Analysis by Its History

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Preface

> . . . that departed from the traditional dry-as-dust mathematics textbook.
> (M. Kline, from the Preface to the paperback edition of Kline 1972)

> Also for this reason, I have taken the trouble to make a great number of drawings. (Brieskorn & Knörrer, *Plane algebraic curves*, p. ii)

> . . . I should like to bring up again for emphasis . . . points, in which my exposition differs especially from the customary presentation in the textbooks:
> 1. Illustration of abstract considerations by means of figures.
> 2. Emphasis upon its relation to neighboring fields, such as calculus of differences and interpolation . . .
> 3. Emphasis upon historical growth.
> It seems to me extremely important that precisely the prospective teacher should take account of all of these. (F. Klein 1908, Engl. ed. p. 236)

Traditionally, a rigorous first course in Analysis progresses (more or less) in the following order:

$$\begin{matrix}\text{sets,}\\ \text{mappings}\end{matrix} \quad\Rightarrow\quad \begin{matrix}\text{limits,}\\ \text{continuous}\\ \text{functions}\end{matrix} \quad\Rightarrow\quad \text{derivatives} \quad\Rightarrow\quad \text{integration.}$$

On the other hand, the historical development of these subjects occurred in reverse order:

$$\begin{matrix}\text{Cantor 1875}\\ \text{Dedekind}\end{matrix} \quad\Leftarrow\quad \begin{matrix}\text{Cauchy 1821}\\ \text{Weierstrass}\end{matrix} \quad\Leftarrow\quad \begin{matrix}\text{Newton 1665}\\ \text{Leibniz 1675}\end{matrix} \quad\Leftarrow\quad \begin{matrix}\text{Archimedes}\\ \text{Kepler 1615}\\ \text{Fermat 1638}\end{matrix}$$

In this book, with the four chapters

Chapter I.	Introduction to Analysis of the Infinite
Chapter II.	Differential and Integral Calculus
Chapter III.	Foundations of Classical Analysis
Chapter IV.	Calculus in Several Variables,

we attempt to restore the historical order, and begin in Chapter I with Cardano, Descartes, Newton, and Euler's famous *Introductio*. Chapter II then presents 17th and 18th century integral and differential calculus "on period instruments" (as a musician would say). The creation of mathematical rigor in the 19th century by Cauchy, Weierstrass, and Peano for one and several variables is the subject of Chapters III and IV.

This book is the outgrowth of a long period of teaching by the two authors. In 1968, the second author lectured on analysis for the first time, at the University of Innsbruck, where the first author was a first-year student. Since then, we have given these lectures at several universities, in German or in French, influenced by many books and many fashions. The present text was finally written up in French for our students in Geneva, revised and corrected each year, then translated into English, revised again, and corrected with the invaluable help of our colleague John Steinig. He has corrected so many errors that we can hardly imagine what we would have done without him.

Numbering: each chapter is divided into sections. Formulas, theorems, figures, and exercises are numbered consecutively in each section, and we also indicate the section number, but not the chapter number. Thus, for example, the 7th equation to be labeled in Section II.6 is numbered "(6.7)". References to this formula in other chapters are given as "(II.6.7)".

References to the bibliography: whenever we write, say, "Euler (1737)" or "(Euler 1737)", we refer to a text of Euler's published in 1737, detailed references to which are in the bibliography at the end of the book. We occasionally give more precise indications, as for instance "(Euler 1737, p. 25)". This is intended to help the reader who wishes to look up the original sources and to appreciate the often elegant and enthusiastic texts of the pioneers. When there is no corresponding entry in the bibliography, we either omit the parentheses or write, for example, "(in 1580)".

Quotations: we have included many quotations from the literature. Those appearing in the text are usually translated into English; the non-English originals can be consulted in the Appendix. They are intended to give the flavor of mathematics as an international science with a long history, sometimes to amuse, and also to compensate those readers without easy access to a library with old books. When the source of a quotation is not included in the bibliography, its title is indicated directly, as for example the book by Brieskorn and Knörrer from which we have quoted above.

Acknowledgments: the text was processed in plain TEX on our Sun workstations at the University of Geneva using macros from Springer-Verlag New York. We are grateful for the help of J.M. Naef, "Mr. Sun" of the "Services Informatiques" of our university. The figures are either copies from old books (photographed by J.M. Meylan from the Geneva University Library and by A. Perruchoud) or have been computed with our Fortran codes and included as Postscript files. The final printing was done on the 1200dpi laser printer of the Psychology Department in Geneva. We also thank the staff of the mathematics department library and many colleagues, in particular R. Bulirsch, P. Deuflhard, Ch. Lubich, R. März, A. Ostermann, J.-Cl. Pont, and J.M. Sanz-Serna for valuable comments and hints. Last but surely not least we want to thank Dr. Ina Lindemann and her *équipe* from Springer-Verlag New York for all her help, competent remarks, and the agreeable collaboration.

March 1995 E. Hairer and G. Wanner.

Preface to the 2nd, 3rd, and 4th Corrected Printings. These new printings allowed us to correct several misprints and to improve the text in many places. In particular, we give a more geometric exposition of Tartaglia's solution of the cubic equation, improve the treatment of envelopes, and give a more complete proof of the transformation formula of multiple integrals. We are grateful to many students and colleagues who have helped us to discover errors and possible improvements, in particular R.B. Burckel, H. Fischer, J.-L. Gaudin, and H.-M. Maire. We would like to address special thanks to Y. Kanie, the translator of the Japanese edition.

March 1997, April 2000, Sept 2007 E. Hairer and G. Wanner.

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