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We study the Hall response of two-dimensional lattice systems of charged fermions in a transverse magnetic field, in the ballistic coherent limit. We identify a setup in which this response vanishes over wide regions of parameter space: the "Landauer-Büttiker" setup commonly studied for coherent quantum transport, consisting of a strip contacted to biased ideal reservoirs of charges. We show that this effect does not rely on particle-hole symmetry, and is robust to a variety of perturbations including variations of the transverse magnetic field, chemical potential, and temperature. We trace this robustness back to a topological property of the Fermi surface: the number of Fermi points with positive velocity of the system. We argue that the mechanism leading to a vanishing Hall response applies to noninteracting and interacting systems alike, which we verify in concrete examples using density-matrix renormalization group simulations.

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Transport properties induced by electromagnetic fields are an active area of study in condensed matter physics. The Hall response, $\sigma_{\rm H}$, is of particular interest: It represents the off-diagonal response of a current density $\mathbf{J}$ to an electric field $\mathbf{E}$, $\sigma_{\rm H} = \varepsilon_{ij}\sigma_{ij}$, where $J_i = \sigma_{ij}E_j$ and ε_{ij} is the Levi-Civita symbol. The Hall response probes important geometric or topological properties of quantum systems: the Fermi-surface curvature of metals under weak magnetic fields [1–3], the Berry curvature of anomalous Hall systems [4], and related topological invariants of band insulators [5,6]. Studies of $\sigma_{\rm H}$ are ubiquitous in fields focused on topological quantum matter [7] and synthetic realizations thereof [8,9].

Scattering is essential in conventional studies of $\sigma_{\rm H}$: in the two-dimensional (2D) Hall effect [10], e.g., Boltzmann-type approaches [11] that reproduce the observed Hall constant, $R_{\rm H}$, for weak magnetic fields B: $R_{\rm H} \equiv -\sigma_{\rm H}/(\sigma_{xx}\sigma_{yy}B) \sim -1/(ne)$, where n is the density of carriers with charge e, and $x(y)$ denote the longitudinal (transverse) direction. Scattering also explains the plateaus of quantized Hall conductance ($\sigma_{\rm H} = \nu e^2/h$ for filling factor ν) appearing in strong-field regimes [12–14].

As ballistic systems become more accessible experimentally [8,15,16], new challenges are emerging for theory beyond Boltzmann-type approaches, despite past efforts in mesoscopic systems [17,18]. For example, σ_{ii} can be infinite in clean interacting systems, even at a finite temperature [19,20]. The connection between Hall response and carrier density is not even clear in the presence of interactions [21–24]. Recent progress was made with the calculation of $R_{\rm H}$ in dissipative metallic systems [25], where σ_{ii} is finite at zero frequency [26]. Nevertheless, the Hall response of coherent ballistic systems remains largely unexplored.

In this Letter, we identify a ballistic setup in which charged fermions in a transverse magnetic field can exhibit a strictly vanishing Hall response. We demonstrate this effect in noninteracting 2D lattice systems at zero temperature, and extend our results to interacting analogs using a density-matrix renormalization group (DMRG). We show that the Hall response vanishes under a wide variety of perturbations: variations in magnetic field, chemical potential, temperature, and particle-hole symmetry breaking. We relate this remarkable robustness to the topological nature of the key property underpinning the effect: the number of Fermi points with positive velocity of the system.

Hall response of ballistic systems.— We consider lattice systems in a 2D strip geometry (Fig. 1), with edges in the y direction. Edges imply that the transverse current J_y vanishes in the low-frequency limit $\omega \to 0$ of the longitudinal electric field E_x. The Hall response is then described by the transverse polarization difference $\Delta P_y(x,t) = \int_{t_0}^t dt' J_y(x,t')$. We set $P_y(x,t_0) = 0$ at time t_0 right before E_x is applied (corresponding to a gauge choice [29–31]), and denote $\Delta P_y(x,t) \equiv P_y(x,t)$.

The relation between P_y and $\sigma_{\rm H}$ can be derived using linear response theory [32]: writing $E_x = -\partial_t A_x$ (with $e = \hbar = c = 1$), one finds

$$P_y(k,\omega) = -\sigma_{\rm H}(k,\omega)A_x(k,\omega), \qquad (1)$$

where k is the crystal momentum along x. This can be seen as a Kubo formula for the polarization induced by a time-dependent vector potential, $P_y(x,t) = i\sum_{x'}\int dt'\theta(t-t')\langle[P_y(x,t), J_x(x',t')]\rangle A_x(x',t')$ [see the Supplemental Material for details [33]].

Equation (1) allows for *very different* Hall responses $\sigma_{\rm H}$, depending on the nature of E_x, for the *same*

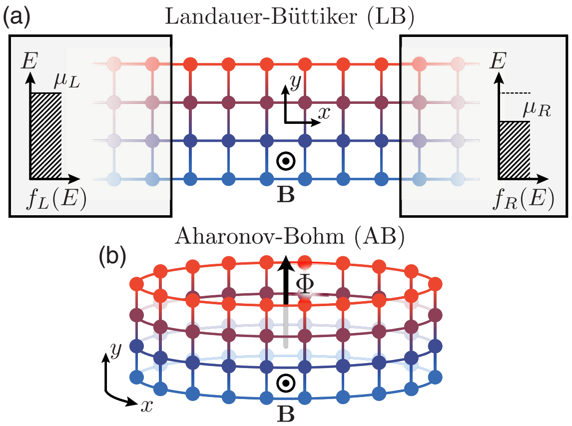

FIG. 1. (a) Landauer-Büttiker setup enabling a vanishing Hall response: a ballistic lattice system is connected to ideal reservoirs (in gray) with weakly biased chemical potentials $\mu_R < \mu_L$ [corresponding to Fermi-Dirac distributions $f_{L/R}(E)$]. (b) Ballistic Aharonov-Bohm setup where the Hall response is, in contrast, generically finite (with persistent current along x induced by a magnetic flux Φ).

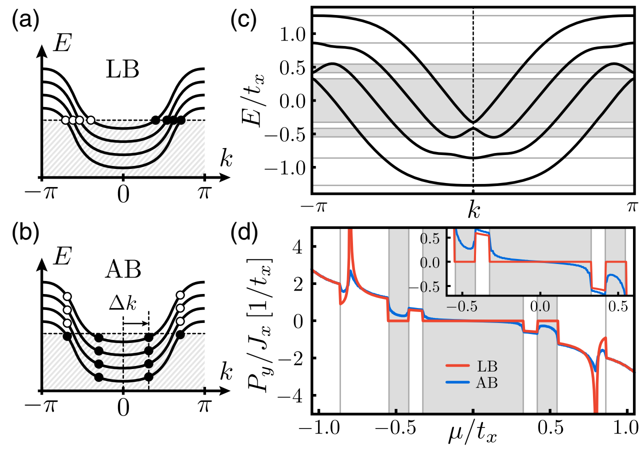

FIG. 2. (a), (b) Schematic band structures showing the key single-particle states for the Hall response: in the LB setup, the current $J_x \neq 0$ is induced by occupied states (full dots) with velocity $v_k > 0$ in a small energy window $[\mu_R, \mu_L]$ around the chemical potential μ (horizontal dashed line). Left-moving states at μ are empty. In the AB setup, instead, $J_x \neq 0$ is induced by the spectral flow $\Delta k = \Phi/N_x$ of *all* states with threaded magnetic flux Φ. (c) Band structure of the HH model computed for $N_y = 4$, $B = 0.7$, and $t_y = 0.5t_x$. Horizontal lines (dark gray) indicate energies at which the number $c(\mu)$ of Fermi points with $v_k > 0$ changes, with $c(\mu) = N_y$ in shaded (light gray) regions. See the Supplemental Material [33] for other parameter regimes including $N_y \to \infty$. (d) Hall response of the system in (c) in LB vs AB setups: when $c(\mu) = N_y$, P_y/J_x strictly vanishes in the LB setup, while it only goes to zero at particle-hole symmetry ($\mu = 0$), in the AB setup (see inset enlargement).

longitudinal current J_x. Here we consider a paradigmatic setup for coherent quantum transport: a system with two ends in the x direction, where E_x (or J_x) is generated by ideal contacts to two external reservoir of charges (left and right) with chemical potentials μ_L and μ_R [Fig. 1(a)]. In this "Landauer-Büttiker" (LB) setup [34], J_x is related to the potential difference $eV \equiv \mu_\mathrm{L} - \mu_\mathrm{R}$ via the conductance G of the system: $J_x = GV$. Without interactions, the polarization $P_y \equiv P_y^\mathrm{LB}$ in this setup can be obtained from conventional scattering theory [35], with conductance G derived from the Landauer formula. Kubo's formalism [Eq. (1)] provides an instructive equivalent approach [36]. As we detail in the Supplemental Material [33], the LB setup can be described by $A_x(x,t) = -Ve^{-i\omega t}\delta(x)$, i.e., by a potential drop of amplitude V at the position $x = 0$ of contact between the system and the left reservoir. Since $A_x(x,t)$ is local, the stationary P_y^LB takes the form of an *integral* of the Hall response *over all momenta* [33]:

$$\frac{P_y^\mathrm{LB}}{J_x} = -G^{-1}\lim_{\omega\to 0}\frac{1}{2\pi}\int dk e^{ikx}\frac{\sigma_\mathrm{H}(k,\omega)}{\omega + i0^+}, \tag{2}$$

where $i0^+$ is a small positive imaginary part.

To illustrate the strong differences that can arise in Hall response between ballistic coherent systems, we consider an additional "Aharonov-Bohm" (AB) setup: a contactless ring where J_x is induced by a time-dependent magnetic flux [Fig. 1(b)]. In that case, $A_x(x,t)$ corresponds to the vector potential describing the inserted flux, i.e., $A_x(x,t) = e^{i\omega t}\Phi/N_x$, where N_x is the number of sites along x. The flux induces a persistent current [37,38] $J_x = D\Phi/N_x$, where D is the Drude weight [39], generating a *reactive* Hall response [40,41] [Fig. 2(b)]. In contrast to Eq. (2), and in agreement with known results [42], the stationary $P_y \equiv P_y^\mathrm{AB}$ found here depends on the *zero-momentum* component of σ_H [33]:

$$\frac{P_y^\mathrm{AB}}{J_x} = -D^{-1}\lim_{\omega\to 0}\sigma_\mathrm{H}(0,\omega). \tag{3}$$

Hall response in the LB setup.—We now detail the LB setup and derive an explicit formula for P_y^LB at zero temperature, in the low-bias limit $\mu_\mathrm{L} \to \mu_\mathrm{R} \equiv \mu$. Our results apply to a broad variety of lattice models. For clarity, however, we take the viewpoint of the Harper-Hofstadter (HH) model [43]. Specifically, we consider fermions on a square lattice with Hamiltonian $H_\mathrm{HH} = -\sum_{x,y}[t_x e^{iBy} c_{x,y}^\dagger c_{x+1,y} + t_y c_{x,y}^\dagger c_{x,y+1}]/2 + \mathrm{H.c.}$, in the Landau gauge, where $c_{x,y}^\dagger$ creates a fermion on site (x,y), B is the magnetic flux per plaquette, and $t_x(t_y)$ is the nearest-neighbor hopping amplitude in the $x(y)$ direction [Fig. 1(a)]. In this minimal model, the system can be seen as N_y coupled longitudinal wires: its spectrum ε_k can be regarded as N_y bands $t_x\cos[k - yB/(N_y - 1)]$, shifted by $yB/(N_y - 1)$ in momentum (with $y = 0, 1, \ldots, N_y - 1$), and hybridized by t_y.

As we demonstrate below, the response P_y^LB vanishes identically when the system's spectrum is symmetric under

$k \to -k$, and the number $c(\mu)$ of Fermi points with velocity $v_k \equiv \partial\varepsilon_k/\partial k > 0$ is equal to N_y. Remarkably, these conditions are satisfied in wide regions of parameter space, for weak and strong B. We distinguish two main scenarios, in particular: (i) the "weak-field" regime ($B \lesssim 1/N_y$), where all bands $t_x \cos[k - yB/(N_y - 1)]$ hybridize in the first Brillouin zone, and (ii) the "strong-field" regime, where bands hybridize after backfolding into the first Brillouin zone. We focus on (i) in what follows, and extend our discussion to (ii) in the Supplemental Material [33].

In the HH model, symmetry under $k \to -k$ arises from a combination of time reversal (TR) and spatial inversion in the y direction. This effective TR symmetry is described by the operator $\Theta = I_y\mathcal{K}$, where I_y permutes positions y around the center of the system, and $\mathcal{K}$ describes complex conjugation. As $[H_{\mathrm{HH}}, \Theta] = 0$, the action of Θ on an eigenstate $|\psi_k(E)\rangle$ of H_{HH} with momentum k and energy E yields a (non-necessarily distinct [44]) eigenstate $\Theta|\psi_{k'}(E)\rangle$ with $k' = -k$ and identical energy. As in Eq. (2) [33], the Hall response can be derived using scattering theory: in the low-bias, zero-temperature limit, the conductance reads $G = G_0 \sum_j T_j$, where $G_0 = e^2/h = 1/(2\pi)$ is the conductance quantum, and T_j is the transmission probability, at the chemical-potential energy μ, of scattering modes $\psi_j(x, y)$ incoming from the left reservoir. We consider infinite reservoirs described by H_{HH} (with chemical potentials μ_L and μ_R, respectively), so that scattering modes have a similar form as the system's eigenmodes. In that case, $T_j = 1$ for all modes available at μ. Relevant modes have an asymptotic form $\psi_j(x \to -\infty, y) = e^{ik_{F,j}x} w_j(y)/v_{F,j}$, where $k_{F,j}$ ($v_{F,j}$) are Fermi momenta (velocities), and $w_j(y)$ transverse wave functions. The conductance reduces to $G = c(\mu)G_0$, where $c(\mu)$ is the number of Fermi points with positive velocity $v_k = \partial\varepsilon_k/\partial k > 0$, as in Fig. 2(a). Equation (2) becomes

$$\frac{P_y^{\mathrm{LB}}(\mu)}{J_x} = \frac{1}{c(\mu)G_0} \sum_{j=1}^{c(\mu)} \sum_y y \frac{w_j(y)^2}{v_{F,j}}, \tag{4}$$

as derived in the Supplemental Material [33] (with more explicit expressions for finite B and $N_y = 2$, or $B \to 0$ and arbitrary $N_y \geq 2$).

We used the simulation package KWANT [46] to verify our formulas, compute P_y^{LB} for arbitrary B and N_y, and compare P_y^{LB} to P_y^{AB}. Our results are illustrated in Fig. 2(d) for the weak-field regime, and in the Supplemental Material [33] for the strong-field regime (including a discussion of the limit $N_y \to \infty$). They demonstrate two key points: first and foremost, P_y^{LB} *vanishes identically* whenever $c(\mu) = N_y$, irrespective of the specific value of B or μ [see, e.g., the region around $\mu/t_x = \pm 0.5$ in Fig. 2(d)], and of particle-hole symmetry (generically absent here). Second, the responses P_y^{LB} to P_y^{AB} are strikingly different, as hinted by Eqs. (2) and (3). Intuitively, this arises from the fact that LB and AB stationary states are different [Figs. 2(a) and 2(b)], with distinct polarizations P_y, though they carry the same current J_x. Specifically, each conduction channel j gives a contribution $P_{y,j} = \mathcal{C}_j J_{x,j}$ to P_y, proportional to the current $J_{x,j}$ carried by the channel [33]. The factor $\mathcal{C}_j$ does not depend on the origin of $J_{x,j}$. Crucially, however, every channel carries a *different* current $J_{x,j} = v_{F,j}\Phi/(\pi N_x)$ in the AB setup, whereas all channels carry the same one in the LB case. Accordingly, the two responses coincide when $c(\mu) = 1$, and generically differ otherwise [33]. They also share the same sign, set by the particle (+) or hole (−) nature of charge carriers, leading to their vanishing at particle-hole symmetry [$\mu = 0$ in Fig. 2(d)]. Moreover, both responses are discontinuous at transitions between distinct $c(\mu)$.

Topological origin of the vanishing Hall response.— We now demonstrate that $P_y^{\mathrm{LB}} = 0$ due to (i) the topological nature of $c(\mu)$, and (ii) the traceless nature of the operator $\hat{P}_y$ describing the polarization. The number $c(\mu)$ of Fermi points with $v_k > 0$ is topological in the sense that it corresponds to the *central charge* of the system (the number of gapless modes with $v_k > 0$, in a Luttinger-liquid interpretation). The polarization operator is $\hat{P}_y = eY$, where $Y \equiv \sum_{x,y} y c_{x,y}^\dagger c_{x,y}$ describes the "center-of-mass" position along y. To ensure that $\langle\psi_i^{\mathrm{LB}}(\mu)|\hat{P}_y|\psi_i^{\mathrm{LB}}(\mu)\rangle = 0$ in the initial state $|\psi_i^{\mathrm{LB}}(\mu)\rangle \equiv |\psi_i\rangle$ with zero bias ($V, J_x = 0$), corresponding to our gauge choice for the polarization, we set $y = 0$ at the center of the system. The operator $\hat{P}_y$ then satisfies $I_y^T \hat{P}_y I_y = -\hat{P}_y$. It is traceless, and $P_y^{\mathrm{LB}} = 0$ at zero bias is ensured by the symmetry between k and $-k$: indeed, $|\psi_i\rangle$ is the many-body ground state of H_{HH} with single-particle states occupied symmetrically around $k = 0$, up to the chemical potential μ. It is symmetric under Θ (i.e., $\Theta|\psi_i\rangle = \pm|\psi_i\rangle$), such that $\langle\psi_i|\hat{P}_y|\psi_i\rangle = \langle\psi_i|\Theta^\dagger\hat{P}_y\Theta|\psi_i\rangle = \langle\psi_i|I_y^T\hat{P}_y I_y|\psi_i\rangle = -\langle\psi_i|\hat{P}_y|\psi_i\rangle$.

When applying a finite bias $V \neq 0$ to generate a stationary current J_x in the "final" state $|\psi_f^{\mathrm{LB}}(\mu)\rangle \equiv |\psi_f\rangle$, the symmetry Θ breaks: the state $|\psi_f\rangle$ is a many-body stationary state with single-particle states occupied symmetrically around $k = 0$ *except* at μ where single-particle states are occupied where $v_k > 0$ only. By symmetry, noncanceling contributions to the polarization can only come from these $c(\mu)$ Fermi points. We index the latter as $j = 1, 2, \ldots, c(\mu)$, and denote by $|j\rangle$ the corresponding single-particle states ($|j\rangle \equiv |k_{F,j}, s_j\rangle$, here, where $k_{F,j}$ and s_j are the Fermi momentum and band index of the Fermi point j). In this picture, the polarization becomes

$$P_y^{\mathrm{LB}}(\mu) = \sum_{j=1}^{c(\mu)} n_j \langle j|\hat{P}_y|j\rangle, \tag{5}$$

where $n_j = 1$ is the occupation of $|j\rangle$.

We can now show that $P_y^{\rm LB}(\mu)$ vanishes when $c(\mu) = N_y$: the states $|j\rangle$ in Eq. (5) belong to the eigenspace of $H_{\rm HH}$ with energy μ, and are characterized by distinct momenta. Since they are not related by symmetry [47], they form a basis for a Hilbert (sub)space of dimension $c(\mu)$. Therefore, when $c(\mu) = N_y$, Eq. (5) becomes

$$P_y^{\rm LB}(\mu)|_{c(\mu)=N_y} = \sum_{j=1}^{N_y} \langle j|\hat{P}_y|j\rangle = {\rm Tr}\hat{P}_y = 0. \qquad (6)$$

This demonstrates our main result: the existence of a conservation law for the Hall response of the LB setup. Note that other, potentially observable conservation laws can be derived from the tracelessness of $\hat{P}_y$: in particular, replacing the set $\{|j\rangle\}$ by a basis of Bloch eigenstates $\{|k,s\rangle\}$ (with momentum k and band index $s = 1, 2, \ldots, N_y$), one finds $P_y(k) \equiv \sum_{s=1}^{N_y} \langle k,s|\hat{P}_y|k,s\rangle = 0$, meaning that the Hall response of a system with N_y bands vanishes in momentum sectors k where bands are equally occupied. This conservation law corresponds to the known zero-sum rule for the Berry curvature of all eigenstates of a Hamiltonian [48]. Equation (6) can be seen as an analog with fixed energy, instead of fixed k.

Robustness to perturbations.— The vanishing of $P_y^{\rm LB}$ at $c(\mu) = N_y$ is protected against temperature by an energy gap $\Delta\mu$ corresponding to the smallest chemical-potential variation required to change $c(\mu)$. More precisely, the Hall response is suppressed as $e^{-\beta|\Delta\mu|}$ at finite temperature $T = 1/\beta$ [33]. We emphasize that the gap $\Delta\mu$ need not close with increasing N_y. In fact, in the above HH model, the gap around $\mu = 0$ is $\Delta\mu \approx (t_x - t_y) - |\mu|$ approximately independent of N_y when $t_y \lesssim t_x$ [33].

Deviations from a strictly vanishing Hall response are expected in the presence of generic disorder, as this breaks the symmetry Θ connecting momentum sectors k and $-k$. Disorder in quasi-1D systems generally leads to Anderson localization [49]. Nevertheless, if the scattering region connecting the reservoirs is shorter than the localization length (scaling as $N_y t_x^2/W^2$ with disorder strength W [50]), disorder remains a weak perturbation. In that case, deviations of the disorder-averaged polarization $\langle P_y\rangle$ from zero scale as W^2/t_x^2, with large fluctuations around the average (as do conductance fluctuations in disordered systems [51]); see the Supplemental Material for details [33].

Generalization to interacting systems.—Equation (6) applies whenever the current J_x is carried by $c = N_y$ independent, equally occupied fermionic channels, regardless of interactions. To demonstrate this, we consider the HH model on a two-leg ladder ($N_y = 2$), with additional intra- and interleg interactions described by Hamiltonian terms $U_\parallel \sum_{x,y=\pm1} n_{x,y} n_{x+1,y} + U_\perp \sum_x n_{x,1} n_{x,-1}$, where $n_{x,y}$ is the density on site (x, y). To simulate transport in the LB setup, we evolve the system with reservoirs described by a quenched steplike potential $-\epsilon \sum_{x<L_{\rm res},y} n_{x,y} + \epsilon \sum_{x>L_{\rm sys}+L_{\rm res},y} n_{x,y}$, where $L_{\rm sys/res}$ denotes the length of the system and reservoirs. We set $\epsilon = 0$ and prepare the full system in its ground state using DMRG [52,53]. We then switch $\epsilon > 0$, at time $\tau = 0$, and evolve the system using time-dependent DMRG (TDMRG) [53] and the ITensor library [54]. We set $L_{\rm sys} = 2$, for simplicity [55], and compute the Hall response $P_y^{\rm LB}/J_x$ in the middle of the system at times $1 \lesssim \tau/t_x \lesssim L_{\rm res}$ [56], averaging over a time window where J_x is approximately stationary. Figure 3 shows typical results for $U_\parallel = U_\perp = t_x/2$. For comparison, we simulate transport in the AB setup by quenching, instead, a small linear potential $-(\epsilon/N_x)\sum_{x,y} x n_{x,y}$. While J_x increases linearly in time in that case [Fig. 3(b)], the ratio $P_y^{\rm AB}/J_x$ oscillates around a constant value corresponding to the stationary Hall response [41].

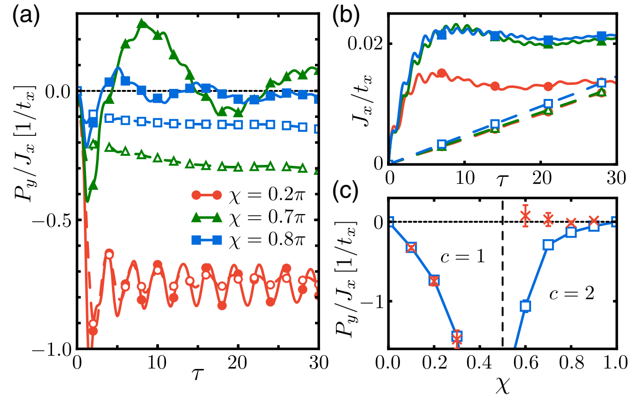

FIG. 3. Numerical TDMRG estimates of the LB and AB Hall responses of the interacting HH model, with $t_x = t_y = 1$, $U_\parallel = U_\perp = 1/2$, and $\epsilon = 0.01$, for 10 fermions in a system of length $L_x = 60$. (a) Evolution of $P_y^{\rm LB}/J_x$ (filled symbols) and $P_y^{\rm AB}/J_x$ (empty symbols) for a magnetic flux $\chi \equiv B/N_x = 0.2\pi$ (Luttinger-liquid phase with $c = 1$), and 0.7π and 0.8π ($c = 2$). Lines interpolate more data points than shown. (b) Time evolution of J_x for parameters as in (a). (c) Average of $P_y^{\rm LB}/J_x$ (×) and $P_y^{\rm AB}/J_x$ (□) over times $10 < \tau < 30$. The dashed line indicates the estimated transition between $c = 1$ and $c = 2$. Averages coincide for $\chi = 0.4\pi$, while no stationary regime was reached for 0.5π.

The results shown in Fig. 3 are consistent with our theoretical analysis: LB and AB Hall responses are identical (with time averaging, within errorbars) when the initial ground state is characterized by a central charge $c = 1$ [33,57]. More importantly, they strongly differ when $c = 2 = N_y$ [Fig. 3(c)], with large oscillations of $P_y^{\rm LB}/J_x$ around an average consistent with $P_y^{\rm LB}/J_x = 0$, and a finite $P_y^{\rm AB}/J_x$. Our results (including additional data presented in the Supplemental Material [33]) fully support our theoretical result that $P_y^{\rm LB}/J_x$ vanishes when $c = 2 = N_y$.

Discussion.— The conservation law found in this work exemplifies the rich and sometimes counterintuitive phenomena that can occur in ballistic coherent systems. Solid-state and synthetic-matter experiments would be well suited to observe it [8,15,16]. In fact, a platform for realizing the LB setup has recently been proposed [58]. We emphasize that our results extend to bosons: a vanishing transverse polarization would be observed in photonic systems [59], e.g., by selectively populating the $c = N_y$ states in Eq. (6) [60].

Our results provide additional clues to better understand the Hall response of strongly correlated (non-Fermi-liquid) systems, for which low-energy quasiparticle descriptions of quantum transport inexorably fail. Presently, they raise intriguing questions regarding the behavior of the transverse polarization P_y of interacting systems at finite temperatures: Although a transition to dissipative or metallic regimes is expected, explicit calculations of P_y remain challenging [25]. Recent studies have shown the persistence of ballistic and superdiffusive behavior in specific cases [19,20]. It will be interesting to investigate analogs in quasi-1D lattice systems.

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