



Chapitre d'actes

2006

Published version

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### How to cite

LEUTENEGGER, Francia et al. The teacher's action, the researcher's conception in mathematics. In: Proceedings of the 4th Congress of the European society for Research in Mathematics Education (ERME Congress). Bosch, M. (Ed.). Barcelona. [s.l.] : [s.n.], 2006. p. 1588–1597.

This publication URL: <https://archive-ouverte.unige.ch/unige:180458>

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Last deposit update in Archive ouverte UNIGE on 30.10.2024 11:13

## THE TEACHER'S ACTION, THE RESEARCHER'S CONCEPTION IN MATHEMATICS

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**Abstract:** *Our intention is to contrast the epistemological positions of teachers and researchers, by the means of their respective actions in a research process. Based on a threefold descriptive model of the teacher's action, our analyses examine the nature of the teaching techniques enacted about a given mathematical situation, the "Race to 20" (pupils aged 9-11 years) and the discourses of two teachers about this lesson. Our findings indicate that the teachers are primarily concerned with the educational coherence of the teaching process, using non specifically knowledge-related teaching techniques that researchers explain by some generic overdeterminations, from the didactical point of view. This gap has to be taken into account in further collaborative research, in order to make teacher's developing specific teaching techniques to foster the mathematical sense-making of the students.*

**Keywords:** Mathematics teaching, Race to 20, Teacher's action, Cognitive values, Mathematical knowledge-related teaching techniques, Epistemological gap.

This paper investigates the epistemological gap existing between two experienced teachers and a team of researchers, both involved in a particular research project. This gap is to be considered through the teacher's practices and discourses on one hand, and through the researchers' expectations and interpretations on the other hand. This study comes out as a side question in the main stream of our work on the teacher's action in the "Race to 20" mathematical situation.

### A TEACHER'S ACTION MODEL AS THE MAIN FRAMEWORK

In order to understand the teacher's action while he carries out mathematics lessons, we designed a threefold model based upon didactical categories (Brousseau 1997 ; Chevallard, 1992). The first level of this theoretical frame comprises a set of *micro* teaching techniques, that were spotted several times among the interactions patterns in a given mathematical situation (Sensevy, Mercier, Schubauer-Leoni 2000). These techniques enable the researcher to describe in depth and precisely how a given piece of knowledge is handled by both the teacher and the students. A second level is also

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defined to gather the *macro* shifts that can be observed in the didactical contract<sup>1</sup>, and therefore it sums up the main teacher's intentions about the knowledge. Then, a third level is to take into account the teachers' general practices and educational beliefs through actions observed during the lesson, and/or comments we may get from the teachers during interviews. Moving up the levels in the model shows a "gradient" in the teaching techniques: at the bottom, are the most knowledge-linked techniques and at the top, are the less specific techniques. Actually, the two first levels are the core of our theoretical point of view, as we defend that the knowledge specificities and the corresponding teaching situations in mathematics are very likely to constrain the didactical interactions. In contrast to other pedagogical models designed to indicate what the teacher should do to achieve a "good" teaching practice, our model is built up empirically from classroom observations of teaching techniques, taking into account the didactical constraints of a given situation. As a descriptive tool at this stage, it is exclusively used for research analyses. Our comparative research project in didactics should contribute to sort out the generic or specific nature of the teaching techniques we identify.

In order to foster the validity of our empirical model, we conceived and carried out a research setup (Sensevy, Mercier, Schubauer-Leoni, Ligozat & Perrot, 2005) in which the didactic situation "Race to 20"<sup>2</sup> is still an experimental paradigm for the studying of teacher's work. It involved two teachers at the fifth grade of French primary school. The teachers were first trained to the mathematical issues of the "Race to 20" game and secondly, they were asked to carry out the "Race to 20" situation as a lesson with their respective classes. The teachers were free to plan the lessons as they wished.. A third phase consisted in the teachers' self-analyzing their first lesson<sup>3</sup>, based on a video recording of the lesson. A fourth phase consisted in the teacher's cross-analyzing their first lesson: T2 analyzed T1's lesson on the video recording (in presence of T1) and reciprocally. In this setup, the teachers are not part of the research team. They know each others quite well due to partnerships developed in other professional circumstances<sup>4</sup>, and we can say that interviews were carried out in a confident atmosphere. The researcher's work started afterwards, to carry out clinical analyses of the teaching techniques related to the "Race to 20" situation, encountered in the

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<sup>1</sup> This concept was developed by Brousseau (1997) to describe the reciprocal expectations of the teachers towards the students and *vice versa*, about the meaning of a mathematical situation in which a knowledge is the stake.

<sup>2</sup> This fundamental situation in Brousseau's work (1997) is based upon a game which opposes two players. The first player says a natural number  $X_1$  that is less than 3 (1, for example). The second player says a natural number  $Y_1$  obtained by adding 1 or 2 to  $X_1$  (for example, he says 3, a number obtained by adding 2 to 1). The first player then says a natural number  $X_2$ , obtained by adding 1 or 2 to  $Y_1$  (for example, he adds 1 and says 4), etc. The player who is the first to say 20 is the winner. There are numbers that it are sufficient to say in order to win: 2, 5, 8, 11, 14, 17, 20.

<sup>3</sup> T1 and T2 gave more than one lesson on this subject.

<sup>4</sup> T1 and T2 are classroom-based teacher trainers, whereas both the researchers involved, teach mathematic education in pre-service teacher education college.

observations of both of the teachers. The main part of this work is detailed in Sensevy & al (2004).

## PRESENT WORK

The purpose of this paper is the confrontation of the two specific logics enacted in the teacher's action / justification and the researcher's action / interpretation, based on the teacher's action model. This research question was born out of an unexpected difference in appraisal on some teaching techniques by teachers on one side and researcher's on the other side. In this paper, the teaching episodes are selected among the existing materials from the setup described above, as particular features that need to be explained into a widest layout. We attempt to proceed in the same way as some historians (Ginzburg, 1989) using a clue-based evidentiary paradigm, to build a comprehensive reality that could not be experienced or questioned directly, because we work afterwards. After a brief description of the particular techniques encountered in T1' practice, we present T1's point of view on his techniques and then T2's analyses of T1's teaching techniques. The last part of this paper is an attempt to understand the two different systems of meaning and the two different epistemologies enacted in the different actions and discourses.

## A DESCRIPTION OF TWO TECHNIQUES USED BY T1

After the analysis of the first lesson of T1, the researchers agree on the following point: two techniques used by T1 seem rather rare<sup>5</sup> and unexpected. Indeed, two original ways of acting are used by T1:

- in the beginning of the lesson, he asks students to question him questions about the topic of this lesson,
- in the pair work that he scheduled for this lesson, a third student has to watch the game played by the two others as a referee.

### *The "questions" technique*

T1 begins the lesson by questioning the students about the meaning of the words "Race to 20".

*1-T1 : Today we are going to work on the race to twenty. It's a mathematical game. From the expression "race to twenty, what can you already tell me?*

*2-Student : (...) we jump from 20 by 20*

*[...]*

*3-Student : Maybe we are going to count from twenty to the next twenty more.*

*4-T1 : Counting from twenty to the next twenty more. Yes.*

*5-Student : Its a race. We have to be quick...*

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<sup>5</sup> Among the tenth of teachers that the research team studied in this situation, it was the first time that such techniques were observed.

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**6-T1** : *Yes. The race. The idea is to be quick. So there is velocity, since we are in a race. Or else we would have called that the walk to twenty. Maybe. Then twenty, so you say "counting twenty by twenty". Can you see another idea?*

**7-Student** : *Its a race involving twenty children, with twenty children who play, who are racing.*

**8-T1** : *Twenty is the number of children taking part to this. Can you see any other thing? Race to twenty. Its true that "to" may be...*

**9-Quentin** : *For example we are going to count three by three up to twenty. And the winner is the first to reach...*

**10-T1** : *And then in that case, what is twenty ? What does it represent? Yes, go on Quentin.*

**11-Quentin** : *Its the number up to which we must go.*

**12-T1** : *The number up to which we must go, the number we must reach. And why are you thinking of counting thee by three?*

**13-Quentin** : *Well, because at the moment we are working a little bit on mental counting, so that counting three by three, we learnt through going backwards.*

**14-Camille** : *Yes, but counting three by tree, if you start to zero up to twenty, we reach thirty but not twenty. It won't be the exact number.*

**15-T1** : *You think we can reach twenty when we start from zero ?*

**16-Camille** : *No, with three...*

**17-T1** : *Jumping three by three...*

**18-Camille** : *Yes, and starting from zero.*

[...]

**19-T1** : *Well, in the race to twenty you suggested several things. The race, effectively, there is an idea of velocity and then, twenty, as Quentin said a few moments ago, you must reach twenty. You must go up to twenty. Another game, we can change it. So, in order to play that game, do you have enough information if I say to you "we are going to play the race to twenty"?*

**20-Student** : *No.*

**21-T1** : *Well, in that case ask me questions!*

In contrast with the other teachers previously studied (and with T2), T1 institutes a "Question-game" (ST<sup>6</sup> 1). This episode lasts almost 20 minutes in which the students try to guess what the Race to 20 is supposed to be, by considering the meaning of the "Race to 20" by considering the meaning of the words. T1 refuses gently the "wrong" answers (e.g. on ST 8). Then he summarizes the students' answers, to emphasize the fact that the students have not "enough information" to play the game. On ST 21, the teacher produces an utterance: "So, ask me questions!", which is emblematic of this "teaching technique". It seems that the division of the activity between the teacher and

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<sup>6</sup> In the following, "ST" stands for "speech turn".

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the students is upside-down: the students ask the “relevant” questions, and the teacher give answers.

### *The “referee” technique*

Ten minutes after the beginning of the lesson, the students are correctly playing the game. T1 organizes the group work.

*39-T1 : We go through the following steps, now you can play, since you seem to have understood. Therefore you are going to play one against one, and one child will be the referee. Because, a few minutes ago I thought there was a mistake and eventually there was none. I had not heard correctly. So, do be careful, don't try to go too fast and pronounce correctly ... sometimes it actually looks like table tennis. It goes very fast. So, do be careful about this, to be well understood. So, you play one against one and the third child is the referee. Remind me, what part does referee Arnaud's play?*

*40-Arnaud : Do the counting up (...).*

*41-T1 : So that's to give an account of the match balance. Who won? Who lost? That's the game, isn't it? Jacques?*

*42-Jacques : Does the referee writes on a sheet ?*

*43-T1 : So, for the first game, we are going to watch very accurately what is happening. But the referee plays another part as well. For instance if a child adds three. Can he do this?*

*44-Student : No, he cannot.*

*45-T1 : If a child gives several times... he says one number then he says another one... in fact one does not know any longer what he said. So, in that case, the referee supervises a little bit the respect... sees to it that the rules be respected. Yes Jacques?*

The teacher institutes the refereeing function of a third student in the group (ST 39). He defines the main features of his role. The referee will be maintained during both lessons. Many times, his task will be discussed in the whole class activity. The studying of the two lessons transcript make us conjecture that this way of acting could be a classroom habit, not specific to mathematics.

### *A first analysis of these techniques*

The two techniques are analyzed by the team research in the same way (for detailed analyses, see Sensevy & al, 2005). Our hypothesis was that these two techniques might be counter-productive from a didactic viewpoint. Indeed, the “questions-game” could slow down the student's activity. The students' attention could be taken off the mathematical aspects of the situation. We conjectured that the “question-game” could work as a metacognitive shift (Brousseau, 1997). In a similar way, the refereeing could affect the involvement of the students in the mathematical tasks. It could also draw their attention to the superficial features of the game (the “basic rules”), and be detrimental to the production of mathematical strategies<sup>7</sup>

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<sup>7</sup> The distinction emphasized by Hintikka (e.-g. 1999) between “definitory rules” and “strategic rules” is useful : one cannot play chess successfully if one knows only the “definitory rules” (how chessmen may be moved, etc...) and does not master the “strategic rules” of this game. In the race to 20, mastering the

## THE TEACHER'S POINT OF VIEW ON THESE TECHNIQUES (T1)

### *About the "questions" technique*

When watching the video recording of his performance, T1 comments the "questions" technique. We can infer from these comments that the "questions-game" is really important in this teacher's practice. It allows the students to find "some coherence" in the learning process. It is obvious that the teaching intentions are far beyond the didactic goal of this lesson. The "questions-game" could be considered as a taken-as-shared way of acting, that seems normative in this classroom community... It is not specifically related to the mathematical knowledge but mostly an educational technique, that could apply to any subject matter area. This technique is produced in order to fulfill some constraints of the didactic process *i.e.* make sure the didactic experience remains coherent for the students, and develop inquiry procedures in the classroom. This argument seems to corroborate our analysis of the different "division of the activity" that this technique entails. In the "question" game, the teacher's role is not so easy: the students have to interpret the teacher's behaviors in the right way.<sup>8</sup>

### *About the "referee" technique*

The teacher's comments make us understand that the "referee technique", as it is enacted in the "Race to 20" lesson, is a frequently used technique (also used, for instance, in Physical Education) that the teacher applied to the mathematical pair-work designed in this situation. In other cases, the referee is said to be useful for the evaluation tasks of the knowledge, but the teacher admits that criteria for assessment are not easy to define. This is interesting because the teacher reveals himself that these technique may not fit with all the class activities.

### *A first interpretation*

In order to understand the teacher's action in this lesson, and particularly in the management of the two techniques that we showed, one has to consider the function of these techniques in the teaching process. T1 is concerned in creating an inquiry-based classroom, and possibly to delegate the assessment task to the students themselves, and this, not only in mathematics or in science, but in a general way, in all the classroom activities. In order to create such a self-directed learning, the teacher calls in some general techniques that can be *replicated* from one situation to another, which bring some coherence in the learning experiences. A didactic analysis make us conjecture that these techniques are not very efficient from a mathematical viewpoint. Nevertheless, the role of the teacher, in primary school, is not only to foster the mathematical thinking and sense-making of the students: it is also to educate them, to

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"strategic rules" implies for instance the discovery of such a "rule" as "17 wins, so the "Race to 20" equals to the Race to 17".

<sup>8</sup> It is interesting to notice that the teacher attempts to apply it to the "Race to 20", where apparently, there is no relevance for links to be found. The mathematical situation is dropped by the researcher in the teaching process of this class, without any peculiar connections with the subjects studied previously.

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give them “cognitive values” (Putnam, 1992) embedded, for instance, in self-questioning or inquiry process.

### THE OTHER TEACHER’S (T2) ANALYSIS OF THESE TECHNIQUES

#### *About the techniques*

When watching the videotape of T1’s lesson, T2 shows interest in the “questions-games” episode.

*1.T2 .. That’s the point I wanted to ask you there. You... you... ask the pupils to speak?*

*2.T1 I wanted to.*

*3.T2 So this means... yes.*

*4.T1 I wanted to start from the race to twenty since... showing people... so there it was a questioning to see which meaning they could give when there was a new discovery.*

*5.T2 Yes I found that it was interesting exactly because they were taking part, I would not have thought at the beginning at first, but I found that it was worth doing it.  
[...]*

*6.T2 When you were having this questioning there, in fact, you felt that it should create links, or was it because you intended simply to explicit the vocabulary?*

*7.T1 Er... no, it was in fact, looking for meaning. Starting from an idea, well, from a proposition, an expression, different meanings, to be able to rebound afterwards a little bit later on. Er... Now we can say that it was rather that way. But that might have been something else. [...]*

*10.T2 Yes but. Your asking a question. Er... On the meaning, and the children. Er... giving an interpretation linking it to something else, I find that, for me, it’s interesting.*

In his comment, T2 grasps T1’s intentions. Notably in ST 6 and 10, T2 stresses that “making links” is important... T2 seems to recognize some “valuable” features in these techniques, that may corroborate our hypothesis of a generic constraint about connections between tasks, that lies upon teachers.

#### *T2 synthetic commentary*

T2 is then asked to give some conclusive comments on T1’s practices :

*T2 About the session, in fact, I notice that we did not at all take the same beginning. There are things which I would never have thought about because I don’t practice them in my class... In fact it gives me ideas, you know, I will try some things. I really enjoy the part of the referee coming from outside, because I do it as well among the groups but it is always within the groups (...) And then there is one thing which I will keep in mind as well: the way you get in the activity with your insisting on the language, the meaning of words ; that I find maybe a way to start. That can be done. That I can do with other activities. But I would never have thought about it, there, for example, and I find it is quite right when starting an activity, to make a link or to avoid disconnections with previous sessions.*

In these comments, we can find arguments that expose very clearly the epistemological gap between the teachers and the researchers. Indeed, T2 appreciates the referee as an outsider. In contrast, researchers analysis show how this referee could be a *mathematical outsider*, who does not mathematically benefit from the situation.

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Similarly, T2 emphasizes, in a very direct way, two fundamental functions of the "questions" technique : to link different activities *a priori* separated ; to avoid the temporal break between the different lessons. There is no consideration for the mathematical content at stake<sup>9</sup>. Finally, the gap is obvious, between researchers who are primarily concerned with the specific mathematical meaning of the situations, and teachers who are primarily concerned with the coherence of the classroom activities and the educational relevance of *replicated forms* of teaching actions.

## TWO DIFFERENT SYSTEMS OF MEANING, TWO DIFFERENT EPISTEMOLOGIES

These two systems can be described as following. The "researchers system" is oriented towards the mathematical content, enacted in specific mathematical practices. The Race to 20 is a mathematical situation that includes several prominent features: the "alternative" (if I play 17, either my opponent plays 18, or he plays 19); "the backwards recurrence" (to play 20, I have to play 17, therefore the race to 20 is a race to 17; to play 17, I have to play 14, therefore the race to 20 is a race to 14... the race to 20 is a race to 2) ; the "methodological" triplet *proof-conjecture-refutation*.. For the researchers, the appropriate didactic contract (Brousseau, 1997) contains these objects, but the teaching practices of T1 and T2 do not include them. Following the distinction coined by Cobb & al (2001), we could say that the researchers put into focus the "mathematical practices" in the classroom. On the contrary, the "teachers system" is based on the generic relevance of some teaching techniques. The appropriate categories are "the development of the student's autonomy", and "the necessity, for the students, of assuming their learning responsibilities"... The generic teaching techniques bring the coherence in the didactic experience of the students. The priority lies, therefore, in fostering "social norms" (Cobb& al, *ibid*) in the classroom. As we find important to explore the ways in which the gap might be bridged, we chose, at least in a first study, to put on hold our theoretical stance<sup>10</sup>. In doing so, we took the opportunity to understand the practical logic of the action. Now, in order to organise the discussion of analyses, we shall come back to the levels of the model of the teacher's action, that we introduced at the beginning. We hypothesize an *overdetermination* of the third level, in which cognitive values and teaching practices are embedded, upon the two *infra* levels, at least at primary school. The generalist task of the primary school teacher may foster this overdetermination, compared with secondary school teachers who are usually in charge of a single subject. However, even within a same subject, we cannot *a priori* minimize this phenomenon. In the same way, some didactical techniques appropriated to teach an arithmetic knowledge may turn to be counterproductive to deal with some geometrical knowledge.

<sup>9</sup> In a more general way, there is very little care, during the teachers' dialogue in the cross-analysis, for the discussion of the mathematical knowledge.

<sup>10</sup> In doing so, we try to avoid the effects of the «scolastic fallacy» (Bourdieu, 1994, p. 112) when social scientists "think that agents involved in action, in practice, in life, think, know, and see... as the scientist whose mode of thought presupposes...distance and freedom from the urgency of the practice"

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If we want to understand the teacher's action, we have to document how the cognitive values, teacher's beliefs and practices can shape the didactical transactions. The techniques used by the teachers appear to be answers to educational constraints (e.g., "try to establish a link between a maximum of activities that, on the face of it, don't have anything to do with each other" or "the assessment tasks must not rely only on me, the teacher"). However, a technique that the teacher replicates in a general way in different activities, could be the answer to a *generic* constraint, from the didactical point of view. Indeed, the didactical theory (Brousseau 1997) shows that pieces of knowledge have intrinsic links between each other. The mathematical situations are designed to help the student in building bridges between different pieces of knowledge. However, the links between situations, or class activities are not obvious and often depends on an *a priori* knowledge organisation made by institutions (curricula, textbooks). Therefore the teacher has to cope with the situations, trying to replicate some teaching techniques, but not very specifically related to the mathematical knowledge, in order to reach an educational achievement. Meanwhile some *generic* didactical constraints exist about the knowledge organization and could be playing in the background. Therefore conflicts may emerge between the teaching techniques and the didactical goal of a mathematical situation, as we saw in T1's lesson. This explanation induces that the teacher may need some specifically knowledge-related techniques, to meet both didactical and educational achievements. Mastering the mathematical content knowledge is of course essential but not sufficient. For example, we can take for granted that T1 is familiar with the Euclidean division through the Race to 20 because he had a three hours mathematical training on this, provided by the research team. However he did not call in the specifically knowledge-related techniques in the classroom (e.g. the *alternative* technique : "if I play 17, the other can only play 18 or 19, and I reach 20" or the *recursive* technique : "17 wins, so the "Race to 20" equals to the Race to 17") that enable the students to identify eventually the Euclidean division in the game. A technical gap has to be overcome between the mathematical content knowledge and knowledge-related techniques.

To conclude, we think that the researchers have to understand the very nature of the teacher's action. That means to identify the different constraints that the teachers have to cope with, in particular the necessity of *educational coherence* for the teachers which that can be explained by some *generic overdetermination, from the didactical point of view*, that we conjecture in this paper. Against such a background, a collaborative research could allow the researchers to acknowledge the multi-determination of the practical logic, and the teachers to analyze the mathematical content in a more efficient way, the collaborative research attempting to answer the following questions: what could be the *specific teaching technique* that the teacher has to produce to foster the mathematical thinking and sense-making of the students? What could be the effective conditions of their productions?

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