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Letters to the Editor

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Hidden Variables Revisited

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In a recent issue of this Journal, D. Bohm and J. Bub have published[1] what they call a "refutation" of a paper published by the present authors[2] on the question whether hidden variables can be excluded in quantum mechanics. The conclusion of our paper was that hidden variables, if they existed, would lead to observable consequences, which would contradict certain known facts on microsystem.

In our view the aforementioned paper by Bohm and Bub does not contain a refutation of the content of our paper. Instead it contains a number of misinterpretations and inaccuracies which might give rise to confusions in a reader not familiar with the problem. Most of these are, however, not germane to the question and we feel therefore not compelled to enter into a detailed discussion of them. This is all the more justified since all of the questions which are raised by Bohm and Bub are discussed in the thesis of one of us.[3]

We therefore content ourselves with a restatement of our result in a nontechnical language. This result is an improvement of a theorem due to von Neumann who claimed to have demonstrated with this theorem the impossibility of hidden variables in quantum mechanics.[4] Later analysis showed that this theorem does not settle the question of hidden variables because some of the assumptions needed for the theorem are too restrictive.

We have therefore examined the problem anew with a view of reducing the assumptions in von Neumann's theorem to the minimum needed to make some valid inferences concerning the hidden variable problem. The improvements with respect to von Neumann's work concerned the following three points:

(i) The validity of quantum mechanics is not assumed. Instead one assumes only a lattice structure of yes–no experiments (called propositions) originating directly from experimental facts.

(ii) It is not assumed that states are linear functionals on the propositions. Indeed, linearity cannot even be expressed since there is no addition defined for propositions.

(iii) It is not assumed that the lattice is coherent thus permitting to make valid inferences for systems with superselection rules.

In common with von Neumann, and all other discussions of the hidden variables problem known to us, we have assumed that hidden variables in quantum mechanics means that every physically realizable state (which usually shows dispersion for some of the measureable quantities) can be represented as a mixture of dispersion-free states.

Physically this means that if one prepares an ensemble under identical relevant conditions, this ensemble can be (in principle, if not in practice) considered as composed of subensembles which are dispersion-free for all physical quantities.

The main result of the paper consists in showing that the existence of hidden variables in this sense would entail certain properties of the lattice of propositions which are incompatible with the facts. We have found nothing in the paper by Bohm and Bub which would invalidate this conclusion.

We join a few remarks which might forstall possible misinterpretations of our result.

We want to emphasize again that, although the lattice structure of yes–no experiments which we have introduced has the mathematical form of a propositional calculus such as it is used in formal logic, it has nothing whatsoever to do with a modification of formal logic. This was already pointed out in the thesis of one of us[3] but apparently not with sufficient emphasis to make the point absolutely clear. A detailed discussion of this point requires considerable technical developments which were not the primary concern of Ref. 3. For the reader who is interested in this point we recommend the excellent and complete discussion in the book by Mittelstaedt[5] with which we fully agree.

Secondly, we should emphasize also that the axiomatic system which we have used is not the only one possible which could be used to construct the physics of microsystems. The best developed other system is the one due to Segal.[6,7] The hidden variable problem poses itself in this new setting again in an entirely different form.

In this setting the problem was recently discussed and settled by Misra.[8] We refer to this paper for further details as well as for a thorough discussion and review of the entire problem.

Finally we should remark that the concept of hidden variables can be broadened by generalizing the notion of states especially that of the hypothetical unobservable dispersion-free states which are used for the hidden variables. We have ourselves drawn attention to this fact at the end of our paper and we have even mentioned a concrete possibility (that of approximate dispersion-free states). This possibility seemed to us to merit a closer study because it is physically more interesting than a drastic ad hoc modification of the notion of states such as it was for instance done in the example of Bell.[9] As Misra has pointed out in particular[8] the quest for hidden variables becomes a meaningful scientific pursuit only if states, even physically non-realizable states, are restricted by physical considerations.

The example of Bell was useful, because it shows that one of the hypotheses of our theorem [condition (4)°] was not only sufficient but also necessary for the affirmation of the theorem.

A similar remark applies to the example of Bohm and Bub.[10] Here we have the additional objection that they postulate a modification of the evolution of states during the process of measurement. This means according to them that all systems evolve with a Schrödinger equation except those which constitute a measurement.

It is contrary to good scientific methodology to modify a generally verified scientific theory for the sole purpose of accomodating hidden variables.

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[4] J. von Neumann, *Mathematische Grundlagen der Quantenmechanik* (Julius Springer-Verlag, Berlin, 1932).
[5] P. Mittelstaedt, *Philosophische Probleme der Modernen Physik* (Mannheim, Germany, 1966), especially Chap. VI.
[6] I. E. Segal, Ann. Math. (2) **48,** 930 (1947).
[7] I. E. Segal, *Mathematical Problems of Relativistic Physics* (American Mathematical Society, Providence, R. I., 1963).
[8] B. Misra, Nuovo Cimento (to be published).
[9] J. S. Bell, Rev. Mod. Phys. **38,** 447 (1966).
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Hidden Variables in Quantum Mechanics Reconsidered

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Recently D. Bohm and J. Bub[1] published a refutation of Jauch and Piron's proof[2] that hidden variables can be excluded in quantum mechanics. The objections upon which this refutation is based can, to a large extent, be overcome. This is accomplished by a change in terminology, a reinterpretation of some of the physical concepts, and a weakening of the axiomatic model used by Jauch and Piron. In any case, whether or not the reader agrees with these new interpretations, this theory extends the class of models for which hidden variables are excluded, beyond those considered by Jauch and Piron.

In the author's opinion the main difficulty encountered in hidden variable arguments is an incorrect phrasing of the problem. One should not ask whether a physical system admits hidden variables or not, but only if a particular model used to describe the system admits hidden variables. It is conceivable that there are many mathematical models for a physical system, some admitting hidden variables and some not. The problem then becomes that of finding which model most closely describes the physical situation. In this note it is shown that a quite large class of mathematical models do not admit hidden variables.

However, it has been demonstrated in the papers of Bell[3], and Bohm and Bub[4] that there are mathematical models for quantum mechanics which do admit hidden variables. The question then is whether the additional complications introduced by these models are justified in terms of new results or predictions not obtainable by the use of simpler models. Thus the author agrees with Bell, Bohm, and Bub to the extent that hidden variables cannot be excluded from quantum mechanics in an absolute sense, but only as far as certain mathematical models are concerned. On the other hand, the author has attempted to show that their objections concerning Jauch and Piron's specific model can be overcome.

Bohm and Bub seem to have three basic objections to Jauch and Piron's proof. The first is the use of the word "proposition" to denote the experimental questions concerning a physical system and the confusion this causes because of its similarity to the logic of thought processes. The second is the interpretation of the concept of compatibility of quantum propositions. The third objection is directed against Jauch and Piron's Axiom (4) [or its weakened form (4°)] which says that it two quantum propositions a and b are true with certainty in some state, then the proposition "a and b" is true with certainty in that state. The first two objections can be overcome by a change of terminology and interpretation. The third, however, seems to be much more serious. As is pointed out by Bohm and Bub,[5] if a and b are incompatible, the quantum proposition "a and b" is in many cases the absurd proposition; and this implies that the quantum proposition a and the quantum proposition b can never be true with certainty in the same state. Bohm and Bub demonstrate that this need not happen in all physical cases.[6] (Bell also objects to this postulate in his recent paper.[3]) This third objection is eliminated by weakening the axiomatic model so that the quantum proposition "a and b" need not exist at all.

The Axiomatic Structure. Let S be a physical system upon which we make laboratory experiments. Let $Q_0 = \{a, b, c, \cdots\}$ be the set of *experimental questions* that can be asked concerning the system S. To be quite explicit, a is an experimental question concerning S if it is possible to construct a definite laboratory experiment (or collection of experiments) on S, the outcome of which is capable of giving both a YES and a NO answer to a. This is a slightly different interpretation than is usually given. Experimental questions are usually assumed to be any meaningful questions one may ask concerning the system. Here we insist that the questions