynthesis around zinc. Even if one mixes these yields with the yields of zero-metallicity $12 - 40M_{\odot}$ models which die as standard Type II SNe

the PISN signature remains and should be observable.

Using those yields, however, [6] have shown that it provides only a very poor fit to the abundances pattern observed in the metal-poor stars known today. The odd-even effect is not observed, and the models significantly over-produce Cr and under-produce V, Co and Zn. The most metal-deficient stars are suggested to have formed in a medium enriched by only one or a few SNe. The absence of the chemical signature of the PISN is a strong argument against their existence. But how could that be?

Simple solutions

The simplest solution to explain this absence is to assume that the mass domain in question was not formed in the primordial clouds. Maybe the primordial IMF was not as top-heavy as current theory suggests, and the most massive stars formed then could very well be too small for such a fate. This solution, however, seems quite unlikely, given the results of the latest works on the primordial star formation (see the contributions in sessions I and II of this conference).

Another possibility is that the signature was very quickly erased by the next generations of stars. Maybe the metal-poor stars we observe are enriched by more SNe than we actually think, and the later contributions are masking the primordial ones. Only the observation of more and more metal-deficient stars will provide an answer to that possibility.

One can also wonder whether there would be a way for those stars to avoid this fate. The simplest solution then is to assume that some mechanism could lead to such a high mass loss that the conditions for pair-instability would no more be met in the central regions. The aim of the present work is to explore this last scenario.

WOULD ROTATION HELP?

We have shown [7] that rotation can change the mass loss history of low metallicity stars in a dramatic way. Two processes are involved:

1. During MS, the low metallicity reduces the radiative winds, so that the star loses very little mass and thus very little angular momentum. As the evolution proceeds, the stellar core contracts and spins up. If a coupling exists between the core and the envelope (i.e., meridional currents or magnetic fields), the surface may be accelerated up to the critical velocity and the star may lose mass by a mechanical wind due to centrifugal acceleration. The matter is launched into an equatorial disk and the question which arises is whether the matter may fall back on the star or may really be considered as lost. According to S. Owocki (private communication), the luminosity of the star is well sufficient to dissipate the disk fast enough for the matter to be lost indeed.
2. Rotation induces an internal mixing which enriches the surface in heavy elements. The effective surface metallicity is enhanced by a factor that has been shown to be very large (up to 10^6 for a $60M_{\odot}$ star at $Z_{\text{ini}} = 10^{-8}$). Rotation also favors a redward evolution after the MS, allowing the star to spend more time in the cooler part of the HR diagram. The opacity of the envelope is increased, and the radiative winds may thus be drastically enhanced.

Both effects add up and lead to strong mass loss at very low metallicity.

Rotation at $Z=0$

But what happens when the metallicity is not very low but strictly zero? All the effects described above are present but their amplitude is moderate (see the contribution of Georges Meynet in these proceedings for a more complete description).

The meridional currents are much smaller at $Z = 0$ than at low or very low Z (Fig. 1). The amplitude of the outer cell of the meridional circulation is lower when the envelope is more compact, and may even change sign (reversing the transport of angular momentum) in some parts of the star. The stars may reach the critical limit, but later in their MS evolution (for a given initial mass and velocity), and the mechanical mass loss remains thus very small.

After the MS, the redward evolution is delayed because of structural differences: the combustion of helium starts smoothly as soon as the central hydrogen content has been exhausted, without core contraction since the

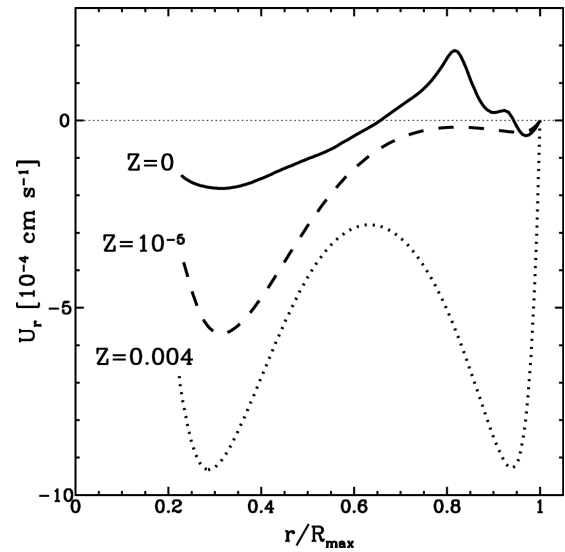


FIGURE 1. Amplitude of the meridional currents in the interior of $20M_{\odot}$ models at the same evolutionary stage ($X_c = 0.40$) but at different metallicities. Continuous line: $Z = 0$; dashed line: $Z = 10^{-5}$; dotted line: $Z = 0.004$.

central temperature is already hot enough to sustain the triple- α reaction during the core H-burning phase. The star remains in the blue part of the HR diagram until some carbon is brought by mixing to the H-shell, driving a flash as the shell turns on the CNO-cycle and becomes suddenly convective. Eventually the star reaches the red part of the HR diagram, but the outer convective zone remains very thin, so the surface enrichment is low. The radiative winds are indeed enhanced by the change in opacity, but not much.

Rotation alone seems thus to fail in providing a way to lose sufficient mass at $Z = 0$ for our purpose.

Two natural effects of rotation

In fact, in previous calculations, we have neglected two mechanisms which arise naturally with rotation and which could change this picture.

The first one is the wind anisotropy [8]: when a star rotates fast, it becomes oblate. As shown by von Zeipel's theorem, the poles become hotter than the equator, and the radiative flux is no more spherically symmetric: it gets much stronger in the polar direction than in the equatorial plane. Since the mass is lost preferentially near the poles, it removes much less angular momentum than in the spherical configuration, and thus the star may reach the critical limit much earlier.

The second mechanism is the magnetic fields. According to [9] the differential rotation may amplify an ex-

isting magnetic field through the Tayler-Spruit dynamo mechanism, and provide a strong core-envelope coupling. This coupling will be able to accelerate the surface very early in the evolution, and maintain the star at critical limit throughout its entire evolution.

MODEL CALCULATIONS

Method

With the help of the two effects mentioned above, we have computed an exploratory model. Its mass has been chosen to be $150M_{\odot}$ and the initial ratio between the equatorial velocity and the critical one to be $v_{\text{ini}}/v_{\text{crit}} = 0.56$. Let us mention that this ratio is higher than the one needed to account for the observed average velocities at solar metallicity, but it is still a “reasonable” value, not an extreme one.

The computation has been accomplished using the Geneva code with up to date nuclear reaction rates obtained with NETGEN (<http://www-astro.ulb.ac.be/Netgen/>). The opacity tables come from OPAL (<http://www-phys.llnl.gov/Research/OPAL/opal.html>) with the extension at low temperature by Ferguson & al. [10]. The initial composition is $X = 0.753$, $Y = 0.247$ and of course $Z = 0$.

The radiative mass loss prescription is an important ingredient for modeling of massive stars. Here we have used [11]. Since this prescription is not aimed at the case $Z = 0$ strictly, we have used the same adaptations as in [12]. The Wolf-Rayet (WR) mass loss rate is taken from [13] with the metallicity scaling from [14]. For the calculation, we have taken the effective surface metallicity $Z_{\text{eff}} = (1 - X - Y)_{\text{surf}}$ so that the enrichment of the surface is accounted for. We must stress that this Z_{eff} is mainly composed by CNO elements but no iron. It is usually considered that WR winds are triggered by Fe lines, whereas the CNO lines determine only v_{∞} , so we expect that no WR winds can take place at $Z = 0$. But [15] have shown that when the metallicity gets really low, the CNO lines take over the role of Fe lines in the line driving, and the metallicity dependence of the mass loss flattens.

When the model reaches the critical limit, the mechanical mass loss has been treated as described in [7]. The treatment of anisotropic winds has been implemented as in [16] and the effect of magnetic fields as in [17].

Evolution and mass loss

In Fig. 2, we present the evolution in the HR diagram (left panel) and the evolution of mass with time (right

panel). The grey line shows a non-rotating model computed with the same physics for comparison.

During its whole evolution up to the end of core He-burning, the non-rotating model loses only $1.37M_{\odot}$. This illustrates the weakness of radiative winds at $Z = 0$.

The evolution of the rotating model (black line) can be described by four distinct stages:

1. (*continuous part*, lower left corner) The model starts its evolution on the MS with only radiative winds, losing only a little more than $0.002M_{\odot}$. During this stage, it accelerates quickly because of the strong coupling exerted by the magnetic fields and the minimal loss of angular momentum in the wind.
2. (*dashed part*) When its central content of hydrogen is still 0.58 in mass fraction, it reaches the critical velocity and starts losing mass by mechanical wind. It remains at the critical limit through the whole MS, but the mechanical wind removes only the most superficial layers that have become unbound, and less than 10% of the initial mass is lost at that stage ($11.44M_{\odot}$). The model becomes also extremely luminous, and reaches the Eddington limit when 10% of hydrogen remains in the core. As a matter of fact, it is not the classical Γ -limit that is reached here, but the so-called $\Omega\Gamma$ -limit. Due to the fast rotation, the maximum Eddington factor allowed is reduced; at the same time, because of the high luminosity, the critical velocity is reduced in comparison with the one derived from the classical Ω -limit (see Maeder & Meynet [18] for details).
3. (*dotted part*) The combustion of helium begins as soon as the hydrogen is exhausted in the core, then the radiative H-burning shell undergoes the CNO flash, setting the model on its redward journey. Whatever the structural modifications, the model remains at the $\Omega\Gamma$ -limit and loses a huge amount of mass: if it contracts slightly, the equatorial velocity gets closer from the Ω -limit and matter is expelled; if it expands slightly, the luminosity gets closer to the Γ -limit, reducing the maximal equatorial velocity allowed, and again matter is expelled. The strong magnetic coupling keeps bringing angular momentum to the surface and even the heavy mass loss is not able to let the model evolve away from the critical limit. The mass lost during that stage amounts to $53.46M_{\odot}$. When the model starts a blue hook in the HR diagram, its surface conditions become those of a WR star: the surface H mass fraction goes down below 0.4 and the T_{eff} gets higher than 10,000 K. The subsequent sudden drop of luminosity takes the model away from the Γ -limit and marks the end of that stage.
4. (*continuous part*) The rest of the core He-burning

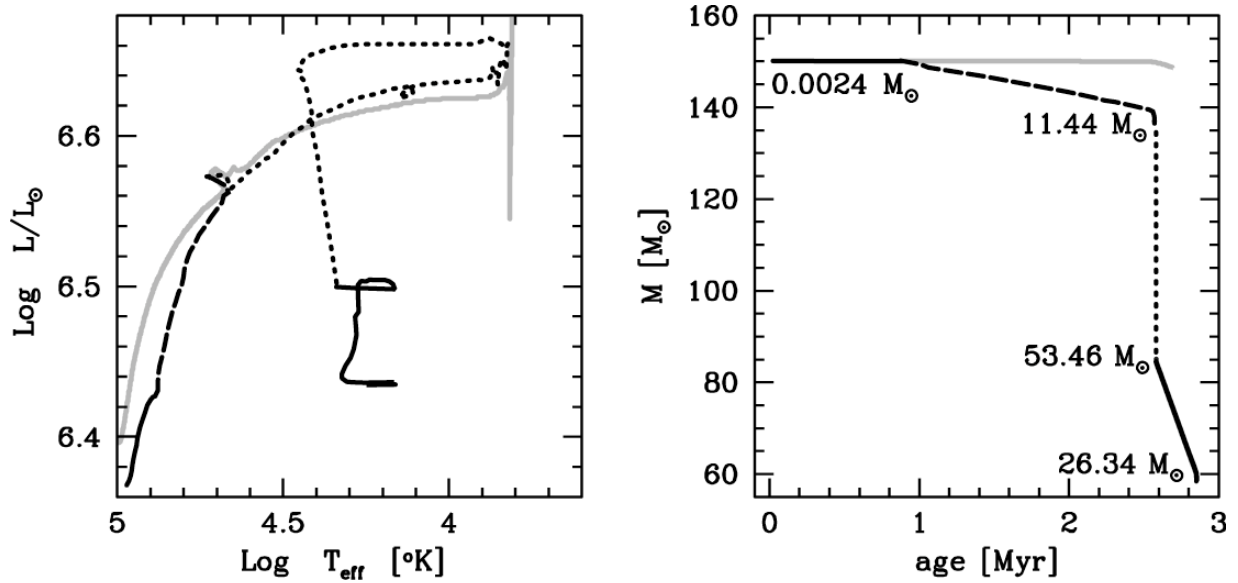


FIGURE 2. Black line: rotating model; *continuous part*: beginning of MS ($X_c = 0.753$ down to 0.58; *dashed part*: rest of the MS; *dotted part*: beginning of core He-burning phase ($Y_c = 1.00$ down to 0.96); *continuous part*: rest of the He-burning. Grey line: non-rotating model for comparison. **Left panel**: evolution in the Hertzsprung-Russell diagram; **Right panel**: evolution of the mass of the model. The mass indicated is the mass lost at each stage, not a summation.

is spent in the WR conditions. The mass loss is strong but less than in the previous stage: another $26.34 M_{\odot}$ are lost.

At the end of core He-burning, the final mass of the model is only $M_{\text{fin}} = 58.05 M_{\odot}$, already below the minimum M_{α} needed for PISN ($M_{\alpha} \geq 64 M_{\odot}$). Note that the contraction of the core after helium exhaustion brings the model back to critical velocity, so this value for M_{fin} must be considered as an upper limit.

SUMMARY

The model we presented here is exploratory. We cannot draw general conclusions from it. But our model shows that heavy mass loss is possible even at $Z = 0$, and the answer to question posed in our title is: *Yes, under certain conditions, very massive stars can indeed avoid PISN.*

Some aspects need yet to be clarified, for example the validity of the WR mass loss rate we have used. Can the CNO lines alone really drive a WR wind? In a more general perspective, we still lack a good mass loss recipe for the strict $Z = 0$ case.

A word of caution must also be cast on the inclusion of magnetic fields. The validity of the Tayler-Spruit dynamo is still under debate, and more work need to be done before we may confidently rely on results that have been obtained with the actual treatment. Magnetic fields do exist, however; they are observed in stars and pulsars,

and the general effect must be a strong core-envelope coupling, no matter how we treat it.

The physics used in the present model is today's "state of the art" and it is interesting to study what can be achieved with it. Our result is encouraging, because the computation has been accomplished with reasonable assumptions:

1. the initial rotation rate used here was fast but not extreme;
2. the mechanisms called upon (anisotropy of the winds and magnetic fields) are not exotic ones, but two natural effects which arise when one treats properly the case of rotation.

After this first step, it will be interesting to explore further the PISN mass range. Higher mass stars should experiment higher mass loss, but will it be strong enough to avoid the pair-instability regime?

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Stellar Evolution at low Metallicity: Mass Loss, Explosions, Cosmology
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Henny J.G.L.M. Lamers, Norbert Langer, Tiit Nugis, Kalju Annuk

Effects of Rotation on Mass Loss for Population III stars

Sylvia Ekström, Georges Meynet and André Maeder

Geneva Observatory, Maillettes 51, CH - 1290 Sauverny, Switzerland

Abstract. The effects of rotation on low-metallicity stellar models are twofold: first, the models reach break-up during main sequence and may lose mass by mechanical process; second, strong internal mixing brings freshly synthesized elements towards the surface and raises the effective metallicity to higher values, so that the initially very low radiative winds are enhanced. Those two effects are also found in $Z = 0$ models, but to a lesser degree because of structural differences. This weak mass loss becomes interesting though in the case of the black hole-doomed stars ($25\text{--}140\text{ M}_\odot$ and $> 260\text{ M}_\odot$) because it allows these stars to contribute to the enrichment of the medium by stellar winds.

1. $Z = 0$ models

The effects of rotation on stellar evolution at low Z are presented in detail by Georges Meynet (see contribution in this volume). We come back here on two aspects that play an important role in $Z = 0$ stellar models. The results presented here have been obtained by computing a small grid of models with initial abundances $X = 0.76$, $Y = 0.24$ and $Z = 0$, and masses ranging from 15 to $200\text{ M}_\odot$. For each mass, we have calculated a rotating and a non-rotating model. The chosen equatorial velocity for the rotating models is $v_{\text{ini}} = 800\text{ km/s}$, which may seem fast but corresponds to a similar fraction of the critical velocity than a velocity of 300 km/s in a solar metallicity model. All the models have been computed with the Geneva code, using the same physical ingredients than in Meynet & Maeder (2002). For the models reaching $\log L/L_\odot > 6$, we have used the radiative mass loss prescription of Kudritzki (2002), with the same adaptations for the $Z = 0$ case than in Marigo, Chiosi, & Kudritzki (2003).

2. First rotation effect: break-up during main sequence

Since the models have very low or no radiative winds at all, they have no way to lose angular momentum. During the main sequence (MS) phase, the contracting core accelerates the envelope through meridional circulation coupling and brings the surface to the break-up limit (see Fig. 1, left).

As can be seen in this figure, all the $Z = 0$ models reach break-up limit. They all remain at break-up until the end of the MS (the 200 and $85\text{ M}_\odot$ models seem to depart from break-up, but remain in fact critical if one takes into account the $\Omega\Gamma$ limit). This critical rotation induces a purely mechanical mass loss due to centrifugal force, with a rather weak amplitude because the expelled layers are very superficial and little dense. If we add just a small fraction of metals in the initial abundance, the effect is slightly stronger because the core-envelope

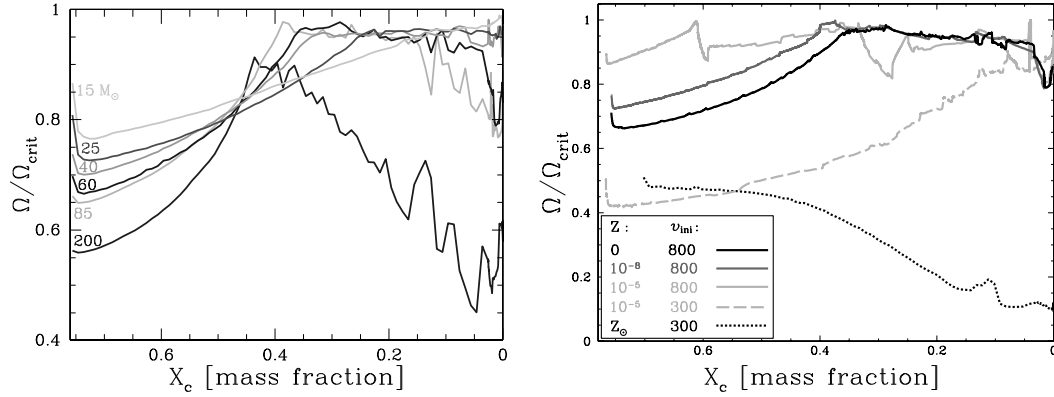


Figure 1. Evolution of the Ω/Ω_c ratio during the main sequence phase. **Left:** all the $Z = 0$ rotating models. **Right:** comparison of several $60 M_\odot$ with different metallicities (light grey: $Z = 10^{-5}$; middle grey: $Z = 10^{-8}$; black: $Z = 0$) and different equatorial velocities (continuous: 800 km/s; dashed: 300 km/s). A $Z = Z_\odot$ stellar model has also been plotted (black dotted) in order to show the effect of strong mass loss on the surface angular velocity.

coupling is a little more efficient and the models reach break-up earlier in their evolution, as can be seen on Fig. 1 (right). Typically, the $60 M_\odot$ at $Z = 0$ loses 3.7% of its initial mass through this mechanism, while the $Z = 10^{-8}$ corresponding model loses 4% and the $Z = 10^{-5}$ model up to 10%.

3. Second rotation effect: strong mixing during core helium burning

We have mentioned that the efficiency of the outer cell of the meridional circulation decreases with decreasing Z , which leads to steep Ω -gradients and thus a strong internal mixing by shear turbulence. Freshly synthesized elements are thus brought to the surface and the effective metallicity increases.

As can be seen in Fig. 2 (left), the surface enrichment of the $Z = 0$ models begins already during the MS phase and reaches non negligible values only at the very end of the core Helium burning phase. Although the surface metallicity increases then up to $Z_{\text{eff}} = 10^{-8}$ or 10^{-4} (depending on the model), the amount of mass lost through this mechanism remains modest because the period of time concerned is short. If we again add a small fraction of metals, the effect becomes very important. In fact rotation induces an early redward evolution in these low Z models, as can be seen in Fig. 2 (right). The increase of the effective surface metallicity takes place earlier and the mass removed by this mechanism is very important. For instance, whereas the $60 M_\odot$ at $Z = 0$ loses only 0.05% of its initial mass during this stage, the $Z = 10^{-5}$ model loses 28% and the $Z = 10^{-8}$ model up to 56%. Let us mention that the fact that the $Z = 10^{-8}$ model loses more mass than the $Z = 10^{-5}$ model is due in part to a blue loop in the HR diagram, at the very end of the core helium burning phase, so the model is brought again at break-up limit (see Heger & Langer 1998).

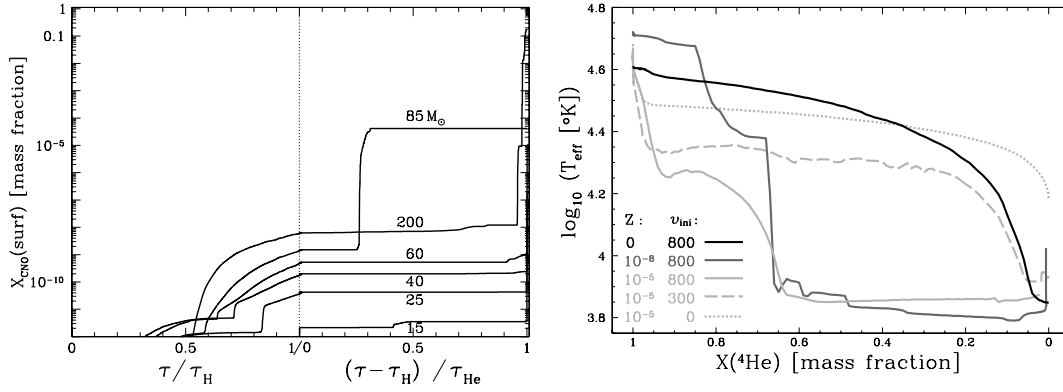


Figure 2. **Left:** evolution of the surface CNO mass fraction with the age of the $Z = 0$ rotating models. The age is expressed as a fraction of the MS time (τ_H) followed by the fraction of τ_{He} . **Right:** redward evolution during core helium burning, comparison of several 60 $M_\odot$ models with different metallicities (light grey: $Z = 10^{-5}$; grey: $Z = 10^{-8}$; black: $Z = 0$) and different equatorial velocities (cont.: 800 km/s; dashed: 300 km/s; dotted: 0 km/s).

4. Total mass loss at $Z = 0$

If we add both effects, we can see that the total mass lost by the $Z = 0$ models remains very modest in the range of masses and velocities considered (see Fig. 3). The 15 $M_\odot$ model loses less than 0.3% of its initial mass, and the 200 $M_\odot$ model only a little more than 10%. This latter result meets the conclusion of Marigo et al. (2003), even though the mass loss found here is a little higher. One can note that for the models ranging up to 40 $M_\odot$, the mass loss is almost only due to the MS mechanical process.

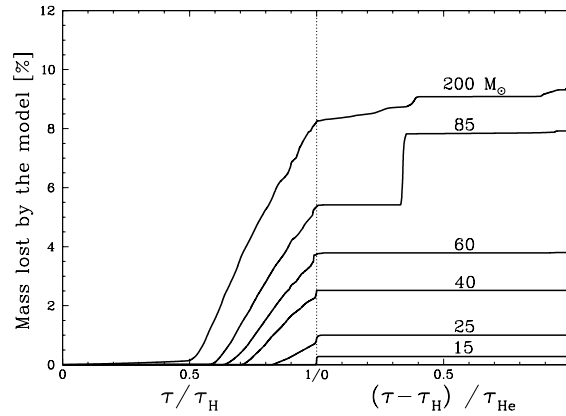


Figure 3. Evolution of the mass lost by the $Z = 0$ rotating models expressed as the percentage of the initial mass. The x axis is the same as in Fig. 2 (left).

Now, if the star explodes at the end of its life (as a Type II supernova for the low mass end of our sample or as pair-instability supernova for the high mass

end) the wind contribution will be totally insignificant in front of the supernova contribution. There is a mass domain where the wind contribution can become non negligible: the mass domain where the stars are supposed to end their life as direct black hole. According to Heger et al. (2003), this mass domain includes stars in the range of 25 to 140 $M_{\odot}$ or above 260 $M_{\odot}$ on the MS. Those stars have long been considered as useless for the chemical evolution of their surroundings. Here we see that they may have a modest, but yet non zero contribution through their winds. If we ponderate the mass ejected by the models with the IMF, and taking into account only the massive stars that may contribute to the chemical evolution at that early time, we see that the part of the wind-only contributors is about 4-9 % of the total ejected mass, depending on the IMF considered (standard Salpeter or double-peaked as in Nakamura & Umemura 2001). We let the reader judge if this is significant or not, but anyway it's a lower limit because we have not study the case of $M > 260 M_{\odot}$, a range of mass that should experience stronger mass loss.

5. Discussion

We have seen that fast rotation enhances mass loss through two mechanisms: the reaching of break-up limit during the main sequence phase and the rising up of the effective metallicity during and after the core helium burning phase. In the strictly $Z = 0$ case, both mechanisms play a role but the effects are not very strong and the mass loss remains low for the initial velocity and mass range considered here. Let us mention though that whereas the evolution is not far from constant mass evolution, the rotational mixing experienced by fast rotating models have strong effects on the abundances profiles and consequently on the chemical yields obtained. In particular, the present population III stellar models produce large amounts of primary nitrogen. In the non-zero but very low-metallicity case, we have seen that both mechanisms lead to very strong effects, changing drastically the vision we may have on the extremely metal-poor stars. The results presented here are qualitative but they show a robust trend through a wide range of masses and metallicities and are based on physical ingredients that have more than once given proof of their validity. For a detailed discussion about the subject, we let the reader refer to Meynet, Ekström, & Maeder (2005).

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CNO production in the first generation stars

Sylvia Ekström*, Georges Meynet & André Maeder

Geneva Observatory, Switzerland

E-mails: sylvia.ekstrom@obs.unige.ch,
georges.meynet@obs.unige.ch,
andre.maeder@obs.unige.ch

Big Bang nucleosynthesis produces only light elements and the very first generation stars are thus formed from metal-free clouds. They start the production of heavy elements during their life, and enrich the interstellar medium through their explosive death. Stellar evolution models show that the treatment of rotation has important effects on the evolution of those metal-free stars: for example, rotating models produce up to five orders of magnitude more primary nitrogen than non rotating models, due to internal mixing. This will have an impact in the composition of the second generation stars, some of which may now be observed in the Galactic halo. In the case Population III stars were very massive and would end up as direct black holes, rotation again have an interesting effect of enhancing mass loss through centrifugal force and surface enrichment. CNO composition patterns observed in ultra metal-poor halo stars may be explained by a 'wind only' contribution.

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1. Primordial stars

1.1 Birth

After the Big Bang, the early Universe is composed only from H and He, a little Be, B and Li, but no heavier elements. At the end of the so-called 'Dark Age', halos of dark matter start to collapse and form stars with the gas trapped within. The star-forming clouds are almost pure H and He, so fragmentation cannot occur, leading to a massive or very massive star formation (see Omukai & Palla 2003 and references therein). The mass domain covered by primordial stars is still very much debated. Numerical simulations including known physics lead to the formation of very massive objects (see Bromm & Larson 2004), with masses up to 500-700 $M_{\odot}$. Stability studies for such extreme stars show that they are stable enough against radial pulsations to evolve until advanced stages (see Baraffe & al. 2001). Observational constraints, though, favor a more standard, slightly top-heavy Larson IMF in the range 1 - 100 $M_{\odot}$ (see Schneider & al. 2006), while the mechanism for producing sufficient fragmentation in primordial clouds is not yet known.

1.2 Evolution

The new star is born metal-free. In its contracting core, the gas heats up until nuclear reactions start burning some H, but the only energy source is provided by pp-chains reactions, so the collapse is only slowed down. The core reaches then a temperature that allows some He burning through 3α reaction, producing some C and allowing the CNO-cycle energy production to stop the collapse. Primordial stars are thus born hotter and more compact, and their metal-free envelope lets them remaining so through the whole main sequence (MS). Since radiative winds scale with the metallicity as $\dot{M} \sim Z^{0.65}$ (see Vink & al. 2001), the star experiences practically no mass loss during its evolution, except if its luminosity is high enough to remove some mass just with H or He lines (see Kudritzki 2002).

1.3 Death

According to Heger & al. (2003), the fate of a massive star depends on the mass of its He core at the end of the evolution. We have just seen that primordial stars evolve at almost constant mass, so they end up their life with a heavy He core. Stars with initial mass ranging from 25 to 140 $M_{\odot}$ are supposed to collapse as direct black hole without ejecting any matter. Between 140 and 260 $M_{\odot}$ the star undergoes pair-creation instability and is supposed to be completely disrupted in the process. Heavier stars again form direct black hole.

2. Rotation at (extremely) low metallicity

A detailed description of the effects of rotation at low or very low metallicity is given in Meynet & al. (2006). We will highlight here two main effects: 1) because of the very low radiative winds, the star cannot lose its angular momentum and is thus prone to reach the break-up velocity during the MS; 2) because of a less efficient meridional circulation, the Ω -profiles are steeper, and thus rotational mixing is stronger (see also the contribution of G. Meynet in this conference).

The first effect leads to enhanced mass loss through purely mechanical process, the uppermost layers of the star being removed by the centrifugal force. The second effect takes place during core

He-burning, enriching the surface with newly synthesized elements and allowing radiative winds to get stronger.

3. Modelization

In order to study the effects of rotation at $Z = 0$, we have computed, with the Geneva stellar evolution code, metal-free models ranging from 9 to 200 $M_{\odot}$ (Fig. 2). For each mass, we have computed a non-rotating model and a differentially rotating one, where the equatorial velocity has been chosen so that $v/v_{\text{crit}} \simeq 0.5$ (depending on the mass, this gives $v_{\text{eq}} \simeq 500 - 800 \text{ km s}^{-1}$), corresponding to the average value observed in massive stars. The initial composition of the models is $X = 0.76$, $Y = 0.24$ and $Z = 0$.

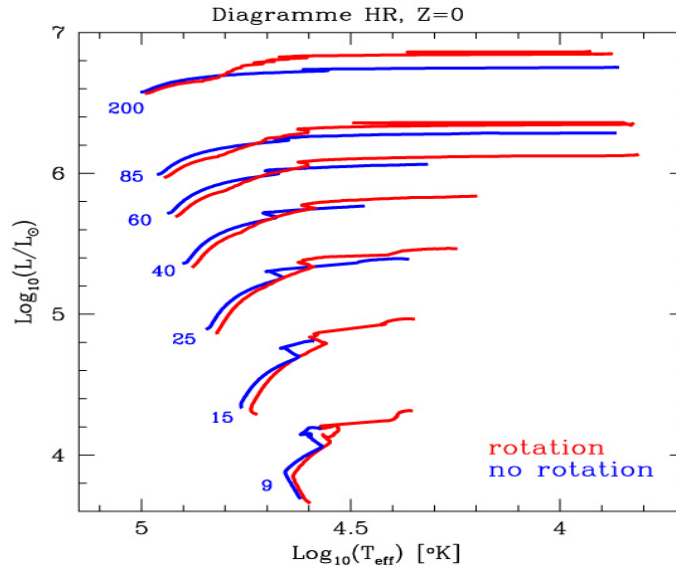


Figure 1: Hertzsprung-Russell diagram of the $Z = 0$ models

The input physics is the same as in Meynet & Maeder (2002), including rotation-induced instabilities as meridional circulation, secular and dynamical shear, and horizontal turbulence. We have used the Schwarzschild criterion for convection, and an overshooting parameter of $0.2H_p$ during core H- and He-burning and 0 otherwise. We have applied the mass loss rates proposed by Kudritzki (2002) for the models reaching a luminosity $L/L_{\odot} \geq 6$, with the same adaptation for the strictly $Z = 0$ case as in Marigo & al. (2003).

4. CNO production

The models have been followed until the end of core O-burning, so the values presented here are not the final yields of the model, but they give insights of the way differential rotation modifies the characteristics of a star. As expected, most of the models have reached the break-up velocity during the MS, some of them very early in their evolution. The mass lost within this process however is not very important because only the uppermost layers of the star are removed, and those

layers have a low density. Rotation favors a redward evolution and, as seen above, an enhancement of the effective surface metallicity through mixing, so the radiative winds are stronger during He-burning. Again, in the $Z = 0$ case, this doesn't lead to strong mass loss because the time involved is short. Our heavier rotating model ($200 M_{\odot}$) loses just a little more than 10% of its initial mass (8.2% during MS, 2.2% after), but its non-rotating counterpart loses all added only 0.5%.

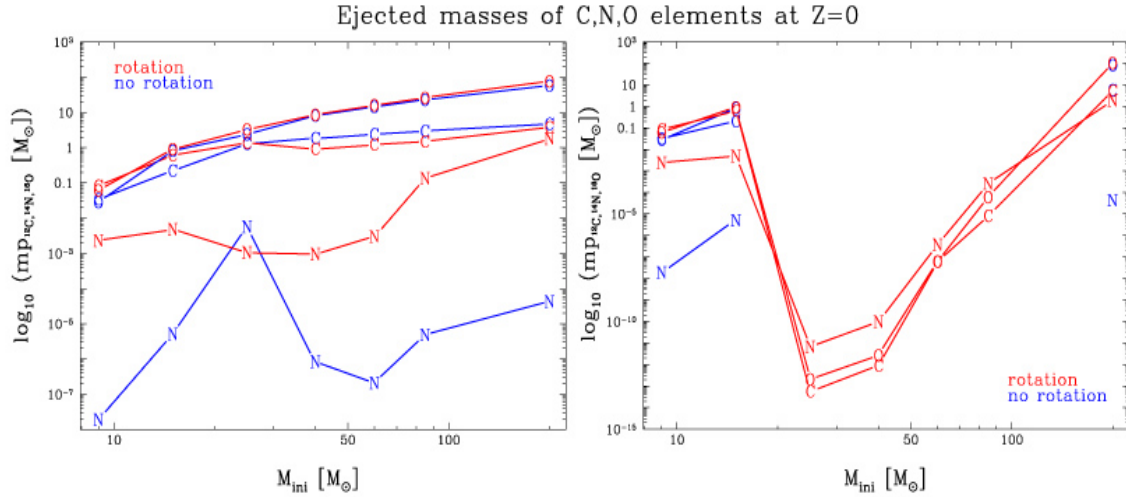


Figure 2: CNO production of $Z = 0$ models. **Left:** assuming a SN explosion with a remnant mass determined by the CO core mass as in Maeder (1992); **right:** following the supposed fate of the star according to Heger & al. (2003).

At the end of the evolution, the CO cores of the rotating models are about 10-20% heavier than the non-rotating ones. Since the bulk of the CO produced comes from the core, the values obtained for both the rotating and non-rotating models are quite similar (see Fig. 2, left).

The case of N is very different. Except for the $25 M_{\odot}$ model, the rotating values are three to five orders of magnitude higher than the non-rotating ones. This is due to the fact that N is mainly produced in the H-burning shell, when some C and O from the He-burning shell is diffused outwards. Rotation-induced mixing favors this process. But for the $25 M_{\odot}$ without rotation, as Limongi & Chieffi (2005), we find a high primary N production about three orders of magnitude higher than the value produced by the other non-rotating models. The value is even slightly larger than the rotating model's value. It can be explained by structure differences: in the non-rotating case, the convective zone associated with the He shell is more extended than in the rotating case, and reaches the H shell, leading to a strong production of primary N.

The picture becomes very different if we take into account the supposed fate of the star according to Heger & al. (2003). In the non-rotating case, the models with masses between 25 and $85 M_{\odot}$ do not contribute at all to the enrichment of the medium, having no winds and ending as direct black holes. In the rotating case, the enrichment is made through a 'wind-only' contribution, leading to low but non-zero contribution of the enrichment of the medium. An important thing to note is that the mass lost through winds has very peculiar abundances compared to the composition of SN-ejecta (see Fig. 2, right), with larger N/O, N/C ratios, and much lower $^{12}\text{C}/^{13}\text{C}$ (not shown).

To widen the discussion, we have also computed fast rotating $60 M_{\odot}$ models at $Z = 10^{-5}$ and 10^{-8} (see Meynet & al. 2006). In those models, the effects described above are very strong, leading to an increase of the effective surface metallicity of 2 to 6 orders of magnitude, and a mass loss of 40-60% of the initial mass. If we compare the wind-only ejecta and the abundances pattern observed in some of the most metal-poor stars of the halo of our Galaxy (see Frebel & al. 2006 for HE 1327-2326 and Plez & Cohen 2005 for G77-61), we get an interesting matching able to reproduce the high C, N, and O at the same time (see Fig. 8 of Meynet & al. 2006).

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Fast rotating first stars

Sylvia Ekström, Georges Meynet and André Maeder

Geneva Observatory, Maillettes 51, CH - 1290 Sauverny, Switzerland
email: sylvia.ekstrom@obs.unige.ch

Abstract. In order to study the effects of fast rotation on primordial stars, we present the evolution and the chemical yields of zero metallicity models with masses between 15 to 200 $M_{\odot}$.

Keywords. Stars: abundances, stars: rotation, early universe

Because of the peculiarities of physics at $Z=0$, primordial stars are more compact than non-zero-metallicity stars, and thus for a given mass the radius is smaller. If one admits that the angular momentum contained in a primordial star-forming cloud has no reason to be lower than the angular momentum observed in present-day star-forming clouds (see Abel *et al.* (2002)), one can suppose that primordial stars will present high equatorial velocities on the ZAMS. It is therefore interesting to study the effects of fast rotation on the first stars of the Universe.

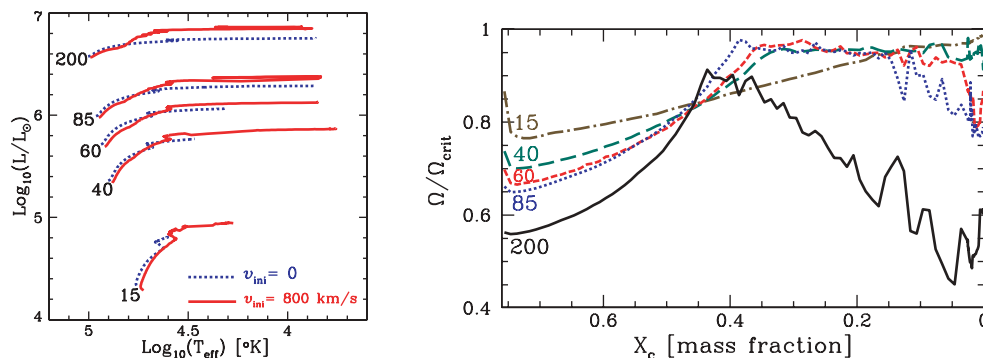


Figure 1. Left: HR diagram for $Z=0$ models. Right: Ω/Ω_c evolution during core H-burning.

For this purpose, we have computed zero-metallicity models with masses ranging from 15 to 200 $M_{\odot}$. For each chosen mass, we have computed a fast rotating model ($v_{\text{ini}}=800$ km/s) and a non rotating one (see Fig. 1 left). The evolution has been followed until core helium exhaustion for the highest masses (85 and 200 $M_{\odot}$) and until core carbon depletion for the others. We refer the reader to Meynet & Maeder (2002) concerning the main physical ingredients of the calculations. In agreement with Meynet & Maeder (2002), all the models reach break-up limit during MS (see Fig. 1 right), but the mechanically experienced mass loss remains modest. Striking differences between rotating and non-rotating models appear in the internal profiles (see Fig. 2). Steep Ω gradients induce very strong internal mixing and early surface enrichment, whereas the surface of non rotating models remains metal-free at least until the end of our calculations.

The ejected masses presented here (see Table 1) have been computed according to the supposed fate of the stars (see Heger & Woosley (2002)). The 200 $M_{\odot}$ models are expected to be totally disrupted by pair-instability supernovae and to return to the ISM all the heavy elements processed during their life. The 85, 60 and 40 $M_{\odot}$ models will be

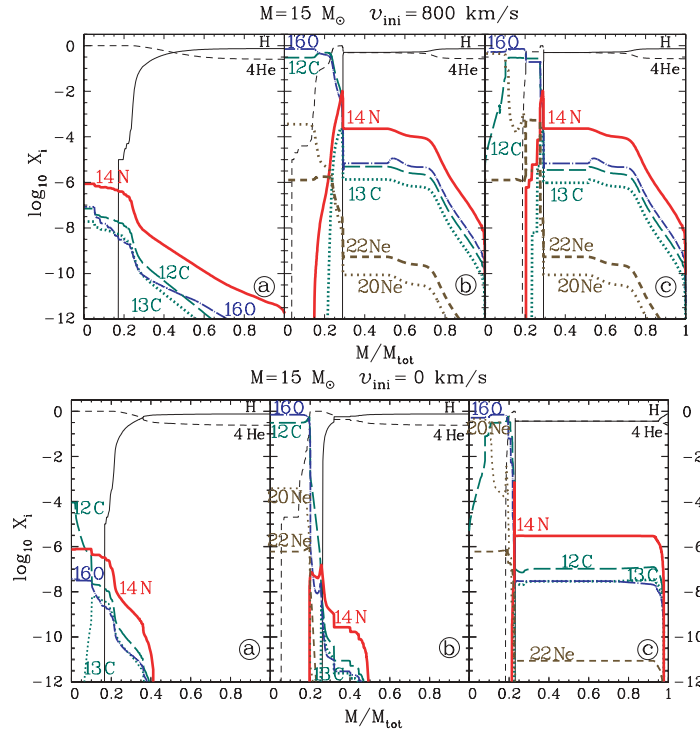


Figure 2. Top: internal abundances profiles at the end of (a) core H-burning, (b) core He-burning and (c) core C-burning in the $15 M_{\odot}$ with $v_{\text{ini}} = 800$ km/s. **Bottom:** for comparison, same as above but in the non-rotating model.

Table 1. Ejected masses of He and CNO elements by the $Z = 0$ models, computed according to the assumed fate of the star (see text).

M_{ini} $M_{\odot}$	v_{ini} km/s	end of He-b		ejected masses in $M_{\odot}$ (end of He-b)				
		M_{fin}	M_{CO}	^4He	^{12}C	^{13}C	^{14}N	^{16}O
200	0	199.95	98.82	57.88	6.29	7.75e-07	4.33e-05	81.45
	800	181.42	121.27	55.31	7.64	5.33e-02	5.17e-01	103.55
85	0	84.94	38.43	1.44e-02	0	0	0	0
	800	78.29	41.09	2.35	2.53e-06	6.33e-07	8.23e-05	1.55e-06
60	0	60.00	25.10	2.40e-05	0	0	0	0
	800	57.72	28.13	0.58	3.56e-12	9.82e-13	3.66e-10	9.57e-12
40	0	40.00	14.66	0	0	0	0	0
	800	38.99	16.33	0.25	6.98e-13	1.92e-13	7.62e-11	2.11e-12
15	0	15.00	2.96	4.63	0.46	2.98e-10	6.61e-08	0.85
	800	14.82	3.38	5.22	0.77	8.52e-05	3.03e-03	0.90

completely swallowed in a black-hole, contributing to the enrichment of the medium only through their winds, if any. In this case, only the rotating models have a contribution, very modest but non zero. The $15 M_{\odot}$ models end their life as SNII and the differences in the rotating and non rotating profiles lead to a difference of 5 orders of magnitude in the values of ^{13}C or ^{14}N pre-SN yields.

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