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How to cite

PIGUET, Olivier, SORELLA, Silvio Paolo. Adler-Bardeen theorem and vanishing of the gauge beta function. In: Nuclear Physics. B, 1993, vol. 395, n° 3, p. 661–671. doi: 10.1016/0550-3213(93)90052-Q

This publication URL: <https://archive-ouverte.unige.ch/unige:119379>

Publication DOI: [10.1016/0550-3213\(93\)90052-Q](https://doi.org/10.1016/0550-3213(93)90052-Q)

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Adler–Bardeen theorem and vanishing of the gauge beta function *

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Received 30 July 1992

(Revised 3 November 1992)

Accepted for publication 8 January 1993

The proof of the non-renormalization theorem for the gauge anomaly of four-dimensional theories is extended to the case of models with a vanishing one-loop gauge beta function.

1. Introduction

The non-renormalization theorem [1–7] of the four-dimensional gauge anomaly [8] is of fundamental importance for the construction of consistent high-energy physics models. This theorem states that the anomaly coefficient vanishes at all orders of perturbation theory if it vanishes in the one-loop approximation.

The original proof of Bardeen [2] of the theorem for the non-abelian gauge anomaly is based on an analysis of Feynman graphs: one shows that if the one-loop triangle anomaly cancels, then there exists a gauge-invariant regularization valid to all orders. Later on it was recognized by Zee [3] in the abelian case, and by Costa et al. [5] in the non-abelian case, that it is possible to give a proof based on the combined use of the gauge (or BRS) Ward identities and of the Callan–Symanzik equation. In the same time Lowenstein and Schroer [4], and later on Bandelloni et al. [6], achieved, with the quantum action principle [9] as the main tool, an algebraic, regularization independent, version of the previous proof.

The main advantage of a regularization independent proof is that it can be naturally extended to more sophisticated theories, e.g. supersymmetric gauge theories and topological theories for which no regularization preserving all the symmetries is available.

The regularization independent proofs given up to the present time [4,6,7], as well as the proof given in ref. [5], based on dimensional regularization, although very general, have their domain of validity restricted by the “technical” assumption

* Supported in part by the Swiss National Science Foundation.

that the one-loop beta function for the gauge coupling [10] should not identically vanish. Even if this assumption covers a very large class of models including the standard model, there is a wide set of interesting theories for which the one-loop gauge beta function do indeed vanish. This set includes in particular gauge models with $N = 1$ supersymmetry which may have some relevance in the construction of grand unified theories [11]. Moreover the supersymmetric gauge models with a vanishing one-loop gauge beta function [12–14] are the starting point towards the construction of ultraviolet finite theories [15]. It is therefore needed to have a proof which also applies to the case of a one-loop vanishing gauge beta function. This is the aim of the present paper.

The demonstration follows the differential geometry setup of the descent equations which are known to characterize the anomaly [16–18]. It is the continuation of a previous work of the authors [7], where a completely algebraic proof of the non-renormalization theorem was given in the case of a non-vanishing one-loop gauge beta function. The main ingredient, as shown in ref. [7], is the vanishing of the anomalous dimensions of the differential form operators which are solutions of the descent equations.

In the proof one has to use the ghost equation shown in ref. [19], which controls the coupling of the Faddeev–Popov ghost c . However this equation holds only in the Landau gauge, and we will therefore have to present our arguments in this particular gauge. The extension of the non-renormalization theorem to a general linear covariant gauge can be easily performed by following the techniques of extended BRS invariance [20], as it was done in ref. [21].

Let us finish this introduction by some remarks.

The proof we are going to present here concerns the non-supersymmetric theories for simplicity, the generalization to the supersymmetric case being apparently straightforward. There is indeed a supersymmetric version of the descent equations which allows for an algebraic set up analog to the non-supersymmetric one and which leads to a unique characterization of the anomaly [22,23].

Our proof covers the cases of theories for which the gauge beta function does not vanish to higher than one-loop order. It does not hold, as it stands, in cases of higher-order vanishing gauge beta function.

This proof in particular would not apply to the topological theories which have vanishing beta functions to all orders [24], but to the present time there is no known example of such a theory having a gauge anomaly, given as a non-trivial solution of the Wess–Zumino consistency conditions [24,25].

It is however relevant for the construction of finite supersymmetric gauge theories [15]. Indeed such a construction starts with a model whose gauge beta function vanishes only in the one-loop approximation and depends on a certain number of *independent* couplings (a gauge and a few Yukawa couplings). It is at this stage that the non-renormalization theorem is needed. The all-order vanishing of the whole set of beta functions is then ensured by requiring the Yukawa

couplings to be a function of the gauge coupling constant, according to the “reduction of coupling constants” theory of Zimmermann [26].

2. Properties of Yang–Mills theories in the Landau gauge

The purpose of this section is to give a brief summary of the algebraic properties which characterize a four-dimensional gauge theory quantized in the Landau gauge [7,19].

Let us consider a massless gauge theory whose complete classical action Σ , using the same notations of ref. [7], reads

$$\Sigma = \Sigma_{\text{inv}} + \Sigma_{\text{gf}} + \Sigma_{\text{ext}}, \quad (2.1)$$

where Σ_{inv} , Σ_{gf} and Σ_{ext} are respectively the gauge-invariant action, the Landau gauge-fixing term and the external field dependent part. They are given by

$$\Sigma_{\text{inv}} = \int d^4x \left(-\frac{1}{4g^2} F^{a\mu\nu} F_{\mu\nu}^a + \mathcal{L}_{\text{matter}}(\phi, D_\mu \phi, \lambda_i) \right), \quad (2.2)$$

$$\Sigma_{\text{gf}} = \int d^4x \left(b^a \partial^\mu A_\mu^a + \bar{c}^a \partial^\mu (D_\mu c)^a \right), \quad (2.3)$$

$$\Sigma_{\text{ext}} = \int d^4x \left(-\Omega^{a\mu} (D_\mu c)^a + \frac{1}{2} \sigma^a f^{abc} c^b c^c - iY c^a T^a \phi \right), \quad (2.4)$$

where f^{abc} are the structure constant of a simple compact gauge group G , T^a are the generators of the matter representation and $\{\lambda_i\}$ denote the self-coupling constants of the matter fields ϕ whose invariant lagrangian $\mathcal{L}_{\text{matter}}$ is restricted by the usual power-counting condition.

The invariance of Σ under the nilpotent BRS transformations [27] (the external fields Ω , σ , Y being kept invariant as usual)

$$\begin{aligned} sA_\mu^a &= -(D_\mu c)^a, \\ sc^a &= \frac{1}{2} f^{abc} c^b c^c, \\ s\bar{c}^a &= b^a, \\ sb^a &= 0, \\ s\phi &= -ic^a T^a \phi, \end{aligned} \quad (2.5)$$

is expressed by the classical Slavnov identity:

$$\mathcal{S}(\Sigma) = \int d^4x \left(\frac{\delta \Sigma}{\delta \Omega^{a\mu}} \frac{\delta \Sigma}{\delta A_\mu^a} + \frac{\delta \Sigma}{\delta \sigma^a} \frac{\delta \Sigma}{\delta c^a} + \frac{\delta \Sigma}{\delta Y} \frac{\delta \Sigma}{\delta \phi} + b^a \frac{\delta \Sigma}{\delta \bar{c}^a} \right) = 0. \quad (2.6)$$

This identity is assumed to be broken at the quantum level by the gauge anomaly [8,28], i.e.

$$\mathcal{S}(\Gamma) = \hbar^n r \mathcal{A} + \mathcal{O}(\hbar^{n+1}), \quad n \geq 2, \quad (2.7)$$

with

$$\mathcal{A} = \varepsilon^{\mu\nu\rho\sigma} \int d^4x \partial_\mu c^a \left(d^{abc} \partial_\nu A_\rho^b A_\sigma^c - \frac{1}{12} \mathcal{D}^{abcd} A_\nu^b A_\rho^c A_\sigma^d \right), \quad (2.8)$$

$$\mathcal{D}^{abcd} = d^{abn} f^{ncd} + d^{acn} f^{nbd} + d^{adn} f^{nbc}, \quad (2.9)$$

where d^{abc} is the totally symmetric invariant tensor and Γ is the vertex functional

$$\Gamma = \Sigma + \mathcal{O}(\hbar). \quad (2.10)$$

One has to note that eq. (2.7) implies that the gauge anomaly is absent at the one-loop level, i.e. we consider the case in which the coefficient r of the one-loop triangle diagram is equal to zero, due to an appropriate choice of the matter field representation [8].

In such a situation the Adler–Bardeen theorem [5–8] states that the coefficient r in (2.7) identically vanishes.

The vertex functional Γ , besides the anomalous Slavnov identity (2.7), is known to obey:

(i) the Landau gauge-fixing condition and the antighost equation [29],

$$\frac{\delta \Gamma}{\delta b^a} = \partial A^a, \quad \frac{\delta \Gamma}{\delta \bar{c}^a} + \partial \frac{\delta \Gamma}{\delta \Omega^a} = 0; \quad (2.11)$$

(ii) the rigid gauge invariance [29],

$$\mathcal{R}_{\text{rig}}^a \Sigma = \sum_{\text{all fields } \varphi} \int d^4x \delta_{\text{rig}}^a \varphi \frac{\delta \Sigma}{\delta \varphi} = 0; \quad (2.12)$$

(iii) the ghost equation [19],

$$\int d^4x \left(\frac{\delta \Gamma}{\delta c^a} + f^{abc} \bar{c}^b \frac{\delta \Gamma}{\delta b^c} \right) = \Delta^a, \quad (2.13)$$

$$\Delta^a = \int d^2x \left(f^{abc} \Omega^{b\mu} A_\mu^c - f^{abc} \sigma^b c^c + i Y T^a \phi \right). \quad (2.14)$$

These conditions, together with (2.7), allow us to write a Callan–Symanzik equation which is Slavnov invariant up to the order \hbar^n [7], i.e.

$$\begin{aligned}\mathcal{C}\Gamma &= \left(\mu \frac{\partial}{\partial \mu} + \hbar \beta_g \frac{\partial}{\partial g} + \hbar \sum_i \beta_i \frac{\partial}{\partial \lambda_i} + \hbar \gamma_A \mathcal{N}_A + \hbar \gamma_\phi \mathcal{N}_\phi \right) \Gamma \\ &= \hbar^{n+1} \Delta_c^{n+1} + \mathcal{O}(\hbar^{n+2}),\end{aligned}\quad (2.15)$$

where μ denotes the renormalization point, Δ_c^{n+1} is an integrated local polynomial, (β_g, β_i) are respectively the beta functions for the gauge and the self matter couplings and $(\mathcal{N}_A, \mathcal{N}_\phi)$ are the Slavnov invariant counting operators:

$$\mathcal{N}_A = \int d^4x \left(A^{a\mu} \frac{\delta}{\delta A^{a\mu}} - b^a \frac{\delta}{\delta b^a} - \bar{c}^a \frac{\delta}{\delta \bar{c}^a} - \Omega^{a\mu} \frac{\delta}{\delta \Omega^{a\mu}} \right), \quad (2.16)$$

$$\mathcal{N}_\phi = \int d^4x \left(\phi \frac{\delta}{\delta \phi} - Y \frac{\delta}{\delta Y} \right). \quad (2.17)$$

The vanishing up to the order \hbar^n of the ghost anomalous dimension, i.e. the absence in (2.15) of the Slavnov invariant counting term

$$\mathcal{N}_c \Gamma = \int d^4x \left(c^a \frac{\delta \Gamma}{\delta c^a} - \sigma^a \frac{\delta \Gamma}{\delta \sigma^a} \right), \quad (2.18)$$

is due to the ghost equation (2.13).

Moreover, as shown in ref. [7], the use of the Landau gauge allows us to define a renormalized anomaly insertion

$$[\mathcal{A} \cdot \Gamma] = \mathcal{A} + \mathcal{O}(\hbar \mathcal{A}), \quad (2.19)$$

which possesses the following properties:

$$\mathcal{C}[\mathcal{A} \cdot \Gamma] = \hbar \mathcal{S}_\Gamma[\hat{\Delta} \cdot \Gamma] + \mathcal{O}(\hbar^{n+1}), \quad (2.20)$$

$$\mathcal{S}_\Gamma[\mathcal{A} \cdot \Gamma] = \mathcal{O}(\hbar^n), \quad (2.21)$$

where \mathcal{S}_Γ is the linearized Slavnov operator

$$\begin{aligned}\mathcal{S}_\Gamma &= \int d^4x \left(\frac{\delta \Gamma}{\delta \Omega^{a\mu}} \frac{\delta}{\delta A_\mu^a} + \frac{\delta \Gamma}{\delta A_\mu^a} \frac{\delta}{\delta \Omega^{a\mu}} + \frac{\delta \Gamma}{\delta \sigma^a} \frac{\delta}{\delta c^a} + \frac{\delta \Gamma}{\delta c^a} \frac{\delta}{\delta \sigma^a} \right. \\ &\quad \left. + \frac{\delta \Gamma}{\delta Y} \frac{\delta}{\delta \phi} + \frac{\delta \Gamma}{\delta \phi} \frac{\delta}{\delta Y} + b^a \frac{\delta}{\delta \bar{c}^a} \right)\end{aligned}\quad (2.22)$$

and

$$\mathcal{S}_\Gamma \mathcal{S}_\Gamma = \mathcal{O}(\hbar^n). \quad (2.23)$$

The non-vanishing right-hand side of the last equation is due to the presence of the gauge anomaly in the Slavnov identity (2.7).

Eqs. (2.20) and (2.21) tell us that the insertion $[\mathcal{A} \cdot \Gamma]$ obeys a Callan–Symanzik equation without anomalous dimension (up to a \mathcal{S}_Γ -variation) till the order \hbar^n , and that it is Slavnov invariant up to the order \hbar^n .

As we will see in the next sections, properties (2.20) and (2.21) will provide a complete algebraic proof of the Adler–Bardeen theorem also in the case of vanishing one-loop gauge beta function.

3. Order \hbar^{n+1}

Following ref. [6], we can extend the anomalous Slavnov identity (2.7) to the order \hbar^{n+1} as

$$\mathcal{S}(\Gamma) = r\hbar^n[\mathcal{A} \cdot \Gamma] + \hbar^{n+1}\mathcal{B} + \mathcal{O}(\hbar^{n+2}), \quad (3.1)$$

where $[\mathcal{A} \cdot \Gamma]$ is the anomaly insertion defined in eqs. (2.20) and (2.21) and \mathcal{B} is an integrated local functional of ultraviolet dimension four and ghost number one.

Applying the Callan–Symanzik operator to both sides of eq. (3.1) and making use of eq. (2.20) and of the algebraic property

$$\mathcal{C}\mathcal{S}(\Gamma) = \mathcal{S}_\Gamma \mathcal{C}\Gamma, \quad (3.2)$$

we get, to the lowest order (i.e. order $n+1$) in \hbar , the equation

$$\left(\beta_g^{(1)} \frac{\partial r}{\partial g} + \sum_i \beta_i^{(1)} \frac{\partial r}{\partial \lambda_i} \right) \mathcal{A} + \mu \frac{\partial \mathcal{B}}{\partial \mu} = \mathcal{S}_\Sigma(\Delta_c^{n+1} - r\hat{\Delta}), \quad (3.3)$$

where \mathcal{S}_Σ is the linearized nilpotent operator corresponding to the classical Slavnov identity (2.6) and $(\beta_g^{(1)}, \beta_i^{(1)})$ are the one-loop beta functions [10].

Taking into account that \mathcal{B} is homogeneous of degree zero in the mass parameter [6], i.e.

$$\mu \frac{\partial \mathcal{B}}{\partial \mu} = 0, \quad (3.4)$$

and that the gauge anomaly \mathcal{A} cannot be written as a local \mathcal{S}_Σ -variation, it follows that (3.3) is equivalent to the two conditions

$$\beta_g^{(1)} \frac{\partial r}{\partial g} + \sum_i \beta_i^{(1)} \frac{\partial r}{\partial \lambda_i} = 0, \quad (3.5)$$

$$\mathcal{S}_\Sigma(\Delta_c^{n+1} - r\hat{\Delta}) = 0. \quad (3.6)$$

In the case in which the one-loop gauge beta function $\beta_g^{(1)}$ does not identically vanish, (3.5) implies the Adler–Bardeen theorem [5–7]. However, for the time being, we keep (3.5) just as an algebraic equation in view of the fact that we will allow the coefficient $\beta_g^{(1)}$ to vanish. In this case eq. (3.5) implies only that r does not depend on the self matter couplings λ_i .

Let us turn now to the analysis of the second condition (3.6). This equation shows that the difference $(\Delta_c^{n+1} - r\hat{\Delta})$, being a Slavnov invariant quantity, can be expanded in terms of the elements of the invariant basis [29]:

$$\left(\frac{\partial \Sigma}{\partial g}, \quad \frac{\partial \Sigma}{\partial \lambda_i}, \quad \mathcal{N}_A \Sigma, \quad \mathcal{N}_\phi \Sigma, \quad \mathcal{N}_c \Sigma \right), \quad (3.7)$$

This amounts to rewrite the Callan–Symanzik equation (2.15) as

$$\mathcal{E}\Gamma + \hbar^{n+1} \gamma_c \mathcal{N}_c \Gamma = r \hbar^{n+1} \hat{\Delta} + \mathcal{O}(\hbar^{n+2}), \quad (3.8)$$

where the ghost-anomalous dimension has reappeared, in agreement with the fact that its absence is ensured only up to the order \hbar^n [7].

Finally, repeating the same argument as in ref. [6], the Callan–Symanzik equation (3.8) extends to the order \hbar^{n+2} as

$$\mathcal{E}\Gamma + \hbar^{n+1} \gamma_c \mathcal{N}_c \Gamma = r \hbar^{n+1} [\hat{\Delta} \cdot \Gamma] + \hbar^{n+2} \Delta_c^{n+2} + \mathcal{O}(\hbar^{n+3}), \quad (3.9)$$

where Δ_c^{n+2} is a local integrated functional. The interesting feature of this equation is that the general local polynomial Δ_c^{n+1} of eq. (2.15) has been replaced by the term $[\hat{\Delta} \cdot \Gamma]$, which is the same quantity as the one appearing in the Callan–Symanzik equation for the anomaly insertion (2.20). This step will turn out to be very useful in the discussion of the model at the order \hbar^{n+2} .

4. Order \hbar^{n+2}

This section is devoted to the analysis of the anomalous Slavnov identity at the order \hbar^{n+2} , i.e. to the algebraic characterization of the local polynomial \mathcal{B} in eq. (3.1).

To do this we will use property (2.21) which shows that the anomaly insertion $[\mathcal{A} \cdot \Gamma]$ is Slavnov invariant up to the order \hbar^n . Applying the linearized operator \mathcal{S}_Γ (2.22) to both sides of eq. (3.1) and making use of (2.21) and of the exact relation

$$\mathcal{S}_\Gamma \mathcal{S}(\Gamma) = 0, \quad (4.1)$$

we find, to the lowest order in \hbar (remember that $n \geq 2$), the equation

$$\mathcal{S}_\Sigma \mathcal{B} = 0. \quad (4.2)$$

This condition implies that the local polynomial \mathcal{B} is Slavnov invariant with ghost number one, and then can be written as

$$\mathcal{B} = \hat{r} \mathcal{A} + \mathcal{S}_\Sigma \mathcal{Q}, \quad (4.3)$$

where \mathcal{A} is the gauge anomaly (2.8), \hat{r} is an arbitrary coefficient and \mathcal{Q} a local integrated polynomial of dimension four and ghost number zero. Moreover, since \mathcal{B} appears in the Slavnov equation (3.1) at the order \hbar^{n+1} , it follows that the cohomological trivial term $\mathcal{S}_\Sigma \mathcal{Q}$ can be reabsorbed in the effective action Γ as a local counterterm without affecting properties (2.20), (2.21) and the Callan–Symanzik equation (3.9).

The Slavnov identity (3.1) becomes then

$$\mathcal{S}(\Gamma) = r \hbar^n [\mathcal{A} \cdot \Gamma] + \hat{r} \hbar^{n+1} \mathcal{A} + \mathcal{O}(\hbar^{n+2}), \quad (4.4)$$

and extends to the order \hbar^{n+2} as

$$\mathcal{S}(\Gamma) = r \hbar^n [\mathcal{A} \cdot \Gamma] + \hat{r} \hbar^{n+1} [\mathcal{A} \cdot \Gamma] + \hbar^{n+2} \hat{\mathcal{B}} + \mathcal{O}(\hbar^{n+3}), \quad (4.5)$$

where $\hat{\mathcal{B}}$ is an integrated local polynomial of ultraviolet dimension four and ghost number one. It is important to note that we cannot iterate the previous arguments to characterize $\hat{\mathcal{B}}$, i.e. property (2.21) allows us to characterize only the order \hbar^{n+1} .

Commuting now the Callan–Symanzik equation (3.9) with the Slavnov identity (4.5) and using eqs. (2.20) and (3.5) and the algebraic relations

$$\mathcal{N}_c \mathcal{S}(\Gamma) = \mathcal{S}_\Gamma \mathcal{N}_c \Gamma, \quad (4.6)$$

$$\mathcal{N}_c [\mathcal{A} \cdot \Gamma] = \mathcal{A} + \mathcal{O}(\hbar), \quad (4.7)$$

we get, to the lowest order (i.e. $n+2$) in \hbar , the equation

$$\begin{aligned} & \left(\beta_g^{(2)} \frac{\partial r}{\partial g} + \sum_i \beta_i^{(2)} \frac{\partial r}{\partial \lambda_i} + \beta_g^{(1)} \frac{\partial \hat{r}}{\partial g} + \sum_i \beta_i^{(1)} \frac{\partial \hat{r}}{\partial \lambda_i} \right) \mathcal{A} + \mu \frac{\partial \hat{\mathcal{B}}}{\partial \mu} \\ &= \mathcal{S}_\Sigma (\Delta_c^{n+2} - \hat{r} \hat{\Delta}), \end{aligned} \quad (4.8)$$

where $(\beta_g^{(2)}, \beta_i^{(2)})$ are the two-loop beta functions.

As in sect. 3, taking into account that $\hat{\mathcal{B}}$ is homogeneous of degree zero in the mass parameter and that the anomaly \mathcal{A} cannot be written as a local \mathcal{S}_Σ -variation, eq. (4.8) splits into the two equations

$$\mathcal{S}_\Sigma(\Delta_c^{n+2} - \hat{r}\hat{\Delta}) = 0, \quad (4.9)$$

$$\left(\beta_g^{(2)} \frac{\partial r}{\partial g} + \sum_i \beta_i^{(2)} \frac{\partial r}{\partial \lambda_i} + \beta_g^{(1)} \frac{\partial \hat{r}}{\partial g} + \sum_i \beta_i^{(1)} \frac{\partial \hat{r}}{\partial \lambda_i} \right) = 0. \quad (4.10)$$

Eq. (4.10), as it will be discussed in sect. 5, allows to control the dependence of the anomaly coefficient r from the coupling constants (g, λ_i) in the case in which $\beta_g^{(1)} = 0$.

As one can easily understand, this is due to the presence in eq. (4.10) of the second-order beta functions.

5. The Adler–Bardeen theorem

As shown in the previous sections, the anomaly coefficient r in (2.7) is constrained by the two conditions (3.5), (4.10); here rewritten for convenience:

$$\beta_g^{(1)} \frac{\partial r}{\partial g} + \sum_i \beta_i^{(1)} \frac{\partial r}{\partial \lambda_i} = 0, \quad (5.1)$$

$$\left(\beta_g^{(2)} \frac{\partial r}{\partial g} + \sum_i \beta_i^{(2)} \frac{\partial r}{\partial \lambda_i} + \beta_g^{(1)} \frac{\partial \hat{r}}{\partial g} + \sum_i \beta_i^{(1)} \frac{\partial \hat{r}}{\partial \lambda_i} \right) = 0. \quad (5.2)$$

To discuss the consequences of these equations on the coefficients (r, \hat{r}) let us consider first the case in which the one-loop gauge beta function $\beta_g^{(1)}$ is non-vanishing.

In this case, as shown in refs. [5–7], eq. (5.1) implies the Adler–Bardeen theorem, i.e. that $r = 0$. Eq. (5.2) reduces to

$$\beta_g^{(1)} \frac{\partial \hat{r}}{\partial g} + \sum_i \beta_i^{(1)} \frac{\partial \hat{r}}{\partial \lambda_i} = 0, \quad (5.3)$$

from which it follows that also \hat{r} vanishes; improving then the validity of the Slavnov identity (4.5) to all orders of perturbation theory by induction.

Let us consider now the case in which

$$\beta_g^{(1)} = 0, \quad \beta_g^{(2)} \neq 0. \quad (5.4)$$

Eqs. (5.1) and (5.2) become

$$\sum_i \beta_i^{(1)} \frac{\partial r}{\partial \lambda_i} = 0, \quad (5.5)$$

$$\beta_g^{(2)} \frac{\partial r}{\partial g} + \sum_i \beta_i^{(2)} \frac{\partial r}{\partial \lambda_i} + \sum_i \beta_i^{(1)} \frac{\partial \hat{r}}{\partial \lambda_i} = 0. \quad (5.6)$$

Eq. (5.5) implies that r is independent from the self matter couplings λ_i . It follows then that eq. (5.6) reads

$$\beta_g^{(2)} \frac{\partial r}{\partial g} + \sum_i \beta_i^{(1)} \frac{\partial \hat{r}}{\partial \lambda_i} = 0, \quad (5.7)$$

which is easily seen to imply that $r = 0$, owing to the fact that r depends only on the gauge coupling g and that the two-loop gauge beta function $\beta_g^{(2)}$ [30] is not identically zero for vanishing self matter couplings.

This concludes the proof of the Adler–Bardeen theorem in the case of vanishing one-loop gauge beta function.

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