Communication cost of simulating entanglement swapping

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Entanglement appears in two different ways in quantum mechanics, namely as a property of states and as a property of measurement outcomes in joint measurements. By combining these two aspects of entanglement, it is possible to generate nonlocality between particles that never interacted, using the protocol of entanglement swapping. We investigate the communication cost of classically simulating this process. While the communication cost of simulating nonlocal correlations of entangled states appears to be generally quite low, we show here that infinite communication is required to simulate entanglement swapping. This result is derived in the scenario of bilocality, where distant sources of particles are assumed to be independent, and takes advantage of a previous result of Massar et al. [Phys. Rev. A **63**, 052305 (2001)]. Our result implies that any classical model simulating entanglement swapping must either assume that (i) infinite shared randomness is available between any two locations in the universe, or that (ii) infinite communication takes place.

By performing suitably chosen local measurements on an entangled quantum state, distant observers can establish nonlocal correlations, as witnessed by the violation of a Bell inequality [1]. This means that quantum statistics cannot be simulated by classically correlated systems, unless some classical communication is added to the model. Although experiments give strong evidence that nature does not use classical communication to establish correlations [2], it is nevertheless interesting from a fundamental perspective to ask how much communication is required to reproduce quantum correlations. Such an approach, generally referred to as classical simulation of entanglement, provides a natural approach to the problem of quantifying quantum nonlocality.

Nonlocality is a fundamental aspect of quantum mechanics, hence quantifying it is much desirable. Besides being one of the most striking and counter-intuitive features of the theory, it is also a powerful resource, allowing for instance for the reduction of communication complexity [3], as well as for information processing in the 'device-independent' setting [4–7], where one wants to achieve an information task and prove its security without any assumption on the devices used in the protocol.

Several works [8–11] underwent the task of estimating how much communication is needed to simulate the correlations of a maximally entangled state of two qubits under all possible projective measurements. This research culminated in 2003, when Toner and Bacon [12] showed that one bit of communication is enough. Importantly this single bit of communication is not an average value, but represents the exact amount that is to be used at each round. Thus the model is said to have bounded communication. The communication costs of other states have been explored as well [13–15]. Notably, Regev and Toner [16] have shown that the correlations obtainable from projective measurements on any bipartite entangled state can be simulated with only two bits of communication — note however, that their protocol does not reproduce the correct marginal distributions. The simulation of multipartite entanglement also attracted some attention [17–20], and two of the authors [21] recently showed that the correlations of equatorial measurements on a tripartite GHZ state can be simulated with 3 bits of communication. Although their model does not work for arbitrary measurements, it does reproduce the Mermin-GHZ paradox which is arguably the strongest demonstration of the nonlocality of this state [22, 23].

From the above results, it is tempting to conclude that simulating entanglement is after all not that expensive in terms of classical communication. However, quantum mechanics allows not only for entangled states of distant systems, but also for entangled measurements, also called joint or coherent measurements. In such a measurement the initial state is arbitrary—it could be entangled or not—but the final state is entangled, that is the eigenstates of the operator that represents such a measurement are entangled. This second aspect of entanglement is in itself independent of nonlocality—although it leads to nonlocality when combined with entangled states [24]. It demonstrates another nonclassical feature of entanglement, which is, loosely speaking, the possibility to ask to two (or more) quantum systems questions about their relations without gaining any information about the individual properties of each subsystem [25].

Here we will see that simulating this second side of entanglement appears to be dramatically more difficult. Specifically, we shall consider the simple scenario of entanglement swapping [26], where quantum particles that never interacted become nonlocally correlated after their twins underwent a joint measurement, and we will show that infinite communication is necessary to simulate this quantum process. In other words, no model with bounded communication can exist. Our result is derived in the scenario of 'bilocality' [29], that is when the two sources of particles are supposed to be independent from each other. This assumption is indeed quite natural: why should initially uncorrelated quantum particles be described by correlated classical variables? Note that experiments with totally independent sources have been realized [27, 28].

The paper is organized as follows. We start by presenting the scenario of entanglement swapping and give the intuition behind our result. After providing a formal proof, we discuss the implications of our result and give some perspectives.

SCENARIO AND INTUITION

We consider three distant parties, Alice, Bob and Charlie. Bob shares two maximally entangled qubit pairs in the state $|\phi_+\rangle$ (see Eq. (1) below) with Alice and Charlie, respectively. The first pair is produced by a source located between Alice and Bob, the second one is produced by an independent source located between Bob and Charlie, see Fig. 1 (i). Accordingly, Alice and Charlie are initially uncorrelated. By performing a Bell state measurement, i.e. a two-qubit joint measurement which features the four maximally entangled Bell states

$$|\phi_\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle) \quad (1)$$

$$|\psi_\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle) \quad (2)$$

as eigenstates, Bob projects Alice and Charlie's particles onto one of the Bell states. Bob can thus 'swap' entanglement to Alice and Charlie. This protocol is essentially identical to the celebrated quantum teleportation protocol [30]; entanglement swapping is basically a *teleportation of entanglement*.

However, onto which Bell state Alice and Charlie's particles are projected depends on the outcome of Bob's Bell state measurement. Bob has no control on this outcome, which ensures that the protocol is non-signaling: if Bob could decide on which Bell state to project Alice and Charlie's particles, this would indeed allow him to signal to the latter, since by coming together Alice and Charlie could determine which Bell state they hold. To complete the protocol, Bob thus needs to communicate the result of his measurement b—by sending two bits of classical communication—to (say) Charlie, who can then apply a suitable unitary local transformation to his qubit to finally share a definite Bell state with Alice. At the end of the protocol, Alice and Charlie thus share a maximally entangled state, say for instance the singlet state $|\psi_-\rangle$. Upon receiving measurement settings $\vec{x}$ and $\vec{z}$ (represented here by 3-dimensional unit vectors on the Bloch sphere) and performing the corresponding projective measurements on their respective qubit, Alice and Charlie obtain binary measurement outcomes $a = \pm 1$ and $c = \pm 1$ respectively. These measurement outcomes exhibit nonlocal correlations of the form

$$E(\vec{x}, \vec{z}) = P(a = c|\vec{x}, \vec{z}) - P(a \neq c|\vec{x}, \vec{z}) = -\vec{x} \cdot \vec{z}. \quad (3)$$

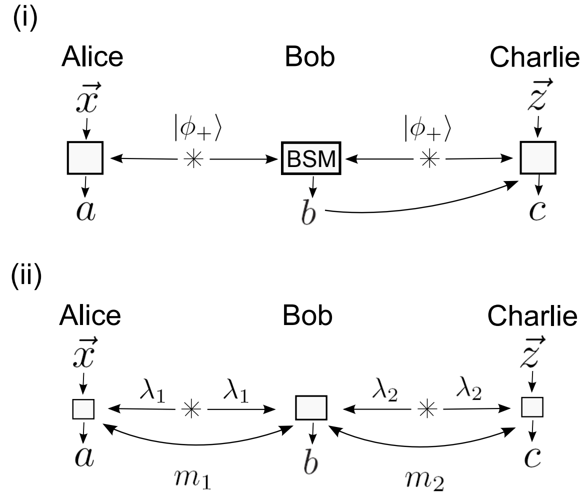

FIG. 1: (i) The scenario of entanglement swapping with two fully independent sources of $|\Phi_+\rangle$ states. Bob sends to Charlie the result b of his Bell state measurement (BSM). Upon receiving these two bits of communication, Charlie can apply the adequate unitary operation to his qubit such that Alice and Charlie finally share a singlet state. (ii) The classical simulation of entanglement swapping in the bilocal scenario. Alice/Bob and Bob/Charlie exchange messages m_1 and m_2 (note that m_2 for instance may contain Bob's result b). When the two sources of shared variables λ_1 and λ_2 are uncorrelated, infinite communication is required, i.e. m_1 and/or m_2 must be unbounded.

We next consider the task of simulating classically this quantum process. We know that separate Von Neumann measurements on the maximally entangled two qubit states shared by Alice/Bob and Bob/Charlie can be simulated by shared randomness augmented with a single bit of classical communication [12]; one may wonder whether such classical resources can also reproduce the Bell state measurement and the full tripartite correlation $P(a, b, c|\vec{x}, \vec{z})$ of the entanglement swapping experiment.

From a physical perspective, it is natural to imagine that the shared randomness is provided by the two sources of particles. After all, instead of emitting entangled pairs of particles, the source could as well produce non-entangled particles carrying classical information. Thus Alice and Bob share the classical variable λ_1, and Bob and Charlie the classical variable λ_2 (see Fig. 1 (ii)). Since the shared classical variables distributed between Alice/Bob and Bob/Charlie originate from independent sources, it is natural also to assume that the variables held by Alice and Charlie, i.e. λ_1 and λ_2 respectively, are initially uncorrelated; this is the so-called bilocality assumption [29]. Assume that upon receiving their measurement settings, the parties are allowed to send classical communication to each other. Denote by m_1 the communication between Alice and Bob, and by m_2 the communication between Bob and Charlie; note that both of these communications can be two-way and sequential, that m_2 may include the two bits b representing the outcome of Bob's joint measurement, and that this scheme could as well include communication between Alice and Charlie, which would transit through Bob.

The crux of our argument is now the following. In any model featuring finite communication between Alice/Bob and Bob/Charlie (or directly between Alice/Charlie), the amount of shared randomness between Alice and Charlie will always be finite. However, Massar, Bacon, Cerf and Cleve [31] have shown that any classical simulation model of a maximally entangled qubit pair requires either infinite communication or infinite shared randomness. Hence, when no shared randomness is available, infinite classical communication is required. Note that this applies also to the case where the shared randomness is finite, since the latter could be included in the communication as well. Now, it is clear that any model simulating entanglement swapping with bounded communication would also allow one to simulate the correlations of the singlet state (3) with bounded communication. Therefore, it follows from the result of Ref. [31] that no classical model with bounded communication can simulate entanglement swapping in the bilocal scenario.

PROOF

We now give a simple proof of the above statement. Specifically, we show that the amount of shared randomness between Alice and Charlie is always finite when the simulation protocol involves only bounded communication.

We need to show that the mutual information between Alice and Charlie is finite, i.e.

$$I(A:C) < \infty \,. \tag{4}$$

The information available to Alice is λ_1 and m_1, while the information available to Charlie is λ_2 and m_2. Thus we have

$$\begin{aligned} I(A:C) &= I(m_1,\lambda_1:m_2,\lambda_2) \\ &= I(m_1,\lambda_1:\lambda_2) + I(m_1,\lambda_1:m_2|\lambda_2) \\ &= I(\lambda_1:\lambda_2) + I(m_1:\lambda_2|\lambda_1) + \\ &\quad\; I(\lambda_1:m_2|\lambda_2) + I(m_1:m_2|\lambda_2,\lambda_1) \end{aligned} \tag{5}$$

where we have used repeatedly the chain rule for mutual information. By assumption of the independence of the two source (the bilocality assumption), we have that

$$I(\lambda_1:\lambda_2) = 0. \tag{6}$$

If both communications m_1 and m_2 are bounded, we have that

$$I(m_1:\lambda_2|\lambda_1) < \infty \tag{7}$$
$$I(\lambda_1:m_2|\lambda_2) < \infty \tag{8}$$
$$I(m_1:m_2|\lambda_2,\lambda_1) < \infty \tag{9}$$

which implies Eq. (4).

Thus from the result of [31], we conclude that no classical bilocal model—i.e. featuring independent sources of shared randomness—with finite communication can reproduce the process of entanglement swapping.

DISCUSSION

We considered the problem of classically simulating quantum entanglement. Entanglement in quantum theory has two sides. On the one hand, it is a property of the joint state of distant systems, leading to nonlocal correlations when suitable local measurements are performed. On the other hand, entanglement is a feature of the eigenstates of a joint measurement, which allows one to measure a global property of two (or more) quantum systems without gaining any information about their individual properties. Whereas the nonlocal correlations of quantum entangled states appear (for some natural cases at least) to be simulable using only very little classical communication, we have shown here that the simulation of a simple protocol involving both aspects of entanglement, in particular involving a joint measurement, requires infinite communication. This illustrates the strength of quantum correlations when entangled states and entangled measurements are combined.

We have worked in the scenario of bilocality, that is under the assumption that, since the two sources of entanglement are independent, they should be simulated using shared classical variables between Alice/Bob and Bob/Charlie that are also independent. Our result can be rephrased in terms of implications for classical simulation models of the entanglement swapping process. More precisely, it implies that any classical simulation of this process must either assume (i) that infinite shared randomness is available between any two locations in the universe, or that (ii) infinite communication between Alice/Bob or Bob/Charlie must take place. We believe that none of these possibilities are satisfactory from a physical point of view. In particular, (i) would prevent us from drawing any conclusion regarding nonlocality in practical Bell tests. Indeed in such tests, it is crucial that the choice of measurement settings is independent of the source of particles itself. In practice this is done using a quantum random number generator which consists of an additional source of particles. Thus if separated sources cannot be considered as independent, the choice of measurement settings could be correlated to the source of particles, which would make it possible for a local model to reproduce quantum nonlocal correlations.

Our result opens several new questions. First, whereas we have focused here on simulation models involving bounded communication, it would be relevant to investigate the average communication cost of simulating entanglement swapping. In Ref. [31] it was shown that the correlations of a Bell state can be reproduced without shared randomness using a model involving finite average communication (less than 20 bits). This model can be trivially adapted to the scenario of entanglement swapping, by having Bob forwarding to Charlie the communication received from Alice, and adding the two bits representing the outcome of his Bell state measurement. It would be interesting to find whether there exists a more economical model.

Another direction worth investigating is the case in which

Alice and Charlie perform only a finite set of measurements. Clearly in this case finite communication is always sufficient since Alice can simply encode her measurement setting and send it to Charlie via Bob. It would be interesting to see if more efficient models can be devised and find bounds on the communication cost which would allow one to quantify quantum nonlocality in real entanglement swapping experiments. Note also that the communication cost of simulating quantum correlations is intimately related to the problem of the detection loophole; it would be interesting to further investigate the latter in the context of entanglement swapping [32] and to explore the implications of the present result.

One may also consider the case of noisy entangled states, for instance by replacing the two maximally entangled qubit pairs by Werner states [33]. A model without communication can be adapted from that of Ref. [33], which allows one to simulate the entanglement swapping process with visibilities $V \leq \frac{1}{4}$ (here V denotes the singlet fraction of the final state shared by Alice and Charlie). With the help of communication, one can adapt the model of Ref. [14]; using two bits of communication (one from Alice to Bob and one from Charlie to Bob), one can simulate visibilities up to $V = \frac{4}{9}$ [34]. It is unclear whether these models are optimal, and how close to a perfect visibility one can get for a given bound on the communication.

Finally it is worth mentioning that, while we have focused here on the amount of classical communication required to simulate both aspects of quantum entanglement, it would also be interesting to show that models with finite-speed communication cannot reproduce quantum correlations in a multipartite scenario. While this conjecture is still not proven [35], recent progress has been made in this direction [36]. This would indeed nicely complement the present result.

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