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Strategies in subtraction problem solving in children

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Abstract

The aim of this study was to investigate the strategies used by third graders in solving the 81 elementary subtractions that are the inverses of the one-digit additions with addends from 1 to 9 recently studied by Barrouillet and Lépine. Although the pattern of relationship between individual differences in working memory, on the one hand, and strategy choices and response times, on the other, was the same in both operations, subtraction and addition differed in two important ways. First, the strategy of direct retrieval was less frequent in subtraction than in addition and was even less frequent in subtraction solving than the recourse to the corresponding additive fact. Second, contrary to addition, the retrieval of subtractive answers is confined to some peculiar problems involving 1 as the subtrahend or the remainder. The implications of these findings for developmental theories of mental arithmetic are discussed.

Keywords: Subtraction; Strategies; Cognitive arithmetic; Number; Cognitive development; Working memory

Introduction

When considering the importance of the four basic operations in mathematics, the psychological studies focusing on subtraction solving are surprisingly scarce compared with those focusing on addition in the literature devoted to cognitive mental arithmetic. The rare studies focusing on subtraction in primary school children (Robinson, 2001) as well as in preschoolers (Siegler, 1987) have reported important individual variability in speed, accuracy, and strategy use. It has been assumed that such variability is comparable to that

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observed in addition (Siegler & Shrager, 1984). Nonetheless, the strategy of direct retrieval of the answer from memory, which is the fastest and most accurate strategy, seems to be used less frequently to solve subtractions than to solve additions (Robinson, 2001). However, so far as we know, no study has systematically compared the strategies used by children to solve the two operations and, more precisely, the extent to which children rely on direct retrieval. The current study fills this gap by exploring the strategies used by children to solve the 81 basic subtractions that are the inverses of the one-digit additions (operands form 1 to 9) recently studied by Barrouillet and Lépine (2005).

Mental subtraction and addition

The first study devoted to the strategies used by children in solving single-digit subtraction problems was conducted by Woods, Resnick, and Groen (1975), who presented second and fourth graders with 54 $m - n = ?$ problems where $0 \leqslant m \leqslant 9$ and $0 \leqslant n \leqslant 8$. Following the method used by Groen and Parkman (1972) in their study of addition, the authors recorded response times and tested several models of problem solving using linear regression analyses. These analyses indicated that children either set the counter to m and decrement n times, with the solution being the final value in the counter, or set the counter to n and increment until m is reached, with the solution being the number of times the counter has been incremented. Interestingly, children chose adaptively between the incrementing and decrementing strategies, opting for whichever one was faster.

It should not be concluded from this seminal study, however, that subtraction solving constantly relies on algorithmic strategies. Using basically the same method of response time analysis in 9- to 11-year-olds, Svenson and Hedenborg (1979) reported that when $m = n$, $n = 1$, and $m = 2n$ (corresponding to additive tie problems), answers are retrieved from memory. Siegler (1987) presented 5-year-old preschoolers and 6-year-old first graders with 25 elementary subtractions where both the subtrahend and the remainder ranged from 1 to 5. Children solved 58% of these small problems using a strategy of retrieval of the answer from memory, and 12% of them used retrieval exclusively. Several other studies have reported a mix of algorithmic and retrieval strategies in children when solving elementary subtractions (Ilg & Ames, 1951; Lankford, 1974; Starkey & Gelman, 1982). Accordingly, studies on subtraction in adults have also reported frequent use of retrieval (Barrouillet & Fayol, 1998; Geary, Frensh, & Wiley, 1993).

However, it seems that the strategy of direct retrieval is relatively rare in subtraction and is less frequent in subtraction than in addition. Some studies report surprisingly low rates of retrieval; for example, Robinson (2001) observed a maximum of 37% in fifth graders for very simple problems (minuend ranging from 5 to 9 and subtrahend ranging from 1 to 3), a result at odds with the comparatively high frequency reported by Siegler (1987) in preschoolers. Moreover, the rate of retrieval reported by Robinson (2001) was also lower than that usually reported for additions (e.g., Barrouillet & Lépine, 2005). This difference between subtraction and addition has also been found in adults. LeFevre, De Stefano, Penner-Wilger, and Daley's (2006) participants reported retrieval on 82% of easy problems (minuend < 10) but on only 42% of hard problems, results echoing those reported by Campbell and Xue (2001), who observed retrieval on 73 and 42% of easy and hard problems, respectively. For sake of comparison, LeFevre, Sadesky, and Bisanz (1996) reported a retrieval rate of 71% in adults solving the 100 additions with operands from 0 to 9. Seyler, Kirk, and Ashcraft (2003) reported higher rates, with 97% of the easy problems and 67% of the hard problems being solved through retrieval, but it should be noted that these authors grouped genuine retrieval of subtractive answers and the use of the basic corresponding additive fact (e.g., solving 7 – 3 by retrieving the additive fact $3 + 4 = 7$) within the same category. Thus, LeFevre and colleagues (2006) concluded that "subtraction appears to be the most procedural of the single-digit arithmetic operations" (p. 214).

Findings and theories

It is worth noting that these facts do not fit very well with the available theories of cognitive arithmetic. The moderate, but not negligible, use of retrieval in subtraction solving contrasts with the results reported in cognitive neuropsychological research and challenges the ensuing theories, which assume that subtractive answers are not systematically stored in long-term memory and, thus, rarely are retrieved. Dehaene and Cohen (1997) reported the case of a patient suffering from pure anarithmetia who exhibited a dissociation between addition and multiplication (which were nearly fully preserved), on the one hand, and subtraction and division (which were found to be nearly impossible), on the other. The authors accounted for this dissociation by assuming that the answers to simple additions and multiplications are stored in rote verbal memory, a memory that would be lacking for subtractions that necessitate the use of an indirect semantic route and reconstructive strategies. More generally, within Dehaene's triple code model, the verbal code is assumed to store problem–answer associations for addition and multiplication. In contrast, subtraction is not normally acquired by rote verbal learning. Thus, subtraction problems would not be systematically memorized and would require the manipulation of quantities represented by the operands (Cohen & Dehaene, 2000; Dehaene, 1992). Accordingly, Lee (2000) identified different cerebral areas activated when solving elementary multiplications and subtractions in a patient with acalculia. Therefore, it can be considered as surprising that adults report retrieval on more than 50% of the basic subtractions.

However, the enduring difference in retrieval use between addition and subtraction questions the available cognitive models. Indeed, although differences have been reported in the acquisition of addition and subtraction (Fuson, 1992), cognitive psychologists usually assume that the processes leading to the storage and retrieval of answers from memory and the mechanisms of strategy choice are the same for both operations. For example, Siegler (1987) modeled children's strategy choice in subtraction solving within the same distribution of associations model used by Siegler and Shrager (1984) to account for additive problem solving. This model suggests that the learning of subtractions involves both the acquisition of problem-solving procedures and the acquisition of associations between problems and answers, with children associating whatever answer they state with the problem on which they state it. Moreover, every time the system advances an answer, the strength of the association between that answer and the problem increases. This learning process results in associations of varying strengths between each problem and possible answers to the problem. Distributions of associations are characterized by their peakedness, depending on the existence of an answer being far more strongly associated with the problem than the other answers. Because the probability for the retrieval of an answer depends on the strength of its association with the problem, and because the associative strength increases with practice, the retrieval strategy is assumed to become more and more frequent during development, at least for the easiest and more frequent problems.

Within this framework, it is difficult to understand why the same mechanisms of learning and strategy selection result in marked differences between addition and subtraction until adulthood.

Thus, neuropsychological and developmental theories fail to account for the fact that subtraction is more procedural than addition but, nevertheless, is frequently solved through direct retrieval by adults and even by children. To shed light on this problem, we explored subtraction solving in children and compared their strategy use with that observed in children of the same age when solving the corresponding additions (e.g., the subtraction corresponding to $8 + 4 = 12$ is $12 - 8 = 4$). For this purpose, we used the material and results described in the study by Barrouillet and Lépine (2005), who presented third graders with the entire set of basic additions involving operands from 1 to 9. As we noted above, Dehaene's and Siegler's accounts of subtraction lead to opposite predictions.

According to Siegler (1987), addition and subtraction rely on the same mechanisms of associative learning and retrieval from distributions of associations. If this is correct, the use of retrieval in both operations should depend on the same problem characteristics and individual capacities. Of course, one could expect less frequent retrievals in subtractions than in additions, for example, because the former operations have been practiced less frequently by children. Nonetheless, the difference should be more quantitative than qualitative, and the distribution of retrieval frequencies among the subtraction problems should reflect what Barrouillet and Lépine (2005) observed for the corresponding additions. In the same way, the processes underpinning the learning of subtractive facts and their retrieval should be the same and, therefore, should be affected by the same individual capacities. Thus, the individual differences in working memory investigated by Barrouillet and Lépine should affect subtraction as they affect addition solving. As these authors observed, children with high working memory capacities should rely on retrieval more often, perform faster retrievals and algorithmic procedures, and exhibit smaller size effects than should children with low working memory capacities. In contrast, according to Dehaene (1992), children rarely should retrieve answers from memory and often should rely on algorithmic procedures or use derived facts from the corresponding addition to solve subtractions.

The current study

The predictions issuing from these models were tested by presenting children with the inverse of the 81 basic addition facts (augend and addend from 1 to 9) studied by Barrouillet and Lépine (2005), with the sum standing for the minuend, the augend standing for the subtrahend, and the addend becoming the remainder; for example, the addition $6 + 3 = 9$ corresponds to $9 - 6 = 3$ with 9 as the minuend, 6 as the subtrahend, and 3 as the remainder. Because there was no difference in strategy use between third and fourth graders in Barrouillet and Lépine's study, the current study focused on third graders only. Moreover, this grade was the level at which Robinson (2001) did not observe any difference in response times between a "no report" condition and the retrospective reports of strategy that we used in this study, suggesting that retrospective reports do not interfere with strategy choice in subtraction at this grade. Because the current study focused on the use of retrieval, no distinction was made between the different algorithmic strategies described by children. In contrast, genuine retrieval of subtractive fact and derived fact from the corresponding addition were scored as two different strategies. Apart from the strategy used, the speed and nature of the response were also recorded. The working memory capacities of the children were assessed using the same working memory span tasks as in Barrouillet and Lépine (2005), the counting span task inspired by Case, Kurland, and Goldberg (1982), and the reading letter span task (Lépine, Barrouillet, & Camos, 2005; Lépine, Bernardin, & Barrouillet, 2005).

Method

Participants

Participants were 48 third graders from suburban French primary schools (28 girls and 20 boys, mean age = 109 months, $SD = 7$ months, range = 99–127). Parental permission and the children's own consent were obtained.

Material and procedure

The material and procedure were essentially the same as those in Barrouillet and Lépine (2005). Children were presented with two working memory span tasks and a subtraction task administered individually using PsyScope software (Cohen, MacWhinney, Flatt, & Provost, 1993). During the first session, they performed the counting span task and solved 20 subtractions. During the second session a week later, they were presented with the reading letter span task and solved a new series of 20 subtractions. Each session lasted approximately 20 min.

Working memory span tasks

In both tasks, each child was presented with increasingly longer series of one to seven items to be remembered (letters or digits). Each item was followed by a series of stimuli to be processed (arrays of dots to be enumerated or series of letters to be read).

In the counting span task, the child memorized series of one to seven letters, with each letter being followed on-screen by an array of red and green dots among which the child needed to count out loud the red ones, pointing at them with a finger. There were 5 to 12 red dots among twice as many green dots (0.6 cm diameter) randomly displayed in a square frame with sides of 14 cm. Each series began with a ready signal that was displayed on the screen for 1 s, and then the first letter to be remembered appeared after a delay of 500 ms. The letter was presented for 1500 ms and was followed, after a delay of 500 ms, by an array of dots that the child needed to enumerate. When the child had completed the counting, the experimenter immediately cleared the screen by pressing a key on the keyboard and a new letter to be remembered was displayed after a delay of 500 ms and so on until the final letter of the list. The child was instructed to recall the letters of the series in their order of appearance when the word "Recall" was displayed on-screen.

In the reading letter span task designed by Lépine et al. (2005), each participant was presented with series of numbers to be remembered. After the presentation of each number, the participant was asked to read out loud a string of 4 to 6 letters successively displayed on-screen. The letters were randomly drawn from the 26 letters of the alphabet provided that a given set never contained the same letter twice or two adjacent letters of the alphabet. The to-be-remembered numbers were randomly drawn from 1 to 16 except for 14, which is a bisyllabic number word in French. Each series of numbers to be remembered started with a 1-s ready signal. After 500 ms, a first number was displayed on-screen for 1500 ms, followed by the first letter to be read. Each letter remained on-screen for 1000 ms after a delay of 350 ms. After a delay of 500 ms, a new number to be remembered appeared for 1500 ms, followed by a new string of letters to be read and so on. The participant was asked to recall the series of numbers presented in the correct order when the word "Recall" appeared on-screen.

Before each working memory span task, two one-item and two two-item training series preceded the experimental series. The experimental series began at length 1, with three series of each length, until the child failed to recall the items of all three series at a particular level. Testing was terminated at this point. Each experimental series correctly recalled was given a score of one-third, and the thirds were added together to provide a span score (Barrouillet, Bernardin, & Camos, 2004; Kemps, De Rammelaere, & Desmet, 2000; Smith & Scholey, 1992). For example, the correct recall of all three series of one and two items and of two three-item series resulted in a span of 2.67 ($[3 + 3 + 2] \times 1/3$).

The subtraction task

Each participant was asked to determine as quickly as possible, without giving up accuracy, the answers to elementary subtractions. The problems were formed by reversing the 81 possible single-digit additions (i.e., from $1 + 1$ to $9 + 9$) used by Barrouillet and Lépine (2005). For each addition, the first operand became the subtrahend s, the second operand became the remainder r, and the sum became the minuend m, resulting in the subtraction $m - s = r$. For example, $4 + 6 = 10$ was reversed into $10 - 4 = 6$. As in Barrouillet and Lépine's study, the subtraction corresponding to $5 + 5 = 10$ (i.e., $10 - 5 = 5$) was used as an example. The 80 remaining problems involved 8 "ties" where the subtrahend and the remainder were equal (e.g., $6 - 3 = 3$), 32 problems where the minuend was less than 10, 32 problems where the minuend was greater than 10, and 8 problems where the minuend was equal to 10. The participant studied one-half of each of these four categories, with the constraint being that a given participant never received the same calculation with the subtrahend and remainder reversed (e.g., $8 - 5$ and $8 - 3$). In the categories where the subtrahend and remainder were different, the subtrahend was larger than the remainder in half of the problems. Half of the resulting 40 problems were presented during the first session, and the other half were presented during the second session. In line with Barrouillet and Lépine, who observed a dramatic difference in response times and use of retrieval for additive problems with a sum greater than 10, on the one hand, and problems with a sum less than or equal to 10, on the other, we considered the corresponding subtractive problems with a minuend greater than 10 as large problems and those with a minuend less than or equal to 10 as small problems (see also Seyler et al., 2003).

Each trial began with a 1-s ready signal and was followed, after a delay of 500 ms, by a problem (e.g., $8 - 5 = ?$) that remained in the center of the screen until the participant's spoken response. A voice key stopped the timer, and the experimenter typed the participant's response as a record of accuracy. The next screen was a picture inviting the child to explain how he or she had reached the result that was given (i.e., picture of a character who is thinking with a "?" on the top), and the experimenter typed whether the response was based on an algorithmic strategy, on retrieval, or on using the corresponding additive fact. A problem was considered as being solved through retrieval when the child reported this strategy (e.g., "I knew that," "I found it in my head") and the experimenter did not notice any overt strategy such as finger counting, whispering, or lip moving. A problem was considered as being solved through additive derived fact when the child stated explicitly that he or she used the retrieval of the corresponding additive fact (e.g., for 8 − 5 = 3, "I know that 8 is 5 + 3") and did not exhibit any behavior suggesting counting as mentioned previously for retrieval. We considered that an algorithmic strategy was being used when the child reported counting to find the answer or when the child's behavior revealed multistep calculation by whispering or keeping track of counting on fingers. The progress of the task was controlled by the participant, who made the next trial appear by clicking on the mouse.

Results

Descriptive data

Among the 40 problems studied by each child, a mean of 8% were lost due to voice key failures, leaving a mean of 36.7 problems per child. Of the 48 children, 3 with rates of correct responses lower than 70% were discarded from the analyses (only 69, 67, and 54% of correct responses). The remaining 45 children achieved a rate of 86.7% ($SD = 6.7$) of correct responses, suggesting that they paid sufficient attention to the arithmetic task. Subtractions proved to be slightly, but significantly, more difficult than additions, eliciting 89.6% ($SD = 7.1$) of correct responses in Barrouillet and Lépine (2005), $t(90) = 2.06$, $p < .05$, $\eta^2 = .05$. The difficulty of the problems depended strongly on the size of both the subtrahend, $r = .632$, $p < .001$, and the minuend, $r = .581$, $p < .001$, but not on the size of the remainder, $r = .189$, $p > .10$. Not surprisingly, children committed far more errors in large problems than in small ones (21 and 7%, respectively), $t(44) = 6.40$, $p < .001$, $\eta^2 = .48$.

So far as the working memory tasks were concerned, the counting span and the reading letter span were very close (Ms = 1.93 and 1.84, SDs = 1.05 and 0.72, respectively) and significantly correlated, $r = .476$, $p < .01$. Thus, we calculated a compound working memory score as the mean of the two z scores. It is worth noting that these working memory spans were very similar to those observed in Barrouillet and Lépine (2005) (Ms = 1.93 and 1.87 for counting span and reading letter span, respectively), suggesting that the two populations are highly comparable.

Strategy choice

Overall, children reported using algorithmic strategies in 53% of the problems, derived facts from the corresponding addition in 28% of the problems, and direct retrieval from memory in only 19% of the problems[1] (Table 1). Strategy choice was strongly influenced

[1] We performed a series of analyses to test the validity of the self-reports. First, algorithmic strategies should be more frequent with large operands. Accordingly, the frequency of use of algorithmic strategies was highly correlated with the size of the minuend, $r = .787$, the subtrahend, $r = .622$, and the remainder, $r = .490$. Conversely, the frequency of use of retrieval was fairly well predicted by the size of the minuend, with retrievals being more rare for larger values, $r = -.737$. As we will see later, the retrieval strategy was used mainly for problems in which either the subtrahend or the remainder was 1. When these problems were set aside, the size of the minuend always was a good predictor of the use of retrieval, $r = -.755$, and was a fairly good predictor of the use of the related additive fact, $r = -.540$. Second, when considering response times, retrievals should be faster than deriving facts from the corresponding addition, which should in turn be faster than algorithmic strategies. Accordingly, the observed mean response times were 2328, 3749, and 7516 ms, respectively (ps < .001). Thus, we assume that the self-reports gathered can be considered as valid.

Table 1
Percentages of use, mean latencies, and accuracy of the three strategies used to solve subtractions and the two strategies for additions studied by Barrouillet and Lépine (2005)

Strategy	Problem type								
	Small			Large			Overall		
	Use (%)	Mean latencies	Accuracy (%)	Use (%)	Mean latencies	Accuracy (%)	Use (%)	Mean latencies	Accuracy (%)
Subtraction									
Algorithmic	37	5399	85	73	9065	74	53	7516	78
Retrieval	31	2274	97	5	3174	81	19	2328	96
Additive fact	33	3109	98	22	4895	94	28	3749	95
Addition[a]			[b]			[b]			[b]
Algorithmic	16	3589	—	62	5043	—	35	4639	—
Retrieval	84	1820	—	38	2406	—	65	1972	—

[a] Data from Barrouillet and Lépine (2005).
[b] Data on accuracy of each strategy were not available.

by the size of the problem. Children used algorithmic strategies more frequently for large problems than for small ones (73 and 37%, respectively), $t(44) = 10.91$, $p < .001$, $\eta^2 = .73$, with the reverse effect being observed for the additive derived fact strategy (33 and 22% for small and large problems, respectively), $t(44) = 3.99$, $p < .001$, $\eta^2 = .27$, and direct retrieval from memory hardly ever being used for large problems (31 and 5% for small and large problems, respectively), $t(44) = 8.89$, $p < .001$, $\eta^2 = .64$. Interestingly, the discontinuous "stair step" function between the values 10 and 11 of the minuend that was reported by Seyler and colleagues (2003) in adults' response times, errors, and use of algorithmic strategies can already be observed in children when considering the use of algorithmic strategies (Fig. 1). Strategy choice was also related to working memory capacities. The propensity to use algorithmic strategies was negatively correlated with the compound working memory score, $r = -.406$, $p < .01$ ($-.359$ and $-.339$, ps $< .05$, with the reading letter span and counting span, respectively). When considering retrieval-based strategies the use of additive derived facts correlated with working memory, $r = .297$, $p < .05$, but the frequency of use of direct retrieval did not, $r = .100$. It is, of course, possible that this lack of a relationship was due to the rare use of this strategy. Overall, higher working memory capacities were associated with a more frequent recourse to a retrieval-based strategy.

Strategy choices for subtractions contrasted with those observed for additions. First, subtraction elicited a more frequent use of algorithmic strategies than did addition in third graders (53 and 35%, respectively, for the 80 additions), $t(90) = 3.65$, $p < .001$, $\eta^2 = .13$. This difference was observed for small problems, where algorithmic strategies were rare for addition but not for subtraction (16 and 37%, respectively), $t(90) = 4.35$, $p < .001$, $\eta^2 = .17$, but also for large problems (62 and 73%, respectively), $t(91) = 1.99$, $p < .05$, $\eta^2 = .04$. Nonetheless, it should be noted that the stair step function discussed above can also be observed in the use of algorithmic strategies for addition (Fig. 2). Thus, it is not a peculiarity of subtraction but rather seems to affect both addition and subtraction in children. Second, there is also a strong difference in the use of direct retrieval of the answer from memory, a strategy used very frequently in additions (84 and 38% of small and large problems, respectively, vs. 31 and 5% for subtractions). Actually, when retrieving a number fact to solve subtractions, children relied more often on the corresponding additive fact than on the subtractive fact (28 and 19%, respectively).

As we stressed above, the main point when comparing retrievals on both operations is not only to notice that retrievals are rarer in subtraction but also to determine whether these retrievals present the same pattern of distribution across problems. Two phenomena suggest that subtraction and addition differ in this respect. First, as Figs. 3 and 4 make clear, most of the direct retrievals from memory reported by the children in subtraction concentrated on peculiar problems where either the subtrahend or the remainder is 1 (i.e., $m - 1 = r$ or $m - s = 1$),[2] a result that was not observed in additions (the corresponding additions are those where the minimum addend is 1). These problems, which represent 21% of the entire set of problems studied, concentrated 58% of the retrievals observed in

[2] The nature of these answers as genuine direct retrievals of associations between problem and answer can be questioned. However, these self-reports seem to be reliable. Indeed, children also reported algorithmic strategies for the problems involving 1. It appears that when children reported counting strategies, the response times were much longer than when children reported retrieving the answer (2125 and 4047 ms for retrieval and counting reports, respectively, on the 17 critical problems), $t(16) = 4.74$, $p < .001$, $\eta^2 = .58$.

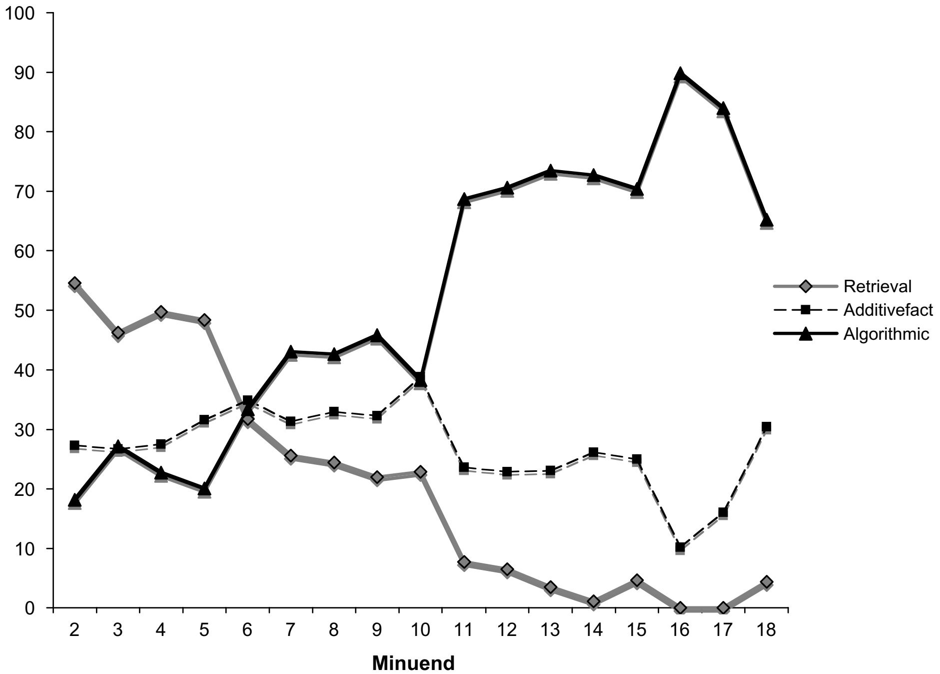

Fig. 1. Percentages of use of algorithmic strategies, of direct retrieval from memory, and of derived fact from the corresponding addition in subtraction solving as a function of the size of the minuend.

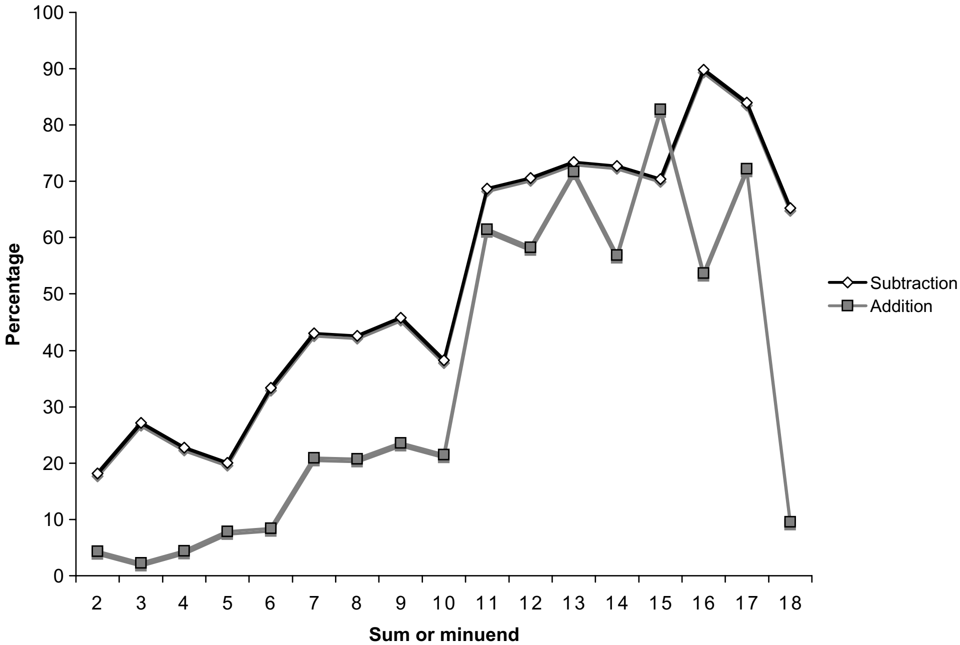

Fig. 2. Comparison of addition and subtraction in the frequency of use of algorithmic strategies as a function of the size of the sum for additions and the minuend for subtractions.

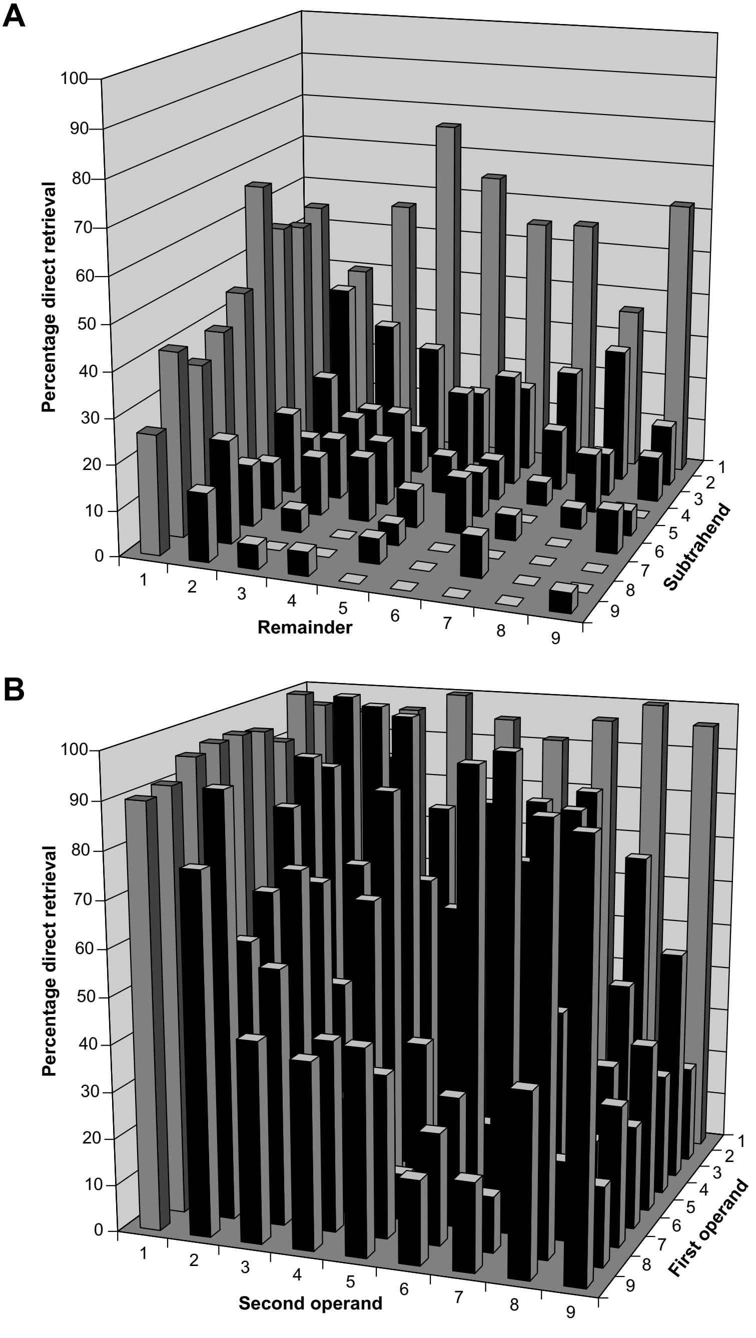

Fig. 3. Percentages of direct retrieval in subtractions (A) and the corresponding additions (B) as a function of the size of the subtrahend and remainder for subtractions and of the first and second addends for additions. Gray bars refer to problems where the subtrahend/first addend or the remainder/second addend equals 1. (Panel B data from Barrouillet and Lépine, 2005).

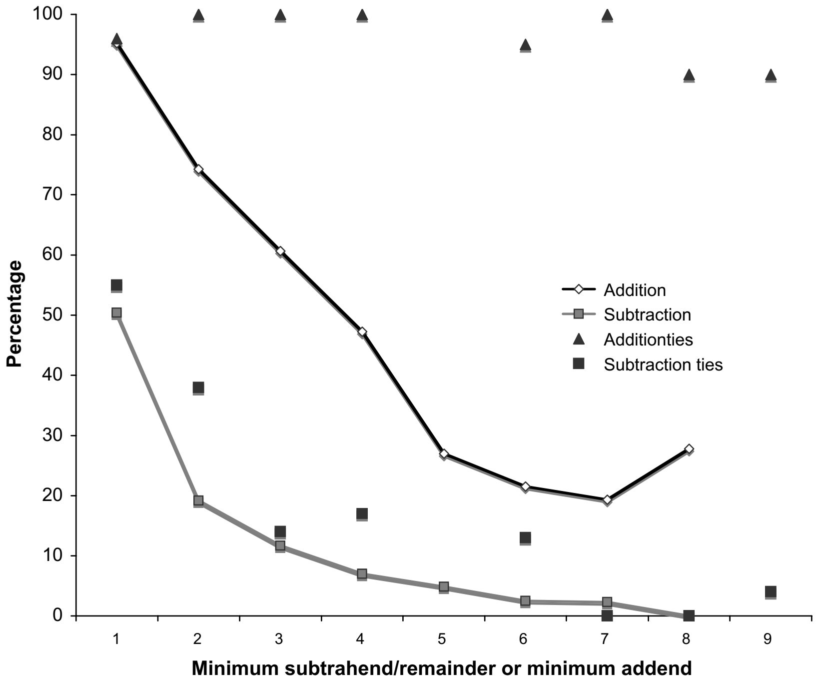

Fig. 4. Percentages of direct retrieval for subtractions and additions with tie problems presented separately as a function of the size of the minimum subtrahend/remainder for subtractions and minimum addend for additions. (Addition data from Barrouillet and Lépine, 2005).

subtraction in comparison with only 32% of those observed in addition. The seemingly smooth decrease of retrieval rate with increasing values of the minuend in Fig. 1 is, therefore, an artifact resulting from the structure of our material. The relative weight of the two critical types of problems decreases progressively when the minuend increases; for minuend 3, all of the problems are of this type (i.e., 3 − 1 and 3 − 2), whereas for minuend 10, only 2 of 9 of the problems are of this type (i.e., 10 − 1 and 10 − 9). Because the subtractive problems we used were reversed single-digit additions, there was no problem with the form $m - 1 = r$ or $m - s = 1$ using minuends larger than 10, hence the quasi-absence of retrieval for these problems.

This difference between the two operations in the distribution of retrieval rates across the problems was apparent even when concentrating on small problems. We systematically compared small subtractions with their corresponding additions; whereas the rates of retrievals for additions with minimum addends of 1, 2, 3, and 4 were 95, 80, 71, and 75%, respectively, the corresponding rates for subtractions were 51, 21, 14, and 13%. There was a significant interaction between the size of the minimum addend (or minimum subtrahend – remainder) and the type of operation, $F(3, 40) = 4.18$, $p = .01$, $\eta_p^2 = .24$.

The second phenomenon concerns "tie" problems. Whereas tie additive problems nearly always were solved by third graders through direct retrieval (96%), the corresponding subtractions (i.e., those where $s = r$) rarely were solved in this way (18%) (Fig. 4), with children relying more often on the corresponding additive fact (43%).

Chronometric analyses

The response times for correct answers revealed the ubiquitous size effect in mental arithmetic. Response times were highly correlated with the sizes of the minuend, $r = .756$, and the subtrahend, $r = .700$, but less strongly with the size of the remainder, $r = .369$. Response times were also correlated, but negatively, with working memory capacities, $r = -.382$, $p < .01$ ($-.354$ and $-.314$, $ps < .05$, with the reading letter span and counting span, respectively). These correlations are not surprising if we consider that an important part of the size effect is due to the frequent use of slow algorithmic strategies for large problems, whereas small problems more frequently elicit a fast strategy, and that children with higher working memory capacities tend to rely more often on a fast retrieval-based strategy. Indeed, the algorithmic strategy was the slowest, whereas direct retrieval, as in other operations, was the fastest strategy (Table 1). Retrieving and using the corresponding additive facts takes longer than retrieving the subtractive fact.

Two main differences appeared when comparing subtraction and addition in response times. First, the algorithmic strategies that are more frequent in subtraction than in addition are also slower (7516 and 4639 ms for subtraction and addition, respectively), $t(89) = 5.32$, $p < .001$, $\eta^2 = .24$, a difference that is present even for small problems (5399 and 3589 ms), $t(71) = 3.63$, $p < .001$, $\eta^2 = .16$, and increases dramatically for large problems (9065 and 5043 ms), $t(89) = 5.51$, $p < .001$, $\eta^2 = .25$. Second, and even more surprising, the responses reported as relying on direct retrieval from memory are slower in subtraction than in addition. The mean retrieval time for addition was 1972 ms compared with 2328 ms for subtraction, $t(84) = 2.81$, $p < .01$, $\eta^2 = .09$. This difference was also observed when the analyses were restricted to small problems in which retrievals were more frequent (1820 and 2274 ms, respectively), $t(84) = 3.64$, $p < .001$, $\eta^2 = .14$. The con-

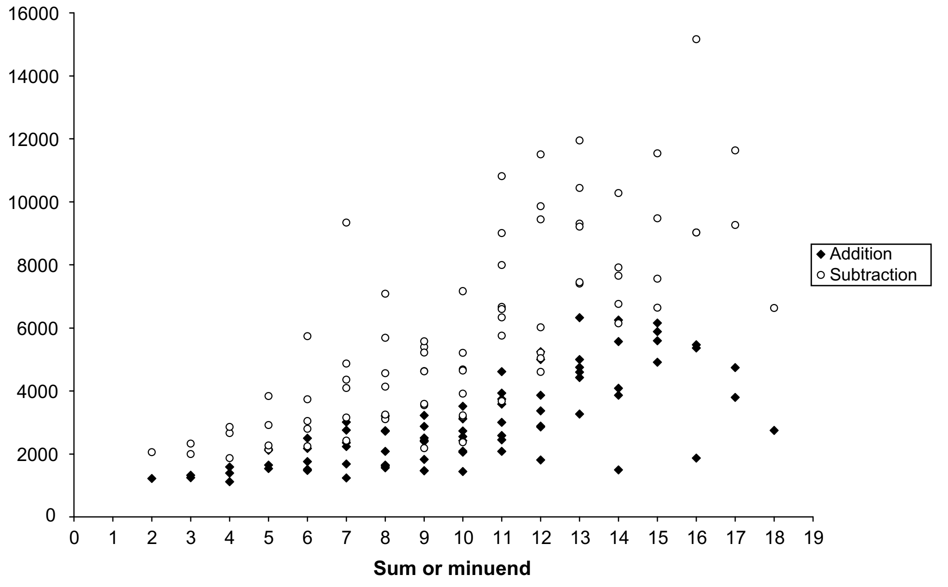

Fig. 5. Comparison of response times for solving addition and subtraction as a function of the size of the sum for additions and the minuend for subtractions. (Addition data from Barrouillet and Lépine, 2005).

clusions remained the same when the analyses concerning subtraction were restricted to those problems that elicited frequent retrievals (i.e., subtractions with a subtrahend or remainder of 1). Direct retrievals always were slower for subtraction than for addition (2072 and 1820 ms, respectively), $t(84) = 2.15$, $p < .05$, $\eta^2 = .05$. Overall, third graders were far slower in solving subtraction than in solving addition (Fig. 5). This difference probably has an impact on the age-related changes in the strategy use, as we argue in the following discussion.

Comparison between children with high and low working memory capacities

The group of 45 children was divided into thirds, with the 15 children with the highest z compound scores being considered as having high working memory capacities (mean $z = .92$) and the 15 children with the lowest z scores being considered as having low working memory capacities. Actually, due to several children having exactly the same score, this latter group was reduced to 14 children (mean $z = -.85$). It is worth noting that these two extreme groups did not differ significantly on the rate of correct responses (88 and 86% for low- and high-span groups, respectively, $t < 1$).

First, we expected that children with high working memory capacities would rely more often on retrieval. Because retrieval-based strategies were divided into direct retrieval and retrieval of the corresponding additive fact, the analyses on algorithmic strategies gives a clearer picture of the results. We performed an analysis of variance (ANOVA) on the frequency of use of algorithmic strategies with the size of the problems (small vs. large) as a within-participant factor and the span level (high vs. low) as a between-participants factor. As was observed with additions, the use of algorithmic strategies was more frequent in children with low working memory span than in children with high working memory span (61 and 40%, respectively), $F(1, 27) = 5.27$, $p < .05$, $\eta_p^2 = .16$. This effect was highly significant in small problems (47 and 23%, respectively), $F(1, 27) = 8.50$, $p < .01$, but not in large problems (78 and 61%), $F(1, 27) = 2.24$, $p = .15$. Nonetheless, there was no significant interaction, $F < 1$, $\eta_p^2 = .02$ (Fig. 6). The same analyses were performed on the frequency of use of direct retrieval and additive derived fact. Although the differences between children with high and low working memory spans were in the expected direction for both strategies (18 and 12% of retrieval and 43 and 27% of additive derived fact for high- and low-span children, respectively), they did not reach significance, $F(1, 27) = 1.34$, $p = .26$, $\eta_p^2 = .05$, and $F(1, 27) = 2.09$, $p = .16$, $\eta_p^2 = .07$, respectively. In summary, children with high working memory capacities relied less frequently than the others on algorithmic strategies, mainly in small problems where they more often used a strategy based on retrieval, either of the corresponding additive facts or of the subtractive answer.

Working memory capacities were also expected to affect response times. We performed an ANOVA on the mean response times with the size of the problems (small vs. large) as a within-participant factor and the span level (high vs. low) as a between-participants factor. Apart from a strong size effect (3634 and 7646 ms for small and large problems, respectively), $F(1, 27) = 70.81$, $p < .001$, $\eta_p^2 = .72$, children with high working memory span were faster than children with low working memory span (3826 and 6545 ms), $F(1, 27) = 16.72$, $p < .001$, $\eta_p^2 = .38$, an effect that was significant in small problems (2773 and 4495 ms), $F(1, 27) = 21.95$, $p < .001$, as well as in large problems (5556 and 9735 ms), $F(1, 27) = 12.83$, $p < .01$. The size of the problems interacted with the groups, $F(1, 27) = 6.64$, $p < .02$, $\eta_p^2 = .20$, testifying for a weaker size effect in children with high working memory span.

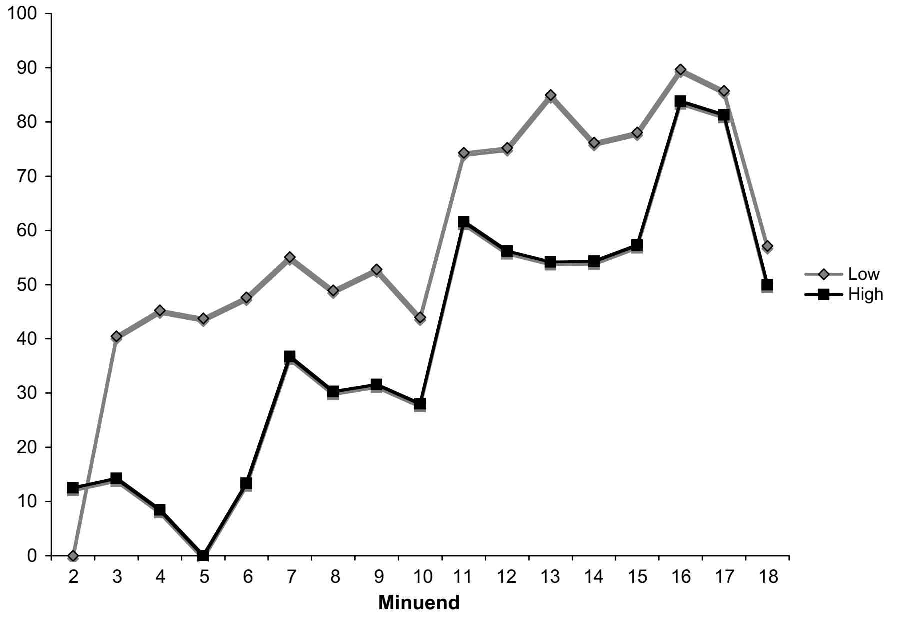

Fig. 6. Rates of algorithmic strategies in subtraction as a function of working memory capacities and the size of the minuend.

In line with Barrouillet and Lépine (2005), it could also be expected that children with high working memory span would achieve faster retrievals than children with low working memory span. In the current study, 2 children with low working memory span and 1 child with high working memory span did not report any direct retrieval from memory and were discarded from the following analysis. Actually, there was no significant difference in response times between children with high and low working memory capacities when they reported a retrieval strategy (2041 and 2440 ms, respectively), $t(24) = 1.69$, $p = .10$, $\eta^2 = .11$, even if this difference failed shortly to reach significance when the analysis was restricted to the small problems (1972 and 2399 ms), $t(24) = 1.88$, $p = .07$, $\eta^2 = .13$. In contrast, working memory had a significant effect on the response times related to the use of additive facts (2965 and 4382 ms for high- and low-span groups, respectively), $t(27) = 3.38$, $p < .01$, $\eta^2 = .30$. This effect was still significant for small problems in which children with high working memory capacities were faster than those with low working memory capacities in retrieving the corresponding additive fact (2529 and 4060 ms, respectively), $t(27) = 4.20$, $p < .001$, $\eta^2 = .40$. The pattern of results was roughly the same for algorithmic strategies. A total of 13 children in both high and low working memory span groups reported algorithmic strategies for small and large problems. An ANOVA with the same design as used previously revealed that children with high working memory capacities were faster than children with low working memory capacities (5580 and 7926 ms, respectively), $F(1, 24) = 6.63$, $p < .02$, $\eta_p^2 = .22$, with the interaction with the size of the problems failing to reach significance, $F(1, 24) = 3.68$, $p = .07$, $\eta_p^2 = .13$.

In summary, children with high working memory capacities were faster in solving subtractive problems and exhibited a weaker size effect. They were also faster in running algorithmic strategies and in producing responses derived from additive facts. In contrast, as we observed for frequency of use, the differences related to working memory in direct retrieval latencies did not reach significance, perhaps because this strategy rarely was used.

Discussion

So far as we know, the current study is the first to systematically investigate the strategies used by children in solving the entire set of elementary subtractions (except those involving 0) and to compare them with the strategies used to solve the corresponding additions. Two main theoretical accounts of the development of these strategies have been suggested. According to Siegler (1987), the same mechanisms are responsible for development and strategy choice in addition and subtraction. Problems initially would be solved through algorithmic strategies such as counting down and counting up, with the stated answers being associated with the problem in long-term memory. Working memory capacities would have an impact on these processes because higher working memory capacities should result in greater efficiency in running algorithmic strategies and in increased probability and speed of retrieval of the relevant problem–answer associations. In contrast, according to Dehaene (1992), subtractive answers would not be systematically memorized, with subtractions necessitating algorithmic strategies or the use of derived facts. Working memory capacities would, of course, have an impact on the efficiency of these two strategies, but they would not affect the construction of problem–answer associations as in additions.

Do our results favor one of these accounts? As Barrouillet and Lépine (2005) observed in addition, children with high working memory capacities were faster in solving subtraction through algorithmic strategies, even for small problems. They also relied less frequently on algorithmic strategies, reporting more frequently the use of arithmetic facts, either additive or subtractive, compared with children with low working memory capacities. The less frequent use of algorithmic strategies combined with faster processing resulted in a smaller size effect in children with high working memory capacities, exactly as was observed in addition problem solving by children of the same age. These findings suggest that the relationship between individual differences in working memory and addition problem solving extend to subtraction. It could be suggested that these results support Siegler's (1987) conception of an identity of mechanisms that underpin the learning and resolution of addition and subtraction.

However, we also observed strong differences between the two operations that could cast doubt on the plausibility of common mechanisms responsible for a shift from reconstructive to reproductive strategies in both operations. First, third graders rarely reported using the direct retrieval of subtraction answers from long-term memory, whereas this was the most frequently reported strategy in addition solving by children of the same age studied by Barrouillet and Lépine (65 and 19% for addition and subtraction, respectively). Moreover, and contrary to addition, there was no clear relation between working memory capacities and the frequency and speed of retrieval in subtraction. Second, investigating the entire set of elementary subtractions revealed that these retrievals present a distribution very different from that observed with addition. Far from being distributed among the different small problems as was observed for addition (see Fig. 3), reported retrievals in subtraction solving concentrated on peculiar problems in which either the subtrahend or the remainder was 1. Moreover, strategies reported by children as retrievals on these problems could correspond to one-step procedures based on browsing the number line using simple rules such as "$m - 1$ is the number preceding m" and "when n and m are successive numbers, $m - n$ is 1." If this is the case, these answers would not be genuine retrievals of stored associations between problems and answers as described by Siegler (1987). The fact that these "retrievals" are restricted to some specific problems, are very rare even for small problems with a subtrahend of 3 (20%) or 4 (16%), do not concern tie problems contrary to additions, and are slower than the retrievals of the corresponding additive facts suggests that children used number line-based rules rather than genuine retrieval in most of the cases. Moreover, it cannot be imagined that children do not use retrieval because this strategy would be superseded by fast and undemanding algorithmic strategies. As we have seen, the algorithmic strategies used to solve subtraction are particularly slow and much slower than strategies used to solve addition. Thus, although it rarely is used, direct retrieval would be particularly beneficial in solving subtraction.

Does this mean that children do not and cannot store subtractive answers? It might be premature to endorse such an extreme conclusion. As we noted above, most of the studies on subtraction in adults have reported the frequent use of retrieval strategies, and in the current study we observed fast and accurate responses for a limited but existing range of problems. However, the low rate of retrievals in subtraction solving deserves an explanation. Our hypothesis is that Siegler (1987) was basically correct in assuming that subtraction solving is based on general mechanisms by which the outcomes of cognitive processing are stored in long-term memory from which they can be reactivated and retrieved. However, the peculiarities of the counting strategies for subtraction would, on the one hand, impair the storage of strong and easily retrievable subtractive facts and, on the other, contribute to reinforce the corresponding additive facts.

So far as the first point is concerned, we have seen that counting strategies for subtraction are especially slow, with mean response times longer than 5 s for small problems and 9 s for large problems. These exaggerated delays between the encoding of the problem and the production of the answer would impair efficient storage of the operand–answer association in long-term memory, as Thevenot, Barrouillet, and Fayol (2001) demonstrated. Indeed, algorithmic processing leads to the manipulation and temporary maintenance of many numbers that are, thus, activated in short-term memory. However, these numbers are irrelevant for operand–answer association and produce representation-based interferences with the memory traces of operands that suffer a time-related decay. As a consequence, the probability that intact memory traces of the operands are still present in working memory when the answer is reached decreases as the time needed to arrive at the answer increases. Thus, it is possible that children are too slow in solving subtraction to ensure operand–answer associations.

So far as the second point is concerned, the nature of the algorithmic strategies could prevent the reinforcement of subtractive facts. Two main algorithmic strategies are used by children to subtract a subtrahend s from a minuend m, namely counting down and counting up (Fuson, 1992). In counting down, children decrement the minuend s times, with the solution being the final value they reach. In contrast, in counting up, children start from s and increment until m is reached, with the solution being the number of times the counter has been incremented. Many studies have shown that counting down is more difficult and error prone than counting up (Baroody, 1984; Baroody & Ginsburg, 1983; Fuson, 1984; Fuson, 1992; Siegler, 1987), with the main reason being that counting backward is more difficult than counting forward (Fuson, Richards, & Briars, 1982). Thus, counting down probably is one of the slowest and least accurate algorithmic strategies that children use to solve arithmetic problems, and it can be imagined that counting down results in weak associations that probably are difficult to retrieve. Nonetheless, operand–answer associations could be stored from the counting up strategy, which is easier and probably faster. However, it is unclear whether this strategy favors the storage of a subtractive fact, as the following example makes clear. Suppose that children are solving $8 - 5$ using the counting up strategy. They start from 5 and increment until 8 while monitoring the number of steps. What is stated using this procedure is that 5 and 3 are 8 rather than that when 5 is subtracted from 8, the remainder is 3. As a consequence, it is possible that the counting up strategy concurs to reinforce additive facts rather than subtractive ones, explaining the frequent use of the retrieval of additive facts to solve subtraction. Thus, both the difficulty and the nature of the algorithmic strategies available to solve subtraction could converge to produce the low rate of retrieval we observed in third graders.

To conclude, the current results demonstrate that strategies used by children to solve subtractions and additions differ not only in frequency of use, as previous studies have reported (Robinson, 2001), but also, and more important, on distribution across the different problems. As we have suggested, the nature and constraints of the algorithmic strategies used by children to solve subtractions probably impede the fast and easy learning of problem–answer associations. This difficulty, the low frequency of retrievals concentrated on few peculiar problems, and the fact that children frequently rely on additive answers question the high rates of direct retrievals reported by some studies on subtraction in adults. Therefore, further developmental research is required to investigate how the use of direct retrieval from memory evolves with age and to determine whether the storage of subtractive facts is developmentally delayed or impossible.

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