- -

Empirical analysis and multiple regression modelling of influential factors behind wheat prices in future market

- -

Köse, Mehmet Berkay

Empirical Analysis and Multiple Regression Modelling of Influential Factors Behind Wheat Prices in Future Market

Master of Science in Commodity Trading

Submitted by
KOSE Mehmet Berkay

Under the supervision of:
Prof. Dr. Craig Pirrong
University of Geneva
Geneva School of Economics and Management

I certify that the work presented here is, to the best of my knowledge and belief, original and the result of my own investigations, except as acknowledged, and has not been submitted, either in part or whole, for a degree at this or any other University.

26/05/2019

ACKNOWLEGMENT

ABSTRACT

This paper aims at pointing out major factors behind wheat prices, clarifying their effects on wheat prices and building a forecasting model to help ones interested in price movement. In the course of study, fundamental factors, macroeconomic indicators, production costs and speculative positions are discovered as four main factors. Under fundamental factors, supply, demand, ending stocks and stock-to-use ratio are examined comprehensively and concluded that ending stocks and stock-to-use ratio are significantly negatively correlated with wheat prices. This clearly demonstrates lower supply means higher prices. Regarding macroeconomic indicators, it is proved that dollar index has strong negative relationship with wheat prices mathematically. It is also observed that political risks & uncertainties are tend to influence wheat prices, particularly, current trade war between the USA and China is likely to cause bearish market environment. Under production costs factors, it is found that oil prices and fertilizer prices have significant positive correlation with wheat prices, which indicates that higher production input costs mean higher wheat prices or vice a versa. In speculative positions, it is determined that managed money net positions in the Commitments of Traders (COT) report, representative of speculative positions, have quite robust positive relationship with wheat prices, which concludes that managed money net positions becoming longer lead to spike in wheat prices or vice a versa. Furthermore, all major factors found in the study have been utilized to build statistical model and it is achieved that all those four major factors operate in the model successfully with conformity. Overall, the study reveals the most substantial factors triggering wheat prices and provides practical and applicable approach to predict wheat prices directionally based on pretty strong multiple linear regression model built in the study.

TABLE OF CONTENTS

GLOSSARY

CBOT: Chicago Board of Trade

SRW: Soft Red Winter Wheat

NYMEX: New York Mercantile Exchange

CFTC: Commodity Futures Trading Commission

COT: The Commitments of Traders

MY: Marketing Years

MMT: Million Metric Tons

TMT: Thousand Metric Tons

USDA: United States Department of Agriculture

DXY: Dollar Index

FED: Federal Reserve

FSI: Food, Seed, Industrial

GDP: Gross Domestic Product

IMF: International Monetary Fund

EU: European Union

MM: Managed Money

CTA: Commodity Trading Advisor

CPA: Commodity Pool Operator

WTI: West Texas Intermediate

LIST OF FIGURES

LIST OF TABLES

1.Introduction

Wheat is listed in one of the most prominent commodities due to its responsibility of feeding human-being for ages. Even in the history wheat was considered sacred in some major civilizations and it was one of the first domesticated crop. Today, it still continues to be irreplaceable for human life and demanded by any people in the world. Wheat is produced in almost any country on the earth and basically it is one of the most widely traded crops, which makes it pretty mobile and shipped from one corner of the world to another one. The main wheat exporters can be classified as big eight: the USA, Canada, Argentina, Australia, European Union (EU), Russia, Ukraine and Kazakhstan. In recent years, wheat production in those countries is substantially growing and those countries are taking lead of wheat exports. So, it can be said that big eight exporters are feeding to all world by exporting significant amount wheat.

It is certain that wheat touches every single human-being's life in almost every single day in a different type. It can be in a format as a flour, bread, pasta, beer or whatever it is, it plays important role to feed to people. Neither there is any alternative for wheat nor any replacing commodities. Therefore, it can be imagined that wheat will continue its critical role to feed human-being as a primary food sources in future. In last couple of decades, world population has been growing remarkably and speedily. This means that it will be needed to have more amount of wheat than ever. Frankly, there is a rising wheat demand depending on increase in world population. On the other hand, wheat production is growing considerably with the help of improved machinery and technology in order to fulfill that accelerating demand.

In this increasing supply and demand environment in the wheat market, one familiar question comes to the minds. "Why wheat prices are changing over time?" Basically, it is not only one popular question but also it is an actual problem, which creates a price risk for all parties taking place in wheat cycle such as producers or farmers, commercial users and consumers. Uncertainty in the prices creates vagueness in the business and in the daily life. Whereas quick upward spike in wheat price can lead to unpleasant conditions for consumers, bearish wheat prices can affect quite negatively to producers. On the other hand, price fluctuation creates opportunity for commodity traders and makes them to take

that price risk from other parties to make profit. Obviously, any parties in wheat cycle, which are producers, commercial users, commodity traders and consumers, try to answer that question in order have an idea about price movement.

The aim of this study to help all those parties who takes places directly or indirectly in wheat cycle understand the major factors and right story behind wheat prices and facilitate them to predict price directions. Basically, in this thesis, three main questions are tried to be answered. The first question is "What are the main drivers affecting wheat prices". To find the correct answer, any potential factors that might influence wheat prices are tried to be analyzed. Main price drivers found in the study are fundamentals, macroeconomic indicators, production costs and speculative positions. The second question tried to be answered is "How those factors impact wheat prices?". So, each of factors or drivers are analyzed comprehensively to point out how they trigger wheat price change over time. Actually, each factors' relationships with wheat prices are researched. The last question is "Can all those factors be utilized together to predict wheat price movements?" In order to give a proper response for this last inquiry, extensive wheat price forecasting model is built successfully in the study. Overall, it can be summarized that the study is targeting to find broad answers of all three questions above in one of most important liquid future markets, namely Chicago Board of Trade (CBOT).

2. Literature Review

Reviewing extensive different papers and researches demonstrates that multiple and dissimilar factors influence commodities and wheat prices in different ways. These factors are namely "Fundamentals", "Macroeconomics Indicators", "Production Costs" and "Speculative Positions".

2.1. Fundamentals

It can be said that fundamentals are the first dynamic that should be considered while looking into commodity price mechanism. Warren Buffet also supported this with his statement of "like most trends, at the beginning it's driven by fundamentals but at some point, speculation takes over."[1] Under fundamentals section, supply, demand and ending stocks (inventory) are deeply examined.

2.1.1. Supply

World wheat supply has outgrown substantially since 1960s, which is more than tripled.[2] As shown in the Figure 1, in last two decades, world's wheat production has grown up predominantly from 587 million metric tons (mmt) on 1999/2000 marketing years (MY) to 758 mmt on 2017/2018 MY. Of big eight exporters, European Union (EU) is the biggest producer; however, Black Sea region countries, Russia and Ukraine, have been getting bigger and significant based on United States Department of Agriculture (USDA) database. Demand of food has been increasing significantly due to strong population and income growth and this creates to pressure on not only wheat but also all other crops which takes responsibility to feed the world. However, through the improved technology, all agricultural crops production as well as wheat production volumes have increased substantially to fulfill solid rising demands.[3] So, improved technology helps farmers or producer considerably to have more efficient production process and obtain significantly bigger crop.
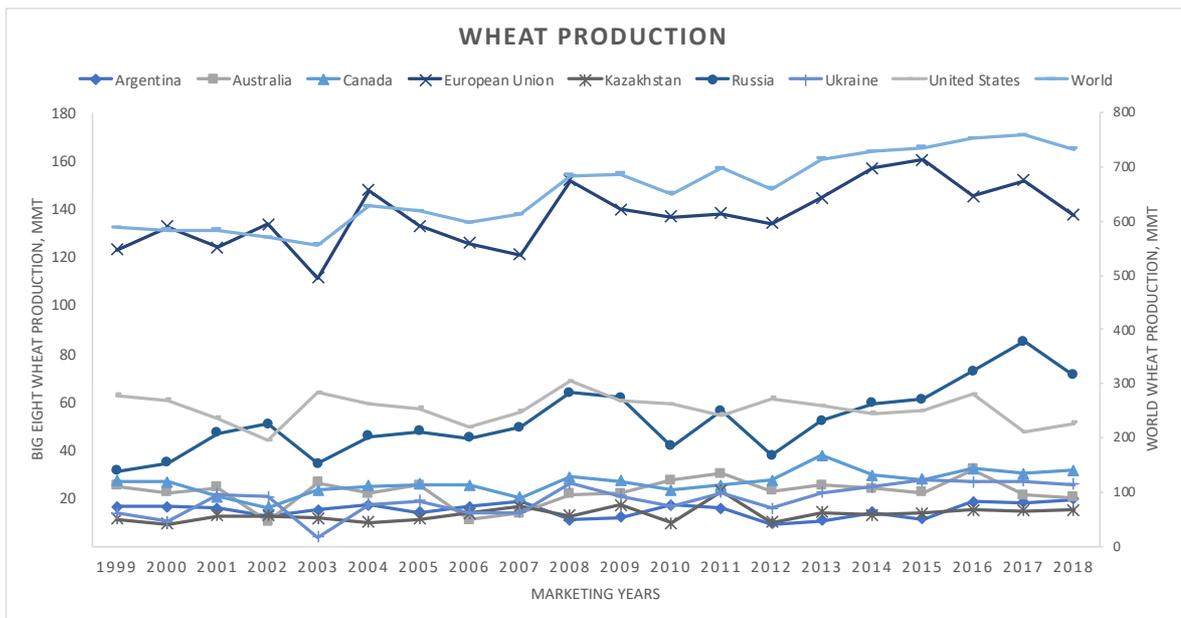
Figure 1. *World and Big Eight Wheat Production between 1999-2018*

It can be claimed that technology is one of the most important elements that influence wheat supply nowadays. Although technology's effect on wheat production is getting more powerful in recent years, crop production or supply is still heavily dependent on weather conditions in wheat as well as in all agricultural commodities. For instance, if the weather conditions are unfavorable for wheat production because of extreme drought or wet conditions, it is very likely to face with production or supply problem. According to IMF Research Department, any supply shock in key wheat exporters driven by weather leads to price spike. For instance, wheat prices went up approximately 85% in two months through 2010's August due to weather-related supply shock in the former Soviet Union countries (Russia, Ukraine, Kazakhstan), which produces around 15% of all global wheat.[4] So that, it is likely to see bullish prices in a potential supply shock as it occurred in 2010 due to drought in former Soviet Union countries.

On the other hand, some researches and authors point out that wheat prices are bearish in an abundant supply environment. According to Thomson Reuters newswire, as of March 1, 2019, wheat has been traded lowest in last eleven months. The reason why is that global and especially Black Sea region's abundant supply pressured Chicago wheat prices.[5] It can be concluded that abundant supply and supply shock in wheat market is pretty impactful on prices.

2.1.2. Demand

Global wheat demand has been rising substantially year by year from 1960 to 2018 because of increase in animal feeding, exports and FSI (Food, seed, industrial) consumption as shown in Figure 2. Basically, FSI consumption has been accelerating more significantly than feed use or export due to robust global population growth. This makes sense since simply more people implies more wheat consumption because of wheat's unending responsibility of feeding human-being.
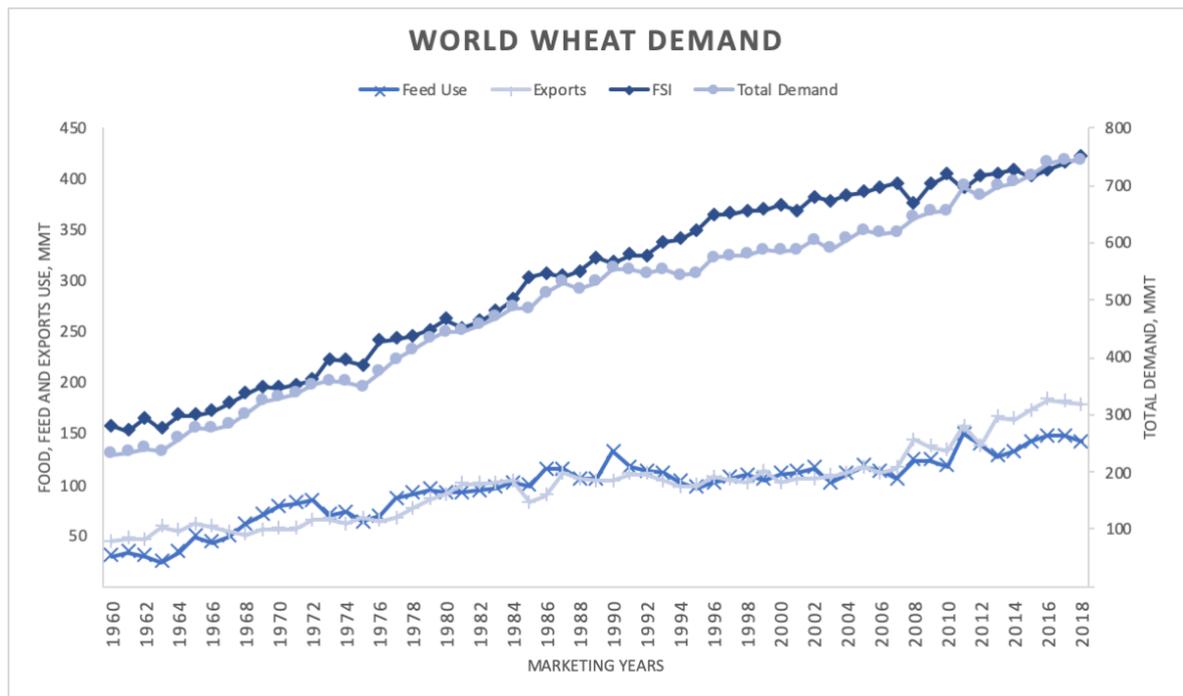
Figure 2. *World Wheat Demand between 1960-2018*

Growing wheat demand is believed to stem from mostly accelerating world population and global economy boom in last decades. Some researchers indicated that widespread global economic growth particularly in major economies such as China and India triggers to increase in global demand. Owing to global demand growth, the commodity prices are likely to rise up.[6] So, in growing demand environment, it is likely to first observe rising wheat animal feeding and FSI consumption as well as increasing trade among countries in wheat market. Then, this rising wheat demand causes upward spike in wheat prices unless supply fulfills that demand promptly.

Two researchers from European Central Bank have argued that global demand pressure is one of the most important explanations of the commodity price fluctuations.[7] Basically,

this argument can be validated by current market situation dominated by trade war between USA and China. Due to trade war, agricultural demand of China, which is one of the biggest agricultural commodity consumers and importers, has switched from USA to other countries. This has created enormous amount pressure on agricultural commodity market and led to crucial price drop in USA. To sum up, it can be claimed that global demand on wheat gives us noteworthy signal about potential price directions in wheat market.

2.1.3. Ending Stocks (Inventory)

Ending stocks (inventory) is an indicator that summarizes all fundamental information inside of it. Actually, it demonstrates the balance between supply and demand. So that, it is very rational to investigate directly to ending stocks instead of looking at supply and demand separately in order to sense price movements.

It can be easily viewed how wheat prices fluctuate year on year while world's ending stocks change in Figure 3. Frankly, it is easily noticed that during abundant supply years with big ending stocks, wheat prices are likely to be lower. For instance, in last couple of years, wheat prices have been depressed and dropping as world ending stocks have been getting bigger. On the other hand, wheat prices are likely to spike in years that there is significant drop in world ending stocks. Therefore, roughly it can be claimed that there is negative relationship between wheat prices and ending stocks. Actually, this view is supported by academia. According to researches, if ending stocks are down year on year, the price is likely to spike.[8] Basically, this view can be validated when looking again Figure 3. Between 2001 and 2005, wheat prices went up due to significant decrease in world wheat stocks. Simply, increase or decrease in wheat inventory year on year implies fluctuations in wheat prices.
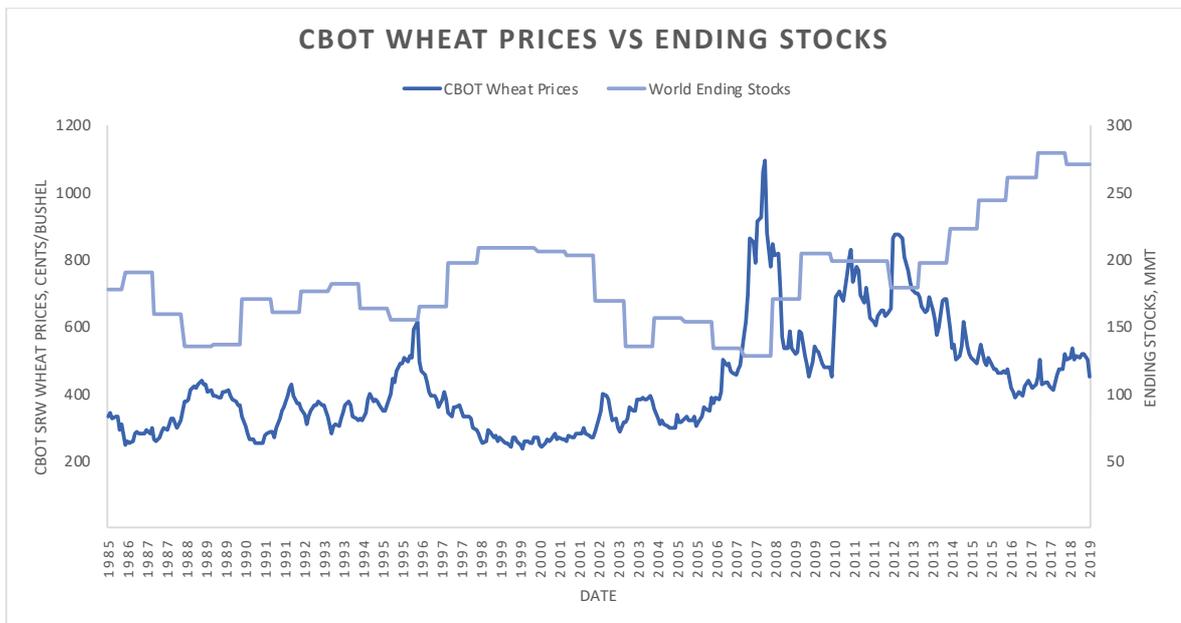
Figure 3. *CBOT Wheat Prices vs. World Ending Stocks between 1985-2019*

Researchers from USDA also supported this view in a different way. Basically, they argued that price-behavior of agricultural commodities are pretty much different in low-inventory (ending stock) environment than it is in high-inventory environment. In low-inventory environment, it is likely to have higher commodity prices for spot or nearby months than the prices for deferred months. So, in tight inventory environment, it might be seen inverted market conditions. They also point out that high inventories create less volatile nearby prices and lead nearby price to maneuver conforming deferred prices.[2] Based on all arguments here, it can be stated here that commodity or specifically wheat prices move in regard to inventory levels.

2.2. Macroeconomics Indicators

It is believed that macroeconomics factors do not only impact financial markets but also commodity markets. According to economics researchers in USDA, during 2007-2008, macroeconomic factors such as depreciation of dollar and switch of the funds from equities and real estate markets to the commodity market assist global commodity prices to spike substantially.[9] This view expresses clearly that commodity market should not be evaluated independently from macroeconomics facts. Thus, macroeconomics factors are decided to be taken into consideration while looking for causes of wheat price movements. Under macroeconomics indicators, two variables: dollar index and political risks and uncertainties are found relevant for wheat market and investigated.

2.2.1. Dollar Index

US Dollar Index (DXY) is an index aiming to measure US dollar's value relative to other foreign currencies (EUR, JPY, GBP, CAD, SEK, CHF).[10] Mostly, it is believed that US dollar is negatively correlated with commodity prices.[11] In order to look at the relation roughly, dollar index and CBOT wheat prices are looked into in Figure 4. It can be quickly interpreted here that wheat prices are prone to be lower while dollar index is higher or vice a versa with simple eyeballing. It can be claimed that US dollar and CBOT wheat prices have an inverse relationship. Some fund managers supported this view and they argued that US dollar has strong impact on all commodities. They indicated that the relation between US dollar and commodity prices is because all commodities are priced in US dollar. Hence, commodity prices decline with strengthening dollar.[12]
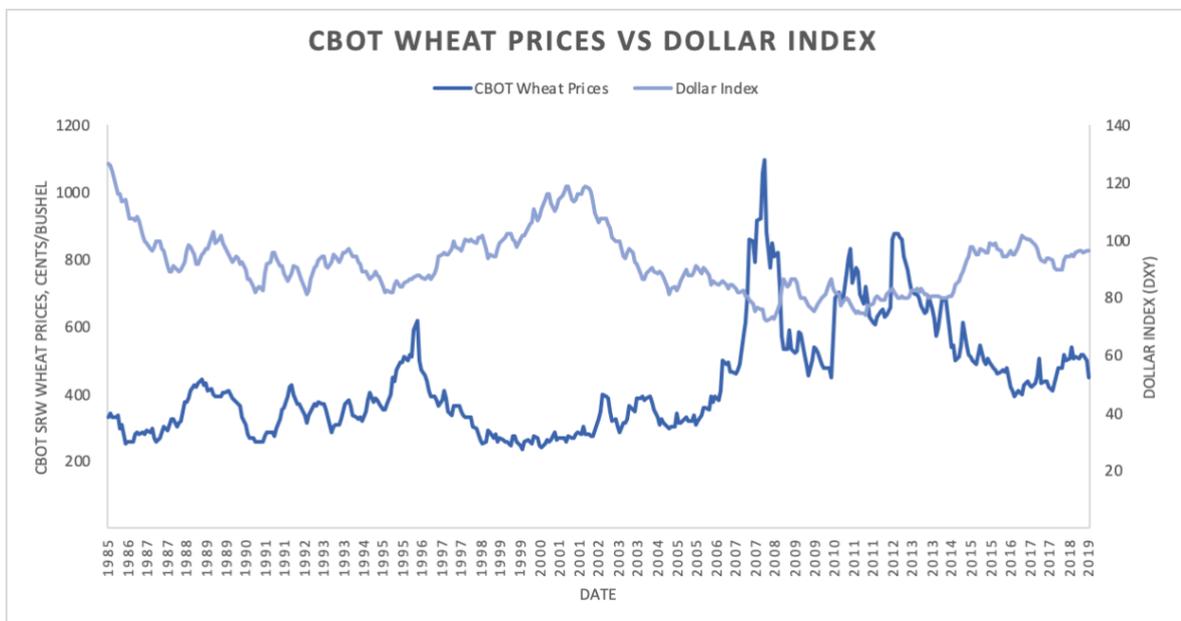
Figure 4. *CBOT Wheat Prices vs. Dollar Index between 1985-2019*

Based on academic researchers, the reason why prominent price escalation during 2008 spikes in most of agricultural and mineral commodities occurred can be clarified by three factors: global economic activity, easy monetary policy and destabilizing speculation.[6] Particularly, easy monetary policy in US means that money supply is boosted by decreasing inflation rates, which has directly impact on dollar index in a negative way but on commodity prices in a positive way.

Overall, almost all commodities are traded in US dollar term and if dollar appreciates against other currencies, it will be more costly or expensive to buy commodity. Hence, any new policy coming up from Federal Reserve (FED) such as easy or tight monetary policy influences US dollar, so does commodity prices. Hence, US dollar index (DXY) is an inevitable variable while looking into commodity price dynamics.

2.2.2. Political Risk and Uncertainty

Albeit it is quite hard to quantify direct impacts of political uncertainty on wheat prices, it is one of the key elements that must be regarded. Researchers claimed that political uncertainty directly alters global commodity prices. Besides, they pointed out that whereas political uncertainty associated with US president elections make commodity prices to drop right before election, political uncertainty associated with national elections in major commodity producer countries causes price spike.[13]

Furthermore, current trade war between the USA and China could be an actual example for political uncertainty. Trade war between the USA and China has directly affected to commodities especially soybeans. As a main soybean producer, the USA was selling most of the soybeans to China; however, due to trade war which imposes 25% extra taxes to USA's export to China, US farmers are not competitive enough to sell their soybeans to China. So that, China's soybean demands change direction from US to South America, mostly Brazil. Therefore, while there is abundant supply in US which makes US soybean prices cheap, there is elevated demand to Brazilian soybeans which makes Brazilian soybean prices expensive. This view is supported by analyst and authors in Bloomberg. According to them, trade war causes biggest drop in commodities in last five months as of July 2018 and one can expect bear commodity market without trade deal.[14]

All in all, what the impact of trade war on wheat is that US farmers are very talented to switch planting among soybeans, corn and wheat. If trade war is extended and continues, it is likely to see switching some amount of planting area from soybeans to corn and wheat, which might lead to abundant supply and bearish market environment in all grains.

2.3. Production Costs

Production costs are natural elements taking a part behind price movements since they are primary cost inputs of production process. Under production costs factors, fertilizer costs and oil prices are discussed.

2.3.1. Fertilizer

Ammonia and urea are the main inputs of a fertilizer and commodities traded all around the world. Since fertilizer is one of the main cost elements for the production process of agricultural commodities, price movement in ammonia or urea impacts production cost of wheat, so does wheat prices. In Figure 5, it can be recognized comfortably that urea price is directionally moving together with wheat prices, which might demonstrate how urea price as a major cost input influence wheat price.
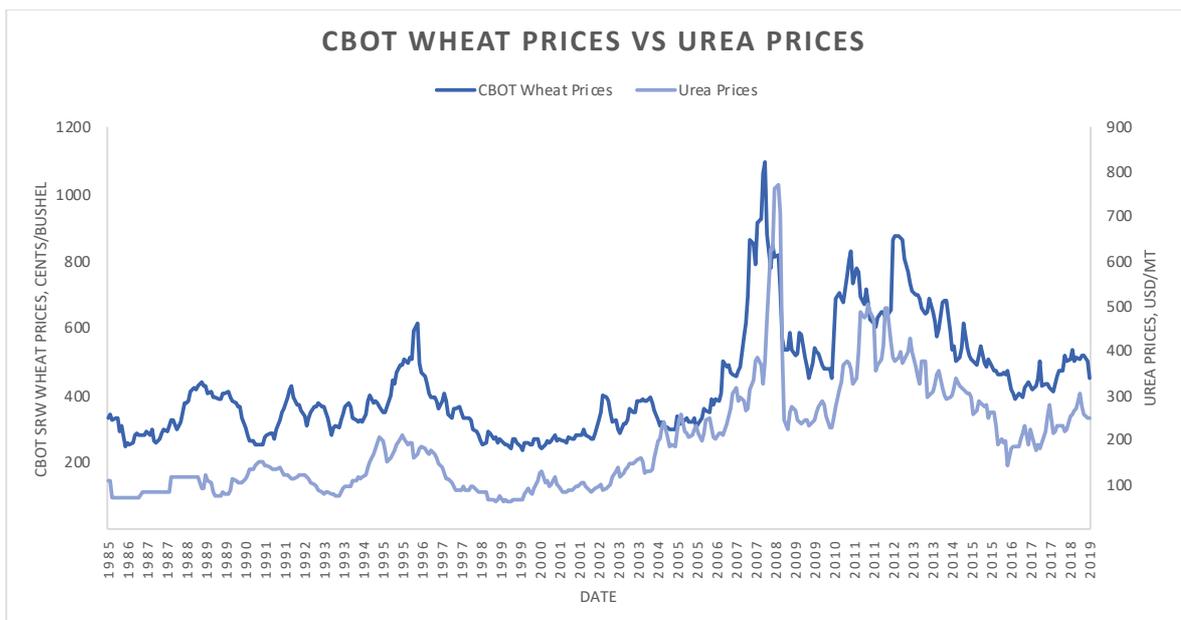
Figure 5. *CBOT Wheat Prices vs. Urea Prices between 1985-2019*

This view is advocated by European Commission researchers. According to them, as fertilizers are key input for agricultural production, they are one of the major reasons behind swing in food prices. Besides, they indicated that fertilizer prices are closely linked to energy prices particularly oil and gas since main inputs of fertilizers are energy sources.[15] In summary, it can be pointed out that fertilizer is a main input for agricultural production that is affected by swing in energy prices but also influences agricultural commodity prices directly.

2.3.2. Oil Prices

Oil prices do not only affect wheat prices but also have impact on almost any prices of every commodities and goods in the world. Everything that people buy in their daily life especially food is affected by crude oil price movement due to the simple fact that oil products are utilized heavily in transportation and food costs are depended on transportation costs.[16] For instance, if crude oil prices rise up, it is expected that cost of transporting fertilizer to the farms and costs of machinery used while planting or harvesting of wheat go up. So that, any fluctuation in oil prices directly has a considerable impact on production costs of wheat so does wheat prices.

According to analysts in Bank of Canada, food commodity prices such as corn, soybeans or cereals prices are directly related to crude oil prices. They point out that spike in crude oil price does not only influence production costs of food but also increases processing, packaging and distribution costs.[17] So, it can be claimed that crude oil is a major cost input for agricultural commodities production.
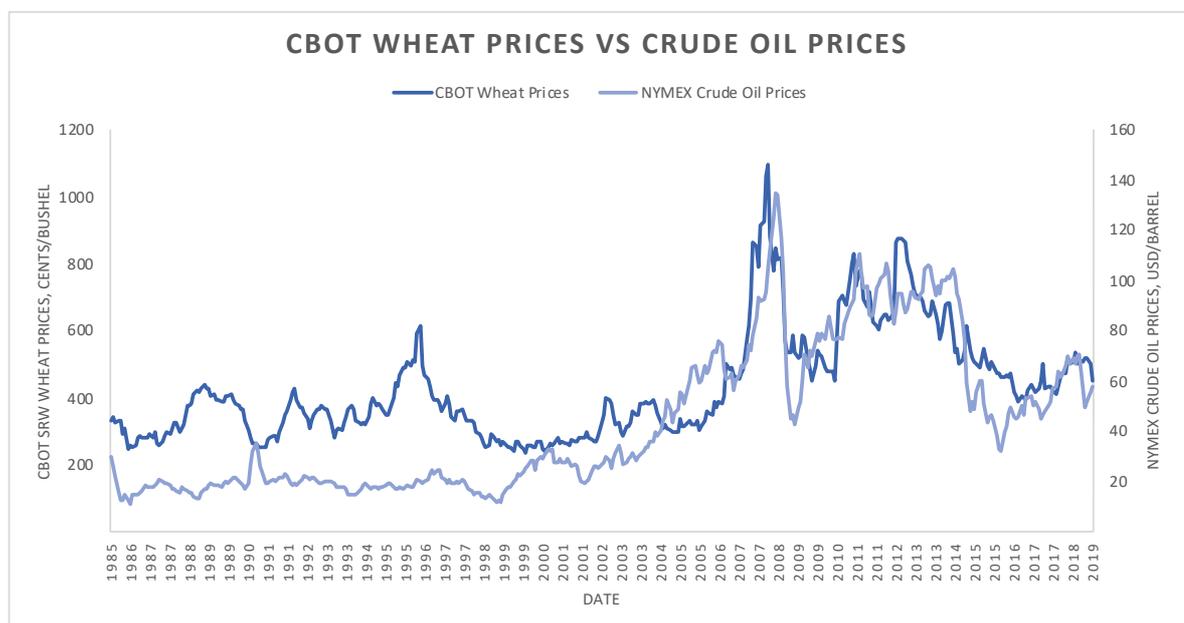
Figure 6. CBOT Wheat Prices vs. NYMEX Crude Oil Prices between 1985-2019

Furthermore, in Figure 6, it is intended to show how wheat price and oil price swing in a specific time period from 1985 to 2019. Therefore, New York Mercantile Exchange (NYMEX) West Texas Intermediate (WTI) crude oil prices and CBOT wheat prices, which is soft red winter wheat (SRW) prices are taken in the figure. It can be eyeballed that oil price and

wheat price are tend to move jointly in a similar direction although there has been some deviation mostly because of seasonality in wheat. In conclusion, crude oil is not only predominant cost inputs for wheat and all agricultural commodities but also financial assets influencing global economy.

2.4. Speculative Positions

Most of researchers and analysts claimed that the reason why observed fluctuation in wheat prices occurs is mainly supply and demand factors in wheat market. On the other hand, some observers point out that excessive speculation stem from massive financial traders makes wheat future prices to deviate from fundamental forces (supply and demand).[2] Namely, it is observed that even fundamentals are accepted by most of scholars as a major price driver for all commodities, they should not be considered independently from speculative positions in future market in order to have complete price discovery.

According to some researchers, speculative positions movement is pretty effective on wheat prices in short-run. Even this effect can be more powerful in economic instability environment and speculations might cause to raise volatility in wheat market.[18] Besides, other researchers and authors supported this view. They state that speculative positions in future market could lead spot price volatility to increase and destabilize the spot markets.[19] Obviously, those two authors point out mutually that speculative positions create extra volatility in the market and causes wheat or commodity prices fluctuate.

In order to gauge speculative positions, The Commitments of Traders (COT) report weekly published by Commodity Futures Trading Commission (CFTC) in United States is a crucial tool utilized by market participants. Despite of being informative tool for speculative positions, COT report does not come out without problems. For example, "commercial" and "non-commercial" categories do not exactly conform to "hedger" and "speculator". So that, some commercial might be speculator.[20] However, still COT report is one of the most powerful tools to interpret speculations in the market in spite of having some issues in reporting.

Comprehending in a right way of the COT report utmost important to grasp approximate speculative positions. Firstly, through COT report it can be seen whether future market is

being manipulated by someone or controlled by someone who is collecting and handling all positions. It might be ended up with very nasty consequences if market is controlled by someone while holding especially short position. Secondly, it is good to know what other market participants are doing. By checking historical extreme long and short position, it can be possible to sense potential price movements.

```
Disaggregated Commitments of Traders- Options and Futures Combined Positions as of October 9, 2018
:                                Reportable Positions                                          :
:------------------------------------------------------------------------------------------- :
: Producer/Merchant :                         :                         :                     :
:  Processor/User   :      Swap Dealers       :      Managed Money      :   Other Reportables :
:  Long  : Short  :  Long  : Short :Spreading:  Long  : Short :Spreading:  Long  : Short :Spreading :
--------------------------------------------------------------------------------------------------
WHEAT-SRW - CHICAGO BOARD OF TRADE   (CONTRACTS OF 5,000 BUSHELS)                            :
CFTC Code #001602                                        Open Interest is    621,138         :
: Positions                                                                                   :
:  89,945  169,337    99,086    5,175   47,118   83,867  100,752   80,222   36,151   27,222  144,145 :
:                                                                                             :
: Changes from:     October 2, 2018                                                           :
:     760   -6,005    -2,554     943    6,636   -4,193     594    1,417    2,350      220    3,679 :
:                                                                                             :
: Percent of Open Interest Represented by Each Category of Trader                             :
:   14.5     27.3      16.0      0.8     7.6     13.5     16.2     12.9     5.8      4.4     23.2 :
:                                                                                             :
: Number of Traders in Each Category                        Total Traders:    450             :
:     98      110        23       7      23       66       62       73       87      63      97 :
--------------------------------------------------------------------------------------------------
```

Figure 7. Disaggregated COT Report of Chicago SRW Wheat

The output of COT report for CBOT wheat in Figure 7 demonstrates that managed money positions are net short with around 17,000 contracts as an example. However, this information alone is not much sufficient to comprehend completely what is going on in the wheat market.
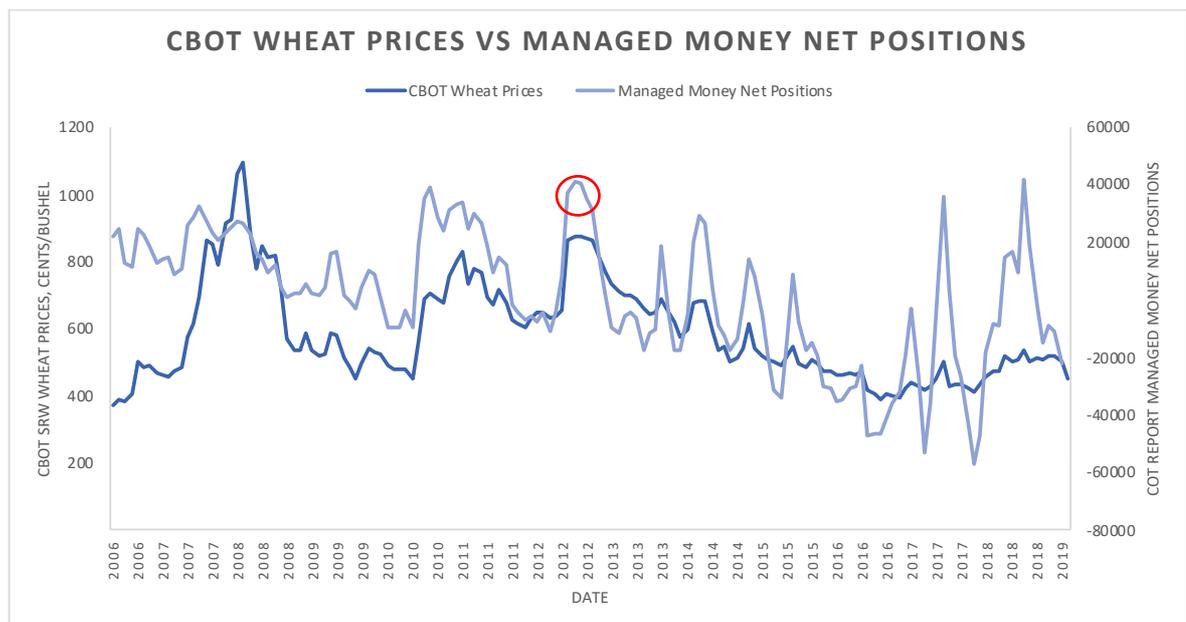
Figure 8. CBOT Wheat Prices vs. Managed Money Net Positions between 2006-2019

To have a better understanding, the historical net managed money positions have been examined with CBOT wheat prices together as in Figure 8. It is important to understand whether managed money net positions hit extreme short or long positions. If net managed money positions are at extreme short, wheat prices place at the bottom level and prone to spike from that level. While net managed money net positions reach to extreme long level, wheat prices hit to peak level and are tend to drop from that level. For instance, in August of 2012, due to extreme drought weather conditions, the market was concern and hit around 41,164 monthly average long contracts, which implies very extreme long positions. In that level, prices attained to peak level, monthly average 877 cents per bushel, and then, they commenced to fall down. To conclude, it can be asserted that managed money net positions, which represent speculative positions, give us some considerable hint about price movement and they are definitely another prominent price driver have an impact on wheat market.

3.Emprical Analysis

3.1. Introduction

Literature review demonstrated that wheat prices are potentially triggered by very various factors: fundamentals, macroeconomics, production costs and speculative positions. Researchers pointed out that all those factors have different impact on price movement. However, they failed to come up with lucid and practical approach to explain the argument. Therefore, it is intended in this section to examine corresponding factors mathematically to prove their influence on wheat prices in a comprehensive way. After that, based on the factors tested, it is aimed to build a statistical extensive model which facilitates to predict monthly average wheat prices.

3.2. Descriptive Statistics

Descriptive statistics gives a general idea about each variable swinging in a certain time period. Table 1 shows that minimum, maximum, average, median and standard deviation of each factors for certain time period. For instance, it can be observed that daily CBOT SRW price was the lowest at 1.14 $/bushel and hit the highest level of 12.80 $/bushel in last 60 years with daily median 3.41 $/bushel.

	MIN	MAX	MEAN	MEDIAN	STD. DEV.	# OBS.	FREQ.	PERIOD
WHEAT PRICES								
SRW - CBOT (1W), $/bu	1.14	12.80	3.59	3.41	1.64	15,045	Daily	1959-2019
HRW - CBOT (1KW), $/bu	3.68	9.88	6.06	5.64	1.55	2,320	Daily	2010-2019
HRS - MGE (1MWE), $/bu	4.71	10.61	6.66	6.17	1.44	2,516	Daily	2010-2019
Matif Wheat (BL2), €/mt	115.75	280.50	186.50	183.75	36.17	2,559	Daily	2009-2019
FUNDAMENTALS								
World Carry Out (mmt)	61	280	149	153	54	59	Annual	1960-2018
World Stock-To-Use Ratio	18%	35%	25%	25%	4%	59	Annual	1960-2018
US All Wheat Carry Out (mmt)	306	1,905	876	876	355	69	Annual	1950-2018
US All Wheat Stock-To-Use Ratio	13%	129%	52%	43%	31%	69	Annual	1950-2018
MACROECONOMICS								
Dollar Index	71.30	128.86	92.12	91.34	10.59	8,512	Daily	1985-2019
Oil Prices (CL2)	10.54	145.86	42.93	29.59	28.79	9,041	Daily	1983-2019
COT REPORTS								
SRW - Managed Money Net	(162,327)	80,827	(18,528)	(11,881)	49,606	667	Weekly	2006-2019
SRW - Producer Net	(208,359)	41,103	(91,874)	(92,755)	56,026	667	Weekly	2006-2019
HRW - Managed Money Net	(51,380)	73,111	15,942	13,879	20,856	667	Weekly	2006-2019
HRW - Producer Net	(129,417)	9,106	(44,051)	(41,082)	25,520	667	Weekly	2006-2019
HRS - Managed Money Net	(12,865)	18,610	4,885	5,054	6,128	667	Weekly	2006-2019
HRS - Producer Net	(26,487)	11,651	(7,418)	(7,089)	7,290	667	Weekly	2006-2019

Table 1. Descriptive Statistics of Major Factors

3.3. Theory and Methodology

It is aimed to express theory used in the study and to discuss the methodology in this section. Essentially, Pearson correlation coefficient test, multiple linear regression and Durbin-Watson autocorrelation test are explained as well as it is discussed how they are utilized and implemented in the study.

3.3.1. Pearson Correlation Coefficient Test

In order to investigate relationship between potential factors and wheat prices, firstly Pearson correlation coefficient test is utilized. This is a method to examine the relationship between two continuous and quantitative variables. Pearson correlation coefficient indicates the measure of strength between two variables and this coefficient ranges from -1 to +1. Whereas -1 and +1 shows perfect negative linear and perfect positive linear relationships respectively, 0 demonstrates that there are no linear relationships between variables. While positive correlation expresses that both variables increase or decrease together, negative correlation signals that variables moves adversely so if one increases or decrease, other decreases or increases.[21] Pearson correlation coefficient formula is stated below as:

$$r = \frac{N \sum xy - (\sum x)(\sum y)}{\sqrt{[N \sum x^2 - (\sum x)^2][N \sum y^2 - (\sum y)^2]}}$$

where

N = number of pairs of scores

$\sum xy$ = sum of the products of paired scores

$\sum x$ = sum of x scores

$\sum y$ = sum of y scores

$\sum x^2$ = sum of squared x scores

$\sum y^2$ = sum of squared y scores

Basically, Pearson test is very practical and useful to see relationship between two variables and it facilitates to see whether variables are positive or negatively influence each other by gauging strengths of the relationship. Therefore, this test is chosen and implemented to analyze each factor's potential relationship with wheat price.

3.3.2. Multiple Linear Regression

Multiple linear regression is used to model the relationship between two or more independent variables and a dependent variable based on the method which fits a linear equation to observed data.[22] It is clear that multiple linear regression is pretty applicable and practical method for our subject since it facilitates to check all factors relationship with dependent variable simultaneously in a single model. It gives opportunity to test each variable to see whether they perform successfully all together in the model. Hence, this approach is selected to build wheat price model.

$$y_i = \beta_{0i} + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_p x_{ip} + \varepsilon_i \qquad \text{for } i = 1,2, \dots n$$

where

y_i : CBOT SRW wheat prices (monthly average)

β_{0i} : Intercept variable

x_{i1} : US stock-to-use ratio

x_{i2} : World stock-to-use ratio

x_{i3} : WTI crude oil prices (monthly average)

x_{i4} : Urea prices (monthly average)

x_{i5} : Dollar index (monthly average)

x_{i6} : Managed money net positions (monthly average)

x_{i7} : Categorical variable for x_{i6}

x_{i8} : Seasonality

x_{i9} : Political risks and uncertainties

x_{i10}: Business cycle

x_{i11}: Dummy variable for removing autocorrelation

The multiple regression model covers thirty-four years starting from 1985 to first quarter of 2019. In this time period, there are 401 data points of which each displays monthly data. The dependent variable of the model is monthly average CBOT SRW wheat prices. There are total eleven explanatory or independent variables in total. Whereas nine of eleven variables are factors considered price dynamics, one variable (x_{i7}) is a categorical variable for x_{i6}, managed money net positions variable, and one variable (x_{i11}) is a dummy variable to remove autocorrelation from the model.

First two variables, US stock-to-use ratio and world stock-to-use ratio, represent the fundamental factors in the US and in the world, which are believed to be two fundamental reasons of price movement. Basically, stock-to-use ratio is a major fundamental variable which illustrates how much wheat stock exists at the end of the marketing year versus how much wheat are consumed during that year in a percentage term. This variable is in annual term since it is known only at the end of the marketing year. The next two variables, WTI crude oil prices and urea prices are plugged in the model due to the fact that they are presumed to be cost drivers of production process of wheat. Both variables' monthly average data is utilized in the model. The fifth variable of the model, dollar index, serves in the model to represent macroeconomics impact on wheat prices and again monthly average of the variable is used in the model.

The next independent variable is managed money net positions, which is taken to capture speculative positions effect on wheat prices. As CFTC commenced to report managed money position in COT report in 2006, there is only available data starting from 2006. Hence, this variable covers only fourteen years period from 2006 to 2019. Because this variable is very significant factor triggering wheat prices, it is regarded in the model despite of having no available data before 2006. In order to introduce managed money net positions variable into the model, one of the methods in literature is use of an additional categorical variable to separate time periods according to data availability. Therefore, categorical variable, which is seventh variable in the model, is used to categorize the model into two periods: from 1985 to 2005 and from 2006 to 2019. This binary variable assign "0" for 1985-2005 and "1" for 2006-2019.

Seasonality is the eighth variable regarded in the model. This variable is utilized to give weight for different month of the year since wheat prices fluctuate based on seasonality pattern. So that, while months in the first quarter of the year have the highest weight, summer months around harvest period have the lowest weight in order to capture seasonality effect on wheat prices. Next two variables, political risks and uncertainties and business cycle variables are intended to count on big macroeconomics event which creates odd market conditions. It is aimed to take consideration of bearish market conditions such as after financial crisis or trade war in political risks and uncertainties variable, which is taken as binary variable. Whereas "0" is assigned to normal usual years, "1" is assigned for bearish years in the model. Nonetheless, in business cycle variable, it is targeted to categorize time intervals based on their own macroeconomic characteristic. thirty-four years are divided into three different time intervals as 1985 to 1998, 1999 to 2006 and 2007 to 2019. Each time intervals are ranked and weighted in the model to indicate their own economical features in the corresponding time period.

All of these ten independent variables perform quite significantly with a harmony in the model. However, it is noticed that model residuals show a little bit positive autocorrelation with the help of Durbin-Watson test. This means that if there is a positive residual coming up from results, the model is prone to continue give positive residuals or vice a versa. This is quite common problem in all sort of financial models. In order to cancel out the positive autocorrelation from the model, one of the methods chosen to follow here is to introduce dummy variable which helps downgrade consecutive higher positive standard error from the data sample. Therefore, eleventh or last variable, which is dummy variable for removing autocorrelation, has been put in the model.

3.3.3. Durbin-Watson Autocorrelation Test

One of the main assumptions of linear regression is having no autocorrelation, serial correlation, in the model residuals. Autocorrelation or serial correlation happens when model residuals are correlated to each other. So, it is mandatory to provide no autocorrelation in the residuals in order to use linear regression or multiple linear regression models. Therefore, in this section, multiple linear regression model built with one dependent and eleven independent variables is contemplated to check for the autocorrelation.

Basically, Durbin-Watson is one famous method which is utilized to test whether the residuals from a linear regression or multiple linear regression are independent or not.[23] So, Durbin-Watson is selected to test model residuals for the autocorrelation. Since most of regression problems having time series data express positive autocorrelation, the hypothesis generally considered in the Durbin-Watson test are

$$H_0 : \rho = 0$$
$$H_1 : \rho > 0$$

The test statistic:

$$d = \frac{\sum_{i=2}^{n}(e_i - e_{i-1})^2}{\sum_{i=1}^{n} e_i^2}$$

where $e_i = y_i - \hat{y}_i$ and y_i and $\hat{y}_i$ are the observed and predicted values of the response variable for individual i respectively. d values get smaller as autocorrelation increases. Upper and lower critical values, d_U and d_L have been categorized for different values of the number of explanatory variables (k) and sample size (n).[23]

If	$d < d_L$	reject $H_0 : \rho = 0$
If	$d > d_U$	do not reject $H_0 : \rho = 0$
If	$d_L < d < d_U$	test is inconclusive.

When the test is used for our model, d is calculated as 1.91.

$$d = \frac{\sum_{i=2}^{401}(e_i - e_{i-1})^2}{\sum_{i=1}^{401} e_i^2} = 1.91$$

There are 11 explanatory variables and an intercept variable and sample size (n) is 401.

If α is chosen as 0.05, then Table 2. gives us critical values for n=400 and 11 explanatory variables as $d_U = 1.893$ and $d_L = 1.780$.

$$d = 1.91 > d_U = 1.89$$

Since d value is bigger than d_U, we do not reject $H_0 : \rho = 0$ and conclude that there is no autocorrelation in the model errors.

Critical Values for the Durbin-Watson Statistic: 5% Significance Level

	k=1		k=2		k=3		k=4		k=5		k=6		k=7		k=8		k=9		k=10		k=11	
n	dL	dU	dL	dU	dL	dU	dL	dU	dL	dU	dL	dU	dL	dU	dL	dU	dL	dU	dL	dU	dL	dU
200	1.758	1.779	1.748	1.789	1.738	1.799	1.728	1.809	1.718	1.820	1.707	1.831	1.697	1.841	1.686	1.852	1.675	1.863	1.665	1.874	1.654	1.885
210	1.764	1.784	1.755	1.793	1.745	1.803	1.735	1.813	1.726	1.823	1.716	1.833	1.706	1.843	1.696	1.854	1.686	1.864	1.675	1.874	1.665	1.885
220	1.770	1.788	1.761	1.798	1.752	1.807	1.742	1.816	1.733	1.826	1.723	1.835	1.714	1.845	1.704	1.855	1.695	1.865	1.685	1.875	1.675	1.885
230	1.775	1.793	1.766	1.802	1.758	1.810	1.749	1.819	1.740	1.829	1.731	1.838	1.722	1.847	1.713	1.856	1.703	1.866	1.694	1.875	1.685	1.885
240	1.780	1.797	1.772	1.805	1.763	1.814	1.755	1.822	1.746	1.831	1.738	1.840	1.729	1.849	1.720	1.858	1.711	1.867	1.702	1.876	1.694	1.885
250	1.785	1.801	1.777	1.809	1.769	1.817	1.760	1.825	1.752	1.834	1.744	1.842	1.736	1.851	1.727	1.859	1.719	1.868	1.710	1.876	1.702	1.885
260	1.789	1.804	1.781	1.812	1.773	1.820	1.766	1.828	1.758	1.836	1.750	1.844	1.742	1.852	1.734	1.860	1.726	1.869	1.717	1.877	1.709	1.885
270	1.793	1.808	1.786	1.815	1.778	1.823	1.771	1.831	1.763	1.838	1.755	1.846	1.748	1.854	1.740	1.862	1.732	1.870	1.724	1.878	1.716	1.886
280	1.797	1.811	1.790	1.818	1.782	1.826	1.775	1.833	1.768	1.841	1.760	1.848	1.753	1.855	1.746	1.863	1.738	1.871	1.731	1.878	1.723	1.886
290	1.801	1.814	1.794	1.821	1.787	1.828	1.780	1.835	1.773	1.843	1.765	1.850	1.758	1.857	1.751	1.864	1.744	1.872	1.737	1.879	1.729	1.887
300	1.804	1.817	1.797	1.824	1.791	1.831	1.784	1.838	1.777	1.845	1.770	1.852	1.763	1.859	1.756	1.866	1.749	1.873	1.742	1.880	1.735	1.887
310	1.807	1.820	1.801	1.827	1.794	1.833	1.788	1.840	1.781	1.847	1.774	1.853	1.768	1.860	1.761	1.867	1.754	1.874	1.748	1.881	1.741	1.888
320	1.810	1.823	1.804	1.829	1.798	1.836	1.791	1.842	1.785	1.848	1.779	1.855	1.772	1.861	1.766	1.868	1.759	1.875	1.753	1.881	1.746	1.888
330	1.813	1.826	1.807	1.832	1.801	1.838	1.795	1.844	1.789	1.850	1.783	1.857	1.776	1.863	1.770	1.869	1.764	1.876	1.757	1.882	1.751	1.889
340	1.816	1.828	1.810	1.834	1.804	1.840	1.798	1.846	1.792	1.852	1.786	1.858	1.780	1.864	1.774	1.870	1.768	1.877	1.762	1.883	1.756	1.889
350	1.819	1.830	1.813	1.836	1.807	1.842	1.802	1.848	1.796	1.854	1.790	1.860	1.784	1.866	1.778	1.872	1.772	1.878	1.766	1.884	1.760	1.890
360	1.822	1.833	1.816	1.838	1.810	1.844	1.805	1.850	1.799	1.855	1.793	1.861	1.788	1.867	1.782	1.873	1.776	1.878	1.770	1.884	1.764	1.890
370	1.824	1.835	1.819	1.840	1.813	1.846	1.808	1.851	1.802	1.857	1.797	1.862	1.791	1.868	1.785	1.874	1.780	1.879	1.774	1.885	1.768	1.891
380	1.826	1.837	1.821	1.842	1.816	1.848	1.810	1.853	1.805	1.858	1.800	1.864	1.794	1.869	1.789	1.875	1.783	1.880	1.778	1.886	1.772	1.891
390	1.829	1.839	1.824	1.844	1.818	1.849	1.813	1.855	1.808	1.860	1.803	1.865	1.797	1.870	1.792	1.876	1.787	1.881	1.781	1.887	1.776	1.892
400	1.831	1.841	1.826	1.846	1.821	1.851	1.816	1.856	1.811	1.861	1.806	1.866	1.800	1.872	1.795	1.877	1.790	1.882	1.785	1.887	1.780	1.893
410	1.833	1.843	1.828	1.848	1.823	1.853	1.818	1.858	1.813	1.863	1.808	1.868	1.803	1.873	1.798	1.878	1.793	1.883	1.788	1.888	1.783	1.893
420	1.835	1.845	1.830	1.849	1.825	1.854	1.821	1.859	1.816	1.864	1.811	1.869	1.806	1.874	1.801	1.879	1.796	1.884	1.791	1.889	1.786	1.894
430	1.837	1.846	1.832	1.851	1.828	1.856	1.823	1.861	1.818	1.865	1.813	1.870	1.809	1.875	1.804	1.880	1.799	1.885	1.794	1.889	1.789	1.894
440	1.839	1.848	1.834	1.853	1.830	1.857	1.825	1.862	1.821	1.867	1.816	1.871	1.811	1.876	1.807	1.881	1.802	1.885	1.797	1.890	1.792	1.895
450	1.841	1.850	1.836	1.854	1.832	1.859	1.827	1.863	1.823	1.868	1.818	1.872	1.814	1.877	1.809	1.882	1.805	1.886	1.800	1.891	1.795	1.895

*Table 2.* *Critical Values for the Durbin-Watson Statistic: 5% Significance Level*

All in all, Durban-Watson test result, which is do not reject $H_0 : \rho = 0$, proves that multiple linear regression method can be used to build a forecasting wheat price model without any doubt for autocorrelation problem.

3.4. Independent Variables

3.4.1. Ending Stocks

Ending stocks or inventory is believed to be the most important fundamental factor which directly impacts commodity prices. Basically, it is like summary component of all supply and demand factors. Hence, it contains the most predominant information per se which is most likely to make an impact on price movement.

In order to understand relations between inventory and wheat prices, Pearson test is conducted. CBOT SRW prices are tested with US wheat inventory. Annual average CBOT wheat prices based on marketing year of wheat, which is from June to July, and ending stocks of corresponding year is taken for the comparison. It is seen that US wheat inventory and CBOT wheat prices have significant negative relationship with Pearson coefficient 0.73 as in Figure 9. Basically, this result indicates that US ending stocks have significant influence on wheat prices and simply implies that higher ending stocks mean lower wheat prices or vice a versa.
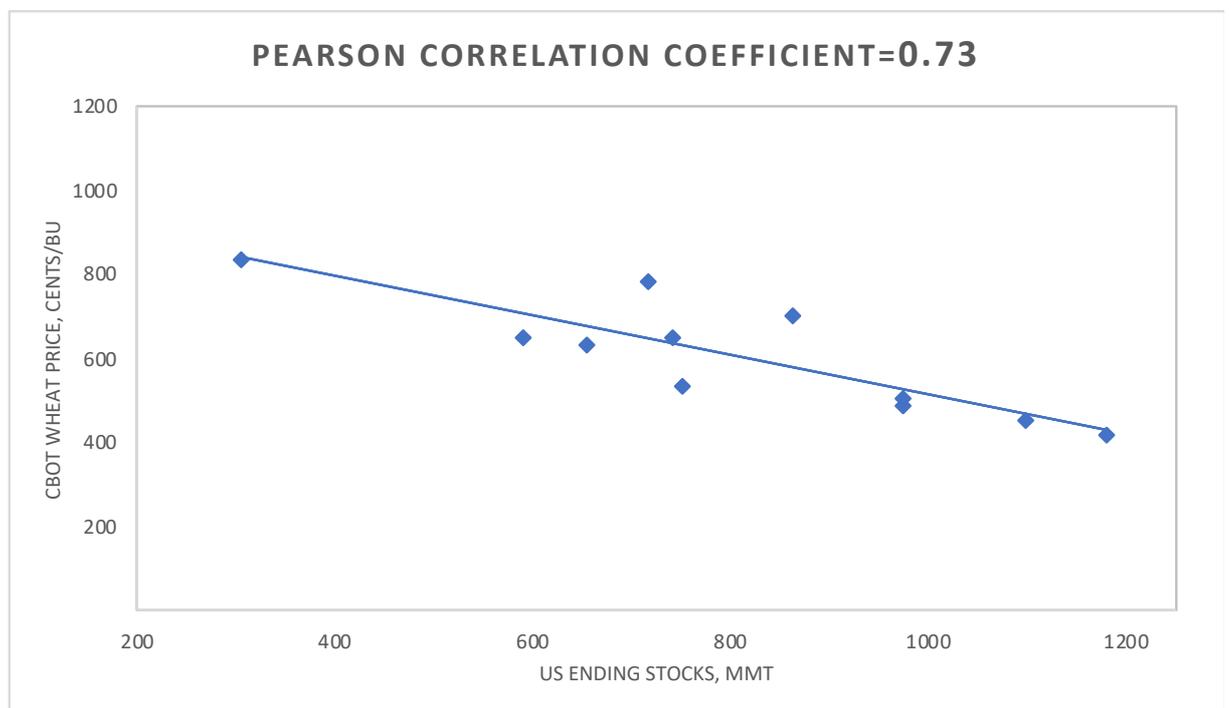
Figure 9. *CBOT Wheat Prices vs. US Ending Stocks between 2007-2017*

Furthermore, the relationship between global world stocks and CBOT wheat prices are investigated. Although almost all countries in the earth are producing wheat and Chicago's

soft red winter wheat production is not big globally, CBOT wheat market is the most liquid future market in the world and CBOT wheat prices are used as benchmark prices in all around the world. Hence, one can expect that global wheat inventory should influence CBOT wheat future market. When looking into relationship through Pearson test, it is seen that global wheat stocks are quite significant. Basically, global wheat stocks are negatively correlated with CBOT future wheat prices with 0.79 correlation coefficient in Figure 10. Again, this demonstrates that higher global ending stocks we have, lower CBOT wheat prices we have or vice a versa.

PEARSON CORRELATION COEFFICIENT=0.79
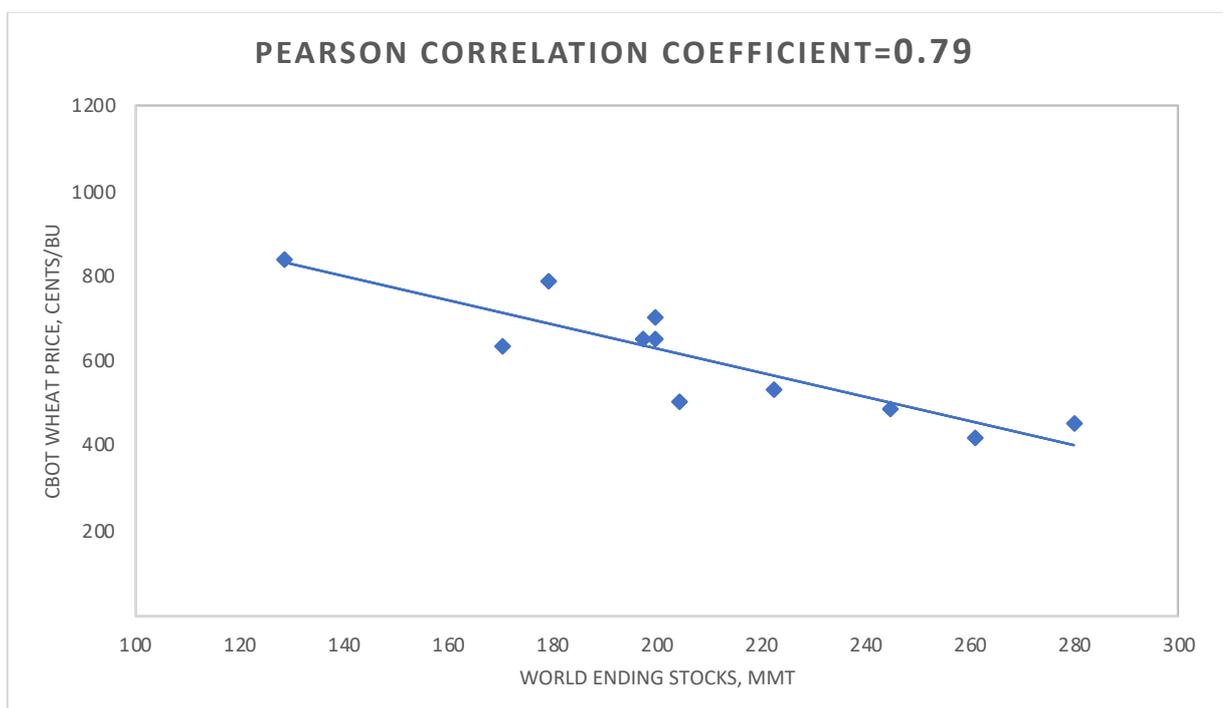
*Figure 10. CBOT Wheat Prices vs. World Ending Stocks between 2007-2017*

All in all, it can be stated that both US and global wheat stocks are negatively correlated to CBOT wheat prices with quite significant relationship. Secondly, CBOT future market is the representative future market for the wheat as it is the most liquid and leading future market. Moreover, CBOT wheat prices are utilized as a benchmark price in all around world.
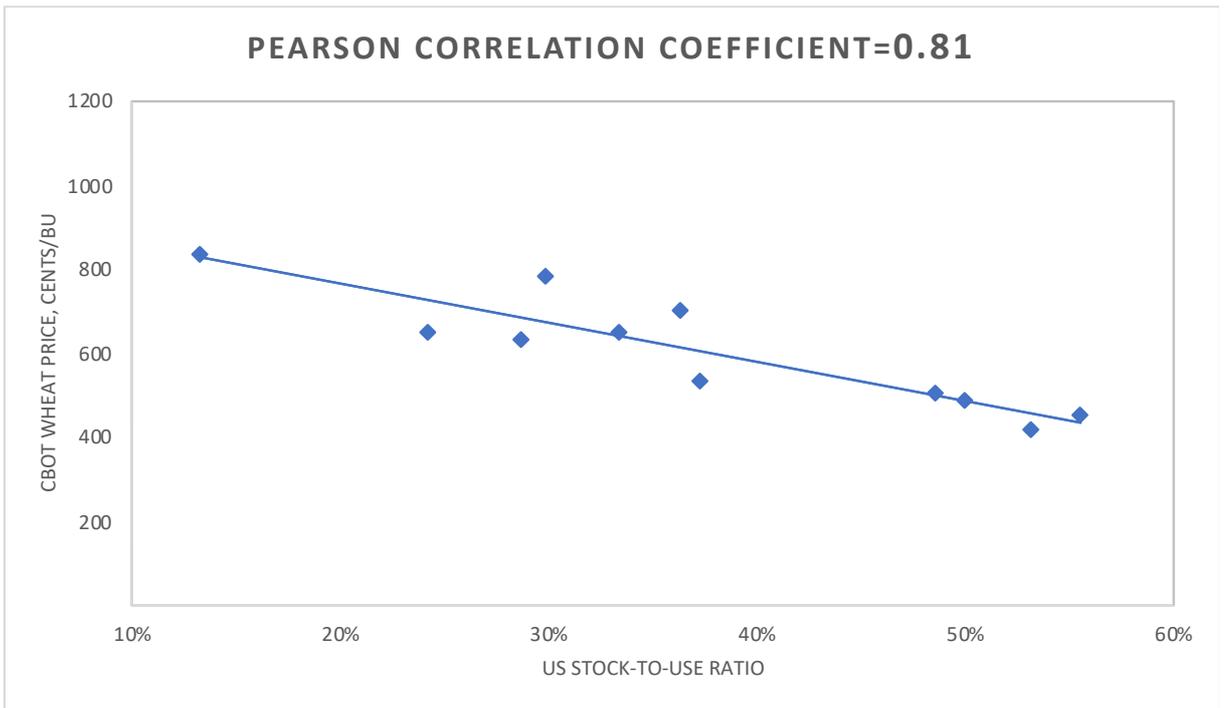
3.4.2. Stock-to-Use Ratio

All fundamental factors, which are namely supply, demand and inventory, are finalized in one bottom line, in stock-to-use ratio. Comparing with ending stocks, stock-to-use ratio facilitates us to gauge ending stocks relative to usage. So that, it makes more sense to utilize stock-to-use ratio instead of absolute value of ending stocks to cancel out year-on-year

trend effect since there is year-on-year growth in all fundamental factors, which is the case in wheat market.

Based on research papers, low stock-to-use ratio causes higher prices since low stocks indicates scarcity in the market.[24] To verify this view, stock-to-use ratio versus wheat prices relation is checked mathematically and it is observed that US stock-to-use ratio has negative linear relationship with wheat prices. So that, if there is a higher stock-to-ratio, one would expect depressed wheat prices because of existence of big stocks with respect to usage. Hence, Pearson test comes up with the negative linear relationship. Moreover, it is seen that when looking into relationship going back to 1960 in Figure 11, the relationship is weaker with 0.24 coefficient than the relationship going back to 2007 with 0.81 coefficient in Figure 12. Although there exist negative linear relationships in both cases, obviously recent years stock-to-use ratio is more influential on wheat prices fluctuation.
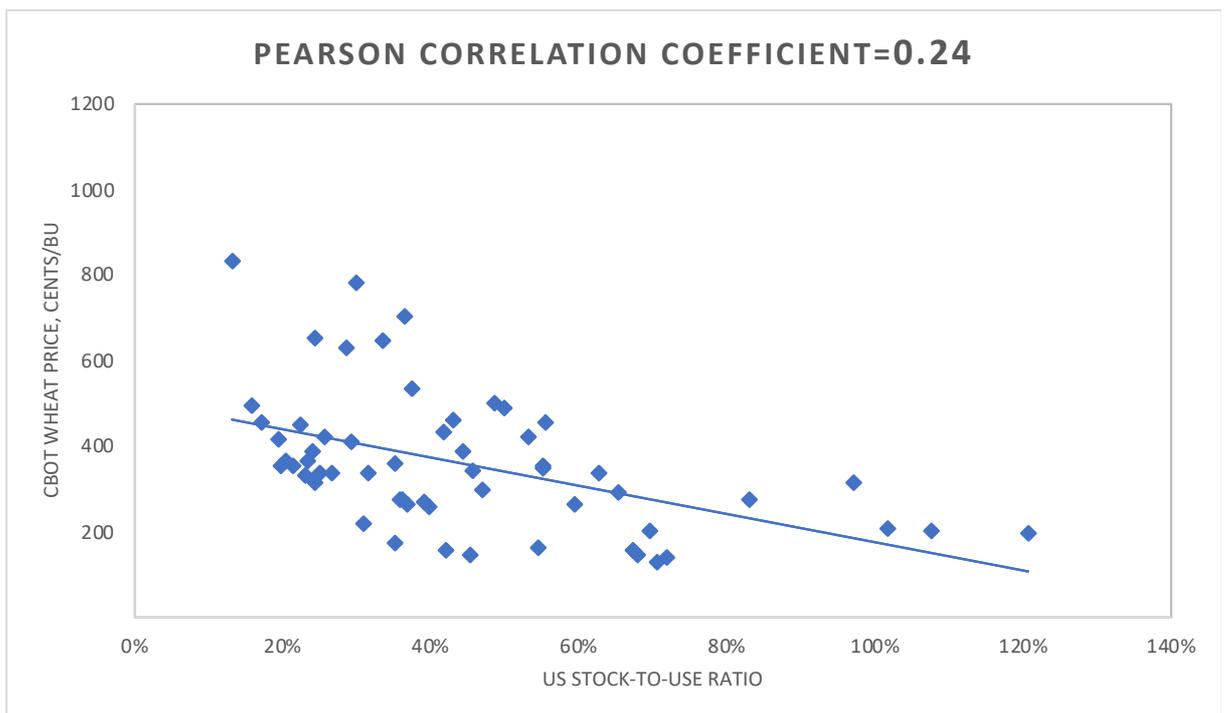
Figure 11. *CBOT Wheat Prices vs. US Stock-to-use Ratio between 1960-2017*

Figure 12. *CBOT Wheat Prices vs. US Stock-to-use Ratio between 2007-2017*

Besides, it is obtained very similar results for the relationship between world's stock-to-use ratio and wheat prices. Certainly, there is a negative linear relationship again and similarly this relationship is stronger in more recent dataset in Figure 14 than one going back more in Figure 13.
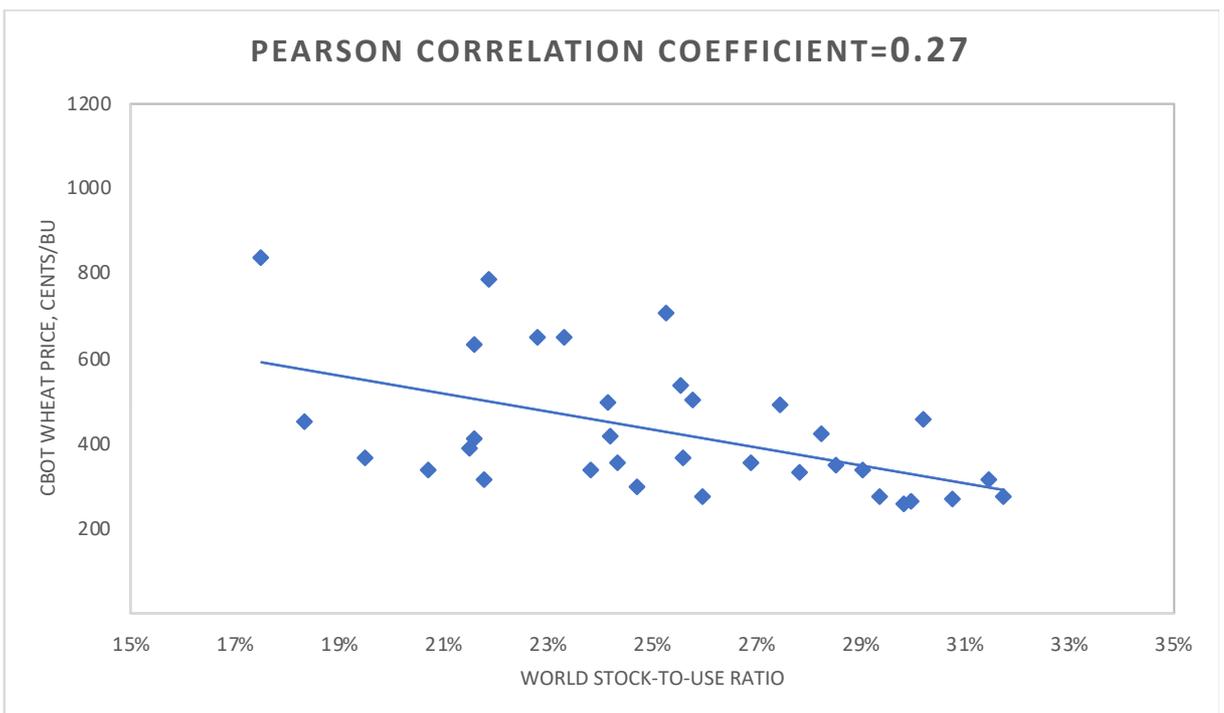
Figure 13. *CBOT Wheat Prices vs. World Stock-to-use Ratio between 1984-2017*

PEARSON CORRELATION COEFFICIENT=0.79
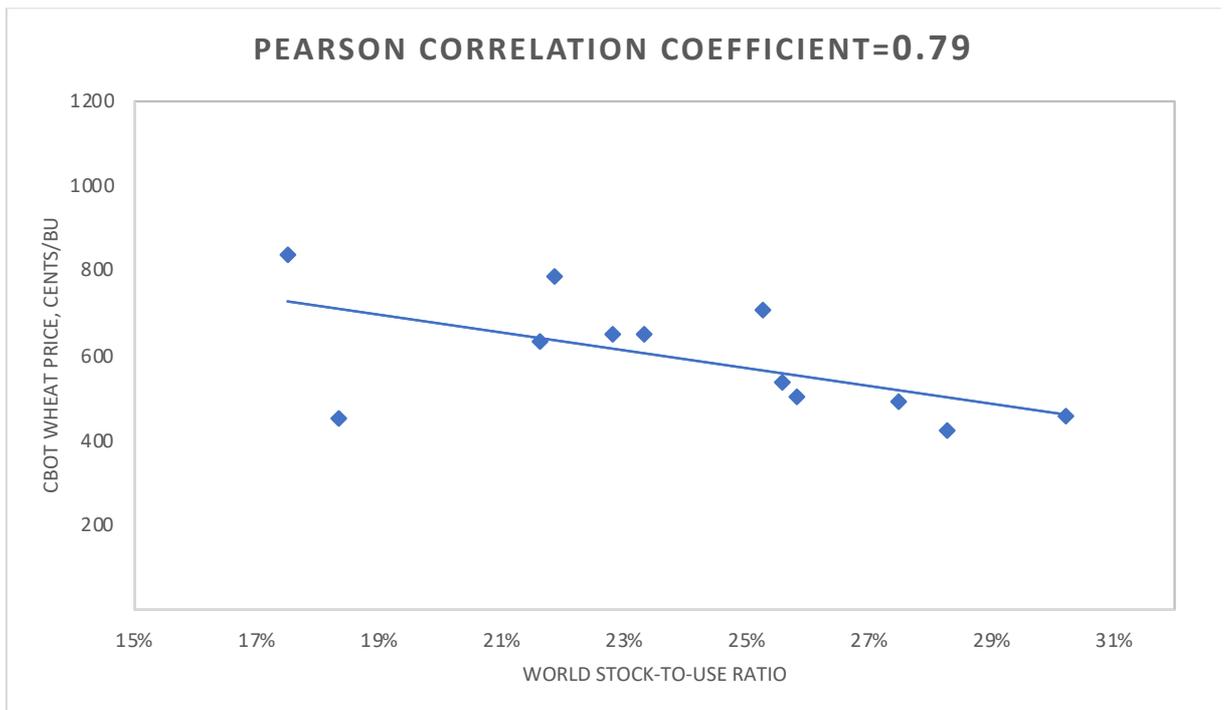
Figure 14. CBOT Wheat Prices vs. World Stock-to-use Ratio between 2007-2017

Overall, it can be concluded here that both US and world's stock-to-use ratios are negatively correlated with wheat prices, which is very much alike conclusion as in ending stocks. However, it should be emphasized that stock-to-use is more proper and powerful factor than ending stocks since it gives ending stocks information relative to usage. Hence, it avoids wrong effect of trend of which fundamental factors (supply and demand) increase year-on-year. So that, stock-to-use ratio is selected in the wheat price model as a representative of fundamental factors.

3.4.3. Oil Prices

Based on academia and literature, it is claimed that crude oil is one of the major effectors on US economy, economic growth and particularly on all other commodity prices.[22] Also, it is seen in the literature review that most researchers and academics assert how undoubtedly crude oil affects wheat prices. So, it is planned to prove the relation mathematically in this section. In order to gauge singular relationship between crude oil prices and wheat prices, Pearson correlation coefficient test is conducted. By using monthly average prices for the period between 1985 and 2019, Pearson's test comes up with 0.81 in Figure 15, which means that WTI crude oil prices and CBOT wheat prices have pretty significant positive linear correlation.

PEARSON CORRELATION COEFFICIENT=0.81
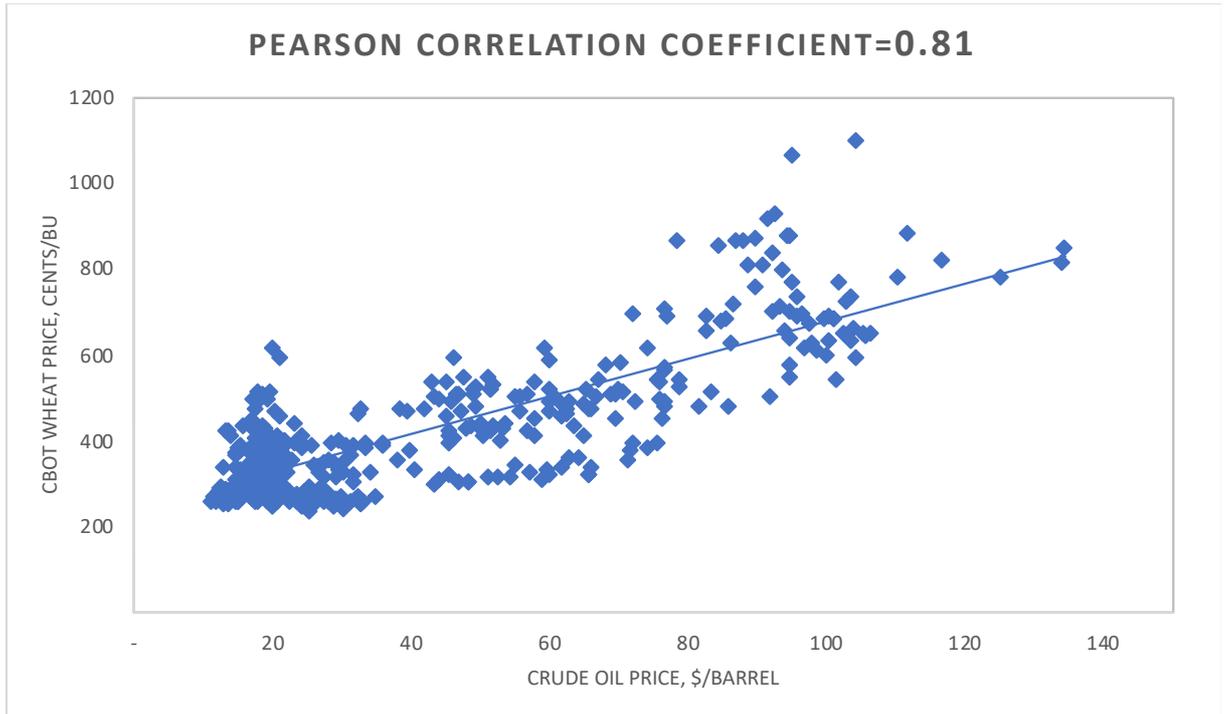
Figure 15. CBOT Wheat Prices vs. WTI Crude Oil Prices between 1985-2017

Essentially, significant relationship between crude oil price and wheat price is detected and mathematically proved here with the help of Pearson test. Logically, it is an inevitable result since not only crude oil triggers world economy and financial sector in a certain way but also it is predominant cost input of any production of goods and services. Hence, it is clear that crude oil influences wheat prices as both production's cost input and macroeconomics driver. So that, it is imminent to acknowledge crude oil's effect on wheat prices and select oil prices as an independent variable in the wheat price model.

3.4.4. Fertilizers

Fertilizers are essential and either natural or chemical substances used for enrichment of soil to improve plant growth of agricultural crops. They are made of three main nutrients: nitrogen, phosphate and potash. Basically, fertilizers are direct cost inputs for production of all agricultural commodities. As fertilizers help farmers to have improved crop yield per acreage, they are inevitable cost inputs for crop production. Therefore, it is key to investigate relationship between wheat prices and fertilizer prices to have a better understanding on drivers of wheat prices.

Urea prices are examined as a representative of all fertilizers and monthly average urea prices have been taken for the period between 1985 and 2019 in Figure 16. It is observed that there is a pretty strong positive correlation between CBOT wheat prices and urea prices according to Pearson test with 0.70 correlation coefficient.
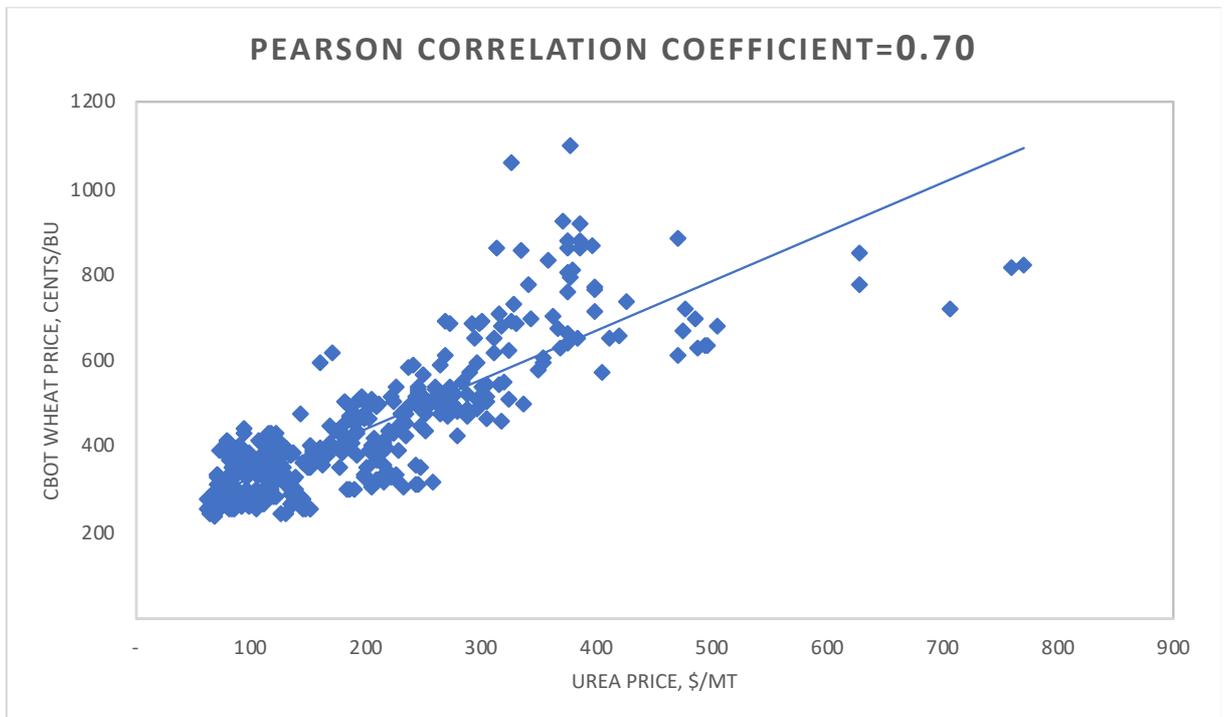
Figure 16. CBOT Wheat Prices vs. Urea Prices between 1985-2019

Frankly, mathematically it is validated that CBOT wheat prices are very likely to ramp up if urea prices rally or vice a versa. To sum up, it can be pointed out that fertilizers are major cost input for wheat production process and one of the main reasons behind fluctuation of wheat prices. So, fertilizer prices have been regarded in the wheat price model as one of the independent variables.

3.4.5. Dollar Index

Almost all commodities in the world are traded on US dollar (USD). Any appreciation in US dollar or increase in dollar index would depress commodity prices as a result of decrease in world demand since countries other than the USA would have weaker currency and lower purchasing power. This indicates that dollar index has a major impact on wheat prices; therefore, any policy changes from Federal Reserve (FED) in the USA which affects dollar index such as monetary policy changes makes an impact on wheat prices. For

instance, rising of interest rates in US as a result of tight monetary policy of FED leads US dollar to strengthen and dollar index to rise up. Then, strong dollar or higher dollar index pressures to all commodity prices as well as wheat prices, so causes decrease in wheat prices.
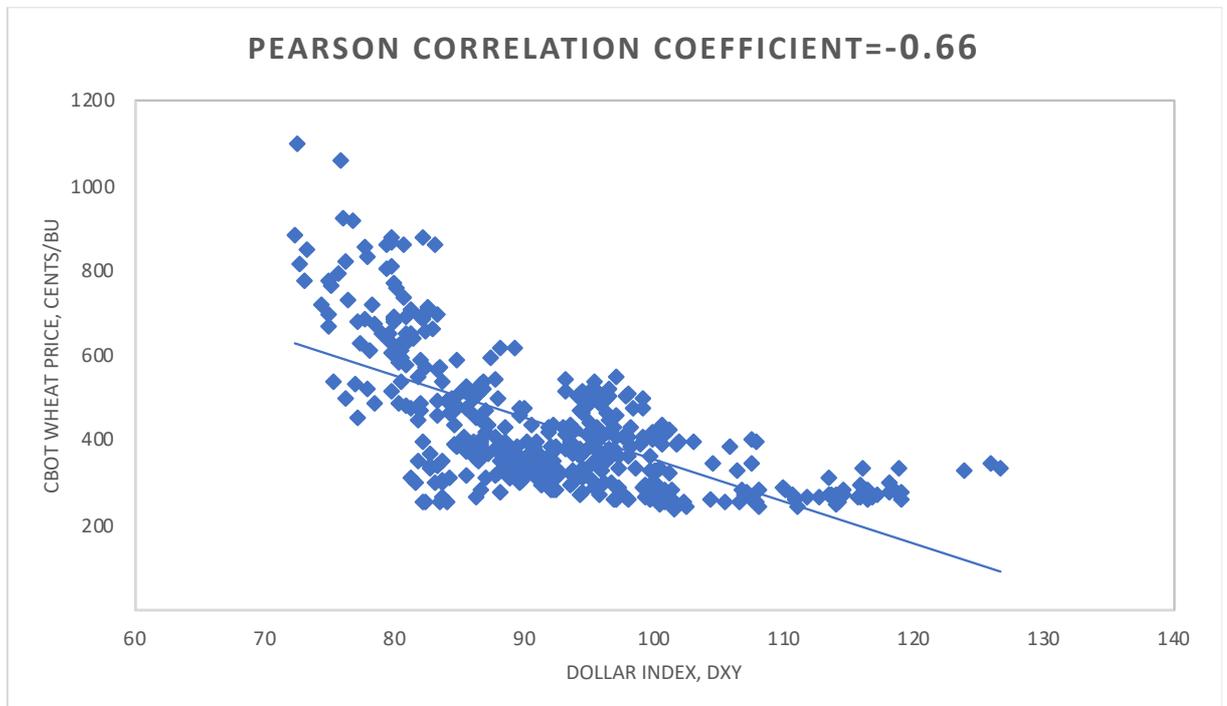
Figure 17. *CBOT Wheat Prices vs. Dollar Index between 1985-2017*

Pearson correlation coefficient test helps us see relationship between them statistically. Basically, it demonstrates very significant negative linear relationship between CBOT wheat prices and dollar index with -0.66 as showed in Figure 17. All in all, it is statistically proved that dollar index has significant negative correlation with wheat prices; therefore, it is picked up as a major independent variable in wheat price model.

3.4.6. Speculative Positions

In order to have a clear idea about speculative positions, COT reports' managed money net positions are examined. Managed money category represents a registered commodity trading advisor (CTA), a registered commodity pool operator (CPO) or an unregistered fund identified by CFTC. All those traders are taking a position in future market on behalf of their clients.[25] It is investigated whether there is a relationship between managed money net positions and wheat prices.

Monthly average managed money net positions and CBOT wheat prices are examined for the period of 2006 to 2017. It is observed that historically CBOT wheat prices are significantly correlated with COT's managed money net positions based on Pearson test with 0.61 correlation coefficient as in Figure 18. This indicates that if managed money net positions are massive short, this depresses wheat prices and leads them to decrease. On the other hand, if market is going mostly long positions and managed money net positions are massive long, this supports wheat prices to spike. Obviously, it can be claimed here that speculative positions are substantial impact on wheat prices.
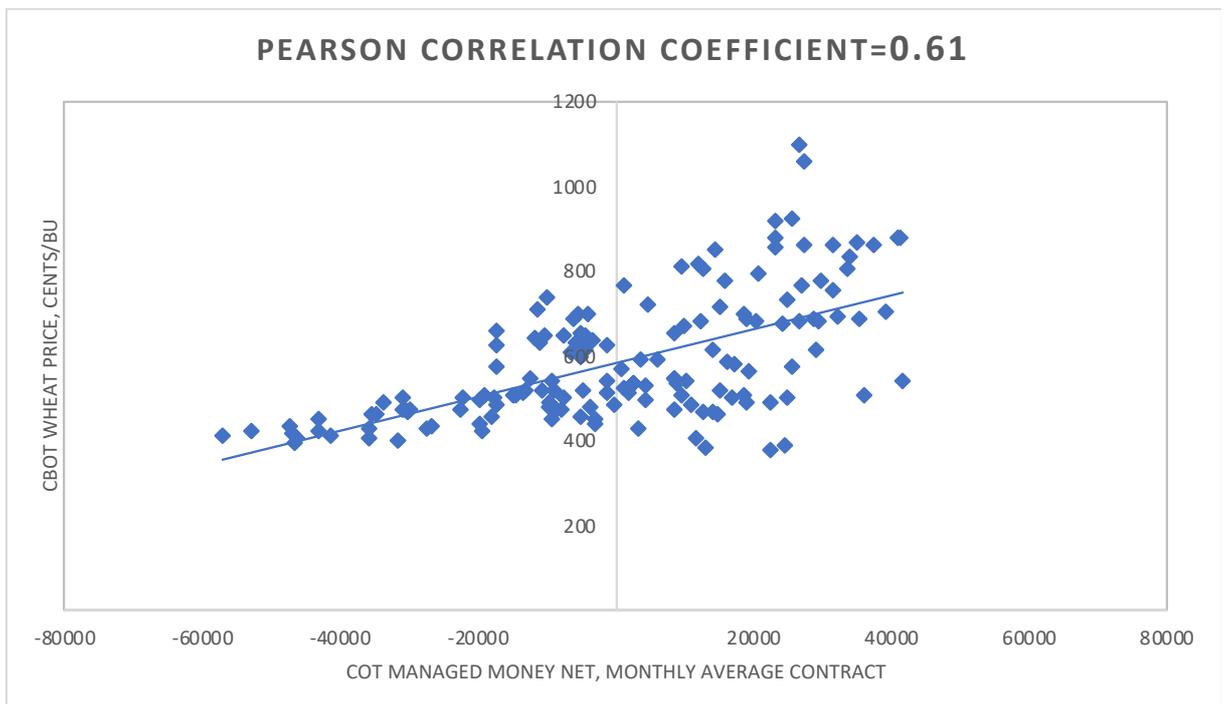
Figure 18. CBOT Wheat Prices vs. COT MM Net Positions between 2006-2017

Besides, it is analyzed relationship between COT producer's net positions and wheat prices for the same period. Producer category in COT report represents actually all producers, merchants, processors and users who significantly engages in the production, processing, packing or handling of a physical commodity and participate in futures markets to hedge risks associated with those activities.[25] Based on Pearson test again, pretty significant negative relationship is observed with -0.52 correlation coefficient as in Figure 18. This demonstrates that if producer category holds net massive short position, there should be very healthy strong demand in the market. Hence, commercials should be selling in future market more than they are receiving from farmers to fulfill that firm demand, which triggers price to go up. So that, it is seen that wheat prices are supposed to be higher

comparing historical prices if producer's net positions stay in the big short which implies robust demand in the market.

PEARSON CORRELATION COEFFICIENT=-0.52
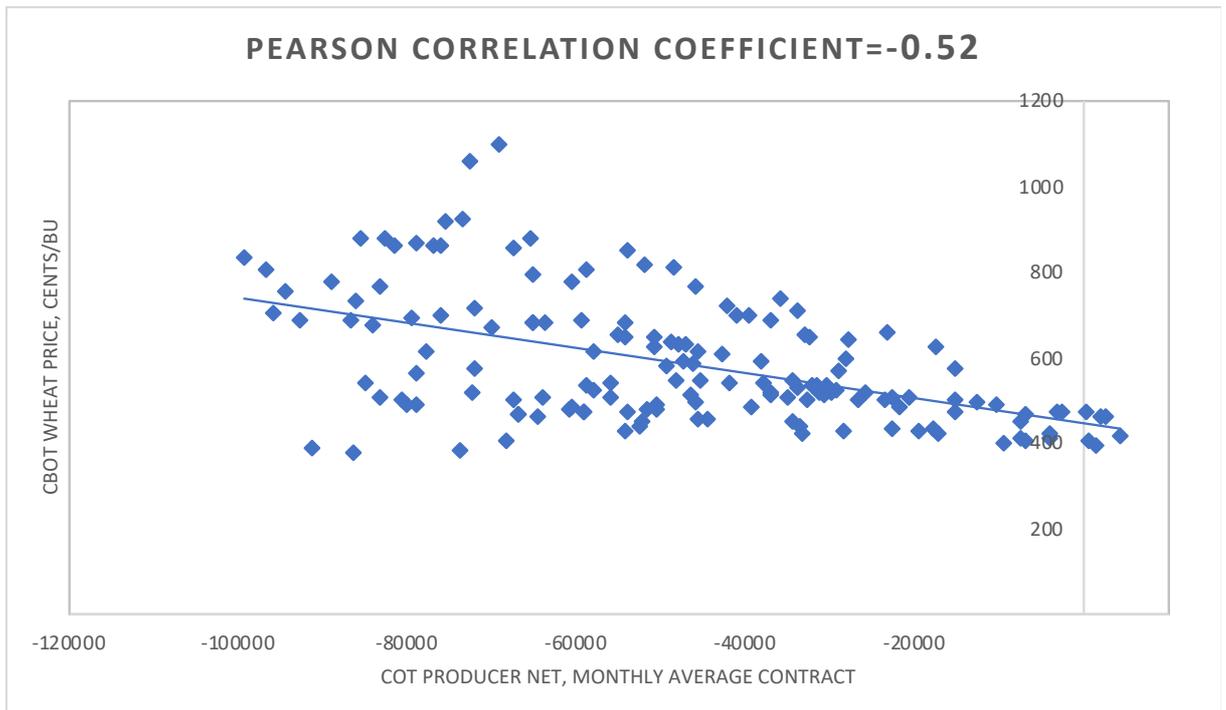
Figure 19. CBOT Wheat Prices vs. COT Producer Net Positions between 2006-2017

To sum up, both managed money and producers net positions have significant impact on wheat prices. Frankly, reaction of market participants on the fundamental or political news in the market leads predominant effect on price movement. It is utmost important to see how market participant behaves based on the changes in factors like fundamentals, economics or politics and to understand whether those changes are already priced in the market or not. Hence, managed money net positions are put into the wheat price model as a speculative positions' variable.

3.4.7. Seasonality

Seasonality is a crucial concept for all agricultural commodities. Although production occurs at specific times of the year, demand spreads throughout the year. So, supply and demand patterns of agricultural commodities differ, which leads to see different price patterns for different months of the year.[26] To monitor to seasonality pattern, historical average of wheat prices for each month are demonstrated for the period of 1960 to 2019 and the period of 1985 to 2019 in Figure 20. It is observed that wheat prices are tend to

peak in first and last quarters of the year whereas they are at the lowest level during summer months such as May, June and July just before harvest period. In order to fine-tune wheat price model, it is quite important to count on seasonality pattern and its effect on wheat prices correctly. Therefore, in the price model, seasonality is regarded carefully and weighted based on price pattern for the corresponding months.
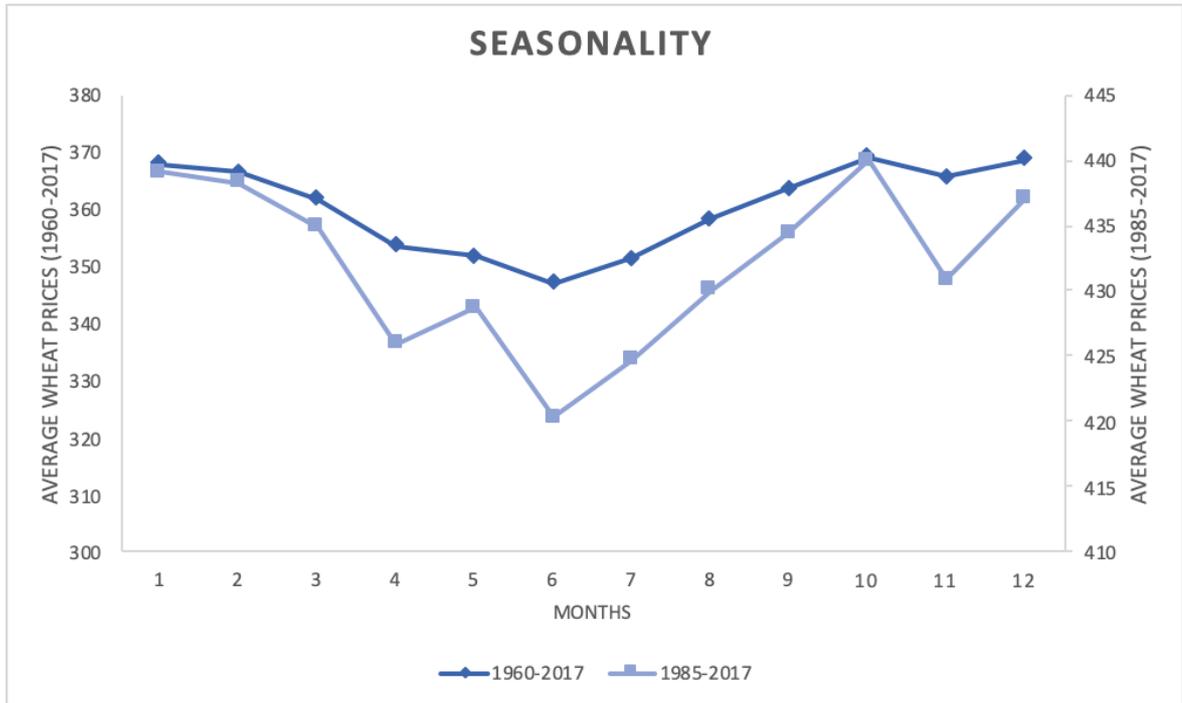
Figure 20. *Seasonality of CBOT Wheat Prices*

3.4.8. Business Cycle

The business cycle explains increase and decrease in production output of goods and services in an economy and it is measured based on increase or decrease in real gross domestic product (GDP) or GDP adjusted for inflation.[27] To understand the fair value of wheat or any commodity, it is crucial to reckon business cycle so the time period that we are in. In the study and model, wheat prices are examined from 1985 to 2019, which is a quite long period of time. In last thirty-four years, globalization has been taking important place, world economy has been expanding and GDP has been getting bigger. So that, it has been seen that too many things were going on in the global economy in this duration so wheat prices have been swinging as in Figure 21.
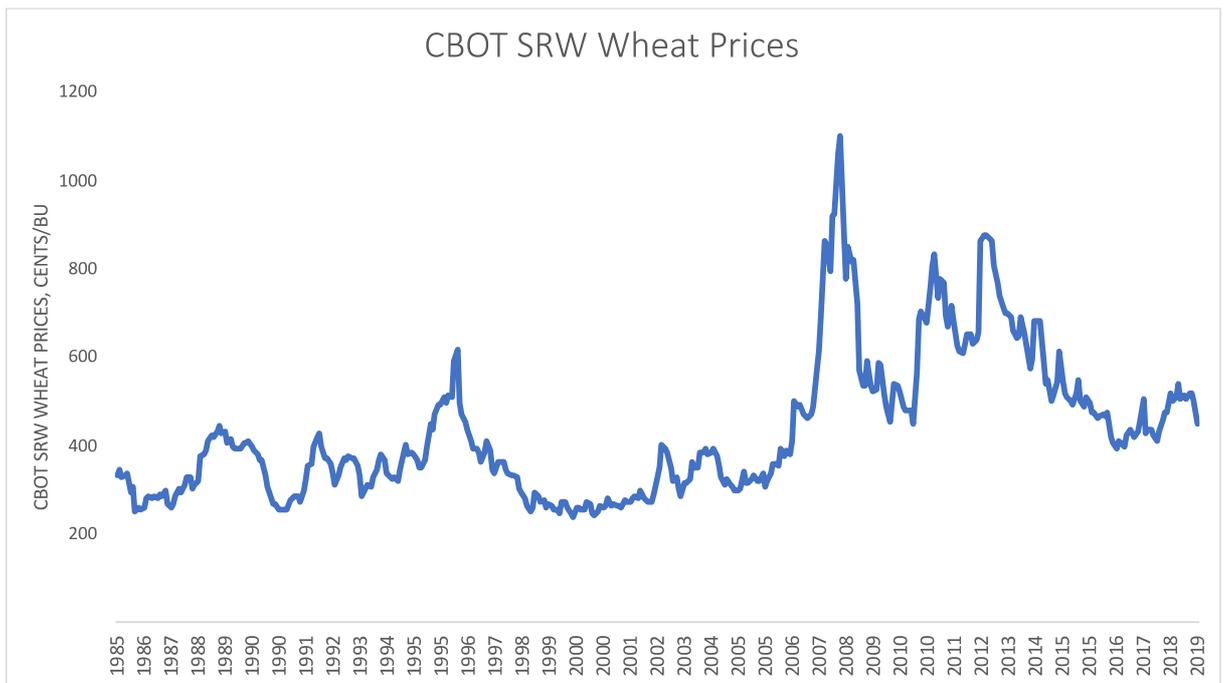
Figure 21. CBOT SRW Wheat Prices between 1985-2019

Global economy has disparate characteristic and pattern for different time-intervals. Therefore, thirty-four years period is divided into different time-intervals as 1985 to 1998, 1999 to 2006 and 2007 to 2019 based on their own aspects. Last time-interval, which is the most recent one, has several bullish events such as economy boom just before 2007-2008 financial crisis and Russia's export ban for wheat due to drought in 2010; hence, it has the highest average wheat price comparing with other time intervals. So that, in the price model, each time-interval is weighted based on their ranking according to average wheat price in corresponding time-interval. Besides, anomaly months which triggered big price spike such as wheat belt drought in May of 1996, first quarter of 2008 and export ban of Russia in August of 2010 are regarded and normalized in the model.

3.4.9. Political Risks & Uncertainties

Political risks and economical uncertainties are factors that influence not only commodity markets but also all any other markets on the earth such as equity market. However, it is very hard to foresee financial crisis and political risks as well as quantify of their exact effects on prices. In order to gauge their effects on wheat prices, dummy variable is introduced into the model. Basically, major events created bearish market environment for wheat prices are regarded in the model. Those events are namely Black Monday in 1987, US Savings and Loan Crisis between 1989 and 1991, Dot-com Bubble between 2000 and

33

2001, Global Financial Crisis between 2007 and 2008 and trade war between the USA and China which has been continuing since early 2018. Mutually, after all those events, there were bear market in the wheat market as well as in other commodity markets. For instance, in Global Financial Crisis or Black Monday, most of the markets collapsed very violently. Recent trade war occurring between the USA and China influences all grain and oilseeds markets and makes those market bearish. According to Jim Rogers, the 75-year-old chairman of Rogers Holding and veteran investor, trade war is making his bearish view even worse and pointing out his extreme concern about trade war.[28] Similarly, many investors, traders and economists are very anxious about potential consequences of trade war and it is believed that trade war might cause shrinkage in all global economy and create bearish environment in most of the markets particularly in agriculture. Basically, trade war between the USA and China are still on and it has already led wheat market down as in Figure 21 since it has started.
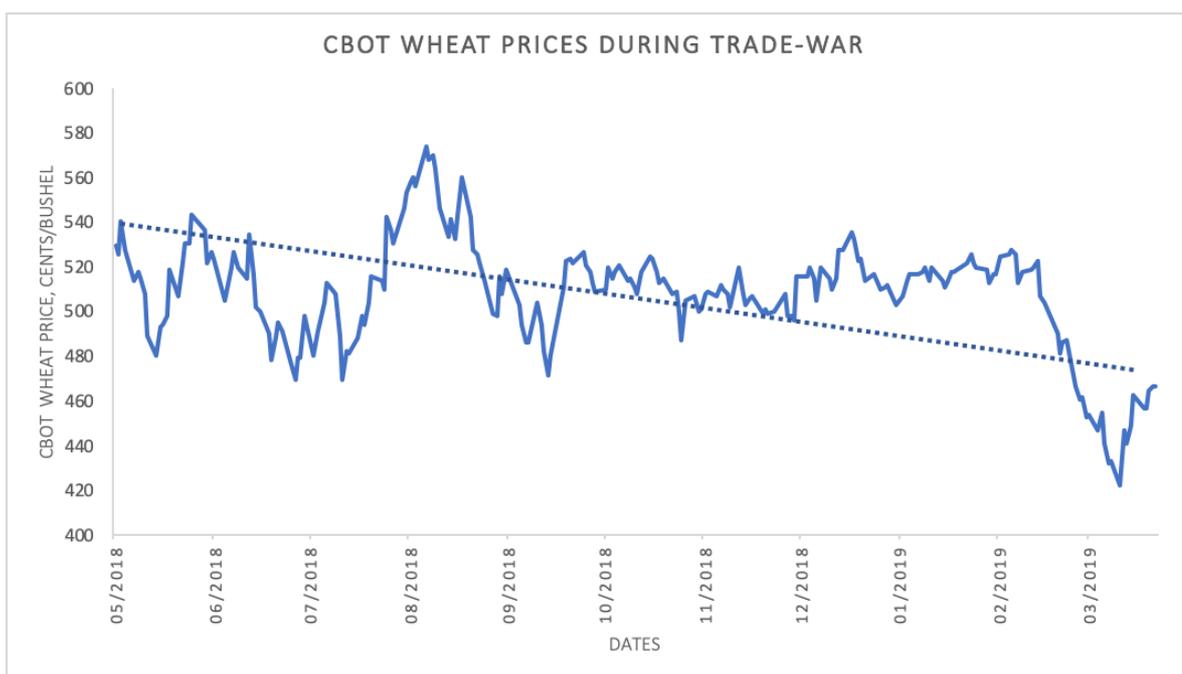
Figure 22. CBOT Wheat Prices During Trade War

To sum up, it can be claimed that political risks and uncertainties that includes financial crisis and trade wars are harmful for the global economy. They create bearish market environment and overrule the other fundamental or economic factors which plays significant role behind price fluctuation. Hence, this variable is believed as one of the major factors behind the story of wheat price fluctuation and added into wheat price model.

3.5. Model

It is argued in this section how multiple linear regression model performs as a whole, how independent variables contribute to the model, how model residuals are distributed and what the model limitations are.

3.5.1. Results and Discussions

Multiple linear regression model has been built to estimate monthly average CBOT wheat prices. The dependent variable, monthly average CBOT wheat price, has been explained in the model with eleven different independent variables. Of eleven independent variables, nine variables are considered major wheat price drivers and investigated for the relationship with wheat prices. In previous sections, it is detected that most of the factors have robust singular correlation with wheat prices. However, the question in the mind is whether all those independent variables together are significant enough to explain wheat price movement in the multiple regression model. Therefore, the model has been run for 401 observations covering time period from 1985 to 2019.

Basically, one of the very first things looked in the result is of course R-square, explaining how closely or well data is fitted in regression line.[29] The model gives us very impressive R-square number, 94.84 %, as in Table 3. Basically, this indicates that the model explains 94.84% of all variability of wheat prices around its mean. R-square is very popular measure to interpret the regression result; however, there are other worthwhile indicators needed to evaluate such as standard error. Standard error is a statistical measure showing accuracy of the sample. It shows deviation of sample mean from actual mean.[30] The standard error of the model is very promising. So that, the model accomplishes to forecast monthly average wheat price with around 37 cents per bushel standard error.

Regression Statistics	
Multiple R	97.39%
R Square	94.84%
Adjusted R Square	94.70%
Standard Error	37.08
Observations	401

Table 3. The Model Statistics

Another important thing for the evaluation of the model result is to have a look whether examine each independent variable is significant or not. To do so, it is necessary to look p-values in Table 4. For the 95 % confidence level, each variable's p-value needs to be less than 5 % to be significant. It is seen that all independent variables have p-value much lower than 5 %. Hence, each variable is pretty significant. This demonstrates that all independent variables tested in the model contribute model result significantly and work with a harmony. This proves again that each variable is significantly correlated with wheat prices in the multiple regression model.

Independant Variables	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
Intercept	2.549655	33.485718	0.076142	0.939346	-63.285981	68.385291
US Stock-to-Use Ratio	-191.377381	17.387015	-11.006914	0.000000	-225.561662	-157.193101
World Stock-to-Use Ratio	-232.296575	81.304356	-2.857123	0.004505	-392.147530	-72.445619
Oil Price	1.805767	0.181689	9.938758	0.000000	1.448551	2.162983
Urea Price	0.388773	0.039214	9.914099	0.000000	0.311675	0.465871
Dollar Index	2.344712	0.318736	7.356270	0.000000	1.718050	2.971373
COT MM Net Positions	0.002924	0.000151	19.351137	0.000000	0.002627	0.003221
Categorical Variable	-27.538859	9.634769	-2.858279	0.004489	-46.481596	-8.596122
Seasonality	2.659613	0.542100	4.906125	0.000001	1.593799	3.725426
Political Risks & Uncertainty	-40.984159	4.376479	-9.364642	0.000000	-49.588672	-32.379646
Business Cycle	109.296046	4.107351	26.609861	0.000000	101.220660	117.371431
Dummy Variable	-78.679068	4.062222	-19.368480	0.000000	-86.665725	-70.692410

Table 4. Independent Variables Statistics

There is also another way of looking for significance of independent variables is to check t statistics. Before looking into t statistics, first degree of freedom should be calculated. Degrees of freedom are the number possessing the freedom to vary.[31] The calculation of it is quite simple and it actually equals to sample size of N minus 1. So that, degrees of freedom are 400-1, which is 400. For the 95% confidence level and 400 degrees of freedom, the critical value of t statistics is 1.966. This means that any independent variable's t statistics should not be between -1.966 and 1.966 in order to be significant variable. Therefore, it can be concluded one more time that all independent variables are quite significant.

Besides, by looking at t statistics, it can be evaluated which variables are more significant than others as well. When regarding to nine price driver factors excluding categorical and dummy variables, it can be noticed that business cycle and COT managed money net positions have the biggest t stat numbers in Table 4, which implies that they are the most

significant variables in the model. It makes sense to have pretty significant business cycle variable since fair value of wheat or any commodity cannot be considered separately from the time period that corresponding commodity is traded in and from important economic events happening in that time period. Moreover, speculative positions variable is pretty significant one. It is lucid as this variable shows behavior of the market participants according to price actions. Another interesting thing to look at here that t stat of two fundamental variables, which are US stock-to-use ratio and world stock-to-use ratio. Basically, the model result suggests that fundamental factors in the United States is more influential on price actions than those in the world. Although CBOT wheat future market is representative market globally for all countries in the world, certainly it is firstly affected by the fundamental factors in the United States then later those in all around the world. It is quite obvious since CBOT future market places in Chicago in the United States.
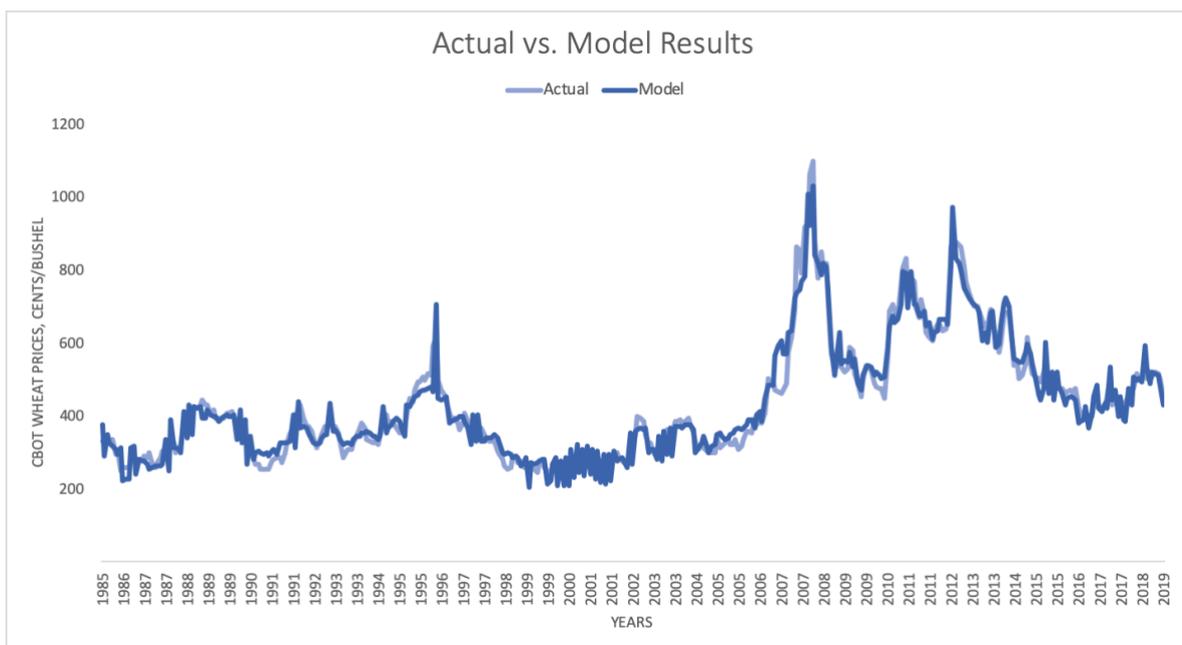
Figure 23. Monthly Average CBOT Wheat Prices Actual vs. Model Output

It is demonstrated how model results perform comparing with actual results in Figure 22. Essentially, the model fits quite well and captures most of the drop and peak of the prices. From starting of the model in 1985 to 2019, there is very smooth model pattern which is very in-line with actual data. It can be interpreted that the model is directionally working very successfully.

Nonetheless, by all means, there are some big model misses for some specific months during exceptional time period such as Global Financial Crisis in 2007 and 2008 or drought in 1996 and 2012 as it is shown in Figure 22. All these big model misses are very extreme and unforeseen events to capture in the model although there exist business cycle and political risks & uncertainty variables in the model, which are helpful to factor in bullish or bearish market environment in the model. When regarding the model's standard error of 37 cents, all these big misses can be tolerated.
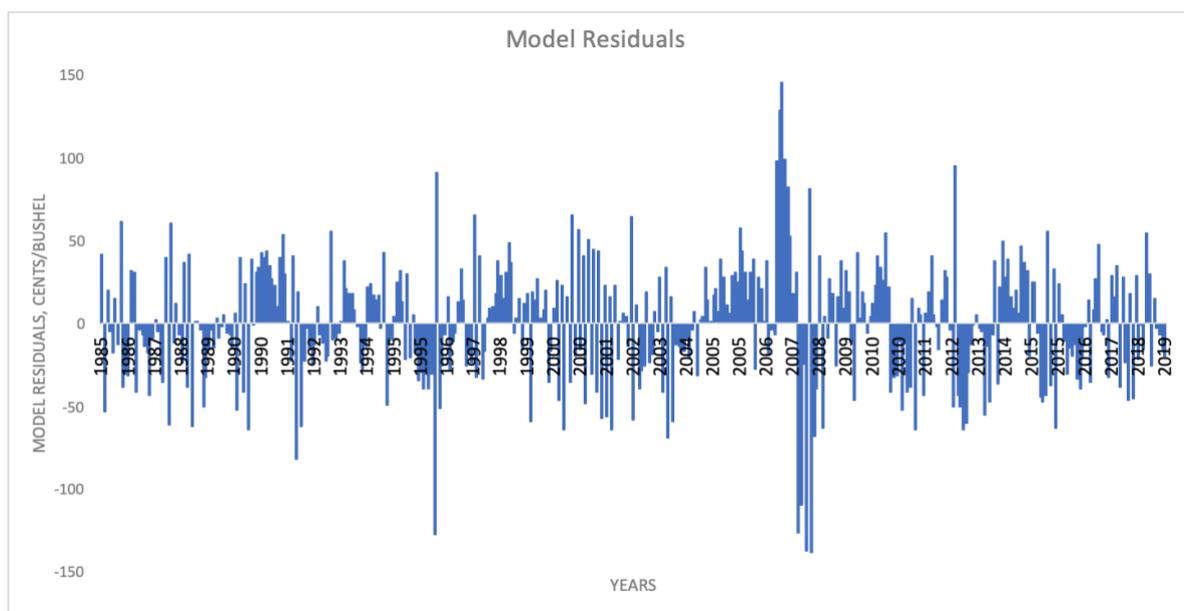
Figure 24. Model Residuals

Overall, the model performs quite robust with 94.8 % R-square and only 37 cents per bushel standard error. All independent variables namely, fundamental factors, production costs variables, macroeconomics indicators and speculative positions operate in the model with a harmony. All those variables are significant enough to contribute to predict wheat prices and each of them has significant correlation with wheat prices.

3.5.1. Limitations

Although the model results are quite healthy and the model is very applicable for practical use, it does not operate without any limits or possible errors. Hence, in this section, the model's limitations are listed.

- Multiple linear regression model is assuming linearity. Therefore, non-linear relationship between dependent and independent variables are missing.
- The study uses published statistics from resources such as USDA, Thomson Reuters. Hence, the study is dependent on those databases and assume that there is not any error or missing data.
- Although model result is pretty robust with very significant R square, there might be some omitted variables not be considered in the model. This might lead model to come up with biased result.
- CFTC was not reporting COT reports before 2006. So that, data of COT managed money net positions is missing before 2006. In order to plug in this variable into the model, categorical dummy variable is introduced together with COT managed money net positions variable.
- Market expectation of stock-to-use ratio throughout the year is missing in the model since there is not historical data of market expectation. So that, only materialized actual stock-to-use ratios are regarded but the market expectation.
- Multiple linear regression model regards historical data of independent variables. Hence, the forecast occurs according to dataset used in the model. However, if there is an unforeseen case which might not place in the model or a new data coming which has never been seen before, the model would fail to capture this surprising event.
- It is intended to count on political risks and uncertainties in the model. However, it could be subjective to interpret whether those events are bearish, bullish or neutral.
- Seasonality pattern is assumed as same for all years between 1985 and 2019. So that, year on year deviation or change in the seasonality such as early or late harvest is not captured.

3.5.2. Future Outlook

Based on the findings and main factors discussed previously, it is targeted to debate about potential predictions of wheat prices in the short run, which covers new crop year from July 2019 to June 2020.

For the time being, in all around the world, wheat crop conditions seem very positive for new crop in July 2019. In Black Sea region, Russia and Ukraine would have perfect conditions for wheat and farmers or producers there have started to expect record high production level for 2019/2020 marketing year. In EU, it is likely to have year on year increase. So that, globally it is expected to have big wheat supply for 2019/2020 marketing year from July to June, which would lead to depress wheat prices. Hence, fundamental factors point out bearish wheat prices.

Secondly, macroeconomic factors are also in more bearish side with expected strong dollar index and continuing trade war between the US and China. Basically, US economy goes well with very low unemployment rate and strong equity market, which would keep dollar index strong in the short run. Also, it looks like that trade war between the US and China is not likely to end any time soon so for the new crop year 2019/2020, it would be logical to expect that it will continue. Even, it is probable to see increase in magnitude of trade war between the US and China and potential new trade wars between Trump administration and other countries.

Thirdly, production costs factors: oil prices and fertilizer (urea) prices are harder to predict for near future since they are driven by many political news. So that, it would be wise to foresee political tensions or events for forthcoming months in order to sense oil prices. It would not be surprised to see soaring political tension between the US and Iran and particularly in Middle East, which would be directly influential on crude oil prices. Therefore, regarding to potential political tension in Middle East which is likely to pop up from Trump administration, it can be expected to have higher crude oil prices so higher production cost inputs. This would cause more bullish impact on wheat prices than bearish impact.

Next one is to interpret wheat prices according to what is going on in speculative positions in near future. It is the hardest one to forecast since managed money positions are tend to alter very quickly and powerfully based on price action. So that, it can be commented here that speculative positions reflect the market expectation on prices regarding more fundamental factors such as expected global big supply and political risks such as trade war. Since fundamental and political factors reveal that it is likely see bearish market environment for 2019/2020, one can foresee that managed money positions would go to more short positions and it would be likely to hit new extreme short managed money net positions in 2019/20, which would weaken wheat prices.

All in all, it can be summarized that most of the factors regarded in the paper roughly lean to imply bearish wheat prices in new crop year of 2019/2020 for the time being despite of having lots of uncertainties to foresee factors precisely.

4.Conclusion

The objective of the study is to first pinpoint the major factors triggering wheat price change, to point out their real effects on price movement and to come up with a predictive wheat price model in order to aid ones who desire forecast wheat prices and comprehend the price directions. In the course of literature review, it is noticed that there exist several potential factors that look like having an impact on wheat price, which are explained in existing literature. Yet, it is seen that existing literature findings mostly failed to demonstrate collective effects of all potential factors on wheat prices mathematically and practically. Therefore, it is targeted to come up with new applicable and businesslike approach to show impacts of potential factors in a statistical model.

First of all, four main factors, namely fundamental factors, macroeconomic indicators, production costs and speculative positions are detected as the potential price drivers in the literature review. After that, each factor is checked mathematically for the possible relationship with wheat prices through Pearson correlation coefficient test. Regarding fundamental factors, ending stocks and stock-to-use ratio are investigated comprehensively as the representatives of supply and demand and concluded that ending stocks and stock-to-use ratio have very significant negative correlation with wheat prices. Obviously, this shows that less supply means more expensive prices or vice a versa. In macroeconomic indicators, it is confirmed that dollar index is significantly negatively correlated with wheat prices statistically, implying that strong US dollar is prone to depress wheat prices or vice a versa. Besides that, it is observed qualitatively that political risks and uncertainties are likely to affect wheat prices and for instance, current trade war between the USA and China is tend to make wheat market bearish. In production costs factors, it is found out that oil prices and fertilizer prices have quite significant positive impact on wheat prices statistically. This reveals that higher production input costs leads to have more expensive wheat prices or vice a versa. Under speculative positions, it is seen that there is quite robust positive relationship between managed money net positions and wheat prices, which concludes that longer managed money net positions indicate higher wheat prices or vice a versa.

In the next step, major factors are taken to test how they operate together in multiple linear regression model to predict wheat prices. Basically, the model has been built for the time period between 1985 and 2019 with eleven independent variables to explain monthly average wheat prices. Of eleven variables, whereas nine variables are considered major price drivers, two variables are supportive variables in the model. Those nine price driver variables are namely, stock-to-use ratios in the US and the world, dollar index, political risks and uncertainties, oil and urea prices, managed money net positions, seasonality and business cycle. It is seen that all variables are quite significant in the model to contribute the price prediction. Besides, it is accomplished that all those major factors operate in the model successfully with conformity, which result in 94.8 % R square and 37 cents per bushel standard error. So, this approach, the multiple linear regression model facilitates us to gauge mathematical impact of possible change in major factors on wheat prices and to predict wheat prices based on factors plugged in the model with quite accurate results.

All in all, the study discloses the major price factors taking place behind the wheat price movement by providing corresponding relationship mathematically and presents applicable approach to predict wheat prices with the help of successfully working multiple linear regression model.

5. List of References

[1] Morrison, K. (2016, May 22). Ever higher': Why commodity bulls are upbeat. Financial Times.

[2] Janzen, J., Carter, C., Smith, A. and Adjemian, M. (2014). Deconstructing Wheat Price Spikes: A Model of Supply and Demand, Financial Speculation, and Commodity Price Comovement. SSRN Electronic Journal.

[3] Institute of Food Technologists. (2014). Agricultural technologies could increase global crop yields up to 67%. [online] Available at: http://www.ift.org/food-technology/daily-news/2014/february/12/agricultural-technologies-could-increase-global-crop-yields-up-to-67.aspx [Accessed 17 May 2019].

[4] IMF. (2019). IMF Survey: Wheat Price Jump a Supply Shock That Should Unwind. [online] Available at: https://www.imf.org/en/News/Articles/2015/09/28/04/53/sonew090110a [Accessed 11 Mar. 2019].

[5] U.S. (2019). GRAINS-Wheat faces biggest weekly slide in 6 mths, abundant supply.... [online] Available at: https://www.reuters.com/article/global-grains/grains-wheat-faces-biggest-weekly-slide-in-6-mths-abundant-supply-drags-idINL3N20O1RO [Accessed 11 Mar. 2019].

[6] Frankel, Jeffrey A., and Andrew K. Rose. 2010 Determinants of Agricultural and Mineral Commodity Prices. HKS Faculty Research Working Paper Series RWP10-038, John F. Kennedy School of Government, Harvard University.

[7] Delle Chiaie, S., Ferrara, L. and Giannone, D. (2017). Common Factors of Commodity Prices. SSRN Electronic Journal.

[8] Hoover, M. and Kenyon, D. (1994). Wheat pricing guide. Blacksburg: Virginia Cooperative Extension.

[9] Childs, N. and Kiawu, J. (2009). Factors Behind the Rise in Global Rice Prices in 2008. [online] ers.usda.gov. Available at: https://www.ers.usda.gov/webdocs/publications/38489/13518_rcs09d01_1_.pdf?v=0 [Accessed 18 May 2019].

[10] De Palmas, C. (2019). Understanding the correlation between USD and Commodities - ForexBoat Trading Academy. [online] ForexBoat Trading Academy. Available at: https://www.forexboat.com/understanding-the-correlation-between-usd-and-commodities/ [Accessed 11 Mar. 2019].

[11] The Balance. (2019). How Impact of the Dollar on Commodity Prices. [online] Available at: https://www.thebalance.com/how-the-dollar-impacts-commodity-prices-809294 [Accessed 11 Mar. 2019].

[12] Baker, A. and Brenner, P. (2019). What effect will the dollar strength have on commodities? - Private Investor - Schroders. [online] Can commodities survive a stronger US dollar?. Available at: https://www.schroders.com/en/uk/private-investor/insights/economics/can-commodities-survive-a-stronger-us-dollar/ [Accessed 11 Mar. 2019].

[13] Hou, K., Tang, K. and Zhang, B. (2017). Political Uncertainty and Commodity Prices. SSRN Electronic Journal.

[14] De Paoli, L. (2018). Trade War Sends Commodities to Biggest Drop in 5 Months. [online] Bloomberg. Available at: https://www.bloomberg.com/news/articles/2018-07-13/commodities-smacked-by-trade-war-with-biggest-drop-in-5-months [Accessed 22 Apr. 2019].

[15] Ott, H. (2012). Fertilizer markets and their interplay with commodity and food prices. Luxembourg: Publications Office.

[16] Amadeo, K. (2019). Crude Oil Impacts Everything You Buy. [online] The Balance. Available at: https://www.thebalance.com/crude-oil-prices-trends-and-impact-on-the-economy-and-you-3305738 [Accessed 22 Apr. 2019].

[17] Baumeister, C. and Kilian, L. (2013). Do oil price increases cause higher food prices?.

[18] Çinar, G., Hushmat, A. and Uzmay, A. (2015). Does Speculation Matters for Wheat Price Shocks?. Theoretical Economics Letters, 05(04), pp.522-530.

[19] Rinden, S. (2015). The impact of speculation in commodity futures on food prices.

[20] Pirrong, S. (2012). Commodity Price Dynamics.

[21] Learntech.uwe.ac.uk. (2019). Data Analysis - Pearson's Correlation Coefficient. [online] Available at: http://learntech.uwe.ac.uk/da/default.aspx?pageid=1442 [Accessed 22 Apr. 2019].

[22] Kilian, L. (2019). WTO | Publications - Oil Price Volatility: Origins and Effects. [online] Wto.org. Available at: https://www.wto.org/english/res_e/publications_e/wtr10_forum_e/wtr10_kilian_e.htm [Accessed 22 Apr. 2019].

[23] Montgomery, D. C., Peck, E. A. and Vining, G. G. (2001). Introduction to Linear Regression Analysis. 3rd Edition, New York, New York: John Wiley & Sons

[24] Daugherty, K., and Kelly, K., (2014). Stocks and the Stocks-to-Use Ratio: Are They Meaningful for Price Determination? GROWMARK Research (2014). pp. 1-32. Available at: http://www.growmark.com/sites/Files/Documents/StocksToUse.pdf [Accessed 18 May 2019].

[25] *Cftc.gov. (2019). Disaggregated Explanatory Notes | U.S. COMMODITY FUTURES TRADING COMMISSION. [online] Available at: https://www.cftc.gov/MarketReports/CommitmentsofTraders/DisaggregatedExplanatory Notes/index.htm [Accessed 11 May 2019].*

[26] Cmegroup.com. (2019). Understanding Seasonality in Grains - CME Group. [online] Available at: https://www.cmegroup.com/education/courses/introduction-to-grains-and-oilseeds/understanding-seasonality-in-grains.html [Accessed 3 May 2019].

[27] Investopedia. (2019). Business Cycle. [online] Available at: https://www.investopedia.com/terms/b/businesscycle.asp [Accessed 3 May 2019].

[28] Bloomberg. (2018). Jim Rogers Says Trade War Is Making His Bearish View Even Darker. [online] Available at: https://www.bloomberg.com/news/articles/2018-03-21/jim-rogers-says-trade-war-is-making-his-bearish-view-even-darker [Accessed 12 Apr. 2019].

[29] Corporate Finance Institute. (2019). R-Squared - Definition, Interpretation, and How to Calculate. [online] Available at: https://corporatefinanceinstitute.com/resources/knowledge/other/r-squared/ [Accessed 24 May 2019].

[30] Kenton, W. (2019). How Standard Errors Work. [online] Investopedia. Available at: https://www.investopedia.com/terms/s/standard-error.asp [Accessed 24 May 2019].

[31] Kenton, W. (2019). Degrees of Freedom. [online] Investopedia. Available at: https://www.investopedia.com/terms/d/degrees-of-freedom.asp [Accessed 24 May 2019].

6. Appendix

Critical Values of t

t Distribution: Critical Values of t

Significance level

Degrees of freedom	Two-tailed test: One-tailed test:	10% 5%	5% 2.5%	2% 1%	1% 0.5%	0.2% 0.1%	0.1% 0.05%
1		6.314	12.706	31.821	63.657	318.309	636.619
2		2.920	4.303	6.965	9.925	22.327	31.599
3		2.353	3.182	4.541	5.841	10.215	12.924
4		2.132	2.776	3.747	4.604	7.173	8.610
5		2.015	2.571	3.365	4.032	5.893	6.869
6		1.943	2.447	3.143	3.707	5.208	5.959
7		1.894	2.365	2.998	3.499	4.785	5.408
8		1.860	2.306	2.896	3.355	4.501	5.041
9		1.833	2.262	2.821	3.250	4.297	4.781
10		1.812	2.228	2.764	3.169	4.144	4.587
11		1.796	2.201	2.718	3.106	4.025	4.437
12		1.782	2.179	2.681	3.055	3.930	4.318
13		1.771	2.160	2.650	3.012	3.852	4.221
14		1.761	2.145	2.624	2.977	3.787	4.140
15		1.753	2.131	2.602	2.947	3.733	4.073
16		1.746	2.120	2.583	2.921	3.686	4.015
17		1.740	2.110	2.567	2.898	3.646	3.965
18		1.734	2.101	2.552	2.878	3.610	3.922
19		1.729	2.093	2.539	2.861	3.579	3.883
20		1.725	2.086	2.528	2.845	3.552	3.850
21		1.721	2.080	2.518	2.831	3.527	3.819
22		1.717	2.074	2.508	2.819	3.505	3.792
23		1.714	2.069	2.500	2.807	3.485	3.768
24		1.711	2.064	2.492	2.797	3.467	3.745
25		1.708	2.060	2.485	2.787	3.450	3.725
26		1.706	2.056	2.479	2.779	3.435	3.707
27		1.703	2.052	2.473	2.771	3.421	3.690
28		1.701	2.048	2.467	2.763	3.408	3.674
29		1.699	2.045	2.462	2.756	3.396	3.659
30		1.697	2.042	2.457	2.750	3.385	3.646
32		1.694	2.037	2.449	2.738	3.365	3.622
34		1.691	2.032	2.441	2.728	3.348	3.601
36		1.688	2.028	2.434	2.719	3.333	3.582
38		1.686	2.024	2.429	2.712	3.319	3.566
40		1.684	2.021	2.423	2.704	3.307	3.551
42		1.682	2.018	2.418	2.698	3.296	3.538
44		1.680	2.015	2.414	2.692	3.286	3.526
46		1.679	2.013	2.410	2.687	3.277	3.515
48		1.677	2.011	2.407	2.682	3.269	3.505
50		1.676	2.009	2.403	2.678	3.261	3.496
60		1.671	2.000	2.390	2.660	3.232	3.460
70		1.667	1.994	2.381	2.648	3.211	3.435
80		1.664	1.990	2.374	2.639	3.195	3.416
90		1.662	1.987	2.368	2.632	3.183	3.402
100		1.660	1.984	2.364	2.626	3.174	3.390
120		1.658	1.980	2.358	2.617	3.160	3.373
150		1.655	1.976	2.351	2.609	3.145	3.357
200		1.653	1.972	2.345	2.601	3.131	3.340
300		1.650	1.968	2.339	2.592	3.118	3.323
400		1.649	1.966	2.336	2.588	3.111	3.315
500		1.648	1.965	2.334	2.586	3.107	3.310
600		1.647	1.964	2.333	2.584	3.104	3.307
∞		1.645	1.960	2.326	2.576	3.090	3.291