Risk-Return Optimization of a Portfolio Containing Crypto Assets: A Return Analysis of Bitcoin in Multi-Asset Portfolios

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Risk-Return Optimization of a Portfolio Containing Crypto Assets: A Return Analysis of Bitcoin in Multi-Asset Portfolios

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by

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Abstract

Over the past few years, Bitcoin has been the subject of increased attention within the investment community as the total market cap of cryptocurrency has grown rapidly. This research examines the potential role of Bitcoin within investment portfolios. Using data from January 2012 to December 2021, the study finds that Bitcoin has exhibited an extraordinary level of return and volatility, as well as risk. However, it also demonstrates a low correlation with other asset classes, justifying a closer examination of its impact on a portfolio’s key metrics, such as the Sharpe ratio and efficient frontier.

This research shows that the strength of the correlation between Bitcoin prices, the stock market, and other commodities varies over time. The study observes that Bitcoin possessed “gold-like” characteristics in the early stage of its development, yet investors' attitude toward Bitcoin as a "gold-like" asset fluctuated and disappeared completely after the outbreak of COVID-19.

Furthermore, the research examines the risk and return characteristics of Bitcoin and investigates its potential impact on the optimization of traditional asset portfolios. The research also explores the degree of correlation between Bitcoin and other assets in different periods. Applying four popular asset allocation strategies to portfolios that include Bitcoin, stocks, and bonds, the research evaluates potential weekly returns and the impact of Bitcoin on portfolio optimization at various levels of risk aversion.

Additionally, the study draws efficient frontier curves of Bitcoin, the S&P 500, and bonds, indicating that Bitcoin can significantly improve portfolio returns. Finally, by constructing portfolios that are rebalanced every month and calculating the expected shortfall and return curves, the paper finds that an investment portfolio comprising the market values of Bitcoin and the S&P 500 can increase investment returns and reduce expected losses. In this portfolio, Bitcoin is weighted 25 times that of the S&P 500 component.

Key words: Cryptocurrencies; Bitcoin; Portfolio; Efficient frontier

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1. Introduction

1.1. Background

Cryptocurrencies are often regarded as a disruptive technology, eliciting both hopes and fears among stakeholders. On the one hand, cryptocurrency bears numerous potential benefits as an innovative, decentralized system of payment. On the other hand, cryptocurrency is also a source of potential risk that could harm investors, consumers, businesses, the financial system at large, and even national security. Academia and the market have mixed views on cryptocurrencies and their future, and this unresolved question remains a driver of excessive volatility in their market value.[1] Now, a growing number of researchers are looking to demystify the complex world of cryptocurrencies.

Bitcoin is one of the most well-known cryptocurrencies, but currently academia has not reached a consensus on the effect of adding cryptocurrencies like Bitcoin to traditional asset portfolios. Ever since Satoshi Nakamoto put forward the concept of Bitcoin in 2008, the utilization of the cryptocurrency has increased at a remarkable rate, with the total market value of Bitcoin in 2021 once exceeding US $1 trillion. As of the end of 2021, the total market value of digital currency market is around US $2 trillion, more than 1/4 of the total market value of gold, the traditional alternative investment. With the expansion of its market value, more investment institutions are accepting Bitcoin and other digital currencies as an alternative to gold and real estate. The Chicago Mercantile Exchange (CME) officially launched Bitcoin futures in 2017, furthering the standardization of digital currency assets.

Furthermore, Bitcoin has disrupted various established domains, drawing attention from computer science and law scholars. However, financial literature on Bitcoin is limited. Although Bitcoin is not currently a viable currency or long-term store of value, Bitcoin shows promise as a digital asset. Studies by Briere, Oosterlinck, and Szafarz (2015) and Eisl, Gasser, and Weinmayer (2015) indicate that adding a small portion of Bitcoin to a diversified

[1] Glaser, Zimmermann, Haferkorn, Weber, and Siering (2014) provide evidence that, at least for Bitcoin, the main reason to purchase a cryptocurrency is speculative investment. Urquhart (2016) shows that Bitcoin returns do not follow a random walk, based on which he concludes that the Bitcoin market exhibits a significant degree of inefficiency, especially in the early years of its existence. Corbet, Meegan, Larkin, Lucey, and Yarovaya, (2018) analyze, in the time and frequency domains, the relationship between the return of three different cryptocurrencies and a variety of other financial assets, showing a lack of relationship between crypto and other assets. Liu and Tsyvinski (2018) invalidate the view of cryptocurrencies as substitutes to monies or as a store of value (like gold) and instead stress that they are assets of their own class.

portfolio can significantly improve risk-return tradeoffs. Given these positive findings, further research on the relationship between Bitcoin and traditional asset portfolios is warranted.

1.2. Literature Review

Academic research on Bitcoin primarily focuses on its properties and definition, as well as its role in portfolio allocation. Various studies have explored the price mechanism of Bitcoin cryptocurrency and its development as an alternative currency system, particularly in relation to currencies, commodities, and alternative assets (Dwyer, 2015; Ciaian, Rajcaniova, & Kancs., 2016) However, Kristoufek (2015) argues that the price of Bitcoin cannot be explained by economic theory and is driven by speculation. Similarly, Yermack (2015) concludes that Bitcoin functions more like a speculative investment than a traditional currency; and therefore, largely fails to fulfill the functions of real money as a medium of exchange, store of value, and unit of account.

In line with these criticisms, opponents of monetary theory often classify Bitcoin as a commodity. While researchers of alternative assets maintain a neutral stance on the question of Bitcoin's value, they *do* acknowledge that digital currencies such as Bitcoin possess semi-monetary characteristics. Dyhrberg (2016) similarly finds that Bitcoin has some similarities with gold and the US dollar, indicating its hedging ability and advantages as a medium of exchange. However, Baur, Dimpfl, and Kuck (2018) argue that Bitcoin exhibits significantly different returns, volatility, and correlation characteristics compared to other assets such as gold and the US dollar. Therefore, Bitcoin may share some similarities with traditional currencies and gold, but there are also many differences to consider.

Cryptocurrencies have attracted the attention of investors seeking alternative investment vehicles that offer diversification or hedging advantages due to their similarities and differences to existing assets. Bitcoin, in particular, has gained interest as a useful tool in portfolio management, similar to commodity trading in the early 2000s, because of its high average returns and low correlation with major financial assets.

For example, Dyhrberg (2016) studied the relationship between Bitcoin and the FTSE 100, as well as the dollar-euro and dollar-sterling exchange rates and found that Bitcoin can be used as a hedging tool to resist stock market fluctuations during periods of economic turbulence. However, there are objections to the concept of using Bitcoin as a hedge: Bouoiyour and Selmi (2017) note that Bitcoin has a decreased hedging capacity in the short and long term.

Moreover, Klein, Thu, and Walther (2018) compared the roles of gold and Bitcoin in asset portfolio management and concluded that Bitcoin does not reflect the obvious characteristics of gold. They found that the value of gold increased rapidly during a market downturn, whereas the trend of Bitcoin correlated positively with a downward trend of the market, suggesting that Bitcoin has no stable hedging capacity in the portfolio.

In fact, Bianchi (2020) showed that there is no significant relationship between returns of cryptocurrencies and global proxies of traditional asset classes, except for a mild correlation with the returns of precious metals. Therefore, while Bitcoin may serve as a useful tool in portfolio management, it is important to consider its limitations and evaluate its role in the context of other assets.

Several studies have delved into Bitcoin's hedging ability, as well as its correlation with gold, and its sensitivity to traditional financial markets and assets. One such study by Zeng and Zeng (2020) used closing price data from 2015 to 2019 for cryptocurrency, stocks, gold, bulk commodities, and monetary assets to establish a GARCH (1, 1) model for the return rate series of all financial assets. The authors found that the dynamic correlation between the cryptocurrency market and traditional financial markets was weak compared to the strong correlation within the traditional financial market, which had a certain spillover effect. They conclude that the dynamic conditional correlation between cryptocurrency and traditional asset returns was sustainable.

Another study by Sachdeva (2021) analyzed the currency risk measurement of Bitcoin and compared it to the futures prices of the S&P 500, the US dollar, the euro, sterling, and gold using a GARCH model. The empirical results of that model showed that, among the financial assets studied, Bitcoin was most sensitive to the return rate of dollar and euro exchange rates.

Regarding the predictability of cryptocurrencies, Daniele et al. (2022) claimed that cryptocurrencies were not systematically predicted by stock market factors, precious metal commodities, or supply factors. Instead, they display significant exposure to investors' attention over time.

In general, there is no consensus among existing studies on the effect of Bitcoin and other digital currencies on asset portfolio optimization. Additionally, there is a lack of agreement on the question of Bitcoin as a currency, commodity, or investment. By examining the relationship between Bitcoin and gold, one can gain a better understanding of Bitcoin's position in the financial market, as gold is one common benchmark of monetary value. This paper analyzes data from 2014 to 2021 to examine the changes in Bitcoin's gold-like characteristics during different periods. This paper also investigates whether Bitcoin can increase the return rates of stock and bond portfolios by using four different portfolio construction methods. The study aims to determine the impact of digital assets, represented by Bitcoin, on the allocation optimization of existing traditional stock and bond portfolios.

2. Methodology

2.1. Data Sources

CATEGORIES	DATA
CRYPTOCURRENCY	Bitcoin
STOCK	S&P 500, VIX
BOND	The U.S. 10-year Treasury note (TNOTE)
COMMODITY	COMEX gold futures, WTI crude oil futures

Table 1: *Data Sources*

This study aims to analyze the potential benefits of adding Bitcoin to investment portfolios. To achieve this goal, daily data of Bitcoin prices are utilized, as well as those of S&P500, TNOTE, COMEX, WTI prices, and VIX from January 2012 to December 2021. This period was chosen because the trading volume of Bitcoin before 2012 was relatively low and the end date marks the point at which the data is collected for analysis.

To assess the impact of Bitcoin on portfolio performance, the S&P500 is used as a benchmark for stocks. Our study seeks to shed light on the role of Bitcoin in portfolio optimization and explore its potential as a unique and relatively new asset class.

2.2. Asset Allocation Strategy

Markowitz Mean Variance Portfolio Optimization

In Markowitz’s (1952) mean-variance portfolio optimization framework, investors aim to optimize the balance between the mean and variance of portfolio returns. Markowitz (2014) highlights that over half a century of research in this field generally favors mean-variance analysis. To compute a vector (x) of portfolio weights, they maximize the following utility function with respect to x, taking into account the sample mean (μ) and the covariance matrix (Σ):

$$\mathrm{U} = \mathrm{x}^T\mu - \frac{\lambda}{2}\mathrm{x}^T\Sigma\mathrm{x}$$

The parameter λ represents the investors' risk aversion. Hence, the optimization problem can be described as:

$$\max_{x}\left\{\mathrm{x}^T\mu - \frac{\lambda}{2}\mathrm{x}^T\Sigma\mathrm{x}\right\}$$

Subject to, $\sum_{i=1}^{N} x_i = 1$

BL Model Combinatorial Optimization

The Black-Litterman method combines two sources of information to generate investment insights. The first source is an investor's subjective return estimate for a particular asset. The second source is a reference (or benchmark) portfolio used to calculate a neutral (or implied) return.

To obtain the implied excess returns, they use the column vector (H), which represents the weight of the benchmark portfolio. According to Black and Litterman's (1992) paper, the formula for the implied excess returns is:

$$\mathrm{H} = \lambda\Sigma\mathrm{x}^{Reference}$$

where the column vector $\mathrm{x}^{\mathrm{Reference}}$ is the weight of the control (benchmark) portfolio.

The Black-Litterman model then computes a posterior estimate of the mean return by using the formula:

$$\mu_{BL} = [(c\Sigma)^{-1} + P^T\Omega^{-1}P]^{-1}[(c\Sigma)^{-1}H + P^T\Omega^{-1}Q]$$

$$\Omega = \frac{1}{\delta}P\Sigma P^T$$

$$\Sigma_{BL} = \Sigma + [(c\Sigma)^{-1} + P^T\Omega^{-1}P]^{-1}$$

By combining an investor's subjective return estimate with a neutral implied return, the Black-Litterman method provides a framework for generating more accurate and informed investment decisions.

Equally Weighted Portfolio

In an equally weighted portfolio, each asset is assigned a weight of 1/N.

Combination of Portfolio Techniques

This method combines three portfolio techniques: equal-weighted portfolios, minimum variance portfolios, and Markowitz mean variance portfolios. All individual portfolios ($\mathrm{x}^{1/N}, \mathrm{x}^{MV}, \mathrm{x}^{Markowitz}$) used in this structure are subject to short selling restrictions and normalization of portfolio weights.

Maximize the following utility function to find the optimal strategy for combining the three strategies:

$$\mathrm{x}^{\frac{1}{N}-MV-TP} = \alpha_1 \mathrm{x}^{1/N} + \alpha_2 \mathrm{x}^{\mathrm{MV}} + \alpha_3 \mathrm{x}^{Markowitz}, \qquad \alpha_1, \alpha_2, \alpha_3 \geq 0$$

2.3. Efficient Frontier

The efficient frontier refers to a collection of optimal portfolios that offer either the highest expected return for a given level of risk or the lowest risk for a specific level of expected return. Portfolios that fall below the efficient frontier are suboptimal because they do not generate enough return for the level of risk they entail. Conversely, portfolios located to the right of the efficient frontier are also suboptimal because they involve a higher level of risk for the same rate of return.

By comparing the efficient frontier curve of a portfolio before and after the addition of Bitcoin, one can intuitively observe the effect of Bitcoin on portfolio optimization. Specifically, the inclusion of Bitcoin can improve the portfolio's return by pushing it closer to the efficient frontier, demonstrating its potential to enhance overall portfolio performance.

The Efficient Frontier theory, which is a cornerstone of Modern Portfolio Theory (MPT), was introduced by Nobel Laureate Harry Markowitz in 1952. The theory rates portfolios (investments) on a scale of return (y-axis) versus risk (x-axis). To measure returns, the compound annual growth rate (CAGR) of an investment is commonly used, while the standard deviation (annualized) depicts the risk metric.

Using the efficient frontier, portfolios that maximize returns for the risk assumed can be graphically represented. Returns are dependent on the investment combinations that make up the portfolio, and a security's standard deviation is synonymous with risk. Ideally, an investor aims to fill a portfolio with securities that offer exceptional returns but with a combined standard deviation that is lower than the standard deviations of the individual securities.

2.4. Conditional Value-at-Risk and Expected Shortfall

CVaR, or Conditional Value-at-Risk, is a probabilistic and statistical approach used for risk measurement, with a crucial role in portfolio optimization, particularly in managing extreme risks. Unlike VaR (Value-at-Risk), which measures the potential loss at a given confidence level, CVaR accounts for all losses below the VaR level and calculates their expected value. Consequently, CVaR is considered a more conservative measure of risk that is better equipped to capture risk in scenarios with extreme losses.

The objective of CVaR optimization is to construct an optimal portfolio by minimizing the portfolio's CVaR. This process involves utilizing optimization algorithms and risk models to build a portfolio that provides the maximum return for an acceptable level of risk. In contrast, VaR optimization minimizes the VaR of a portfolio at a given confidence level. While VaR is a commonly used risk measure, it is not as effective in capturing risk in situations of extreme losses. Therefore, when dealing with extreme risks, CVaR optimization is a more suitable method.

Utilizing VaR and CVaR to construct an optimal portfolio aims to achieve better risk control. VaR is primarily used for routine risk control, while CVaR is more appropriate for extreme risk control. By combining VaR and CVaR, the portfolio can be optimized across different risk levels, resulting in superior risk management and higher returns.

When constructing a portfolio consisting of Bitcoin and the S&P 500, CVaR optimization is particularly effective. These two assets differ significantly in terms of their risk characteristics and return performance. The application of CVaR optimization can better capture these differences and, in extreme scenarios, better control risk. Therefore, constructing a portfolio of Bitcoin and the S&P 500 using CVaR optimization can results in better risk-return tradeoffs.

3. Empirical results

3.1. Data Sample Description

Price Metrics	BTC (USD)	S&P500 (USD)	GOLD (USD)	BOND	WTI (USD)	VIX
mean	8002	2483	1422	2.033	64.71	17.06
p50	984.4	2275	1317	2.078	59.43	15.32
min	4.300	1278	1052	0.512	7.790	9.140
max	67526	4793	2063	3.239	110.3	82.69
variance	2.040e+08	676081	57074	0.375	549.4	46.76
sd	14283	822.2	238.9	0.612	23.44	6.838
skewness	2.456	0.869	0.682	-0.446	0.163	3.405
N	2492	2492	2492	2492	2492	2492

Table 2: *Price statistics of Bitcoin, S&P500, TNOTE, COMEX, WTI, and VIX (2012-2022)*

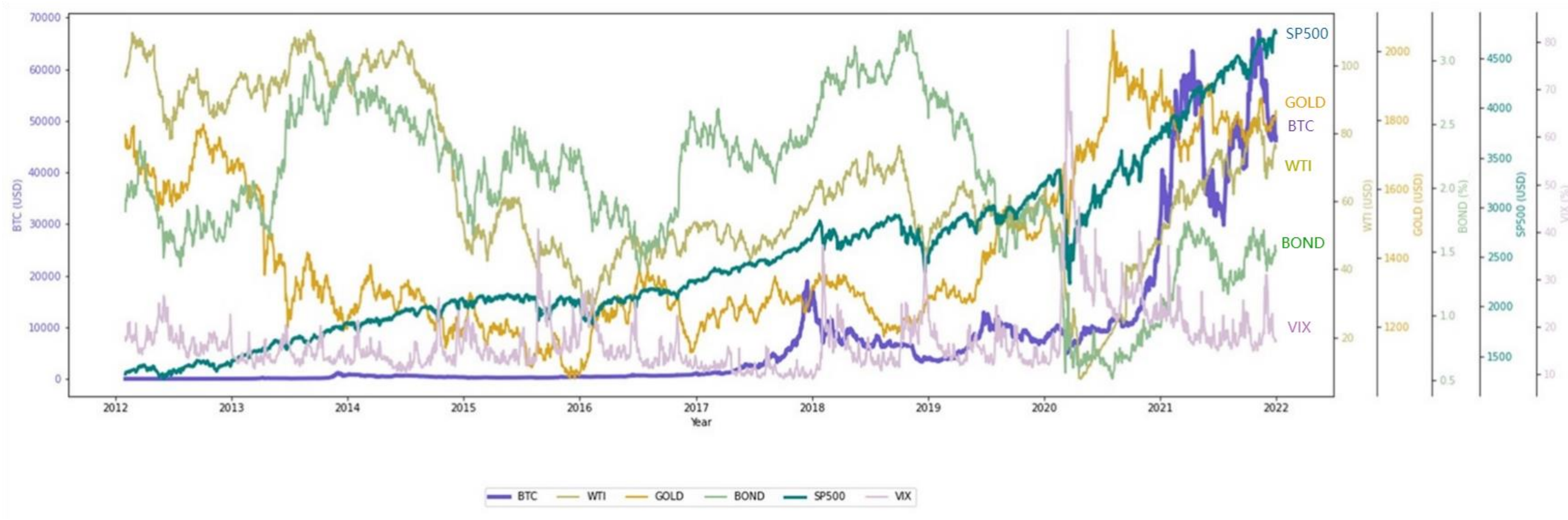


Figure 1: *Time series comparison of Bitcoin and other major asset price index trends（2012-2021）*

In Figure 1, Bitcoin prices exhibited a significant upward trend in late 2017, as well as in the first and second half of 2021. Similarly, the S&P500 and Bitcoin prices both showed an upward trend between 2012 and 2021, but they did not rise in perfect correlation.

Since the trend of asset prices in the specific period of Figure 1 (2012-2021) is not easily discernible, 2017 and 2019 are chosen as key time frames to analyze the trend of asset prices in more detail.

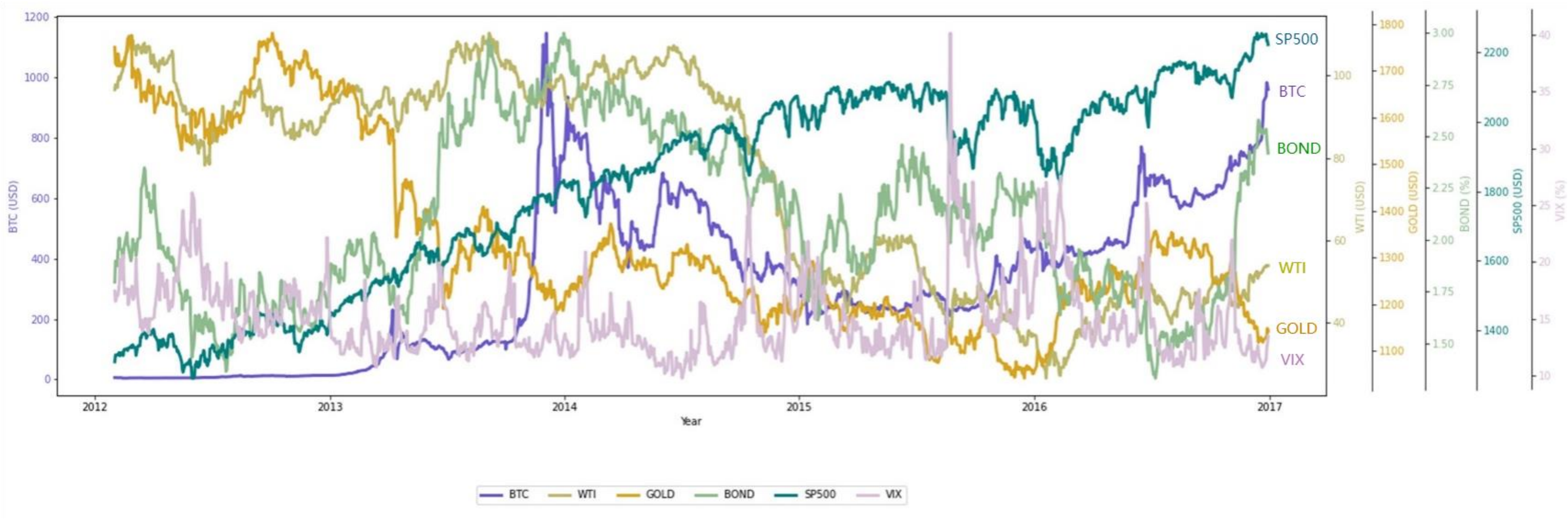


Figure 2: *Time series comparison of Bitcoin and other major asset price index trends（2012-2016）*

The trends before 2017 are shown in *Figure 2*. There is almost no obvious correlation between Bitcoin and the S&P500, WTI, bonds or VIX. However, from around March 2014 to August 2016, Bitcoin and gold appear to be highly correlated.

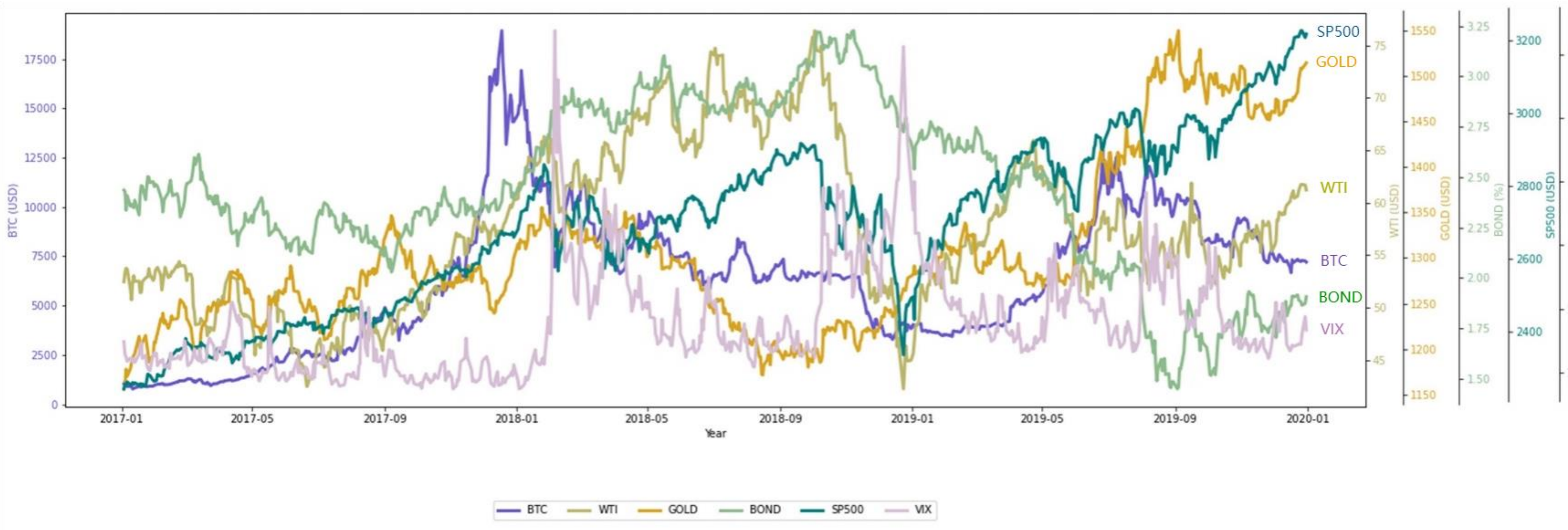


Figure 3:*Time series comparison of Bitcoin and other major asset price index trends（2017-2019）*

Note. the left y-axis represents the price of Bitcoin while the right y-axis represents the price of another major asset price.

Figure 3 shows the trends from 2017 to 2019. Bitcoin appears to correlate with gold to some extent from January 2017 to July 2018. Between November 2017 and February 2018, Bitcoin even showed the opposite price movement compared to other traditional assets, including gold.

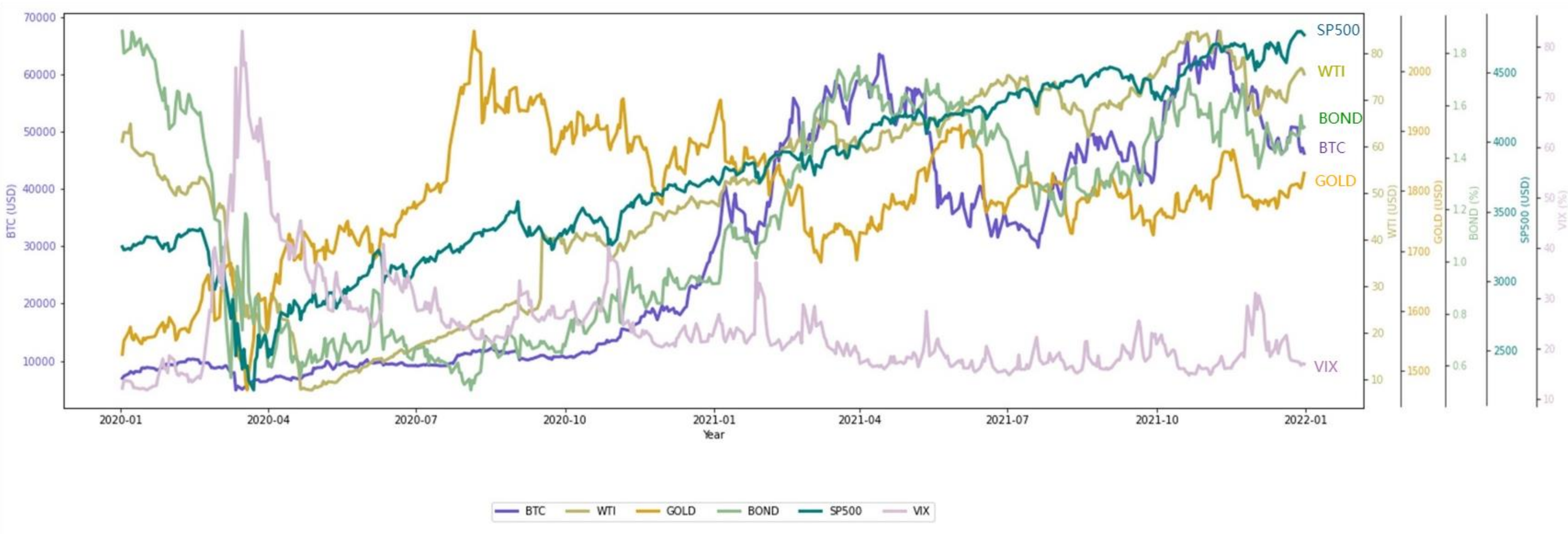


Figure 4: *Time series comparison of Bitcoin and other major asset price index trends（2020-2021）*

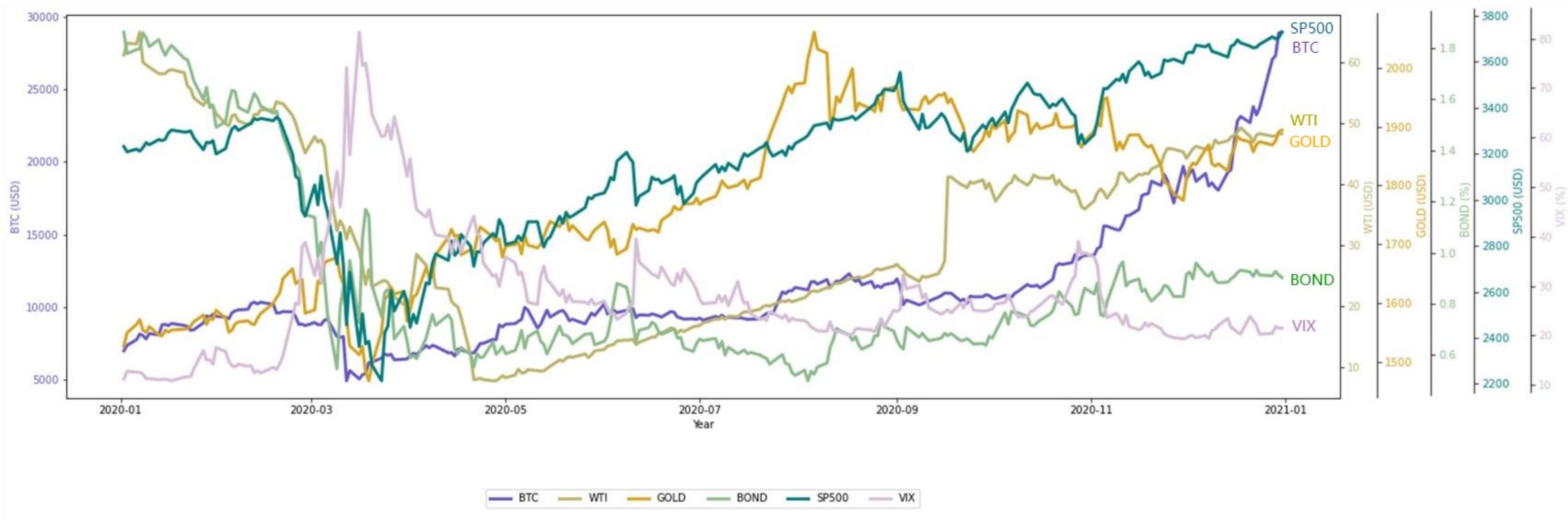


Figure 5: *Time series comparison of Bitcoin and other major asset price index trends（2020）*

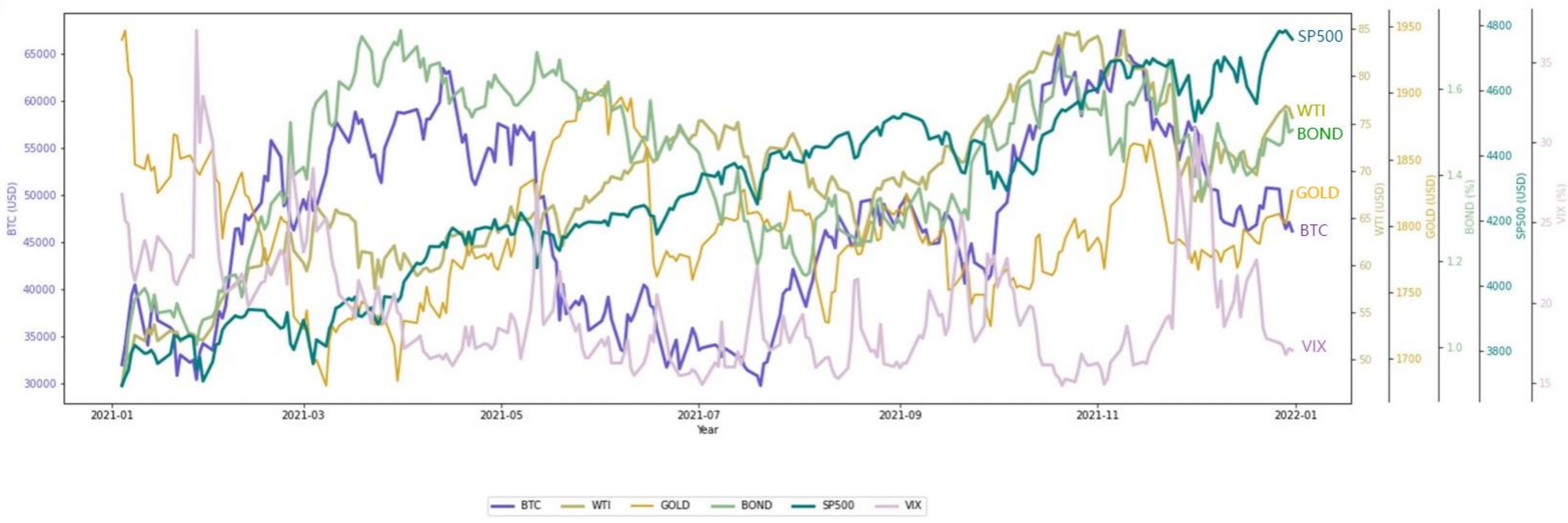


Figure 6: *Time series comparison of Bitcoin and other major asset price index trends（2021）*

Figure 4 shows the trends from 2020 to 2021. For clarity, *Figure 5* and *Figure 6* provide detailed trends for 2020 and 2021. From January to March 2020, with the economic impact of the global spread of COVID-19, asset prices fell precipitously. The VIX rose sharply, while the price of Bitcoin fell by a much smaller amount compared to the immense drop in other asset prices.

3.2. Correlation

Table 3 displays the price relationships among the six assets in the portfolio. The correlation coefficient matrix is generated using the Ledoit-Wolf shrinkage method, which effectively reduces the impact of extreme values. As a result, it provides a more stable and precise representation of the underlying relationships between the assets. By minimizing the influence of outliers and noise in the data, the Ledoit-Wolf shrinkage method can enhance the accuracy of financial models and risk management strategies that rely on covariance matrix estimation. Consequently, this method can offer a more dependable and resilient comprehension of the connections between various assets and their associated risks. The results from the Ledoit-Wolf shrinkage indicate that, throughout the research period, there exists a significant positive correlation between the S&P 500 index and bond prices. In contrast, a considerable negative correlation exists between the S&P 500 index and the VIX. However, the correlations between other asset pairs remain unclear. Of particular interest is the relationship between Bitcoin and the other five assets, which requires further research to determine its correlation.

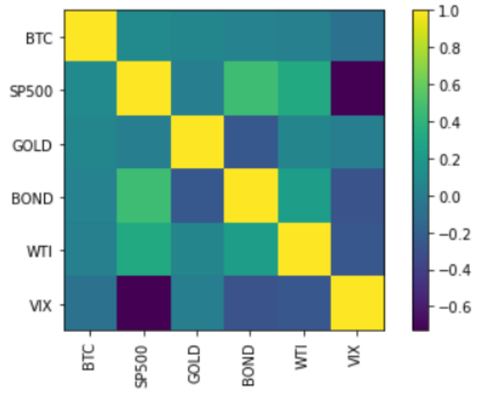


Table 3: *Correlation coefficient matrix with Ledoit-Wolf shrinkage*

	BTC	SP500	GOLD	BOND	WTI	VIX
BTC	6.746258	0.130990	0.074400	0.150841	0.078613	-0.905890
SP500	0.130990	0.266598	0.001922	0.375889	0.222882	-1.611242
GOLD	0.074400	0.001922	0.221861	-0.185745	0.036541	0.020427
BOND	0.150841	0.375889	-0.185745	2.467827	0.486506	-1.943579
WTI	0.078613	0.222882	0.036541	0.486506	1.809540	-1.509081
VIX	-0.905890	-1.611242	0.020427	-1.943579	-1.509081	18.222354

Figure 7: *Covariance matrix with Ledoit-Wolf shrinkage*

This study investigates the correlation between Bitcoin and the S&P500 from 2012 to

2021 using the Pearson correlation matrix. The results suggest a high positive correlation, but this may only reflect the concurrent effect of the economic upswing. However, when examining 2018 and 2019, the correlation coefficient was positive but not significant, and in 2018, it was negative.

Analysis of 2014-2021 indicates a positive correlation coefficient between the S&P500 and Bitcoin due to the overall rise in prices, market capitalization of Bitcoin and the stock market. However, the correlation varies in different stages, suggesting that the general positive correlation may be unreliable. Thus, further exploration is needed to better understand the relationship between the two.

Below are the Pearson correlation matrices for different periods:

Pearson correlation (2012-2021)						
Variables	(1)	(2)	(3)	(4)	(5)	(6)
(1) BTC	1.000					
(2) S&P500	0.857*	1.000				
(3) GOLD	0.571*	0.446*	1.000			
(4) BOND	-0.371*	-0.358*	-0.747*	1.000		
(5) WTI	-0.096	-0.407*	0.036	0.431*	1.000	
(6) VIX	0.255*	0.260*	0.466*	-0.597*	-0.362*	1.000

**** p<0.01, ** p<0.05, * p<0.1*

Pearson correlation (2018)						
Variables	(1)	(2)	(3)	(4)	(5)	(6)
(1) BTC	1.000					
(2) S&P500	-0.306	1.000				
(3) GOLD	0.552	-0.718*	1.000			
(4) BOND	-0.752*	0.431	-0.664	1.000		
(5) WTI	-0.100	0.414	-0.390	0.318	1.000	
(6) VIX	-0.240	-0.606	0.313	0.106	-0.331	1.000

**** p<0.01, ** p<0.05, * p<0.1*

Pearson correlation (2018-2019)						
Variables	(1)	(2)	(3)	(4)	(5)	(6)
(1) BTC	1.000					
(2) S&P500	0.157	1.000				
(3) GOLD	0.438	0.462	1.000			
(4) BOND	-0.385	-0.575*	-0.912*	1.000		
(5) WTI	0.079	0.002	-0.549*	0.588*	1.000	
(6) VIX	-0.131	-0.470	0.175	-0.097	-0.426	1.000

**** p<0.01, ** p<0.05, * p<0.1*

Pearson correlation (2021)						
Variables	(1)	(2)	(3)	(4)	(5)	(6)
(1) BTC	1.000					
(2) S&P500	0.471	1.000				
(3) GOLD	-0.653	-0.334	1.000			
(4) BOND	0.643	0.434	-0.595	1.000		
(5) WTI	0.378	0.881*	-0.431	0.564	1.000	
(6) VIX	-0.193	-0.447	0.270	-0.579	-0.605	1.000

**** p<0.01, ** p<0.05, * p<0.1*

Table 4: *Pearson correlation matrix of different period*

3.3. Bitcoin, Gold and the Stock Market

This subsection is to explore the correlation between Bitcoin, gold, and the stock market, and how Bitcoin's role in the eyes of investors changes with its development stage and shifts in the financial market environment.

To test the co-movement and correlation between Bitcoin, gold, S&P 500, and commodities during different periods, following regression model is established:

$$BTC = \alpha_1 GOLD + \alpha_2 SP500 + \alpha_3 WTI + \beta + \varepsilon$$

BTC represents the daily yield of Bitcoin and is the independent variable; GOLD, S&P500, and WTI represent the daily yield of gold, S&P 500, and crude oil futures, respectively, and are the dependent variables. The variables α_1, α_2, α_3 are the regression coefficients, β is the constant term, and ε is the residual. The data from the same period is used for regression.

1) Early Stage (before 2014)

Linear regression

rBTC	Coef.	St. Err.	t-value	p-value	[95% Conf	Interval]	Sig
rGOLD	.495	.26	1.90	.058	-.016	1.006	*
rS&P500	-.291	.434	-0.67	.502	-1.144	.561	
rWTI	-.185	.243	-0.76	.447	-.662	.293	
Constant	.011	.003	3.68	0	.005	.016	***

Mean dependent var	0.010	SD dependent var	0.067
R-squared	0.008	Number of obs	539
F-test	1.512	Prob > F	0.210
Akaike crit. (AIC)	-1376.566	Bayesian crit. (BIC)	-1359.407

**** p<.01, ** p<.05, * p<.1*

Table 5: *Regression results (before Mar 2014)*

Between 2012 and March 2014, the price of Bitcoin remained low, with a significant increase only occurring at the end of 2013. This period can be seen as the early stage of Bitcoin's development. Table 5 presents the regression results of Bitcoin versus both gold and the S&P 500 during this period, revealing a positive correlation between Bitcoin and gold. This suggests that in the early stage of Bitcoin's development, it shared similar properties with gold to some extent.

2) Initial Development (Mar 2014 – Aug 2016)

***Panel A:** Mar 2014 – Aug 2016*

Linear regression

rBTC	Coef.	St. Err.	t-value	p-value	[95% Conf	Interval]	Sig
rGOLD	-.031	.185	-0.17	.866	-.395	.333	
rS&P500	.172	.213	0.81	.42	-.246	.589	
rWTI	-.135	.071	-1.91	.056	-.274	.003	*
Constant	.001	.002	0.65	.515	-.002	.004	

Mean dependent var	0.001	SD dependent var	0.042
R-squared	0.006	Number of obs	610
F-test	1.310	Prob > F	0.270
Akaike crit. (AIC)	-2127.983	Bayesian crit. (BIC)	-2110.330

**** p<.01, ** p<.05, * p<.1*

Panel B: Mar 2016 – Jun 2016

Linear regression

rBTC	Coef.	St. Err.	t-value	p-value	[95% Conf	Interval]	Sig
rGOLD	1.456	.807	1.80	.079	-.177	3.089	*
rS&P500	.907	1.116	0.81	.421	-1.35	3.165	
rWTI	-.743	.406	-1.83	.075	-1.564	.078	*
Constant	.01	.007	1.36	.18	-.005	.025	

Mean dependent var	0.010	SD dependent var	0.051
R-squared	0.150	Number of obs	43
F-test	2.287	Prob > F	0.094
Akaike crit. (AIC)	-133.749	Bayesian crit. (BIC)	-126.704

**** p<.01, ** p<.05, * p<.1*

***Table 6:** Regression results (Mar 2014 – Aug 2016)*

During the initial development period from March 2014 to August 2016, Bitcoin prices were lower compared to current prices, but grew at a rate greater than its earlier stages. Table 6 Panel A presents the regression results of the return rate of Bitcoin, gold, and the S&P500 during this period. Additionally, Panel B shows a significant positive correlation between Bitcoin and gold yield in Mar-June 2016. This indicates that during the early stage of Bitcoin's development, the movements of Bitcoin resembled those of gold.

3) Volatile Time (Jan 2017-Jul 2019)

Panel A: Jan 2017- Oct 2017

Linear regression

rBTC	Coef.	St. Err.	t-value	p-value	[95% Conf	Interval]	Sig
rGOLD	.496	.606	0.82	.414	-.699	1.691	
rS&P500	1.067	.92	1.16	.248	-.747	2.882	
rWTI	-.225	.241	-0.93	.352	-.7	.251	
Constant	.01	.004	2.50	.013	.002	.017	**

Mean dependent var	0.011	SD dependent var	0.055
R-squared	0.011	Number of obs	210
F-test	0.732	Prob > F	0.534
Akaike crit. (AIC)	-618.915	Bayesian crit. (BIC)	-605.527

*** *p<.01*, ** *p<.05*, * *p<.1*

Panel B: Nov 2017- Nov 2018

Linear regression

rBTC	Coef.	St. Err.	t-value	p-value	[95% Conf	Interval]	Sig
rGOLD	-1.13	.54	-2.09	.037	-2.193	-.067	**
rS&P500	.655	.379	1.73	.085	-.092	1.402	*
rWTI	.167	.211	0.79	.43	-.249	.583	
Constant	-.001	.003	-0.16	.873	-.007	.006	

Mean dependent var	-0.000	SD dependent var	0.055
R-squared	0.032	Number of obs	273
F-test	2.953	Prob > F	0.033
Akaike crit. (AIC)	-811.956	Bayesian crit. (BIC)	-797.518

*** *p<.01*, ** *p<.05*, * *p<.1*

Panel C: Dec 2018-Jul 2019

Linear regression

rBTC	Coef.	St. Err.	t-value	p-value	[95% Conf	Interval]	Sig
rGOLD	1.921	.529	3.63	0	.876	2.965	***
rS&P500	-.823	.426	-1.93	.055	-1.664	.017	*
rWTI	.046	.195	0.24	.814	-.338	.43	
Constant	.005	.004	1.47	.142	-.002	.012	

Mean dependent var	0.007	SD dependent var	0.048
R-squared	0.108	Number of obs	165
F-test	6.488	Prob > F	0.000
Akaike crit. (AIC)	-544.296	Bayesian crit. (BIC)	-531.872

*** $p<.01$, ** $p<.05$, * $p<.1$

Table 7: *Regression results (2017-Jul 2019)*

During this period (Jan 2017-Jul 2019), the relationship between Bitcoin and gold was volatile. Panel A of Table 7 shows that from January to October 2017, there was no clear correlation between Bitcoin, gold, and S&P 500 returns. However, from November 2017 to November of the following year (Panel B), the returns of all three assets exhibited a significant correlated relationship. Unlike the previous trend where Bitcoin and gold moved in the same direction, there was now a negative correlation between Bitcoin and gold, while Bitcoin and the S&P 500 showed a positive correlation.

Between December 2018 and July 2019 (Panel C, Table 7), Bitcoin's correlation with gold returned to a significant positive relationship, while its correlation with the S&P 500 was negative. During this time, the return rates of Bitcoin and gold showed a significant and sustained negative correlation for the first time but returned to a positive correlation after December 2018. The market's perception of Bitcoin fluctuated between a "gold-like" safe-haven asset and a traditional financial asset during this period.

4) After the COVID-19 (After Nov 2019)

Panel A: Nov 2019-Apr 2020

Linear regression

rBTC	Coef.	St. Err.	t-value	p-value	[95% Conf	Interval]	Sig
rGOLD	.809	.342	2.36	.02	.131	1.486	**
rS&P500	.865	.155	5.60	0	.559	1.172	***
rWTI	.102	.067	1.52	.132	-.031	.234	
Constant	.002	.004	0.38	.701	-.007	.01	

Mean dependent var	0.001	SD dependent var	0.053
R-squared	0.310	Number of obs	124
F-test	17.964	Prob > F	0.000
Akaike crit. (AIC)	-415.535	Bayesian crit. (BIC)	-404.254

*** *p<.01*, ** *p<.05*, * *p<.1*

Panel B: May 2020-Apr 2021

Linear regression

rBTC	Coef.	St. Err.	t-value	p-value	[95% Conf	Interval]	Sig
rGOLD	.755	.254	2.97	.003	.254	1.255	***
rS&P500	.722	.246	2.94	.004	.238	1.206	***
rWTI	-.007	.069	-0.10	.92	-.142	.128	
Constant	.007	.003	2.66	.008	.002	.013	***

Mean dependent var	0.008	SD dependent var	0.044
R-squared	0.077	Number of obs	252
F-test	6.902	Prob > F	0.000
Akaike crit. (AIC)	-875.747	Bayesian crit. (BIC)	-861.629

*** *p<.01*, ** *p<.05*, * *p<.1*

Panel C: May 2021-Aug 2021

Linear regression

rBTC	Coef.	St. Err.	t-value	p-value	[95% Conf	Interval]	Sig
rGOLD	-1.588	.719	-2.21	.03	-3.019	-.157	**
rS&P500	2.776	.914	3.04	.003	.958	4.595	***
rWTI	-.381	.304	-1.25	.214	-.986	.224	
Constant	-.003	.006	-0.47	.638	-.014	.008	

Mean dependent var	-0.001	SD dependent var	0.053
R-squared	0.122	Number of obs	85
F-test	3.769	Prob > F	0.014
Akaike crit. (AIC)	-261.597	Bayesian crit. (BIC)	-251.826

**** p<.01, ** p<.05, * p<.1*

Panel D: Sep 2021-Dec 2021

Linear regression

rBTC	Coef.	St. Err.	t-value	p-value	[95% Conf	Interval]	Sig
rGOLD	.323	.542	0.60	.553	-.756	1.402	
rS&P500	1.245	.598	2.08	.04	.056	2.435	**
rWTI	.149	.223	0.67	.508	-.296	.593	
Constant	-.001	.004	-0.13	.895	-.009	.008	

Mean dependent var	0.001	SD dependent var	0.039
R-squared	0.114	Number of obs	85
F-test	3.460	Prob > F	0.020
Akaike crit. (AIC)	-311.959	Bayesian crit. (BIC)	-302.188

**** p<.01, ** p<.05, * p<.1*

Table 8: *Regression results (Nov 2019-Dec 2021)*

From November 2019 to April 2020 (Panel A, Table 8), Bitcoin showed a positive correlation with gold and the S&P 500. However, during this period, the COVID-19 pandemic severely affected global financial markets, and the unity of all types of assets was severely impacted. Thus, the correlation between these three assets was primarily caused by external economic conditions.

From May 2020 to April 2021, Bitcoin maintained a positive correlation with gold and the stock markets (Table 8 Panel B). This could be attributed to the rapid growth of Bitcoin's price and the gradual recovery of the global financial market.

In Panel C of Table 8 (May 2021 to August 2021), Bitcoin displayed a positive correlation with the S&P 500 and a negative correlation with gold. During this period, the return of Bitcoin was more correlated with the stock market and negatively correlated with the return of gold, gradually breaking away from the most primitive gold-like characteristics.

From September 2021 to the end of 2021, Bitcoin lost its apparent correlation with gold. Therefore, it can be concluded that Bitcoin's gold-like characteristics began to fluctuate after the COVID-19 pandemic, and its role in financial markets continued to change. This suggests that Bitcoin cannot be considered a gold-like asset. Nonetheless, Bitcoin's impact on portfolio returns is different from stock assets and deserves further discussion.

3.4. Returns, Efficient Frontier, Sharpe Ratio and Other Indicators of the Portfolio of Four Bitcoin Allocation Strategies

This study examines the out-of-sample returns of four different Bitcoin allocation strategies in equity and bond portfolios, employing various levels of risk aversion in our asset allocation strategies. Table 9 presents the results of the 52-week extended window and shows the mean and variance-covariance matrices for each of the four asset strategies. The efficient frontier for each of the four strategies is constructed and the Sharpe ratio is calculated for each allocation. The performance of Bitcoin as an asset class is evaluated by comparing the risk and return trade-offs for each strategy in the following sections.

Portfolio	Metric	$\lambda = 2$		$\lambda = 5$		$\lambda = 10$	
		benchmark	(+Bitcoin)	benchmark	(+Bitcoin)	benchmark	(+Bitcoin)
Markowitz	Excess Return	0.0462	0.4922	0.0458	0.2608	0.0407	0.1427
	Std	0.0395	0.3233	0.0390	0.1581	0.0342	0.0812
	Sharp Ratio	1.1163	1.5133	1.1180	1.6300	1.1277	1.7176
Black-Litterman	Excess Return	0.0387	0.2323	0.0248	0.1858	0.0211	0.1486
	Std	0.0347	0.1371	0.0223	0.1103	0.0197	0.0889
	Sharp Ratio	1.0657	1.6709	1.0269	1.6541	1.0272	1.6354
1/N	Excess Return	0.0015	0.0089	0.0015	0.0089	0.0015	0.0089
	Std	0.0013	0.0047	0.0013	0.0047	0.0013	0.0047
	Sharp Ratio	1.0348	1.6240	1.0348	1.6240	1.0348	1.6240
3-Fund	Excess Return	0.0453	0.4846	0.0431	0.2587	0.0367	0.1397
	Std	0.0387	0.3187	0.0367	0.1568	0.0312	0.0799
	Sharp Ratio	1.1155	1.5115	1.1163	1.6307	1.1235	1.7104

Table 9: *Results for mean and variance covariance matrices.*

Incorporating Bitcoin into a portfolio of traditional assets has the potential to increase both excess returns and standard deviation of the new investment portfolio. In fact, including Bitcoin has been shown to significantly improve the Sharpe ratio and excess returns of both equity and bond portfolios. Although Bitcoin has shown a positive correlation with the S&P 500 over time in our chosen study period (2012-2021), this relationship appears to be largely driven by the overall trends of Bitcoin price growth and stock market expansion. Nevertheless, integrating Bitcoin into a portfolio can still improve risk-adjusted performance.

The Efficient Frontier refers to the portfolio that achieves the highest expected return for a given level of risk. It is a key concept in portfolio theory, whereby one aims to optimize the balance of risk and return. Essentially, the Efficient Frontier helps us determine the optimal allocation of assets to achieve the best possible return at various levels of risk.

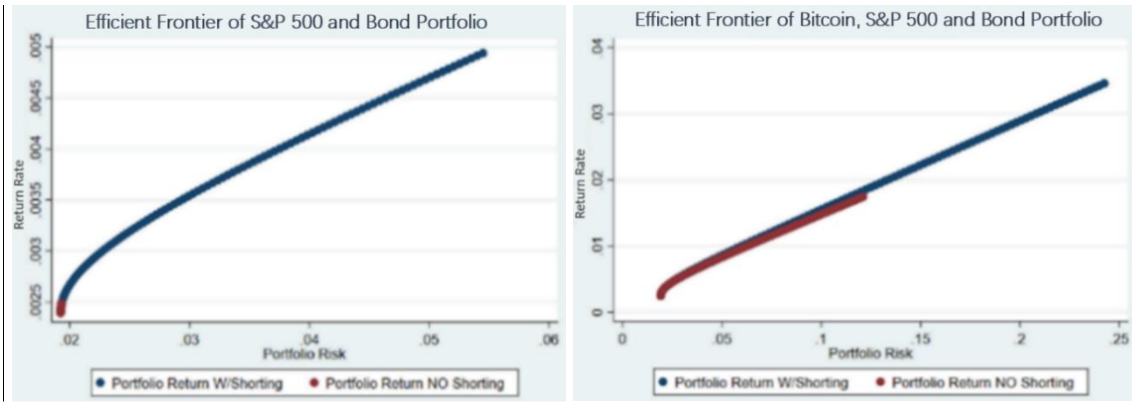


Figure 8: *Figure 8: Efficient Frontier of S&P 500 and Bond Portfolio (Left) & Efficient Frontier of Bitcoin, S&P 500, and Bond Portfolio (Right) - With and Without Short Sale Constraints*

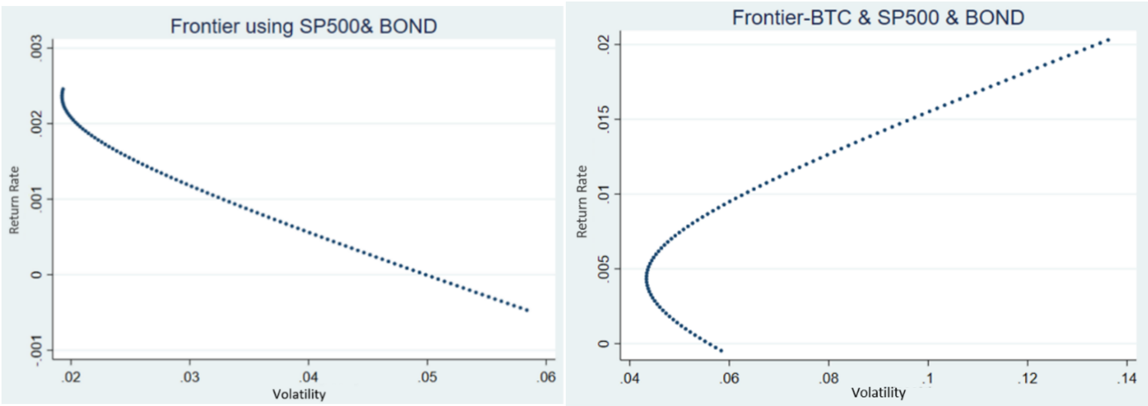


Figure 9: *Efficient Frontier of S&P 500 and Bond Portfolio (Left) & Efficient Frontier of Bitcoin, S&P 500, and Bond Portfolio (Right) - Analysis Without Short Sale Constraints*

Figures 8 and 9 provide a detailed examination of the efficient frontiers of different portfolio compositions, considering the impact of short sale constraints. Short sale constraints are limitations that hinder investors from selling an asset they do not own in the hope of buying it back later at a reduced price. These constraints can substantially affect portfolio optimization and risk management.

Figure 8 presents the efficient frontiers for portfolio compositions under two scenarios, both with and without short sale constraints. The left panel in this figure illustrates the efficient frontier of a portfolio composed solely of traditional assets, specifically the S&P 500 and bonds. On the other hand, the right panel displays the efficient frontier after integrating Bitcoin into the portfolio.

By comparing these two scenarios, it's evident that the inclusion of Bitcoin has led to an upward shift in the efficient frontier, regardless of whether short sale constraints are in place or not. This shift suggests potential enhancements in the risk-return trade-off, thus underscoring the potential benefits of incorporating Bitcoin into conventional asset portfolios.

Figure 9 complements this analysis by focusing exclusively on the scenario without short sale constraints. This figure provides a more comprehensive view of the efficient frontier under this condition, demonstrating that the absence of short sale constraints potentially leads to a more favorable efficient frontier. In such a scenario, investors can fully capitalize on market opportunities arising from asset price fluctuations, further emphasizing the potential merits of Bitcoin inclusion.

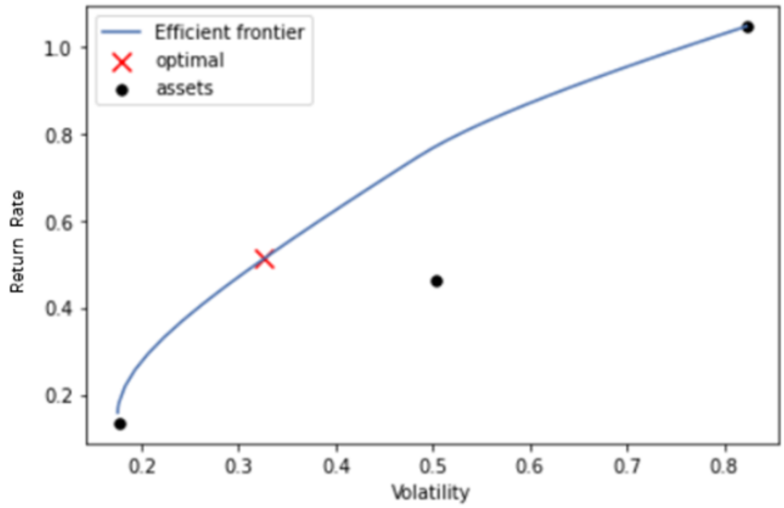


Figure 10: *Unconstrained efficient frontier*

Figure 10 displays the efficient frontier curve under unconstrained conditions. Here, the maximum expected annual return of 51.2% is achieved with an annual volatility of 32.6%, resulting in a Sharpe ratio of 1.51. The Sharpe ratio is a measure used by investors to understand the return of an investment compared to its risk. A higher Sharpe ratio implies better risk-adjusted return.

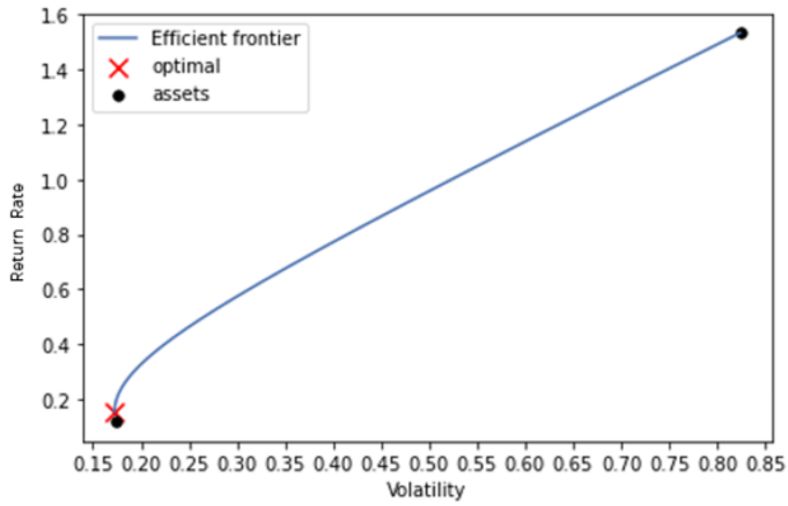


Figure 11: *CVaR (ES) effective frontier*

Figure 11 focuses on the concept of CVaR, or Conditional Value at Risk, also known as Expected Shortfall (ES). This measure is more sensitive to the shape of the loss distribution in the tails, making it a more comprehensive risk metric than VaR (Value at Risk). The optimal shortfall moving downward indicates that when the portfolio is optimized for the least CVaR, it becomes more risk-averse, which may lead to a lower expected return but with lower potential for extreme losses.

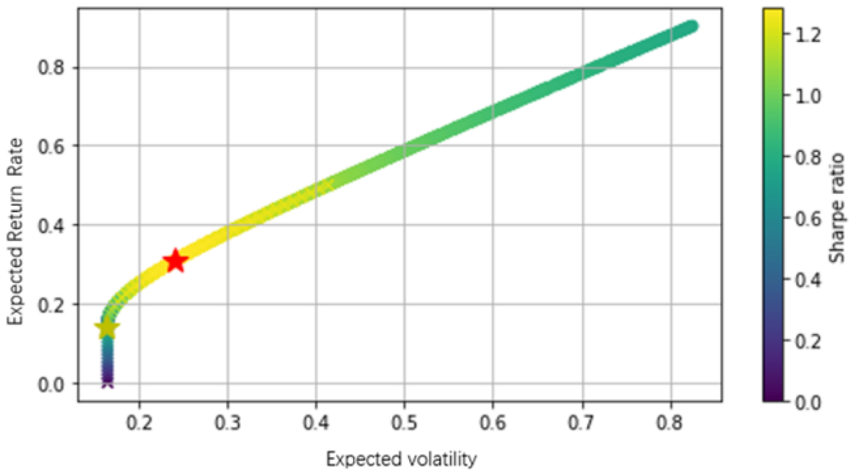


Figure 12: *Efficient frontier with short-sale constraints*

Figure 12 shows the efficient frontier under the constraint of no short sales. Short selling is a strategy where an investor sells securities that they do not own, typically with the expectation that the price will fall and they can buy it back at a lower price for a profit. This strategy can increase potential returns but also introduces additional risks. When short selling is prohibited, the efficient frontier shifts down, leading to a lower optimal return. This demonstrates that the constraint of no short selling can limit the portfolio's ability to maximize returns.

The red star represents the portfolio with the highest Sharpe ratio, at [0.232, 0.768], which means this portfolio provides the best risk-adjusted returns under the no-short-selling condition. On the other hand, the yellow star symbolizes the portfolio with the least variance, which implies it has the lowest risk. Its expected return, volatility, and Sharpe Ratio are 0.143, 0.164, and 0.873 respectively, suggesting a more conservative investment strategy that prioritizes stability over high returns.

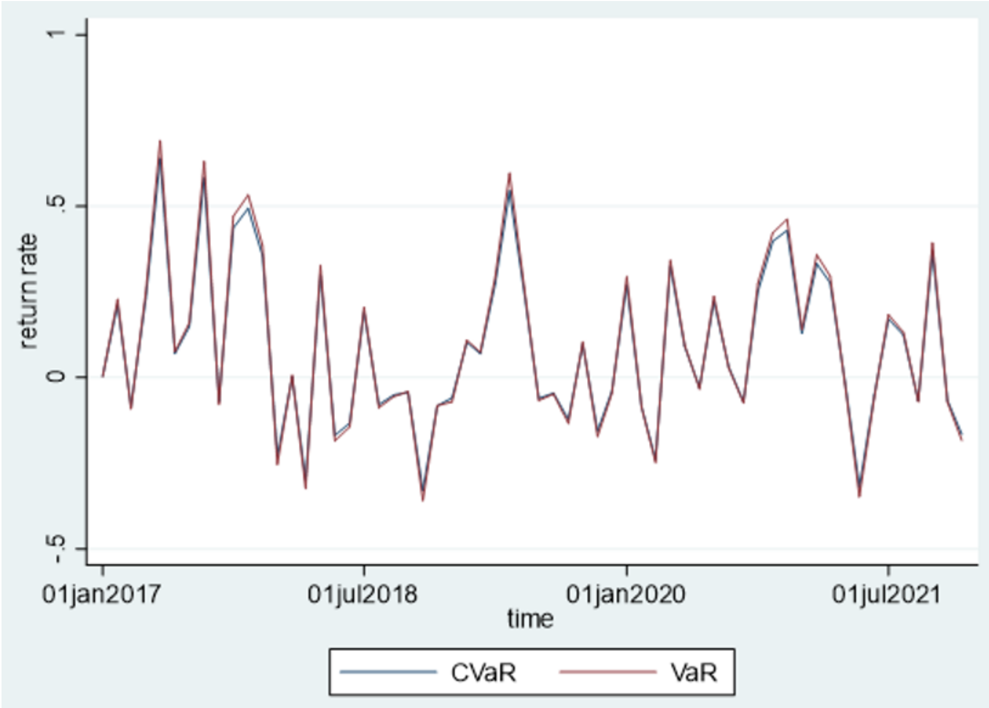


Figure 13: *Optimal CVaR (ES) and optimal VaR under the Bitcoin and S&P 500 portfolio return curve.*

Note. In the following analysis, the investment returns of a portfolio consisting of Bitcoin and the S&P 500 is examined. The data used for this analysis is from 2017 onwards. Thus, this time period is specifically chosen due to the significant difference in market capitalization between Bitcoin and the S&P 500 before 2017.

Figure 13 is depicting the return curves of a portfolio composed of Bitcoin and S&P 500, which are optimized under two different risk measures: Conditional Value at Risk (CVaR, also known as Expected Shortfall or ES) and Value at Risk (VaR). These two measures are represented by blue and red lines respectively.

Both VaR and CVaR are widely used in finance to measure the risk of loss for investments. However, they assess risk differently: VaR estimates the maximum loss over a given time period at a certain confidence level, while CVaR measures the expected loss given that the loss is beyond the VaR.

The return trends of the two portfolios being quite similar suggests that, in terms of expected returns, there may not be a significant difference whether the portfolio is optimized under CVaR or VaR. This can be the case when the underlying distribution of the returns is normal or close to normal, where the distinction between VaR and CVaR becomes less significant.

However, the fact that the magnitude of losses based on the CVaR portfolio consistently remains smaller than that of VaR's implies that CVaR portfolio may be a more risk-averse strategy. This is because CVaR, by taking into account the expected value of extreme losses, tends to mitigate the potential for large losses. This suggests that the portfolio optimized under CVaR might be more resilient during market downturns and can provide investors with a higher level of protection against extreme market events compared to the VaR optimized portfolio. Therefore, investors who are more risk-averse might prefer the CVaR optimization strategy over VaR.

4. Investment in Bitcoin and the S&P 500 Monthly Rebalancing

4.1. Return from Monthly Portfolio Rebalancing.

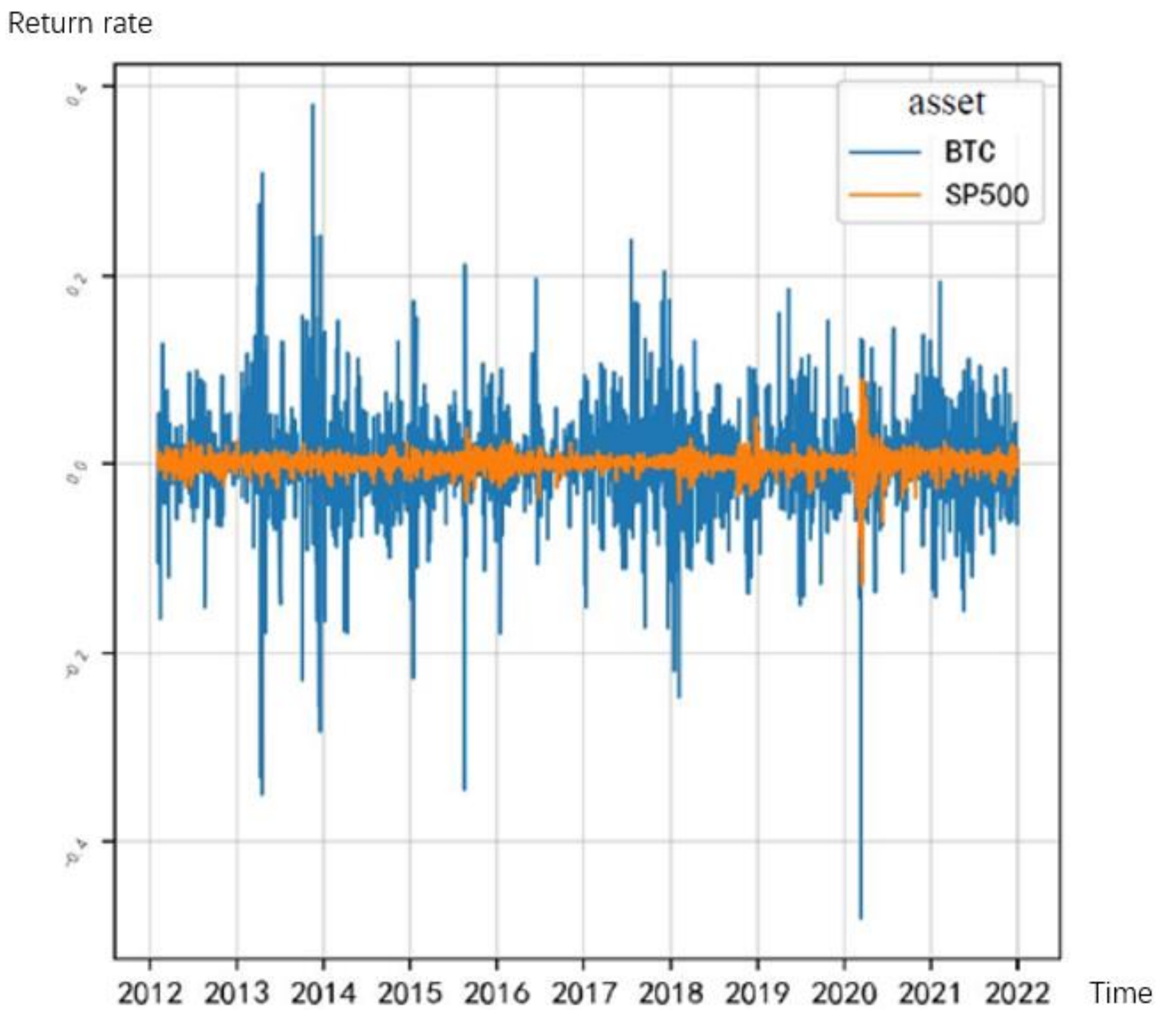


Figure 14: *Bitcoin's return on the S&P 500*

Figure 14 depicts the daily return rate of Bitcoin and S&P 500 from January 2012 to December 2022. Within this time frame, Bitcoin's yield fluctuated between plus and minus 40 percent.

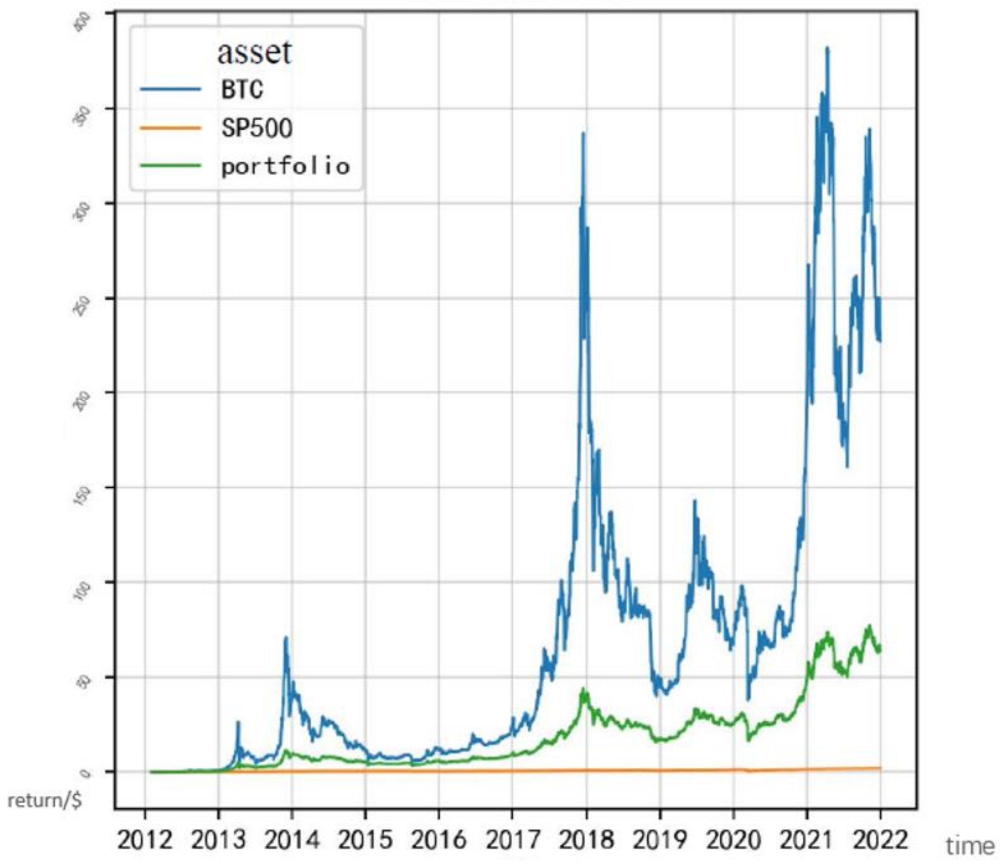


Figure 15: *Monthly rebalancing of equal-weighted portfolio returns.*

Note. the portfolio curve is an equal-weighted portfolio of Bitcoin and the S&P 500

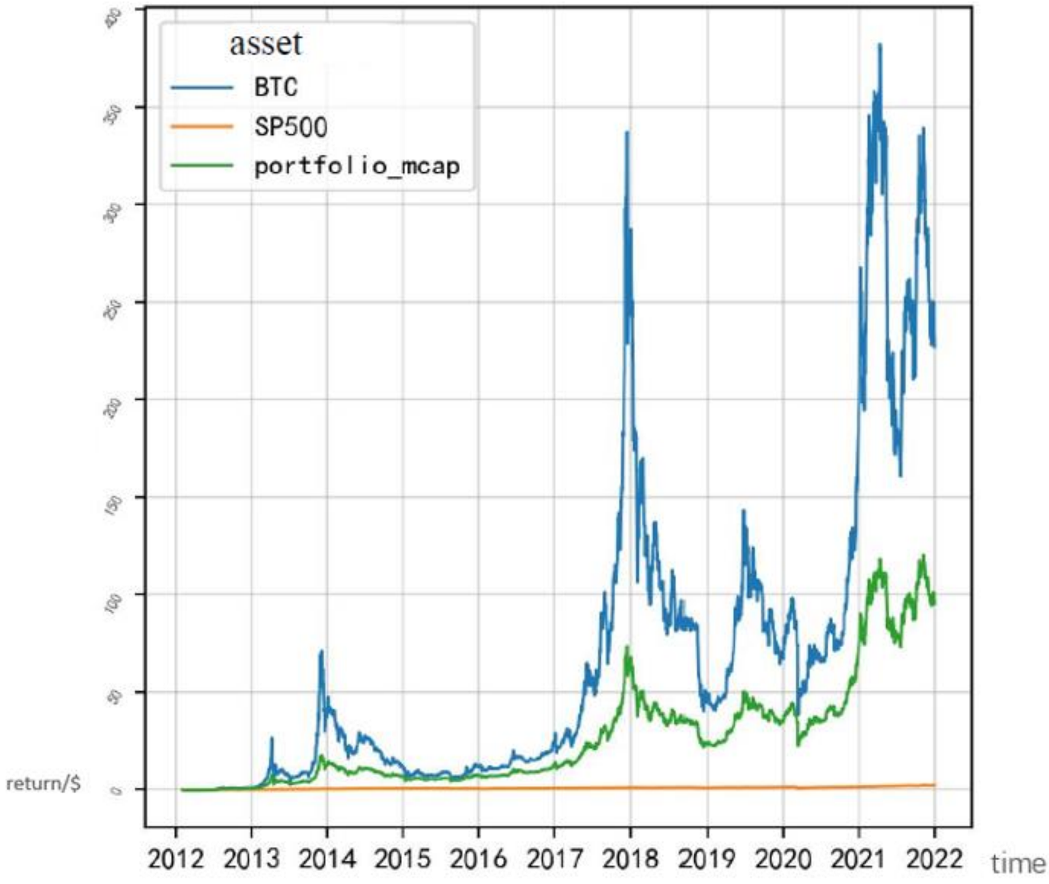


Figure 16: *Monthly rebalancing of value-weighted portfolio returns.*

Note. the portfolio curve is a value-weighted portfolio of Bitcoin and the S&P 500

To evaluate portfolio performance, Figures 15 and 16 showcase the results of investing $100 in equal-weighted and value-weighted portfolios using daily data from 2012 to 2021 for both Bitcoin and S&P 500. The portfolios are rebalanced monthly, and all assets are sold at the end of each month. The analysis shows that the portfolio returns under both strategies fall between the returns of buying only a single asset. Additionally, the equal-weighted portfolio returns exhibit better stability than the value-weighted portfolio.

To examine the impact of Bitcoin price surging after 2017, data from 2017 to 2021 is used in the below portfolios.

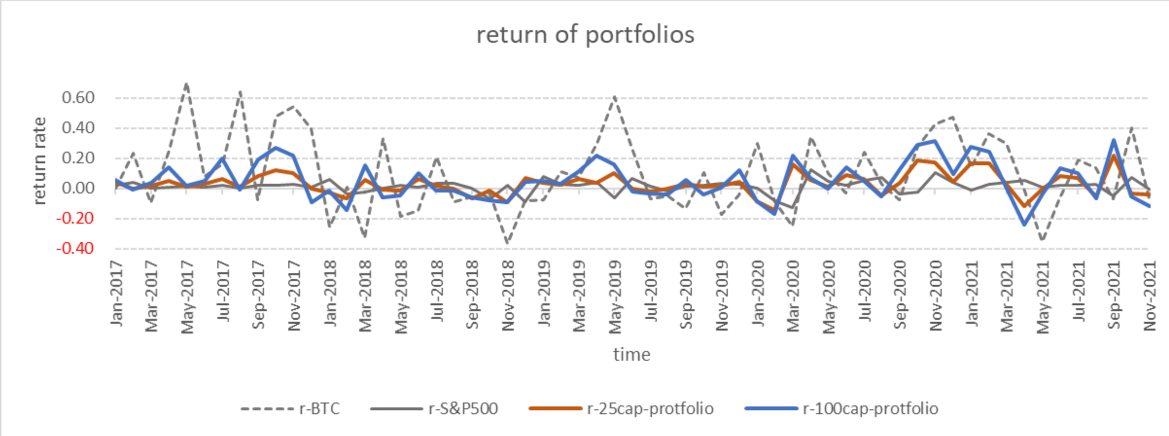


Figure 17: *Returns of monthly rebalanced value-weighted portfolios (2017-2021)*

Figure 17 compares the performance of the S&P 500 index, Bitcoin, and the two investment portfolios they constitute starting from 2017, assuming no transaction costs. In these portfolios, the weight of Bitcoin's market value is scaled up by a factor of 25 (r-25cap-portfolio) and 100 (r-100cap-portfolio), respectively. The assets are sold at the end of each month and bought at the beginning of the following month. It can be seen that as the weight of Bitcoin in the portfolio increases, the volatility of the portfolio returns increases.

The portfolios below were initiated on January 1, 2017, with an initial investment of $100 allocated according to the ratio of different multiples of the market capitalization of Bitcoin and 1 time the market capitalization of S&P 500. They were sold and rebalanced at the end of each month.

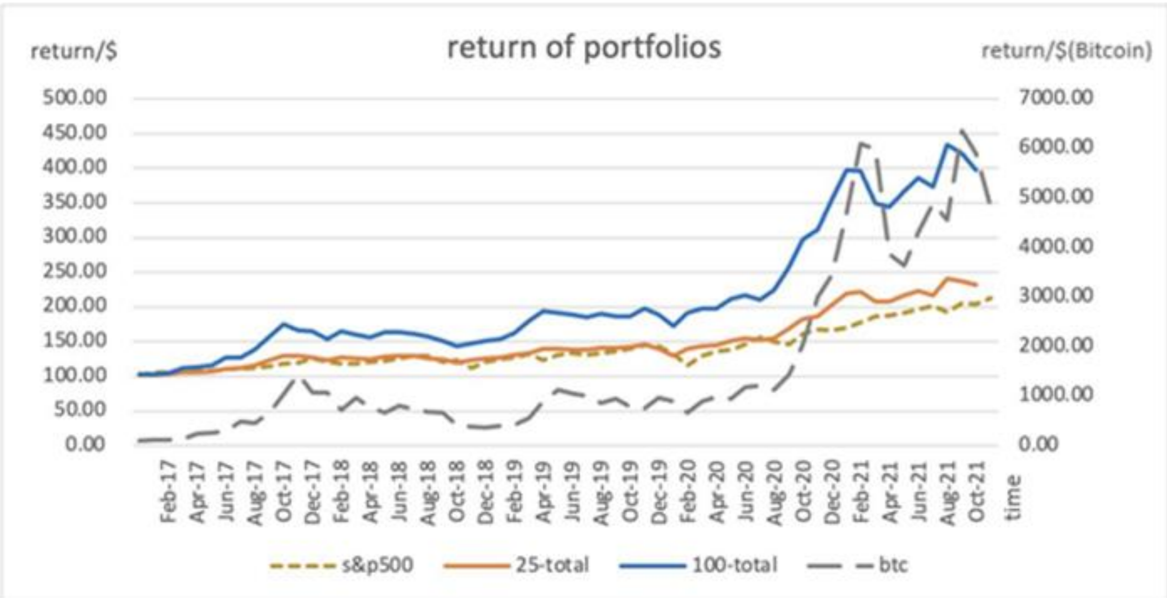


Figure 18: *Monthly equal-weighted Portfolio Return by Rebalancing (2017-2021)*

Note. The right axis is set to be the dollar return of investing only in Bitcoin, it is important to note that Bitcoin returns also begin at $100. This setting also applies to panel A in Figure 22.

In Figure 18, the investment return trend is shown for a $100 investment that is rebalanced at the end of each month. The chart considers four investment options:

1) an investment solely in the S&P 500 (s&p 500),
2) an investment solely in Bitcoin (btc),
3) a value-weighted portfolio comprising 25x Bitcoin market cap and S&P 500 market cap (25-total),
4) a value-weighted portfolio comprising 100x Bitcoin market cap and S&P 500 market cap (100-total).

The results show notable variations in the returns and degrees of fluctuation for portfolios weighted according to different multiples of Bitcoin market capitalization.

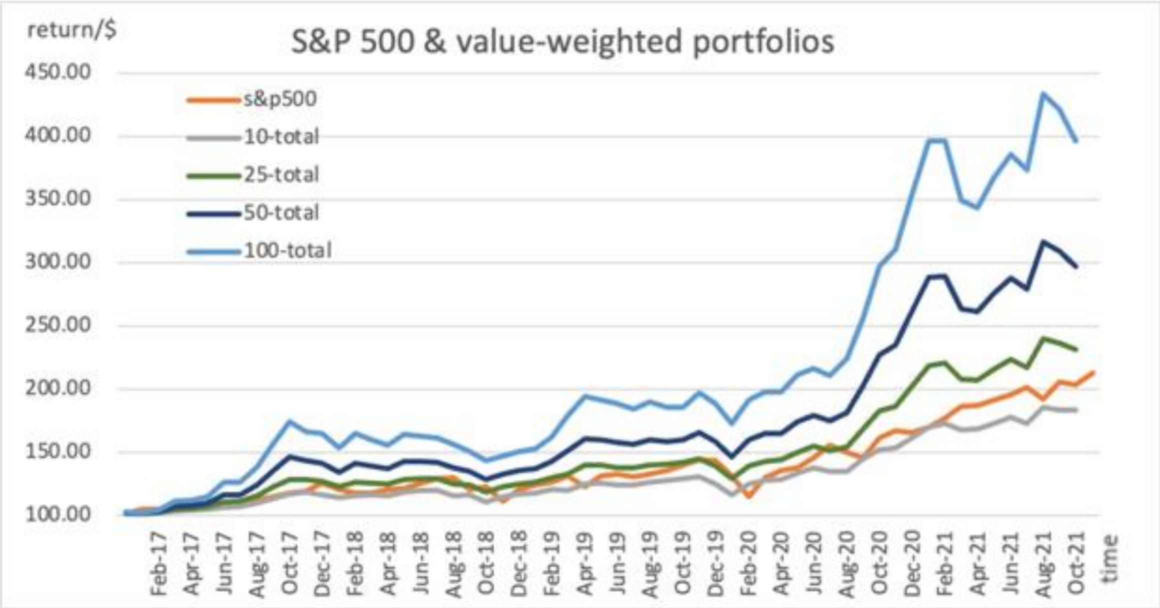


Figure 19: *Monthly market-cap weighted portfolio return (100 times Bitcoin market cap)*

Figure 19 displays the return curves of five portfolios:

1) an investment solely in the S&P 500 (s&p 500)
2) a value-weighted portfolio comprising 10x Bitcoin market cap and S&P 500 market cap (10-total)
3) a value-weighted portfolio comprising 25x Bitcoin market cap and S&P 500 market cap (25-total)
4) a value-weighted portfolio comprising 50x Bitcoin market cap and S&P 500 market cap (50-total)
5) a value-weighted portfolio comprising 100x Bitcoin market cap and S&P 500 market cap (100-total)

As the proportion of Bitcoin in the portfolio increases with the market value multiple, the yield curve of the portfolio ranges from gray to orange to green, dark blue, and light blue. The portfolio yield curve becomes more similar to the Bitcoin yield curve trend. Specifically, the portfolio with a market cap multiple of 10x (the gray curve) yields less than the S&P 500, while the portfolios with market cap multiples of 50x and 100x (the red and purple curves) exhibit significantly higher volatility. The portfolio with a market cap multiple of 25x (the green curve) delivers relatively low volatility and higher returns compared to the S&P 500.

4.2. Differences in Fitting Returns of iInvestment Portfolios

Assessing asset value based solely on returns is insufficient. To fully evaluate the value of an asset, it's important to consider the downside risks and trends associated with it, as well as the underlying market conditions. The historical performance of Bitcoin, the S&P 500, and portfolios containing those assets is more closely examined to this end.

In Figure 20, a monthly-rebalanced, equal-weighted portfolio consisting of Bitcoin and the S&P 500 is presented, along with the corresponding scatter OLS regression fitting curve. Figure 21 displays the fitting curve of the portfolio's return alongside the return predicted through machine learning of past data. It's worth noting that the actual portfolio return, as shown in Figure 20, deviates significantly from the standard value estimated through regression. The green area below the fitted curve accounts for 52.21% of the total shaded area (orange and green). However, the degree of dispersion is relatively low for the return predicted by the machine learning model in Figure 21. The green area below the fitted curve represents 37.14% of the total shaded area.

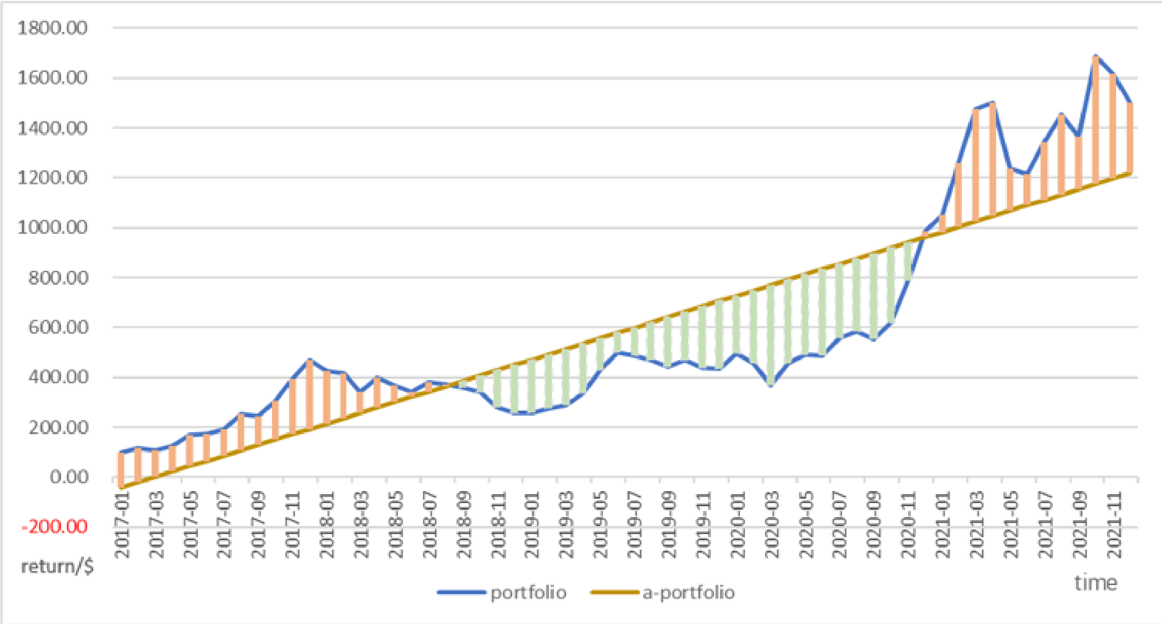


Figure 20: *Monthly balanced equal-weight portfolio returns differential curve (OLS regression)*

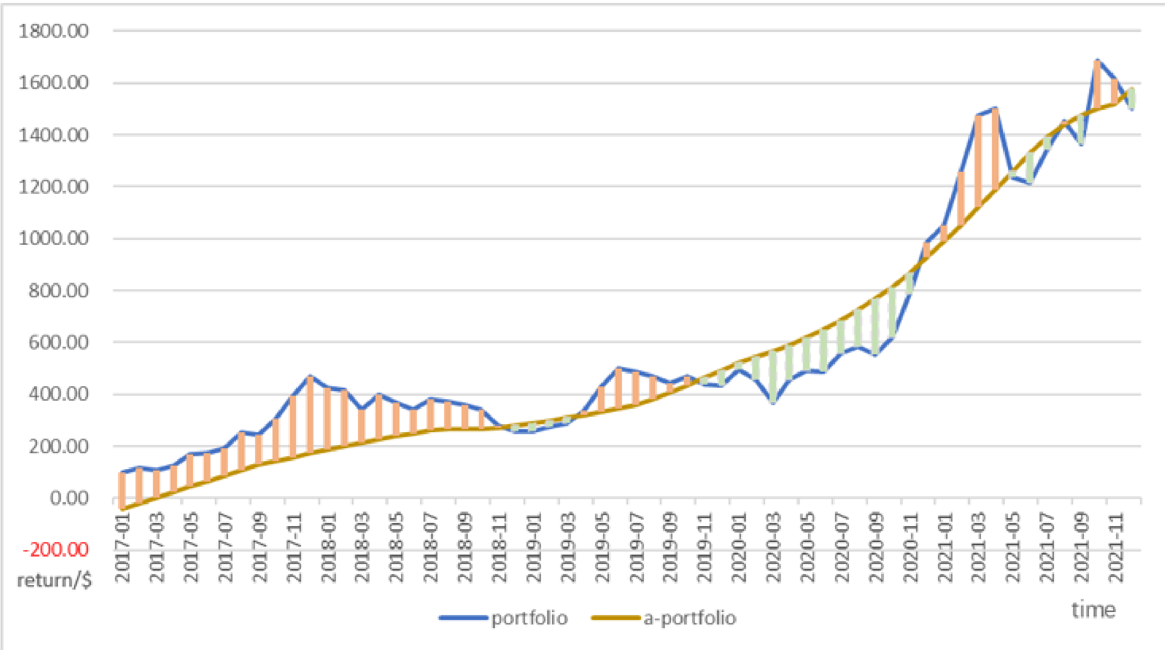
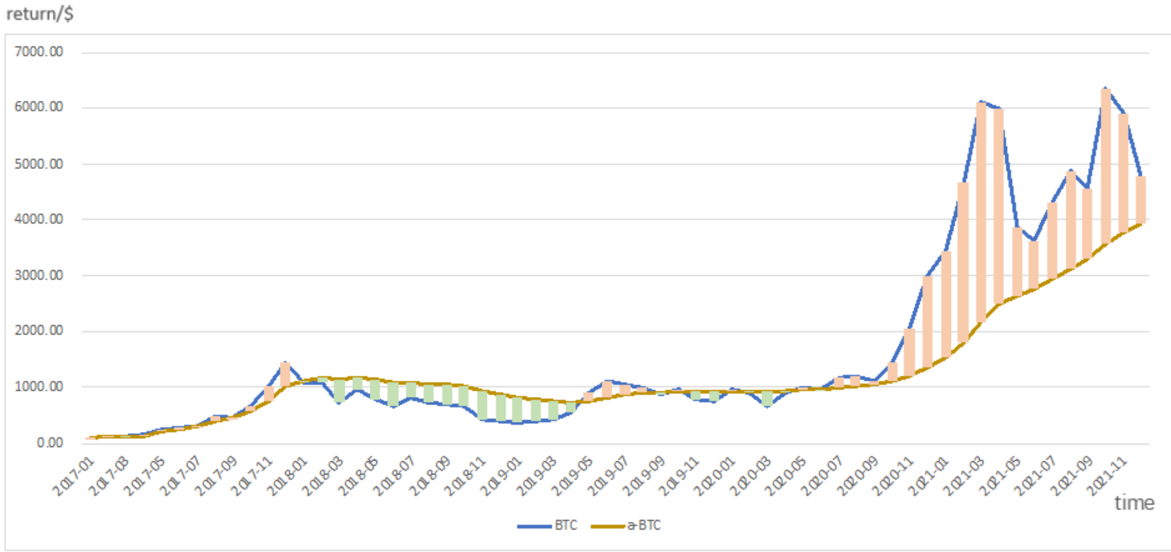
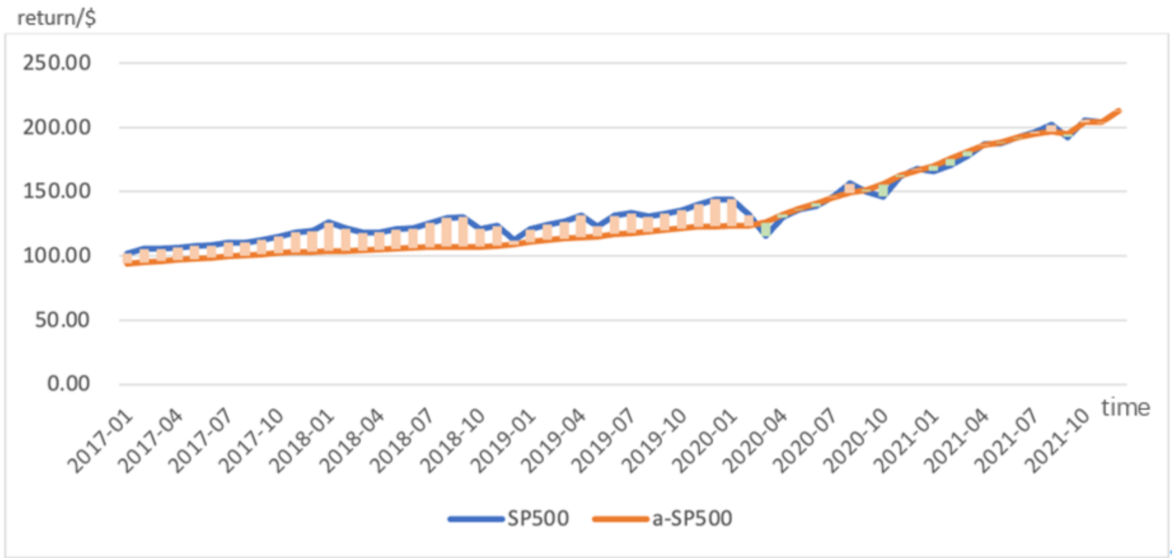


Figure 21: *Monthly rebalanced equal-weight portfolio returns differential curve (regression forecast)*



Panel A: Return and forecast curve for investing only in Bitcoin.



Panel B: Return and forecast curve for investing only in S&P 500

Figure 22: *Quarterly returns (USD) of the Bitcoin and S&P 500 portfolios (2017-2022)*

To make informed investment decisions, it is crucial to consider not only the potential returns but also the associated downside risks. In order to assess these risks, Figure 22 depicts the deviation between the actual return curves and the forecast curves for investments in Bitcoin (panel A) and S&P 500 (panel B). It is worth noting that both panels exhibit a smaller percentage of the green shaded area below the expected return line when compared to Figure 21, indicating a lower level of downside risk. However, the returns gap for Bitcoin in panel A is considerably larger than that seen in Figure 21, resulting in a much larger absolute area of the green portion (approximately 17 times greater than in Figure 21). Despite this discrepancy, the equal-weight portfolio demonstrates a lower downside risk compared to investing solely in Bitcoin.

4.3. Measuring the Downside Risk of Portfolios

Expected Shortage (ES) is a widely used risk measure that calculates the average value of tail loss, also known as Conditional Value at Risk (VaR) or Expected Tail Loss. In this study, ES and VaR of Bitcoin (in panel A), S&P 500 (in panel B), and five portfolios (in panels C to G) are analyzed using various weighting schemes. In this sample 67% of the data before April 2020 are in-sample data and the remaining 33% after May 2020 are out-of-sample data.

The weighting schemes include: 1) equal weighting of Bitcoin and S&P 500, 2) 10x Bitcoin market cap weighted by the S&P 500 market cap value, 3) 25x Bitcoin market cap weighted by the S&P 500 market cap value, 4) 50x Bitcoin market cap weighted by the S&P 500 market cap value, and 5) 100x Bitcoin market cap weighted by the S&P 500 market cap value. VaR and ES are calculated at three confidence levels, which are 95%, 90%, and 85%.

Panel A: VaR & ES of Bitcoin (In sample)		
Confidence Level	VaR	ES
95%	32.51%	34.93%
90%	18.84%	29.08%
85%	14.55 %	25.05%

Panel B: VaR & ES of S&P 500 (In sample)		
Confidence Level	VaR	ES
95%	8.34%	20.46%
90%	4.68%	8.06%
85%	2.76%	6.55%

Panel C: VaR & ES of Equal-Weighted Portfolio (In sample)		
Confidence Level	VaR	ES
95%	17.78%	18.25%
90%	9.96%	16.00%
85%	8.47%	13.53%
VaR & ES of Equal-Weighted Portfolio (Out of sample)		
Confidence Level	VaR	ES

95%	16.89%	17.40%
90%	7.07%	12.30%
85%	5.87%	10.17%

Panel D: VaR & ES of 10*BTC Value-Weight Portfolio (In sample)		
Confidence Level	VaR	ES
95%	9.11%	11.18%
90%	6.56%	9.39%
85%	3.29%	3.51%
VaR & ES of 10*BTC Value-Weight Portfolio (Out of sample)		
Confidence Level	VaR	ES
95%	5.67%	5.67%
90%	5.20%	5.44%
85%	4.17%	5.01%

Panel E: VaR & ES of 25*BTC Value-Weight Portfolio (In sample)		
Confidence Level	VaR	ES
95%	9.07%	11.60%
90%	6.73%	9.60%
85%	2.94%	8.00%
VaR & ES of 25*BTC Value-Weight Portfolio (Out of sample)		
Confidence Level	VaR	ES
95%	11.79%	11.79%
90%	5.61%	11.79%
85%	4.48%	8.70%

Panel F: VaR & ES of 50*BTC Value-weight Portfolio (In sample)		
Confidence Level	VaR	ES
95%	9.79%	12.58%
90%	8.25%	10.66%
85%	3.85%	8.80%
VaR & ES of 50*BTC Value-weight Portfolio (Out of sample)		
Confidence Level	VaR	ES
95%	17.82%	17.82%
90%	7.97%	12.90%
85%	5.99%	10.59%

Panel G: VaR & ES of 100*BTC Value-Weight Portfolio (In sample)

Confidence Level	VaR	ES
95%	13.97%	15.67%
90%	8.87%	12.30%
85%	7.75%	10.96%
VaR & ES of 100*BTC Value-Weight Portfolio (Out of sample)		
Confidence Level	VaR	ES
95%	23.76%	23.76%
90%	11.72%	17.74%
85%	6.36%	13.95%

Table 10: *VaR & ES of different portfolios*

Table 10 presents the in-sample data of VaR (value at risk) and ES (expected shortfall) of Bitcoin and S&P 500 at various confidence levels in Panel A and B, respectively. Panels C to G display the VaR and ES of five different portfolios at different confidence levels, both in-sample (IS) and out-of-sample (OOS).

In Panel A of Table 10, the VaR value at the 95% confidence level for Bitcoin is 32.51% in-sample and 34.93% out-of-sample respectively, implying that there is a 5% chance of losing more than 32.51% in-sample and 34.93% out-of-sample in the next trading day. In addition, the table also provides the ES value at each confidence level, which represents the expected loss over VaR.

Similarly, Panel B of Table 10 shows that the VaR value at the 95% confidence level for S&P 500 is 8.34% and the ES value at 20.46%, which is substantially lower than the VaR of Bitcoin. The ES value of the S&P 500 is only larger than that of the 10-times Bitcoin market-cap weighted portfolio.

Panels C to G present the VaR and ES of various portfolios. For instance, Panel C illustrates that the equally weighted portfolio has greater VaR and ES than the S&P 500 at all confidence levels, indicating that the equally weighted portfolio is riskier than the S&P 500. Panels D to G display that the VaR and ES of portfolios with various multiples of Bitcoin market capitalization and S&P 500 market capitalization can differ significantly. The 100*BTC value-weighted portfolio has the highest VaR and ES among all portfolios, implying that this portfolio has the most risk.

The results suggest that the VaR of Bitcoin is significantly higher than that of the S&P 500 across all three alpha levels, as shown in Panels A and B. Additionally, any portfolio containing Bitcoin has a higher VaR value than the S&P 500. However, including Bitcoin in an S&P 500 portfolio does not necessarily result in worse performance than investing solely in the S&P 500. In fact, the inclusion of Bitcoin in a diversified portfolio can actually result in higher returns and lower volatility, especially during times of economic uncertainty. As a result, investors and financial analysts should carefully evaluate the potential advantages and disadvantages of adding Bitcoin to their investment portfolio and establish a solid investment strategy that is consistent with their investment goals and risk tolerance. While the S&P 500 has a lower Value at Risk (VaR), its Expected Shortfall (ES) at a 95% confidence level is higher, at 20.46%, surpassing that of five other portfolios. This means that even though the S&P 500 incurs lower losses at the same confidence level (i.e., a lower VaR), once a loss happens, its expected average loss is higher than a portfolio that includes Bitcoin (i.e., a higher ES). This is why we not only compared the VaR of portfolios but also computed the expected loss, as VaR does not account for extreme tail risk once a loss has occurred. Despite Bitcoin's return volatility being greater compared to the S&P 500 index (larger VaR at the same confidence level), as discussed in Chapter Three, Bitcoin may exhibit properties similar to gold - its price has a low correlation with the S&P 500, showing characteristics of a hedging asset relative to the stock market. Consequently, the expected shortfall of a portfolio that includes both Bitcoin and the S&P 500 index could be lower than that of the S&P 500 alone.

Regarding the five portfolios analyzed, there is minimal variation between the VaR and ES values calculated using in-sample and out-of-sample data. A portfolio that weighs the market capitalizations of Bitcoin and the S&P 500 differently outperforms an equally weighted portfolio in terms of VaR and ES at different alpha levels. The investment portfolio that weights the Bitcoin market value 100 times exhibited VaR and ES levels that were similar to those of the portfolio where assets have equal weights. This is due to the significant difference in market capitalization between Bitcoin and the S&P 500. When the market value of Bitcoin is scaled up by a factor of 100, it reaches a level that's comparable to the market value of the S&P 500. In doing so, Bitcoin shows similar Value at Risk (VaR) and Expected Shortfall (ES) levels to the S&P 500, thus making the risk profiles of the two assets more comparable.

Expected Shortfall (ES) is considered a better measure than Value at Risk (VaR) for risk management due to its ability to provide a more accurate and comprehensive assessment of potential losses. This conclusion is supported by the out-of-sample data presented, which indicates that portfolios with a market cap weight of 10 to 25 times that of Bitcoin perform best with ES. Although the 10x Bitcoin market cap portfolio displays a lower overall loss, the 25x Bitcoin market cap portfolio demonstrates a better ES level at an 85% confidence interval. Additionally, Figure 18 illustrates that a 25x Bitcoin portfolio has a higher yield curve than the S&P 500, while a 10x Bitcoin portfolio has a lower yield curve. Thus, a portfolio with a 25x Bitcoin market cap is preferred for investors who aim to manage risk effectively while also potentially generating higher returns. In conclusion, ES is a superior risk management tool to VaR since it not only estimates potential losses but also provides a more accurate assessment of the expected magnitude of those losses, which is crucial for making informed investment decisions.

5. Conclusion

In this study, we utilize a dataset spanning from January 2012 to December 2021 to examine the potential of Bitcoin in enhancing the performance of traditional asset portfolios that include the S&P 500 stock index. Our correlation analysis initially suggests a positive relationship between Bitcoin prices and those of stocks and other commodities. However, a detailed review of the data, considering the time factor, concludes that the correlation between Bitcoin and other assets is not significant.

Despite this, we observed that Bitcoin bears some similarities to gold-like assets, but this relationship is dynamic. In the early stages of Bitcoin's development, the asset did not exhibit strong gold-like characteristics, but over time, a positive correlation with gold emerged. From 2017 onwards, investors' attitudes towards Bitcoin and gold became more erratic, and the COVID-19 pandemic significantly impacted Bitcoin's gold-like performance. Consequently, Bitcoin gradually lost its correlation with gold and exhibited some correlation with the stock market. Yet, it remains uncertain whether Bitcoin will evolve into a separate asset class from gold and equity assets or remain similar to one of them.

After constructing portfolios for Bitcoin and the S&P 500 using the minimum VaR and CVaR (Expected Shortfall) methods, it was found that the portfolio created using CVaR had a smaller risk gap. This consistency extends to later findings, indicating that a portfolio optimized under CVaR exhibits more resilience during market downturns and provides investors with a higher level of protection against extreme market events compared to a portfolio optimized under VaR. These findings are particularly relevant for risk-averse investors.

Furthermore, our study explores the potential impact of incorporating Bitcoin into conventional stock and bond portfolios through the application of four widely used asset allocation techniques. We graphed the efficient frontier curve of Bitcoin alongside the S&P 500 and bonds, revealing compelling evidence that Bitcoin can significantly enhance portfolio returns.

Finally, a portfolio consisting of Bitcoin and the S&P 500, rebalanced on a monthly basis from January 2017 to December 2021, was constructed to analyze the return series and downside risk. The results indicate that combining Bitcoin and the S&P 500 can yield a portfolio with a lower expected shortfall compared to solely investing in the S&P 500, despite Bitcoin's inherent volatility. By utilizing a value-weighted approach based on a 25x Bitcoin market cap and S&P 500 market cap, returns can be enhanced without raising the expected shortfall of an individual asset.

In conclusion, our study indicates that Bitcoin, despite its volatility and dynamic correlation with other assets, can deliver substantial improvements to portfolio performance under specific conditions and allocation strategies. This is especially true when risk is measured and optimized using CVaR.

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