Is it Alpha or Beta? Decomposing Hedge Fund Returns When Models are Misspecified

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Is it Alpha or Beta? Decomposing Hedge Fund Returns When Models are Misspecified[☆]

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Abstract

We develop a novel approach to separate alpha and beta under model misspecification. It comes with formal tests to identify less misspecified models and sharpen the return decomposition of individual funds. Our hedge fund analysis reveals that: (i) prominent models are as misspecified as the CAPM, (ii) several factors (time-series momentum, variance, carry) capture alternative strategies and lower performance in all investment categories, (iii) fund heterogeneity in alpha and beta is large—an important result for fund selection and models of active management, (iv) performance is increasingly similar to mutual funds, (v) fund valuation is sensitive to investor sophistication.

Keywords: Hedge fund returns, alpha, beta, model misspecification, large cross-section
JEL : C55, C58, G11, G12, G23

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I. Introduction

The average return that any hedge fund i delivers to investors is the sum of two components. The alpha component ac_i^* (or alpha) is based on private information—it captures the return that the fund earns by exploiting its unique investment abilities. The beta component bc_i^* is based on public information—it captures the return that the fund earns by following mechanical trading strategies. The economic importance of these two components is potentially large. A commonly held view is that hedge fund managers deliver positive alphas because they are more sophisticated, less constrained, and more incentivized than mutual fund managers. In addition, the literature consistently emphasizes that hedge funds increase their returns by using alternative strategies that are weakly correlated with the equity market (*e.g.*, Carhart et al., 2014; Pedersen, 2015).

Decomposing returns is key for evaluating the performance and risk profile of hedge funds. It is therefore important for researchers, investors, and policymakers alike. This decomposition is likely to exhibit substantial variation across funds as they follow many investment strategies and private information signals (*e.g.*, Lhabitant, 2007). As a result, averaging across funds provides limited information about the entire fund population. For example, it does not reveal how many funds deliver positive alphas—an important quantity for testing the predictions of equilibrium models of active asset management. Capturing fund heterogeneity is also crucial for hedge fund investors because they can only invest in a handful of funds (Bollen, Joenväärä, and Kauppila, 2021). These arguments call for the estimation of the entire distributions of the alpha and beta components characterized by the densities ϕ_{ac}^* and ϕ_{bc}^*.

The estimation of these distributions is hampered by misspecification. Capturing all the strategies followed by hedge funds is notoriously difficult, which implies that any chosen model k is misspecified—that is, it omits relevant factors that drive the beta components of individual funds. In this case, we cannot infer the true components ac_i^* and bc_i^*. Instead, we can only observe the estimated components $\hat{ac}_i^k$ and $\hat{bc}_i^k$, which serve as inputs to compute the distributions $\hat{\phi}_{ac}^k$ and $\hat{\phi}_{bc}^k$. These distributions are imperfect and noisy versions of ϕ_{ac}^* and ϕ_{bc}^*. They are imperfect because the alphas absorb the average returns of the factors omitted by the model. Put differently, high alphas are simply hidden betas. The distributions are also noisy because of the sampling variation of the omitted factors. This misspecification-driven variation, which affects all funds, does not vanish even when the fund population grows large.

We propose a new methodology to address the challenges of misspecification. We develop a nonparametric approach for estimating the distributions ϕ^k_{ac} and ϕ^k_{bc} for any chosen model. Its key feature is to explicitly account for the dual impact of misspecification. First, it provides a framework for comparing models and identifying the ones with greater ability to capture alternative strategies. Using models less prone to misspecification produces a sharper identification of the true alpha and beta components ac^*_i and bc^*_i. Second, it comes with a full-fledged asymptotic theory that incorporates the estimation noise due to misspecification. This contribution is crucial to conduct proper statistical inference on the fund alphas and betas and perform valid model comparison tests.

Our approach contributes to previous studies which measure the distribution of fund alphas under correct specification (*e.g.*, Barras, Gagliardini, and Scaillet, 2022; Harvey and Liu, 2018). In contrast, we account for the impact of misspecification, which is essential for decomposing hedge fund returns. Our approach also provides the first formal comparison of misspecified models in a large population of funds. It therefore departs from previous tests based on a small number of assets (*e.g.*, Kan and Robotti, 2009; Kan, Robotti, and Shanken, 2013). Such tests cannot be applied here because they require the inversion of the entire return covariance matrix—an operation that cannot be performed because the number of funds is larger than the number of return observations.

To demonstrate the benefits of our approach, we apply it to a set of nine diverse models. In addition to the CAPM, we include four standard models commonly used in previous work: the models of Carhart (1997), Fama and French (2015), Fung and Hsieh (2004), and Asness, Moskowitz, and Pedersen (2013). We examine the two machine learning models of Kozak, Nagel, and Santosh (2020) trained on 50 characteristic-based equity portfolios. Finally, we consider two models formed with the following factors (called additional factors hereafter): marketwide illiquidity, betting-against-beta (BAB), variance (short position), carry, and time-series (TS) momentum.[1] The first model by Joenväärä et al. (2021) (JKKT) is a Carhart model with illiquidity, BAB, and TS momentum. Based on the work by Carhart et al. (2014) and Pedersen (2015), we form another model (CP) that includes all five additional factors (with market and size).

We conduct our analysis between 1994 and 2020 using monthly data on 5,231 hedge funds. To construct this sample, we follow Joenväärä et al. (2021) and carefully aggregate four different

[1] As discussed in Section IV.B, the illiquidity, BAB, carry, and TS momentum factors are constructed by Pástor and Stambaugh (2003), Frazzini and Pedersen (2014), Koijen et al. (2018), and Moskowitz, Ooi, and Pedersen (2012).

databases to mitigate the various biases that affect hedge fund reporting (backfill, selectivity, and survivorship). For each model k, the estimation of the alpha and beta distributions ϕ_{ac}^k and ϕ_{bc}^k requires as inputs the estimated components $\hat{ac}_i^k$ and $\hat{bc}_i^k$ for each fund. To this end, we run a regression of the excess return (net of fees) of each fund on the excess returns of the factors included in model k. We then compute the two components as $\hat{ac}_i^k = \hat{\alpha}_i^k$ and $\hat{bc}_i^k = \hat{\mu}_i - \hat{ac}_i^k$, where $\hat{\alpha}_i^k$ and $\hat{\mu}_i$ are the estimated alpha and the average return of the fund.

Our formal comparisons uncover sharp differences in misspecification between models. For these comparisons, we use as reference the CAPM—a model ill-equipped to capture hedge fund returns (beyond equity risk). If a proposed model delivers the same alphas as the CAPM, it is therefore unable to capture any of the alternative strategies followed by hedge funds. This conclusion holds for the standard and machine learning models. In contrast, the JKKT and CP models are able to capture the premia on alternative factors. This is particularly the case for the CP model, whose alpha distribution departs markedly from that of the CAPM. This difference is stable over time and robust—it holds with factor trading costs and alternative filters for reducing data biases.

These results affect the fund return decomposition. Consistent with previous studies, the standard models produce large alpha components—more than 70% of the funds have a positive alpha equal to 2.7% per year on average.[2] At the same time, the average beta component due to the non-market factors barely reaches 0.2% per year. Therefore, hedge funds deliver superior performance, while being immune to alternative sources of risk. The CP model reverses this conclusion. We find that only 50% of the funds deliver positive alphas. The average alpha drops to 0.4% per year, whereas the non-market factors become the main contributors to average returns (2.9% per year).

The five additional factors are economically important. The majority of the funds load positively on each of them, which supports the view that hedge funds follow alternative strategies to boost returns (*e.g.*, Carhart et al., 2014). Out of the five factors, the CP model includes the three most relevant ones—TS momentum, variance, and carry—whose respective return contributions are equal to 1.1%, 0.8%, and 0.4% per year. This finding explains the edge of the CP model over the JKKT model. It is also in line with the evidence that hedge funds follow trends, take short option positions, and buy cheap assets with high carry (Asness et al., 2015; Pedersen, 2015).

[2] A non-exhaustive list of papers that document positive average alphas under the standard models includes Avramov, Barras, and Kosowski (2013), Buraschi, Kosowski, and Trojani (2014), Capocci and Hübner (2004), Chen, Cliff, and Zhao (2017), Diez de los Rios and Garcia (2010), Kosowski, Naik, and Teo (2007).

Next, we turn to the analysis of three investment categories: (i) equity funds, (ii) macro funds, and (iii) arbitrage funds. We find that the superiority of the CP model applies to each category. The average alpha is equal to 0.6% and 0.9% per year for equity and arbitrage funds (versus 2.4% and 2.8% under the CAPM) and reaches the negative value of -0.4% for macro funds (versus 3.7% under the CAPM). The widespread reduction in alphas arises because the additional factors transcend style boundaries. For instance, we find that carry contributes to fund returns in all three categories. This result is consistent with economic intuition—carry is widely used by hedge funds when they pursue currency carry trades, buy high-yielding bonds, or favor value stocks (Pedersen, 2015). At the same time, some factors primarily matter for specific styles. This is the case of TS momentum, which captures the trend-following strategies of macro funds.[3]

The analysis of investment styles reveals a large fund heterogeneity. Under the CP model, the cross-sectional volatilities in the alpha and beta components range between 4.9% and 11% per year. Therefore, sorting funds based on style does not produce homogenous groups with similar performance and risk profiles. This result invalidates the common practice of benchmarking funds uniformly using hedge fund style indices. We also find that the two components are negatively correlated, as low alphas come with high betas. In other words, the worst funds load heavily on the five additional factors to boost returns—possibly to hide their lack of skills.

This heterogeneity introduces substantial uncertainty in the fund selection process. To reduce this uncertainty, we examine whether fund characteristics that proxy for managerial incentives and flexibility (*e.g.*, performance fees, lockup period) could be used as initial filters. We find that this is the case—the proportion of positive-alpha funds is systematically higher among funds with higher incentives/flexibility. The observed heterogeneity also has theoretical implications. The Berk and Green (2004) model predicts that all funds deliver zero alphas. Whereas this prediction holds relatively well for mutual funds, it is at odds with our empirical results. In contrast, the model of Gârleanu and Pedersen (2018) predicts heterogeneity as investors need to be compensated for search costs—an intuitive explanation given the complex hedge fund selection process.

Our analysis uncovers notable time trends between 1994 and 2020. Hedge funds remain unique in their exposure to the alternative strategies captured by the CP model. However, they converge

[3]This result suggests that adjusting models to a particular category by adding style-specific factors may lower misspecification and sharpen the return decomposition. We elaborate on this point in Section V.B.3.

towards mutual funds along two dimensions. First, their performance becomes increasingly similar as the gap in alphas reaches its lowest value of 1.6% in 2020. Second, they load increasingly on the equity market after the 2008 crisis. The equity market is now the most prevalent source of risk—its average contribution is equal to 2.3% per year, which represents 44% of the total beta component.

Finally, our approach sheds light on the impact of investor sophistication on hedge fund valuation. Whereas the CP alpha gives the valuation of an investor able to replicate the five additional factors, the CAPM alpha determines the valuation of a less sophisticated investor who can only invest in the market. We find that the average valuation gap is large (2.6% per year) as the CAPM investor not only values the alpha component but also the beta component due to the alternative strategies she cannot replicate. Examining fund flows, we find that real-world hedge fund investors are closer to the CP investor in terms of sophistication. Decomposing returns across funds with low and high flows, we find that flows primarily respond to the alpha component obtained with the CP model but not to the beta component (as would a CAPM investor do).

The remainder of the paper is as follows. Section II presents the framework for decomposing hedge fund returns. Section III describes the methodology. Section IV presents the hedge fund dataset and the models. Section V contains the empirical analysis, and Section VI concludes. The appendix provides additional information on the methodology, the data, and the empirical results.

II. Decomposing Hedge Fund Returns Under Misspecification

II.A. Theoretical Framework

II.A.1. Definition of the Alpha and Beta Components

We consider a population of n funds over T periods and denote by $r_{i,t}$ the excess net-of-fee return that fund i ($i = 1, \ldots, n$) delivers to investors at time t ($t = 1, \ldots, T$). Our objective is to decompose the average fund return $E[r_{i,t}]$ into alpha and beta components. The definition of each component hinges on its information source. The alpha component ac_i^* is based on private information—it captures the return that the fund produces from exploiting its superior information. The beta component bc_i^* is based on public information—it captures the return that the fund produces by following mechanical trading strategies. Put differently, bc_i^* represents the return component that can be replicated by sophisticated investors with access to public information.

To formalize this intuition, we denote the excess return vector of the mechanical strategies by f_t and its average value by $E[f_t] = \lambda$. The dimension of f_t is potentially large as it includes the returns of all the alternative strategies followed by hedge funds (*e.g.*, option and dynamically managed portfolios). If the factors f_t are known, we can decompose the average fund return as

$$E[r_{i,t}] = ac_i^* + bc_i^* = \alpha_i^* + b_i^{*\prime}\lambda, \tag{1}$$

where ac_i^* is given by the fund alpha α_i^* and $bc_i^* = b_i^{*\prime}\lambda$ corresponds to the average return of a benchmark portfolio with the same betas on the mechanical strategies as the fund.[4] This decomposition assumes that f_t includes the dynamic strategies followed by hedge funds. Therefore, the use of constant betas is not restrictive for capturing factor-timing strategies based on public information (see, *e.g.*, Ferson and Schadt, 1996; Haddad, Kozak, and Santosh, 2020; Kelly and Pruitt, 2013). To elaborate, suppose that hedge funds commonly change their market betas after observing a public signal z_{t-1} that predicts the equity market return $r_{m,t}$. Equation (1) absorbs the time-variation in betas by including the scaled factor $z_{t-1}r_{m,t}$ in the vector f_t.[5]

The magnitude of the beta component depends on the factor premia λ. Consistent with standard asset pricing models, these premia can be a compensation for fundamental sources of risk in the economy. They can also reflect limits to arbitrage when hedge funds bear nonfundamental risk to correct mispricing caused by behavioral biases (Gromb and Vayanos, 2010). Finally, they can be the outcome of market segmentation when hedge funds require a premium for holding stocks that constrained investors cannot buy (*e.g.*, Merton, 1987). We remain agnostic on the drivers of the premia—bc_i^* simply controls for strategies that investors can replicate using public information.

II.A.2. Remarks About the Alpha and Beta Components

The return decomposition in Equation (1) calls for several comments. First, the alpha component ac_i^* is a measure of performance, not skill. Whereas the two notions are commonly used interchangeably, they differ in important ways (Barras, Gagliardini, and Scaillet, 2022; Berk and van

[4]A key requirement is that all elements in f_t are tradable factors (*i.e.*, excess returns with zero prices). Otherwise, $\lambda \neq E[f_t]$, implying that the difference $E[r_{i,t}] - b_i^{*\prime}E[f_t]$ cannot be interpreted as the alpha α_i^* (see Ferson, 2013).

[5]This reasoning is also valid if the public signal is observed multiple times between $t-1$ and t. For instance, suppose that we observe fund returns at a monthly frequency and that hedge funds change their market betas using daily signals $z_{j,t-1}^d$ ($d = 1, 2, ...$). In this case, we include the scaled factor $\sum_d z_{t-1}^d r_{m,t}^d$ (ignoring compounding for simplicity), where $r_{m,t}^d$ denotes the daily stock return (Patton and Ramadorai, 2013).

Binsbergen, 2015). Skill determines whether funds are able to create value by exploiting superior information. Performance determines whether investors hold sufficient bargaining power to receive some of this value. Positive performance implies positive skill, but not vice-versa.

Second, Equation (1) ignores the short-term variations in alpha (around its average α_i^*) that could arise from changing economic conditions, industry competition, or aggregate mispricing (*e.g.*, Avramov, Barras, and Kosowski, 2013; Pástor, Stambaugh, and Taylor, 2015, 2017). As discussed below, measuring α_i^* for hedge funds is a challenging task. Modelling its conditional variation via a proper choice of predictors and functional forms makes the estimation even more difficult (see, *e.g.*, Bakalli, Guerrier, and Scaillet, 2021, for individual stocks).

Third, Equation (1) does not require that we model the determinants of ac_i^* and bc_i^* across funds. For instance, performance may depend on fund characteristics such as its managerial incentives (Agarwal, Daniel, and Naik, 2009), its bagaining power over investors (Glode and Green, 2011; Pástor and Stambaugh, 2012), or its leverage policy.[6] In this case, we can simply interpret ac_i^* and bc_i^* as fund-specific functions of the these characteristics.[7]

II.A.3. The Distributions of the Alpha and Beta Components

Hedge funds follow diverse alternative strategies and rely on multiple information signals to create value. It is therefore likely that the alpha and beta components vary across funds. This heterogeneity cannot be captured with a simple average—instead, it requires that we estimate the entire cross-sectional distributions characterized by their densities ϕ_{ac}^* and ϕ_{bc}^*.

Measuring fund heterogeneity is important for several reasons. The alpha distribution ϕ_{ac}^* determines how many funds deliver positive alphas—a key quantity to test the equilibrium predictions of asset management models. It is also useful for hedge fund investors who only select a handful of funds because of multiple frictions (Bollen, Joenväärä, and Kauppila, 2021). The alpha distribution allows these investors to determine the range of performance outcomes when selecting funds. In other words, ϕ_{ac}^* has a natural Bayesian interpretation as it provides prior information about individual fund alphas (*e.g.*, Jones and Shanken, 2005; Pástor and Stambaugh, 2002).

[6] To see how leverage changes the alpha and beta components, consider a set of hedge funds with access to the same active strategy whose average net exess return is equal to $\alpha^* + b^{*\prime}\lambda = ac^* + bc^*$. For each fund i, both ac_i^* and bc_i^* scale up with its leverage ratio π_i: $ac_i^* = \pi_i ac^*$ and $bc_i^* = \pi_i bc^*$.

[7] Understanding the determinants of ac_i^* and bc_i^* is important for forming hedge fund portfolios with high alpha or high exposures to specific strategies. We examine this issue in more detail in Section V.C.1.

The distribution ϕ^*_{bc} measures how many funds follow mechanical strategies. It also determines whether funds with the same investment style have similar risk profiles. While this information obviously matters for researchers and investors, it is also relevant for regulators. As discussed by Brown, Lynch, and Petajisto (2010, ch. 12), hedge funds can contribute to systemic risk when they liquidate positions, reduce liquidity provision, or impose losses on counterparties. Contrary to a simple average, ϕ^*_{bc} can identify clusters of funds with strong exposures to similar factors.

II.B. The Misspecification of Hedge Fund Models

II.B.1. The Prevalence of Misspecification

The theoretical decomposition in Equation (1) is important for defining the alpha and beta distributions ϕ^*_{ac} and ϕ^*_{bc}. However, it does not provide guidance for estimating these distributions for two reasons. First, we do not know the identity of the factors in the vector f_t. Second, data limitation favors models with a parsimonious number of factors.

Choosing appropriate factors is a daunting task as hedge funds invest in many countries and asset classes (Lhabitant, 2007). They take nonlinear option positions that are difficult to capture with a limited set of option factors (Karehnke and de Roon, 2020). Hedge funds also follow multiple dynamic strategies in which the portfolio composition or leverage ratio respond to changing economic conditions (Avramov, Barras, and Kosowski, 2013; Bollen and Whaley, 2009; Ang, Gorovyy, and van Inwegen, 2011). Based on this analysis, there is no hope of capturing the average returns of all the alternative hedge fund strategies. As a result, the empirical return decomposition is based on a misspecified model—that is, a model that omits some relevant factors in f_t.[8]

Misspecification also arises if we want to measure how different types of investors value the alpha and beta components. Whereas the most sophisticated investors only care about ac^*_i, less sophisticated investors also value bc^*_i because it provides an exposure to alternative strategies that are difficult to replicate (*e.g.*, Agarwal, Green, and Ren, 2018). To evaluate hedge funds from the viewpoint of a particular investor, we need a model that only includes the factors that she can replicate. This model is therefore misspecified because it only includes a subset of f_t.

[8]One could use holdings-based benchmarks to avoid specifying the factors (*e.g.*, Grinblatt and Titman, 1993; Lo, 2008). However, these measures are still subject to misspecification when the fund exhibits time-varying betas (Ferson and Khang, 2002). Another issue is that hedge funds generally do not disclose their portfolio weights.

II.B.2. The Dual Impact of Misspecification

Misspecification has a dual impact on the estimation of the distributions ϕ^*_{ac} and ϕ^*_{bc}. First, the estimation is imperfect. To see this point, suppose that we use a misspecified model k that only includes the factors $f^k_{I,t}$, but omits the factors $f^k_{O,t}$ (with $f_t = (f^{k\prime}_{I,t}, f^{k\prime}_{O,t})'$). From Equation (1), we have $E[r_{i,t}] = ac^*_i + bc^*_{i,I} + bc^*_{i,O}$, where $bc^*_{i,I} = b^{*\prime}_{i,I}\lambda^k_I$ and $bc^*_{i,O} = b^{*\prime}_{i,O}\lambda^k_O$ are the beta components due to the included and omitted factors under the true model (*i.e.*, $b^*_{i,I}$ and $b^*_{i,O}$ are the true fund exposures to $f^k_{I,t}$ and $f^k_{O,t}$). The key question is how the omitted beta component $bc^*_{i,O}$ affects the return decomposition obtained with the misspecified model, which we write as $E[r_{i,t}] = ac^k_i + bc^k_i$.

Regressing the omitted factors on the included factors, we have $f^k_{O,t} = \alpha^k_O + \Psi^k_{O,I} f^k_{I,t} + u^k_{O,t}$ and $\lambda^k_O = \alpha^k_O + \Psi^k_{O,I}\lambda^k_I$, where α^k_O is the vector of factor alphas, $\Psi^k_{O,I}$ is the matrix of slope coefficients, and $u^k_{O,t}$ is the vector of errors. We can split the omitted beta component into two parts: $bc^*_{i,O} = b^{*\prime}_{i,O}\alpha^k_O + b^{*\prime}_{i,O}\Psi^k_{O,I}\lambda^k_I$, where the split depends on the correlation between the included and omitted factors (captured by $\Psi^k_{O,I}$). The first part, which arises from the component of $f^k_{O,t}$ that is orthogonal to $f^k_{I,t}$, is absorbed by ac^k_i. The second part, which arises from the component of $f^k_{O,t}$ that is spanned by $f^k_{I,t}$, is absorbed by bc^k_i. As a result, we have:

$$ac^k_i = ac^*_i + b^{*\prime}_{i,O}\alpha^k_O = \alpha^k_i\,, \tag{2}$$

$$bc^k_i = bc^*_i - b^{*\prime}_{i,O}\alpha^k_O = b^{k\prime}_{i,I}\lambda^k_I\,, \tag{3}$$

where α^k_i and $b^k_{i,I} = b^*_{i,I} + \Psi^{k\prime}_{O,I} b^*_{i,O}$ are the coefficients from the regression of the fund return $r_{i,t}$ on the factors $f^k_{I,t}$ included in model k (and a constant). Equations (2)–(3) reveal that ac^k_i and bc^k_i are informative about the true components ac^*_i and bc^*_i. However, this information is polluted—if the fund loads on alternative strategies with positive alphas, $b^{*\prime}_{i,O}\alpha^k_O$ is positive and the model-implied alpha component is inflated ($ac^k_i > ac^*_i$ and $bc^k_i < bc^*_i$). Misspecification produces an imperfect estimation because we can only infer the pseudo-true distributions ϕ^k_{ac} and ϕ^k_{bc}, but not the true ones ϕ^*_{ac} and ϕ^*_{bc} (Gourieroux, Monfort, and Trognon, 1984; White, 1982).

The second impact of misspecification pertains to estimation noise. When model k is misspecified, the densities ϕ^k_{ac} and ϕ^k_{bc} are estimated with substantial noise. The simplest way to illustrate this point is to focus on the average alpha $M^k_{1,ac} = \int_{-\infty}^{+\infty} x\phi^k_{ac}(x)dx$, where $\phi^k_{ac}(x)$ is the density evaluated at x. To estimate $M^k_{1,ac}$, we run the linear regression under model k for each fund:

$$r_{i,t} = \alpha^k_i + b^{k\prime}_{i,I} f^k_{I,t} + \varepsilon^k_{i,t}\,. \tag{4}$$

We then compute the cross-sectional mean $\hat{M}^k_{1,ac} = \frac{1}{n}\sum_i \hat{ac}^k_i = \frac{1}{n}\sum_i \hat{\alpha}^k_i$.

Contrary to the correctly specified case, the error terms $\varepsilon^k_{i,t}$ in Equation (4) are strongly cross-correlated because they all depend on the common omitted factors $f^k_{O,t}$. Formally, we have $\varepsilon^k_{i,t} = \varepsilon^*_{i,t} + b^{*\prime}_{i,O} u^k_{O,t}$, where $u^k_{O,t}$ is the error vector of $f^k_{O,t}$. Even if the hedge fund population size n is large, the information is limited because the values of $\hat{ac}^k_i$ ($i = 1, ..., n$) are all impacted by the random realizations of the omitted component $u^k_{O,t}$. It implies that $\hat{M}^k_{1,ac}$ is estimated with substantial noise, as discussed in detail in Section III.

II.C. A New Approach for Addressing Misspecification

II.C.1. Overview of the Approach

We develop a novel approach to estimate the entire alpha and beta distributions for any model. A key feature of our approach is to explicitly account for the dual impact of misspecification. First, it allows for an examination of multiple models to address the imperfect separation between alpha and beta. Given the diversity in hedge fund strategies, several models can reasonably be used for decomposing returns. By designing formal comparison tests, we identify models less prone to misspecification and more able to capture the true distributions ϕ^*_{ac} and ϕ^*_{bc}.[9]

Second, our approach comes with a full-fledged inferential theory to incorporate the estimation noise caused by misspecification. We derive the asymptotic properties of the estimated distributions in a setting that accounts for the large population of hedge funds observed in the data (*i.e.*, we let n to grow large). Using these results, we (i) conduct proper statistical inference on the distributions for each model, and (ii) obtain valid tests for comparing models.

Our approach departs from previous studies that estimate the alpha distribution (*e.g.*, Chen, Cliff, and Zhao, 2017; Harvey and Liu, 2018). First, it does not assume that the model is correctly specified—an important feature given the difficulty in modeling hedge fund returns. Second, it provides a framework for estimating both the alpha and beta distributions. Third, it is flexible

[9] Another benefit of this comparison analysis is to measure how investors with different sophistication levels value hedge fund investments—an issue that we examine empirically in Section V.E.

because it imposes no restrictions on the shape of these distributions. Fourth, it is simple and fast as it does not rely on sophisticated and computer-intensive Gibbs sampling and expectation maximization methods. Last but not least, it allows us to conduct statistical inference and formal testing guided by econometric theory.

II.C.2. Comparing Models using the CAPM

The comparison of models ultimately hinges on their ability to capture alternative strategies. These strategies, which are weakly correlated with the equity market, represent a defining feature of hedge funds.[10] Superior models capture the positive premia earned on alternative strategies and thus deliver alpha components close to zero. In contrast, highly misspecified models are unable to do so and produce larger alphas. To distinguish between these models, we use the CAPM as a reference—that is, we formally compare the alphas under any proposed model k and the CAPM.

The CAPM is a natural reference for two reasons. First, it controls for the equity market risk taken by hedge funds—an opener evaluation practice among academics and practitioners. Second, it is the simplest and thus the least equipped model for capturing alternative strategies. To formalize this point, we refer to the CAPM as model 0 and denote by $f_{I,t}^0$ the market factor and by $f_{O,t}^0$ the set of alternative strategies. If $f_{I,t}^0$ and $f_{O,t}^0$ are uncorrelated ($\Psi_{O,I}^0 = 0$), we have $\alpha_O^0 = \lambda_O^0$. As a result, the CAPM alpha entirely absorbs the beta component $bc_{i,O}^*$ due to the alternative strategies:

$$ac_i^0 = ac_i^* + b_{i,O}^{*\prime}\alpha_O^0 = ac_i^* + b_{i,O}^{*\prime}\lambda_O^0 = ac_i^* + bc_{i,O}^* \,. \tag{5}$$

Building on this insight, our comparison analysis delivers a strong message. If the alpha distributions ϕ_{ac}^k and ϕ_{ac}^0 are identical, it implies that model k is as misspecified as the CAPM. In other words, model k is unable to capture any of the alternative strategies followed by hedge funds.[11]

II.C.3. A Simple Illustrative Example

Before presenting our approach in more detail, we briefly illustrate its usefulness using a simple example. We assume that the correct hedge fund model includes the market and three uncorrelated

[10] For instance, Carhart et al. (2014) define alternative (exotic) betas as exposures to risk factors that are uncorrelated with global equity markets and have positive expected returns. They also note that "it is fair to characterize hedge funds as providing considerable exposure to equity market risk and to alternative risk premiums."

[11] Whereas we use the CAPM for the baseline analysis, our comparison tests can be applied to any pair of models. We can therefore replace the CAPM with any model commonly used for performance evaluation (see Section V.A.1).

alternative factors: $E[r_{i,t}] = \alpha_i^* + b_{i,m}^* \lambda_m + b_{i,1}^* \lambda_1 + b_{i,2}^* \lambda_2 + b_{i,3}^* \lambda_3$, where λ_m is the equity premium, and λ_j denotes the premium of each alternative factor j ($j = 1, 2, 3$), which we set equal to λ_m. For each fund, the alpha component α_i^* is drawn from a normal $N(\mu_\alpha^*, \sigma_\alpha^{*2})$, $b_{i,m}^*$ from a normal $N(\mu_b^*, \sigma_b^{*2})$, and $b_{i,j}^*$ from a normal $N(\mu_{b_j}^*, \sigma_b^{*2})$, where $\mu_{b_j}^*$ is positive to capture the view that hedge funds load on alternative strategies (each draw is mutually independent). We further assume that the first factor is more important by setting $\mu_{b_1}^* = \mu_b^*$ and $\mu_{b_2}^* = \mu_{b_3}^* = \mu_b^*/3$.

We consider a misspecified hedge fund (HF) model (model 1) that includes the market and two out of the three alternative factors (f_1 and f_2). We compare this model with the CAPM (model 0), which only includes the market. For each model, we assume that we observe the fund alpha and beta components without errors, leaving aside estimation noise for now. Given the above assumptions, the alpha and beta densities are normal. Under the CAPM, we have

$$ac_i^0 \sim N(\mu_\alpha^* + (\mu_{b_1}^* + \mu_{b_2}^* + \mu_{b_3}^*)\lambda, \sigma_\alpha^{*2} + 3\sigma_b^{*2}\lambda^2)\,, \quad (6)$$

$$bc_i^0 \sim N(\mu_b^*\lambda, \sigma_b^{*2}\lambda^2)\,, \quad (7)$$

while the HF model produces the following densities:

$$ac_i^1 \sim N(\mu_\alpha^* + \mu_{b_3}^*\lambda, \sigma_\alpha^{*2} + \sigma_b^{*2}\lambda^2)\,, \quad (8)$$

$$bc_i^1 \sim N((\mu_b^* + \mu_{b_1}^* + \mu_{b_2}^*)\lambda, 3\sigma_b^{*2}\lambda^2)\,. \quad (9)$$

To begin, we plot in Panel A of Figure 1 the two alpha densities using the following parameter values: $\mu_\alpha^* = 0\%$, $\sigma_\alpha^* = 1.4\%$, $\lambda = 7.5\%$, $\mu_b^* = 0.3$, and $\sigma_b^* = 0.4$.[12] The comparison of ϕ_{ac}^0 and ϕ_{ac}^1 reveals that the magnitude of the alpha components decreases substantially under the HF model (ϕ_{ac}^1 moves to the left towards zero). This difference arises because the HF model is less misspecified than the CAPM—by capturing two out of the three alternative strategies, it produces a sharper identification of the true components ac_i^* and bc_i^*. The implications for performance evaluation are economically important. Under the CAPM, the average alpha reaches 3.8% per year, and more than 75% of the funds deliver positive alphas. In contrast, the average alpha drops

[12] We express μ_α^*, σ_α^*, and λ in percent per year. For simplicity, we set μ_α^* equal to zero and calibrate σ_α^* using the the value reported by Barras, Gagliardini, and Scaillet (2022, Table VI). We further set λ equal to the average market return and μ_b, σ_b equal to the cross-sectional mean and volatility of the market betas in our sample of hedge funds.

to 0.8% per year under the HF model and moves closer to the true average μ_α^* equal to zero.

In addition to reducing the magnitude of the alphas, the HF model also produces a lower dispersion as it absorbs the variation due to factors 1 and 2 (the term $2\sigma_b^{*2}\lambda^2$). However, this result depends on the specific assumptions in our simple example—in particular, we assume that funds choose similar factor exposures regardless of their true alphas (*i.e.*, we set $corr[\alpha_i^*, b_{i,j}^*] = 0$). If we relax this assumption, a lower average does not necessarily come with a lower dispersion. In the empirical analysis, we simply let the data speak—our approach is nonparametric and thus does not require any assumptions on the joint distributions of α_i^* and $b_{i,j}^*$.

Next, we repeat the analysis for the two beta densities ϕ_{bc}^0 and ϕ_{bc}^1. Consistent with intuition, Panel B is the mirror image of Panel A as the magnitude of the beta components rises under the HF model (ϕ_{bc}^1 moves to the right away from zero). We can go one step further and decompose bc_i^1 as $bc_{i,m}^1 + bc_{i,1}^1 + bc_{i,2}^1$, where $bc_{i,m}^1$ is the beta component due to the market, and $bc_{i,1}^1$, $bc_{i,2}^1$ are the beta components due to the two additional factors in the HF model. We can then determine the economic importance of these factors using their densities $\phi_{bc_1}^1$ and $\phi_{bc_2}^1$, which are both normal: $bc_{i,1}^1 \sim N(\mu_{b_1}^*\lambda, \sigma_b^{*2}\lambda^2)$ and $bc_{i,2}^1 \sim N(\mu_{b_2}^*\lambda, \sigma_b^{*2}\lambda^2)$. This analysis identifies factor 1 as the most relevant—its contribution to hedge fund returns equals 2.3% per year on average and is positive for 77% of the funds. In contrast, these values are only equal to 0.8% and 60% for factor 2.

Please insert Figure 1 here

III. Methodology

III.A. Estimation Procedure

We consider a total of K models indexed by k ($k = 0, \ldots, K-1$), where model 0 denotes the CAPM. Each model k is allowed to be misspecified as it includes the factors $f_{I,t}^k$ but omits the factors $f_{O,t}^k$. We are interested in (i) the density ϕ_{ac}^k of the alpha component, (ii) the density ϕ_{bc}^k of the beta component, and (iii) the density $\phi_{bc,j}^k$ of the beta component due to each factor j included in model k. We summarize the shape of each of these densities with the following characteristics: (i) the cross-sectional mean and standard deviation, denoted by M_1^k and M_2^k, (ii) the proportion of funds with a return component below a given value a, denoted by $P^k(a)$, and (iii) the quantile at a given percentile level u, denoted by $Q^k(u) = (P^k)^{-1}(u)$.

To estimate the distribution characteristics of each density, we need to estimate the fund components $ac_i^k = \alpha_i^k$, $bc_i^k = b_{i,I}^{k\prime}\lambda_I^k$, and $bc_{i,j}^k = b_{i,I,j}^k\lambda_{I,j}^k$, where $b_{i,I,j}^k$ and $\lambda_{I,j}^k$ is the beta and premium associated with factor j in model k. For each fund, we compute these values by running the time-series regression in Equation (4). We interpret this regression as a random coefficient model (*e.g.*, Hsiao, 2003) in which α_i^k and $b_{i,I}^k$ are not fixed parameters, but random realizations from a continuum of funds in order to invoke cross-sectional limits.[13] We also assume that at least one omitted factor is strong in the sense that it has a pervasive impact on the cross-section of fund returns.[14] This mild assumption, which is satisfied by all the models we examine, delivers a well-defined convergence rate for the distribution characteristics. The least-square estimate $\hat{\gamma}_i^k = (\hat{\alpha}_i^k, \hat{b}_{i,I}^{k\prime})'$ is given by

$$\hat{\gamma}_i^k = (\hat{Q}_{x,i}^k)^{-1}\frac{1}{T_i}\sum_t I_{i,t}x_t^k r_{i,t}\,, \tag{10}$$

where $I_{i,t}$ is an indicator variable equal to one if $r_{i,t}$ is observable, $T_i = \sum_t I_{i,t}$, $x_t^k = (1, f_{I,t}^{k\prime})'$, and $\hat{Q}_{x,i}^k = \frac{1}{T_i}\sum_t I_{i,t}x_t^k x_t^{k\prime}$. We then compute the alpha and beta components for each fund as

$$\hat{ac}_i^k = \hat{\alpha}_i^k\,, \tag{11}$$

$$\hat{bc}_i^k = \hat{\mu}_i - \hat{\alpha}_i^k\,, \tag{12}$$

$$\hat{bc}_{i,j}^k = \hat{b}_{i,I,j}^k\hat{\lambda}_{I,j}^k\,, \tag{13}$$

where $\hat{bc}_i^k$ is the empirical counterpart of $bc_i^k = E[r_{it}] - ac_i^k$, $\hat{\mu}_i = \frac{1}{T_i}\sum_t I_{i,t}r_{i,t}$ is the average fund return, and $\hat{\lambda}_{I,j}^k = \frac{1}{T_i}\sum_t I_{i,t}f_{I,j,t}^k$ is the empirical average of $f_{I,j,t}^k$.

Next, we account for the unbalanced nature of the hedge fund sample. Following Barras, Gagliardini, and Scaillet (2022) and Gagliardini, Ossola, and Scaillet (2016), we introduce a formal selection rule $\mathbf{1}_i^\chi$ equal to one if the following conditions are met: $\mathbf{1}_i^\chi = \mathbf{1}\left\{\tau_{i,T} \leq \chi_{1,T}, CN_i \leq \chi_{2,T}\right\}$, where $\tau_{i,T} = T/T_i$, $CN_i = \sqrt{eig_{\max}\left(\hat{Q}_{x,i}^k\right)/eig_{\min}\left(\hat{Q}_{x,i}^k\right)}$ is the condition number of $\hat{Q}_{x,i}^k$, and $\chi_{1,T}$, $\chi_{2,T}$ denote the two threshold values. The first condition $\tau_{i,T} \leq \chi_{2,T}$ excludes funds for which the sample size is too small. The second condition $CN_i \leq \chi_{1,T}$ excludes funds for which the time-

[13] Barras, Gagliardini, and Scaillet (2022) and Gagliardini, Ossola, and Scaillet (2016) use the same sampling scheme to measure mutual fund performance and test the arbitrage pricing theory in a large cross-section of assets.

[14] More formally, an omitted factor is strong if the largest eigenvalue of the residual covariance matrix of hedge fund returns does not vanish as the population size n grows large. In contrast, a factor is weak if its loading vanishes at a rate $1/\sqrt{T}$ (Gagliardini, Ossola, and Scaillet, 2019). Our formulation allows for weak factors in both $f_{I,t}^k$ and $f_{O,t}^k$.

series regression is subject to multicollinearity problems (*e.g.*, Belsley, Kuh, and Welsch, 2004). Both thresholds $\chi_{1,T}$ and $\chi_{2,T}$ increase with the sample size T—with more return observations, we estimate the fund coefficients with greater accuracy, which allows for a less stringent selection rule. Applying this selection rule, we work with a population size equal to $n_\chi = \sum_{i=1}^{n} \mathbf{1}_i^\chi$.

The final step is to compute the distribution characteristics using the vector of estimated components. We compute the mean, standard deviation, proportion, and quantile of the alpha distribution as $\hat{M}_1^k = \frac{1}{n_\chi}\sum_i \mathbf{1}_i^\chi \hat{ac}_i^k$, $\hat{M}_2^k = \left(\frac{1}{n_\chi}\sum_i \mathbf{1}_i^\chi (\hat{ac}_i^k)^2 - (\frac{1}{n_\chi}\sum_i \mathbf{1}_i^\chi \hat{ac}_i^k)^2\right)^{1/2}$, $\hat{P}^k(a) = \frac{1}{n_\chi}\sum_i \mathbf{1}_i^\chi \mathbf{1}\{\hat{ac}_i^k \leq a\}$, and $\hat{Q}^k(u) = (\hat{P}^k)^{-1}(u)$. To obtain the characteristics of the distributions of the beta components, we use the same formulas after replacing $\hat{ac}_i^k$ with $\hat{bc}_i^k$ or $\hat{bc}_{i,j}^k$.

III.B. Properties of the Distribution Characteristics

We begin our theoretical analysis by examining the properties of the distribution characteristics $\hat{M}_1^k$, $\hat{M}_2^k$, $\hat{P}^k(a)$, and $\hat{Q}^k(u)$. The following proposition derives the asymptotic distribution of the estimated characteristics of the alpha distribution ϕ_{ac}^k as the numbers of funds n and observations T grow large. To capture the large cross-sectional dimension of the hedge fund population observed in the data, we require that n is larger than T.

Proposition 1. *As $n, T \to \infty$, such that $T/n \to 0$, we obtain the following properties for the estimated characteristics of ϕ_{ac}^k under the misspecified model k:*

$$\sqrt{T}\left(\hat{M}_s^k - M_s^k\right) \to_d N(0, V[M_s^k])\,, \tag{14}$$

$$\sqrt{T}\left(\hat{P}^k(a) - P^k(a)\right) \to_d N(0, V[P^k(a)])\,, \tag{15}$$

$$\sqrt{T}\left(\hat{Q}^k(u) - Q^k(u)\right) \to_d N(0, V[Q^k(u)])\,, \tag{16}$$

where $s \in \{1,2\}$ and $\to_d$ denotes convergence in distribution. The variance terms are given by

$$V[M_s^k] = \left(\eta_{M_s}^{k\prime} \otimes E_1'(Q_x^k)^{-1}\right)\Omega_{ux}^k\left(\eta_{M_s}^k \otimes (Q_x^k)^{-1}E_1\right)\,, \tag{17}$$

$$V[P^k(a)] = \left(\eta_{P(a)}^{k\prime} \otimes E_1'(Q_x^k)^{-1}\right)\Omega_{ux}^k\left(\eta_{P(a)}^k \otimes (Q_x^k)^{-1}E_1\right)\,, \tag{18}$$

$$V[Q^k(u)] = V[P^k(Q^k(u))]/\phi_{ac}^k(Q^k(u))^2\,, \tag{19}$$

where $\eta_{M_s}^k = E\left[\left(\frac{\partial M_s^k}{\partial E[g_i^k]}\right)' \frac{\partial g_i^k}{\partial ac_i^k} b_{i,O}^*\right]$, $E[g_i^k]$ is the vector of uncentered moments with $g_i^k = (ac_i^k, (ac_i^k)^2)'$, $\otimes$ denotes the Kronecker product, $Q_x^k = E[x_t^k x_t^{k\prime}]$, $\Omega_{ux}^k = \lim_{T\to\infty} V\left[\frac{1}{\sqrt{T}}\sum_t u_{O,t}^k \otimes x_t^k\right]$, $\eta_{P(a)}^k = E[b_{i,O}^* | ac_i^k = a]\phi_{ac}^k(a)$, $b_{i,O}^*$ and $u_{O,t}^k$ denote the vectors of betas and residuals associated with the omitted factors $f_{O,t}^k$, and $\phi_{ac}^k(a)$ is the probability density evaluated at a.

Proof. *See the appendix.*

To save space, we refer the reader to the appendix for the theoretical analysis of the distributions ϕ^k_{bc} and $\phi^k_{bc,j}$ whose estimated characteristics have the same properties as in Proposition 1. These results allow for formal tests on the shape of the alpha or beta distribution. Denoting the generic estimated characteristic by $\hat{C}^k \in \{\hat{M}^k_s, \hat{P}^k(a), \hat{Q}^k(u)\}$, we can test the null hypothesis:

$$H_0 : C^k = \upsilon, \tag{20}$$

where υ is a given scalar. For instance, we can test whether the proportion of positive-alpha funds equals 50% ($\upsilon = 0.5$), or whether the average beta component due to any factor j is null ($\upsilon = 0$).

Proposition 1 reveals two important insights about inference under misspecification. First, the estimated characteristics converge towards their respective parameter values. Asymptotically, we are able to estimate the alpha distribution ϕ^k_{ac} under model k without bias, even though we use as inputs noisy versions of the fund components (*i.e.*, we use $\hat{ac}^k_i$ instead of ac^k_i). Under misspecification, Proposition 1 therefore provides a theoretical justification for the common practice of reporting cross-sectional summary statistics (*e.g.*, boxplots) based on estimated coefficients without any bias adjustment.[15] Second, the variance of the estimators is large because the convergence rate is equal to $1/\sqrt{T}$ (and not $1/\sqrt{n}$)—a result that formalizes our previous point that misspecification amplifies estimation noise. This result is a priori surprising because the characteristics are all computed as cross-sectional averages (we sum across n funds, not across T periods).

None of these properties hold when the model is correctly specified—a setting examined in detail by Barras, Gagliardini, and Scaillet (2022). In this case, the estimated distribution characteristics must be adjusted for the error-in-variable (EIV) bias that arises because we use as inputs noisy versions of the fund coefficients. In addition, the characteristics are estimated with greater precision because the convergence rate is equal to $1/\sqrt{n}$ (instead of $1/\sqrt{T}$).

The strong impact of misspecification on inference stems from the properties of the fund error terms. The estimation error on the alpha component $\hat{ac}^k_i$ involves the term $\bar{\varepsilon}^k_i = \bar{\varepsilon}^*_i + b^{*\prime}_{i,O}\bar{u}^k_O$, where $\bar{\varepsilon}^k_i$, $\bar{\varepsilon}^*_i$, and $\bar{u}^k_O$ denote the time-series averages of $\varepsilon^k_{i,t}$, $\varepsilon^*_{i,t}$, and $u^k_{O,t}$. The average $\bar{\varepsilon}^*_i$ obtained with the correct model is weakly correlated across funds, which implies that its impact on the estimated characteristics vanishes with the population size n. It is not the case for the average error term $\bar{u}^k_O$

[15] For instance, Almeida, Ardison, and Garcia (2020), Capocci and Hübner (2004), and Kosowski, Naik, and Teo (2007) follow this practice when reporting the characteristics of the distribution of hedge fund alphas.

due to the omitted factors because it affects all funds simultaneously. This term is noisy because it converges to zero at the rate equal to $1/\sqrt{T}$. As a result, the noise contained in $\bar{u}_O^k$ (i) slows down the convergence rate of the estimated characteristics from $1/\sqrt{n}$ to $1/\sqrt{T}$, and (ii) dominates in magnitude the EIV bias, which makes any bias adjustment unnecessary.[16]

III.C. Comparison Tests Between Models

We now turn to the comparison tests based on the alpha distributions. We compare the distributions ϕ_{ac}^k and ϕ_{ac}^l between two models k and l ($k, l = 0, ..., K-1$). Our comparison framework is highly flexible because we can apply it to both nested and non-nested models (models are nested if one is included in the other). When we set $l = 0$, the comparison is made with respect to the CAPM. We compute the differences in distribution characteristics between the two models as $\Delta\hat{M}_1 = \hat{M}_1^k - \hat{M}_1^l$, $\Delta\hat{M}_2 = \hat{M}_2^k - \hat{M}_2^l$, $\Delta\hat{P}(a) = \hat{P}^k(a) - \hat{P}^l(a)$, and $\Delta\hat{Q}(u) = \hat{Q}^k(u) - \hat{Q}^l(u)$. Proposition 2 derives the asymptotic distribution of each estimated difference as the numbers of funds n and observations T grow large.

Proposition 2. *As $n, T \to \infty$ such that $T/n \to 0$, we obtain the following properties for the differences in the estimated characteristics of ϕ_{ac}^k and ϕ_{ac}^l under the misspecified models k and l:*

$$\sqrt{T}\left(\Delta\hat{M}_s - \Delta M_s\right) \to_d N\left(0, V[\Delta M_s]\right) , \tag{21}$$

$$\sqrt{T}\left(\Delta\hat{P}(a) - \Delta P(a)\right) \to_d N\left(0, V[\Delta P(a)]\right) , \tag{22}$$

$$\sqrt{T}\left(\Delta\hat{Q}(u) - \Delta Q(u)\right) \to_d N\left(0, V[\Delta Q(u)]\right) , \tag{23}$$

where $s \in \{1, 2\}$. The variance of the characteristic differences are equal to

$$V[\Delta M_s] = V[M_s^k] + V[M_s^l] - 2Cov[M_s^k, M_s^l] , \tag{24}$$

$$V[\Delta P(a)] = V[P^k(a)] + V[P^l(a)] - 2Cov[P^k(a), P^l(a)] , \tag{25}$$

$$V[\Delta Q(u)] = V[Q^k(u)] + V[Q^l(u)] - 2Cov[Q^k(u), Q^l(u)] , \tag{26}$$

where $V[M_s^k]$, $V[M_s^l]$, $V[P^k(a)]$, $V[P^l(a)]$, $V[Q^k(u)]$, and $V[Q^l(u)]$ are obtained from Proposition 1. The covariance terms are given by

$$Cov[M_s^k, M_s^l] = \left((\eta_{M_s}^{k\prime} \otimes E_1'(Q_x^k)^{-1}\right)\Omega_{ux}^{kl}\left(\eta_{M_s}^l \otimes (Q_x^l)^{-1}E_1\right) , \tag{27}$$

$$Cov[P^k(a), P^l(a)] = \left(\eta_{P(a)}^{k\prime} \otimes E_1'(Q_x^k)^{-1}\right)\Omega_{ux}^{kl}\left(\eta_{P(a)}^l \otimes (Q_x^l)^{-1}E_1\right) , \tag{28}$$

$$Cov[Q^k(u), Q^l(u)] = \frac{Cov[P^k(Q^k(u)), P^l(Q^l(u))]}{\phi_{ac}^k(Q^k(u))\phi_{ac}^l(Q^l(u))} , \tag{29}$$

[16] Our Monte Carlo simulations calibrated on our sample of funds confirm these results (see the appendix). When the model is misspecified, the mean squared error (MSE) of each characteristic estimator (i) is primarily driven by the variance (and not by the finite-sample bias), and (ii) decreases with T, but not with n.

where $\Omega_{ux}^{kl} = \lim_{T\to\infty} Cov\left[\frac{1}{\sqrt{T}}\sum_t u_{O,t}^k \otimes x_t^k, \frac{1}{\sqrt{T}}\sum_t u_{O,t}^l \otimes x_t^l\right]$.

Proof*. See the appendix.*

The results in Proposition 2 provide simple comparison tests for each pair of models k and l. We denote the generic estimated characteristic difference by $\Delta\hat{C} \in \{\Delta\hat{M}_s, \Delta\hat{P}(a), \Delta\hat{Q}(u)\}$. We can then test the null hypothesis that each characteristic difference equals zero:

$$H_0 : \Delta C = 0. \tag{30}$$

Misspecification arises naturally in pairwise comparisons because competing models cannot be all correct. Therefore, the impact of misspecification discussed in Proposition 1 carries over to model comparisons. Proposition 2 shows that the comparison tests inherit the high estimation noise caused by misspecification as each characteristic difference converges at a slow rate equal to $1/\sqrt{T}$. In other words, the bar for detecting significant differences between models is considerably higher when the tests are properly adjusted for misspecification.

Our approach departs from previous studies (*e.g.*, Kan and Robotti, 2009; Kan, Robotti, and Shanken, 2013) which derive comparison tests for misspecified models under a fixed number of assets n (single asymptotics with n fixed and $T \to \infty$). These tests require the inversion of the entire covariance matrix of returns—an operation that cannot be performed with thousands of hedge funds, but only hundreds of observations. To address this issue, we derive comparison tests in which n grows large (double asymptotics with n and $T \to \infty$). Another important difference is that we do not focus on a single aggregate measure of asset alphas (*e.g.*, Hansen-Jagannathan distance), but instead focus on the disaggregated distribution to capture fund heterogeneity.

III.D. Estimation of the Asymptotic Variance Terms

Conducting inference requires consistent estimators of the variance terms in Propositions 1 and 2. For each distribution characteristic, the variance depends on the error term $u_{O,t}^k$ and betas $b_{i,O}^*$ associated with the omitted factors. For instance, the estimated average $\hat{M}_1^k$ is more volatile when the variance of the factor residuals $V[u_{O,t}^k]$ and the magnitude of the average betas $E[b_{i,O}^*]$ increase. Because $u_{O,t}^k$ and $b_{i,O}^*$ are not observable, estimating V is not trivial.

To address this issue, we derive a consistent variance estimator based on the observed fund residuals of each model $\hat{\varepsilon}_{i,t}^k = r_{i,t} - x_t^{k\prime}\hat{\gamma}_i^k$. The estimators of the asymptotic variances of $\sqrt{T}(\hat{C}^k -$

C^k) and $\sqrt{T}(\Delta\hat{C} - \Delta C)$ are given by

$$\hat{V}[\hat{C}^k] = \frac{1}{n_\chi^2 T} \sum_i \sum_j \sum_t \mathbf{1}_i^\chi \tau_{i,T} I_{i,t} \mathbf{1}_j^\chi \tau_{j,T} I_{j,t} \hat{a}_{i,t}(\hat{C}^k) \hat{a}_{j,t}(\hat{C}^k)\,, \quad (31)$$

$$\hat{V}[\Delta\hat{C}] = \frac{1}{n_\chi^2 T} \sum_i \sum_j \sum_t \mathbf{1}_i^\chi \tau_{i,T} I_{i,t} \mathbf{1}_j^\chi \tau_{j,T} I_{j,t} \hat{a}_{i,t}^\Delta(\Delta\hat{C}) \hat{a}_{j,t}^\Delta(\Delta\hat{C})\,, \quad (32)$$

where the terms $\hat{a}_{i,t}(\hat{C}^k)$ and $\hat{a}_{i,t}^\Delta(\Delta\hat{C})$ are functions of $\hat{C}^k$ and $\Delta\hat{C}$ (see the appendix). The following proposition shows that $\hat{V}[\hat{C}^k]$ and $\hat{V}[\Delta\hat{C}]$ are consistent variance estimators as the numbers of funds n and observations T grow large.

Proposition 3. *As $n, T \to \infty$ such that $T/n \to 0$, we have*

$$\hat{V}[\hat{C^k}] \to_p V[\hat{C^k}]\,, \quad (33)$$

$$\hat{V}[\Delta\hat{C}] \to_p V[\Delta\hat{C}]\,, \quad (34)$$

where $\to_p$ denotes convergence in probability.

Proof. *See the appendix.*

IV. Data and Model Construction

IV.A. Hedge Fund Dataset

We collect the monthly net-of-fee returns of hedge funds between January 1994 and December 2020. We combine four databases (Barclayhedge, HFR, Morningstar, TASS) to mitigate the selection bias that arises from the voluntary nature of information disclosure by hedge funds. We remove the first 12 months of data for each fund to control for backfill bias. The appendix provides more detail on the construction of the dataset, which largely follows Joenväärä et al. (2021). For comparison purposes, we also collect the monthly net-of-fee returns of actively managed U.S. equity funds over the same period following Barras, Gagliardini, and Scaillet (2022).

Table I reports summary statistics for the equal-weighted portfolio of all hedge funds in our sample, as well as three investment styles: (i) equity funds (long-short and market neutral), which rely on discretionary or quantitative analysis to detect mispriced stocks, (ii) macro funds (global macro and managed futures), which take directional bets across asset classes using broad economic and financial indicators, and (iii) arbitrage funds (relative value and event driven), which exploit various sources of mispricing primarily in the debt market. Overall, the results are similar to those reported by Getmansky, Lee, and Lo (2015) between 1996 and 2014.

To obtain reliable estimates of ac_i^k, bc_i^k, and $bc_{i,j}^k$ in the unbalanced panel of hedge funds, we apply the selection rule of Section III.A. Taking the same thresholds as Barras, Gagliardini, and Scaillet (2022), we set the minimum number of return observations equal to 60 and the minimum condition number for each model equal to 15. Empirically, the second threshold plays no role, which implies that the sample of selected funds is identical across all models ($n_\chi = 5{,}231$ funds). To address the concern that the results depend on the construction of the hedge fund database, we consider alternative filters in the appendix.[17] Consistent with intuition, we find that these changes have the same impact across models. As a result, they leave the model comparison unchanged.

Please insert Table I here

IV.B. Hedge Fund Models

We apply our methodology to nine models examined in previous work. These models only include tradable factors, which allows us to use the fund-by-fund regression in Equation (4) to compute the alpha and beta components. Our selection focuses on omnibus models that aim at explaining the return of any given hedge fund. We therefore do not include models whose factors vary with each individual fund (*e.g.*, Bollen and Whaley, 2009; O'Doherty, Savin, and Tiwari, 2016).

In addition to the CAPM, we consider four standard models. The chosen set is by no means exhaustive but provides a good representation of the models commonly used for performance evaluation. We select the Carhart (1997) model and the Five-Factor model of Fama and French (2015), which includes the market, size, value, momentum, profitability, and investment factors. We examine the well-known model of Fung and Hsieh (2004), which includes two equity factors (market and size), two bond factors (term and default), and three option straddles (bond, commodity, and currency). Finally, we consider the model of Asness, Moskowitz, and Pedersen (2013), which adds to the CAPM two global value and momentum factors across international asset classes.

Next, we consider the models of Kozak, Nagel, and Santosh (2020) based on machine learning. These authors apply lasso and ridge penalizations to form models with the highest out-of-sample ability to explain the average returns of 50 characteristic-based equity portfolios.[18] We use the two

[17] We change the fund selection rule by imposing 36 or 84 minimum return observations. We also apply the more stringent backfill bias correction of Joenväärä et al. (2021), eliminating all the observations before the fund listing date. Finally, we use the five filters proposed by Straumann (2009) to remove errors in reported fund returns.

[18] We thank Serhiy Kozak for providing us with the data and code. In principle, we could expand the set of candidate factors using machine learning techniques. However, these techniques require a considerably large number of return observations (Gu, Kelly, and Xiu, 2020), which is not the case for hedge fund datasets.

models presented in their Table 4, which impose sparsity by selecting five factors only. The first one is formed with the portfolios themselves, while the second one is formed with their principal components. We also add the market to both models for consistency with the other models.

Finally, we consider two models that include five factors (referred to as additional factors): illiquidity, betting against beta (BAB), variance (short position), carry, and time-series (TS) momentum. These factors are based on economic intuition and plausibly capture several strategies followed by hedge funds.[19] We consider the model of Joenväärä et al. (2021, JKKT), which extends the Carhart (1997) model by including the illiquidity, BAB, and TS momentum factors. Building on the work of Carhart et al. (2014) and Pedersen (2015), we examine another model (CP) which replaces the five non-equity factors of Fung and Hsieh (2004) (bond factors and straddles) with the five additional factors. Measuring the trading costs of these factors is difficult because it requires timely price information on a wide range of markets. Given this uncertainty, we conduct our baseline analysis using the original factor returns.

Table II reports summary statistics for the excess returns of the factors.[20] We find that all but one factor (bond term) deliver positive premia. Therefore, hedge funds boost their average returns when they increase their betas. Unreported results also show that the factors capture distinct strategies—only 11 pairwise correlations out of 135 are above 0.5 (in absolute value). Table III provides a complete list of the nine models with the factors they include (see the appendix for details on the data sources of the factors).

Please insert Tables II and III here

IV.C. Misspecification Statistics

Before moving to the main empirical results, we examine the misspecification of the nine models. An intuitive statistic is the adjusted R^2 of the fund-by-fund regression in Equation (4). A high R^2 is a signal of low misspecification because the omitted factors must have implausibly high

[19] The illiquidity factor of Pástor and Stambaugh (2003) captures marketwide changes in market liquidity. The BAB strategy of Frazzini and Pedersen (2014) exploits the price distortions caused by leverage-constrained investors on low- and high-beta stocks. The variance factor tracks the realized variance of the S&P 500. The global carry and TS momentum factors of Koijen et al. (2018) and Moskowitz, Ooi, and Pedersen (2012) invest in assets with high carry and positive 12-month returns across international asset classes. TS momentum departs from traditional cross-sectional momentum, which only invests in assets with past returns higher than the cross-sectional average.

[20] We define the variance and straddle factors as short positions to obtain positive premia (*i.e.*, these short positions perform poorly in bad times when realized variance is high (Bakshi and Kapadia, 2003)).

premia to substantially affect the fund return decomposition (see Cochrane, 2005, ch. 9). Table IV shows that the average R^2 is relatively low across all models—the values range between 20.4% and 31.0%, leaving plenty of room for omitted factors.

In theory, a correct model could deliver a low R^2 if hedge funds take large idiosyncratic positions to exploit their private information. In this case, a low R^2 measures skill, not misspecification (Titman and Tiu, 2011). To address this issue, we compute the criterion of Gagliardini, Ossola, and Scaillet (2019). As n and T converge to infinity, it is positive with probability one if (i) the model is misspecified, and (ii) at least one omitted factor is strong (see the appendix). We find that this criterion is systematically positive. This result confirms that all the models are misspecified. It also validates our estimation assumption that one of the omitted factors is strong.

While misspecified, the nine models are likely to produce different return decompositions. Table IV provides preliminary evidence consistent with this view. For each model k, we report the relative importance of each component of the average fund return: (i) the alpha component $\hat{ac}_i^k$, (ii) the beta component due to the market computed as $\hat{bc}_{i,m}^k = \hat{b}_{i,m}^k \hat{\lambda}_m$, and (iii) the beta component due to the non-market (nm) factors (all factors but the market) computed as $\hat{bc}_{i,I_{nm}}^k = \hat{b}_{i,I_{nm}}^{k\prime} \hat{\lambda}_{I_{nm}}^k$. For instance, the relative importance of the alpha component is equal to 47.1% on average under the Carhart model but drops to 18.0% under the JKKT model. Interpreting these differences requires formal comparison tests—a point we examine below.

Looking at mutual funds, we see that the average R^2 is substantially higher (above 80%). In addition, all models lead to the same conclusion that average returns are driven by a single component—the equity market. In short, these results confirm that accounting for misspecification is important for hedge funds but not for mutual funds.

Please insert Table IV here

V. Empirical Results

V.A. Hedge Fund Return Decomposition

V.A.1. Formal Model Comparison

We begin the empirical analysis with a formal model comparison. This analysis determines whether some models are less prone to misspecification and more able to separate alpha and beta.

As discussed in Section II.C, we compare the alpha distributions under each proposed model and the CAPM. We apply our methodology to test for differences in the mean and standard deviation, the proportions of negative- and positive-alpha funds, and the quantiles at 10% and 90%. To compute the standard deviation of the estimated differences, we replace T with $T_\chi = \frac{1}{n_\chi}\sum_{i=1}^{n} \mathbf{1}_i^\chi T_i$, where T and T_χ are equal to 324 and 125 observations. As a result, we account for the increased noise due to the unbalanced nature of the hedge fund panel.

The comparison tests reported in Table V reveal several new insights. We observe a striking similarity between the CAPM and the four standard models (Carhart, Five-Factor, Fung-Hsieh, and Asness-Moskowitz-Pedersen). Only one of the 24 characteristic differences is statistically significant. Properly accounting for misspecification is key when interpreting these differences. Suppose that we naively use the convergence rate of $1/\sqrt{n_\chi}$, instead of the appropriate rate of $1/\sqrt{T_\chi}$ under misspecification. In this case, we find that 75% of the 24 characteristic differences are significant at the 5% level. Applying the correct procedure, we conclude that the standard models are no better than the CAPM at capturing alternative hedge fund strategies.

Similarly, the machine learning models produce the same alpha distributions as the CAPM. Whereas these models do a great job at explaining the average returns of characteristic-based equity portfolios (Kozak, Nagel, and Santosh, 2020), they are not trained on strategies beyond the equity space. Their limited success in the hedge fund population highlights the importance of accounting for other asset classes, such as bonds, currencies, and commodities.

The only exceptions are the JKKT and CP models. The tests reveal highly significant differences relative to the CAPM. These differences hold after replacing the CAPM with any of the standard and machine learning models. They also hold when accounting for factor trading costs using estimates from previous studies (see the appendix). In short, the JKKT and CP models are better equipped to capture alternative strategies and sharpen the fund return decomposition.

Please insert Table V here

V.A.2. Magnitude of the Alpha and Beta Components

We now turn to the return decomposition for the entire population. For each model k, we apply our methodology to infer the main characteristics of (i) the distribution of the alpha components $\hat{\phi}_{ac}^k$, and (ii) the distribution of the beta components $\hat{\phi}_{bc}^k$. We report these results in Table VI.

Both the CAPM and the standard models produce large alpha components. Panel A shows that the average alpha clusters around 2.8% per year, and the proportion of positive-alpha funds is always above 70%. These results are in line with the previous literature, which, by and large, finds that hedge funds deliver superior performance (*e.g.*, Buraschi, Kosowski, and Trojani, 2014; Diez de los Rios and Garcia, 2010; Kosowski, Naik, and Teo, 2007). Another takeaway from these models is that hedge funds are only exposed to the market factor. Panel B shows that the average beta component equals 2.6% under the CAPM and remains largely unchanged as we include profitability, investment, bond, or straddle factors. In short, hedge funds deliver high alphas to investors, while being immune to alternative sources of risk.

The JKKT and CP models reverse these conclusions. For instance, the CP model delivers an average beta component of 5.2%, which is twice as large as under the CAPM. At the same time, it brings the average alpha down to 0.4% per year and the proportion of positive-alpha funds down to 53.0%. These results highlight the importance of the additional factors included in the JKKT and CP models. As discussed by Asness, Moskowitz, and Pedersen (2013) and Pedersen (2015), there is ample anecdotal evidence that hedge funds hold illiquid assets, take levered positions, trade equity options, buy cheap assets with high carry, and follow trends in asset prices.

To visualize these results, we plot in Figure 2 the densities of the two components for the CAPM, the JKKT model, and the CP model. In line with our illustrative example in Figure 1, these two models capture the returns of hedge fund strategies and thus produce (i) a shift of the alpha distribution towards zero and (ii) a shift of the beta distribution away from zero.

Please insert Table VI and Figure 2 here

V.A.3. Economic Importance of Each Factor

Our previous analysis shows the key role played by the additional factors. Motivated by these results, we measure their economic importance separately. To this end, we break down the beta component obtained with the CP model into the respective contributions due to market, size, illiquidity, BAB, variance, carry, and TS momentum. Using these estimated quantities, we then apply our methodology to infer the main characteristics of the beta distribution $\phi_{bc,j}^{k}$ for each factor.

Table VII shows that the equity market is the most prevalent source of risk as 78% of the funds have positive market betas. On average, the market contribution equals 2.3% per year, representing

44% of the total beta component. We also find that a majority of funds load positively on each of the five additional strategies. They are therefore widely used by hedge funds to boost their returns. Interestingly, individual funds do not load on all of them simultaneously. As shown in Panel B, the pairwise correlations between the factor components only range between -18% and 14%.

TS momentum, variance, and carry are the most important additional factors. Their average contributions are all statistically significant and equal to 1.1%, 0.8%, and 0.4% per year. In the top decile of funds with the highest exposures, these contributions reach 4.5%, 4.5%, and 2.7% per year. The main difference between the CP and JKKT models is the inclusion of variance and carry. The ability of both factors to capture hedge fund returns explains why the CP model has an edge over the JKKT model. It consistently delivers lower average alphas and lower proportions of positive-alpha funds (both in the population and for all investment categories). For this reason, we primarily focus on the CP model for the rest of our analysis.

Please insert Table VII here

V.B. Hedge Fund Investment Styles

V.B.1. A Closer Look at Equity, Macro, and Arbitrage Funds

An important question is whether the superiority of the CP model is driven by particular hedge fund styles. To examine this issue, we re-estimate the alpha and beta distributions within each investment category (equity, macro, and arbitrage) for the CAPM, the JKKT model, and the CP model. The other models deliver the same results as the CAPM and are thus not shown.

Table VIII uncovers the same patterns as in the entire population. Among equity and arbitrage funds, the average alphas under the CP model are equal to 0.6% and 0.9% per year (versus 2.4% and 2.8% for the CAPM). In the macro category, the difference is even larger as the majority of funds deliver a negative alpha equal to -0.4% on average (versus 3.7% for the CAPM). For completeness, we examine multi-strategy funds and funds of funds, which could be more difficult to model given their diversity. In both categories, the appendix shows that the CP model still delivers a sharp reduction in alphas.

These results suggest that the additional factors in the CP model transcend style boundaries. Carry provides a good illustration of this phenomenon. This strategy invests in cheap assets with a high difference between spot and forward prices. As discussed by Pedersen (2015), carry is routinely implemented by hedge funds, regardless of their investment styles. This is the case when they buy value stocks, high interest rate currencies, or backwarded commodities. Consistent with this analysis, Table IX reveals that carry matters for all three categories. Its average beta component is always positive and ranges between 0.34% (equity) and 0.56% (arbitrage) per year.

Table IX shows that variance also transcends style boundaries. Its average return contribution among equity and arbitrage funds is equal to 1.1% per year. The economic importance of variance reflects the option strategies followed by these funds. For instance, mortgage, fixed income volatility, and merger arbitrage activities all involve taking short option positions (Duarte, Longstaff, and Yu, 2006; Mitchell and Pulvino, 2001). In addition, the variance factor captures unexpected increases in asset correlations (Driessen, Maenhout, and Vilkov, 2009). As such, it signals crisis times when equity and arbitrage funds suffer from less effective hedging strategies and tighter funding constraints (Buraschi, Kosowski, and Trojani, 2014).

At the same time, some factors in the CP model cater to specific styles. For instance, TS momentum plays a key role for macro funds—its average contribution reaches 3.10% per year. This result is consistent with the intuition that global funds rely on past returns to determine their asset allocation and exploit trends caused by behavioral biases, frictions, or slow-moving capital.[21] Another example is BAB, which primarily matters for arbitrage funds. These funds extensively use leverage to exploit price distortions in capital markets (Ang, Gorovyy, and van Inwegen, 2011). It is therefore plausible that some of these distortions originate from the leverage constraints faced by traditional investors (mutual and pension funds) and captured by the BAB strategy.

Please insert Table VIII and Table IX here

V.B.2. The Heterogeneity Across Funds

The return decomposition within styles in Table VIII uncovers a large fund heterogeneity. Starting with the alpha distribution, we see that the CP model produces a cross-sectional volatility between 6.8% (arbitrage) and 11.0% (macro) per year. Similarly, the beta distribution covers a wide interval as its standard deviation ranges between 4.9% (arbitrage) and 9.6% (macro) per year. These values are similar to those obtained with the entire population—Table VI reports a volatility of 9.1% and

[21] Managed futures funds (a subset of macro funds) are well known for following trends. Unsurprisingly, the appendix shows that the average contribution of TS momentum for these funds is even higher (3.7% per year).

7.6% per year for the alpha and beta distributions. Forming style groups is therefore insufficient to identify funds with similar performance and risk levels. This result reflects the difficulty in forming homogeneous groups given the diversity in hedge fund strategies.[22]

It might be surprising that the CP model always produces a higher dispersion in alphas than the CAPM. A priori, the CAPM should generate a higher cross-sectional variance because its alpha absorbs the dispersion due to the omitted factors (the term $2\sigma_b^{*2}\lambda^2$ in our example in Figure 1). Whereas this effect is at play, it is more than offset by the reduction in variance due to the negative correlation between the alpha and beta components. This correlation arises because poorly performing funds load aggressively on alternative strategies—possibly to hide their lack of skill.

To elaborate, consider the worst-performing macro funds under the CP model. These funds load heavily on the five additional factors to boost their returns. Unreported results show that their average beta component is three times larger than in the population (16.7% versus 5.2% per year). By controlling for these factor exposures, the CP model uncovers the strong underperformance of these funds—the 10%-quantile equals -8.2% per year (versus -4.0% only under the CAPM).

V.B.3. Benchmarking Using Style Information

The results in Table VIII use the same models for all categories. Therefore, a natural question is whether we can sharpen the return decomposition by incorporating style information. A common approach for using this information is peer benchmarking. The basic idea is to use as a benchmark a style index that includes all funds in a given investment category (*e.g.*, Hunter et al., 2014; Buraschi, Kosowski, and Sritrakul, 2014). Applying this approach to hedge funds is problematic for two reasons. First, the style index is not investible given the numerous constraints that prevent investors from forming diversified hedge fund portfolios. Therefore, it cannot be used to passively replicate hedge funds.[23] Second, the style index produces substantial misspecification because it imposes a constant beta component for all funds within the same style. This restriction is inconsistent with the large heterogeneity in Table VIII. Any fund with a beta component lower than

[22]Part of this heterogeneity may be driven by misclassification. As we aggregate multiple databases, it is difficult to define a common style classification (see the appendix). In addition, the reported styles are rarely updated and may not always closely reflect the actual fund strategies. To form homogeneous groups, one potential solution is to use clustering to identify funds with similar alphas and factor exposures (see Hastie, Tibshirani, and Friedman, 2009).

[23]Investability is less problematic for mutual funds. As noted by Kandel, Hunter, and Wermers (2014): "Even the least sophisticated investor always has a fallback strategy of equally-weighting (or value-weighting) all funds in the group every period; this tradeable strategy is quite simple."

average is mechanically credited with a lower alpha.

A better approach is to construct style-based models that include style factors. Adding these factors potentially reduces model misspecification but raises the risk of data mining when the set of candidate factors is large. Therefore, style-based models are likely to perform better on investment categories with a small number of well-defined strategies. For instance, Duarte, Longstaff, and Yu (2006) show the benefits of using these models for capturing the strategies of fixed income arbitrage funds (*e.g.*, swap spread, yield curve arbitrage). In contrast, finding appropriate style factors to cover the three broad categories examined here is more challenging. As a simple illustration, we consider a style-based CP model in which we replace the carry and TS momentum factors with their style-specific counterparts.[24] For each category, the appendix shows that these replacements decrease the ability of the CP model to capture hedge fund returns.

V.C. The Implications of Fund Heterogeneity

V.C.1. Hedge Fund Selection

Our previous analysis reveals that the dispersion in alphas is large. Fund heterogeneity has important implications for hedge fund selection. Because investors are constrained to invest in a handful of funds, they face substantial uncertainty in their selection process. This uncertainty provides a strong rationale for conducting due diligence to avoid the worst-performing funds. In particular, investors could use fund characteristics as initial filters in the selection process.

To examine this issue, we consider two sets of characteristics. The first set proxies for managerial incentives and includes management fees, performance fees, and a high-water mark dummy. The second set proxies for managerial flexibility and includes a lockup period dummy and a notice period dummy. For each characteristic, we sort funds in two groups and apply our methodology to estimate the alpha distributions under the CP model.

Panel A of Table X reports the results for the first set of characteristics. We find that sorting funds on each proxy for managerial incentives increases the likelihood of selecting positive-alpha funds. For instance, close to 60.0% of the funds that charge high performance fees produce a positive alpha equal to 1.1% per year on average. In contrast, the average alpha is negative among funds with low performance fees (-1.5%) as only 39.2% of them deliver positive alphas.

[24]We use equity carry (TS momentum) for equity funds, the equal-weighted average of currency and commodity carry (TS momentum) for macro funds, and bond carry (TS momentum) for arbitrage funds.

Panel B documents similar patterns for lockup and notice periods. Funds with higher flexibility tend to perform better, possibly because they can invest in illiquid assets and exploit arbitrage opportunities that take time to be profitable. In short, all five characteristics are useful for partially discriminating between inferior and superior funds. This result emphasizes the importance of managerial incentives and flexibility as determinants of fund performance, which is in line with previous studies by Agarwal, Daniel, and Naik (2009), Aragon (2007), and Joenväärä et al. (2021).

Please insert Table X here

V.C.2. Models of Active Management

The observed heterogeneity also has implications for the models of Berk and Green (2004) and Gârleanu and Pedersen (2018)—two popular models of active management. In the model of Berk and Green (2004), skilled funds have bargaining power because they are in short supply. As investors compete for performance, the alphas of all funds are null, and the heterogeneity disappears (after an adjustment period due to learning). This prediction holds quite well for mutual funds—Barras, Gagliardini, and Scaillet (2022) find that the standard deviation of the alpha distribution only equals 1.4% per year. However, it is at odds with the volatility observed for hedge funds.

In contrast, the dispersion in alphas is consistent with the model of Gârleanu and Pedersen (2018). In this model, skilled funds deliver positive alphas because they need to compensate investors for their search costs. At the same time, unskilled funds deliver negative alphas as they charge fees to unsophisticated investors. While other economic mechanisms could produce fund heterogeneity, introducing search costs is intuitive given the complex process for evaluating hedge funds (*e.g.*, Gârleanu and Pedersen, 2018; Lhabitant, 2007).

V.D. Return Decomposition Over Time

V.D.1. Hedge Funds versus Mutual Funds

The universe of hedge funds has expanded substantially since 1994. As a result, the average return decomposition may be subject to notable time trends. Using the CP model, we track the evolution of (i) the average alpha component $\hat{M}^k_{1,ac} = \frac{1}{n_\chi}\sum_i^{n_\chi} \hat{ac}^k_i$, (ii) the average beta component due to the market $\hat{M}^k_{1,bc,m} = \frac{1}{n_\chi}\sum_i^{n_\chi} \hat{bc}^k_{i,m}$, and (iii) the average beta component due to the non-market (nm) factors (all factors but the market) $\hat{M}^k_{1,bc,I_{nm}} = \frac{1}{n_\chi}\sum_i^{n_\chi} \hat{bc}^k_{i,I_{nm}}$. We start the analysis at the end of 2003 and estimate each cross-sectional average using the entire return history for each fund up to that point in time. As we move forward in time, we expand the set of return observations and add new hedge funds once they satisfy the fund selection rule. For comparison purposes, we conduct the same analysis for mutual funds using the traditional Carhart model.

Figure 3 identifies two sources of convergence between hedge funds and mutual funds. First, performance becomes increasingly similar—at the end of 2020, the gap in average alphas drops to 1.6%. Understanding the drivers of this trend is an important topic for hedge fund researchers. One intuitive explanation is the presence of scalability constraints. As a result of the growth of the hedge fund industry, it becomes increasingly difficult to maintain the same performance level. Bollen, Joenväärä, and Kauppila (2021) find support for this explanation but also suggest that central bank interventions might have reduced the profitability of hedge fund strategies.

Second, hedge funds load increasingly on the equity market. At the end of 2020, $\hat{M}^{k}_{1,bc,m}$ reaches its highest level at 2.3% per year. Whereas it remains smaller than for mutual funds (7.8%), it follows the same trend after the 2008 crisis. This trend is relevant for investors eager to capture hedge fund alphas. Given the sizable market exposure of hedge funds, they need a proper hedging strategy to offset this risk and maintain an optimal allocation to the equity market.

Please insert Figure 3 here

V.D.2. Differences Between Models

Next, we examine whether the superiority of the CP model is stable over time. For sake of brevity, we compare it with the CAPM and the Fung-Hsieh model. For each of these models, we compute the difference in average alpha relative to the CP model between 2003 and 2020. Figure 4 reveals that the CP model always delivers smaller alphas as hedge funds consistently use non-market factors to boost their returns. The average contribution of these factors captured by $\hat{M}^{k}_{1,bc,I_{nm}}$ represents at least 52% of the average hedge fund return during the period.

The gap between models slightly narrows over time as $\hat{M}^{k}_{1,bc,I_{nm}}$ falls from 4.4% to 2.9% per year. We find no evidence that hedge funds reduce their factor exposures over time. Instead, the decrease in $\hat{M}^{k}_{1,bc,I_{nm}}$ is caused by the reduction in alternative factor premia.[25] An open question is whether this trend will continue in the future. On the one hand, variance, carry, and TS momentum

[25]For instance, the premium on carry drops from 8.8% to 6.8% per year between 2003 and 2020.

have high Sharpe ratios—an observation made by Dew-Becker et al. (2017), Koijen et al. (2018), and Moskowitz, Ooi, and Pedersen (2012). As a result, investors should have incentives to exploit these factors and drive down their returns. On the other hand, Ilmanen et al. (2021) go back to 1926 and find little evidence of arbitrage activities that permanently reduce alternative factor premia.

The large difference between the Fung-Hsieh and CP models may be surprising as one might expect the straddles to overlap with the variance factor. Conceptually, there are two reasons why this is not the case. First, a straddle position benefits from trends, but not necessarily from variance. For example, consider a highly volatile period for an asset during which the ending price is the same as the starting price. In this case, the payoff of the straddle is zero, whereas the payoff of the variance factor is large. Second, the straddles in the Fung-Hsieh model cover the bond, currency, and commodity markets. In contrast, the variance factor focuses on the equity market.

Please insert Figure 4 here

V.E. The Sophistication of Hedge Fund Investors

V.E.1. Hedge Fund Valuation

In addition to sharpening the estimation of the distributions ϕ^*_{ac} and ϕ^*_{bc}, our comparison approach measures how investors with different sophistication value hedge funds. We first consider a sophisticated investor able to replicate all five additional factors (illiquidity, BAB, variance, carry, TS momentum). The valuation of this hypothetical investor is given by the alpha under the CP model. We can formalize this intuition using the stochastic discount factor (SDF) framework. Writing the investor SDF m^k_t as a linear function of CP factors, we have $\alpha^k_i = (1 + r_f)E[m^k_t r_{i,t}]$, where r_f is the risk-free rate. A positive α^k_i signals that the investor can increase his overall utility by investing in the fund (*e.g.*, Chen and Knez, 1996; Ferson, 2013).[26] Next, we consider a less sophisticated investor who can only invest in the equity market. His hedge fund valuation is then given by the CAPM alpha: $\alpha^0_i = (1 + r_f)E[m^0_t r_{i,t}]$, where m^0_t is a linear function of the market.

Table VI shows that the average valuation is close to zero for the CP investor (0.4% per year), but substantially higher for the CAPM investor (2.9%). This valuation gap is consistent with intuition. Like the CP investor, the CAPM investor values the alpha component ac^*_i. In addition,

[26]See also Almeida, Ardison, and Garcia (2020) and Karehnke and de Roon (2020) for a recent application of the SDF framework in which investors have nonlinear preferences (*i.e.*, m_t is a nonlinear function of the factors).

he values the beta component due to all the nonreplicable strategies. This component, denoted by $bc^*_{i,O}$ in Equation (5), includes the contributions of the additional factors in the CP model.[27]

V.E.2. Measuring Sophistication Using Flows

As a final exercise, we study the sophistication of real-world hedge fund investors. Our analysis builds on the premise that investors learn about funds by observing past returns. As they update their valuation over time, they reallocate capital accordingly. If this mechanism is at play, fund flows contain information about investor preference for alpha and beta. With misspecified models, the learning process is likely to be noisier than for mutual funds. To address this issue, we compare the entire distributions of the alpha and beta components between low- and high-flow funds.[28]

We proceed in three steps. First, we follow Barras, Scaillet, and Wermers (2010) and partition our data into non-overlapping subperiods of five years, beginning with 1996 to 2000 and ending with 2016 to 2020. For each subperiod, we include all funds that pass the fund selection rule and compute their average monthly flows and return decomposition obtained with the CP model ($\hat{ac}_i^k$, $\hat{bc}_{i,m}^k$, $\hat{bc}_{i,I_{nm}}^k$). Second, we sort funds into flow quintiles (from low to high) and pool these five-year records together across all time periods. Third, we apply our methodology to compute the distributions of the alpha and beta components for each (pooled) flow quintile.

Panel A of Table XI reveals that flows are primarily directed into funds with positive contemporaneous alphas. In the high-flow group, the alpha is equal to 3.3% per year on average and positive for 70.4% of the funds. In the low-flow group, these numbers drop substantially (-0.6% and 45.2%). In spite of these differences, the evidence is consistent with our premise that learning about fund alphas is quite noisy—we observe a large distribution overlap.

If investors are unsophisticated, they not only chase alphas, but also past or market-adjusted returns. In other words, they also direct capital into funds that load aggressively on the market and alternative factors (*i.e.*, funds with high $\hat{bc}_{i,m}^k$ and $\hat{bc}_{i,I_{nm}}^k$). Panels B and C do not support this interpretation. For the two sets of factors, the average beta components are actually larger in the low-flow group than in the high-flow group. In terms of sophistication, real-world investors are

[27]This point is well summarized by Cochrane (2011): "I tried telling a hedge fund manager, 'You don't have alpha. Your returns can be replicated with a value-growth, momentum, currency and term carry, and short-vol strategy.' He said, 'Exotic beta is my alpha. I understand those systematic factors and know how to trade them. My clients don't.'"

[28]An alternative approach, which does not capture fund heterogeneity, imposes a panel structure in which fund flows are regressed on fund return components (Barber, Huang, and Odean, 2016). Ben-David et al. (2022) argue that this approach produces spurious results because it overestimates the importance of alpha in driving flows.

therefore closer to the CP investor than the CAPM investor.

Please insert Table XI here

VI. Conclusion

Decomposing hedge fund returns is challenging because factor models are likely misspecified—that is, they omit relevant factors for capturing hedge fund strategies. Model misspecification makes the estimation of the alpha and beta components both imperfect and noisy. To mitigate these challenges, we develop a new approach to estimate and compare the distributions of the alpha and beta components across models. Our approach improves the imperfect separation between alpha and beta by identifying models less prone to misspecification. It also explicitly accounts for estimation noise based on a full-fledged asymptotic theory in a large cross-section of funds.

Our comparison analysis yields several insights. We find that the standard models are similar to the CAPM and thus ill-equipped for separating alpha and beta. In contrast, several economically motivated factors—primarily, TS momentum, variance, and carry—capture the returns of hedge funds in all investment categories. Including these factors increases the relative importance of the beta components and uncovers a gradual convergence in performance towards mutual funds. Another important finding is the large fund heterogeneity in alpha and beta. This dispersion substantially increases the uncertainty faced by investors during the fund selection process. It is also consistent with equilibrium models of active management that feature search costs.

Our methodology is flexible and can be applied in other situations where models are misspecified. For instance, it can be used to further improve the decomposition of hedge fund returns across specific investment categories. It could also be applied to international mutual funds for which the set of trading strategies is substantially larger than for traditional US equity funds.

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TABLE I. Summary Statistics for the Equal-Weighted Portfolio of Hedge Funds

This table provides summary statistics for the equal-weighted portfolio of all existing funds at the start of each month in the entire population and three investment categories: (i) equity funds (long-short, market neutral), (ii) macro funds (global macro, managed futures), and (iii) arbitrage funds (relative value, event driven). We report the annualized mean and standard deviation, skewness, kurtosis, and quantiles at 10% and 90% of the portfolio excess return. The statistics are computed using monthly data between January 1994 and December 2020.

	Moments				Quantiles	
	Mean	Std Dev.	Skewness	Kurtosis	10%	90%
All Funds	5.45	5.49	-0.31	4.51	-1.58	2.34
Equity	6.73	8.70	-0.47	5.32	-2.56	3.30
Long-Short	7.24	9.68	-0.44	5.31	-2.90	3.68
Market Neutral	3.07	2.72	-0.58	5.75	-0.66	1.13
Macro	5.05	5.79	0.24	3.30	-1.66	2.50
Global Macro	4.40	6.12	0.40	3.57	-1.68	2.59
Managed Futures	4.09	6.73	0.47	3.74	-1.79	2.92
Arbitrage	5.40	4.96	-2.16	14.57	-0.97	1.83
Relative Value	4.90	4.47	-2.46	18.03	-0.78	1.60
Event Driven	6.08	6.11	-1.74	11.79	-1.30	2.35

TABLE II. Summary Statistics for the Hedge Fund Factors

This table provides summary statistics for the factors used in the construction of the nine hedge fund models. Panel A reports the statistics for the US equity market and the standard factors used for performance evaluation. The size, value, momentum, investment, and profitability factors are computed using US equity data. The global value and momentum factors are constructed across multiple international asset classes. The term and default factors are computed using US bond data. The bond, commodity, and currency straddles are computed using option data on bonds, commodities, and currencies. Panel B reports the statistics for the the additional hedge fund factors. The illiquidity and betting-against-beta (BAB) factors are computed using US equity data. The variance factor is computed using index option data on the S&P500. The carry and time-series (TS) momentum factors are constructed across multiple international asset classes. We define the straddle and variance factors as short positions to obtain positive premia. We report the annualized mean and standard deviation, skewness, kurtosis, and quantiles at 10% and 90% of the excess returns of the factors. The statistics are computed using monthly data between January 1994 and December 2020.

Panel A: Market and Standard Factors	Moments				Quantiles	
	Mean	Std Dev.	Skewness	Kurtosis	10%	90%
Market	8.81	15.48	-0.64	4.26	-5.15	6.02
Size	1.61	10.83	0.38	7.54	-3.49	3.69
Value	0.20	10.88	0.05	5.84	-3.30	3.47
Momentum	4.67	17.15	-1.42	12.88	-5.07	5.50
Investment	2.11	7.04	0.67	5.09	-2.14	2.70
Profitability	3.77	9.38	-0.47	13.49	-2.03	2.90
Global Value	1.30	6.19	-0.64	12.25	-1.72	1.76
Global Momentum	3.26	7.55	-0.30	5.46	-2.25	2.69
Term	-0.18	0.89	-0.03	4.22	-0.35	0.29
Default	0.01	0.77	1.90	17.66	-0.20	0.19
Straddle on Bonds	16.00	57.86	-1.85	8.99	-19.68	17.73
Straddle on Commodities	2.99	50.38	-1.32	6.07	-19.71	15.74
Straddle on Currencies	11.27	69.02	-1.56	6.64	-23.36	20.15

Panel B: Alternative Factors	Moments				Quantiles	
	Mean	Std Dev.	Skewness	Kurtosis	10%	90%
Iliquidity	6.52	12.97	-0.27	4.49	-3.60	4.99
BAB	8.58	13.77	-0.36	6.10	-3.58	5.21
Variance	38.10	23.80	-4.57	29.76	-1.03	7.61
Carry	6.85	4.96	0.01	3.98	-1.10	2.34
TS Momentum	11.11	12.56	0.16	3.16	-3.54	5.71

TABLE III. The Set of Hedge Fund Models

This table summarizes the set of nine hedge fund models chosen for the empirical analysis. This set includes the CAPM as reference model as well as three distinct groups. The first group includes the standard models used in previous work, namely the Carhart, Five-Factor, Fung-Hsieh, and Asness-Moskowitz-Pedersen (AMP) models. The second group includes the two machine learning models of Kozak, Nagel, and Shantosh (KNS) formed with either five characteristic-based equity portfolios, or five principal components of these portfolios. The final group includes two models formed with the additional hedge fund factors. The first one is the model of Joenväärä et al. (2021) (JKKT). The second one combines the factors proposed by Carhart et al. (2014) and Pedersen (2018) (CP).

Model	List of Included Factors
Reference Model	
CAPM	Market
Standard Models	
Carhart	Market, Size, Value, Momentum
Five-Factor	Market, Size, Value, Investment, Profitability
Fung-Hsieh	Market, Size, Term, Default, Straddles (Bonds, Commodities, Currencies)
AMP	Market, Global Value and Momentum
Machine Learning Models	
KNS1	Market, Five Characteristic-Based Equity Factors
KNS2	Market, Five Principal Component Equity Factors
Additional Models	
JKKT	Market, Size, Value, Momentum, Illiquidity, BAB, TS Momentum
CP	Market, Size, Illiquidity, BAB, Variance, Carry, TS Momentum

TABLE IV. Model Misspecification Statistics

This table provides misspecification statistics for the CAPM, the four standard models (Carhart, Five-Factor, Fung-Hsieh, AMP), the two machine learning models (KNS1, KNS2), and the two models with the additional factors (JKKT and CP). For each model, we measure the relative importance of the three components of the average fund return: (i) the alpha component (Alpha), (ii) the beta component due to the market (Beta Mkt), and the beta component due to the non-market factors (Beta Non-Mkt). These proportions, which are averaged across all funds, sum up to 100%. We also compute the average adjusted R^2 of the time-series regression of the fund return on the factor returns. We conduct this analysis for the entire hedge fund population (first four columns) and for the entire mutual fund population (last four columns).

	Hedge Funds				Mutual Funds			
	Relative Importance (%)				Relative Importance (%)			
	Alpha	Beta Mkt	Beta Non-Mkt	R^2	Alpha	Beta Mkt	Beta Non-Mkt	R^2
CAPM	52.75	47.25	0.00	20.43	-15.23	115.23	0.00	81.34
Carhart	47.14	46.60	6.26	25.28	-17.16	110.57	6.58	88.98
Five-Factor	49.04	46.65	4.31	24.85	-16.38	110.27	6.11	88.99
Fung-Hsieh	54.10	40.56	5.34	30.24	-14.96	107.78	7.18	85.88
AMP	47.08	47.37	5.55	24.36	-15.37	115.01	0.36	84.55
KNS1	52.52	48.24	-0.76	24.47	-17.41	114.58	2.84	83.40
KNS2	64.01	51.03	-15.04	26.39	-6.27	115.99	-9.73	87.09
JKKT	17.97	44.28	37.75	31.01	-20.01	110.20	9.81	89.65
CP	6.64	41.19	52.17	30.26	-24.33	109.06	15.27	87.15

TABLE V. Formal Model Comparisons

This table compares the CAPM with the four standard models (Carhart, Five-Factor, Fung-Hsieh, AMP), the two machine learning models (KNS1, KNS2), and the two models with the additional factors (JKKT and CP). We compute the differences in characteristics between the cross-sectional distributions of the alpha components under the CAPM and each model. We report the differences in the annualized mean and standard deviation, the proportions of funds with negative and positive alphas, and the annualized quantiles at 10% and 90%. Figures in parentheses denote the standard deviation of the estimated differences. ***, **, * indicate that the null hypothesis of equal characteristics is rejected at the 1%, 5%, and 10% levels. Lack of differences signals that the model is as misspecified as the CAPM and thus ill-equipped to capture any alternative hedge fund strategies.

	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
Carhart	-0.31 (0.40)	-0.19 (0.31)	2.27 (2.50)	-2.27 (2.50)	-0.02 (0.40)	-0.60* (0.34)
Five-Factor	-0.21 (0.46)	0.02 (0.34)	2.41 (3.01)	-2.41 (3.01)	-0.01 (0.50)	-0.18 (0.40)
Fung-Hsieh	0.08 (0.60)	-0.20 (0.35)	0.52 (3.88)	-0.52 (3.88)	0.28 (0.56)	-0.15 (0.47)
AMP	-0.31 (0.42)	0.06 (0.37)	2.50 (2.44)	-2.50 (2.44)	-0.17 (0.37)	-0.34 (0.45)
KNS1	-0.01 (0.39)	0.43 (0.29)	1.59 (1.78)	-1.59 (1.78)	-0.26 (0.29)	0.19 (0.38)
KNS2	0.63 (0.49)	0.20 (0.31)	-2.47 (2.65)	2.47 (2.65)	0.58 (0.41)	0.65 (0.47)
JKKT	-1.93*** (0.67)	0.41 (0.50)	14.76*** (4.01)	-14.76*** (4.01)	-2.32*** (0.66)	-1.98*** (0.68)
CP	-2.56*** (0.76)	2.13*** (0.50)	19.82*** (4.27)	-19.82*** (4.27)	-4.22*** (0.73)	-1.42* (0.76)

TABLE VI. Decomposition of Average Fund Returns

This table shows the decomposition of average fund returns under the CAPM, the four standard models (Carhart, Five-Factor, Fung-Hsieh, AMP), the two machine learning models (KNS1, KNS2), and the two models with the additional factors (JKKT and CP). Panel A reports the characteristics of the cross-sectional distribution of the alpha components under each model. We report the annualized mean and standard deviation, the proportions of funds with negative and positive alphas, and the annualized quantiles at 10% and 90%. Figures in parentheses denote the standard deviation of the estimated characteristics. Panel B reports the characteristics of the cross-sectional distribution of beta components under each model.

Panel A: Distribution of the Alpha Components

	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
CAPM	2.93 (0.94)	7.01 (0.48)	27.15 (5.49)	72.85 (5.49)	-3.95 (0.67)	10.06 (0.67)
Carhart	2.62 (0.84)	6.82 (0.31)	29.42 (5.08)	70.58 (5.08)	-3.97 (0.57)	9.46 (0.55)
Five-Factor	2.72 (0.89)	7.03 (0.31)	29.55 (5.23)	70.45 (5.23)	-3.96 (0.63)	9.87 (0.54)
Fung-Hsieh	3.01 (0.74)	6.81 (0.29)	27.66 (3.60)	72.34 (3.60)	-3.67 (0.47)	9.90 (0.51)
AMP	2.62 (0.92)	7.08 (0.28)	29.65 (5.62)	70.35 (5.62)	-4.12 (0.59)	9.71 (0.54)
KNS1	2.92 (0.87)	7.44 (0.42)	28.73 (5.13)	71.27 (5.13)	-4.21 (0.63)	10.24 (0.60)
KNS2	3.56 (0.86)	7.21 (0.37)	24.68 (4.33)	75.32 (4.33)	-3.37 (0.55)	10.70 (0.62)
JKKT	1.00 (0.73)	7.42 (0.31)	41.90 (4.80)	58.10 (4.80)	-6.27 (0.60)	8.07 (0.38)
CP	0.37 (0.86)	9.15 (0.40)	46.97 (5.31)	53.03 (5.31)	-8.17 (0.77)	8.64 (0.44)

Panel B: Distribution of the Beta Components

	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
CAPM	2.62 (0.83)	4.37 (0.67)	22.54 (9.57)	77.46 (9.57)	-0.71 (0.61)	8.01 (0.73)
Carhart	2.94 (0.81)	4.30 (0.52)	17.99 (5.68)	82.01 (5.68)	-0.65 (0.42)	8.20 (0.72)
Five-Factor	2.83 (0.84)	4.48 (0.51)	20.61 (5.62)	79.39 (5.62)	-1.06 (0.41)	8.32 (0.75)
Fung-Hsieh	2.55 (0.78)	4.39 (0.52)	22.58 (5.98)	77.42 (5.98)	-1.23 (0.44)	7.96 (0.67)
AMP	2.94 (0.84)	4.66 (0.49)	18.98 (5.41)	81.02 (5.41)	-0.96 (0.43)	8.56 (0.73)
KNS1	2.64 (0.80)	5.14 (0.57)	23.04 (5.71)	76.96 (5.71)	-1.37 (0.56)	8.38 (0.86)
KNS2	2.00 (0.82)	4.47 (0.49)	27.53 (7.96)	72.47 (7.96)	-1.61 (0.59)	7.28 (0.69)
JKKT	4.56 (0.83)	5.90 (0.42)	13.90 (2.20)	86.10 (2.20)	-0.50 (0.28)	11.12 (0.74)
CP	5.19 (0.93)	7.64 (0.49)	15.91 (2.17)	84.09 (2.17)	-1.17 (0.35)	12.78 (1.11)

TABLE VII. Economic Importance of the Hedge Fund Factors

This table measures the economic importance of each factor in the CP model as a driver of fund returns. Panel A reports the characteristics of the cross-sectional distribution of the beta components due to each factor. By construction, the average total beta component is equal to the sum of the average factor beta components. We report the annualized mean and standard deviation, the proportions of funds with negative and positive contributions, and the annualized quantiles at 10% and 90%. Figures in parentheses denote the standard deviation of the estimated characteristics. Panel B reports the cross-sectional correlation between the beta components for each pair of factors.

Panel A: Distribution of the Beta Components for Each Factor

	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
Market	2.29 (1.27)	3.84 (1.19)	21.62 (6.13)	78.38 (6.13)	-0.55 (0.15)	6.99 (2.05)
Size	0.23 (0.26)	1.01 (0.32)	38.23 (4.26)	61.77 (4.26)	-0.31 (0.03)	1.02 (0.53)
Illiquidity	0.05 (0.07)	1.26 (0.29)	44.94 (3.25)	55.06 (3.25)	-0.75 (0.24)	0.85 (0.38)
Betting Against Beta	0.35 (0.25)	2.22 (0.69)	35.14 (3.80)	64.86 (3.80)	-1.30 (0.44)	2.23 (0.90)
Variance	0.78 (0.25)	5.17 (0.70)	36.74 (1.89)	63.26 (1.89)	-3.08 (0.54)	4.53 (0.87)
Carry	0.43 (0.17)	2.56 (0.46)	39.21 (2.38)	60.79 (2.38)	-1.89 (0.37)	2.73 (0.51)
Time-Series Momentum	1.06 (0.32)	4.03 (0.78)	42.86 (1.51)	57.14 (1.51)	-1.35 (0.34)	4.49 (0.72)

Panel B: Correlations Between the Beta Components

	Size	Illiquidity	BAB	Variance	Carry	TS Mom
Market	-0.08	-0.01	-0.01	-0.03	-0.02	-0.18
Size		-0.07	-0.00	-0.06	0.06	0.05
Illiquidity			-0.09	0.14	-0.02	0.08
Betting Against Beta				-0.07	0.03	-0.05
Variance					0.04	-0.09
Carry						-0.13

TABLE VIII. Decomposition of Average Fund Returns – Investment Categories

This table shows the decomposition of average fund returns under the CAPM and the two models with the additional factors (JKKT and CP) across investment styles. Panel A reports the characteristics of the cross-sectional distribution of the alpha and beta components across equity funds (long-short, market neutral). We report the annualized mean and standard deviation, the proportions of funds with negative and positive alphas, and the annualized quantiles at 10% and 90%. Figures in parentheses denote the standard deviation of the estimated characteristics. Panels B and C repeat the analysis for macro funds (global macro, managed futures) and arbitrage funds (relative value, event driven).

Panel A: Equity Funds						
	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
Distribution of the Alpha Components						
CAPM	2.40 (1.06)	7.09 (0.48)	30.23 (6.31)	69.77 (6.31)	-4.52 (0.83)	9.54 (0.96)
JKKT	1.12 (0.81)	6.98 (0.31)	42.98 (5.48)	57.02 (5.48)	-5.73 (0.75)	7.77 (0.66)
CP	0.58 (0.96)	9.08 (0.49)	47.38 (5.46)	52.62 (5.46)	-7.84 (1.02)	8.53 (0.79)
Distribution of the Beta Components						
CAPM	4.18 (1.16)	5.19 (0.70)	12.94 (8.25)	87.06 (8.25)	-0.14 (0.74)	10.66 (1.39)
JKKT	5.46 (1.18)	5.75 (0.54)	10.59 (2.37)	89.41 (2.37)	-0.10 (0.43)	12.19 (1.41)
CP	6.00 (1.31)	7.58 (0.61)	13.69 (2.77)	86.31 (2.77)	-0.75 (0.56)	13.71 (1.77)

Panel B: Macro Funds						
	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
Distribution of the Alpha Components						
CAPM	3.71 (1.72)	8.04 (0.78)	25.73 (6.47)	74.27 (6.47)	-4.26 (0.89)	12.41 (2.14)
JKKT	-0.14 (1.42)	9.14 (0.65)	51.52 (7.66)	48.48 (7.66)	-9.25 (1.43)	8.70 (0.75)
CP	-0.39 (1.65)	11.04 (0.70)	51.83 (7.02)	48.17 (7.02)	-10.91 (1.58)	10.26 (1.00)
Distribution of the Beta Components						
CAPM	1.01 (1.08)	3.70 (0.68)	43.44 (21.78)	56.56 (21.78)	-1.76 (1.29)	5.44 (0.66)
JKKT	4.87 (1.11)	7.49 (0.74)	20.32 (3.33)	79.68 (3.33)	-1.57 (0.44)	12.86 (1.30)
CP	5.11 (1.33)	9.62 (0.79)	23.74 (3.89)	76.26 (3.89)	-2.60 (0.63)	14.18 (1.35)

Panel C: Arbitrage Funds						
	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
Distribution of the Alpha Components						
CAPM	2.81 (1.24)	5.61 (0.44)	24.74 (9.71)	75.26 (9.71)	-2.79 (1.31)	8.67 (0.63)
JKKT	1.97 (0.89)	5.70 (0.29)	31.03 (8.00)	68.97 (8.00)	-3.66 (0.92)	7.60 (0.50)
CP	0.87 (1.06)	6.81 (0.44)	41.64 (9.30)	58.36 (9.30)	-5.48 (1.20)	7.62 (0.49)
Distribution of the Beta Components						
CAPM	2.31 (0.98)	3.02 (0.66)	13.63 (8.51)	86.37 (8.51)	-0.08 (0.45)	5.88 (1.22)
JKKT	3.14 (0.81)	3.57 (0.42)	11.60 (3.04)	88.40 (3.04)	-0.09 (0.32)	7.28 (1.03)
CP	4.25 (0.98)	4.90 (0.56)	10.86 (2.29)	89.14 (2.29)	-0.10 (0.33)	9.42 (1.37)

TABLE IX. Economic Importance of the Hedge Fund Factors – Investment Categories

This table measures the economic importance of each factor in the CP model as a driver of fund returns across investment styles. Panel A reports the characteristics of the cross-sectional distributions of the beta components due to each factor across equity funds (long-short, market neutral). By construction, the average total beta component is equal to the sum of the average factor beta components. We report the annualized mean and standard deviation, the proportions of funds with negative and positive contributions, and the annualized quantiles at 10% and 90%. Figures in parentheses denote the standard deviation of the estimated characteristics. Panel B and C repeat the analysis for macro funds (global macro, managed futures) and arbitrage funds (relative value, event driven).

Panel A: Equity Funds	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
Market	3.67 (2.07)	4.53 (1.22)	14.54 (8.16)	85.46 (8.16)	-0.19 (0.61)	9.33 (3.18)
Size	0.42 (0.57)	1.32 (0.42)	32.68 (11.70)	67.32 (11.70)	-0.35 (0.11)	1.65 (1.04)
Illiquidity	0.14 (0.10)	1.49 (0.35)	44.13 (2.66)	55.87 (2.66)	-0.85 (0.25)	1.27 (0.48)
Betting Against Beta	0.25 (0.21)	2.66 (0.80)	38.48 (3.90)	61.52 (3.90)	-1.71 (0.55)	2.55 (1.05)
Variance	0.67 (0.25)	5.06 (0.70)	38.58 (2.19)	61.42 (2.19)	-3.12 (0.54)	4.46 (0.85)
Carry	0.34 (0.17)	2.80 (0.46)	44.78 (2.67)	55.22 (2.67)	-2.11 (0.41)	2.73 (0.52)
Time-Series Momentum	0.51 (0.17)	2.83 (0.55)	42.08 (2.04)	57.92 (2.04)	-1.56 (0.46)	3.05 (0.74)
Panel B: Macro Funds	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
Market	1.18 (0.62)	3.47 (1.10)	35.11 (4.40)	64.89 (4.40)	-1.40 (0.35)	4.87 (1.62)
Size	0.07 (0.09)	0.83 (0.20)	50.59 (9.02)	49.41 (9.02)	-0.43 (0.28)	0.64 (0.20)
Illiquidity	-0.08 (0.10)	1.26 (0.31)	54.07 (3.37)	45.93 (3.37)	-0.90 (0.37)	0.75 (0.28)
Betting Against Beta	0.21 (0.19)	1.99 (0.65)	40.83 (4.63)	59.17 (4.63)	-1.30 (0.47)	1.84 (0.76)
Variance	0.22 (0.35)	6.56 (0.87)	49.41 (3.11)	50.59 (3.11)	-4.73 (0.92)	4.55 (0.76)
Carry	0.41 (0.26)	2.79 (0.57)	41.89 (3.33)	58.11 (3.33)	-2.43 (0.58)	3.23 (0.64)
Time-Series Momentum	3.10 (1.11)	5.83 (1.23)	24.43 (3.26)	75.57 (3.26)	-0.73 (0.12)	10.22 (2.20)
Panel C: Arbitrage Funds	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
Market	1.68 (0.90)	2.52 (0.75)	16.96 (6.38)	83.04 (6.38)	-0.19 (0.21)	4.40 (1.44)
Size	0.16 (0.20)	0.60 (0.17)	32.82 (8.47)	67.18 (8.47)	-0.11 (0.04)	0.56 (0.45)
Illiquidity	0.07 (0.12)	0.87 (0.18)	36.89 (7.20)	63.11 (7.20)	-0.40 (0.06)	0.60 (0.32)
Betting Against Beta	0.61 (0.39)	1.76 (0.58)	25.35 (4.61)	74.65 (4.61)	-0.68 (0.14)	2.15 (0.95)
Variance	1.47 (0.42)	3.34 (0.55)	21.90 (2.39)	78.10 (2.39)	-0.78 (0.08)	4.59 (0.97)
Carry	0.56 (0.20)	1.91 (0.34)	29.67 (3.66)	70.33 (3.66)	-0.86 (0.17)	2.37 (0.52)
Time-Series Momentum	-0.30 (0.19)	1.53 (0.37)	62.12 (4.12)	37.88 (4.12)	-1.64 (0.57)	0.82 (0.17)

TABLE X. Fund Characteristics and Alpha

This table examines how the alpha component varies across groups of funds sorted on fund characteristics. Panel A reports the characteristics of the cross-sectional distribution of the alpha component for groups sorted on proxies of managerial incentives, which are management fees, performance fees, and a dummy that takes a value of one if the fund has a high water mark provision. We set the cutoff levels at the usual 2% for management fees and 20% for performance fees. We report the annualized mean and standard deviation, the proportions of funds with negative and positive contributions, and the annualized quantiles at 10% and 90%. Figures in parentheses denote the standard deviation of the estimated characteristics. Panel B repeats the analysis using proxies of managerial flexibility, which are two dummies that take a value of one if the fund imposes a lockup period or a notice period.

Panel A: Managerial Incentives	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
Low Management Fees	0.01 (0.81)	8.03 (0.38)	49.04 (5.91)	50.96 (5.91)	-7.63 (0.84)	7.39 (0.48)
High Management Fees	1.28 (1.27)	11.41 (0.59)	41.72 (4.99)	58.28 (4.99)	-10.08 (1.32)	11.31 (0.67)
Low Performance Fees	-1.53 (0.98)	7.30 (0.41)	60.77 (7.43)	39.23 (7.43)	-8.17 (1.18)	5.44 (0.50)
High Performance Fees	1.11 (0.90)	9.70 (0.41)	41.44 (5.08)	58.56 (5.08)	-8.20 (0.80)	9.85 (0.51)
No High Water Mark	-1.44 (1.35)	9.97 (0.74)	58.38 (5.48)	41.62 (5.48)	-10.23 (1.20)	7.48 (0.83)
High Water Mark	1.17 (0.87)	8.82 (0.43)	41.55 (5.58)	58.45 (5.58)	-7.39 (0.89)	9.36 (0.46)

Panel B: Managerial Flexibility	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
No Lockup Period	0.03 (0.85)	9.23 (0.39)	48.77 (5.27)	51.23 (5.27)	-8.52 (0.87)	8.12 (0.51)
Lockup Period	1.77 (1.00)	8.86 (0.50)	37.40 (6.06)	62.60 (6.06)	-6.26 (1.17)	10.08 (0.58)
No Notice Period	-0.62 (1.45)	9.72 (0.56)	56.50 (6.22)	43.50 (6.22)	-9.48 (1.22)	8.72 (1.11)
Notice Period	0.73 (0.87)	9.03 (0.41)	43.67 (5.63)	56.33 (5.63)	-7.72 (0.90)	8.71 (0.49)

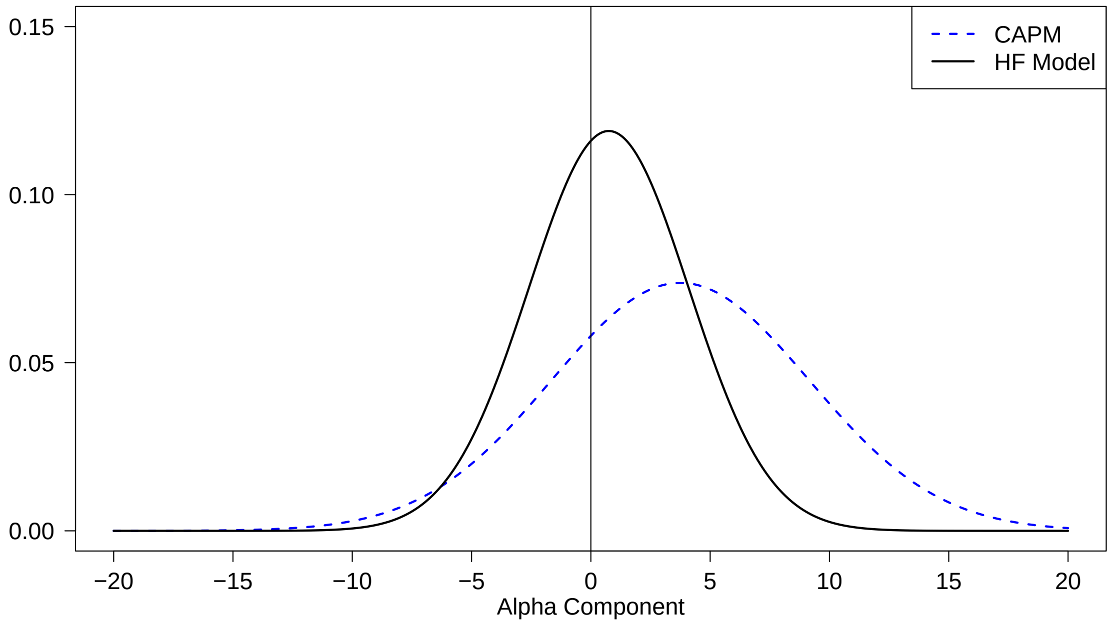
TABLE XI. Fund Flows and Investor Sophistication

This table measures how investor flows contemporaneously respond to the different components of the average fund return under the CP model: (i) the alpha component, (ii) the beta component due to the market, and (iii) the beta component due to the non-market factors (size, illiquidity, BAB, variance, carry, TS momentum). For each of the non-overlapping five-year periods between 1996 and 2020, we measure the three components for all funds. We then rank them according to their average monthly net flows and group them into quintiles (low, 2, 3, 4, high). Panel A reports the characteristics of the cross-sectional distribution of the alpha components (pooled over all five-year periods) for the five flow groups. We report the annualized mean and standard deviation, the proportions of funds with negative and positive alphas, and the annualized quantiles at 10% and 90%. Figures in parentheses denote the standard deviation of the estimated characteristics. Panels B and C repeat the analysis for the beta components due to the market and the non-market factors.

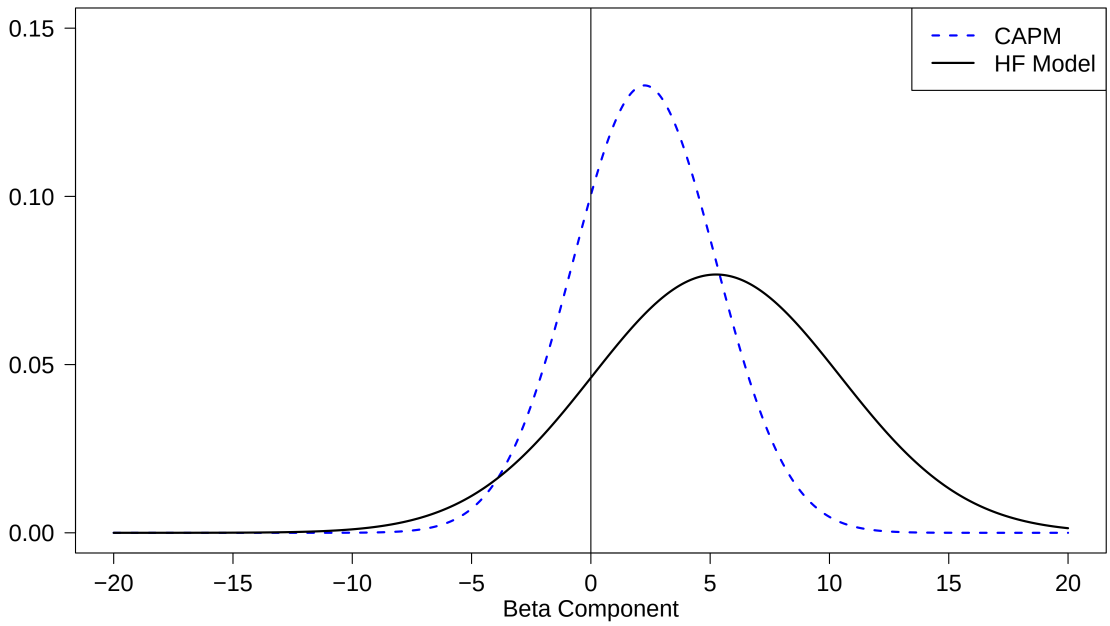
	Panel A: Distribution of the Alpha Components					
	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
Low	-0.57 (1.54)	10.71 (0.79)	54.84 (5.88)	45.16 (5.88)	-10.53 (1.33)	9.55 (0.74)
2	0.39 (1.33)	9.03 (0.72)	47.82 (5.66)	52.18 (5.66)	-9.01 (0.98)	9.15 (0.83)
3	1.52 (1.32)	8.42 (0.76)	41.49 (6.10)	58.51 (6.10)	-7.18 (0.90)	10.33 (0.65)
4	2.92 (1.44)	8.85 (0.82)	34.57 (6.90)	65.43 (6.90)	-5.97 (1.18)	13.00 (0.56)
High	3.25 (1.24)	9.87 (0.61)	29.61 (5.31)	70.39 (5.31)	-4.92 (0.98)	13.18 (0.74)
	Panel B: Distribution of the Beta Components (Market Factor)					
	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
Low	2.32 (1.98)	4.19 (1.75)	18.19 (19.41)	81.81 (19.41)	-0.53 (0.84)	8.37 (3.00)
2	3.15 (2.45)	5.09 (1.75)	14.25 (18.87)	85.75 (18.87)	-0.17 (1.19)	10.34 (4.27)
3	2.91 (2.26)	4.94 (1.84)	15.85 (18.60)	84.15 (18.60)	-0.17 (1.13)	9.48 (4.06)
4	2.40 (1.84)	4.42 (1.74)	19.96 (15.33)	80.04 (15.33)	-0.33 (0.71)	8.29 (3.30)
High	1.95 (1.45)	3.90 (1.61)	23.07 (13.06)	76.93 (13.06)	-0.46 (0.45)	7.31 (2.63)
	Panel C: Distribution of the Beta Components (Non-Market Factors)					
	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
Low	2.38 (0.90)	8.18 (3.58)	32.13 (8.54)	67.87 (8.54)	-4.43 (1.93)	10.44 (3.66)
2	2.13 (0.83)	7.36 (3.03)	35.93 (12.43)	64.07 (12.43)	-4.26 (1.59)	9.60 (3.57)
3	2.14 (0.81)	6.94 (3.28)	32.07 (11.95)	67.93 (11.95)	-4.23 (2.02)	9.34 (3.59)
4	2.30 (0.78)	7.16 (3.19)	34.30 (8.29)	65.70 (8.29)	-3.88 (1.79)	9.63 (3.59)
High	2.37 (0.81)	7.82 (3.09)	33.70 (6.23)	66.30 (6.23)	-3.81 (1.70)	8.65 (3.46)

Figure 1. Distributions of the Alpha and Beta Components – A Simple Example

This table compares the average return decomposition obtained with a candidate hedge fund (HF) model and the CAPM. Panel A plots the cross-sectional distributions of the alpha components (annualized) under both models. In this simple example, the average fund returns are explained by four factors (the market and three alternative factors 1, 2, and 3 with similar premia), and hedge funds load more aggressively on factor 1 than on factors 2 and 3. Whereas the CAPM omits factors 1, 2, and 3, the HF model only omits factor 3. Panel B repeats the analysis for the beta components.

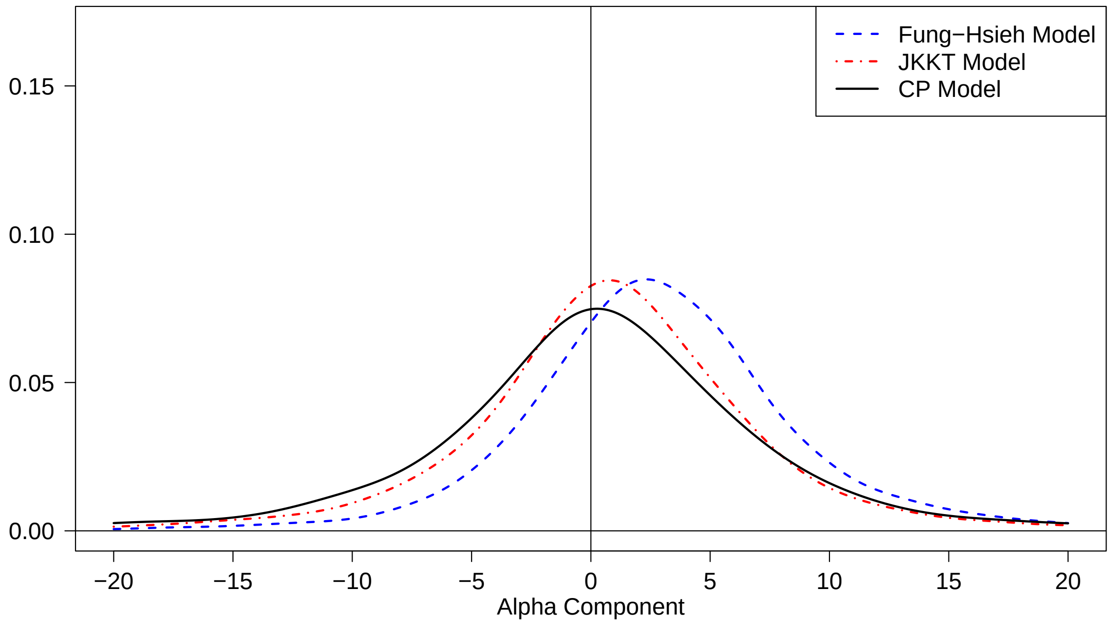


(a) Distribution of the Alpha Components

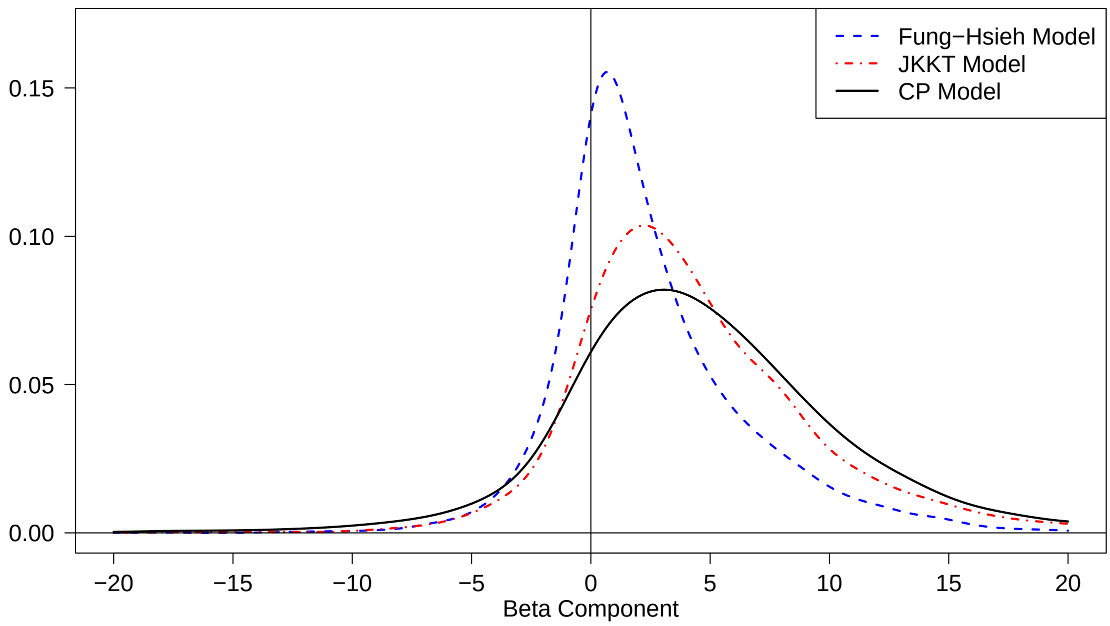


(b) Distribution of the Beta Components

Figure 2. Distributions of the Alpha and Beta Components
This figure provides a visual comparison of the Fung-Hsieh model and the two models with the additional factors (JKKT and CP). Panel A plots the cross-sectional distributions of the alpha components (annualized) under the three models. Panel B repeats the analysis for the beta components.



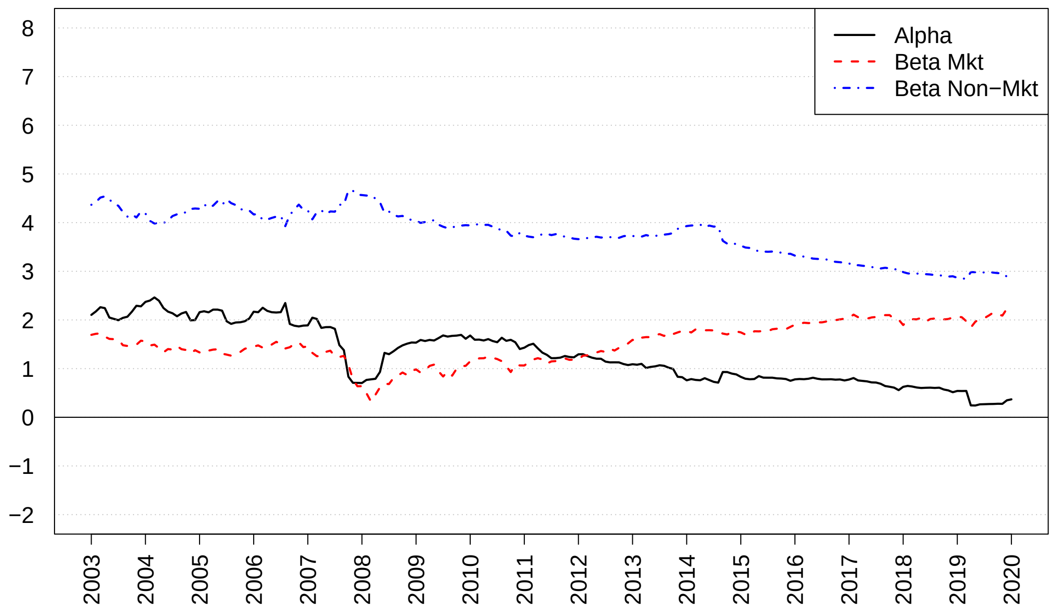
(a) Distributions of the Alpha Components



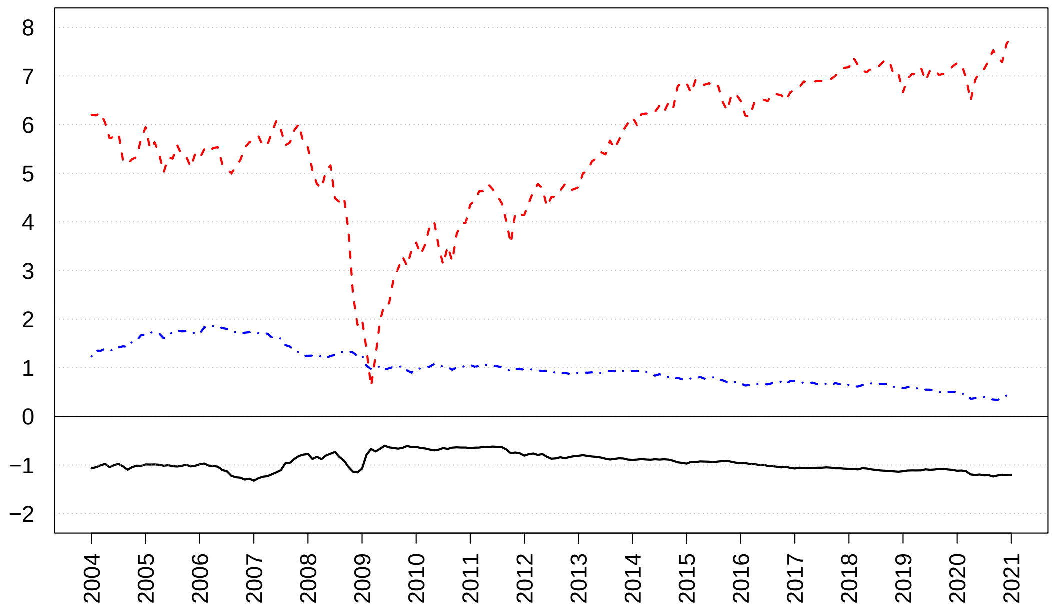
(b) Distributions of the Beta Components

Figure 3. Time-Variation in the Average Return Components

This figure compares the return decomposition for hedge funds and mutual funds. Panel A plots the evolution of the average return components (annualized) under the CP model: (i) the alpha component (Alpha), (ii) the beta component due to the market (Beta Mkt), and the beta component due to the non-market factors (Beta Non-Mkt), which are size, illiquidity, BAB, variance, carry, and TS momentum. At the end of each month, we measure the three components of each fund using its entire return history up to that point and take cross-sectional averages across all existing funds. The initial estimates cover the period 1994 to 2003, while the last ones cover the entire period 1994 to 2020. Panel B repeats the analysis for mutual funds using the traditional Carhart (1997) model, which includes size, value, and momentum as non-market factors.



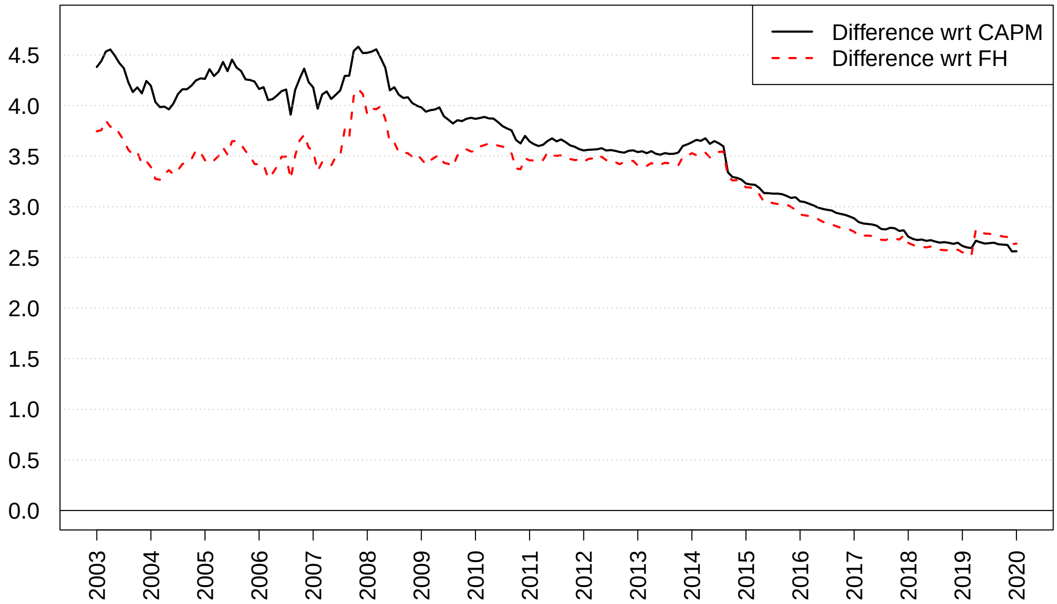
(a) Hedge Funds



(b) Mutual Funds

Figure 4. Time-Variation in the Differences between Models

This figure plots the evolution of the differences in average alphas (annualized) between: (i) the CAPM and the CP model, and (ii) the Fung-Hsieh model and the CP model. At the end of each month, we measure the alpha of each fund under all three models, take cross-sectional averages across all existing funds, and then compute the differences between models. The initial estimates cover the period 1994 to 2003, while the last ones cover the entire period 1994 to 2020. The differences in the average beta components are not reported because they are identical to the ones obtained with the alpha component (with opposite signs).



— Internet Appendix —

Is it Alpha or Beta? Decomposing Hedge Fund Returns When Models are Misspecified

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This appendix is divided into four sections. Section I contains the proofs of the propositions discussed in the paper (including the regularity assumptions), provides the list of the terms for computing the asymptotic variance of the different estimators, and shows how to extend the methodology from the distribution of the alpha components to the distribution of the beta components. Section II presents the Monte-Carlo analysis for examining the properties of the estimators. Section III describes the construction of the hedge fund dataset and the different factors. Section IV reports additional empirical results on (i) the misspecification diagnostic criterion, (ii) model comparisons with different data filters, (iii) model comparisons with alternative reference models, (iv) the impact of factor trading costs, (v) the return decomposition for multi-strategy funds and funds of funds, (vi) the economic importance of the hedge fund factors across investment subcategories, and (vii) the return decomposition under a style-based version of the CP model.

I. Methodology

I.A. Regularity Assumptions

To begin, we list the required assumptions underlying the results of Propositions 1 to 3. In particular, we need assumption A.7 to obtain non-zero asymptotic variances for the different estimators and guarantee that the limiting Gaussian distributions are all well-defined. We use a generic univariate function $g = g(\alpha_i)$ to simplify the presentation and avoid vectorial notations, and apply the compact notation $g^{(1)}$ and $g^{(2)}$ for its first- and second-order derivatives. We also omit the superscript k to lighten the notation when clarity permits.

Assumption A.1. *The individual effects* $\gamma_i^* = \left(\alpha_i^*, b_{i,I}^{*\prime}, b_{i,O}^{*\prime}\right)'$, *with* $i = 1, ..., n$, *are i.i.d. with continuous distribution,* $E[\|b_{i,O}^*\|^2] < \infty$, *and are independent of the factors and the errors.*

Assumption A.2. *The observability indicator processes* $(I_{i,t})$, *with* $i = 1, ..., n$, *are i.i.d., such that* $(I_{i,t})$ *is strictly stationary with mean* τ_i^{-1} *for any given* i, *and independent of the individual effects, the factors, and the error processes.*

Assumption A.3. *The factor process* $f_t = (f_{I,t}', f_{O,t}')'$ *is strictly stationary, such that* $E[\|f_{I,t}\|^8] < \infty$ *and satisfies the central limit theorem (CLT):* $\frac{1}{\sqrt{T}}\sum_t (u_{O,t} \otimes x_t) \to_d N(0, \Omega_{ux})$, *as* $T \to \infty$, *where* $\Omega_{ux} = \lim_{T\to\infty} V\left[\frac{1}{\sqrt{T}}\sum_t u_{O,t} \otimes x_t\right]$.

Assumption A.4. *The error process* $(\varepsilon_{i,t}^*)$ *is such that* $\frac{1}{nT}\sum_{i,j}\sum_{t,s} E\left[E[\varepsilon_{i,t}^*\varepsilon_{j,s}^*|f_{\underline{T}}, \gamma_i^*, \gamma_j^*]^2\right]^{1/2} \leq C$, *for a constant* C *and all* n, T *where* $f_{\underline{t}} = \{f_t, f_{t-1}, \ldots\}$.

Assumption A.5. *The trimming sequences $\chi_{1,T}$ and $\chi_{2,T}$ are such that $\chi_{1,T} = O((\log T)^{\kappa_1})$ and $\chi_{2,T} = O((\log T)^{\kappa_2})$, for $\kappa_1, \kappa_2 > 0$.*

Assumption A.6. *The function g is twice differentiable, and such that $E[|g(\alpha_i)|^2] < \infty$, $E[|g^{(1)}(\alpha_i)|^8] < \infty$, and $E[|g^{(2)}(\alpha_i)|^4] < \infty$.*

Assumption A.7. *For any pair of models k and l ($k, l = 0, ..., K-1$), we have: (1) $\eta^k_{M_s} \neq 0$ and $\eta^l_{M_s} \neq 0$, (2) $\eta^k_P(a) \neq 0$ and $\eta^l_P(a) \neq 0$, (3) $\eta^k_{M_s}(u^k_{O,t} \otimes x^k_t) - \eta^l_{M_s}(u^l_{O,t} \otimes x^l_t)$ is not the zero process, and (4) $\eta^k_P(a)(u^k_{O,t} \otimes x^k_t) - \eta^l_P(a)(u^l_{O,t} \otimes x^l_t)$ is not the zero process.*

I.B. Proofs of Propositions 1 and 2

We now prove Proposition 1 on the estimated characteristics of the alpha distribution under a given model k. To simplify notation, we drop the superscript k for the proof. We have

$$\begin{aligned} \hat{\alpha}_i &= E_1' \hat{Q}_{x,i}^{-1} \frac{1}{T_i} \sum_t I_{i,t} x_t r_{i,t} = \alpha_i + E_1' \hat{Q}_{x,i}^{-1} \frac{1}{T_i} \sum_t I_{i,t} x_t \varepsilon_{i,t} \\ &= \alpha_i + \frac{\tau_{i,T}}{\sqrt{T}} E_1' \hat{Q}_{x,i}^{-1} \left(\frac{1}{\sqrt{T}} \sum_t I_{i,t} x_t \varepsilon^*_{i,t} \right) + \frac{\tau_{i,T}}{\sqrt{T}} E_1' \hat{Q}_{x,i}^{-1} \left(\frac{1}{\sqrt{T}} \sum_t I_{i,t} x_t u_t' \right) b^*_{i,O} \\ &=: \alpha_i + \frac{1}{\sqrt{T}} \eta_{i,T}. \end{aligned} \tag{A1}$$

By a second-order Taylor expansion,

$$g(\hat{\alpha}_i) = g(\alpha_i) + \frac{1}{\sqrt{T}} g^{(1)}(\alpha_i) \eta_{i,T} + \frac{1}{2T} g^{(2)}(\bar{\alpha}_i) \eta^2_{i,T}, \tag{A2}$$

where $\bar{\alpha}_i$ is between $\hat{\alpha}_i$ and α_i for all i. Thus, we get

$$\sqrt{T} \left(\frac{1}{n} \sum_i g(\hat{\alpha}_i) \mathbf{1}^\chi_i - E\left[g(\alpha_i)\right] \right) \tag{A3}$$

$$= \sqrt{T} \left(\frac{1}{n} \sum_i g(\alpha_i) - E\left[g(\alpha_i)\right] \right) - \sqrt{T} \frac{1}{n} \sum_i g(\alpha_i) \left(1 - \mathbf{1}^\chi_i\right) \tag{A4}$$

$$+ \frac{1}{n} \sum_i \mathbf{1}^\chi_i g^{(1)}(\alpha_i) \tau_{i,T} E_1' \hat{Q}_{x,i}^{-1} \left(\frac{1}{\sqrt{T}} \sum_t I_{i,t} x_t \varepsilon^*_{i,t} \right) \tag{A5}$$

$$+ \frac{1}{n} \sum_i \mathbf{1}^\chi_i g^{(1)}(\alpha_i) \tau_{i,T} E_1' \hat{Q}_{x,i}^{-1} \left(\frac{1}{\sqrt{T}} \sum_t I_{i,t} x_t u'_{O,t} \right) b^*_{i,O} \tag{A6}$$

$$+ \frac{1}{2\sqrt{T} n} \sum_i \mathbf{1}^\chi_i g^{(2)}(\bar{\alpha}_i) \eta^2_{i,T} =: I_1 + I_2 + I_3 + I_4 + I_5. \tag{A7}$$

We control the five terms separately, the leading term being the fourth one and the others being asymptotically negligible, *i.e.*, of probability order $o_p(1)$.

i) Proof that $I_1 = o_p(1)$. By Assumptions A.1 and A.6, and the standard CLT, we have $I_1 = O_p(\sqrt{T/n})$. By using $T/n = o(1)$, it follows $I_1 = o_p(1)$.

ii) Proof that $I_2 = o_p(1)$. We have $E[|I_2|] \leq \sqrt{T}E[|g(\alpha_i)|(1-\mathbf{1}_i^{\chi})] \leq \sqrt{T}E[|g(\alpha_i)|^2]^{1/2}P[\mathbf{1}_i^{\chi} = 0]^{1/2}$, by the Cauchy-Schwarz inequality. By Lemma 7 in Gagliardini, Ossola, and Scaillet (2016), $P[\mathbf{1}_i^{\chi} = 0] = O(T^{-\bar{b}})$, for any $\bar{b} > 0$. From Assumption A.6, $E[|I_2|] = o(1)$.

iii) Proof that $I_3 = o_p(1)$. We have

$$E[I_3^2|f_{\underline{T}}, \gamma_i^*, I_{i,\underline{T}}, i = 1, ..., n] \tag{A8}$$

$$= \frac{1}{n^2T}\sum_{i,j}\sum_{t,s}\mathbf{1}_i^{\chi}\mathbf{1}_j^{\chi}g^{(1)}(\alpha_i)g^{(1)}(\alpha_j)\tau_{i,T}\tau_{j,T}E_1'\hat{Q}_{x,i}^{-1}I_{i,t}x_tE_1'\hat{Q}_{x,j}^{-1}I_{i,s}x_sE[\varepsilon_{i,t}^*\varepsilon_{j,s}^*|f_{\underline{T}}, \gamma_i^*, \gamma_j^*]. \tag{A9}$$

By using $\mathbf{1}_i^{\chi}\tau_{i,T} \leq \chi_{2,T}$ and $\mathbf{1}_i^{\chi}\|\hat{Q}_{x,i}^{-1}\| \leq C\chi_{1,T}$ for a generic constant C (see Gagliardini, Ossola, and Scaillet (2016), proof of Lemma 3), we get

$$E[I_3^2|f_{\underline{T}}, \gamma_i^*, I_{i,\underline{T}}, i = 1, ..., n] \leq \frac{C\chi_{1,T}^2\chi_{2,T}^2}{n^2T}\sum_{i,j}\sum_{t,s}|g^{(1)}(\alpha_i)|\,|g^{(1)}(\alpha_j)|\,\|x_t\|\|x_s\|\,|E[\varepsilon_{i,t}^*\varepsilon_{j,s}^*|f_{\underline{T}}, \gamma_i^*, \gamma_j^*]|. \tag{A10}$$

By the Cauchy-Schwarz inequality, we get

$$E[I_3^2] \leq C\chi_{1,T}^2\chi_{2,T}^2E[|g^{(1)}(\alpha_i)|^8]^{1/4}E[\|x_t\|^8]^{1/4}\frac{1}{n^2T}\sum_{i,j}\sum_{t,s}E\left[|E[\varepsilon_{i,t}^*\varepsilon_{j,s}^*|f_{\underline{T}}, \gamma_i^*, \gamma_j^*]|^2\right]^{1/2}. \tag{A11}$$

From Assumptions A.3-A.6, we get $E[I_3^2] = o(1)$.

iv) Proof that $I_4 \to_d N\left(0, V_g\right)$. We have

$$I_4 = \frac{1}{n}\sum_i g^{(1)}(\alpha_i)\tau_iE_1'Q_x^{-1}\left(\frac{1}{\sqrt{T}}\sum_t I_{i,t}x_tu_{O,t}'\right)b_{i,O}^* + o_p(1) \tag{A12}$$

$$= E_1'Q_x^{-1}\frac{1}{\sqrt{T}}\sum_t x_tu_{O,t}'\left(\frac{1}{n}\sum_i I_{i,t}\tau_ig^{(1)}(\alpha_i)b_{i,O}^*\right) + o_p(1), \tag{A13}$$

where $\tau_i = E[I_{i,t}|\gamma_i^*]^{-1}$ by Assumption A.2. By Assumptions A.1 and A.2, the cross-sectional average $\frac{1}{n}\sum_i I_{i,t}\tau_ig^{(1)}(\alpha_i)b_{i,O}^*$ converges in probability to the expectation $E[I_{i,t}\tau_ig^{(1)}(\alpha_i)b_{i,O}^*]$. Moreover, we have the chain of equalities: $E[I_{i,t}\tau_ig^{(1)}(\alpha_i)b_{i,O}^*] = E[E[I_{i,t}|\gamma_i^*]\tau_ig^{(1)}(\alpha_i)b_{i,O}^*] =$

$E[g^{(1)}(\alpha_i)b^*_{i,O}]$. This expectation is finite by Assumptions A.1 and A.6. Thus, we get

$$I_4 = E_1' Q_x^{-1} \frac{1}{\sqrt{T}} \sum_t x_t u_{O,t}' E[g^{(1)}(\alpha_i)b^*_{i,O}] + o_p(1). \tag{A14}$$

Now, we use $x_t u_{O,t}' E[g^{(1)}(\alpha_i)b^*_{i,O}] = \left(E[g^{(1)}(\alpha_i)b^*_{i,O}]' \otimes I_{d+1}\right)(u_{O,t} \otimes x_t)$, where d denotes the number of factors included in the model. Thus, we get

$$\left(E[g^{(1)}(\alpha_i)b^*_{i,O}]' \otimes E_1' Q_x^{-1}\right) \frac{1}{\sqrt{T}} \sum_t u_{O,t} \otimes x_t \to_d N(0, V_g), \tag{A15}$$

by Assumption A.3, and the result follows.

v) Proof that $I_5 = o_p(1)$. We have

$$\frac{1}{n} \sum_i \mathbf{1}_i^{\chi} g^{(2)}(\bar{\alpha}_i)\eta_{i,T}^2 = E[g^{(2)}(\alpha_i)\eta_{i,T}^2] + o_p(1), \tag{A16}$$

from Assumptions A.1-A.6, and the result follows.

For the proportion and quantile estimators, we proceed in a similar manner even if the indicator $g(\alpha) = \mathbf{1}\{\alpha \leq a\}$ is not differentiable. To understand intuitively the asymptotic variance of the proportion estimator, we can consider that the derivative of the indicator function is minus the Dirac function $g^{(1)}(\alpha) = -\delta(\alpha - a)$ in the sense of distribution theory. Thus, we have $E[g^{(1)}(\alpha_i)b^*_{i,O}] = -\int \delta(\alpha - a)m(\alpha)d\alpha = -m(a)$, where $m(a) = E[b^*_{i,O}|\alpha_i = a]\phi_{ac}(a)$. By plugging this expression in Equation (A15), we obtain the asymptotic distribution of the proportion estimator. The asymptotic distribution of the quantile estimator is derived from that of the cdf estimator by means of the Bahadur (1966) representation $\hat{Q}(u) - Q(u) = -\frac{1}{\phi_{ac}(Q(u))}\left(\hat{P}(Q(u)) - u\right) + o_p(1)$.

Next, we turn to the proof of Proposition 2, which examines the difference in characteristics between models. Whereas our empirical analysis focuses on the comparison between various models and the CAPM, the results in Proposition 2 are general and apply to any pair of models k and l (which can be nested or non-nested). We can view Proposition 2 as a corollary of Proposition 1. For each characteristic (moments, proportion, quantile), we simply need to work with $g(\hat{\alpha}_i^k) - g(\hat{\alpha}_i^l)$ substituted for $g(\hat{\alpha}_i)$, and apply the delta method to obtain the results.

I.C. Proof of Proposition 3

In this section, we provide the theoretical arguments to show that the asymptotic variance estimators are consistent. We focus on the estimation of the asymptotic variance of Proposition 1, namely

$$V_g = \left(E[g^{(1)}(\alpha_i)b_{i,O}^*]' \otimes E_1'Q_x^{-1}\right)\Omega_{ux}\left(E[g^{(1)}(\alpha_i)b_{i,O}^*] \otimes Q_x^{-1}E_1\right), \tag{A17}$$

using the notations of appendix I.A. Because the arguments are similar for the consistency of the estimators of the asymptotic variances in Proposition 2, we omit their lengthy developments.

The asymptotic variance V_g depends on the omitted factors and their loadings. We can still estimate it without knowing them through the pseudo-residuals defined as $\hat{\varepsilon}_{i,t} = r_{i,t} - \hat{\gamma}_i'x_t$, where $\hat{\gamma}_i = (\hat{Q}_{x,i})^{-1}\frac{1}{T_i}\sum_t I_{i,t}x_t r_{i,t}$ is the vector of coefficients of the time-series regression in Equation (4) of the paper. We build

$$\hat{V}_g = \frac{1}{n^2T}\sum_i\sum_j\sum_t \mathbf{1}_i^\chi \tau_{i,T}I_{i,t}\mathbf{1}_j^\chi \tau_{j,T}I_{j,t}\hat{a}_{i,t}\hat{a}_{j,t}, \tag{A18}$$

where $\hat{a}_{i,t} = E_1'\hat{Q}_x^{-1}g^{(1)}(\hat{\alpha}_i)\hat{\varepsilon}_{i,t}x_t$, $i = 1, ..., n$.[1] To simplify the presentation, we assume a scalar omitted factor, and we treat vector x_t as a scalar in some terms.[2] The pseudo-residuals are

$$\hat{\varepsilon}_{i,t} = r_{i,t} - \hat{\gamma}_i'x_t = \varepsilon_{i,t}^* + b_{i,O}^*u_{O,t} - (\hat{\gamma}_i - \gamma_i)'x_t. \tag{A19}$$

Then, we have $\hat{V}_g = E_1'\hat{Q}_x^{-1}I_6\hat{Q}_x^{-1}E_1$, where

$$I_6 := \frac{1}{n^2T}\sum_i\sum_j\sum_t \mathbf{1}_i^\chi \tau_{i,T}I_{i,t}\mathbf{1}_j^\chi \tau_{j,T}I_{j,t}g^{(1)}(\hat{\alpha}_i)\hat{\varepsilon}_{i,t}g^{(1)}(\hat{\alpha}_j)\hat{\varepsilon}_{j,t}x_tx_t'. \tag{A20}$$

By using Equation (A19) of the pseudo-residuals, we can decompose I_6 into six terms, the leading

[1] In the empirical analysis of the paper, we replace n with n_χ to obtain conservative estimators of the variance. This replacement has no effect on the asymptotic properties of the variance estimator derived in this section.

[2] For expository purpose, we only develop the case where both the error terms and the factors are independent across time. When the error terms and/or the factors are correlated across time, we need to modify the estimator by including weighted cross-terms at different dates (Newey and West, 1987).

term being the second one and the other five ones being asymptotically negligible

$$I_6 = \frac{1}{n^2T}\sum_i\sum_j\sum_t \mathbf{1}_i^{\chi}\tau_{i,T}I_{i,t}\mathbf{1}_j^{\chi}\tau_{j,T}I_{j,t}g^{(1)}(\hat{\alpha}_i)\varepsilon_{i,t}^*g^{(1)}(\hat{\alpha}_j)\varepsilon_{j,t}^*x_tx_t' \tag{A21}$$

$$+\frac{1}{T}\sum_t\left(\frac{1}{n}\sum_i \mathbf{1}_i^{\chi}\tau_{i,T}I_{i,t}g^{(1)}(\hat{\alpha}_i)b_{i,O}^*\right)^2 u_{O,t}^2x_tx_t' \tag{A22}$$

$$+\frac{1}{T}\sum_t\left(\frac{1}{n}\sum_i \mathbf{1}_i^{\chi}\tau_{i,T}I_{i,t}g^{(1)}(\hat{\alpha}_i)(\hat{\gamma}_i-\gamma_i)\right)^2 x_t^4 \tag{A23}$$

$$+\frac{2}{n^2T}\sum_i\sum_j\sum_t \mathbf{1}_i^{\chi}\tau_{i,T}I_{i,t}\mathbf{1}_j^{\chi}\tau_{j,T}I_{j,t}g^{(1)}(\hat{\alpha}_i)\varepsilon_{i,t}^*g^{(1)}(\hat{\alpha}_j)b_{j,O}^*u_{O,t}x_tx_t' \tag{A24}$$

$$-\frac{2}{n^2T}\sum_i\sum_j\sum_t \mathbf{1}_i^{\chi}\tau_{i,T}I_{i,t}\mathbf{1}_j^{\chi}\tau_{j,T}I_{j,t}g^{(1)}(\hat{\alpha}_i)\varepsilon_{i,t}^*g^{(1)}(\hat{\alpha}_j)(\hat{\gamma}_j-\gamma_j)x_t^3 \tag{A25}$$

$$-\frac{2}{T}\sum_t\left(\frac{1}{n}\sum_i \mathbf{1}_i^{\chi}\tau_{i,T}I_{i,t}g^{(1)}(\hat{\alpha}_i)b_{i,O}^*\right)\left(\frac{1}{n}\sum_i \mathbf{1}_i^{\chi}\tau_{i,T}I_{i,t}g^{(1)}(\hat{\alpha}_i)(\hat{\gamma}_i-\gamma_i)\right)u_{O,t}x_t^3 \tag{A26}$$

$$=: I_{61}+I_{62}+I_{63}+I_{64}+I_{65}+I_{66}\,. \tag{A27}$$

We control the six terms separately.

i) Proof that $I_{61}=o_p(1)$. We have

$$I_{61} = \frac{1}{n^2}\sum_i\sum_j \tau_i\tau_jg^{(1)}(\alpha_i)g^{(1)}(\alpha_j)\frac{1}{T}\sum_t I_{i,t}I_{j,t}\varepsilon_{i,t}^*\varepsilon_{j,t}^*x_t^2+o_p(1) \tag{A28}$$

$$= \frac{1}{n^2}\sum_i\sum_j g^{(1)}(\alpha_i)g^{(1)}(\alpha_j)\frac{1}{T}\sum_t E[\varepsilon_{i,t}^*\varepsilon_{j,t}^*x_t^2|\gamma_i^*,\gamma_j^*]+o_p(1) =: I_{611}+o_p(1)\,. \tag{A29}$$

From the Cauchy-Schwarz inequality and the law of iterated expectations, we have $E[\varepsilon_{i,t}^*\varepsilon_{j,t}^*x_t^2|\gamma_i^*,\gamma_j^*] = E[E[\varepsilon_{i,t}^*\varepsilon_{j,t}^*|f_{\underline{T}},\gamma_i^*,\gamma_j^*]x_t^2|\gamma_i^*,\gamma_j^*] \leq E[E[\varepsilon_{i,t}^*\varepsilon_{j,t}^*|f_{\underline{T}},\gamma_i^*,\gamma_j^*]^2|\gamma_i^*,\gamma_j^*]^{1/2}E[\|x_t\|^4]^{1/2}$. Thus, we get:

$$|I_{611}| \leq E[\|x_t\|^4]^{1/2}\frac{1}{n^2T}\sum_i\sum_j\sum_t |g^{(1)}(\alpha_i)||g^{(1)}(\alpha_j)|E[E[\varepsilon_{i,t}^*\varepsilon_{j,t}^*|f_{\underline{T}},\gamma_i^*,\gamma_j^*]^2|\gamma_i^*,\gamma_j^*]^{1/2}. \tag{A30}$$

By applying again the Cauchy-Schwarz inequality, we get

$$E[|I_{611}|] \leq E[\|x_t\|^4]^{1/2}E[|g^{(1)}(\alpha_i)|^4]^{1/2}\frac{1}{n^2T}\sum_i\sum_j\sum_t E[E[\varepsilon_{i,t}^*\varepsilon_{j,t}^*|f_{\underline{T}},\gamma_i^*,\gamma_j^*]^2]^{1/2}. \tag{A31}$$

From Assumptions A.3, A.4, and A.6, we get $E[|I_{611}|] = o(1)$ and thus $I_{611} = o_p(1)$.

ii) Proof that $I_{62} = E[g^{(1)}(\alpha_i)b^*_{i,O}]^2 E[u^2_{O,t}x_t x'_t] + o_p(1)$. We have

$$I_{62} = \frac{1}{T}\sum_t \left(\frac{1}{n}\sum_i \tau_i I_{i,t} g^{(1)}(\alpha_i) b^*_{i,O}\right)^2 u^2_{O,t}x_t x'_t + o_p(1) \tag{A32}$$

$$= \frac{1}{T}\sum_t E[\tau_i I_{i,t} g^{(1)}(\alpha_i) b^*_{i,O}]^2 u^2_{O,t}x_t x'_t + o_p(1), \tag{A33}$$

from Assumptions A.1 and A.2. Now, we have $E[\tau_i I_{i,t} g^{(1)}(\alpha_i) b^*_{i,O}] = E[g^{(1)}(\alpha_i) b^*_{i,O}]$, and this expectation is finite by Assumptions A.1 and A.6. Further, $\frac{1}{T}\sum_t u^2_{O,t}x_t x'_t = E[u^2_{O,t}x_t x'_t] + o_p(1)$, and the conclusion follows.

iii) Proof that $I_{63} = o_p(1)$. We have

$$I_{63} = \frac{1}{T}\sum_t \left(\frac{1}{n}\sum_i \tau_i I_{i,t} g^{(1)}(\alpha_i)(\hat{\gamma}_i - \gamma_i)\right)^2 x^4_t + o_p(1) \tag{A34}$$

$$= \frac{1}{T}\sum_t \left(\frac{1}{n}\sum_i \tau_i I_{i,t} g^{(1)}(\alpha_i) Q^{-1}_{x,i}\frac{1}{T}\sum_s I_{i,s}x_s\varepsilon_{i,s}\right)^2 x^4_t + o_p(1), \tag{A35}$$

since $\sup_i \mathbf{1}^\chi_i \|\hat{Q}^{-1}_{x,i} - Q^{-1}_{x,i}\| = O_p(T^{-c})$ for some $c > 0$ under Assumption A.5 (see Gagliardini, Ossola, and Scaillet, 2016, proof of Lemma 3 (iii), Equation (38)). Now, from Assumptions A.1 and A.2, we have $\frac{1}{n}\sum_i \tau_i I_{i,t} g^{(1)}(\alpha_i) Q^{-1}_{x,i}\frac{1}{T}\sum_s I_{i,s}x_s\varepsilon_{i,s} = E[\tau_i I_{i,t} g^{(1)}(\alpha_i) Q^{-1}_{x,i}\frac{1}{T}\sum_s I_{i,s}x_s\varepsilon_{i,s}] + o_p(1)$. By applying the law of iterated expectations, the conclusion comes from $E\left[\tau_i I_{i,t} g^{(1)}(\alpha_i) Q^{-1}_{x,i}\frac{1}{T}\sum_s I_{i,s}x_s\varepsilon_{i,s}\right] = 0$, since $E[\varepsilon_{i,s}|x_{\underline{T}}, I_{i,\underline{T}}, \gamma^*_i] = 0$.

iv) Proof that $I_{64} = o_p(1)$. We have

$$I_{64} = \frac{2}{n^2}\sum_i\sum_j \tau_i\tau_j g^{(1)}(\alpha_i) g^{(1)}(\alpha_j) b^*_{j,O}\frac{1}{T}\sum_t I_{i,t}I_{j,t}\varepsilon^*_{i,t}u_{O,t}x_t x'_t + o_p(1) \tag{A36}$$

$$= \frac{2}{n^2}\sum_i\sum_j g^{(1)}(\alpha_i) g^{(1)}(\alpha_j) b^*_{j,O}\frac{1}{T}\sum_t E[\varepsilon^*_{i,t}u_{O,t}x_t x'_t|\gamma^*_i, \gamma^*_j] + o_p(1). \tag{A37}$$

The result follows from the law of iterated expectations, and Assumption A.1 implying $E[\varepsilon^*_{i,t}|f_t, \gamma^*_i, \gamma^*_j] = E[\varepsilon^*_{i,t}|f_t] = 0$.

v) Proof that $I_{65} = o_p(1)$. We have

$$I_{65} = -\frac{2}{n^2}\sum_i\sum_j \tau_i\tau_j g^{(1)}(\alpha_i)g^{(1)}(\alpha_j)(\hat{\gamma}_j - \gamma_j)\frac{1}{T}\sum_t I_{i,t}I_{j,t}\varepsilon^*_{i,t}x_t^3 + o_p(1) \tag{A38}$$

$$= -\frac{2}{n^2}\sum_i\sum_j g^{(1)}(\alpha_i)g^{(1)}(\alpha_j)(\hat{\gamma}_j - \gamma_j)\frac{1}{T}\sum_t E[\varepsilon^*_{i,t}x_t^3|\gamma_i^*, \gamma_j^*] + o_p(1). \tag{A39}$$

The result follows from the law of iterated expectations, and Assumption A.1 implying $E[\varepsilon^*_{i,t}|f_t, \gamma_i^*, \gamma_j^*] = E[\varepsilon^*_{i,t}|f_t] = 0$.

vi) Proof that $I_{66} = o_p(1)$. We have

$$I_{66} = -\frac{2}{T}\sum_t \left(\frac{1}{n}\sum_i \tau_i I_{i,t}g^{(1)}(\alpha_i)b^*_{i,O}\right)\left(\frac{1}{n}\sum_i \tau_i I_{i,t}g^{(1)}(\alpha_i)(\hat{\gamma}_i - \gamma_i)\right)u_{O,t}x_t^3 + o_p(1) \tag{A40}$$

$$= -\frac{2}{T}\sum_t E[g^{(1)}(\alpha_i)b^*_{i,O}]E\left[\tau_i I_{i,t}g^{(1)}(\alpha_i)Q_{x,i}^{-1}\frac{1}{T}\sum_s I_{i,s}x_s\varepsilon_{i,s}\right]u_{O,t}x_t^3 + o_p(1). \tag{A41}$$

The conclusion comes from $E\left[\tau_i I_{i,t}g^{(1)}(\alpha_i)Q_{x,i}^{-1}\frac{1}{T}\sum_s I_{i,s}x_s\varepsilon_{i,s}\right] = 0$, by applying the law of iterated expectations and $E[\varepsilon_{i,s}|x_{\underline{T}}, I_{i,\underline{T}}, \gamma_i^*] = 0$.

Finally, by using $\hat{Q}_x^{-1} = Q_x^{-1} + o_p(1)$ and $E_1'Q_x^{-1}E[g^{(1)}(\alpha_i)b^*_{i,O}]^2E[u_{O,t}^2x_tx_t']Q_x^{-1}E_1 = V_g$, we deduce that $\hat{V}_g = V_g + o_p(1)$.

I.D. List of Terms for the Asymptotic Variance Estimators

To obtain the variance estimator for each characteristic in Proposition 1, we simply need to plug the correct $\hat{a}_{i,t}$ in Equation (A18). To mitigate the impact of outliers, we also winsorize the observed values $\hat{a}_{i,t}$ at 99%.

For the mean M_1, we have

$$\hat{a}_{i,t} = E_1'\hat{Q}_{x,i}^{-1}\hat{\varepsilon}_{i,t}x_t. \tag{A42}$$

For the standard deviation, we need to apply the delta method. Let us denote the derivative of the standard deviation M_2, w.r.t. $E[\alpha_i^j]$ by $\nabla_j M_2$. We get $\nabla_2 M_2 = (2M_2)^{-1}$, $\nabla_1 M_2 = -M_1/M_2$. For the second moment $E[\alpha_i^2]$, we have $\hat{a}_{i,t} = 2\hat{\alpha}_i E_1'\hat{Q}_x^{-1}\hat{\varepsilon}_{i,t}x_t$. We can build an estimate of the variance of the standard deviation from a weighted sum of the contributions corresponding to the moments of orders 2 and 1:

$$\hat{a}_{i,t} = \left(\widehat{\nabla_2 M_2} \times 2\hat{\alpha}_i + \widehat{\nabla_1 M_2}\right) E_1' \hat{Q}_{x,i}^{-1} \hat{\varepsilon}_{i,t} x_t, \tag{A43}$$

where $\widehat{\nabla_2 M_2}$ is a plug-in estimate of the derivative of M_2 w.r.t. $E[\alpha_i^2]$.

For the proportion $P(a)$ at point a, we can approximate the Dirac function by a smooth bump, namely a kernel function K, and take a vanishing bandwidth h. Hence, we can use

$$\hat{a}_{i,t} = -h^{-1} K((\hat{\alpha}_i - a)/h) E_1' \hat{Q}_{x,i}^{-1} \hat{\varepsilon}_{i,t} x_t, \tag{A44}$$

where K is a kernel function such that $K \geq 0$, $\int K(u)du = 1$, $\int uK(u)du = 0$, and $\int u^2 K(u)du < \infty$. In practice, we use a Gaussian kernel corresponding to the Gaussian density, and take the Silverman rule of thumb for the bandwidth selection, namely $h = 1.06 \hat{M}_2 n_\chi^{-1/5}$.

For the quantile $Q(u)$ of level u, we can rely on the Bahadur (1966) representation, and use

$$\hat{a}_{i,t} = -h^{-1} K((\hat{\alpha}_i - \hat{Q}(u))/h) \hat{\phi}_{ac}(\hat{Q}(u))^{-1} E_1' \hat{Q}_{x,i}^{-1} \hat{\varepsilon}_{i,t} x_t. \tag{A45}$$

Extending the above analysis to the characteristic differences in Proposition 2 is straightforward. For each estimated difference, we simply plug the appropriate $\hat{a}_{i,t}$ redefined as $\hat{a}_{i,t} = \hat{a}_{i,t}^k - \hat{a}_{i,t}^l$, where we obtain $\hat{a}_{i,t}^k$ and $\hat{a}_{i,t}^l$ for models k and l from the previous expressions in Equations (A42)-(A45).

I.E. Application of the Methodology to the Beta Component

Whereas our description of the methodology focuses on the distribution of the alpha component, we can apply the same arguments to the distribution of the beta component. For each fund, we simply need to replace the estimated alpha component $\hat{ac}_i^k$ with the estimated beta component $\hat{bc}_i^k = \hat{\mu}_i - \hat{ac}_i^k$, where $\hat{\mu}_i = \frac{1}{T_i} \sum_t I_{i,t} r_{i,t}$. Using this information, we can then compute the different characteristics of the cross-sectional distribution (mean, standard deviation, proportion, and quantile). We omit the detailed technical derivation of the asymptotic properties for the beta component since it parallels closely the lines and arguments used in the previous subsections for the alpha component. We can also adapt the regularity assumptions (Section I.A) to the case of the beta component in a straightforward manner. The same remarks apply to the estimate $\hat{bc}_{i,j}^{k}$ of the contribution associated with each factor j included in model k analysed the next section.

To compute the asymptotic variance terms for each distribution characteristic, we proceed as follows. Since the residuals $\hat{\varepsilon}_{i,t}^{k}$ are centered (their time series average is zero), we get the identity $0 = \frac{1}{T_i}\sum_t I_{i,t}r_{i,t} - \hat{ac}_i^k - \hat{bc}_i^k$, and thus $\hat{bc}_i^k = \frac{1}{T_i}\sum_t I_{i,t}r_{i,t} - \hat{ac}_i^k$. Hence, we can use this expression to get $\hat{bc}_i^k - bc_i^k = (\frac{1}{T_i}\sum_t I_{i,t}r_{i,t} - E[r_{it}]) - (\hat{ac}_i^k - ac_i^k)$. We then compute the term $\hat{a}_{i,t}$ to estimate the asymptotic variance of each estimated characteristic (as per Equation (A18)).

We then use the following estimated quantities $\hat{a}_{i,t}$ to build an estimate of the asymptotic variance based on the the average return and the pseudo-residuals $\hat{\varepsilon}_{i,t}^{k}$ inferred from the least-squares regression:

$$\hat{a}_{i,t} = g^{(1)}(\hat{bc}_i^k)r_{i,t} - E_1'(\hat{Q}_x^k)^{-1}g^{(1)}(\hat{bc}_i^k)\hat{\varepsilon}_{i,t}^k x_t^k \, . \tag{A46}$$

I.F. Application of the Methodology to the Factor Contribution

We can further apply our methodology to the cross-sectional distribution of the contribution associated with each factor j included in model k. For each fund, we simply need to replace the estimated alpha component $\hat{ac}_i^k$ with the estimated factor contribution $\hat{bc}_{i,j}^k = \hat{b}_{i,I,j}^k \hat{\lambda}_{I,j}^k$, where $\hat{\lambda}_{I,j}$ is the empirical average of $f_{I,t,j}^k$. Using this information, we can then compute the different characteristics of the cross-sectional distribution (mean, standard deviation, proportion, and quantile).

To compute the asymptotic variance terms, we apply the delta method to obtain $(\hat{b}_{i,I,j}^k - b_{i,I,j}^k)\lambda_{I,j}^k + b_{i,I,j}^k(\hat{\lambda}_{I,j}^k - \lambda_{I,j}^k)$. We then compute the term $\hat{a}_{i,t}$ to estimate the asymptotic variance of each estimated characteristic (as per Equation (A18)). This term depends on the residual $\hat{\varepsilon}_{i,t}^k$ obtained from the regression of the fund return on the factors included in model k. Formally, we have

$$\hat{a}_{i,t} = \hat{\lambda}_{I,j}^k E_{j+1}'(\hat{Q}_x^k)^{-1}g^{(1)}(\hat{bc}_{i,j}^k)\hat{\varepsilon}_{i,t}^k x_t^k + \hat{b}_{i,I,j}^k g^{(1)}(\hat{bc}_{i,j}^k) f_{I,t,j}^k \, , \tag{A47}$$

where E_{j+1} is a vector with one in the $j+1$ entry and zeros elsewhere.

II. Monte Carlo Analysis

II.A. Setup

We now conduct a Monte-Carlo analysis to evaluate the finite-sample properties of the estimated characteristics of the alpha distribution when the model is misspecified. We consider a hypothetical population of n funds with T return observations (n=1,000, 2,500, 5,000, 7,500, and 10,000; T=50, 100, 250, 500, and 1,000). Building on our example in Section II.C.3 of the paper, we model the fund excess return as

$$r_{i,t} = \alpha_i^* + b_{i,m}^* r_{m,t} + b_{i,1}^* f_{1,t} + b_{i,2}^* f_{2,t} + b_{i,3}^* f_{3,t} + \varepsilon_{i,t}^* \,, \tag{A48}$$

where $r_{m,t}$ is the market excess return, $f_{1,t}$, $f_{2,t}$, and $f_{3,t}$ denote the excess returns of three uncorrelated factors that track alternative strategies, and $\varepsilon_{i,t}^*$ is the fund residual. For each fund, the true alpha α_i^* is drawn from a normal $N(\mu_\alpha^*, \sigma_\alpha^{*2})$, $b_{i,m}^*$ from a normal $N(\mu_b^*, \sigma_b^{*2})$, and $b_{i,j}^*$ from a normal $N(\mu_{b_j}^*, \sigma_b^{*2})$, where $\mu_{b_j}^*$ is positive to capture the exposure of hedge funds to alternative strategies. We further assume that the first factor is a more important driver of hedge fund returns by setting $\mu_{b_1}^* = \mu_b^*$ and $\mu_{b_2}^* = \mu_{b_3}^* = \mu_b^*/3$.

To construct the return time-series for each iteration, we need to draw values for the factors and the fund residuals. We draw the market return $r_{m,t}$ from a normal $N(\lambda_m, \sigma_m^2)$, and the returns of the each alternative factor $f_{j,t}$ ($j = 1, 2, 3$) from a normal $N(\lambda_j, \sigma_j^2)$, where we set $\lambda_j = \lambda_m$ and $\sigma_j^2 = \sigma_m^2$ for simplicity. Finally, we draw $\varepsilon_{i,t}^*$ from a normal $N(0, \sigma_\varepsilon^{*2})$.

We use our monthly dataset to calibrate the parameters of the model. We set λ_m and σ_m^2 equal to the empirical average and variance of the equity market. We set μ_b^* and σ_b^* equal to the cross-sectional average and volatility of the fund market betas. Finally, we calibrate μ_α^*, σ_α^*, and σ_ε^{*2} using the values reported for mutual funds by Barras, Gagliardini, and Scaillet (2022).[3] This calibration yields the following values on a monthly basis: $\lambda = 0.63\%$, $\sigma_m = 4.36\%$, $\mu_b^* = 0.3$, $\sigma_b^* = 0.4$, $\sigma_\varepsilon^* = 1.67\%$, $\mu_\alpha^* = 0\%$, and $\sigma_\alpha^* = 0.13\%$.

In our simulations, we evaluate hedge fund performance using the CAPM. Given the above assumptions, the CAPM is misspecified because it does not include the three alternative factors (we have $f_{I,t} = r_{m,t}$ and $f_{O,t} = (f_{1,t}, f_{2,t}, f_{3,t})'$). We conduct a total of $S = 1{,}000$ simulation iterations. For each iteration s ($s = 1, ..., S$), we follow the following steps. First, we draw values for $\alpha_i^*(s), b_{i,m}^*(s), b_{i,1}^*(s), b_{i,2}^*(s)$, and $b_{i,3}^*(s)$ for each fund i ($i = 1, ..., n$). Second, we draw values

[3] The rationale for calibrating the values under the correct model using mutual fund data is that the issue of misspecification is far less severe than for hedge funds. We find that choosing alternative values does not change the finite-sample properties of the estimators.

for the factors

$$f_t(s) = (r_{m,t}(s), f_{1,t}(s), f_{2,t}(s), f_{3,t}(s))', \tag{A49}$$

for $t = 1, ..., T$ and the fund residuals $\varepsilon^*_{i,t}(s)$ for $i = 1, ..., n$ and $t = 1, ..., T$. Third, we construct the return time-series of each fund as

$$r_{i,t}(s) = \alpha^*_i(s) + b^*_{i,m}(s) r_{m,t}(s) + b^*_{i,1}(s) f_{1,t}(s) + b^*_{i,2}(s) f_{2,t}(s) + b^*_{i,3}(s) f_{3,t}(s) + \varepsilon^*_{i,t}(s) \,. \tag{A50}$$

Fourth, we estimate the CAPM alphas for each fund by regressing its return on the market:

$$\hat{\alpha}_i(s) = E_1'(\hat{Q}_{x,i}(s))^{-1} \frac{1}{T} \sum_t x_t(s) r_{i,t}(s) \,, \tag{A51}$$

where E_1 is a vector with one in the first position, $x_t(s) = (1, r_{m,t}(s))'$, and $\hat{Q}_{x,i}(s) = \frac{1}{T} \sum_t x_t(s) x_t(s)'$. Finally, we apply our approach to compute the distribution characteristics of the CAPM alpha distribution using as inputs the estimated alphas across funds $\hat{\alpha}_i(s)$ ($i = 1, ..., n$). We compute (i) the cross-sectional mean and standard deviation, $\hat{M}_1(s)$ and $\hat{M}_2(s)$, (ii) the proportion of funds with negative alphas $\hat{P}(0)(s)$, and (iii) the quantiles at 10% and 90%, $\hat{Q}(0.1)(s)$ and $\hat{Q}(0.9)(s)$.[4]

For each estimated characteristic $\hat{C} \in \{\hat{M}_1, \hat{M}_2, \hat{P}(0), \hat{Q}(0.1), \hat{Q}(0.9)\}$, we compute the mean squared error (MSE) as

$$\mathit{MSE}(\hat{C}) = \mathit{bs}^2(\hat{C}) + \sigma^2(\hat{C}) \,, \tag{A52}$$

where $\mathit{bs}(\hat{C})$ and $\sigma^2(\hat{C})$ denote the bias and variance of the estimator $\hat{C}$. These terms are given by

$$\mathit{bs}(\hat{C}) = \frac{1}{S} \sum_s \hat{C}(s) - C \,, \tag{A53}$$

$$\sigma^2(\hat{C}) = \frac{1}{S} \sum_s \left(\hat{C}(s) - \frac{1}{S} \sum_s \hat{C}(s) \right)^2 \,, \tag{A54}$$

where the population value C for each characteristic can be easily computed because the CAPM alpha distribution is normally distributed.

[4] We do not examine the estimated proportion of positive alpha funds whose properties are identical to $\hat{P}(0)(s)$.

II.B. Main Results

In Table AI, we report the MSE, bias, and standard deviation of the five estimated characteristics for the different combinations of T and n. We express the MSE in squared percent per month (multiplied by 100). We express the bias and standard deviation in percent per year for the mean, standard deviation, and quantiles, and in percent for the proportion of negative-alpha funds.

The simulation results are in line with the theoretical analysis in Proposition 2. First, the convergence rate of each estimator depends on T and not on n. As shown in the rightmost columns, the standard deviation decreases when the sample period increases. In contrast, increasing the population size does not produce more precise estimators because the omitted factors $f_{1,t}$, $f_{2,t}$, and $f_{3,t}$ have an impact on the estimated alphas of all funds simultaneously.

Second, the bias of each estimator vanishes relatively quickly as we increase the sample sizes n and T. As a result, it is smaller in magnitude than the standard deviation. To illustrate, we consider the proportion estimator under the scenario where $n = 5{,}000$ and $T = 100$, which provides a conservative analysis of our actual sample after trimming (*i.e.*, we have $n_\chi = \sum_{i=1}^{n} \mathbf{1}_i^\chi = 5{,}231$ and $T_\chi = \frac{1}{n_\chi} \sum_{i=1}^{n} \mathbf{1}_i^\chi T_i = 125$). Whereas the bias of the estimated proportion equals 2.9%, its standard deviation is around two times larger (5.7%).

Consistent with these results, we find that the MSE of the estimators (i) decreases with the number of observations T, and (ii) is primarily driven by the standard deviation, and not the bias. This analysis departs significantly from the well-specified case examined by Barras, Gagliardini, and Scaillet (2022). In their Monte-Carlo simulations reported in the appendix, we see that the standard deviation of the estimators decreases with the number of funds n. In addition, the bias dominates the standard deviation and thus requires an error-in-variable bias adjustment procedure.

Please insert Table AI here

III. Data Description

III.A. Construction of the Hedge Funds Dataset

We use monthly net-of-fee returns of individual funds (including dead funds) across four data providers (Barclayhedge, HFR, Morningstar, and TASS). The initial sample shown in Panel A of Table AII contains 65,142 funds that classify themselves across four investment categories: equity (long-short and market neutral), macro (global macro and managed futures), arbitrage (relative value and event driven), and other (multi-strategy and funds of funds). To map the specific investment styles used by each database into one of the four categories above, we apply the mapping proposed by Joenväärä et al. (2021).[5] We convert the fund returns into USD using the exchange rates at the end of the month retrieved from Bloomberg and remove monthly returns lower than -90% and above 300%.

We apply a set of filters to the initial population. For each database, we include the fund if it: (i) has more than 12 observations (in order to compute return correlations), (ii) reports continuously to the database, (iii) exhibits less than three consecutive zero returns, (iv) has a non-zero return volatility, and (v) reports in USD, EUR, GBP, or JPY. As shown in Panel B, these filters reduce the total size of the population to 40,169 funds.

Next, we remove the duplicates for each database. We use the fund manager ID to cluster funds based on a string matching approach based on the Jaro-Winkler distance (see Joenväärä, Kosowski, and Tolonen, 2016). Within each of these clusters, we identify funds with pairwise return correlations above 0.99, and keep one fund using the following priority rule: (i) maximum number of observations, (ii) largest average size, (iii) USD as reporting currency, and (iv) onshore.[6] Panel C shows that removing the duplicates reduces the total number of funds to 30,734.

Finally, we remove the duplicates across all four databases. To this end, we compute the pairwise correlations across all funds in the aggregated dataset to identify groups of funds with correlations above 0.99. For each group, we then keep one fund using the following priority rule: (i) maximum number of observations, (ii) largest average size, (iii) USD as reporting currency, and (iv) onshore. As shown in Panel D, the final sample size includes a total of 21,293 funds.

Please insert Tables AII here

III.B. Data Sources for the Factors

In this section, we provide additional information on the factors included in the standard models. We download the market, size, value, momentum, investment, and profitability factors from Ken

[5]We also use an earlier version of their paper (Joenväärä, Kosowski, and Tolonen, 2016) to obtain the mapping for long-short and market neutral funds.

[6]TASS does not provide information about the fund manager ID. To remove the duplicates in this database, we therefore conduct a correlation analysis on the entire population to detect funds with correlations above 0.99.

French's website. For the bond factors, we use the FRED database. The term factor is defined as the monthly change in the 10-year treasury constant maturity yield, and the default factor is defined as the monthly change in the Moody's Baa yield less the 10-year Treasury constant maturity yield. These two series capture changes in yields and thus provide an approximation of the return of the term and default strategies (using the duration formula). Data on the bond, currency, and commodity straddles are obtained from David Hsieh's website.

Turning to the description of the additional factors, we obtain the time series of the traded liquidity factor from Lubos Pastor's website. We obtain the return of the BAB strategy from the website of AQR. For the variance factor, we do not directly observe quotes of traded variance swaps on the S&P 500. Therefore, we use the FRED database to compute the difference between the monthly sum of the daily squared S&P 500 returns and the squared VIX (at the start of the month), divided by the squared VIX.[7] In the presence of jumps, our computation provides an approximation of the return of variance swaps (Martin, 2017). This approximation is quite accurate given that our summary statistics are in line with those reported by Dew-Becker et al. (2017) using actual swap quotes.[8] We download the return of the time-series momentum strategy from the website of AQR. For carry, we download the return time-series of the carry factors for equity, bonds (level and slope), currency, and commodity from Ralph Koijen's website. We then compute the average return of these five strategies (scaled by their volatility) to obtain the carry factor.[9]

IV. Additional Results

IV.A. Misspecification Diagnostic Criterion

We now provide additional information on the misspecification diagnostic proposed by Gagliardini, Ossola, and Scaillet (2019). This criterion computed for each model k is defined as

$$GOS^k = \mu_1^k(\hat{V}) - g(n_\chi, T)\,. \tag{A55}$$

[7]To compute the annualized statistics in Table II, we further divide the variance return by 10 to obtain similar magnitude as the other factors.

[8]They find that the average monthly excess return of the one-month swap is equal to -25.7% over the period 1995-2013 (see their Table II). We find a monthly average of -31.7% over the period 1994-2020.

[9]Carhart et al. (2014) and Pedersen (2015) also consider factors for real asset, quality, credit, and catastrophe bonds. We do not include them in the list of additional factors either because they are closely related to other factors (*e.g.*, quality is similar to profitability) or difficult to construct (*e.g.*, catastrophe bonds).

The first term μ_1^k is the largest eigenvalue of the matrix $\hat{V} = \frac{1}{n_\chi T} \sum_i \mathbf{1}_i^\chi \bar{\varepsilon}_i^k \bar{\varepsilon}_i^{k'}$, where $\bar{\varepsilon}_i^k$ is of size T and gathers the values $\bar{\varepsilon}_{i,t}^k / \sqrt{\frac{1}{T} \sum_t (\bar{\varepsilon}_{i,t}^k)^2}$ with $\bar{\varepsilon}_{i,t} = I_{i,t} \hat{\varepsilon}_{i,t}^k$. The second term $g(n_\chi, T)$ is the penalization equal to $\frac{n_\chi + T}{n_\chi T} \ln(\frac{n_\chi T}{n_\chi + T})$. As n and T converge to infinity, the criterion is positive with probability one if the model is misspecified and omits a strong factor. We find that all the models are misspecified because the value of GOS^k is always positive, both in the entire population and in each investment category (equity, macro, and arbitrage).

IV.B. Impact of Data Filters on Model Comparisons

In this section, we examine how different data filters impact the comparisons of models. We begin our analysis by changing the minimum number of observations. Our initial sample is free of survivorship bias because it includes both living and dead funds. However, our fund selection rule requires that each fund has a minimum number of return observations T_{min} to estimate its alpha. Our results could therefore be subject to survivorship bias if negative-alpha funds disappear early (*i.e.*, the reported alphas could be too high). At the same time, choosing a small T_{min} increases the severity of the reverse survivorship bias (Linnainmaa, 2013), which arises because some positive-alpha funds may perform unexpectedly poorly and disappear early (*i.e.*, the reported alphas could be too low). To examine these issues, we repeat our CAPM-based comparison using two alternative thresholds for T_{min} equal to 36 and 84.

Next, we use different filters to construct the hedge fund database. We apply the backfill bias correction proposed by Joenväärä et al. (2021), which eliminates all the return observations before the fund listing date.[10] We also apply the five filters proposed by Straumann (2009) and applied by Almeida, Ardison, and Garcia (2020) to remove errors in reported hedge fund returns. These filters are based on the number of returns equal to zero, the proportion of unique values, the repetition of identical values, the occurrence of identical sequences of returns, and the presence of rounding errors. Applying the filters of Straumann (2009) leads to a reduction in the number of selected funds in the three main categories (equity, macro, arbitrage) from 15,567 to 13,877.

For each of these changes, we formally compare the alpha distribution of each proposed model with that of the CAPM. The results in Table AIII show that the CAPM-based comparisons remain

[10]Whereas this alternative procedure provides a more stringent control of the backfill bias, it potentially discards important information about the fund performance by eliminating a large number of observations—in some cases, more than five years of data (see Aggarwal and Jorion, 2010; Fung and Hsieh, 2009, for a discussion)

robust to all these changes. Whereas the standard and machine learning models are similar to the CAPM, the JKKT and CP models produce sharp differences. These results are consistent with intuition—changing the data filters affects all models uniformly. It therefore leaves their differences unchanged.

Please insert Table AIII here

IV.C. Model Comparisons with Alternative Reference Models

In our baseline analysis, we use the CAPM as the reference model for the formal model comparisons. We now show that the superiority of the JKKT and CP holds when we replace the CAPM with any of the four standard models and the two machine learning models. The outcome of these formal comparisons is reported in Table AIV. For instance, Panel C shows that the CP model produces a highly significant reduction in the average alpha and in the proportion of positive-alpha funds relative to the Fung-Hsieh model (respectively equal to 2.6% per year and 19.3%).

Please insert Table AIV here

IV.D. Factor Trading Costs

In our baseline comparisons, we do not include the costs of trading the five alternative factors. To address this issue, we approximate these costs using estimates from previous studies. The costs of trading illiquidity, carry, and TS momentum are modest because these strategies are rebalanced annually or implemented in futures markets. For illiquidity, we use a value of 4.5 bps equal to the average cost estimate for size and value (Novy-Marx and Velikov, 2016). For carry and TS momentum, we choose a value of 9.7 bps, which is equal to the average estimated costs of rolling futures positions (Bollerslev et al., 2018). In contrast, the costs of trading the BAB and variance factors are significantly higher. For BAB, we take the estimate of Novy-Marx and Velikov (2022) equal to 60 bps. For variance, we use a value of 75 bps, which corresponds to the costs of trading variance swaps (Dew-Becker et al., 2017).

Consistent with intuition, Table AV shows that accounting for trading costs increases the alpha components. However, this increase is generally modest—the average alphas under the JKKT and CP models equal 1.3% and 1.0% per year (versus 1.0% and 0.4% without trading costs). As a result, these models still produce alpha distributions that depart significantly from the CAPM.

Please insert Table AV here

IV.E. Return Decomposition for Multi-Strategy Funds and Funds of Funds

We estimate the distributions of the alpha and beta components for two additional categories—multi-strategy funds and funds of funds. Consistent with our baseline results, Table AVI reveals that the JKKT and CP model produce a sizable reduction in the alpha component and a sizable increase in the beta component. In both categories, the decrease in fund alphas is particularly strong under the CP model. For multi-strategy funds, the average alpha is 0.1% and only 50.7% of the funds deliver positive alphas. Among funds of funds, the performance is even lower (-2.5% for the average alpha and 23.1% for the proportion of positive-alpha funds). Whereas it is well known that the alpha of these funds is hampered by their additional fees (*e.g.*, Agarwal, Mullally, and Naik, 2015), we find that the underperformance is worse than previously documented.

Please insert Table AVI here

IV.F. Economic Importance of Hedge Fund Factors within Investment Categories

We deepen the analysis of the economic importance of the hedge fund factors by splitting each investment category into two subcategories. For the equity category, we have long-short and market neutral funds. For the macro category, we have macro and managed futures funds. For the arbitrage category, we have relative value and event driven funds. For each subcategory, we apply our approach to estimate the cross-sectional distribution of the beta components due to each factor included in the CP model.

Consistent with our baseline results, Table AVII provides substantial evidence that hedge funds follow alternative strategies to boost their returns (*e.g.*, Carhart et al., 2014). Across the six categories and the five alternative factors, the proportion of funds with positive betas is above 50% in all but seven cases. The variation in factor loadings across subcategories is largely in line with economic intuition. Managed futures funds, which are known to exploit market trends, load extensively on TS momentum—its average contribution to the beta component reaches 3.7% per year. The BAB factor is particularly important among market neutral funds as it allows them to take advantage of leverage flexibility, while maintaining a neutral exposure to the market and various industries (see Pedersen, 2015). Long-short equity funds are exposed to the variance factor, possibly because it reduces the effectiveness of their hedging strategies (Buraschi, Kosowski, and Trojani, 2014). This is also the case for relative value funds, which commonly use option-based strategies (Duarte, Longstaff, and Yu, 2006), and for event-driven funds, which take short put positions when they engage in merger arbitrage (Mitchell and Pulvino, 2001).

Please insert Table AVII here

IV.G. The CP Model With Style-Specific Factors

In this section, we examine whether a style-based version of the CP model can do a better job than the original CP model at capturing hedge fund returns. To this end, we follow a simple approach which replaces the global carry and TS momentum factors with their style-specific counterparts. Implementing these changes is straightforward because the website of AQR provides detailed data on the TS momentum strategy across four asset classes (equity, bonds, currency, commodity). Similarly, the return time-series of the carry factors for equity, bonds (level and slope), currency, and commodity are available from Ralph Koijen's website.

We construct the equity model by replacing the global carry and TS momentum factors with the equity carry and TS momentum factors. The macro model replaces the global carry and TS momentum factors with the equal-weighted average of the currency and commodity carry and TS momentum factors. Finally, the arbitrage model replaces the global carry and TS momentum factors with the fixed income carry (level and slope) and TS momentum factors.

The results in Table AVIII reveal that the style-based model does not perform as well as the original CP model. In all three categories, we generally observe an increase in the fund alpha component (at the expense of the beta component). For instance, the average alphas are equal to 0.8%, 1.2%, and 1.0% per year (versus 0.6%, -0.4%, and 0.9% per year under the CP model). These results suggest that the three categories examined here are too broad to successfully fit a style-based version of the CP model.

Please insert Table AVIII here

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TABLE AI. Finite-Sample Properties of the Estimated Characteristics of the Alpha Distribution
This table reports the Mean Squared Error (MSE), bias, and standard deviation of the different characteristic estimators under the CAPM for different combinations for the numbers of funds n and return observations T. In the simulations, the average fund returns are explained by four factors (the market and three alternative factors 1, 2, and 3). The CAPM is misspecified because it omits factors 1, 2, and 3. We examine a total of five characteristics, which are the mean, standard deviation, proportions of funds with negative alphas, and quantiles at 10% and 90%. The bias and standard deviation are expressed in percent per year for the mean, standard deviation, and quantiles, and in percent for the proportion of negative-alpha funds.

Mean (True Value 3.75%)

MSE (x100)						Bias (Annualized)						Standard Deviation (Annualized)					
n\T	50	100	250	500	1000	n\T	50	100	250	500	1000	n\T	50	100	250	500	1000
1000	3.93	2.12	0.95	0.43	0.25	1000	0.13	-0.01	0.02	-0.02	0.02	1000	2.38	1.75	1.17	0.79	0.60
2500	4.11	2.19	0.87	0.45	0.22	2500	-0.01	-0.02	-0.04	0.02	0.04	2500	2.43	1.78	1.12	0.80	0.56
5000	4.24	2.03	0.94	0.45	0.24	5000	-0.01	-0.02	0.03	0.04	0.04	5000	2.47	1.71	1.16	0.80	0.59
7500	4.73	2.12	0.94	0.45	0.23	7500	0.02	0.06	0.02	0.02	0.04	7500	2.61	1.75	1.16	0.81	0.58
10000	4.39	2.31	0.81	0.42	0.23	10000	-0.13	-0.05	-0.04	0.03	0.01	10000	2.51	1.82	1.08	0.77	0.57

Volatility (True Value 7.35%)

MSE (x100)						Bias (Annualized)						Standard Deviation (Annualized)					
n\T	50	100	250	500	1000	n\T	50	100	250	500	1000	n\T	50	100	250	500	1000
1000	5.26	2.03	0.77	0.34	0.18	1000	2.00	1.00	0.45	0.19	0.11	1000	1.89	1.39	0.95	0.67	0.50
2500	4.97	2.08	0.67	0.34	0.18	2500	1.90	0.97	0.38	0.23	0.14	2500	1.88	1.44	0.91	0.66	0.48
5000	5.19	2.08	0.79	0.34	0.17	5000	1.96	1.01	0.45	0.22	0.12	5000	1.90	1.41	0.97	0.66	0.49
7500	5.42	2.17	0.77	0.34	0.18	7500	1.96	1.08	0.44	0.20	0.14	7500	1.99	1.40	0.96	0.67	0.48
10000	4.95	2.11	0.64	0.33	0.17	10000	1.83	0.98	0.39	0.22	0.12	10000	1.94	1.44	0.87	0.66	0.48

Probability $<$ 0 (True Value 30.5%)

MSE (x100)						Bias (Annualized)						Standard Deviation (Annualized)					
n\T	50	100	250	500	1000	n\T	50	100	250	500	1000	n\T	50	100	250	500	1000
1000	0.79	0.45	0.18	0.09	0.05	1000	4.38	2.90	1.31	0.70	0.23	1000	7.75	6.04	4.04	2.85	2.26
2500	0.92	0.46	0.17	0.08	0.04	2500	4.93	2.94	1.40	0.63	0.24	2500	8.22	6.07	3.86	2.69	1.93
5000	0.89	0.42	0.17	0.07	0.04	5000	5.01	2.95	1.28	0.53	0.22	5000	8.03	5.76	3.90	2.63	1.93
7500	1.00	0.45	0.17	0.07	0.04	7500	5.07	2.80	1.32	0.58	0.25	7500	8.60	6.07	3.91	2.65	1.91
10000	0.99	0.50	0.16	0.06	0.04	10000	5.35	3.14	1.42	0.58	0.34	10000	8.36	6.32	3.75	2.47	1.86

10th Percentile (True Value -5.67%)

MSE (x100)						Bias (Annualized)						Standard Deviation (Annualized)					
n\T	50	100	250	500	1000	n\T	50	100	250	500	1000	n\T	50	100	250	500	1000
1000	6.43	2.26	0.61	0.30	0.20	1000	-2.42	-1.28	-0.53	-0.25	-0.11	1000	1.85	1.27	0.77	0.60	0.52
2500	6.50	2.14	0.59	0.22	0.13	2500	-2.45	-1.26	-0.53	-0.26	-0.15	2500	1.84	1.22	0.75	0.50	0.40
5000	6.82	2.13	0.56	0.20	0.10	5000	-2.53	-1.31	-0.55	-0.25	-0.11	5000	1.86	1.16	0.71	0.47	0.36
7500	6.67	2.22	0.54	0.19	0.10	7500	-2.49	-1.33	-0.54	-0.24	-0.14	7500	1.85	1.20	0.70	0.46	0.36
10000	6.70	2.17	0.51	0.18	0.09	10000	-2.47	-1.31	-0.54	-0.25	-0.14	10000	1.88	1.19	0.67	0.45	0.33

90th Percentile (True Value -13.16%)

MSE (x100)						Bias (Annualized)						Standard Deviation (Annualized)					
n\T	50	100	250	500	1000	n\T	50	100	250	500	1000	n\T	50	100	250	500	1000
1000	18.60	8.70	3.89	1.76	0.98	1000	2.67	1.25	0.57	0.21	0.15	1000	4.43	3.31	2.30	1.58	1.18
2500	18.12	9.18	3.38	1.81	0.93	2500	2.44	1.21	0.44	0.31	0.20	2500	4.49	3.43	2.16	1.59	1.14
5000	18.72	8.77	3.94	1.83	0.96	5000	2.50	1.26	0.61	0.32	0.19	5000	4.55	3.32	2.30	1.59	1.16
7500	20.60	9.10	3.87	1.84	0.96	7500	2.53	1.43	0.58	0.28	0.21	7500	4.82	3.33	2.29	1.60	1.16
10000	18.32	9.35	3.20	1.75	0.94	10000	2.22	1.21	0.46	0.31	0.16	10000	4.63	3.46	2.10	1.56	1.15

TABLE AII. Construction of the Hedge Fund Dataset

This table summarizes the different steps for forming the consolidated hedge fund dataset. Panel A shows the total number of funds in each database. Panel B provides the same information after imposing the filters on each database. Panel C provides the same information after removing the duplicates within each database. Panel D provides the same information after removing the duplicates across all databases.

	BarclayHedge	HFR	Morningstar	TASS	Aggregate
Panel A: Raw Databases					
All Funds	22,315	15,275	6,120	21,432	65,142
Equity					
Long/Short	4,401	3,639	1,467	4,886	14,393
Market Neutral	687	931	227	873	2,718
Macro					
Global Macro	1,834	2,499	235	1,228	5,796
CTA/Managed Futures	3,944	750	260	3,071	8,025
Arbitrage					
Relative Value	3,431	2,674	1,536	904	8,545
Event Driven	1,104	1,204	192	948	3,448
Other					
Fund of Funds	5,813	3,578	383	6,908	16,682
Multi-Strategy	1,101	0	1,820	2,614	5,535
Panel B: Filtered Databases					
	BarclayHedge	HFR	Morningstar	TASS	Aggregate
All Funds	17,815	13,456	1,669	7,229	40,169
Equity					
Long/Short	3,622	3,168	427	1,874	9,091
Market Neutral	561	795	61	352	1,769
Macro					
Global Macro	1,380	2,183	106	360	4,029
CTA/Managed Futures	3,048	664	156	1,426	5,294
Arbitrage					
Relative Value	2,748	2,356	462	370	5,936
Event Driven	966	1,050	98	413	2,527
Other					
Fund of Funds	4,619	3,240	172	2,008	10,039
Multi-Strategy	871	0	187	426	1,484
Panel C: Filtered and Duplicate-Free Databases					
	BarclayHedge	HFR	Morningstar	TASS	Aggregate
All Funds	12,381	11,229	1,116	6,008	30,734
Equity					
Long/Short	2,505	2,688	329	1,576	7,098
Market Neutral	379	686	39	297	1,401
Macro					
Global Macro	1,002	1,870	68	308	3,248
CTA/Managed Futures	2,550	602	100	1,297	4,549
Arbitrage					
Relative Value	1,773	1,984	246	300	4,303
Event Driven	663	870	71	350	1,954
Other					
Fund of Funds	2,938	2,529	142	1,525	7,134
Multi-Strategy	571	0	121	355	1,047
Panel D: Filtered, Duplication-Free, and Merged Databases					
	BarclayHedge	HFR	Morningstar	TASS	Aggregate
All Funds	9,070	7,730	638	3,855	21,293
Equity					
Long/Short	1,736	1,936	177	914	4,763
Market Neutral	273	492	26	162	953
Macro					
Global Macro	811	1,163	34	180	2,188
CTA/Managed Futures	1,991	390	43	1,007	3,431
Arbitrage					
Relative Value	1,306	1,384	161	159	3,010
Event Driven	407	606	32	177	1,222
Other					
Fund of Funds	2,136	1,759	81	1,030	5,006
Multi-Strategy	410	0	84	226	720

TABLE AIII. Formal Model Comparisons – Impact of Data Filters

This table compares the CAPM with the four standard models (Carhart, Five-Factor, Fung-Hsieh, AMP), the two machine learning models (KNS1, KNS2), and the two models with the additional factors (JKKT and CP) using different data filters. We compute the difference in characteristics between the cross-sectional distributions of the alpha components under the CAPM and each model. Panel A reports the differences in the annualized mean and standard deviation, the proportions of funds with negative and positive alphas, and the annualized quantiles at 10% and 90% after imposing a minimum number of 36 observations. Figures in parentheses denote the standard deviation of the estimated differences. ***, **, * indicate that the null hypothesis of equal characteristics is rejected at the 1%, 5%, and 10% levels. Lack of differences signals that the model is as misspecified as the CAPM and thus ill-equipped to capture any alternative hedge fund strategies. Panels B to D repeat the analysis after imposing (i) a minimum of 84 observations,(ii) a more stringent backfill bias procedure, and (iii) filters to eliminate reporting errors.

Panel A: Minimum Number of 36 Observations						
	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
Carhart	-0.35 (0.44)	-0.33 (0.31)	2.00 (2.46)	-2.00 (2.46)	0.00 (0.41)	-0.46 (0.32)
Five-Factor	-0.17 (0.50)	0.04 (0.33)	1.95 (2.89)	-1.95 (2.89)	-0.15 (0.50)	-0.21 (0.39)
Fung-Hsieh	0.18 (0.63)	-0.12 (0.35)	-0.38 (3.78)	0.38 (3.78)	0.38 (0.57)	0.06 (0.47)
AMP	-0.26 (0.45)	-0.01 (0.35)	1.95 (2.41)	-1.95 (2.41)	-0.24 (0.38)	-0.22 (0.43)
KNS1	0.06 (0.42)	0.69** (0.29)	0.89 (1.84)	-0.89 (1.84)	-0.41 (0.29)	0.56 (0.37)
KNS2	0.55 (0.53)	0.51* (0.30)	-1.67 (2.68)	1.67 (2.68)	0.32 (0.42)	0.98** (0.46)
JKKT	-1.77** (0.69)	0.26 (0.49)	12.51*** (3.87)	-12.51*** (3.87)	-2.25*** (0.67)	-1.75*** (0.67)
CP	-2.42*** (0.78)	2.47*** (0.49)	16.86*** (4.06)	-16.86*** (4.06)	-4.34*** (0.74)	-0.89 (0.73)

Panel B: Minimum Number of 84 Observations						
	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
Carhart	-0.32 (0.36)	-0.17 (0.28)	1.81 (2.43)	-1.81 (2.43)	-0.15 (0.39)	-0.72** (0.35)
Five-Factor	-0.28 (0.42)	0.10 (0.31)	2.37 (2.94)	-2.37 (2.94)	-0.15 (0.51)	-0.34 (0.42)
Fung-Hsieh	0.08 (0.55)	-0.10 (0.34)	0.22 (3.70)	-0.22 (3.70)	0.11 (0.59)	-0.19 (0.49)
AMP	-0.40 (0.41)	0.03 (0.33)	2.62 (2.41)	-2.62 (2.41)	-0.28 (0.38)	-0.44 (0.52)
KNS1	-0.17 (0.34)	0.18 (0.26)	2.09 (1.70)	-2.09 (1.70)	-0.39 (0.30)	0.02 (0.39)
KNS2	0.61 (0.47)	0.07 (0.29)	-2.90 (2.54)	2.90 (2.54)	0.48 (0.40)	0.54 (0.51)
JKKT	-2.08*** (0.64)	0.39 (0.48)	15.91*** (4.00)	-15.91*** (4.00)	-2.72*** (0.67)	-2.24*** (0.70)
CP	-2.64*** (0.73)	1.89*** (0.46)	22.03*** (4.38)	-22.03*** (4.38)	-4.20*** (0.74)	-1.71** (0.81)

Panel C: Stringent Backfill Bias Procedure						
	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
Carhart	-0.29 (0.42)	-0.16 (0.35)	2.18 (2.70)	-2.18 (2.70)	0.13 (0.48)	-0.76** (0.34)
Five-Factor	-0.14 (0.48)	-0.02 (0.38)	2.01 (3.25)	-2.01 (3.25)	0.09 (0.58)	-0.26 (0.37)
Fung-Hsieh	0.10 (0.62)	-0.26 (0.40)	-0.56 (4.14)	0.56 (4.14)	0.45 (0.65)	-0.32 (0.50)
AMP	-0.26 (0.44)	0.07 (0.41)	2.07 (2.53)	-2.07 (2.53)	-0.03 (0.45)	-0.46 (0.47)
KNS1	0.03 (0.39)	0.41 (0.32)	1.21 (1.84)	-1.21 (1.84)	-0.21 (0.34)	0.21 (0.36)
KNS2	0.61 (0.51)	0.26 (0.34)	-2.87 (2.64)	2.87 (2.64)	0.74 (0.49)	0.56 (0.49)
JKKT	-1.91*** (0.69)	0.37 (0.54)	14.05*** (4.19)	-14.05*** (4.19)	-2.17*** (0.72)	-2.13*** (0.71)
CP	-2.54*** (0.75)	2.27*** (0.51)	19.21*** (4.26)	-19.21*** (4.26)	-3.99*** (0.77)	-1.52** (0.75)

Panel D: Filters for Removing Reporting Errors						
	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
Carhart	-0.30 (0.39)	-0.21 (0.31)	2.30 (2.50)	-2.30 (2.50)	-0.13 (0.40)	-0.58* (0.33)
Five-Factor	-0.23 (0.46)	-0.03 (0.34)	2.54 (2.97)	-2.54 (2.97)	-0.17 (0.52)	-0.32 (0.39)
Fung-Hsieh	0.06 (0.60)	-0.16 (0.35)	0.59 (3.85)	-0.59 (3.85)	0.20 (0.58)	-0.15 (0.50)
AMP	-0.30 (0.42)	0.06 (0.38)	2.45 (2.52)	-2.45 (2.52)	-0.30 (0.36)	-0.25 (0.48)
KNS1	-0.04 (0.39)	0.38 (0.29)	1.36 (1.86)	-1.36 (1.86)	-0.37 (0.30)	0.18 (0.38)
KNS2	0.60 (0.50)	0.14 (0.33)	-2.38 (2.69)	2.38 (2.69)	0.47 (0.44)	0.61 (0.48)
JKKT	-1.94*** (0.68)	0.29 (0.51)	14.77*** (4.06)	-14.77*** (4.06)	-2.46*** (0.68)	-2.03*** (0.72)
CP	-2.56*** (0.76)	1.84*** (0.49)	20.39*** (4.41)	-20.39*** (4.41)	-4.36*** (0.73)	-1.39* (0.78)

TABLE AIV. Formal Model Comparisons – Alternative Reference Models

This table compares the two models with the additional factors (JKKT and CP) with a set of alternative reference models, which are the four standard models (Carhart, Five-Factor, Fung-Hsieh, AMP) and the two machine learning models (KNS1, KNS2). Panel A computes the differences in characteristics between the cross-sectional distributions of the alpha components under the Carhart model and each model (JKKT and CP). We report the differences in the annualized mean and standard deviation, the proportions of funds with negative and positive alphas, and the annualized quantiles at 10% and 90%. Figures in parentheses denote the standard deviation of the estimated differences. ***, **, * indicate that the null hypothesis of equal characteristics is rejected at the 1%, 5%, and 10% levels. Panels B to E repeat the analysis using the other models.

	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
Panel A: Carhart Model as Reference						
JKKT	-1.62*** (0.52)	0.60* (0.32)	12.48*** (3.23)	-12.48*** (3.23)	-2.30*** (0.45)	-1.39*** (0.47)
CP	-2.25*** (0.67)	2.33*** (0.43)	17.55*** (4.25)	-17.55*** (4.25)	-4.20*** (0.71)	-0.82 (0.54)
Panel B: Five-Factor Model as Reference						
JKKT	-1.73*** (0.60)	0.39 (0.33)	12.35*** (3.69)	-12.35*** (3.69)	-2.31*** (0.51)	-1.80*** (0.47)
CP	-2.36*** (0.73)	2.11*** (0.45)	17.42*** (4.58)	-17.42*** (4.58)	-4.21*** (0.75)	-1.24** (0.55)
Panel C: Fung-Hsieh Model as Reference						
JKKT FH	-2.01*** (0.60)	0.61* (0.36)	14.24*** (3.79)	-14.24*** (3.79)	-2.60*** (0.54)	-1.83*** (0.54)
CP FH	-2.64*** (0.72)	2.33*** (0.46)	19.31*** (4.47)	-19.31*** (4.47)	-4.50*** (0.72)	-1.27** (0.58)
Panel D: Asness-Moskowitz-Pedersen Model as Reference						
JKKT	-1.62** (0.65)	0.35 (0.32)	12.25*** (3.96)	-12.25*** (3.96)	-2.14*** (0.60)	-1.64*** (0.50)
CP	-2.25*** (0.73)	2.07*** (0.42)	17.32*** (4.36)	-17.32*** (4.36)	-4.05*** (0.74)	-1.08** (0.52)
Panel E: Kozak, Nagel, and Shantosh (KNS1) Model as Reference						
JKKT	-1.92*** (0.65)	-0.01 (0.48)	13.17*** (3.91)	-13.17*** (3.91)	-2.05*** (0.64)	-2.17*** (0.64)
CP	-2.55*** (0.74)	1.71*** (0.48)	18.24*** (4.25)	-18.24*** (4.25)	-3.95*** (0.74)	-1.61** (0.63)
Panel E: Kozak, Nagel, and Shantosh (KNS2) Model as Reference						
JKKT	-2.56*** (0.68)	0.21 (0.43)	17.22*** (3.87)	-17.22*** (3.87)	-2.90*** (0.63)	-2.63*** (0.65)
CP	-3.19*** (0.72)	1.93*** (0.46)	22.29*** (3.93)	-22.29*** (3.93)	-4.80*** (0.71)	-2.07*** (0.64)

TABLE AV. Factor Trading Costs

This table measures the impact of the costs of trading the additional factors (illiquidity, BAB, variance, carry, TS momentum). Panel A reports the differences in characteristics between the cross-sectional distributions of the alpha components under the CAPM and each model (JKKT and CP) after accounting for trading costs. We report the differences in the annualized mean and standard deviation, the proportions of funds with negative and positive alphas, and the annualized quantiles at 10% and 90%. Figures in parentheses denote the standard deviation of the estimated differences. ***, **, * indicate that the null hypothesis of equal characteristics is rejected at the 1%, 5%, and 10% levels. Panels B and C report the characteristics of the cross-sectional distribution of the alpha and beta components under the JKKT and CP models. We report the annualized mean and standard deviation, the proportions of funds with negative and positive alphas, and the annualized quantiles at 10% and 90%. Figures in parentheses denote the standard deviation of the estimated characteristics.

Panel A: Comparison With the CAPM						
	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
JKKT	-1.59** (0.70)	0.23 (0.53)	11.11** (4.44)	-11.11** (4.44)	-1.84*** (0.70)	-1.86*** (0.67)
CP	-1.96** (0.77)	1.79*** (0.52)	14.57*** (4.58)	-14.57*** (4.58)	-3.26*** (0.77)	-1.13 (0.75)
Panel B: Distribution of the Alpha Components						
	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
CAPM	2.93 (0.94)	7.01 (0.48)	27.15 (5.49)	72.85 (5.49)	-3.95 (0.67)	10.06 (0.67)
JKKT	1.34 (0.72)	7.24 (0.29)	38.25 (4.58)	61.75 (4.58)	-5.79 (0.58)	8.19 (0.40)
CP	0.97 (0.87)	8.80 (0.33)	41.71 (5.26)	58.29 (5.26)	-7.21 (0.74)	8.93 (0.51)
Panel C: Distribution of the Beta Components						
	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
CAPM	2.62 (0.83)	4.37 (0.67)	22.54 (9.57)	77.46 (9.57)	-0.71 (0.61)	8.01 (0.73)
JKKT	4.22 (0.83)	5.71 (0.41)	14.83 (2.88)	85.17 (2.88)	-0.58 (0.31)	10.78 (0.69)
CP	4.59 (0.94)	7.20 (0.47)	16.44 (2.71)	83.56 (2.71)	-1.18 (0.36)	11.91 (1.00)

TABLE AVI. Decomposition of Average Fund Returns – Multi-Strategy and Fund of Funds
This table shows the decomposition of average fund returns under the CAPM and the two models with the additional factors (JKKT and CP) across multi-strategy funds and funds of funds. Panel A reports the characteristics of the cross-sectional distributions of the alpha and beta components across multi-strategy funds. We report the annualized mean and standard deviation, the proportions of funds with negative and positive alphas, and the annualized quantiles at 10% and 90%. Figures in parentheses denote the standard deviation of the estimated characteristics. Panel B repeats the analysis for funds of funds.

	Panel A: Multi-Strategy					
	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
	Distribution of the Alpha Components					
CAPM	2.48 (1.09)	6.68 (0.54)	32.23 (7.02)	67.77 (7.02)	-4.29 (1.40)	9.15 (1.00)
JKKT	0.49 (0.83)	7.41 (0.53)	46.45 (4.67)	53.55 (4.67)	-6.83 (1.23)	8.01 (0.80)
CP	-0.11 (0.87)	7.79 (0.60)	49.29 (4.83)	50.71 (4.83)	-8.06 (1.22)	7.66 (0.80)
	Distribution of the Beta Components					
CAPM	1.90 (0.81)	3.44 (0.44)	19.43 (13.61)	80.57 (13.61)	-0.37 (0.76)	4.74 (1.14)
JKKT	3.89 (0.74)	4.65 (0.40)	10.43 (4.04)	89.57 (4.04)	0.00 (0.51)	8.13 (0.97)
CP	4.49 (0.80)	4.68 (0.52)	8.53 (3.46)	91.47 (3.46)	0.19 (0.55)	9.65 (1.09)

	Panel B: Fund of Funds					
	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
	Distribution of the Alpha Components					
CAPM	1.22 (1.39)	4.16 (0.26)	31.82 (16.99)	68.18 (16.99)	-2.76 (1.23)	4.93 (1.11)
JKKT	-1.28 (1.03)	4.42 (0.17)	66.08 (12.46)	33.92 (12.46)	-5.39 (0.95)	2.74 (0.76)
CP	-2.46 (1.08)	5.41 (0.28)	76.88 (7.96)	23.12 (7.96)	-7.18 (1.18)	2.29 (0.70)
	Distribution of the Beta Components					
CAPM	1.75 (0.99)	2.56 (0.34)	14.15 (22.75)	85.85 (22.75)	-0.14 (1.01)	4.37 (0.80)
JKKT	4.25 (0.94)	3.40 (0.18)	3.24 (1.28)	96.76 (1.28)	1.35 (0.67)	7.67 (0.84)
CP	5.42 (1.04)	4.49 (0.26)	3.52 (1.04)	96.48 (1.04)	1.66 (0.68)	9.49 (1.14)

TABLE AVII. Economic Importance of Hedge Fund Factors – Investment Subcategories

This table measures the economic importance of each factor in the CP model as a driver of fund returns across investment subcategories. Panel A reports the characteristics of the cross-sectional distributions of the beta components due to each factor across long-short funds. By construction, the average total beta component is equal to the sum of the average factor beta components. We report the annualized mean and standard deviation, the proportions of funds with negative and positive contributions, and the annualized quantiles at 10% and 90%. Figures in parentheses denote the standard deviation of the estimated characteristics. Panel B to D repeat the analysis for market neutral, global macro, managed futures, relative value, and event driven funds.

	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
Panel A: Long-Short						
Market	4.10 (2.30)	4.62 (1.14)	11.51 (8.94)	88.49 (8.94)	-0.10 (0.87)	9.64 (3.33)
Size	0.47 (0.63)	1.39 (0.43)	30.94 (12.81)	69.06 (12.81)	-0.35 (0.14)	1.77 (1.15)
Illiquidity	0.15 (0.11)	1.56 (0.37)	43.93 (2.68)	56.07 (2.68)	-0.90 (0.26)	1.37 (0.51)
Betting Against Beta	0.22 (0.19)	2.79 (0.83)	39.32 (3.83)	60.68 (3.83)	-1.92 (0.62)	2.63 (1.07)
Variance	0.77 (0.28)	5.25 (0.73)	36.92 (2.36)	63.08 (2.36)	-3.14 (0.52)	4.64 (0.89)
Carry	0.34 (0.18)	2.94 (0.49)	45.24 (2.87)	54.76 (2.87)	-2.23 (0.44)	2.84 (0.54)
Time-Series Momentum	0.51 (0.17)	2.99 (0.58)	44.10 (2.11)	55.90 (2.11)	-1.66 (0.47)	3.19 (0.77)
Panel B: Market Neutral Funds						
Market	0.63 (0.34)	2.06 (0.55)	36.18 (4.04)	63.82 (4.04)	-0.73 (0.06)	2.21 (0.81)
Size	0.05 (0.11)	0.49 (0.21)	45.12 (6.31)	54.88 (6.31)	-0.35 (0.13)	0.53 (0.24)
Illiquidity	0.06 (0.05)	0.74 (0.27)	45.53 (3.44)	54.47 (3.44)	-0.67 (0.29)	0.73 (0.33)
Betting Against Beta	0.48 (0.31)	1.42 (0.53)	32.52 (5.10)	67.48 (5.10)	-0.53 (0.06)	1.96 (0.80)
Variance	-0.08 (0.18)	3.32 (0.49)	50.41 (2.31)	49.59 (2.31)	-3.04 (0.57)	2.16 (0.33)
Carry	0.35 (0.14)	1.52 (0.31)	41.46 (2.33)	58.54 (2.33)	-1.36 (0.28)	1.96 (0.47)
Time-Series Momentum	0.58 (0.21)	1.24 (0.35)	27.64 (3.30)	72.36 (3.30)	-0.61 (0.13)	2.13 (0.58)
Panel C: Macro Funds						
Market	1.97 (0.99)	3.87 (1.24)	27.58 (5.24)	72.42 (5.24)	-0.97 (0.13)	6.79 (2.31)
Size	0.08 (0.04)	0.72 (0.22)	49.85 (5.26)	50.15 (5.26)	-0.44 (0.22)	0.54 (0.20)
Illiquidity	-0.01 (0.07)	0.85 (0.28)	50.29 (3.22)	49.71 (3.22)	-0.71 (0.34)	0.71 (0.29)
Betting Against Beta	0.29 (0.21)	1.89 (0.63)	38.20 (4.36)	61.80 (4.36)	-1.19 (0.40)	1.91 (0.78)
Variance	0.56 (0.27)	5.68 (0.73)	44.10 (2.60)	55.90 (2.60)	-3.83 (0.70)	4.14 (0.69)
Carry	0.38 (0.23)	2.54 (0.54)	39.23 (2.89)	60.77 (2.89)	-2.24 (0.51)	3.20 (0.67)
Time-Series Momentum	2.29 (0.82)	4.50 (1.08)	26.55 (2.87)	73.45 (2.87)	-0.67 (0.08)	7.85 (1.91)
Panel D: CTA/Managed Futures Funds						
Market	0.60 (0.37)	3.03 (0.92)	40.60 (3.95)	59.40 (3.95)	-1.77 (0.54)	3.22 (1.30)
Size	0.07 (0.14)	0.91 (0.19)	51.13 (11.18)	48.87 (11.18)	-0.42 (0.26)	0.73 (0.18)
Illiquidity	-0.13 (0.12)	1.49 (0.36)	56.82 (3.54)	43.18 (3.54)	-1.06 (0.37)	0.79 (0.27)
Betting Against Beta	0.14 (0.18)	2.06 (0.66)	42.75 (4.81)	57.25 (4.81)	-1.35 (0.45)	1.81 (0.73)
Variance	-0.02 (0.45)	7.13 (0.98)	53.28 (3.46)	46.72 (3.46)	-5.30 (1.00)	4.72 (0.71)
Carry	0.42 (0.30)	2.95 (0.60)	43.82 (4.08)	56.18 (4.08)	-2.51 (0.65)	3.26 (0.66)
Time-Series Momentum	3.70 (1.32)	6.56 (1.34)	22.88 (3.73)	77.12 (3.73)	-0.79 (0.21)	11.31 (2.84)
Panel E: Relative Value Funds						
Market	1.49 (0.76)	2.41 (0.72)	19.94 (5.58)	80.06 (5.58)	-0.28 (0.15)	3.97 (1.34)
Size	0.10 (0.11)	0.49 (0.15)	36.93 (4.35)	63.07 (4.35)	-0.12 (0.01)	0.42 (0.29)
Illiquidity	0.07 (0.12)	0.76 (0.18)	34.72 (7.32)	65.28 (7.32)	-0.33 (0.06)	0.59 (0.35)
Betting Against Beta	0.58 (0.39)	1.69 (0.64)	27.61 (4.65)	72.39 (4.65)	-0.71 (0.15)	2.19 (1.01)
Variance	1.32 (0.40)	3.13 (0.58)	23.45 (2.30)	76.55 (2.30)	-0.93 (0.10)	4.35 (1.00)
Carry	0.73 (0.25)	1.76 (0.33)	22.81 (3.99)	77.19 (3.99)	-0.48 (0.06)	2.53 (0.58)
Time-Series Momentum	-0.27 (0.20)	1.48 (0.37)	61.59 (4.68)	38.41 (4.68)	-1.52 (0.60)	0.82 (0.15)
Panel F: Event Driven Funds						
Market	2.05 (1.17)	2.68 (0.77)	10.97 (8.55)	89.03 (8.55)	-0.02 (0.45)	5.02 (1.86)
Size	0.28 (0.39)	0.76 (0.18)	24.54 (17.15)	75.46 (17.15)	-0.09 (0.14)	0.77 (0.59)
Illiquidity	0.07 (0.12)	1.06 (0.19)	41.26 (6.95)	58.74 (6.95)	-0.48 (0.04)	0.64 (0.25)
Betting Against Beta	0.67 (0.40)	1.88 (0.49)	20.82 (5.90)	79.18 (5.90)	-0.56 (0.09)	2.03 (0.80)
Variance	1.78 (0.48)	3.71 (0.50)	18.77 (3.32)	81.23 (3.32)	-0.51 (0.17)	4.87 (0.97)
Carry	0.20 (0.17)	2.14 (0.34)	43.49 (4.29)	56.51 (4.29)	-1.75 (0.33)	1.80 (0.38)
Time-Series Momentum	-0.34 (0.19)	1.64 (0.36)	63.20 (3.71)	36.80 (3.71)	-1.87 (0.57)	0.92 (0.18)

TABLE AVIII. Decomposition of Average Fund Returns – Style-Specific Factors

This table shows the decomposition of average fund returns under a style-specific version of the CP model across investment styles. The equity model replaces the global carry and TS momentum factors with the equity carry and TS momentum factors. The macro model replaces the global carry and TS momentum factors with the equal-weighted average of the currency and commodity carry and TS momentum factors. The arbitrage model replaces the global carry and TS momentum factors with the fixed income carry and TS momentum factors. Panel A reports the characteristics of the cross-sectional distributions of the alpha and beta components across equity funds (long-short, market neutral). We report the annualized mean and standard deviation, the proportions of funds with negative and positive alphas, and the annualized quantiles at 10% and 90%. Figures in parentheses denote the standard deviation of the estimated characteristics. Panels B and C repeat the analysis for macro funds (global macro, managed futures) and arbitrage funds (relative value, event driven).

	Panel A: Equity Funds					
	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
Alpha Component	0.80 (0.91)	8.96 (0.47)	45.98 (5.28)	54.02 (5.28)	-7.44 (1.05)	8.74 (0.77)
Beta Component	5.78 (1.25)	7.54 (0.62)	14.59 (2.89)	85.41 (2.89)	-1.20 (0.53)	13.35 (1.81)
	Panel B: Macro Funds					
	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
Alpha Component	1.16 (1.95)	10.58 (0.54)	43.19 (7.83)	56.81 (7.83)	-8.58 (1.44)	11.16 (1.48)
Beta Component	3.56 (1.58)	8.47 (0.63)	27.72 (6.14)	72.28 (6.14)	-3.10 (0.80)	11.24 (1.29)
	Panel C: Arbitrage Funds					
	Moments (Ann.)		Proportions (%)		Quantiles (Ann.)	
	Mean	Std Dev.	Negative	Positive	10%	90%
Alpha Component	0.97 (1.17)	6.82 (0.56)	41.21 (9.36)	58.79 (9.36)	-5.60 (1.52)	7.86 (0.52)
Beta Component	4.15 (1.04)	4.74 (0.68)	11.60 (2.29)	88.40 (2.29)	-0.19 (0.32)	9.18 (1.60)