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Meson Theory of the Magnetic Moment of Proton and Neutron

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The magnetic moment of proton and neutron is determined by a second-order perturbation calculation. A new limiting process secures the convergence of the integration in momentum space. The result for both pseudoscalar and vector meson is the same and leads to a positive magnetic moment for the neutron if the magnetic moment of the vector meson is assumed to be one meson magneton ($e\hbar/2\mu c$). Agreement with the known values of the magnetic moment of the heavy particles can only be obtained if we give the vector meson an anomalous magnetic moment of about three meson magnetons.

1.

THE relativistic wave equation of particles with spin $\frac{1}{2}$ leads in a natural way to a value of the magnetic moment of one magneton ($e\hbar/2mc$), where m is the mass and e the charge of the particle. In the case of proton and neutron the magnetic moments differ appreciably from the values which they should have according to this theory. Measured in units of nuclear magnetons the experimental values for these moments are:

$$\mu_p = +2.785 \pm 0.02$$

for the proton[1] and

$$\mu_n = -1.935 \pm 0.02$$

for the neutron.[2] The minus sign in the case of the neutron indicates that the spin points in the opposite direction of the magnetic moment.

It was pointed out by Kemmer[3] and others[4] that the meson theory of nuclear forces would also lead to anomalous magnetic moments of the nucleons, due to the virtual emission and reabsorption of mesons by the nuclear particles. Unfortunately a second-order perturbation calculation leads to divergent integrals. This divergence difficulty was dodged by cutting off the integrals suitably, which is equivalent to introducing a finite size of the nucleon.

This procedure, however, is highly unsatisfactory for two reasons:

(1) It introduces an arbitrary constant into the theory, the physical significance of which is not quite clear.

(2) It destroys the relativistic invariance of the theory.

In spite of these grave objections one feels that the meson theory of the magnetic moment is essentially correct and that a theory of interaction of nucleons with the meson field, formulated in such a way that the divergence difficulties disappear, should give the correct value for these moments.

Recently a theory was proposed by Dirac[5] for the interaction of an electron with the electromagnetic field, which avoids all divergence difficulties for that case. The theory consists of two essentially different parts. The first part is the so-called λ-limiting process, first investigated by Wentzel.[6] It is essentially a classical procedure and consists of using the difference of retarded and advanced potential at the position of the particle. Dirac showed[7] that the λ process may best be introduced in quantum mechanics by a change in the commutators of the field quantities entering into the interaction energy.[8] In this form the procedure may easily be generalized to charged fields with rest mass different from zero.

The integral which causes the divergence of the meson theory of the magnetic moment is the integral

$$\int_0^\infty k^4/(k^2+\mu^2)^2 dk$$

[1] Kellogg, Rabi, Ramsey, and Zacharias, Phys. Rev. **56**, 728 (1939).
[2] L. W. Alvarez and F. Bloch, Phys. Rev. **57**, 111 (1940).
[3] N. Kemmer, Proc. Roy. Soc. **A166**, 127 (1938).
[4] Fröhlich, Heitler, and Kemmer, Proc. Roy. Soc. **A166**, 154 (1938). H. Fröhlich, Phys. Rev. **62**, 180L (1942). The result announced in this note, that the pseudoscalar meson gives no magnetic moment, is incorrect. I am indebted to Professor Fröhlich for correspondence on this point.
[5] P. A. M. Dirac, Proc. Roy. Soc. **A180**, 1 (1942).
[6] G. Wentzel, Zeits. f. Physik **86**, 479, 635 (1933); **87**, 726 (1934).
[7] P. A. M. Dirac, Am. de l'Inst. H. Poincare, **9**, 13 (1939).
[8] For a detailed account see an article by W. Pauli, to be published in the Rev. Mod. Phys.

with μ = meson mass. This integral will be made convergent with the λ process. We must, however, remark that this integral occurs only if a second-order perturbation theory is applied and if the mass m of the heavy particle is infinitely large. In higher approximations in the coupling constant and in μ/m other integrals will appear in addition to this which no longer converge with the λ process alone. In general only integrals which for large values of k behave like

$$\int k^{2n} dk$$

with n integer will be made to converge with this method. A limitation to a second-order perturbation treatment would not be justified if such higher order divergent terms remain.

In order to make the other integrals converge, Dirac has introduced another kind of field oscillator corresponding to states with negative energy and quantized according to an indefinite metric in Hilbert space.[8] Dirac has shown that the two methods lead to the elimination of all divergencies in the quantum theory of fields. Serious interpretation difficulties still remain, however.

It may easily be seen that the higher order terms lead always to integrals of the form[9]

$$\int_0^\infty R(k_0, k) dk$$

with a rational function $R(k_0, k)$ for the integrand containing only even powers of k. The introduction of the negative energy oscillators changes this integral into the form

$$\tfrac{1}{2}\int_0^\infty [R(k_0, k)+R(-k_0, k)]dk$$

and this is sufficient to make all the remaining integrals occurring in this perturbation treatment converge.

The calculation of the magnetic moment due to the meson field with this method leads to a positive value for the neutron both in the pseudoscalar and the vector theory. This is in disagreement with the experimental result. Only by making use of the possibility that the vector meson may have a magnetic moment different from one meson magneton can agreement with the experiment be obtained. The contribution of the meson field to the magnetic moment has then a certain value M which is positive for the proton and negative for the neutron, so that we shall have for the magnetic moment of these particles

$$\mu_p = 1+M, \quad \mu_n = -M.$$

By choosing $M \sim +1.9$, agreement with experiment will be obtained within the limits of our approximation.

The objection may be raised that this assumption of an anomalous magnetic moment of the vector meson introduces a new constant into the theory and we stand where we were before. The situation is, however, not quite so bad. We must bear in mind that we have introduced the new parameter in a way which does not destroy the relativistic invariance of the theory. Moreover, the constant has a definite physical meaning, namely, the magnetic moment of the vector meson, which may be checked with an experiment, deciding for or against the theory here presented.

2. THE PSEUDOSCALAR THEORY

In the following we often use one single letter "a" for a four vector with components a_μ. Here $\mathbf{a} = a^k$ is the space part of the vector. The invariant scalar product of two four vectors shall be denoted by $(a, b) = (\mathbf{a}, \mathbf{b}) - a^0 b^0 = a_\mu a^\mu$. $(a_0 = -a^0.)$

Charged particles of spin zero are described by a complex field $\psi(x)$ satisfying the field equation

$$\Box\psi - \mu^2\psi = 0,$$

where μ is the meson mass in units $\hbar/c$ and $\Box = \partial^2/\partial x^\mu \partial x_\mu$. It is quantized according to the commutation rules

$$i[\psi(x), \psi^*(x')] = D(x-x'), \quad (1)$$

where $D(x)$ is the invariant D function defined by

$$D(\mathbf{x}, x^0) = \frac{1}{(2\pi)^3}\int \frac{d^3k}{k^0} \exp[i(\mathbf{k}, \mathbf{x})] \sin k^0 x^0, \quad (2)$$
$$k^0 = +(k^2+\mu^2)^{\frac{1}{2}}.$$

The operator for the current density vector is

$$s_\mu = i\left(\frac{\partial\psi^*}{\partial x^\mu}\psi - \frac{\partial\psi}{\partial x^\mu}\psi^*\right) \quad (3)$$

[9] This is not true for the theory of holes, where integrals of the form $\int dk/(k^2+m^2)^{\frac{1}{2}}$ occur, which are logarithmically divergent, even in this theory.

and the total magnetic moment of the field

$$M_{\mu\nu}=\frac{e}{2}\int(x_\mu s_\nu - x_\nu s_\mu)d^3x, \quad (4)$$

where the positive number "e" is the magnitude of the charge of the mesons in units $(\hbar c)^{\frac{1}{2}}$.

For our calculations it is somewhat more convenient to introduce two real fields ψ_1 and ψ_2:

$$\psi=\frac{1}{\sqrt{2}}(\psi_1-i\psi_2), \quad \psi^*=\frac{1}{\sqrt{2}}(\psi_1+i\psi_2),$$

for which we have the commutation rules

$$i[\psi_\alpha(x), \psi_\beta(x')]=\delta_{\alpha\beta}D(x-x'). \quad (5)$$

The spin dependent interaction of the heavy particle with the coordinates z may be written[10]

$$H'=g(4\pi)^{\frac{1}{2}}\tau_\alpha(\boldsymbol{\sigma}, \text{grad})\psi_\alpha(z). \quad (6)$$

The τ_α represent the matrices for the isotopic spin and $\boldsymbol{\sigma}=(\sigma_1, \sigma_2, \sigma_3)$ is the ordinary spin vector. They satisfy the equation

$$\tau_1\tau_2=-\tau_2\tau_1=i\tau_3, \cdots \sigma_1\sigma_2=-\sigma_2\sigma_1=i\sigma_3, \cdots \quad \tau_1^2=\cdots=\sigma_3^2=1, \quad (7)$$

together with all the other equations which can be obtained from them by cyclic permutation of the indices. The summation over α is extended over the two indices $\alpha=1, 2$. In the symmetrical theory there is a third real field ψ_3 present representing the neutral mesons, and the summation goes then over all three indices. Since neutral particles do not contribute anything to the current and to the magnetic moment, we can restrict ourselves to the first two indices.

The interaction energy contains only the nonrelativistic part of the heavy particles. This approximation can be made since we are only interested in the effects which are of highest order in μ/m, and they are independent of m. This amounts to setting the mass of the heavy particle infinitely large, and then relativistic effects of the heavy particle are of course unimportant. The mesons, however, are treated relativistically all through the calculations. The interaction term in the Hamiltonian leads to an additional term in the current so that the total current now is[11]

$$s_\mu=(\partial\psi_1/\partial x^\mu)\psi_2-(\partial\psi_2/\partial x^\mu)\psi_1 + g(4\pi)^{\frac{1}{2}}\sigma_\mu(\tau_2\psi_1-\tau_1\psi_2)\delta(\mathbf{x}-\mathbf{z}). \quad (8)$$

We develop the field operators into eigenstates in momentum space

$$\psi_\alpha(x)=\frac{1}{\sqrt{2}(2\pi)^{\frac{3}{2}}}\int\frac{d^3k}{k^0}\{\varphi_\alpha(\mathbf{k})e^{i(k,x)} + \varphi_\alpha^*(\mathbf{k})e^{-i(k,x)}\}. \quad (9)$$

This definition of the $\varphi_\alpha(k)$ and $\varphi_\alpha^*(k)$ is useful since they are constant operators if there is no interaction between heavy particles and field. In case of an interaction they are slowly varying according to the law[12]

$$d\varphi_\alpha/dx^0=i[H', \varphi_\alpha]. \quad (10)$$

From (1), (2), and (9) it follows that the commutation rules for the $\varphi_\alpha(k)$ are

$$[\varphi_\alpha(\mathbf{k}), \varphi_\beta^*(\mathbf{k}')]=k^0\delta_{\alpha\beta}\delta(\mathbf{k}-\mathbf{k}'), \quad (11)$$

from which it follows in the well-known way that

$$N_\alpha(\mathbf{k})=(1/k^0)\varphi_\alpha^*(\mathbf{k})\varphi_\alpha(\mathbf{k})$$

has the eigenvalues 0, 1, 2, $\cdots$ and represents the number of mesons in the state $(\alpha, \mathbf{k})$.

The interaction energy is

$$H'=if\tau_\alpha\int\frac{d^3k}{k^0}(\boldsymbol{\sigma}, \mathbf{k}) \times\{\varphi_\alpha(\mathbf{k})e^{i(k,z)}-\varphi_\alpha^*(\mathbf{k})e^{-i(k,z)}\} \quad (12)$$

with

$$f=g/2\pi.$$

The λ process may be introduced, as Dirac[13] has shown, by a change in the commutation rules of the field operators which enter into the interaction energy. In x space these commutation rules may be written

$$i[\tilde{\psi}_\alpha(x), \tilde{\psi}_\beta(x')]=\delta_{\alpha\beta}D_\lambda(x-x'), \quad (13)$$

where

$$D_\lambda(x)=\tfrac{1}{2}\{D(x+\lambda)+D(x-\lambda)\}. \quad (14)$$

$\lambda=\{\lambda, \lambda^0\}$ is a time-like four vector, $(\lambda, \lambda)<0$, (15)

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[10] In the complex notation this term corresponds to the expression

$$H'=g(8\pi)^{\frac{1}{2}}[\tau_-(\boldsymbol{\sigma}, \text{grad})\psi^*+\tau_+(\boldsymbol{\sigma}, \text{grad})\psi]$$

with $\tau_\pm=1/2(\tau_1\pm i\tau_2)$. Our g is therefore $\sqrt{2}$ times the coupling constant which is usually used in the literature.

[11] See W. Pauli and S. M. Dancoff, Phys. Rev. **62**, 105 (1942). There would be a third term in the total current due to the proton which, however, in our approximation is zero, since the velocity (although not the momentum) of the nucleons is zero.

[12] See W. Pauli, Rev. Mod. Phys. **13**, 210 (1942).

[13] P. A. M. Dirac, Ann. de l'Inst. Henri Poincaré **9**, 13 (1939).

which ultimately goes to zero in such a way that the inequality (15) is always satisfied. If λ transforms like a four vector, then the scheme is evidently relativistically invariant. In k space the introduction of the λ process shows itself in the commutation rules

$$[\tilde{\varphi}_\alpha(\mathbf{k}),\, \tilde{\varphi}_\beta^*(\mathbf{k}')] = k^0\delta_{\alpha\beta}\delta(\mathbf{k}-\mathbf{k}')\cos(k,\lambda) \tag{16}$$

which follow from (13).

A possible representation of the $\tilde{\varphi}_\alpha(\mathbf{k})$ in terms of the $\varphi_\alpha(\mathbf{k})$ is given by

$$\tilde{\varphi}_\alpha(\mathbf{k}) = \varphi_\alpha(\mathbf{k})\left\{\cos\frac{(k,\lambda)}{2} - \sin\frac{(k,\lambda)}{2}\right\},$$

$$\tilde{\varphi}_\alpha^*(\mathbf{k}) = \varphi_\alpha^*(\mathbf{k})\left\{\cos\frac{(k,\lambda)}{2} + \sin\frac{(k,\lambda)}{2}\right\}.$$

The quantities which depend on the field alone remain unchanged.[14] For instance, the magnetic moment may be written

$$M_{\mu\nu} = M_{\mu\nu}^{(1)} + M_{\mu\nu}^{(2)}, \tag{17}$$

$$\begin{aligned} M_{\mu\nu}^{(1)} &= \frac{e}{2}\int\left\{x_\mu\left(\frac{\partial\psi_1}{\partial x^\nu}\psi_2 - \frac{\partial\psi_2}{\partial x^\nu}\psi_1\right) - x_\nu\left(\frac{\partial\psi_1}{\partial x^\mu}\psi_2 - \frac{\partial\psi_2}{\partial x^\mu}\psi_1\right)\right\}d^3x \\ &= \frac{ei}{4(2\pi)^3}\int d^3x\left\langle x_\mu\int\int\frac{d^3l}{l_0}\frac{d^3l'}{l_0'} \times[\{\varphi_1(1)e^{i(l,x)} - \varphi_1^*(1)e^{-i(l,x)}\} \times\{\varphi_2(1')e^{i(l',x)} + \varphi_2(1')e^{-i(l',x)}\} - \text{sym. }(12)] - \text{sym. }(\mu\nu)\right\rangle, \end{aligned} \tag{18}$$

where sym. $(\mu\nu)$ stands for the preceding expression with the indices μ and ν interchanged.

$$M_{\mu\nu}^{(2)} = \frac{e}{2}g(4\pi)^{\frac{1}{2}}(z_\mu\sigma_\nu - z_\nu\sigma_\mu)\frac{1}{\sqrt{2}(2\pi)^{\frac{3}{2}}} \times\int\frac{d^3k}{k^0}[\tau_2\{\tilde{\varphi}_1(\mathbf{k})e^{i(k,z)} + \tilde{\varphi}_1^*(\mathbf{k})e^{-i(k,z)}\} - \text{sym. }(1,2)]. \tag{19}$$

The first of these two expressions may be simplified by replacing x_μ by a differentiation of the exponential and subsequent integration by parts

$$M_{\mu\nu}^{(1)} = -\frac{e}{4(2\pi)^3}\int d^3x\int\frac{d^3l}{l_0}\frac{d^3l'}{l_0'} \times\left\langle l_\nu\left[\left\{\frac{\partial\varphi_1(1)}{\partial l^\mu}e^{i(l,x)} + \frac{\partial\varphi_1^*(1)}{\partial l^\mu}e^{-i(l,x)}\right\} \times\{\varphi_2(1')e^{i(l',x)} + \varphi_2^*(1')e^{-i(l',x)}\} - \text{sym. }(12)\right] - \text{sym. }(\mu\nu)\right\rangle. \tag{20}$$

Using

$$\frac{1}{(2\pi)^3}\int\exp[i(1,\mathbf{x})]d^3x = \delta(1)$$

we find

$$M_{\mu\nu}^{(1)} = \frac{e}{4}\int\frac{d^3l}{l_0^2}\left[l_\nu\left\{\left(\frac{\partial\varphi_2(1)}{\partial l^\mu}\varphi_1(-1)e^{-2il_0x_0} + \frac{\partial\varphi_2^*(1)}{\partial l^\mu}\varphi_1(1) + \text{compl. conj.}\right) - \text{sym. }(1,2)\right\} - \text{sym. }(\mu\nu)\right]. \tag{21}$$

The Schrödinger functional Ω satisfies the Schrödinger equation

$$i(\partial\Omega/\partial x^0) = H'\Omega. \tag{22}$$

We look for stationary solutions of this equation which are correct up to the second order in g. For that purpose we write

$$\Omega = \Omega_0 + \Omega_1 + \Omega_2 + \cdots,$$

where Ω_0, Ω_1, Ω_2, $\cdots$ represent those parts of the functional with 0, 1, 2, $\cdots$ mesons present. The leading terms of these functionals will then be of zeroth, first, second, $\cdots$ order in the coupling constant g.

Let u represent the state of the nucleon which we choose in such a way that $\sigma_3 u = u$. The spin then points in the positive z direction. Furthermore, let ω represent that state of the field in which no mesons are present, so that

$$\varphi_\alpha(\mathbf{k})\omega = 0. \tag{23}$$

Then we have for $\Omega_0 = u\omega$.

We then write the equations for the Ω's

$$\begin{aligned} i(d\Omega_1/dx^0) &= H'\Omega_0, \\ i(d\Omega_2/dx^0) &= H'\Omega_1. \end{aligned} \tag{24}$$

[14] This definition of the adjoint operators of the $\tilde{\varphi}$ is a generalization of the Hermitian conjugate, similar to the one described by Pauli in a paper to be published in the Rev. Mod. Phys.

If we can integrate these equations, we can use the Ω's to calculate the expectation value of $M_{12}\equiv M_3$.

$$\bar{M}_3=\Omega^*M_3\Omega=2\Omega_0^*M_3^{(1)}\Omega_2+\Omega_1^*M_3^{(1)}\Omega_1+2\Omega_0M_3^{(2)}\Omega_1 \quad (25)$$

since M_3 is Hermitian.

It is seen from the last equation that only terms of zero order in g which occur in Ω_0 will contribute anything to the magnetic moment in our approximation. With this remark in mind we can integrate the equations (24) approximately. The first one with the help of (22) and (23) reads

$$i\frac{d}{dx^0}\Omega_1=-if\tau_\alpha\int\frac{d^3k}{k^0}(\boldsymbol{\sigma},\mathbf{k})\tilde{\varphi}_\alpha^*(\mathbf{k})e^{-i(k,z)}u\omega$$

or

$$\Omega_1=if\tau_\alpha\int\frac{d^3k}{k_0^2}(\boldsymbol{\sigma},\mathbf{k})\tilde{\varphi}_\alpha^*(\mathbf{k})e^{-i(k,z)}u\omega \quad (26)_1$$

and from this and the second equation of (22) we get

$$i\frac{d}{dx^0}\Omega_2=f^2\tau_\alpha\tau_\beta\int\frac{d^3k'}{k^{0\prime}}(\boldsymbol{\sigma},\mathbf{k}')\tilde{\varphi}_\alpha^*(\mathbf{k}')e^{-i(k',z)}\times\int\frac{d^3k}{k_0^2}(\boldsymbol{\sigma},\mathbf{k})e^{-i(k,z)}u\omega,$$

$$\Omega_2=-\tau_\alpha\tau_\beta f^2\int\frac{d^3k'}{k_0'(k_0+k_0')}\frac{d^3k}{k_0^2}(\boldsymbol{\sigma},\mathbf{k}')(\boldsymbol{\sigma},\mathbf{k})\times\tilde{\varphi}_\alpha^*(\mathbf{k}')\tilde{\varphi}_\beta^*(\mathbf{k})e^{-i(k'+k,z)}u\omega. \quad (26)_2$$

These integrations are only approximately correct since the operators φ_α and φ_α^* are slowly varying according to (10). But this variation would lead to a correction term to Ω_0 of the second order and a similar term in Ω_1 of the third order. Another term of the second order has been neglected by including in the expression $H'\Omega_1$ only those terms which correspond to two mesons present. In view of the above remark, we may, for our approximation, neglect all these terms. With these wave functions we can now calculate the expectation value of the magnetic moment. First we prove that the last term in (25) does not contribute anything in this approximation. This term is proportional to

$$u^*\omega^*(z_1\sigma_2-z_2\sigma_1)\int\frac{d^3k}{k^0}\{\tau_2\tilde{\varphi}_1(\mathbf{k})e^{i(k,z)}-\text{sym. }(12)\}\times\tau_\alpha\int\frac{d^3k'}{k_0'^2}(\boldsymbol{\sigma},\mathbf{k}')\tilde{\varphi}_\alpha^*(\mathbf{k}')e^{-i(k',z)}u\omega,$$

which is equal to

$$-iu^*(z_1\sigma_2-z_2\sigma_1)\tau_3\int\frac{d^3k}{k_0^2}(\boldsymbol{\sigma},\mathbf{k})u\cos(k,\lambda)$$

when we use the commutation relations. The integral vanishes and therefore

$$\Omega_0^*M_3^{(2)}\Omega_1=0.$$

Of the remaining terms let us first calculate $\Omega_1^*M_3\Omega_1=\bar{M}_3^{\mathrm{I}}$. We notice at once that only the part in M_3 which contains an emission and an absorption operator can give a contribution to this expression. The remaining part may be simplified by adding to it the expression

$$\left(k_2\frac{\partial}{\partial k_1}-k_1\frac{\partial}{\partial k_2}\right)\{\varphi_1^*\varphi_2+\varphi_1\varphi_2^*\}$$

which vanishes when integrated over k space. In this way we obtain for the part M_3^{I} of M_3 [15]

$$M_3^{\mathrm{I}}=\frac{e}{2}\int\frac{d^3k}{k_0^2}\left[k_y\left(\varphi_1^*(\mathbf{k})\frac{\partial\varphi_2(\mathbf{k})}{\partial k_x}+\varphi_1(\mathbf{k})\frac{\partial\varphi_2^*(\mathbf{k})}{\partial k_x}\right)-\text{sym. }(xy)\right],$$

$$\bar{M}_3^{\mathrm{I}}=\frac{e}{2}f^2u^*\omega^*\tau_\alpha\tau_\beta\int\int\frac{d^3k}{k_0^2}\frac{d^3k'}{k_0'^2}(\boldsymbol{\sigma},\mathbf{k})(\boldsymbol{\sigma},\mathbf{k}')\times e^{i(k-k',z)}\tilde{\varphi}_\alpha(k)\int\frac{d^3l}{l_0^2}\left[l_y\left(\varphi_1^*(\mathrm{l})\frac{\partial\varphi_2(\mathrm{l})}{\partial l_x}+\varphi_1(\mathrm{l})\frac{\partial\varphi_2^*(\mathrm{l})}{\partial l_x}\right)-\text{sym. }(xy)\right]\times\tilde{\varphi}_\beta^*(\mathbf{k}')u\omega. \quad (27)$$

This expression may be simplified by using the commutation rules (11). We have

$$\omega^*\tau_\alpha\tau_\beta\tilde{\varphi}_\alpha(\mathbf{k})\left[l_y\left(\varphi_1^*(\mathrm{l})\frac{\partial\varphi_2(\mathrm{l})}{\partial l_x}+\varphi_1(\mathrm{l})\frac{\partial\varphi_2^*(\mathrm{l})}{\partial l_x}\right)-\text{sym. }(xy)\right]\tilde{\varphi}_\beta^*(\mathbf{k}')\omega$$

$$=\tau_1\tau_2l_0^2\left[l_y\left(\delta(\mathbf{k}-\mathrm{l})\frac{\partial}{\partial l_x}\delta(\mathrm{l}-\mathbf{k}')-\delta(\mathrm{l}-\mathbf{k}')\frac{\partial}{\partial l_x}\delta(\mathbf{k}-\mathrm{l})-\text{sym. }(xy)\right]\cos(l,\lambda),$$

[15] In order to avoid ambiguity we have put $\mu=x$, $\nu=y$.

so that after a partial integration[16]

$$\bar{M}_3{}^{\mathrm{I}} = -if^2u^*\tau_3\int\frac{d^3k}{k_0{}^4}\{\sigma_1k_2-\sigma_2k_1\}(\boldsymbol{\sigma},\mathbf{k})u\cos(k,\lambda).$$

If we choose a special coordinate system for which the space part of λ is zero, we may first carry out the integration over the angles and obtain

$$\bar{M}_3{}^{\mathrm{I}} = \frac{8\pi}{3}f^2u^*\tau_3u\int\left(\frac{k}{k_0}\right)^4\cos k_0\lambda_0dk.$$

The integral may be evaluated in the following way

$$\int\left(\frac{k}{k_0}\right)^4\cos k_0\lambda_0dk = \int\cos k_0\lambda_0dk - \int\frac{k_0{}^4-k^4}{k_0{}^4}\cos k_0\lambda_0dk.$$

The first integral is zero in the limit $\lambda_0=0$, as is shown in the appendix, and for the second we get $-(3\pi/4)\mu$, by going to the limit $\lambda_0=0$ before carrying out the integration so that finally

$$\bar{M}_3{}^{\mathrm{I}} = \mp 2\pi^2f^2e\mu = \mp(g\mu)^2(e/2\mu). \quad (28)$$

The upper sign is for the proton and the lower for the neutron.

For the calculation of $2\Omega_0{}^*M_3\Omega_2$ we need only to consider that part in M_3 which contains two absorption operators

$$M_3{}^{\mathrm{II}} = \frac{e}{4}\int\frac{d^3k}{k_0{}^2}\left[k_y\left(\varphi_1(-\mathbf{k})\frac{\partial\varphi_2(\mathbf{k})}{\partial k_x}-\mathrm{sym.}\ (12)\right)-\mathrm{sym.}\ (xy)\right]e^{-2ik_0x_0},$$

$$\bar{M}_3{}^{\mathrm{II}} = -\frac{e}{2}f^2u^*\omega^*\tau_\alpha\tau_\beta\int\frac{d^3l}{l_0{}^2}\left[l_y\left(\varphi_1(-1)\frac{\partial\varphi_2(1)}{\partial l_x}\times e^{-2il_0x_0}-\mathrm{sym.}\ (12)\right)-\mathrm{sym.}\ (xy)\right]$$
$$\times\int\int\frac{d^3k'}{k_0(k_0+k_0')}\frac{d^3k}{k_0{}^2}(\boldsymbol{\sigma},\mathbf{k}')(\boldsymbol{\sigma},\mathbf{k})\times\tilde{\varphi}_\alpha{}^*(\mathbf{k}')\tilde{\varphi}_\beta{}^*(\mathbf{k})e^{-i(k+k',\,z)}u\omega.$$

[16] We have dropped terms which arise from the differentiation of the cos-factor and which tend to zero as λ goes to zero and terms coming from the differentiation of the exponential function which leads to an expression of the form $\sigma_1z_2-\sigma_2z_1$ and which is zero after averaging over z.

Again applying the commutation rules we have

$$u^*\omega^*\tau_\alpha\tau_\beta\left[l_y\left(\varphi_1(-1)\frac{\partial\varphi_2(1)}{\partial l_x}-\mathrm{sym.}\ (12)\right)-\mathrm{sym.}\ (xy)\right]\varphi_\alpha{}^*(\mathbf{k}')\varphi_\beta{}^*(\mathbf{k})u\omega$$
$$=2iu^*\tau_3l_0{}^2\left[\delta(1+\mathbf{k}')\left(l_y\frac{\partial}{\partial l_x}\delta(1-\mathbf{k})-l_x\frac{\partial}{\partial l_y}\delta(1-\mathbf{k})\right)-\delta(1+\mathbf{k}')\left(l_y\frac{\partial}{\partial l_x}\delta(1-\mathbf{k}')-l_y\frac{\partial}{\partial l_x}\delta(1-\mathbf{k}')\right)\right]\cos(k,\lambda),$$

$$\bar{M}_3{}^{\mathrm{II}} = 2ief^2u^*\tau_3\int\int\frac{d^3k'}{k_0(k_0+k_0')}\frac{d^3k}{k_0{}^2}\times(\sigma_1k_2'-\sigma_2k_1')(\boldsymbol{\sigma},\mathbf{k})\delta(\mathbf{k}+\mathbf{k}')u$$
$$=\frac{8\pi}{3}ef^2\int\left(\frac{k}{k_0}\right)^4\cos(k,\lambda)dku^*\tau_3u$$
$$=\mp(g\mu)^2\frac{e}{2\mu}. \quad (29)$$

The total magnetic moment due to the pseudoscalar meson field is

$$\bar{M}_3 = \mp 2(g\mu)^2(e/2\mu) \quad (30)$$

for the proton and neutron, respectively.

3. THE VECTOR THEORY

The mesons of spin 1 are described by a complex vector field ψ_μ satisfying the equations

$$\Box\psi_\mu-\mu^2\psi_\mu=0, \quad \partial\psi_\mu/\partial x_\mu=0. \quad (31)$$

It is quantized according to

$$i[\psi_\mu(x),\psi_\nu{}^*(x')] = \left(g_{\mu\nu}-\frac{1}{\mu^2}\frac{\partial^2}{\partial x^\mu\partial x^\nu}\right)\times D(x-x'). \quad (32)$$

The current is defined in terms of the quantities ψ_μ and $\psi_{\mu\nu}=(\partial\psi_\nu/\partial x^\mu)-(\partial\psi_\mu/\partial x^\nu)$.

$$s_\mu = i(\overset{*}{\psi}_{\mu\nu}\psi^\nu-\psi^{*\nu}\psi_{\mu\nu}) \quad (33)$$

and the magnetic moment is then again given by Eq. (4) of the preceding section.

We decompose the field again in the real fields by

$$\psi_\mu=\frac{1}{\sqrt{2}}(\psi_\mu{}^1-i\psi_\mu{}^2), \quad \psi_{\mu\nu}=\frac{1}{\sqrt{2}}(\overset{1}{\psi}_{\mu\nu}-i\overset{2}{\psi}_{\mu\nu}).$$

We put $\boldsymbol{\psi}=\{\psi_k\}(k=1, 2, 3)$ and write for the interaction energy

$$H'=g(4\pi)^{\frac{1}{2}}\tau_\alpha(\boldsymbol{\sigma}, \text{curl})\boldsymbol{\psi}^\alpha(z), \quad (34)$$

with this interaction present the current is

$$s_\mu=(\overset{1}{\psi}_{\mu\nu}\psi^{2\nu}-\overset{2}{\psi}_{\mu\nu}\psi^{1\nu}) +g(4\pi)^{\frac{1}{2}}\{\boldsymbol{\sigma}\times(\boldsymbol{\psi}^1\tau_2-\boldsymbol{\psi}^2\tau_1)\}_\mu\delta(\mathbf{x}-\mathbf{z}). \quad (35)$$

The second part corresponds to the second part in (8) in the scalar theory. It will give rise to an additional term in the magnetic moment. A consideration similar to the one given above will show that this term does not contribute anything to the expectation value of the magnetic moment within the limits of our approximation.

We develop $\psi_\mu{}^\alpha$ and $\overset{\alpha}{\psi}_{\mu\nu}$ in eigenstates in momentum space

$$\psi_\mu{}^\alpha(x)=\frac{1}{\sqrt{2}(2\pi)^{\frac{3}{2}}}\int\frac{d^3k}{k^0} \times\{\varphi_\mu{}^\alpha(\mathbf{k})e^{i(k,x)}+\varphi_\mu{}^{*\alpha}(\mathbf{k})e^{-i(k,x)}\} \quad (36)$$

and

$$\psi_{\mu\nu}{}^\alpha(x)=\frac{1}{\sqrt{2}(2\pi)^{\frac{3}{2}}}\int\frac{d^3k}{k^0} \times\{(k_\mu\varphi_\nu{}^\alpha(\mathbf{k})-k_\nu\varphi_\mu{}^\alpha(\mathbf{k}))e^{i(k,x)} -(k_\mu\varphi_\nu{}^{*\alpha}(\mathbf{k})-k_\nu\varphi_\mu{}^{*\alpha}(\mathbf{k}))e^{-i(k,x)}. \quad (37)$$

Because of the subsidiary condition $\partial\psi_\mu/\partial x^\mu=0$ the $\varphi_\mu{}^\alpha(\mathbf{k})$ satisfy the equations

$$(\varphi^\alpha, k)=(\varphi^{*\alpha}, k)=0.$$

From the commutation relations (32) it follows that

$$[\varphi_\mu{}^\alpha(\mathbf{k}), \varphi_\nu{}^{*\beta}(\mathbf{k}')] =k^0\left(g_{\mu\nu}+\frac{1}{\mu^2}k_\mu k_\nu\right)\delta(\mathbf{k}-\mathbf{k}')\delta_{\alpha\beta}. \quad (38)$$

The $\varphi_\mu{}^\alpha$, $\varphi_\mu{}^{*\alpha}$ again are defined in such a way that they are slowly varying according to

$$\frac{d\varphi_\mu{}^\alpha}{dx^0}=i[H', \varphi_\mu{}^\alpha]. \quad (39)$$

For H' we get from (34) in momentum space

$$H'=if\tau_\alpha\int\frac{d^3k}{k^0}\{(\boldsymbol{\sigma}, [\mathbf{k}\times\boldsymbol{\phi}^\alpha])e^{i(k,z)} -(\boldsymbol{\sigma}, [\mathbf{k}\times\boldsymbol{\phi}^\alpha])e^{-i(k,z)}\}, \quad (40)$$

$$f=g/2\pi.$$

The λ process is again introduced by defining the operators $\tilde{\varphi}_\mu{}^\alpha$ and $\tilde{\varphi}_\mu{}^{*\alpha}$ as was done in the scalar theory. As before this is equivalent to replacing the D function in (32) by the D_λ function. This alters the commutation relations (38) in momentum space to

$$[\tilde{\varphi}_\mu{}^\alpha(\mathbf{k}), \tilde{\varphi}_\nu{}^{*\beta}(\mathbf{k}')]=k^0\delta_{\alpha\beta}\left(g_{\mu\nu}+\frac{1}{\mu^2}k_\mu k_\nu\right) \times\delta(\mathbf{k}-\mathbf{k}')\cos(k, \lambda). \quad (41)$$

The magnetic moment operator

$$M_{\mu\nu}=\frac{e}{2}\int\{x_\mu(\overset{1}{\psi}_{\nu\rho}\psi^{2\rho}-\overset{2}{\psi}_{\nu\rho}\psi^{1\rho}) -x_\nu(\overset{1}{\psi}_{\mu\rho}\psi^{2\rho}-\overset{2}{\psi}_{\mu\rho}\psi^{1\rho})\}d^3x \quad (42)$$

may be transformed with a calculation exactly analogous to that which led to the expression (21) in the scalar theory to give

$$M_{\mu\nu}=\frac{e}{4}\int\frac{d^3l}{l_0{}^2}\left\langle\left\{\left[\left(l_\nu\frac{\partial\varphi_\rho{}^2(1)}{\partial l^\mu}-l_\rho\frac{\partial\varphi_\nu{}^2(1)}{\partial l^\mu}\right)\times\varphi^{1\rho}(-1)e^{-2il_0x_0} +\left(l_\nu\frac{\partial\varphi_\rho{}^{*2}(1)}{\partial l^\mu}-l_\rho\frac{\partial\varphi_\nu{}^{*2}(1)}{\partial l^\mu}\right)\varphi^{1\rho}(1) +\text{compl. conj.}\right]-\text{sym.}(12)\right\}-\text{sym.}(\mu\nu)\right\rangle. \quad (43)$$

The solutions of the equations

$$i(d/dx^0)\Omega_1=H'\Omega_0, \quad i(d/dx^0)\Omega_2=H'\Omega_1, \quad (44)$$

corresponding to the solutions (26) in the scalar theory are then

$$\Omega_0=u\omega,$$
$$\Omega_1=if\tau_\alpha\int\frac{d^3k}{k_0{}^2}(\boldsymbol{\sigma}, [\mathbf{k}\times\tilde{\boldsymbol{\phi}}^{*\alpha}])e^{-i(k,z)}u\omega,$$
$$\Omega_2=-f^2\tau_\alpha\tau_\beta\int\frac{d^3k}{k_0(k_0+k_0')}\frac{d^3k}{k_0'^2}(\boldsymbol{\sigma}, [\mathbf{k}\times\tilde{\boldsymbol{\phi}}^{*\alpha}]) \times(\boldsymbol{\sigma}, [\mathbf{k}'\times\tilde{\boldsymbol{\phi}}^{*\beta}])e^{-i(k+k',z)}u\omega. \quad (45)$$

The part $\bar{M}_3{}^{\text{I}}$ corresponding to (27) is

$$\bar{M}_3{}^{\text{I}}=\tfrac{1}{2}ef^2u^*\omega^*\tau_\alpha\tau_\beta\int\int\frac{d^3k}{k_0{}^2}\frac{d^3k'}{k_0{}'^2}\times(\boldsymbol{\sigma},[\mathbf{k}\times\tilde{\boldsymbol{\phi}}^\alpha])e^{i(k-k',z)}\int\frac{d^3l}{l_0{}^2}\times\left\{\left[\left(l_y\frac{\partial\varphi_\rho{}^{*2}}{\partial l_x}-l_\rho\frac{\partial\varphi^{*2}}{\partial l_x}\right)\varphi^{1\rho}+\text{compl. conj.}\right]-\text{sym. }(xy)\right\}\times(\boldsymbol{\sigma},[\mathbf{k}'\times\tilde{\boldsymbol{\phi}}^{*\beta}])u\omega. \quad (46)$$

Writing

$$(\boldsymbol{\sigma},[\mathbf{k}\times\tilde{\boldsymbol{\phi}}^\alpha])=-i\sigma_r\sigma_sk_r\tilde{\varphi}_s{}^\alpha\quad(r\neq s)$$

we can simplify the expression (46) considerably by using the commutation rules (37)

$$-\omega^*\tau_\alpha\tau_\beta\sigma_r\sigma_sk_rk_s\varphi_s{}^\alpha\left\{\left[\left(l_y\frac{\partial\varphi_\rho{}^{*2}}{\partial l_x}-l_\rho\frac{\partial\varphi_y{}^{*2}}{\partial l_x}\right)\varphi^{1\rho}+\text{compl. conj.}\right]-\text{sym. }(xy)\right\}\sigma_t\sigma_hk_t'\varphi_h{}^{*\beta}\omega\quad\begin{matrix}r\neq s\\t\neq h\end{matrix}$$

$$=-2\tau_\alpha\tau_\beta\sigma_r\sigma_sk_rk_0\left\{\left[l_y\delta_{\alpha2}\left(\delta_{s\rho}+\frac{1}{\mu^2}k_sk_\rho\right)\frac{\partial}{\partial l_x}\times\delta(\mathbf{k}-\mathbf{l})-l_\rho\delta_{\alpha2}\left(\delta_{sy}+\frac{1}{\mu^2}k_sk_y\right)\frac{\partial}{\partial l_x}\delta(\mathbf{k}-\mathbf{l})\right]\times l_0\delta_{1\beta}\left(\delta_{\rho h}+\frac{1}{\mu^2}l_\rho l_h\right)-\text{sym. }(xy)\right\}\times\delta(\mathbf{l}-\mathbf{k}')\sigma_t\sigma_hk_t'\cos(k,\lambda).\quad\begin{matrix}r\neq\rho\\t\neq\rho\end{matrix}$$

After multiplying with

$$(d^3k'/k_0{}'^2)\exp[i(k-k',z)]$$

and integrating over k' we get by using

$$\sum_{r\neq s}\sigma_r\sigma_sk_rk_s=0$$

$$2i\tau_3(\sigma_rk_r)\frac{k_0}{l_0}\left[(l_y\sigma_\rho-l_\rho\sigma_y)(\sigma_tl_t)\sigma_\rho\frac{\partial}{\partial l_x}\delta(\mathbf{k}-\mathbf{l})-\text{sym. }(xy)\right]e^{i(k-l,z)}\cos(k,\lambda),\quad\begin{matrix}r\neq\rho\\t\neq\rho\end{matrix}$$

so that (46) becomes

$$eif^2u^*\tau_3\int\frac{d^3k}{k_0}\frac{d^3l}{l_0{}^3}(\sigma_rk_r)\left[(l_y\sigma_\rho-l_\rho\sigma_y)(\sigma_tl_t)\sigma_\rho\times\frac{\partial}{\partial l_x}\delta(\mathbf{k}-\mathbf{l})-\text{sym. }(xy)\right]e^{i(k-l,z)}\cos(k,\lambda).\quad(47)$$
$$r\neq\rho,\ t\neq\rho$$

Integrating by parts over $\mathbf{l}$ first we get a term from the differentiation of $1/l_0{}^3$ which, however, is zero just as in the scalar theory because it contains a factor $\sigma_t\sigma_\rho l_tl_\rho=0$ since $t\neq\rho$.

The other part gives a term

$$(\sigma_2k_2+\sigma_3k_3)\sigma_2(\sigma_2k_2+\sigma_3k_3)\sigma_1-(\sigma_1k_1+\sigma_3k_3)\sigma_1(\sigma_1k_1+\sigma_3k_3)\sigma_2+(\sigma_1k_1+\sigma_2k_2)(k_3\sigma_2-k_2\sigma_3)\sigma_1\sigma_3-(\sigma_1k_1+\sigma_2k_2)(k_3\sigma_1-k_1\sigma_3)\sigma_2\sigma_3.$$

Most of these terms will give zero after the integration over the angles. The only term different from zero is

$$-i\sigma_3(k_1{}^2+k_2{}^2);$$

introducing this in (47) we get

$$\bar{M}_3{}^{\text{I}}=\pm ef^2\frac{8\pi}{3}\int\left(\frac{k}{k_0}\right)^4\cos k_0\lambda_0dk=\mp2\pi^2ef^2\mu.$$

The term $\bar{M}_3{}^{\text{II}}$ is

$$\bar{M}_3{}^{\text{II}}=-f^2eu^*\omega^*\tau_\alpha\tau_\beta\int\int\frac{d^3k}{k_0(k_0+k_0')}\frac{d^3k'}{k_0{}'^2}\times e^{-i(k+k',z)}\int\frac{d^3l}{l_0{}^2}\left[\left(l_y\frac{\partial\varphi_\rho{}^2}{\partial l_x}-l_\rho\frac{\partial\varphi_y{}^2}{\partial l_x}\right)\times\varphi^{1\rho}(-\mathbf{l})e^{-2il_0x_0}-\text{sym. }(xy)\right]\times(\boldsymbol{\sigma},[\mathbf{k}\times\tilde{\boldsymbol{\phi}}^{*\alpha}])(\boldsymbol{\sigma},[\mathbf{k}'\times\tilde{\boldsymbol{\phi}}^{*\beta}]).$$

A calculation similar to the one above leads to

$$2eif^2u^*\tau_3\int\int\frac{d^3kd^3l}{k_0(k_0+l_0)l_0{}^2}(\sigma_rk_r)\times\left[(l_2\sigma_\rho-l_\rho\sigma_2)(\sigma_tl_t)\sigma_\rho\frac{\partial}{\partial l_1}\delta(\mathbf{k}-\mathbf{l})-\text{sym. }(12)\right]\cos(k,\lambda).\quad\begin{matrix}r\neq\rho\\t\neq\rho\end{matrix}$$

In comparing this with (47) we find that again we have

$$\bar{M}_3{}^{\text{II}}=\mp2\pi^2ef^2\mu,$$

so that the total magnetic moment in the vector theory is

$$\bar{M}_3=\bar{M}_3{}^{\text{I}}+\bar{M}_3{}^{\text{II}}=\mp2(g\mu)^2(e/2\mu).$$

The upper sign is for the proton and the lower for the neutron. This is the same result as for the pseudoscalar meson.

4. A VECTOR MESON WITH AN ARBITRARY MAGNETIC MOMENT

The Lagrangian for a vector meson field is not unique insofar as it is possible to introduce terms in the Lagrange-function which are proportional to the external electromagnetic field.[17] Such a term makes itself felt in an altered expression for the current density and consequently a change in the magnetic moment. The additional current density is

$$s_\mu' = -iK\frac{\partial}{\partial x_\rho}(\psi_\rho{}^*\psi_\mu - \psi_\mu{}^*\psi_\rho), \tag{48}$$

where K is a dimensionless and arbitrary constant. The change in the magnetic moment operator is then from Eq. (4)

$$\begin{aligned} M'_{\mu\nu} &= ieK\int(\psi_\mu{}^*\psi_\nu - \psi_\nu{}^*\psi_\mu)d^3x \\ &= eK\int(\psi_\mu{}'\psi_\nu{}^2 - \psi_\nu{}'\psi_\mu{}^2)d^3x \\ &= \tfrac{1}{2}eK\int\frac{d^3k}{k_0{}^2}\{[(\varphi_\mu{}^1(\mathbf{k})\varphi_\nu{}^2(-\mathbf{k})e^{-2ik_0x_0} \\ &\qquad + \varphi_\mu{}^1(\mathbf{k})\varphi_\nu{}^{*2}(\mathbf{k})) - \text{compl. conj.}] \\ &\qquad - \text{sym. }(\mu\nu)\}. \end{aligned} \tag{49}$$

We have, therefore,

$$\begin{aligned} \bar{M}_3{}'^{\mathrm{I}} &= 2eKf^2u^*\omega^*\tau_\alpha\tau_\beta\int\frac{d^3k}{k_0{}^2}\frac{d^3k'}{k_0'^2}(\boldsymbol{\sigma}, [\mathbf{k}\times\tilde{\boldsymbol{\phi}}^\alpha]) \\ &\quad \times e^{i(k-k',z)}\int\frac{d^3l}{l_0{}^2}\varphi_1{}^1(\mathrm{l})\varphi_2{}^{*2}(\mathrm{l}) \\ &\quad \times[\boldsymbol{\sigma}, [\mathbf{k}'\times\tilde{\boldsymbol{\phi}}^{*\beta}])u\omega, \\ &= -2eKf^2u^*\tau_3\sigma_3\frac{4\pi}{3}\int\left(\frac{k}{k_0}\right)^4\cos(k_0\lambda_0)dk \\ &= \pm K(g\mu)^2(e/2\mu). \end{aligned} \tag{50}$$

A similar calculation for $\bar{M}_3{}'^{\mathrm{II}} = 2\Omega_0{}^*M_3\Omega_2$ leads to the same result so that

$$\bar{M}_3{}' = \bar{M}_3{}'^{\mathrm{I}} + \bar{M}_3{}'^{\mathrm{II}} = \pm 2K(g\mu)^2(e/2\mu). \tag{51}$$

The total magnetic moment in the vector theory is then

$$\bar{M}_3 = \pm 2(K-1)(g\mu)^2(e/2\mu) \tag{52}$$

for the proton and neutron, respectively.

Taking the two theories together and assuming for the vector meson twice the mass of the scalar meson, but the same coupling constant, we find

$$M = \pm 2(g\mu)^2\{2(K-1)-1\}(e/2\mu) \tag{53}$$

for $(g\mu)^2 \sim 0.1$, $\mu \sim \frac{1}{10}m$, $K \sim 2$,

$$M \sim \pm 2,$$

for the magnetic moment of proton and neutron, respectively, due to the meson field only. If we add to that the magnetic moment of the nucleons (0 for the neutron and +1 for the proton), we find agreement with the experiment within the limits of our calculation.

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APPENDIX

The integral $I = \int_0^\infty \cos k_0\lambda_0 dk$ may be evaluated in the following way. With the substitution $k = \mu i \sin v\lambda_0\mu = \alpha$, we have

$$I = \frac{i\mu}{4}\int_{-i\infty}^{+i\infty}\cos v(e^{i\alpha\cos v} + e^{-i\alpha\cos v})dv.$$

For the second integral we substitute v by $\pi - v$.

$$I = \frac{i\mu}{4}\left\{\int_{-i\infty}^{+i\infty}\cos v e^{i\alpha\cos v}dv + \int_{+i\infty+\pi}^{-i\infty+\pi}\cos v e^{i\alpha\cos v}dv\right\}.$$

The paths of integration have to be taken in the regions for which $Im(\cos v) > 0$, in order to secure the convergence of the integrals. They may be chosen in such a way that the contribution from the branches leading to infinity cancel, and we are then left with the expression

$$I = -\frac{i\mu}{2}\int_0^{2\pi}\cos v e^{i\alpha\cos v}dv = -\frac{\mu}{2\pi}J_1(\alpha) \qquad \lim_{\alpha\to 0} I(\alpha) = 0.$$

[17] See H. C. Corben and J. Schwinger, Phys. Rev. **58**, 953 (1940).