# Existence of *d*-wave and *s*-wave solutions of Eliashberg equations

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# Existence of *d*-wave and *s*-wave solutions of Eliashberg equations

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We solve the Eliashberg equations for different models of electron-phonon coupling. In particular we study a coupling which is different on different parts of the Fermi surface in order to look for possible conditions for *d*-wave pairing. This is of some importance since many recent experiments claim *d*-wave pairing in YBCO. We find that a very localized interaction (i.e. a strong coupling within certain Fermi surface pieces, but weak coupling between different pieces) together with a Coulomb repulsion is favorable for *d*-wave pairing. The connection between the models and the calculated coupling and Fermi surface of real copper oxide systems is discussed.

There is experimental evidence for *d*-wave pairing symmetry in high-$T_c$ cuprates [1], and many other features are not BCS-like in these compounds. It is usually considered that a phonon-mediated interaction can only lead to *s*-wave symmetry of the gap parameter [2]. Nevertheless, in this paper, we show that some of the "non-BCS" features, as *d*-wave pairing symmetry, can be obtained from Eliashberg equations by choosing suitable parameters for the electron-phonon coupling and the Coulomb repulsion.

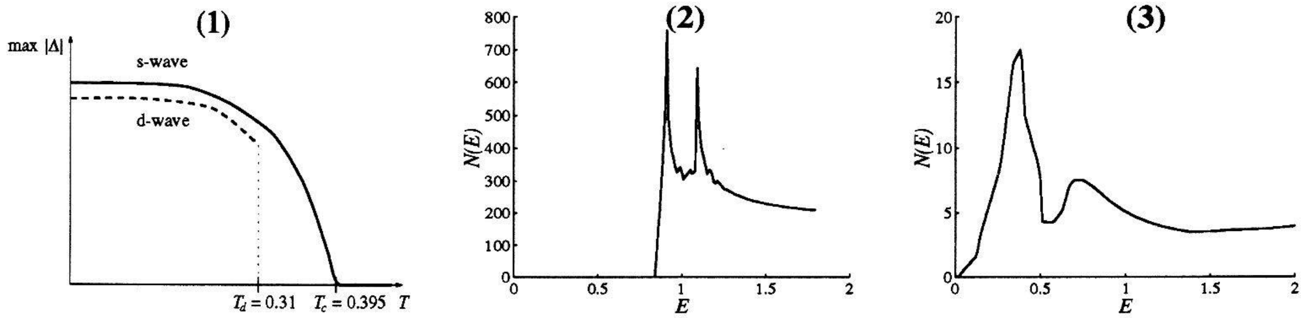
We solve self-consistently the "renormalized" Eliashberg equations. This is a set of 3 coupled non-linear equations [3]. The equation for the gap parameter $\Phi$ (corresponding to the BCS gap equation) can be written, in the Matsubara representation ($\omega_n = (2n+1)\pi T$)

$$\Phi(\mathbf{k}, i\omega_n) = T \sum_{n'} \iint d^2\mathbf{k}' G(\mathbf{k}', i\omega_{n'})[D(\mathbf{k}, \mathbf{k}', i\omega_n - i\omega_{n'}) - \mu(\mathbf{k}, \mathbf{k}')]\Phi(\mathbf{k}', i\omega_{n'}) \tag{1}$$

where $G$ is the renormalized electronic Green function, $D$ is the phononic interaction kernel and $\mu$ represents the Coulomb potential.

Calculations and measurements describe the Fermi surface (FS) as 2-dimensional in the cuprates. Here we separate the momentum $\mathbf{k} = (q, k)$ in parallel ($q$) and perpendicular ($k$) components with respect to the FS. We divide the FS in several pieces ($q$), and we set the coupling parameters in such a way that the interaction is highly attractive (due to $D$) for electrons belonging to the same piece, and weakly repulsive (due to $\mu$) between most remote pieces. This kind of interaction should be favorable to *d*-wave pairing. In previous works, we considered a strong $k$-dependence of the interaction, in particular a cutoff in $|k - k'|$ (i.e. the coupling falls to zero if $|k - k'| > \xi$), and we showed that this could explain some deviations from the BCS behaviour as the diminution of the isotopic effect and the decrease of the Hebel-Slichter peak [3].

The present calculations were done with and without the cutoff. We find that a *d*-wave solution is possible in both cases. The *d*-wave solution is obtained for a coupling which is strong only when $q$ and $q'$ are close to each other. However, the initial condition of the iterative procedure is important. Starting with *s*-like gap symmetry with the same coupling




**(1)** Temperature dependence of the maximum of the gap $\Delta$ (being the renormalized $\Phi$). **(2)** Density of states (DOS) for a case without cutoff, 12 FS pieces and $s$-wave solution. **(3)** DOS for a case with cutoff, 3 FS pieces and $d$-wave solution. (All energies are in units of the typical phonon frequency, and the DOS is calculated through an approximation valid for low energies).

parameters gives an $s$-solution. Below the temperature $T_d$, both solutions can exist, whereas above $T_d$, only the $s$-wave solution exists (Fig.1). Free energies are not calculated, but the $d$-wave might be metastable for temperatures lower than $T_d$, since the gap is smaller for the $d$-wave than for the $s$-wave. If the parameters are changed to a strong coupling between different $q$-pieces, the gap converges always to the $s$-wave, even if the initial condition has $d$-wave symmetry.

Recent experimental works on cuprates by photoemission and tunneling [5] show a second attenuated peak in the density of states. A similar behavior is found from the anisotropic $s$-wave solution of Eliashberg equation (Fig. 2) where 1/6 of the FS pieces have strong and the rest have weak coupling. The sharpness of the peaks is due to our neglect of lifetime effects. A different calculation with a cutoff gives a similar, second peak (Fig. 3). In the first case, the 2 peaks are due to the fact that there are mainly 2 different values of the gap, whereas in the second, they are due to the $k$-dependence of the energy of the quasiparticles created by the cutoff. In this case, the so-called "in-gap" states come from the $d$-wave symmetry.

The Eliashberg equations have a richness of solution that has still to be explored. We have shown here that some aspects of high-$T_c$ can be accounted for with this theory. It is then premature to eliminate phonon-based mechanism to explain high-$T_c$ superconductivity.

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