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Pre-big bang on the brane

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Abstract

The equations of motion and junction conditions for a gravi-dilaton brane world scenario are studied in the string frame. It is shown that they allow Kasner-like solutions on the brane, which makes the dynamics of the brane very similar to the low curvature phase of pre-big bang cosmology. Analogies and differences of this scenario with the Randall-Sundrum one and with the standard bulk pre-big bang dynamics are also discussed.

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I. INTRODUCTION

The pre-big bang model [1] is an attempt at constructing a nonsingular cosmological scenario based on string theory. A decade of work [2] has produced a model which contains inflation, is nonsingular, and has a number of distinctive phenomenological predictions.

This model has some drawbacks, too. On the phenomenological side, it predicts a generally blue spectrum of cosmological perturbations [3]; this feature, which is very welcome for what concerns the possible detection of gravitational waves of cosmological origin [4], determines on the other hand a huge suppression of scalar perturbations at large scale. Some modifications of the minimal version of the model have been recently proposed in order to render its predictions for the scalar perturbation spectrum compatible with the large scale inhomogeneities observed in the cosmic microwave background radiation (CMBR) [5].

On the theoretical side, a very serious obstacle is related to the study of the high curvature, strongly coupled phase, and to the dilaton stabilization: henceforth I will refer to all these theoretical challenges as the graceful exit problem. Though some physical mechanisms (such as the inclusion of α' corrections and loop terms, or entropy considerations) for resolving this problem have been found and studied, also with encouraging results, it is still true that a full comprehension of the “stringy” phase of the model has not yet been established, due mainly to the poor technical control one has on this phase.

Since no significant progress has been made in the last few years for solving the graceful exit problem, it is unlikely that the techniques used so far will help further. Rather, a new physical mechanism, or a new framework, or at least a change of perspective could be quite helpful at this point. This is exactly the spirit of this paper, in which the pre-big bang dynamics will be reproduced in a brane-world scenario. The general idea (which goes beyond the purpose of the present work) is that the previously mentioned problems could assume a different form in a brane world context, and maybe become less difficult if treated in terms of brane world-related techniques (such as AdS-CFT). In this sense, the approach of this paper could be considered similar to the one taken in the so-called ekpyrotic model [6].

Started as an alternative to compactification [7], brane world models have been extensively studied recently: new explicit solutions were discovered and the original Randall-Sundrum picture was extended to more general situations like the inclusion of matter and

of a cosmological constant on the brane (see [8] and references therein), and the relation of this scenario with the AdS-CFT conjecture has been further investigated [9]. This scenario being inspired by string theory, a further natural extension has been the inclusion of a scalar field, both in the Einstein [10] and in the string [11] frame.

In the present paper I will work exclusively in the string frame: since the goal is to find out whether, and under which conditions, a pre-big bang phase can take place on the brane, I find that a string frame treatment is the most appropriate one.

The action will be displayed and the bulk and boundary equations will be written. Then, following a standard technique, the bulk equations will be projected on the brane and studied. It will turn out that, despite the presence of the dilaton (which induces energy nonconservation on the brane), it is possible in some situations to study the brane dynamics in a rather self-contained way, without needing to know the full bulk solution.

Then an homogeneous and isotropic ansatz will be made on the brane, and the relative equations of motion will be written. They are found to be very similar to ordinary bulk vacuum gravi-dilaton equations, and indeed they admit similar solutions, which are written, displayed and commented. Some final remarks conclude this work.

II. THE EQUATIONS IN THE STRING FRAME

Let us consider a D dimensional spacetime, in which one dimension has been compactified on a S^1/Z_2 orbifold (it is assumed that the other $10-D$ spatial dimensions are small, compact and frozen). This orbifold has the topology of a segment, and the Z_2 symmetric points are effectively the boundaries of spacetime; I will assume that one brane is located at one of these points. This is the simplest and most common choice in brane world scenarios, though more general configurations, involving more complicated junction conditions, can also be considered (see [12] for more details). The action governing the dynamics of such a system is

$$\mathcal{S} = \int d^D x \sqrt{-g} e^{-\phi} \left[\mathcal{R} + (\mathcal{D}\phi)^2 + \mathcal{L}_{bulk} \right] - \int d^{D-1} y \sqrt{-h} e^{-\phi} \left[2K^\pm + \mathcal{L}_{brane} \right]. \quad (1)$$

Throughout the paper I will denote with \mathcal{R} and \mathcal{D}_μ , respectively, the D -dimensional Ricci tensor and the covariant derivative with respect to the bulk metric $g_{\mu\nu}$, while the symbols R and ∇_μ will indicate the Ricci tensor of the brane and the covariant derivative with

respect to the induced metric $h_{\mu\nu}$. Moreover, the second fundamental form $K_{\mu\nu}$ is defined as $K_{\mu\nu} = h_\mu^\rho h_\nu^\sigma \mathcal{D}_\rho n_\sigma$, with n_μ a unitary vector field pointing *into* the bulk, orthogonal to the boundary at the boundary location, and defined in the bulk by the condition $n^\rho \mathcal{D}_\rho n_\mu = 0$ (this definition is always valid at least in the neighborhoods of the boundary); with these definitions, one has $g_{\mu\nu} = h_{\mu\nu} + n_\mu n_\nu$.

Having clarified the notations, one can read in (1) that the bulk part is the usual low energy effective action for gravity and dilaton in the string frame plus a generic bulk matter contribution denoted as \mathcal{L}_{bulk} , while the boundary, as well as the Gibbons-Hawking term K^\pm (in which the \pm means that one must take the sum of the value of that term at both sides of the brane), contains also the brane-matter lagrangian \mathcal{L}_{brane} .

Variation of (1) with respect to the bulk metric and to the dilaton gives directly the bulk equations of motion

$$\mathcal{R}_{\mu\nu} + \mathcal{D}_\mu \mathcal{D}_\nu \phi = T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T^\phi, \quad (2)$$

$$\mathcal{R} - (\mathcal{D}\phi)^2 + 2\mathcal{D}^2\phi = -T^\phi, \quad (3)$$

and the boundary junction conditions (here specialized to the Z_2 symmetric case)

$$2K_{\mu\nu} = K_{\mu\nu}^\pm = \tau_{\mu\nu} - \frac{\tau^\phi}{2} h_{\mu\nu}; \quad 2^\perp \mathcal{D}\phi \equiv (n \cdot \mathcal{D}\phi)^\pm = \tau - \frac{D-2}{2} \tau^\phi. \quad (4)$$

In the previous equations $T_{\mu\nu} = -\frac{1}{\sqrt{-g}} \frac{\delta \sqrt{-g} \mathcal{L}_{bulk}}{\delta g^{\mu\nu}}$, $T^\phi = -e^\phi \frac{\delta e^{-\phi} \mathcal{L}_{bulk}}{\delta \phi}$, and analogous definitions hold for $\tau_{\mu\nu}$ and τ^ϕ , with \mathcal{L}_{bulk} and $\sqrt{-g}$ replaced by \mathcal{L}_{brane} and $\sqrt{-h}$, respectively. The junction conditions look quite different from the usual Einstein frame ones for a gravi-scalar system; this is due to the fact that in the string frame the equations for $K_{\mu\nu}$ and ϕ are mixed together. A derivation of the junction conditions (4) in the string frame is presented in the Appendix.

It is convenient to split the quantities appearing in the bulk equations (2,3) into their components parallel and orthogonal to the brane. Thus, given a symmetric tensor $S_{\mu\nu}$, I define $\parallel S_{\mu\nu} \equiv h_\mu^\rho h_\nu^\sigma S_{\rho\sigma}$ and $^\perp S \equiv n^\rho n^\sigma S_{\rho\sigma}$, so that the trace of S can be written as $S_{\mu\nu} g^{\mu\nu} = ^\perp S + \parallel S$. Using these definitions, the equations of motion can be projected and rewritten as follows:

$$\parallel \mathcal{R}_{\mu\nu} + \parallel \mathcal{D}_\mu \mathcal{D}_\nu \phi = \parallel T_{\mu\nu} - \frac{1}{2} T^\phi h_{\mu\nu}, \quad (5)$$

$$^\perp \mathcal{R} + ^\perp \mathcal{D}^2 \phi = ^\perp T - \frac{1}{2} T^\phi, \quad (6)$$

$$\parallel \mathcal{R} + ^\perp \mathcal{R} - \left(^\perp \mathcal{D}\phi \right)^2 - h^{\mu\nu} \mathcal{D}_\mu \phi \mathcal{D}_\nu \phi + 2^\perp \mathcal{D}^2 \phi + 2 \parallel \mathcal{D}^2 \phi = -T^\phi \quad (7)$$

(there is also a mixed $\perp\parallel$ component which has not been reported here, since it will not be needed in the following). This form is convenient for studying the system from the point of view of the brane. Indeed, one can express the $(D - 1)$ -dimensional Ricci tensor and the $(D - 1)$ -dimensional double covariant derivative of ϕ on the brane in terms of the various projections of the corresponding D -dimensional quantities and of the projection of the Weyl tensor, $E_{\mu\nu} = \mathcal{C}_{\alpha\beta\gamma\delta}n^\alpha n^\gamma h_\mu^\beta h_\nu^\delta$:

$$R_{\mu\nu} = \frac{D-3}{D-2} \parallel \mathcal{R}_{\mu\nu} - \frac{h_{\mu\nu}}{(D-2)(D-1)} \left[(D-2)^\perp \mathcal{R} - \parallel \mathcal{R} \right] + K K_{\mu\nu} - K_{\mu\alpha} K_\nu^\alpha - E_{\mu\nu} \quad [\text{Gauss-Codacci}], \quad (8)$$

$$\nabla_\mu \nabla_\nu \phi = \parallel \mathcal{D}_\mu \mathcal{D}_\nu \phi - K_{\mu\nu}^\perp \mathcal{D} \phi. \quad (9)$$

Now, from (8) one can write the brane Einstein tensor in terms of bulk curvature tensors and of $K_{\mu\nu}$, then one uses the projected equations of motion to express the bulk curvature tensors in terms of dilaton derivatives and of bulk sources, and at last decomposes the dilaton derivatives into their brane and bulk part thanks to eq. (9). Finally, after having moved on the l. h. s. all the brane tensors, one is left with

$$\begin{aligned} R_{\mu\nu} &- \frac{h_{\mu\nu}}{2} \left[R + 2 \frac{D-3}{D-2} \nabla^2 \phi - \frac{D-3}{D-1} (\nabla \phi)^2 \right] + \frac{D-3}{D-2} \nabla_\mu \nabla_\nu \phi = \\ &= \frac{D-3}{D-2} \left(\parallel T_{\mu\nu} - \frac{h_{\mu\nu}}{D-1} \parallel T \right) + \frac{D-3}{D-1} h_{\mu\nu}^\perp T - \frac{D-3}{2(D-1)} h_{\mu\nu} \left(\perp \mathcal{D} \phi \right)^2 + \\ &+ \left(h_\mu^\rho h_\nu^\sigma - \frac{1}{2} h^{\rho\sigma} h_{\mu\nu} \right) \left(K K_{\rho\sigma} - K_\rho^\alpha K_{\sigma\alpha} \right) - \frac{D-3}{D-2} \left(K_{\mu\nu} - h_{\mu\nu} K \right)^\perp \mathcal{D} \phi - E_{\mu\nu}. \end{aligned} \quad (10)$$

If the dilaton derivatives are neglected in the previous expression, one is formally left with the usual brane world Einstein equations (see, for example, the second reference of [8] for a comparison), in which the brane curvature tensors are expressed in terms of the bulk and brane sources (the latter enter through the extrinsic curvature tensor because of the junction conditions) and of $E_{\mu\nu}$, which has the peculiarity of not being determined by the bulk equations of motion. In general the form of the projected Weyl tensor can be inferred from the symmetries of the system, and thus the brane world Einstein equations can be studied in a rather self-contained way. The addition of the dilaton does not change (so far) this state of things, since also the terms containing $^\perp \mathcal{D} \phi$ can be expressed in terms of brane sources via the junction conditions (4).

It is interesting to take the trace of this equation, which gives

$$R - (\nabla \phi)^2 + 2 \nabla^2 \phi = -2^\perp T + K^2 - K_{\mu\nu} K^{\mu\nu} - 2K^\perp \mathcal{D} \phi + \left(\perp \mathcal{D} \phi \right)^2. \quad (11)$$

The structure tensor of the l. h. s. of this equation is formally the same as the usual bulk dilaton equation of motion in the string frame (3); this fact will make our purpose of looking for pre-big bang-like solutions on the brane easier.

Since eqns. (10,11) are not independent, I need another scalar equation of motion; this can be found, for example, by taking the trace of (5) and then using eqns. (8,9), as well as the other bulk equations of motion, in order to finally reach the following result:

$$R + \nabla^2\phi = \parallel T - \perp T - \frac{D-2}{2}T^\phi + K^2 - K_{\alpha\beta}K^{\alpha\beta} - K\perp\mathcal{D}\phi + \perp\mathcal{D}^2\phi. \quad (12)$$

As before, the structure tensor on the l. h. s. is the same of the bulk gravi-dilaton equation (2). What makes this equation qualitatively different from (11) is the presence of the term $\perp\mathcal{D}^2\phi$. Contrarily to all the other terms on the r. h. s., this is not expressible in terms of sources because it is not determined by the junction conditions. Rather, in order to know the value of $\perp\mathcal{D}^2\phi$ at the brane location, one should in principle solve the full set of bulk equations. Thus, differently from what happens in the brane Einstein equations, for a gravi-dilaton system generally it is not possible to study the brane evolution in a self-contained way, without knowing the bulk dynamics. This fact has already been pointed out in the first works on gravi-scalar brane models [10] and it is related to the fact that in such systems the dilaton dynamics generally induces energy-momentum nonconservation on the brane.

However it will be shown in the next section that in some specific situations the term $\perp\mathcal{D}^2\phi$ can be determined *at the brane* without solving the bulk equations, thus allowing a selfcontained description of the brane evolution.

Before proceeding further, it is perhaps worth pointing out that, despite the fact that there is some freedom in choosing the form of the evolution equations for our brane world model (and indeed this freedom was used to write the equations in a way that will result the most appropriate for the purposes of this work), this does not mean that there is any arbitrariness in them. That is, there are not other independent ways to write down combinations of the brane curvature tensors and of the derivative of the dilaton on the brane in terms of the bulk and brane sources, of $E_{\mu\nu}$ and of $\perp\mathcal{D}^2\phi$.

III. LOOKING FOR PRE-BIG BANG SOLUTIONS

A. Specification of the model

Let us now choose the matter content of the model. For the bulk part I take $\mathcal{L}_{bulk} = -2\Lambda$, so that the matter term is given simply by a cosmological constant:

$$T_{\mu\nu} = -\Lambda g_{\mu\nu}; \quad T^\phi = -2\Lambda. \quad (13)$$

As to the brane matter part, the following term is considered:

$$\mathcal{L}_{brane} = 2\lambda e^{\xi\Phi+\phi}, \quad (14)$$

where Φ is the 10-dimensional dilaton, which can be expressed in terms of the D -dimensional one ϕ by means of the relation $e^{-\phi} = e^{-\Phi}\mathcal{V}$, \mathcal{V} being the volume (in string units) of the compact $(10-D)$ -dimensional space, which is supposed to be a number of order 1. The parametrization has been chosen in such a way that for $\xi = -1/2$ one recovers the usual D-brane tension behavior $\sim e^{-\Phi/2} \sim 1/g_{10}$, while for $\xi = -1$ the brane tension term has the same dilaton prefactor $e^{-\phi}$ as all the other terms in the action, thus making this case the most appropriate one for a comparison with the standard Randall-Sundrum scenario.

Given eq. (14), the brane matter tensors are

$$\tau_{\mu\nu} = \lambda h_{\mu\nu} e^{(\xi+1)\phi} \mathcal{V}^\xi, \quad \tau^\phi = -2\lambda \xi e^{(\xi+1)\phi} \mathcal{V}^\xi, \quad (15)$$

and the junction conditions (4) read:

$$2K_{\mu\nu} = (\xi + 1)\lambda h_{\mu\nu} e^{(\xi+1)\phi} \mathcal{V}^\xi, \quad 2^\perp \mathcal{D}\phi = [(D - 2)(\xi + 1) + 1]\lambda e^{(\xi+1)\phi} \mathcal{V}^\xi. \quad (16)$$

Moreover, I have in mind to study the most symmetric configuration, in which the brane metric and the dilaton are homogeneous on the brane. In this case only three equations are needed. The 00 component of (10), which does not contain second time derivatives, gives a constraint on the initial conditions, as usually happens in general relativity. The remaining two equations, which contain second time derivatives of the dilaton and of the scale factor on the brane, are given by eq. (12) and by any of the ij components of (10) or, better, by its trace, eq. (11).

One can now rewrite the full set of equations, after having expressed everything in terms of Λ and λ :

$$\begin{aligned} R_{\mu\nu} - \frac{h_{\mu\nu}}{2} \left[R + 2\frac{D-3}{D-2}\nabla^2\phi - \frac{D-3}{D-1}(\nabla\phi)^2 \right] + \frac{D-3}{D-2}\nabla_\mu\nabla_\nu\phi = \\ = -\frac{(D-3)}{D-1} \left[\Lambda + \frac{\lambda^2}{8}F_\xi(D)\mathcal{V}^{2\xi}e^{2(\xi+1)\phi} \right] h_{\mu\nu} - E_{\mu\nu} \quad (00 \text{ component}), \end{aligned} \quad (17)$$

$$R + 2\nabla^2\phi - (\nabla\phi)^2 = 2\Lambda + \frac{\lambda^2}{4}F_\xi(D)\mathcal{V}^{2\xi}e^{2(\xi+1)\phi}, \quad (18)$$

$$R + \nabla^2\phi = \mathcal{D}^2\phi - \frac{\lambda^2}{4}(D-1)(\xi+1)\mathcal{V}^{2\xi}e^{2(\xi+1)\phi}, \quad (19)$$

with $F_\xi(D) = \xi^2 - (D-1)(\xi+1)^2$.

For $\xi > -1$ (which includes also the D-brane case, $\xi = -1/2$), the brane matter terms are multiplied by a positive power of the D -dimensional coupling constant $g_D = e^{\phi/2}$; thus they should be considered strong coupling corrections, and can be disregarded in the small coupling limit. On the contrary, for $\xi = -1$, these terms are as relevant as the others and should be taken into account. The case $\xi < -1$ will not be considered here.

After having noticed that each of the different bulk sources appears in only one equation (the projection of the Weyl tensor only in the constraint equation, and the term $\mathcal{D}^2\phi$ only in the last one), one observes also that the same combination of Λ and λ appears in the first two equations.

One can take advantage of this fact by tuning the parameters of the model in such a way that this particular combination vanishes. This requirement is analogous to the condition of vanishing cosmological constant on the brane, which is usually imposed in pure gravity brane world models.

For $\xi = -1$ one has $F_\xi(D) = 1$ and thus the requirement is $\lambda^2 = -8\mathcal{V}^2\Lambda$. It should be noted that, even for $\mathcal{V} = 1$, this condition is slightly different from the one found in the Randall-Sundrum model, which was $\lambda^2 = -\frac{8(D-2)}{D-1}\Lambda$. This could be surprising in light of the previous comment about the fact that eq. (10) reduces to the standard brane world case if the dilaton is neglected, a fact which does not seem to happen in eq. (17), nor for $\xi = -1$. The explanation is that the latter (and its trace, eq. (18)) has been obtained after making use of the junction conditions (4). As a matter of fact, one of the differences between the present system and the pure gravity one is that here the dilaton couples to the brane; because of this, the formal limit of neglecting the dilaton derivatives cannot be done after

one has already used the junction conditions. In other words, here a different condition is obtained because the physical system is different.

Coming to the case $\xi > -1$, all the λ terms should be neglected in the small coupling limit, and thus the vanishing brane cosmological constant requirement reduces to $\Lambda = 0$. Under such a condition, there is not confinement of a $(D-1)$ -dimensional graviton on the brane, and this should certainly be considered a problem if one wants to recover Einstein gravity on the brane. However here the goal is to find a pre-big bang solution, which does not need to satisfy such a constraint, and so the requirement $\Lambda = 0$ (which is, in my opinion, slightly less awkward than the RS one) is not phenomenologically problematic.

Coming back to the equations, it has just been shown that one can tune the parameters in order to make the r. h. s. of eq. (18) vanish. Now let us push things further by considering the particular case $E_{00} = 0$: in this case the only source term we are left with is the ${}^{\perp}\mathcal{D}^2\phi$ term that appears in eq. (19) (the λ term in that equation is absent even for $\xi = -1$ because its coefficient vanishes).

To summarize, after these assumptions on the sources one remains with a system of equations which has many similarities with the usual *vacuum* bulk gravi-dilaton system, the only differences being: (i) the different tensor structure of the constraint equation, and (ii) the presence of a source term in eq. (19). It will now be shown that these two “problems” are actually one the “solution” of the other.

B. Homogeneous solution

It is now time to look for an explicit solution of this system. The following ansatz is made for the induced metric and for the dilaton on the brane:

$$ds_{brane}^2 = -dt^2 + a^2(t)d\vec{x}^2, \quad \phi_{brane} = \varphi(t), \quad (20)$$

and these expressions are substituted into eqns. (17, 18, 19) thus giving, after some rearrangements in the two dynamical equations:

$$(D-2)(D-1)H^2 + \dot{\varphi}^2 - 2(D-1)\dot{\varphi}H = 0, \quad (21)$$

$$\ddot{\varphi} = \dot{\varphi}^2 - (D-2)\dot{\varphi}H + {}^{\perp}\mathcal{D}^2\phi, \quad (22)$$

$$2(D-2)\dot{H} = \dot{\varphi}^2 - (D-2)(D-1)H^2 + 2{}^{\perp}\mathcal{D}^2\phi, \quad (23)$$

with $H \equiv \dot{a}/a$ and $\dot{\varphi} \equiv d\varphi/dt$.

A crucial observation should be made at this point: despite the fact that we cannot know *a priori* what is the behavior of ϕ in the bulk, we know that the constraint (21) must be satisfied on the brane. This information can be used to express the value at the brane of ${}^{\perp}\mathcal{D}^2\phi$ in terms of $\dot{\varphi}$ and H only. This is done by taking the time derivative of the constraint equation and by using the dynamical equations to express $\ddot{\varphi}$, \dot{H} in terms of $\dot{\varphi}$, H and ${}^{\perp}\mathcal{D}^2\phi$, thus obtaining the following condition:

$$2\dot{\varphi}{}^{\perp}\mathcal{D}^2\phi = (D-3)\dot{\varphi}^3 - [(D-2)(D-1)]^2 H^3 + (D-2)(3D-5)H\dot{\varphi}[(D-1)H - \dot{\varphi}]. \quad (24)$$

This condition can now be used to re-express the dynamical equations in terms of H and φ only. As it happens in the ordinary bulk vacuum case, the resulting equations are easily solved by means of the following ansatz, which corresponds to a Kasner-like solution

$$\begin{cases} \dot{H} = \beta^{\pm} H^2 \\ \dot{\varphi} = \alpha^{\pm} H \end{cases} \Rightarrow \begin{cases} H = -\frac{1}{\beta^{\pm}(t-t_0)} \\ \dot{\varphi} = -\frac{\alpha^{\pm}}{\beta^{\pm}} \frac{1}{t-t_0} \end{cases}. \quad (25)$$

The two D -dependent couples of coefficients $(\alpha^{\pm}, \beta^{\pm})$ correspond to two different branches (conventionally indicated as (\pm) according to the sign of β) of the solution, just like in the bulk vacuum string cosmology equations. In the latter case it was $\alpha^{\pm} = d \pm \sqrt{d}$, $\beta^{\pm} = \pm\sqrt{d}$, with d the number of spatial dimensions (for a comparison with our case, one should take $d = D - 2$). In terms of d , the coefficients for the brane world solution (25) turn out to be only slightly different from the bulk case:

$$\alpha^{\pm} = d + 1 \pm \sqrt{d+1}, \quad \beta^{\pm} = \pm\sqrt{d+1}. \quad (26)$$

The solutions (25) are plotted in fig. 1. As can be seen, the bulk and brane world behaviors are practically the same: in both cases H and $\dot{\varphi}$ are always increasing [decreasing] in the $(+)$ [$(-)$] branch, and they are positive [negative] for $t < t_0$ and viceversa. If one chooses to look at the $(+)$ branch for $t < t_0$ the initial state is flat and weakly coupled, and evolves towards a singularity going through a period of inflationary expansion and increasing coupling: this is the pre-big bang phase. If one considers the $(-)$ branch for $t > t_0$, one sees a period of decelerated expansion which originates from a singularity, just like in standard cosmology.

At t approaches t_0 , the solution is not reliable anymore because it goes beyond the validity regime of the low energy effective action (1): at large curvature the role of α' corrections

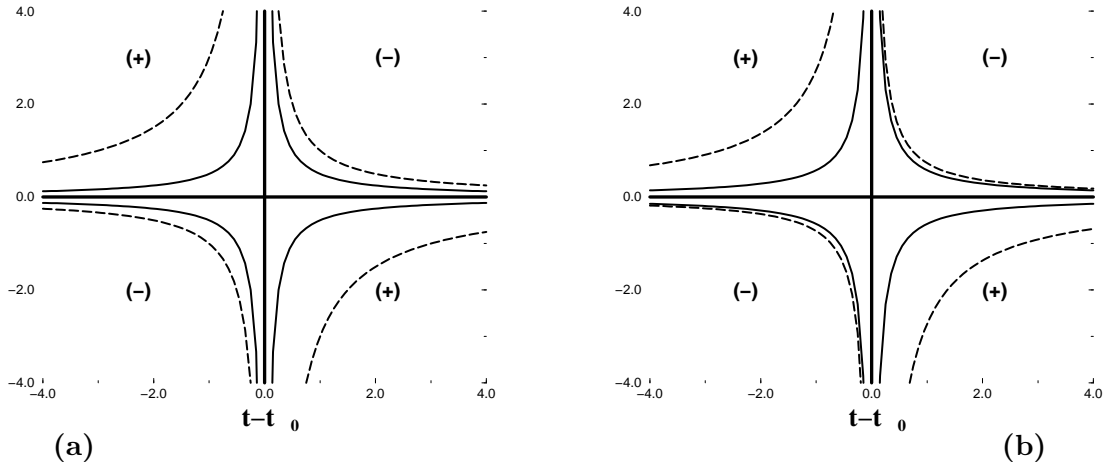


FIG. 1: On the left (a) the solutions (25) for H (solid line) and φ (dashed); both the (+) and (-) branches are displayed. On the right (b) the analogous pre- and post-big bang solutions for the bulk vacuum equations, in the same number of spatial dimensions. The data displayed in these graphs correspond to the case $d = 3$; for $d > 3$ the solutions are qualitatively the same.

should be taken into account, while as φ increases the dynamics is certainly modified by perturbative (loops) and nonperturbative strong coupling effects among which, in the case $\xi > -1$, one should also include the terms proportional to $e^{(\xi+1)\phi}$ that appear in eqns. (17, 18, 19). The graceful exit problem in pre-big bang cosmology consists of connecting the two zeroth order branches of the solution into a whole nonsingular expanding evolution. As mentioned in the introduction, this problem has not yet found a satisfactory solution.

The fact that the vacuum pre-big bang dynamics have been reproduced in a brane world model shows that the graceful exit problem can be rephrased in a brane world context. For instance, the inclusion of α' corrections is expected to damp the singularity also in the brane world case (some indications have already been found in [13]). In the same spirit, in the present context the eventual presence of a singularity in the bulk at a finite distance from the brane should be regarded as irrelevant, since at high curvature the solution found from the low energy action is not valid anymore.

Finally, for what concerns the dilaton stabilization, its realization in this context corresponds to the brane moving through a region of constant dilaton. Of course this issue should also be studied in detail in order to understand whether or not there are specific technical

obstacles to the realization of this scenario.

IV. CONCLUSIONS

I have just shown that a pre-big bang like dynamics can take place, under rather mild assumptions, also in a brane world model. The equations for such a model have been written in the string frame, and their tensor structure reminds that of the well known equations for a bulk gravi-dilaton system. For a particular choice of the matter content of the model the similarity becomes even more evident and, if the bulk Weyl tensor is set to zero, the system has the same behavior of the vacuum bulk pre-big bang cosmology.

Further investigation of the analogies and differences between the bulk and brane realization of pre-big bang, with particular reference to the high curvature and strong coupling phase and to the graceful exit problem, are left for future work.

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APPENDIX

Here the junction conditions for the string frame action (1) are derived. The derivation presented here is a modification of the Einstein frame one given in [14], to which the reader is addressed for further details.

The variation of the action is the following:

$$\begin{aligned} \delta\mathcal{S} = & \int d^D x e^{-\phi} \left[\sqrt{-g} \left(\left\{ \mathcal{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \left[\mathcal{R} - (\mathcal{D}\phi)^2 + 2\mathcal{D}^2\phi \right] + \mathcal{D}_\mu \mathcal{D}_\nu \phi - T_{\mu\nu} \right\} \delta g^{\mu\nu} + \right. \right. \\ & - \left. \left\{ \mathcal{R} - (\mathcal{D}\phi)^2 + 2\mathcal{D}^2\phi + T \right\} \delta\phi \right) + \\ & + \left. \left(e^{-\phi} \sqrt{-g} \left\{ g^{\mu\nu} \delta\Gamma_{\mu\nu}^\rho - g^{\mu\rho} \delta\Gamma_{\mu\nu}^\nu + [g^{\nu\rho} \mathcal{D}^\mu \phi - g^{\mu\nu} \mathcal{D}^\rho \phi] \delta g_{\mu\nu} + [2\mathcal{D}^\rho \phi] \delta\phi \right\} \right)_{,\rho} \right] + \\ & - \int d^{D-1} y \sqrt{-h} e^{-\phi} \left[2\delta K^\pm + K^\pm h^{\mu\nu} \delta g_{\mu\nu} - \tau_{\mu\nu} \delta g^{\mu\nu} - (2K^\pm + \tau^\phi) \delta\phi \right]. \end{aligned} \quad (27)$$

The first two lines give the bulk equations of motion. As to the third line, it can be transformed into an integral on the boundary by making use of the Gauss theorem which, with our definition of n_μ , can be written as

$$\int d^D x \left(\sqrt{-g} V^\rho \right)_{,\rho} \longrightarrow - \int d^{D-1} y \sqrt{-h} (n_\rho V^\rho)^\pm. \quad (28)$$

After some rearrangements in order to express the $\delta\Gamma$'s in terms of derivatives of δg , one finds that the third line becomes

$$- \int d^{D-1} y \sqrt{-h} e^{-\phi} \left[n^\rho h^{\mu\nu} (\mathcal{D}_\mu \delta g_{\rho\nu} - \mathcal{D}_\rho \delta g_{\mu\nu}) + (n^\mu \mathcal{D}^\nu \phi - g^{\mu\nu} n_\rho \mathcal{D}^\rho \phi) \delta g_{\mu\nu} + 2n_\rho \mathcal{D}^\rho \phi \delta\phi \right]^\pm \quad (29)$$

Coming to the fourth line of equation (27), it can be computed by knowing the definition of $K_{\mu\nu}$ in terms of n_μ , and by recalling that the normalization condition for n_μ implies

$$\delta n_\mu = \frac{1}{2} n_\mu n^\rho n^\sigma \delta g_{\rho\sigma}, \quad (30)$$

thus giving

$$\delta K = K_{\mu\nu} \delta g^{\mu\nu} + \frac{1}{2} K n^\rho n^\sigma \delta g_{\rho\sigma} - h^{\mu\nu} n^\rho \left(\mathcal{D}_\mu \delta g_{\nu\rho} - \frac{1}{2} \mathcal{D}_\rho \delta g_{\mu\nu} \right). \quad (31)$$

Now everything can be put together and the boundary part of $\delta\mathcal{S}$ becomes:

$$\begin{aligned} \delta\mathcal{S}_b = & \int d^{D-1}y\sqrt{-\bar{h}}e^{-\phi} \left[(h^{\mu\nu}n^\rho\mathcal{D}_\mu\delta g_{\nu\rho})^\pm + (n_\mu\mathcal{D}_\nu\phi - g_{\mu\nu}n_\rho\mathcal{D}^\rho\phi + Kg_{\mu\nu} - 2K_{\mu\nu})^\pm \delta g^{\mu\nu} \right. \\ & \left. + \tau_{\mu\nu}\delta g^{\mu\nu} + 2(K - n_\rho\mathcal{D}^\rho\phi)^\pm \delta\phi + \tau^\phi\delta\phi \right]. \end{aligned} \quad (32)$$

The first term can be rewritten in the following way:

$$\sqrt{-\bar{h}} \left[\mathcal{D}_\mu \left(e^{-\phi} h^{\mu\nu} n^\rho \delta g_{\nu\rho} \right) - \delta g_{\nu\rho} \mathcal{D}_\mu \left(e^{-\phi} h^{\mu\nu} n^\rho \right) \right]^\pm, \quad (33)$$

and the first part vanishes because $\mathcal{D}_\mu V^\mu \equiv \nabla_\mu V^\mu + n^\mu n^\nu \mathcal{D}_\mu V_\nu$ when V_μ is tangential to the brane, and because the boundary of the boundary is zero.

After having expanded the derivative of the second term of (33), one is finally left with

$$\begin{aligned} \delta\mathcal{S}_b = & \int d^{D-1}y\sqrt{-\bar{h}}e^{-\phi} \left\{ \left[K^\pm h_{\mu\nu} - K_{\mu\nu}^\pm - h_{\mu\nu} (n_\rho\mathcal{D}^\rho\phi)^\pm + \tau_{\mu\nu} \right] \delta g^{\mu\nu} + \right. \\ & \left. + \left[2(K - n_\rho\mathcal{D}^\rho\phi)^\pm + \tau^\phi \right] \delta\phi \right\}, \end{aligned} \quad (34)$$

from which the junction conditions (4) can be straightforwardly derived.