Study of mems-based inertial sensors operating in dynamic conditions

Stebler, Yannick Sébastien; Guerrier, Stéphane; Skaloud, Jan; Molinari, Roberto Carlo; Victoria-Feser, Maria-Pia

How to cite

STEBLER, Yannick Sébastien et al. Study of mems-based inertial sensors operating in dynamic conditions. In: Position, Location and Navigation Symposium - PLANS 2014. Monterey (CA, USA). [s.l.] : IEEE, 2014. doi: 10.1109/PLANS.2014.6851497

This publication URL: https://archive-ouverte.unige.ch/unige:95183
Publication DOI: 10.1109/PLANS.2014.6851497

Study of MEMS-based Inertial Sensors Operating in Dynamic Conditions

Yannick Stebler, Stéphane Guerrier, Jan Skaloud, Roberto Molinari, Maria-Pia Victoria-Feser

***Abstract*—This paper aims at studying the behaviour of the errors coming from inertial sensors when measured in dynamic conditions. After proposing a method for constructing the error process, the properties of these errors are estimated via the Generalized Method of Wavelets Moments methodology. The developed model parameters are compared to those obtained under static conditions. Finally an attempted is presented to find the link between the encountered dynamic of the vehicle and error-model parameters.**

***Keywords*—Estimation, Errors, IMU, Modelling, Integration.**

I. Introduction

The integration of a strapdown *Inertial Navigation System* (INS) with satellite-based systems (GNSS) using Bayesian techniques, usually the *Extended Kalman Filter* (EKF), is a standard approach to reliably estimate the navigation states of a vehicle at any time (i.e. position, velocity and attitude in space). The INS comprises an *Inertial Measurement Unit* (IMU) formed by a triad of usually orthogonally mounted gyroscopes and accelerometers observing angular rate or change, and specific force, respectively. After initialization, these signals are integrated with respect to time to yield attitude, velocity and finally position. During periods of poor GNSS signal quality or when no GNSS solution can be computed by the receiver, inertial navigation operates in coasting mode, i.e. the navigation states are estimated independently from the satellite data. In such cases, the overall navigation performance becomes strongly dependent on the errors corrupting inertial signals. These errors are integrated in the INS making their impact grow with time. Correct error modelling and estimation of their systematic part is thus essential for enhancing and correctly predicting its quality.

Inertial sensors are corrupted by errors (scale factors, biases) of deterministic and stochastic nature. For the latter errors, deterministic models do not apply and stochastic processes must be considered and which are assessed and estimated, in the context of this paper, by the Generalized Method of Wavelet Moments (GMWM) recently introduced in [1]. However, it is generally assumed that the error process is independent of the dynamics of the platforms on which the inertial sensors are positioned. To the authors' knowledge, not much work has been developed concerning dynamic-dependent error analysis in navigation, aside some preliminary work in [2] in which some dynamics dependent error behaviour affecting a tactical-grade and low-cost IMU was highlighted. The aim of this paper is to develop a technique for observing the errors coming from the low-cost MEMS inertial sensors in dynamic conditions and, secondly, estimate their characteristics (i.e. model parameters) and study the relation between error processes and the dynamics of the platform.

Yannick Stebler is a R&D Engineer at u-blox AG, Thalwil, Switzerland (E-mail: yannick.stebler@gmail.com). **Stéphane Guerrier** is a Visiting Assistant Professor in the Department of Statistics & Applied Probability, University of California, Santa Barbara, CA, USA (E-mail: guerrier@pstat.ucsb.edu). **Jan Skaloud** is a Senior Scientist in the Geodetic Engineering Laboratory, Swiss Federal Institute of Technology Lausanne (EPFL), Switzerland (E-mail: jan.skaloud@epfl.ch). **Roberto Molinari** is a PhD student and **Maria-Pia Victoria-Feser** is a Professor in the Research Center for Statistics, University of Geneva, Geneva, Switzerland (E-mail: Roberto.Molinari@unige.ch; Maria-Pia.VictoriaFeser@unige.ch).

II. Methodology and Experimental framework

A. Error construction

To investigate the impact of the dynamic conditions on the modelling of the error process it is of course necessary to construct the error process itself. To do so, a *Reference Inertial Measurement Unit* (R-IMU) was rigidly mounted on the same platform as the IMU under study (S-IMU). The data was collected from three functioning *XSens MTx* MEMS-based IMUs and the reference signals were issued from an *Ixsea Airins* navigation-grade INS logging at 100 Hz (noise < 0.0015 deg/$\sqrt{\text{Hr}}$, drift < 0.01 deg/Hr), combined with a geodetic grade *Javad Alpha* L1/L2 GPS rover receiver (sampling at 10 Hz), and a *Topcon Hiper Pro* L1/L2 GPS base receiver (sampling at 5 Hz), both used for computing a double-differenced carrier-phase GPS solutions. The reference data was post-processed through Kalman filtering yielding compensated inertial signals.

One of the main issues that is addressed within this study is the estimation of the spatial relations between the triads forming the IMUs. Consider the R-IMU and the S-IMU rigidly mounted on the same platform. Assume that the b-frame is equivalent to the R-IMU instrumental frame, and that the S-IMU provides observations in his instrumental s-frame. The b-frame and s-frame origins are separated by a vector $\mathbf{r}^b_{b\to s}$, called *inter-IMU leverarm*, and their relative orientation is expressed by the $\mathbf{C}^s_b$ direction cosine matrix, called *inter-IMU boresight*. The relationship between $\mathbf{C}^s_b$ and the estimated boresight, denoted as $\hat{\mathbf{C}}^s_b$, may be expressed in terms of misalignment errors as

$$\mathbf{C}^s_b = (\mathbf{I} + \boldsymbol{\Psi})\,\hat{\mathbf{C}}^s_b$$

where $\boldsymbol{\Psi} = \left[\boldsymbol{\psi}^b_{b\to s}\times\right]$ is a skew-symmetric matrix containing the misalignment error angles $\boldsymbol{\psi}^b_{b\to s} = \left[\psi_x \quad \psi_y \quad \psi_z\right]^T$ between the b-frame and the s-frame. The (3×1) observed S-IMU angular rate $\boldsymbol{\omega}^s_{is}$ and specific force $\mathbf{f}^s$ vectors must be corrected by boresight $\mathbf{C}^s_b$ and lever-arm $\mathbf{r}^b_{b\to s}$ effects. Both quantities can either be known *a priori* or estimated. Considering possible determination of $\mathbf{r}^b_{b\to s}$ by other means (e.g. calliper), the $\mathbf{C}^s_b$ needs to be estimated indirectly. For instance, an EKF-based estimation techniques can be considered with the EKF navigation states defined as

$$\mathbf{x} = \begin{bmatrix} \delta\mathbf{r}^l_e & \delta\mathbf{v}^l_e & \boldsymbol{\varepsilon}^l & \delta\mathbf{f}^b & \delta\boldsymbol{\omega}^b_{ib} & \delta\mathbf{x}_c \end{bmatrix}^T$$

with $\delta\mathbf{r}^l_e = [\delta\phi \quad \delta\lambda \quad \delta h]^T$ the errors in latitude ϕ, longitude λ and height h, $\delta\mathbf{v}^l_e = [\delta v_N \quad \delta v_E \quad \delta v_D]^T$ the errors in North, East and Down (NED) velocity components, and the misalignment angles $\boldsymbol{\varepsilon}^l = [\varepsilon_N \quad \varepsilon_E \quad \varepsilon_D]^T$ expressing the attitude errors with respect to the NED l-frame. The $\delta\mathbf{f}^b$ and $\delta\boldsymbol{\omega}^b_{ib}$ are vectors of size (3×1) each accounting for accelerometer and gyroscope biases, respectively. The $\delta\mathbf{x}_c$ vector contains inter-IMU calibration states that will be described in the next two sections.

The position error model is:

$$\delta\dot{\mathbf{r}}^l_e = -\boldsymbol{\omega}^l_{el}\times\mathbf{r}^l_e + \delta\boldsymbol{\theta}\times\mathbf{v}^l_e + \delta\mathbf{v}^l_e$$

with $\delta\boldsymbol{\theta}$ the misalignment vector of computer c-frame [3] with respect to l-frame as a consequence of position error:

$$\delta\boldsymbol{\theta} = \begin{bmatrix} \delta\lambda\cos\phi & -\delta\phi & -\delta\lambda\sin\phi \end{bmatrix}^T.$$

The velocity error model is:

$$\begin{aligned}\delta\dot{\mathbf{v}}^l_e = &-\mathbf{f}^l\times\boldsymbol{\varepsilon}^l - \left(\boldsymbol{\omega}^l_{ie}+\boldsymbol{\omega}^l_{il}\right)\times\delta\mathbf{v}^l_e \\ &-\left(\delta\boldsymbol{\omega}^l_{ie}+\delta\boldsymbol{\omega}^l_{il}\right)\times\mathbf{v}^l_e + \mathbf{C}^l_b\delta\mathbf{f}^b + \delta\mathbf{g}^l.\end{aligned}$$

The attitude error model is

$$\dot{\boldsymbol{\phi}}^l = -\boldsymbol{\omega}^l_{il}\times\boldsymbol{\varepsilon}^l + \delta\boldsymbol{\omega}^l_{il} - \mathbf{C}^l_b\delta\boldsymbol{\omega}^b_{ib}.$$

B. Bore-sight estimation via inertial measurement aiding

One possibility to estimate the components and parameters of these models (especially $\mathbf{C}^s_b$), is to feed the Kalman Filter of MEMS-IMU with attitude updates from the reference system [4]. If the S-IMU is of poor quality, as it is our case, the solution provided by the S-IMU/GNSS filter may be considerably affected by the imperfections of initialization stage. For this reason we suggest to work with the R-IMU/GNSS filter and use less precise $\boldsymbol{\omega}^s_{is}$ and $\mathbf{f}^s$ measurements from S-IMU to jointly estimate the $\mathbf{C}^s_b$ and $\mathbf{r}^b_{b\to s}$.

The relation between $\boldsymbol{\omega}^s_{is}$ and $\boldsymbol{\omega}^b_{ib}$ is given by

$$\boldsymbol{\omega}^s_{is} = \mathbf{C}^s_b\boldsymbol{\omega}^b_{ib} \tag{1}$$

which is true under the conditions that $\dot{\mathbf{C}}^s_b = 0$ and $\dot{\mathbf{r}}^b_{b\to s} = 0$. If $\boldsymbol{\Omega}^s_{is} = [\boldsymbol{\omega}^s_{is}\times]$ and $\boldsymbol{\Omega}^b_{ib} = [\boldsymbol{\omega}^b_{ib}\times]$, the relation between $\mathbf{f}^b$ and $\mathbf{f}^s$ can be written as [5]

$$\mathbf{f}^s = \mathbf{C}^s_b\left(\mathbf{f}^b + \dot{\boldsymbol{\Omega}}^b_{ib}\mathbf{r}^b_{b\to s} + \boldsymbol{\Omega}^b_{ib}\boldsymbol{\Omega}^b_{ib}\mathbf{r}^b_{b\to s}\right) \tag{2}$$

in which $\dot{\boldsymbol{\Omega}}^b_{ib}\mathbf{r}^b_{b\to s}$ and $\boldsymbol{\Omega}^b_{ib}\boldsymbol{\Omega}^b_{ib}\mathbf{r}^b_{b\to s}$ represent centrifugal and Coriolis forces, respectively.

Under the condition that ψ_x, ψ_y and ψ_z are small, we have $\mathbf{C}^s_b \approx \mathbf{I} - \boldsymbol{\Psi}$. By denoting $\boldsymbol{\omega}^s_{is}$ and $\mathbf{f}^s$ respectively as $\mathbf{z}_{\boldsymbol{\omega}}$ and $\mathbf{z}_{\mathbf{f}}$,

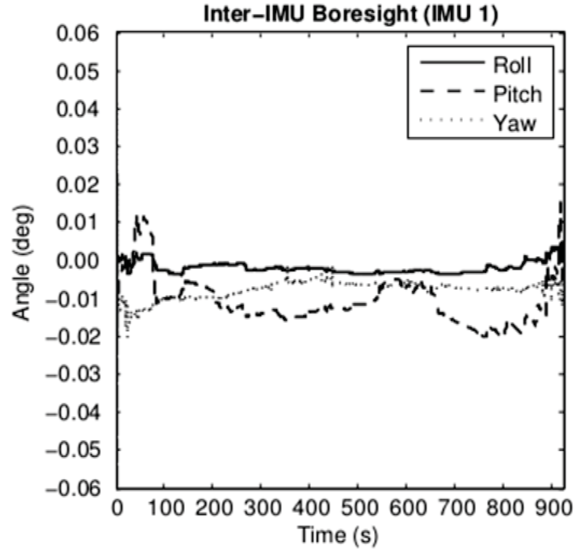

Fig. 1: Result of the inter-IMU boresight angle estimation for one *XSens MTx* IMU using inertial measurements from an *Ixsea Airins* navigation-grade IMU.

the linearized measurement models given in Eq. (1) and (2) are

$$\mathbf{z}_{\boldsymbol{\omega}} = h(\mathbf{x}_c) + \boldsymbol{\nu}_{\boldsymbol{\omega}} \approx \boldsymbol{\Omega}^b_{ib}\boldsymbol{\psi}^b_{b\to s} + \boldsymbol{\nu}_{\boldsymbol{\omega}} \tag{3}$$

and

$$\begin{aligned}\mathbf{z}_{\mathbf{f}} &= h(\mathbf{x}_c) + \boldsymbol{\nu}_{\mathbf{f}} \\ &\approx -\left([\mathbf{f}^b\times] + \boldsymbol{\Omega}^b_{ib}\boldsymbol{\Omega}^b_{ib}[\mathbf{r}^b_{b\to s}\times] + \dot{\boldsymbol{\Omega}}^b_{ib}[\mathbf{r}^b_{b\to s}\times]\right)\boldsymbol{\psi}^b_{b\to s} \\ &\quad + \mathbf{C}^s_b\left(\boldsymbol{\Omega}^b_{ib}\boldsymbol{\Omega}^b_{ib} + \dot{\boldsymbol{\Omega}}^b_{ib}\right)\delta\mathbf{r}^b_{b\to s} + \boldsymbol{\nu}_{\mathbf{f}}\end{aligned} \tag{4}$$

where

$$\mathbf{x}_c = \begin{bmatrix} \left(\boldsymbol{\psi}^b_{b\to s}\right)^T & \left(\mathbf{r}^b_{b\to s}\right)^T \end{bmatrix}^T$$

are the augmented calibration states. The design matrix $\mathbf{H}_{\mathbf{f}}$ for the whole state vector can be deduced from Eq. (3) and (4), yielding

$$\mathbf{H}_{\mathbf{f}} = \left[\begin{array}{c|c} \mathbf{0}_{3\times 15} & \mathbf{0}_{3\times 15} \\ \left(\boldsymbol{\Omega}^b_{ib}\right)^T & \left(-[\mathbf{f}^b\times] - \boldsymbol{\Omega}^b_{ib}\boldsymbol{\Omega}^b_{ib}[\mathbf{r}^b_{b\to s}\times] - \dot{\boldsymbol{\Omega}}^b_{ib}[\mathbf{r}^b_{b\to s}\times]\right)^T \\ \mathbf{0}_{3\times 3} & \left(\mathbf{C}^s_b\left(\boldsymbol{\Omega}^b_{ib}\boldsymbol{\Omega}^b_{ib} + \dot{\boldsymbol{\Omega}}^b_{ib}\right)\right)^T \end{array}\right]^T \tag{5}$$

Fig. 1 depicts the estimated smoothed inter-IMU angles $\hat{\boldsymbol{\psi}}^b_{b\to s_i}$ with $i = 1, 2$ for one of the *XSens MTx* IMUs (measuring in its s_i instrumental frame) with respect to the reference signals provided by a the *Ixsea Airins* IMU. The data were collected on a vehicle during a 15 minutes long trajectory. The final boresight angles are estimated as a weighted mean (considering the smoothed variances).

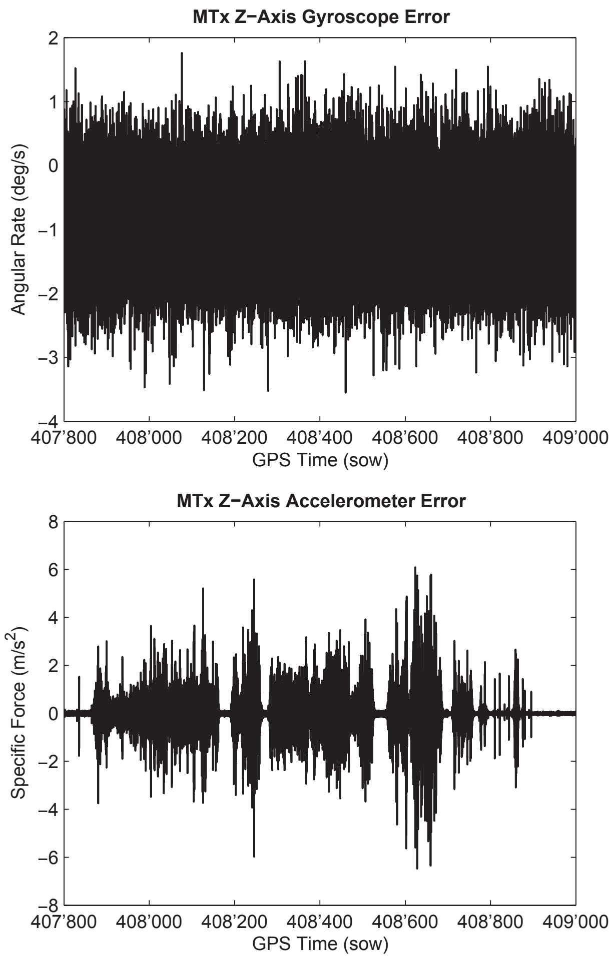

Fig. 2: Typical *XSens MTx* gyroscope (top panel) and accelerometer (bottom panel) error signals computed from the reference IMU under dynamics.

Having checked for time alignment, the sensor error is simply the difference between the transformed S-IMU signals and the reference signals. The error of the Z-axis gyroscope and accelerometer over the complete run is depicted as a representative example in Fig. 2.

C. Analysis of the IMU errors

Once the error process from the IMUs is constructed, a first step which can be taken in investigating the behaviour of the errors with respect to the dynamics is a basic scatterplot. The scatterplot in Fig. 3 shows some of the errors measured on the x-axis of the gyroscope and on the z-axis of the accelerometer against some of the quantities representative of dynamics such as acceleration and jerk. The errors indeed seem to form some ellipsoids with the latter measures, indicating that there appears to be a correlation between these quantities.

To confirm this analysis, a linear regression was performed on the gyroscope errors to test the statistical significance

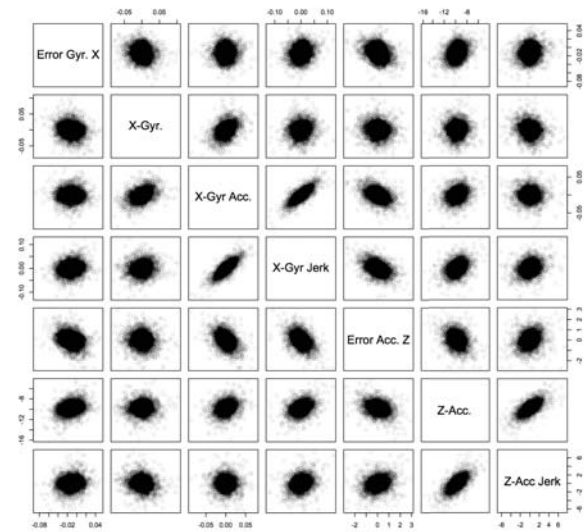

Fig. 3: “Scatter” plot between the errors of the gyroscope on the axis x, the angular velocity, acceleration and jerk on the same axis, the errors of the accelerometer on the axis z and the linear acceleration and linear jerk for the same axis.

of these linear relationships. Denoting by $\mathbf{\Omega}_k$ the vector of dynamics measurements on all three axes (i.e. angular rate, angular acceleration, angular jerk, specific force, linear jerk) for the k-th error y_k, the linear model was the following

$$y_k = \mathbf{\Omega}_k^T \boldsymbol{\beta} + \varepsilon_k \tag{6}$$

with $\boldsymbol{\beta}$ being the 15 coefficients related to each dynamic measurement and ε_k being the residual error following a Gaussian distribution with null expectation and a certain variance σ_ε^2. Table I collects the general results of this regression analysis and shows, for each regression, the coefficient of determination (i.e. R^2) and p-value associated to the F-test based on the following hypotheses:

$$\begin{aligned} \mathrm{H}_0 : &\quad \boldsymbol{\beta} = \mathbf{0} \\ \mathrm{H}_\alpha : &\quad \boldsymbol{\beta} \neq \mathbf{0}. \end{aligned}$$

It can be observed that H_α is accepted for the three axes considered in this analysis. This clearly demonstrates that the dynamics has a statistically significant influence of the errors of the considered IMU.

D. Model parameter estimation

Given the above results, the following step was to find an adequate stochastic model to explain the error process and estimate its parameters. To this purpose, a newly proposed method called the GMWM was used. This method was introduced by [1] and uses the quantity called the Wavelet Variance (WV) to estimate the parameters of time series. Let (y_t), $t = 1, \ldots, T$ be a realisation of the process (Y_t), $t \in \mathbb{Z}$ associated to the parametric model $F_{\boldsymbol{\theta}}$, with $\boldsymbol{\theta} \in \mathbf{\Theta} \subseteq \mathbb{R}^p$. The GMWM takes

TABLE I: General results from linear regression fit of the errors using dynamic measurements as covariates.

	Error Gyr. X	Error Gyr. Y	Error Gyr. Z
R^2	15.42%	36.73%	2.23%
F-statistic	2.60×10^3	7.71×10^3	3.25×10^2
p-value	≈ 0	≈ 0	≈ 0

TABLE II: Comparison between the *XSens MTx* gyroscope model constructed on a signal acquired in non-moving conditions (static model) and in moving conditions (dynamic model).

Process	Parameter	Unit	Static Model	Dynamic Model
Gaussian White Noise	σ^2_{WN}	(deg/s)^2	0.50552 ± 0.00035	0.65382 ± 0.00001
First-order Gauss-Markov #1	β_1	1/s	0.00492 ± 0.01892	0.00155 ± 0.00082
	$\sigma^2_{GM,1}$	(deg/s)^2	0.00142 ± 0.00001	0.00981 ± 0.00005
First-order Gauss-Markov #2	β_2	1/s	113.51523 ± 0.00404	1.68012 ± 0.04980
	$\sigma^2_{GM,2}$	(deg/s)^2	0.05528 ± 0.00051	0.00168 ± 0.00005

advantage of the implicit link between the WV $\nu^2(\tau_j)$ and the parameters $\boldsymbol{\theta}$ given by

$$\nu^2(\tau_j) = \int_{-1/2}^{1/2} S_{W_j}(f)df = \int_{-1/2}^{1/2} |\overline{H}_j(f)|^2 S_{F_{\boldsymbol{\theta}}}(f)df \qquad (7)$$

where the index j indicates the scale at which the WV is considered, W are the wavelet coefficients coming from a wavelet decomposition, $S_{F_{\boldsymbol{\theta}}}(f)$ is the power spectral density function associated to model $F_{\boldsymbol{\theta}}$ at frequency f and $|\overline{H}(f)|$ is the modulus of the transfer function of the wavelet filters.

The idea behind the GMWM is therefore to estimate the WV $\nu^2(\tau_j)$ from the observed signal (y_t) and estimate the parameters $\boldsymbol{\theta}$ which minimize a certain distance between the WV implied by $F_{\boldsymbol{\theta}}$, denoted as $\boldsymbol{\phi}(\boldsymbol{\theta})$, and the estimated WV $\hat{\boldsymbol{\phi}}$. Hence, the proposed methodology to estimate $\boldsymbol{\theta}$ is given by the solution to the following Generalized Least Squares problem

$$\hat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta} \in \boldsymbol{\Theta}}{\operatorname{argmin}} \left(\hat{\boldsymbol{\phi}} - \boldsymbol{\phi}(\boldsymbol{\theta})\right)^T \boldsymbol{\Omega} \left(\hat{\boldsymbol{\phi}} - \boldsymbol{\phi}(\boldsymbol{\theta})\right) \qquad (8)$$

where $\boldsymbol{\Omega}$ is a positive definite weighting matrix (see [1] for more details).

The GMWM is a methodology which can reliably estimate the parameters of simple processes or composite stochastic processes (i.e. processes made of the sum of different simple processes).

Exploiting this methodology, a series of models were considered using the sum of a white noise process with a sum of Gauss-Markov processes since a combination of multiple Gauss-Markov processes can approximate many random processes. Using the WV plots to assess the goodness-of-fit of the different models, finally a white noise process summed with two Gauss-Markov processes was chosen for both the errors measured in static condition and those measured in dynamic ones. Fig. 4 shows how the chosen model seems to fit the estimated WV quite well. Once the models were chosen, the estimated parameters for both types of errors (static and dynamic) were collected and compared.

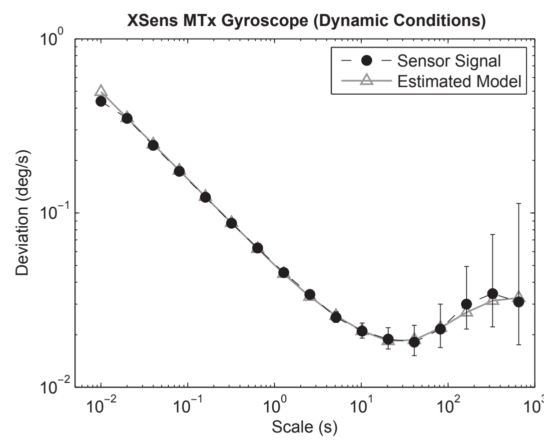

Fig. 4: Wavelet variance sequence computed on the gyroscope error signal acquired in moving conditions (black circles), together with the wavelet variance issued from an estimated model.

Table II allows this comparison and highlights how the gyroscope noise structure does not change but the magnitude of the white noise and Gauss-Markov processes differ significantly since the confidence intervals do not overlap. Hence, it appears evident that the calibration of inertial sensors should take into account how the dynamics in which the measurements are made can influence the error processes and, consequently, the precision of the navigation system.

III. Conclusion

We have presented a methodology that allows reliable estimation of spatial differences between the reference and tested IMUs even if the latter is of a poor quality. Applying this method within a car experiment allowed constructing error signal that underwent the following analysis: first we have shown that the correlation between the observed errors and the vehicle dynamics is statistically significant. We have then constructed a model structure and estimate model parameters via the GMWM methodology. When comparing this model to that obtained under static conditions we conclude that although the model structure did not change the model parameters are significantly different. In future we will investigate the relevance of the obtained model-parameter value (e.g. as used within the Kalman Filter) on the quality of the integrated navigation.

REFERENCES

[1] Guerrier, S., Skaloud, J., Stebler, Y., Victoria-Feser, M.P.: Wavelet-variance-based estimation for composite stochastic processes. Journal of the American Statistical Association **108**(503), 1021–1030 (2013)

[2] Wis, M., Colomina, I.: Dynamic Dependency of Inertial Sensor Errors and its Application to INS/GNSS Navigation. In: Proceedings of the NAVITEC 2010 Congress. Noordvijk, Netherlands (2010)

[3] Scherzinger, B.: Inertial Navigator Error Models for Large Heading Uncertainty. Position Location and Navigation Symposium, 1996 IEEE/ION p. 477484 (1996)

[4] Waegli, A.: Trajectory Determination and Analysis in Sports by Satellite and Inertial Navigation. Ph.D. thesis, Lausanne, Switzerland (2009)

[5] Farrell, J.: Aided Navigation: GPS with High Rate Sensors. McGraw-Hill New York, NY, USA: (2008)