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FALSE DISCOVERIES IN MUTUAL FUND PERFORMANCE : MEASURING LUCK IN ESTIMATED ALPHAS

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False Discoveries in Mutual Fund Performance: Measuring Luck in Estimated Alphas

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ABSTRACT

This paper develops a simple technique that controls for “false discoveries,” or mutual funds that exhibit significant alphas by luck alone. Our approach precisely separates funds into (1) unskilled, (2) zero-alpha, and (3) skilled funds, even with dependencies in cross-fund estimated alphas. We find that 75% of funds exhibit a zero alpha (net of expenses), consistent with the Berk and Green (2004) equilibrium. Further, we find a significant proportion of skilled (positive alpha) funds prior to 1996, but almost none by 2006. We also show that controlling for false discoveries substantially improves the ability to find funds with persistent performance.

Investors and academic researchers have long searched for outperforming mutual fund managers. Although several researchers document negative average fund alphas, net of expenses and trading costs (e.g., Jensen (1968), Elton et al. (1993), and Carhart (1997)), recent papers indicate that some fund managers have stock-selection skills. For instance, Kosowski, Timmermann, Wermers, and White (2006; KTW) use a bootstrap technique to document outperformance by some funds, while Baks, Metrick, and Wachter (2001), Pastor and Stambaugh (2002b), and Avramov and Wermers (2006) illustrate the benefits of investing in actively-managed funds from a Bayesian perspective. While these papers are useful in uncovering whether, on the margin, outperforming mutual funds exist, they are not particularly informative regarding their prevalence in the entire fund population. For instance, it is natural to wonder how many fund managers possess true stockpicking skills, and where these funds are located in the cross-sectional estimated alpha distribution. From an investment perspective, precisely locating skilled funds maximizes our chances of achieving persistent outperformance.¹

Of course, we cannot observe the *true* alpha of each fund in the population. Therefore, a seemingly reasonable way to estimate the prevalence of skilled fund managers is to simply count the number of funds with sufficiently high estimated alphas, $\hat{\alpha}$. In implementing such a procedure, we are actually conducting a multiple-hypothesis test, since we simultaneously examine the performance of all funds in the population (instead of just one fund).² However, it is clear that this simple count of significant-alpha funds does not properly adjust for luck in such a multiple test setting—many of the funds will have significant estimated alphas by luck alone (i.e., their true alphas are zero). To illustrate, consider a population of funds with skills just sufficient to cover trading costs and expenses (truly zero-alpha funds). With the usual chosen significance level of 5%, we should expect that 5% of these zero-alpha funds will have significant estimated alphas—some of them will be unlucky (significant with $\hat{\alpha} < 0$) while others are lucky (significant with $\hat{\alpha} > 0$), but all will be “false discoveries”—funds with significant *estimated* alphas, but zero *true* alphas.

This paper implements a new approach to controlling for false discoveries in such a multiple fund setting. Our approach much more precisely estimates (1) the proportions of unskilled and skilled funds in the population (those with *truly* negative and positive alphas, respectively), and (2) their respective locations in the left and right tails of the cross-sectional *estimated* alpha (or estimated alpha *t*-statistic) distribution. One main virtue of our approach is its simplicity—to determine the frequency of false discoveries, the only parameter needed is the proportion of zero-alpha funds in the population, π_0 .

Rather than arbitrarily imposing a prior assumption on π_0 , our approach estimates it with a straightforward computation that uses the p -values of individual fund estimated alphas—no further econometric tests are necessary. A second advantage of our approach is its accuracy. Using a simple Monte-Carlo experiment, we demonstrate that our approach provides a much more accurate partition of the universe of mutual funds into zero-alpha, unskilled, and skilled funds than previous approaches that impose an a priori assumption about the proportion of zero-alpha funds in the population.

Another important advantage of our approach to multiple testing is its robustness to cross-sectional dependencies among fund estimated alphas. Prior literature has indicated that such dependencies, which exist due to herding and other correlated trading behaviors (e.g., Wermers (1999)), greatly complicate performance measurement in a group setting. With our approach, the computation of the proportions of unskilled and skilled funds only requires the (alpha) p -value for each fund in the population, and not the estimation of the cross-fund covariance matrix. Indeed, the large cross-section of funds in our database makes these estimated proportions very accurate estimators of the true values, even when funds are cross-sectionally correlated. We confirm, with Monte Carlo simulations, that our simple approach is quite robust to cross-fund dependencies.

We apply our novel approach to the monthly returns of 2,076 actively managed U.S. open-end, domestic-equity mutual funds that exist at any time between 1975 and 2006 (inclusive), and revisit several important themes examined in the previous literature. We start with an examination of the long-term (lifetime) performance of these funds, net of trading costs and expenses. Our decomposition of the population reveals that 75.4% are zero-alpha funds—funds having managers with some stockpicking abilities, but that extract all of the rents generated by these abilities through fees. Further, 24.0% of the funds are unskilled (true $\alpha < 0$), while only 0.6% are skilled (true $\alpha > 0$)—the latter being statistically indistinguishable from zero. While our empirical finding that the majority are zero-alpha funds is supportive of the long-run equilibrium theory of Berk and Green (2004; BG), it is surprising that we find so many truly negative-alpha funds—those that overcharge relative to the skills of their managers. Indeed, we find that such unskilled funds underperform for long time periods, indicating that investors have had some time to evaluate and identify them as underperformers. Across the investment subgroups, Aggressive Growth funds have the highest proportion of skilled managers, while no Growth & Income funds exhibit skills.

We also uncover some notable time trends in our study. Specifically, we observe that the proportion of skilled funds decreases from 14.4% in 1990 to 0.6% in 2006, while the proportion of unskilled funds increases from 9.2% to 24.0%. Thus, although

the number of actively managed funds dramatically increases over this period, skilled managers (those capable of picking stocks well enough, over the long-run, to overcome their trading costs and expenses) have become exceptionally rare.

Motivated by the possibility that funds may outperform over the short-run, before investors compete away their performance with inflows (as modeled by BG), we conduct further tests over five-year subintervals—treating each five-year fund record as a separate “fund.” Here, we find that the proportion of skilled funds equals 2.4%, implying that a small number of managers have “hot hands” over short time periods. These skilled funds are concentrated in the extreme right tail of the cross-sectional estimated alpha distribution, which indicates that a *very* low p -value is an accurate signal of short-run fund manager skill (relative to pure luck). Further analysis indicates that larger and older funds consist of far more unskilled funds than smaller and newer funds, and that high inflow funds exhibit the highest proportion of skilled funds (18%) during the five years ending with the flow year, but the largest reduction in skilled funds during the five years subsequent to the flow year (from 18% to 2.4%). Conversely, funds in the lowest flow quintile exhibit high proportions of unskilled funds prior to the measured flows, but lower proportions afterwards (perhaps due to a change in strategy or portfolio manager in response to the outflows; Lynch and Musto (2003)). These results are generally consistent with the predictions of the BG model.

The concentration of skilled funds in the extreme right tail of the estimated alpha distribution suggests a natural way to choose funds in seeking out-of-sample persistent performance. Specifically, we form portfolios of right-tail funds that condition on the frequency of “false discoveries”—during years when our tests indicate higher proportions of lucky, zero-alpha funds in the right tail, we move further to the extreme tail to decrease such false discoveries. Forming this “false discovery” controlled portfolio at the beginning of each year from January 1980 to 2006, we find a four-factor alpha of 1.45% per year, which is statistically significant. Notably, we show that this luck-controlled strategy outperforms prior persistence strategies used by Carhart (1997) and others, where constant top-decile portfolios of funds are chosen with no control for luck.

Our final tests examine the performance of fund managers before expenses (but after trading costs) are subtracted. Specifically, while fund managers may be able to pick stocks well enough to cover their trading costs, they usually do not exert direct control over the level of fund expenses and fees—management companies set these expenses, with the approval of fund directors. We find, on a pre-expense basis, a much higher incidence of funds with positive alphas—9.6%, compared to our above-mentioned finding of 0.6% after expenses. Thus, almost all outperforming funds appear to capture (or

waste through operational inefficiencies) the entire surplus created by their portfolio managers. It is noteworthy that the proportion of skilled managers (before expenses) declines substantially over time, again indicating that portfolio managers with skills have become increasingly rare. We also observe a large reduction in the proportion of unskilled funds when we move from net alphas to pre-expense alphas (from 24.0% to 4.5%), indicating a big role for excessive fees (relative to manager stockpicking skills in excess of trading costs) in underperforming funds. Although industry sources argue that competition among funds has reduced fees and expenses substantially since 1980 (Rea and Reid (1998)), our study indicates that a large subgroup of investors appear to either be unaware that they are being overcharged (Christoffersen and Musto (2002)), or are constrained to invest in high-expense funds (Elton, Gruber, and Blake (2007)).

The remainder of the paper is as follows. The next section explains our approach to separating luck from skill in measuring the performance of asset managers. Section II presents the performance measures, and describes the mutual fund data. Section III contains the results of the paper, while Section IV concludes.

I The Impact of Luck on Mutual Fund Performance

A Overview of the Approach

A.1 Luck in a Multiple Fund Setting

Our objective is to develop a framework to precisely estimate the fraction of mutual funds in a large group that truly outperform their benchmarks. To begin, suppose that a population of M actively managed mutual funds is composed of three distinct performance categories, where performance is due to stock-selection skills. We define such performance as the ability of fund managers to generate superior model alphas, net of trading costs as well as all fees and other expenses (except loads and taxes). Our performance categories are defined as follows:

- **Unskilled funds:** funds having managers with stockpicking skills insufficient to recover their trading costs and expenses, creating an “alpha shortfall” ($\alpha < 0$),
- **Zero-alpha funds:** funds having managers with stockpicking skills sufficient to just recover trading costs and expenses ($\alpha = 0$),
- **Skilled funds:** funds having managers with stockpicking skills sufficient to provide an “alpha surplus,” beyond simply recovering trading costs and expenses ($\alpha > 0$).

Note that our above definition of skill is one that is relative to expenses, and not in an absolute sense. This definition is driven by the idea that consumers search for

actively managed mutual funds that deliver surplus alpha, net of all expenses.³

Of course, we cannot observe the true alphas of each fund in the population. Therefore, how do we best infer the prevalence of each of the above skill groups from performance estimates for individual funds? First, we use the t -statistic, $\hat{t}_i = \hat{\alpha}_i / \hat{\sigma}_{\hat{\alpha}_i}$, as our performance measure, where $\hat{\alpha}_i$ is the estimated alpha for fund i , and $\hat{\sigma}_{\hat{\alpha}_i}$ is its estimated standard deviation—KTWW show that the t -statistic has superior statistical properties relative to alpha, since alpha estimates have differing precision across funds with varying lives and portfolio volatilities. Second, after choosing a significance level, γ (e.g., 10%), we observe whether \hat{t}_i lies outside the thresholds implied by γ (denoted by t_γ^- and t_γ^+), and label it “significant” if it is such an outlier. This procedure, simultaneously applied across all funds, is a multiple-hypothesis test (for several null hypotheses, $H_{0,i}$, and alternative hypotheses, $H_{A,i}$, $i = 1, \dots, M$):

$$\begin{aligned} H_{0,1} & : \alpha_1 = 0, & H_{A,1} & : \alpha_1 \neq 0, \\ \dots & : \dots \\ H_{0,M} & : \alpha_M = 0, & H_{A,M} & : \alpha_M \neq 0. \end{aligned} \tag{1}$$

To illustrate the difficulty of controlling for luck in this multiple test setting, Figure 1 presents a simplified hypothetical example that borrows from our empirical findings (to be presented later) over the last five years of our sample period. In Panel A, individual funds within the three skill groups—unskilled, zero alpha, and skilled—are assumed to have true annual four-factor alphas of -3.2%, 0%, and 3.8%, respectively (the choice of these values is explained in Appendix B—available online at www.afa.jof.org).⁴ The individual fund t -statistic distributions shown in the panel are assumed to be normal for simplicity, and are centered at -2.5, 0, and 3.0 (which correspond to the prior-mentioned assumed true alphas; see Appendix B online).⁵ The t -distribution shown in Panel B is the cross-section that (hypothetically) would be observed by a researcher. This distribution is a mixture of the three skill-group distributions in Panel A, where the weight on each distribution is equal to the proportion of zero-alpha, unskilled, and skilled funds in the population, denoted by π_0 , π_A^- , and π_A^+ , respectively (specifically, $\pi_0 = 75\%$, $\pi_A^- = 23\%$, and $\pi_A^+ = 2\%$; see Appendix B online).

Please insert Figure 1 here

To illustrate further, suppose that we choose a significance level, γ , of 10% (corresponding to $t_\gamma^- = -1.65$ and $t_\gamma^+ = 1.65$). With the test shown in expression (1), the researcher

would expect to find 5.6% of funds with a positive and significant t -statistic.⁶ This proportion, denoted by $E(S_\gamma^+)$, is represented by the shaded region in the right tail of the cross-sectional t -distribution (Panel B). Does this area consist merely of skilled funds, as defined above? Clearly not, because some funds are just lucky; as shown in the shaded region of the right tail of Panel A, zero-alpha funds can exhibit positive and significant estimated t -statistics. By the same token, the proportion of funds with a negative and significant t -statistic (the shaded region in the left-tail of Panel B) overestimates the proportion of unskilled funds, because it includes some unlucky zero-alpha funds (the shaded region in the left tail of Panel A). Note that we have not considered the possibility that skilled funds could be *very* unlucky, and exhibit a negative and significant t -statistic. In our example of Figure 1, the probability that the estimated t -statistic of a skilled fund is lower than $t_\gamma^- = -1.65$ is less than 0.001%. This probability is negligible, so we ignore this pathological case. The same applies to unskilled funds that are very lucky.

The message conveyed by Figure 1 is that we measure performance with a limited sample of data, therefore, unskilled and skilled funds cannot easily be distinguished from zero-alpha funds. This problem can be worse if the cross-section of actual skill levels has a complex distribution (and not all fixed at the same levels, as assumed by our simplified example), and is further compounded if a substantial proportion of skilled fund managers have low levels of skill, relative to the error in estimating their t -statistics. To proceed, we must employ a procedure that is able to precisely account for “false discoveries,” i.e., zero-alpha funds that falsely exhibit significant estimated alphas in the face of these complexities.

A.2 Measuring Luck

How do we measure the frequency of “false discoveries” in the tails of the cross-sectional (alpha) t -distribution? At a given significance level γ , it is clear that the probability that a zero-alpha fund (as defined in the last section) exhibits luck equals $\gamma/2$ (shown as the dark shaded region in Panel A of Figure 1)). If the proportion of zero-alpha funds in the population is π_0 , the expected proportion of “lucky funds” (zero-alpha funds with positive and significant t -statistics) equals

$$E(F_\gamma^+) = \pi_0 \cdot \gamma/2. \tag{2}$$

To illustrate, if we take our previous example with $\pi_0 = 75\%$ and $\gamma = 0.10$, we find using Equation (2) that $E(F_\gamma^+) = 3.75\%$. Now, to determine the expected proportion of

skilled funds, $E(T_\gamma^+)$, we simply adjust $E(S_\gamma^+)$ for the presence of these lucky funds:

$$E(T_\gamma^+) = E(S_\gamma^+) - E(F_\gamma^+) = E(S_\gamma^+) - \pi_0 \cdot \gamma/2. \quad (3)$$

From Figure 1, we see that $E(S_\gamma^+) = 5.6\%$ (the shaded region in the right-tail of Panel B). By subtracting $E(F_\gamma^+) = 3.75\%$, the expected proportion of skilled funds, $E(T_\gamma^+)$, amounts to 1.85%.

Since the probability of a zero-alpha fund being unlucky is also equal to $\gamma/2$ (i.e., the grey and black areas in Panel A of Figure 1 are identical), $E(F_\gamma^-)$, the expected proportion of “unlucky funds,” is equal to $E(F_\gamma^+)$. As a result, the expected proportion of unskilled funds, $E(T_\gamma^-)$, is similarly given by

$$E(T_\gamma^-) = E(S_\gamma^-) - E(F_\gamma^-) = E(S_\gamma^-) - \pi_0 \cdot \gamma/2. \quad (4)$$

The significance level, γ , chosen by the researcher determines the segment of the tail examined for lucky versus skilled (or unlucky versus unskilled) mutual funds, as described by Equations (3) and (4). This flexibility in choosing γ provides us with opportunities to make important insights into the merits of active fund management. One objective of this paper—estimating the proportions of unskilled and skilled funds in the entire population, π_A^- and π_A^+ —is achieved only by choosing an appropriately large value for γ . Ultimately, as we increase γ , $E(T_\gamma^-)$ and $E(T_\gamma^+)$ converge to π_A^- and π_A^+ , thus minimizing Type II error (failing to locate truly unskilled or skilled funds).

Another objective of this paper—determining the *location* of truly skilled (or unskilled) funds in the tails of the cross-sectional t -distribution—can only be achieved by evaluating Equations (3) and (4) at several different values of γ . For instance, if the majority of skilled funds lie in the extreme right tail, then increasing the value of γ from 0.10 to 0.20 in Equation (3) would result in a very small increase in $E(T_\gamma^+)$, the proportion of truly skilled funds, since most of the additional significant funds, $E(S_\gamma^+)$, would be lucky funds. Alternatively, if skilled funds are dispersed throughout the right tail, then increases in γ would result in larger increases in $E(T_\gamma^+)$.

To illustrate the impact of fund location, consider two different fund populations (A and B) identical to the one shown in Figure 1 (with $\pi_0 = 75\%$, $\pi_A^- = 23\%$, and $\pi_A^+ = 2\%$), except that the (true) annual alpha of the skilled funds is equal to 3.8% in A (t -mean of 3.0) and 1.9% in B (t -mean of 1.5). Although these two populations have the same proportion of skilled funds ($\pi_A^+ = 2\%$), their locations differ, since the skilled funds in A are more concentrated in the extreme right tail. This information is useful for investors trying to form portfolios with skilled managers, since, in population A , the

skilled funds can be more easily distinguished from the zero-alpha funds. For instance, by forming a portfolio of the significant funds in A at $\gamma = 0.05$ ($t_\gamma^+ = 1.96$), the investor would obtain an expected alpha of 1.8% per year, as opposed to only 45 basis points in population B .⁷ Our approach to fund selection presented later (in Section III.C), explicitly accounts for fund location in order to choose the significance level γ used to construct the portfolio.

A.3 Estimation Procedure

The key to our approach to measuring luck in a group setting, as shown in Equation (2), is the estimator of the proportion, π_0 , of zero-alpha funds in the population. Here, we turn to a recent estimation approach developed by Storey (2002)—called the “False Discovery Rate” (FDR) approach. The FDR approach is very straightforward, as its sole inputs are the (two-sided) p -values associated with the (alpha) t -statistics of each of the M funds. By definition, zero-alpha funds satisfy the null hypothesis, $H_{0,i} : \alpha_i = 0$, and, therefore, have p -values that are uniformly distributed over the interval $[0, 1]$.⁸ On the other hand, p -values of unskilled and skilled funds tend to be very small because their estimated t -statistics tend to be far from zero (see Panel A of Figure 1). We can exploit this information to estimate π_0 without knowing the exact distribution of the p -values of the unskilled and skilled funds.

To explain further, a key intuition of the FDR approach is that it uses information from the center of the cross-sectional t -distribution (which is dominated by zero-alpha funds) to correct for luck in the tails. To illustrate the FDR procedure, suppose we randomly draw 2,076 t -statistics (the number of funds in our study), each from one of the three t -distributions in Panel A of Figure 1—with probability according to our estimates of the proportion of unskilled, zero-alpha, and skilled funds in the population, $\pi_0 = 75\%$, $\pi_A^- = 23\%$, and $\pi_A^+ = 2\%$, respectively. Thus, our draw of t -statistics comes from a known frequency of each type (75%, 23%, and 2%, respectively). Next, we apply the FDR technique to estimate these frequencies—from the sampled t -statistics, we compute two-sided p -values for each of the 2,076 funds, then plot them in Figure 2.

Please insert Figure 2 here

Given the sampled p -values, we estimate π_0 as follows. First, we know that the vast majority of p -values larger than a sufficiently high threshold, λ^* (e.g., $\lambda^* = 0.6$, as shown in the figure), come from zero-alpha funds. Accordingly, after choosing λ^* , we measure the proportion of the total area that is covered by the four lightest grey bars to the right

of λ^* , $\widehat{W}(\lambda^*)/M$ (where $\widehat{W}(\lambda^*)$ equals the number of funds having p -values exceeding λ^*). Note the nearly uniform mass of sampled p -values in intervals between 0.6 and 1—each interval has a mass close to 0.075. Extrapolating this area over the entire region between 0 and 1,

$$\widehat{\pi}_0(\lambda^*) = \frac{\widehat{W}(\lambda^*)}{M} \cdot \frac{1}{(1 - \lambda^*)}, \quad (5)$$

indicates that our estimate of the proportion of zero-alpha funds, $\widehat{\pi}_0(\lambda^*)$, is close to 75%, which is the true (but unknown to the researcher) value of π_0 (since the 75% proportion of zero-alpha funds have uniformly distributed p -values).⁹

To select λ^* , we apply a simple bootstrap procedure introduced by Storey (2002), which minimizes the estimated mean-squared error (MSE) of $\widehat{\pi}_0(\lambda)$ (see Appendix A online).¹⁰ While the main advantage of this procedure is that it is entirely data-driven, we find that $\widehat{\pi}_0(\lambda^*)$ is not overly sensitive to the choice of λ^* . For instance, a simple approach which fixes the value of λ^* to intermediate levels (such as 0.5 or 0.6) produces similar estimates (see Appendix D online).

Substituting the resulting estimate, $\widehat{\pi}_0$, in Equations (2), (3), and replacing $E(S_\gamma^+)$ with the observed proportion of significant funds in the right tail, \widehat{S}_γ^+ , we can easily estimate $E(F_\gamma^+)$ and $E(T_\gamma^+)$ corresponding to any chosen significance level, γ . The same approach can be used in the left tail by replacing $E(S_\gamma^-)$ in Equation (4) with the observed proportion of significant funds in the left tail, \widehat{S}_γ^- . This implies the following estimates of the proportions of unlucky and lucky funds:

$$\widehat{F}_\gamma^- = \widehat{F}_\gamma^+ = \widehat{\pi}_0 \cdot \gamma/2. \quad (6)$$

Using Equation (6), the estimated proportions of unskilled and skilled funds (at the significance level γ) are, respectively, equal to

$$\begin{aligned} \widehat{T}_\gamma^- &= \widehat{S}_\gamma^- - \widehat{F}_\gamma^- = \widehat{S}_\gamma^- - \widehat{\pi}_0 \cdot \gamma/2, \\ \widehat{T}_\gamma^+ &= \widehat{S}_\gamma^+ - \widehat{F}_\gamma^+ = \widehat{S}_\gamma^+ - \widehat{\pi}_0 \cdot \gamma/2. \end{aligned} \quad (7)$$

Finally, we estimate the proportions of unskilled and skilled funds in the entire population as

$$\widehat{\pi}_A^- = \widehat{T}_{\gamma^*}^-, \quad \widehat{\pi}_A^+ = \widehat{T}_{\gamma^*}^+, \quad (8)$$

where γ^* is a sufficiently high significance level—similar to the choice of λ^* , we select γ^* with a bootstrap procedure which minimizes the estimated MSE of $\widehat{\pi}_A^-$ and $\widehat{\pi}_A^+$ (see Appendix A online). While this method is entirely data-driven, there is some flexibility

in the choice of γ^* , as long as it sufficiently high. In Appendix D online, we find that simply setting γ^* to prespecified values (such as 0.35 or 0.45) produces similar estimates.

B Comparison of Our Approach with Existing Methods

The previous literature has followed two alternative approaches when estimating the proportions of unskilled and skilled funds. The “full luck” approach proposed by Jensen (1968) and Ferson and Qian (2004) assumes, a priori, that all funds in the population have zero alphas ($\pi_0 = 1$). Thus, for a given significance level, γ , this approach implies an estimate of the proportions of unlucky and lucky funds equal to $\gamma/2$.¹¹ At the other extreme, the “no luck” approach reports the observed number of significant funds (for instance, Ferson and Schadt (1996)) without making a correction for luck ($\pi_0 = 0$).

What are the errors introduced by assuming, a priori, that the proportion of zero-alpha funds, π_0 , equals 0 or 1, when it does not accurately describe the population? To address this question, we compare the bias produced by these two approaches relative to our FDR approach across different possible values for π_0 ($\pi_0 \in [0, 1]$) using our simple framework of Figure 1. Our procedure consists of three steps. First, for a chosen value of π_0 , we create a simulated sample of 2,076 fund t -statistics (corresponding to our fund sample size) by randomly drawing from the three distributions in Panel A of Figure 1 in the proportions π_0 , π_A^- , and π_A^+ . For each π_0 , the ratio π_A^-/π_A^+ is held fixed to 11.5 (0.23/0.02), as in Figure 1, to assure that the proportion of skilled funds remains low compared to the unskilled funds. Second, we use these sampled t -statistics to estimate the proportion of unlucky ($\alpha = 0$, significant with $\hat{\alpha} < 0$), lucky ($\alpha = 0$, significant with $\hat{\alpha} > 0$), unskilled ($\alpha < 0$, significant with $\hat{\alpha} < 0$), and skilled ($\alpha > 0$, significant with $\hat{\alpha} > 0$) funds under each of the three approaches—the “no luck,” “full luck,” and FDR techniques.¹² Third, under each approach, we repeat these first two steps 1,000 times, then compare the average value of each estimator with its true population value.

Please insert Figure 3 here

Specifically, Panel A of Figure 3 compares the three estimators of the expected proportion of unlucky funds. The true population value, $E(F_\gamma^-)$, is an increasing function of π_0 by construction, as shown by Equation (2). While the average value of the FDR estimator closely tracks $E(F_\gamma^-)$, this is not the case for the other two approaches. By assuming that $\pi_0 = 0$, the “no luck” approach consistently underestimates $E(F_\gamma^-)$ when the true proportion of zero-alpha funds is higher ($\pi_0 > 0$). Conversely, the “full luck” approach, which assumes that $\pi_0 = 1$, overestimates $E(F_\gamma^-)$ when $\pi_0 < 1$. To illustrate

the extent of the bias, consider the case where $\pi_0 = 75\%$. While the “no luck” approach substantially underestimates $E(F_\gamma^-)$ (0% instead of its true value of 7.5%), the “full luck” approach overestimates $E(F_\gamma^-)$ (10% instead of its true 7.5%). The biases for estimates of lucky funds, $E(F_\gamma^+)$, in Panel B are exactly the same, since $E(F_\gamma^+) = E(F_\gamma^-)$.

Estimates of the expected proportions of unskilled and skilled funds, $E(T_\gamma^-)$ and $E(T_\gamma^+)$, provided by the three approaches are shown in Panels C and D, respectively. As we move to higher true proportions of zero-alpha funds (a higher value of π_0), the true proportions of unskilled and skilled funds, $E(T_\gamma^-)$ and $E(T_\gamma^+)$, decrease by construction. In both panels, our FDR estimator accurately captures this feature, while the other approaches do not fare well due to their fallacious assumptions about the prevalence of luck. For instance, when $\pi_0 = 75\%$, the “no luck” approach exhibits a large upward bias in its estimates of the total proportion of unskilled and skilled funds, $E(T_\gamma^-) + E(T_\gamma^+)$ (37.3% rather than the correct value of 22.3%). At the other extreme, the “full luck” approach underestimates $E(T_\gamma^-) + E(T_\gamma^+)$ (17.3% instead of 22.3%).

Panel D reveals that the “no luck” and “full luck” approaches also exhibit a nonsensical positive relation between π_0 and $E(T_\gamma^+)$. This result is a consequence of the low proportion of skilled funds in the population. As π_0 rises, the additional lucky funds drive the proportion of significant funds up, making the “no luck” and “full luck” approaches wrongly indicate that more skilled funds are present. Further, the excessive luck adjustment of the “full luck” approach produces estimates of $E(T_\gamma^+)$ below zero.

In addition to the bias properties exhibited by our FDR estimators, their variability is low because of the large cross-section of funds ($M = 2,076$). To understand this, consider our main estimator $\hat{\pi}_0$ (the same arguments apply to the other estimators). Since $\hat{\pi}_0$ is a proportion estimator that depends on the proportion of p -values higher than λ^* , the Law of Large Numbers drives it close to its true value with our large sample size. For instance, taking $\lambda^* = 0.6$ and $\pi_0 = 75\%$, the standard deviation of $\hat{\pi}_0$, σ_{π_0} , is as low as 2.5% with independent p -values ($1/30^{th}$ the magnitude of π_0).¹³ In Appendix B online, we provide further evidence of the remarkable accuracy of our estimators using Monte-Carlo simulations.

C Cross-Sectional Dependence among Funds

Mutual funds can have correlated residuals if they “herd” in their stockholdings (Wermers (1999)) or hold similar industry allocations. In general, cross-sectional dependence in fund estimated alphas greatly complicates performance measurement. Any inference test with dependencies becomes quickly intractable as M rises, since this requires the estimation and inversion of an $M \times M$ residual covariance matrix. In a Bayesian frame-

work, Jones and Shanken (2005) show that performance measurement requires intensive numerical methods when investor prior beliefs about fund alphas include cross-fund dependencies. Further, KTWW show that a complicated bootstrap is necessary to test the significance of performance of a fund located at a particular alpha rank, since this test depends on the joint distribution of all fund estimated alphas–cross-correlated fund residuals must be bootstrapped simultaneously.

An important advantage of our approach is that we estimate the p -value of each fund in isolation—avoiding the complications that arise because of the dependence structure of fund residuals. However, high cross-sectional dependencies could potentially bias our estimators. To illustrate this point with an extreme case, suppose that all funds produce zero alphas ($\pi_0 = 100\%$), and that fund residuals are perfectly correlated (perfect herding). In this case, all fund p -values would be the same, and the p -value histogram would not converge to the true p -value distribution, as shown in Figure 2. Clearly, we would make serious errors no matter where we set λ^* .

In our sample, we are not overly concerned with dependencies, since we find that the average correlation between four-factor model residuals of pairs of funds is only 0.08. Further, many of our funds do not have highly overlapping return data, thus, ruling out highly correlated residuals by construction. Specifically, we find that 15% of the funds pairs do not have a single monthly return observation in common; on average, only 55% of the return observations of fund pairs is overlapping. Therefore, we believe that cross-sectional dependencies are sufficiently low to allow consistent estimators.¹⁴

However, in order to explicitly verify the properties of our estimators, we run a Monte-Carlo simulation. In order to closely reproduce the actual pairwise correlations between funds in our dataset, we estimate the residual covariance matrix directly from the data, then use these dependencies in our simulations. In further simulations, we impose other types of dependencies, such as residual block correlations or residual factor dependencies, as in Jones and Shanken (2005). In all simulations, we find both that average estimates (for all of our estimators) are very close to their true values, and that confidence intervals for estimates are comparable to those that result from simulations where independent residuals are assumed. These results, as well as further details on the simulation experiment are discussed in Appendix B online.

II Performance Measurement and Data Description

A Asset Pricing Models

To compute fund performance, our baseline asset pricing model is the four-factor model proposed by Carhart (1997):

$$r_{i,t} = \alpha_i + b_i \cdot r_{m,t} + s_i \cdot r_{smb,t} + h_i \cdot r_{hml,t} + m_i \cdot r_{mom,t} + \varepsilon_{i,t}, \quad (9)$$

where $r_{i,t}$ is the month t excess return of fund i over the riskfree rate (proxied by the monthly 30-day T-bill beginning-of-month yield); $r_{m,t}$ is the month t excess return on the CRSP NYSE/AMEX/Nasdaq value-weighted market portfolio; and $r_{smb,t}$, $r_{hml,t}$, and $r_{mom,t}$ are the month t returns on zero-investment factor-mimicking portfolios for size, book-to-market, and momentum obtained from Kenneth French’s website.

We also implement a conditional four-factor model to account for time-varying exposure to the market portfolio (Ferson and Schadt (1996)),

$$r_{i,t} = \alpha_i + b_i \cdot r_{m,t} + s_i \cdot r_{smb,t} + h_i \cdot r_{hml,t} + m_i \cdot r_{mom,t} + B' (z_{t-1} \cdot r_{m,t}) + \varepsilon_{i,t}, \quad (10)$$

where z_{t-1} denotes the $J \times 1$ vector of predictive variables measured at the end of month t (minus their mean values over 1975 to 2006), and B is the $J \times 1$ vector of coefficients. The four predictive variables are the one-month T-bill yield; the dividend yield of the CRSP value-weighted NYSE/AMEX stock index; the term spread, proxied by the difference between yields on 10-year Treasurys and three-month T-bills; and the default spread, proxied by the yield difference between Moody’s Baa-rated and Aaa-rated corporate bonds. We have also computed fund alphas using the CAPM and the Fama-French (1993) models. These results are summarized in Section III.D.2.

To compute each fund t -statistic, we use the Newey-West (1987) heteroscedasticity and autocorrelation consistent estimator of the standard deviation, $\hat{\sigma}_{\hat{\alpha}_i}$. Further, KTWW find that the finite-sample distribution of the t -statistic is non-normal for approximately half of the funds. Therefore, we use a bootstrap procedure (instead of asymptotic theory) to compute fund p -values for the two-sided tests with equal-tail significance level, $\gamma/2$ (see Appendix A online). In order to estimate the distribution of the t -statistic for each fund i under the null hypothesis $\alpha_i = 0$, we use a residual-only bootstrap procedure, which draws with replacement from the regression estimated residuals $\{\hat{\varepsilon}_{i,t}\}$.¹⁵ For each fund, we implement 1,000 bootstrap replications. The reader is referred to KTWW for details on this bootstrap procedure.

B Mutual Fund Data

We use monthly mutual fund return data provided by the Center for Research in Security Prices (CRSP) between January 1975 and December 2006 to estimate fund alphas. Each monthly fund return is computed by weighting the net return of its component shareclasses by their beginning-of-month total net asset values. The CRSP database is matched with the Thomson/CDA database using the MFLINKs product of Wharton Research Data Services (WRDS) in order to use Thomson fund investment-objective information, which is more consistent over time. Wermers (2000) provides a description of how an earlier version of MFLINKS was created. Our original sample is free of survivorship bias, but we further select only funds having at least 60 monthly return observations in order to obtain precise four-factor alpha estimates. These monthly returns need not be contiguous. However, when we observe a missing return, we delete the following-month return, since CRSP fills this with the cumulated return since the last non-missing return. In unreported results, we find that reducing the minimum fund return requirement to 36 months has no material impact on our main results, thus, we believe that any biases introduced from the 60-month requirement are minimal.

Our final universe has 2,076 open-end, domestic equity mutual funds existing for at least 60 months between 1975 and 2006. Funds are classified into three investment categories: Growth (1,304 funds), Aggressive Growth (388 funds), and Growth & Income (384 funds). If an investment objective is missing, the prior non-missing objective is carried forward. A fund is included in a given investment category if its objective corresponds to the investment category for at least 60 months.

Table I shows the estimated annualized alpha as well as factor loadings of equally-weighted portfolios within each category of funds. The portfolio is rebalanced each month to include all funds existing at the beginning of that month. Results using the unconditional and conditional four-factor models are shown in Panels A and B, respectively.

Please insert Table I here

Similar to results previously documented in the literature, we find that unconditional estimated alphas for each category are negative, ranging from -0.45% to -0.60% per annum. Aggressive Growth funds tilt toward small capitalization, low book-to-market, and momentum stocks, while the opposite holds for Growth & Income funds. Introducing time-varying market betas provides similar results (Panel B). In tests available upon request, we find that all results to be discussed in the next section are qualitatively similar whether we use the unconditional or conditional version of the four-factor model. For

brevity, we present only results from the unconditional four-factor model.

III Empirical Results

A The Impact of Luck on Long-Term Performance

We begin our empirical analysis by measuring the impact of luck on long-term mutual fund performance, measured as the lifetime performance of each fund (over the period 1975-2006) using the monthly four-factor model of Equation (9). Panel A of Table II shows estimated proportions of zero-alpha, unskilled, and skilled funds in the population ($\hat{\pi}_0$, $\hat{\pi}_A^-$, and $\hat{\pi}_A^+$), as defined in Section I.A.1, with standard deviations of estimates in parentheses. These point estimates are computed using the procedure described in Section I.A.3, while standard deviations are computed using the method of Genovese and Wasserman (2004)—which is described in Appendix A (available online).

Please insert Table II here

Among the 2,076 funds, we estimate that the majority—75.4%—are zero-alpha funds. Managers of these funds exhibit stockpicking skills just sufficient to cover their trading costs and other expenses (including fees). These funds, therefore, capture all of the economic rents that they generate—consistent with the long-run prediction of Berk and Green (2004; BG).

Further, it is quite surprising that the estimated proportion of skilled funds is statistically indistinguishable from zero (see “Skilled” column). This result may seem surprising in light of some prior studies, such as Ferson and Schadt (1996), which find that a small group of top mutual fund managers appear to outperform their benchmarks, net of costs. However, a closer examination—in Panel B—shows that our adjustment for luck is key in understanding the difference between our study and prior research.

To be specific, Panel B shows the proportion of significant alpha funds in the left and right tails (\hat{S}_γ^- and \hat{S}_γ^+ , respectively) at four different significance levels ($\gamma = 0.05, 0.10, 0.15, 0.20$). Similar to past research, there are many significant alpha funds in the right tail— \hat{S}_γ^+ peaks at 8.2% of the total population (170 funds) when $\gamma = 0.20$ (i.e., these 170 funds have a positive estimated alpha with a two-sided p -value below 20%). However, of course, “significant alpha” does not always mean “skilled fund manager.” Illustrating this point, the right side of Panel B decomposes these significant funds into the proportions of lucky zero-alpha funds and skilled funds (\hat{F}_γ^+ and \hat{T}_γ^+ , respectively), using the technique described in Section I.A.2. Clearly, we cannot reject that all of the

right tail funds are merely lucky outcomes among the large number of zero-alpha funds (1,565), and that none of these right-tail funds have truly skilled fund managers (i.e., \widehat{T}_γ^+ is not significantly different from zero for any significance level, γ).

It is interesting (Panel A) that 24% of the population (499 funds) are truly unskilled fund managers—unable to pick stocks well enough to recover their trading costs and other expenses.¹⁶ In untabulated results, we find that left-tail funds, which are overwhelmingly comprised of unskilled (and not merely unlucky) funds, have a relatively long fund life—12.7 years, on average. And, these funds generally perform poorly over their entire lives, making their survival puzzling. Perhaps, as discussed by Elton, Gruber, and Busse (2004), such funds exist if they are able to attract a sufficient number of unsophisticated investors, who are also charged higher fees (Christoffersen and Musto (2002)).

The bottom of Panel B presents characteristics of the average fund in each segment of the tails. Although the average estimated alpha of right-tail funds is somewhat high (between 4.8% and 6.5% per year), this is simply due to very lucky outcomes for a small proportion of the 1,565 zero-alpha funds in the population. It is also interesting that expense ratios are higher for left-tail funds, which likely explains some of the underperformance of these funds (we will revisit this issue when we examine pre-expense returns in a later section), while turnover does not vary systematically among the various tail segments.

In Appendix C (available online), we repeat the long-term performance test described above for investment-objective subgroups—Growth, Aggressive-Growth, and Growth & Income categories. The overall results are as follows. Growth funds show similar results to the overall universe of funds: 76.5% have zero alphas, 23.5% are unskilled, while none are skilled. Performance is somewhat better for Aggressive-Growth funds, as 3.9% of them show true skills. Finally, Growth & Income funds consist of the largest proportion of unskilled funds (30.7%), but have no skilled funds. The long-term existence of this category of actively-managed funds, which includes “value funds” and “core funds” is remarkable in light of these poor results.

As noted by Wermers (2000), the universe of U.S. domestic equity mutual funds has expanded substantially since 1990. Accordingly, the proportions of unskilled and skilled funds estimated over the entire period 1975-2006 may not accurately describe the performance generated by the industry before this rapid expansion. To address this issue, we next examine the evolution of the long-term proportions of unskilled and skilled funds over time. At the end of each year from 1989 to 2006, we estimate the proportions of unskilled and skilled funds ($\widehat{\pi}_A^-$ and $\widehat{\pi}_A^+$, respectively) using the entire return history for each fund up to that point in time. As we move forward in time, we add new

mutual funds once they exhibit a 60-month record. To illustrate, our initial estimates, on December 31, 1989, cover the first 15 years of the sample, 1975-1989 (427 funds), while our final estimates, on December 31, 2006, are based on the entire 32 years, 1975-2006 (2,076 funds; these are the estimates shown in Panel A of Table II).¹⁷ The results in Panel A of Figure 4 show that the proportion of funds with non-zero alphas (equal to the sum of the proportions of skilled and unskilled funds) remains fairly constant over time. However, there are dramatic changes in the relative proportions of unskilled and skilled funds from 1989 to 2006. Specifically, the proportion of skilled funds declines from 14.4% to 0.6%, while the proportion of unskilled funds rises from 9.2% to 24.0% of the entire universe of funds. These changes are also reflected in the population average estimated alpha, shown in Panel B, which drops from 0.16% to -0.97% per year over the same period.

Please insert Figure 4 here

Further, Panel B shows the yearly count of funds included in the estimated proportions of Panel A. From 1996 to 2005, there are more than 100 additional actively managed domestic-equity mutual funds (having a 60-month history) per year. Interestingly, this coincides with the time-variation in the proportions of unskilled and skilled funds shown in Panel A—which can be attributed to two distinct sources. First, new funds created during the 1990’s generate very poor performance, as we find (in untabulated tests) that 24% of them are unskilled, while none are skilled (i.e., $\hat{\pi}_A^- = 24.0\%$ and $\hat{\pi}_A^+ = 0\%$). Since these 1,328 new funds account for more than 60% of the total population (2,076), they greatly contribute to the performance decline shown in Panel A. Second, our results suggest that the growth in the industry has also affected the alpha of the older funds created before January 1990. While many of these 748 funds exhibit truly positive performance up to December 1996 ($\hat{\pi}_A^+ = 14.4$, see Panel A), the decline is breathtaking afterwards. Specifically, we estimate that, during 1997-2006, 34.8% of these older funds are truly unskilled, while none produce truly positive alphas (i.e., $\hat{\pi}_A^- = 34.8\%$, $\hat{\pi}_A^+ = 0\%$).¹⁸ Either the growth of the fund industry has coincided with greater levels of stock market efficiency, making stockpicking a more difficult and costly endeavor, or the large number of new managers simply have inadequate skills. It is also interesting that, during our period of analysis, many fund managers with good track records left the sample to manage hedge funds (as shown by Kostovetsky (2007)), and that indexed investing increased substantially.

Although increased competition may have decreased the average level of alpha, it is also possible that funds do not achieve superior performance in the long run because

flows compete away any alpha surplus. However, we might find evidence of funds with superior short-term alphas, before investors become fully aware of such outperformers due to search costs. Since our long-term performance estimates average alphas over time, they are not able to detect such dynamics. To address this issue, we investigate, in the next section, whether funds exhibit superior alphas over the short run.¹⁹

B The Impact of Luck on Short-Term Performance

To test for short-run mutual fund performance, we partition our data into six non-overlapping subperiods of five years, beginning with 1977-1981 and ending with 2002-2006. For each subperiod, we include all funds having 60 monthly return observations, then compute their respective alpha p -values—in other words, we treat each fund during each five-year period as a separate “fund.” We pool these five-year records together across all time periods to represent the average experience of an investor in a randomly chosen fund during a randomly chosen five-year period. After pooling, we obtain a total of 3,311 p -values from which we compute our different estimators. The results are shown in Table III.

Please insert Table III here

First, Panel A of Table III shows that a small fraction of funds (2.4% of the population) exhibit skill over the short-run (with a standard deviation of 0.7%). Thus, short-term superior performance is rare, but does exist, as opposed to long-term performance. Second, these skilled funds are located in the extreme right tail of the cross-sectional t -distribution. Panel B of Table III shows that, with a γ of only 10% (i.e., funds having a positive estimated alpha with a two-sided p -value below 10%), we capture almost all skilled funds, as \widehat{T}_γ^+ reaches 2.3% (close to its maximum value of 2.4%). Proceeding toward the center of the distribution (by increasing γ to 0.10 and 0.20) produces almost no additional skilled funds, and almost entirely additional zero-alpha funds that are lucky (\widehat{F}_γ^+). Thus, skilled fund managers, while rare, may be somewhat easy to find, since they have extremely high t -statistics (extremely low p -values)—we will use this finding in our next section, where we attempt to find funds with out-of-sample skills.

In the left tail, we observe that the great majority of funds are unskilled, and not merely unlucky zero-alpha funds. For instance, in the extreme left tail (at $\gamma = 0.05$), the proportion of unskilled funds, \widehat{T}_γ^- , is roughly five times the proportion of unlucky funds, \widehat{F}_γ^- (9.4% versus 1.8%). Here, the short-term results are similar to the prior-discussed long-term results—the great majority of left-tail funds are truly unskilled. It is also interesting that true short-term skills seem to be inversely related to turnover, as

indicated by the substantially higher levels of turnover of left-tail funds (which are mainly unskilled funds). Unskilled managers apparently trade frequently, in the short-run, to appear skilled, which ultimately hurts their performance. Perhaps poor governance of some funds (Ding and Wermers (2009)) explains why they end up in the left tail (net of expenses)—they overexpend on both trading costs (through high turnover) and other expenses relative to their skills.

In Appendix C (available online), we repeat the short-term performance test for investment-objective subgroups (Growth, Aggressive Growth, and Growth & Income funds). We find that the proportions of unskilled funds across the three categories are similar to that of the entire universe (from Table III). While Aggressive-Growth funds exhibit somewhat higher skills ($\hat{\pi}_A^+ = 4.2\%$) than Growth funds ($\hat{\pi}_A^+ = 2.6\%$), no Growth & Income funds are able to produce positive short-term alphas.

Since we find evidence of short-term fund manager skills that disappear in the long-term, it is interesting to further examine the mechanism through which skills disappear. The model of BG provides guidance for how this process may unfold. Specifically, if competing fund investors chase winning funds (which have higher proportions of truly skilled funds), then superior fund management companies (which are in scarce supply) may capture the majority of the rents they produce. We examine this conjecture in Table IV. Specifically, at the beginning of each (non-overlapping) five-year period from 1977 to 2006 (similar to Table III), we rank funds into quintiles based on their (1) size (total net assets under management), (2) age (since first offered to the public), and (3) prior-year flows, as a percentage of total net assets. Then, we measure the proportions of zero-alpha, unskilled, and skilled funds ($\hat{\pi}_0$, $\hat{\pi}_A^-$, and $\hat{\pi}_A^+$, respectively) within each fund size quintile (Panel A), fund age quintile (Panel B), and fund flow quintile (Panels C and D).

The BG model implies that larger and older funds should exhibit lower alphas, since they have presumably grown (or survived) to the point where they provide no superior alphas, net of fees—partly due to flows that followed past superior performance. Smaller and newer funds, on the other hand, may exhibit some skills before investors learn about their superior abilities. Consistent with this conjecture, Panels A and B show that larger and older funds are populated with far more unskilled funds than smaller and newer funds.

Perhaps more directly, BG also implies that flows should disproportionately move to truly skilled funds, and that these funds should exhibit the largest reduction in future skills. Panel C shows, for each past-year flow quintile, the proportions of each fund type during the five years ending with the flow-measurement year, while Panel D shows

similar statistics for these quintiles during the following five years. Here, the results are strongly supportive of the BG model. Specifically, the highest flow quintile exhibits the highest proportion of skilled funds (18%) during the five years prior to the flow year, and the largest reduction in skilled funds during the five years subsequent to the flow year (from 18% to 2.4%). Conversely, funds in the lowest flow quintile exhibit high proportions of unskilled funds prior to the flow year, but appear to improve their skills during the following years (perhaps due to a change in strategy or portfolio manager in response to the outflows). However, consistent with prior research (e.g., Sirri and Tufano (1998)), it appears that investors should have withdrawn even more money from these funds, as they continue to exhibit poor skills (27% are unskilled, compared to 17% for high inflow funds). Although the BG model does not capture the behavior of these apparently irrational investors, our results are generally consistent with the predictions of their model.

Please insert Table IV here

C Performance Persistence

Our previous analysis reveals that only 2.4% of the funds are skilled over the short-term. Can we detect these skilled funds over time, in order to capture their superior alphas? Ideally, we would like to form a portfolio containing only the truly skilled funds in the right tail; however, since we only know which segment of the tails in which they lie, but not their identities, such an approach is not feasible.

Nonetheless, the reader should recall from the last section that skilled funds are located in the extreme right tail. By forming portfolios containing all funds in this extreme tail, we stand a greater chance of capturing the superior alphas of the truly skilled ones. For instance, Panel B of Table III shows that, when the significance level γ is low ($\gamma = 0.05$), the proportion of skilled funds among all significant funds, $\widehat{T}_\gamma^+ / \widehat{S}_\gamma^+$, is about 50%, which is much higher than the proportion of skilled funds in the entire universe, 2.4%.

In order to choose the significance level, γ , that determines the significant funds, S_γ^+ , included in the portfolio, we explicitly account for the location of the skilled funds by using the False Discovery Rate in the right tail, FDR^+ . The FDR_γ^+ is defined as the expected proportion of lucky funds included in the portfolio at the significance level γ :

$$FDR_\gamma^+ = E \left(\frac{F_\gamma^+}{S_\gamma^+} \right). \quad (11)$$

The FDR^+ makes possible a simple portfolio formation rule.²⁰ When we set a low FDR^+ target, we allow only a small proportion of lucky funds (“false discoveries”) in the chosen portfolio. Specifically, we set a sufficiently low significance level, γ , so as to include skilled funds along with a small number of zero-alpha funds that are extremely lucky. Conversely, increasing the FDR^+ target has two opposing effects on a portfolio. First, it decreases the portfolio expected future performance, since the proportion of lucky funds in the portfolio is higher. However, it also increases its diversification, since more funds are selected—reducing the volatility of the portfolio out-of-sample performance. Accordingly, we examine five FDR^+ target levels, z^+ , in our persistence test: $z^+ = 10\%$, 30% , 50% , 70% , and 90% .²¹

The construction of the portfolios proceeds as follows. At the end of each year, we estimate the alpha p -values of each existing fund using the previous five-year period. Using these p -values, we estimate the FDR_γ^+ over a range of chosen significance levels ($\gamma = 0.01, 0.02, \dots, 0.60$). Following Storey (2002), we implement the following straightforward estimator of the FDR_γ^+ :

$$\widehat{FDR}_\gamma^+ = \frac{\widehat{F}_\gamma^+}{\widehat{S}_\gamma^+} = \frac{\widehat{\pi}_0 \cdot \gamma / 2}{\widehat{S}_\gamma^+}, \quad (12)$$

where $\widehat{\pi}_0$ is the estimator of the proportion of zero-alpha funds described in Section I.A.3. For each FDR^+ target level z^+ , we determine the significance level, $\gamma(z^+)$, that provides a $\widehat{FDR}_{\gamma(z^+)}^+$ as close as possible to this target. Then, only funds with p -values smaller than $\gamma(z^+)$ are included in an equally-weighted portfolio. This portfolio is held for one year, after which the selection procedure is repeated. If a selected fund does not survive after a given month during the holding period, its weight is reallocated to the remaining funds during the rest of the year to mitigate survival bias. The first portfolio formation date is December 31, 1979 (after five years of returns have been observed), while the last is December 31, 2005.

In Panel A of Table V, we show the FDR level ($\widehat{FDR}_{\gamma(z^+)}^+$) of the five portfolios, as well as the proportion of funds in the population that they include ($\widehat{S}_{\gamma(z^+)}^+$) during the five-year formation period, averaged over the 27 formation periods (ending from 1979 to 2005)—and, their respective distributions. First, we observe (as expected) that the achieved FDR increases with the FDR target assigned to a portfolio. However, the average $\widehat{FDR}_{\gamma(z^+)}^+$ does not always match its target. For instance, $FDR_{10\%}$ achieves an average of 41.5%, instead of the targeted 10%—during several formation periods, the proportion of skilled funds in the population is too low to achieve a 10% FDR target.²² Of course, a higher FDR target means an increase in the proportion of funds included

in a portfolio—as shown in the rightmost columns of Panel A—since our selection rule becomes less restrictive.

In Panel B, we present the average out-of-sample performance (during the following year) of these five “false discovery” controlled portfolios, starting January 1, 1980 and ending December 31, 2006. We compute the estimated annualized alpha, $\hat{\alpha}$, along with its bootstrapped p -value; annualized residual standard deviation, $\hat{\sigma}_\varepsilon$; information ratio, $IR = \hat{\alpha}/\hat{\sigma}_\varepsilon$; four-factor model loadings; annualized mean return (minus T-bills); and annualized time-series standard deviation of monthly returns. The results reveal that our *FDR* portfolios successfully detect funds with short-term skills. For example, the portfolios *FDR10%* and *30%* produce out-of-sample alphas (net of expenses) of 1.45% and 1.15% per year (significant at the 5% level). As the *FDR* target rises to 90%, the proportion of funds in the portfolio increases, which improves diversification ($\hat{\sigma}_\varepsilon$ falls from 4.0% to 2.7%). However, we also observe a sharp decrease in the alpha (from 1.45% to 0.39%), reflecting the large proportion of lucky funds contained in the *FDR90%* portfolio.

Please insert Table V here

Panel C examines portfolio turnover—we determine the proportion of funds which are still selected using a given false discovery rule 1, 2, 3, 4, and 5 years after their initial inclusion. The results sharply illustrate the short-term nature of truly outperforming funds. After 1 year, 40% or fewer funds remain in portfolios *FDR10%* and *30%*, while after 3 years, these percentages drop below 6%.

Finally, we examine, in Figure 5, how the estimated alpha of the portfolio *FDR10%* evolves over time using expanding windows. The initial value, on December 31, 1989, is the yearly out-of-sample alpha, measured over the period 1980 to 1989, while the final value, on December 31, 2006, is the yearly out-of-sample alpha, measured over the entire period 1980-2006 (i.e., this is the estimated alpha shown in Panel B of Table V). Again, these are the entire history of persistence results that would be observed by a researcher at the end of each year. The similarity with Figure 4 is striking. While the alpha accruing to the *FDR10%* portfolio is impressive at the beginning of the 1990s, it consistently declines thereafter. As the proportion, π_A^+ , of skilled funds falls, the *FDR* approach moves much further to the extreme right tail of the cross-sectional t -distribution (from 5.7% of all funds in 1990 to 0.9% in 2006) in search of skilled managers. However, this change is not sufficient to prevent the performance of *FDR10%* from dropping substantially.

Please insert Figure 5 here

It is important to note the differences between our approach to persistence and that of the previous literature (e.g., Hendricks, Patel, and Zeckhauser (1993); Elton, Gruber, and Blake (1996); and Carhart (1997)). These prior papers generally classify funds into fractile portfolios based on their past performance (past returns, estimated alpha, or alpha t -statistic) over a previous ranking period (one to three years). The proportionate size of fractile portfolios (e.g., deciles) are held fixed, with no regard to the changing proportion of lucky funds within these fixed fractiles. As a result, the signal used to form portfolios is likely to be noisier than our *FDR* approach. To compare these approaches with ours, Figure 5 displays the performance evolution of top decile portfolios which are formed based on ranking funds by their alpha t -statistic, estimated over the previous one and three years, respectively.²³ Over most years, the *FDR* approach performs much better, consistent with the idea that it much more precisely detects skilled funds. However, this performance advantage declines during later years, when the proportion of skilled funds decreases substantially, making them much tougher to locate. Therefore, we find that the superior performance of the *FDR* portfolio is tightly linked to the prevalence of skilled funds in the population.

D Additional Results

D.1 Performance Measured with Pre-Expense Returns

In our baseline framework described previously, we define a fund as skilled if it generates a positive alpha net of trading costs, fees, and other expenses. Alternatively, skill could be defined, in an absolute sense, as the manager’s ability to produce a positive alpha before expenses are deducted. Measuring performance on a pre-expense basis allows one to disentangle the manager’s stockpicking skills, net of trading costs, from the fund’s expense policy—which may be out of the control of the fund manager. To address this issue, we add monthly expenses (1/12 times the most recent reported annual expense ratio) to net returns for each fund, then revisit the long-term performance of the mutual fund industry.²⁴

Panel A of Table VI contains the estimated proportions of zero-alpha, unskilled, and skilled funds in the population ($\hat{\pi}_0$, $\hat{\pi}_A^-$, and $\hat{\pi}_A^+$), on a pre-expense basis. Comparing these estimates with those shown in Table II, we observe a striking reduction in the proportion of unskilled funds—from 24.0% to 4.5%. This result indicates that only a small fraction of fund managers have stockpicking skills that are insufficient to at least compensate for their trading costs. Instead, mutual funds produce negative net-of-expense alphas chiefly because they charge excessive fees, in relation to the selection

abilities of their managers. In Panel B, we further find that the average expense ratio across funds in the left tail is slightly lower when performance is measured prior to expenses (1.4% versus 1.5% per year), indicating that high fees (potentially charged to unsophisticated investors) are one reason why funds end up in the extreme left tail, net of expenses. In addition, there is no reliable relation between turnover and pre-expense performance, indicating that some unskilled managers trade too much, relative to their abilities, although it is also possible that some skilled managers trade too little.

Please insert Table VI here

In the right tail, we find that 9.6% of fund managers have stockpicking skills sufficient to more than compensate for trading costs (Panel A). Since 75.4% of funds produce zero net-of-expense alphas, it seems surprising that that we do not find more pre-expense skilled funds. However, this is due to the relatively small impact of expense ratios on the performance of funds located in the center of the cross-sectional t -distribution. Adding back these expenses leads only to a marginal increase in the alpha t -statistic, making it difficult to detect the presence of skill.²⁵

Finally, in untabulated tests, we find that the proportion of pre-expense skilled funds in the population decreases from 27.5% to 10% between 1996 and 2006. This implies that the decline in net-expense skills noted in Figure 4 is mostly driven by a reduction in stockpicking skills over time (as opposed to an increase in expenses for pre-expense skilled funds).

On the contrary, the proportion of pre-expense unskilled funds remains equal to zero until the end of 2003. Thus, poor stockpicking skills (net of trading costs) cannot explain the large increase in the proportion of unskilled funds (net of both trading costs and expenses) from 1996 onwards. This increase is likely to be due to rising expenses charged by funds with weak stock-selection abilities, or the introduction of new funds with high expense ratios and marginal stockpicking skills.

D.2 Performance Measured with Other Asset Pricing Models

Our estimation of the proportions of unskilled and skilled funds, $\hat{\pi}_A^-$ and $\hat{\pi}_A^+$, obviously depends on the choice of the asset pricing model. To examine the sensitivity of our results, we repeat the long-term (net of expense) performance analysis using the (unconditional) CAPM and Fama-French models. Based on the CAPM, we find that $\hat{\pi}_A^-$ and $\hat{\pi}_A^+$ are equal to 14.3% and 8.6% respectively, which is much more supportive of active management skills, compared to Section III.A.1. However, this result may be due

to the omission of the size, book-to-market, and momentum factors. This conjecture is confirmed in Panel A of Table VII: the funds located in the right tail (according to the CAPM) have substantial loadings on the size and the book-to-market factors, which carry positive risk premia over our sample period (3.7% and 5.4% per year, respectively).

Please insert Table VII here

Turning to the Fama-French (1993) model, we find that $\hat{\pi}_A^-$ and $\hat{\pi}_A^+$ amount to 25.0% and 1.7%, respectively. These proportions are very close to those obtained with the four-factor model, since only one factor is omitted. As expected, the 1.1% difference in the estimated proportion of skilled funds between the two models (1.7%-0.6%) can be explained by the momentum factor. As shown in Panel B, the funds located in the right tail (according to the Fama-French model) have substantial loadings on the momentum factor, which carries a positive risk premium over the period (9.4% per year).

D.3 Bayesian Interpretation

Although we operate in a classical frequentist framework, our new FDR measure, FDR^+ , also has a natural Bayesian interpretation.²⁶ To see this, we denote, by G_i , a random variable which takes the value of -1 if fund i is unskilled, 0 if it has zero alpha, and +1 if it is skilled. The prior probabilities for the three possible values (-1, 0, +1) are given by the proportion of each skill group in the population, π_A^- , π_0 , and π_A^+ . The Bayesian version of our FDR^+ measure, denoted by fdr_γ^+ , is defined as the posterior probability that fund i has a zero alpha given that its t -statistic, denoted by T_i , is positive and significant: $fdr_\gamma^+ = \text{prob}(G_i = 0 | T_i \in \Gamma^+(\gamma))$, where $\Gamma^+(\gamma) = (t_\gamma^+, +\infty)$. Using Bayes theorem, we have:

$$fdr_\gamma^+ = \frac{\text{prob}(T_i \in \Gamma^+(\gamma) | G_i = 0) \cdot \text{prob}(G_i = 0)}{\text{prob}(T_i \in \Gamma^+(\gamma))} = \frac{\gamma/2 \cdot \pi_0}{E(S_\gamma^+)}. \quad (13)$$

Stated differently, the fdr_γ^+ indicates how the investor changes his prior probability that fund i has a zero alpha ($G_i = 0$) after observing that its t -statistic is significant. In light of Equation (13), our estimator $\widehat{FDR}_\gamma^+ = (\gamma/2 \cdot \hat{\pi}_0) / \widehat{S}_\gamma^+$ can therefore be interpreted as an empirical Bayes estimator of fdr_γ^+ , where π_0 and $E(S_\gamma^+)$ are directly estimated from the data.²⁷

In the recent Bayesian literature on mutual fund performance (e.g., Baks, Metrick, and Wachter (2001) and Pastor and Stambaugh (2002a)), attention is given to the posterior distribution of the fund alpha, α_i , as opposed to the posterior distribution of

G_i . Interestingly, our approach also provides some relevant information for modeling the fund alpha prior distribution in an empirical Bayes setting. The parameters of the prior can be specified based on the relative frequency of the three fund skill groups (zero-alpha, unskilled, and skilled funds). In light of our estimates, an empirically-based alpha prior distribution is characterized by a point mass at $\alpha = 0$, reflecting the fact that 75.4% of the funds yield zero alphas, net of expenses. Since $\hat{\pi}_A^-$ is higher than $\hat{\pi}_A^+$, the prior probability of observing a negative alpha is higher than that of observing a positive alpha. These empirical constraints yield an asymmetric prior distribution. A tractable way to model the left and right parts of this distribution is to exploit two truncated normal distributions in the same spirit as in Baks, Metrick, and Wachter (2001). Further, we estimate that 9.6% of the funds have an alpha greater than zero, before expenses. While Baks, Metrick, and Wachter (2001) set this probability to 1% in order to examine the portfolio decision made by a skeptical investor, our analysis reveals that this level represents an overly skeptical belief.

Finally, we can also interpret the mutual fund selection (Section III.C) from a Bayesian perspective. In her attempt to determine whether to include fund i ($i = 1, \dots, M$) in her portfolio, the Bayesian investor is subject to two sorts of misclassification. First, she may wrongly include a zero-alpha fund in the portfolio (i.e., falsely rejecting H_0). Second, she may fail to include a skilled fund in the portfolio (i.e., falsely accepting H_0). Following Storey (2003), the investor's loss function, BE , can be written as a weighted average of each misclassification type:

$$BE(\Gamma^+) = (1 - \psi) \text{prob}(T_i \in \Gamma^+) \cdot fdr_\gamma^+(\Gamma^+) + \psi \cdot \text{prob}(T_i \notin \Gamma^+) \cdot fnr_\gamma^+(\Gamma^+), \quad (14)$$

where $fnr^+(\Gamma^+) = \text{prob}(G_i = +1 | T_i \notin \Gamma^+)$ is the "False Nondiscovery Rate" (i.e., the probability of failing to detect skilled funds), and ψ is a cost parameter which can be interpreted as the investor's regret after failing to detect skilled funds.²⁸ The decision problem consists in choosing the significance threshold, $t^+(\psi)$, such that $\Gamma^+(\psi) = (t^+(\psi), +\infty)$ minimizes Equation (14) (equivalently, we could work with p -values, and determine the optimal significance level, $\gamma(\psi)$). Contrary to the frequentist approach used in the paper, the Bayesian analysis requires an extensive parameterization, which includes, among others, the exact specification of the null and alternative distributions of T_i , as well as the cost parameter, ψ (see Efron et al. (2001) for an application in genomics).

If we agree to make this additional parameterization, we can determine the optimal Bayesian decision implied by the FDR^+ targets used in our persistence tests ($z^+ = 10\%$,

30%, 50%, 70%, and 90%). One way to do this is to consider our simple example shown in Figure 1, where the null and alternative distributions of T_i are assumed to be normal. We find that a high FDR^+ target z^+ (such as 90%) is consistent with the behavior of a Bayesian investor with a high cost of regret, ψ ($\psi(90\%) = 0.997$). Therefore, she chooses a very high significance level, γ ($\gamma(90\%) = 0.477$), in order to include the vast majority of the skilled funds in the portfolio. On the contrary, a low FDR^+ target z^+ (such as 10%) implies a lower regret, ψ ($\psi(10\%) = 0.318$), and a lower significance level, γ ($\gamma(10\%) = 0.003$) (further details can be found on Appendix D online).

IV Conclusion

In this paper, we apply a new method for measuring the skills of fund managers in a group setting. Specifically, the “False Discovery Rate” (FDR) approach provides a simple and straightforward method to estimate the proportion of skilled funds (those with a positive alpha, net of trading costs and expenses), zero-alpha funds, and unskilled funds (those with a negative alpha) in the entire population. Further, we use these estimates to provide accurate counts of skilled funds within various intervals in the right tail of the cross-sectional alpha distribution, as well as unskilled funds within segments of the left tail.

We apply the FDR technique to show that the proportion of skilled fund managers has diminished rapidly over the past 20 years, while the proportion of unskilled fund managers has increased substantially. Our paper also shows that the long-standing puzzle of actively managed mutual fund underperformance is due to the long-term survival of a minority of truly underperforming funds. Most actively managed funds provide either positive or zero net-of-expense alphas, putting them at least on par with passive funds. Still, it is puzzling why investors seem to increasingly tolerate the existence of a large minority of funds that produce negative alphas, when an increasing array of passively managed funds have become available (such as ETFs).

While our paper focuses on mutual fund performance, our approach has potentially wide applications in finance. It can be used to control for luck in any setting in which a multiple-hypothesis test is run and a large sample is available. This is for instance the case when we assess the performance of the myriad of trading rules used in technical trading (e.g., Sullivan, Timmermann, and White (1999)), or when we determine how many individual stocks have a commonality in liquidity (e.g., Chordia, Roll, and Subrahmanyam (2000)). With our approach, controlling for luck in multiple testing is trivial: the only input required is a vector of p -values, one for each individual test.

REFERENCES

- Avramov, Doron, and Russ Wermers, 2006, Investing in Mutual Funds When Return are Predictable, *Journal of Financial Economics* 81, 339-377.
- Baks, Klaas P., Andrew Metrick, and Jessica Wachter, 2001, Should Investors Avoid all Actively Managed Mutual Funds? A Study in Bayesian Performance Evaluation, *Journal of Finance* 56, 45-85.
- Bell, David E., 1982, Regret Theory in Decision Making Under Uncertainty, *Operations Research* 30, 961-981.
- Benjamini, Yoav, and Yosef Hochberg, 1995, Controlling the False Discovery Rate: A Practical and Powerful Approach to Multiple Testing, *Journal of the Royal Statistical Society* 57, 289-300.
- Berk, Jonathan B., and Richard C. Green, 2004, Mutual Fund Flows and Performance in Rational Markets, *Journal of Political Economy* 112, 1269-1295.
- Carhart, Mark M., 1997, On Persistence in Mutual Fund Performance, *Journal of Finance* 52, 57-82.
- Chordia, Tarun, Richard Roll, and Avanidhar Subrahmanyam, 2000, Commonality in Liquidity, *Journal of Financial Economics* 56, 3-28.
- Christoffersen, Susan E. K., and David K. Musto, 2002, Demand Curves and the Pricing of Money Management, *Review of Financial Studies* 15, 1495-1524.
- Ding, Bill, and Russ Wermers, 2009, Mutual Fund Performance and Governance Structure: The Role of Portfolio Managers and Boards of Directors, Working Paper.
- Efron, Bradley, and Robert Tibshirani, 2002, Empirical Bayes Methods and False Discovery Rates for Microarrays, *Genetic Epidemiology* 23, 70-86.
- Efron, Bradley, Robert Tibshirani, John D. Storey, and Virginia Tusher, 2001, Empirical Bayes Analysis of a Microarray Experiment, *Journal of the American Statistical Association* 96, 1151-1160.
- Elton, Edwin J., Martin J. Gruber, Sanjiv Das, and Matthew Hlavka, 1993, Efficiency with Costly Information: A Reinterpretation of Evidence from Managed Portfolios, *Review of Financial Studies* 6, 1-22.

- Elton, Edwin J., Martin J. Gruber, Christopher R. Blake, 1996, The Persistence of Risk-Adjusted Mutual Fund Performance, *Journal of Business* 69, 133-157.
- Elton, Edwin J., Martin J. Gruber, and Jeffrey Busse, 2004, Are Investors Rational? Choices among Index Funds, *Journal of Finance* 59, 261-288.
- Elton, Edwin J., Martin J. Gruber, and Christopher R. Blake, 2007, Participant Reaction and the Performance of Funds Offered by 401(k) Plans, *Journal of Financial Intermediation* 16, 249-271.
- Fama, Eugene F., and Kenneth R. French, 1993, Common Risk Factors in the Returns on Stocks and Bonds, *Journal of Financial Economics* 33, 3-56.
- Ferson, Wayne E., and Rudi W. Schadt, 1996, Measuring Fund Strategy and Performance in Changing Economic Conditions, *Journal of Finance* 51, 425-461.
- Ferson, Wayne E., and Meijun Qian, 2004, Conditional Performance Evaluation Revisited, in *Research Foundation Monograph of the CFA Institute*.
- Genovese, Christopher, and Larry Wasserman, 2004, A Stochastic Process Approach to False Discovery Control, *Annals of Statistics*, 32, 1035-1061.
- Grinblatt, Mark, and Sheridan Titman, 1989, Mutual Fund Performance: An Analysis of Quarterly Portfolio Holdings, *Journal of Business* 62, 393-416.
- Hall Peter, Joel L. Horowitz, and Bing-Yi Jing, 1995, On Blocking Rules for the Bootstrap with Dependent Data, *Biometrika* 82, 561-574.
- Hamilton, James D., 1994, *Times-Series Analysis*, Princeton University Press, Princeton.
- Hendricks, Darryll, Jayendu Patel, and Richard Zeckhauser, 1993, Hot Hands in Mutual Funds: The Persistence of Performance, 1974-88, *Journal of Finance* 48, 93-130.
- Jensen, Michael C., 1968, The Performance of Mutual Funds in the Period 1945-1964, *Journal of Finance* 23, 389-416.
- Jones, Christopher S., and Jay Shanken, 2005, Mutual Fund Performance with Learning across Funds, *Journal of Financial Economics* 78, 507-552.

- Kosowski, Robert, Allan Timmermann, Russ Wermers, and Halbert White, 2006, Can Mutual Fund "Stars" Really Pick Stocks? New Evidence from a Bootstrap Analysis, *Journal of Finance* 61, 2551-2595.
- Kostovetsky, Leonard, 2007, Brain Drain: Are Mutual Funds Losing their Best Minds? Working Paper, Princeton University.
- Loomes, Graham, and Robert Sugden, 1982, Regret Theory: An Alternative Theory of Rational Choice Under Uncertainty, *Economic Journal* 92, 805-824.
- Lynch, Anthony W., and David Musto, 2003, How Investors Interpret Past Fund Returns, *Journal of Finance* 58, 2033-2058.
- Newey, Whitney K., and Kenneth D. West, 1987, A Simple, Positive Semi-Definite, Heteroscedasticity and Autocorrelation Consistent Covariance Matrix, *Econometrica* 55, 703-708.
- Pastor, Lubos, and Robert F. Stambaugh, 2002a, Mutual Fund Performance and Seemingly Unrelated Assets, *Journal of Financial Economics* 63, 315-349.
- Pastor, Lubos, and Robert F. Stambaugh, 2002b, Investing in Equity Mutual Funds, *Journal of Financial Economics* 63, 351-380.
- Rea, John D., and Brian K. Reid, 1998, Trends in the Ownership Cost of Equity Mutual Funds, *Investment Company Institute Perspective*, November.
- Romano, Joseph P., Azeem M. Shaikh, and Michael Wolf, 2008, Formalized Data Snooping on Generalized Error Rates, *Econometric Theory* 24, 404-447.
- Sirri, E., and P. Tufano, 1998, Costly Search and Mutual Fund Flows, *Journal of Finance* 53, 1589-1622.
- Storey, John D., 2002, A Direct Approach to False Discovery Rates, *Journal of the Royal Statistical Society* 64, 479-498.
- Storey, John D., 2003, The Positive False Discovery Rate: A Bayesian Interpretation and the q-Value, *Annals of Statistics* 31, 2013-2035.
- Storey, John D., Jonathan E. Taylor, and David Siegmund, 2004, Strong Control, Conservative Point Estimation and Simultaneous Conservative Consistency of False Discovery Rates: A Unified Approach, *Journal of the Royal Statistical Society* 66, 187-205.

- Sullivan Ryan, Allan Timmermann, and Halbert White, 1999, Data-Snooping, Technical Trading Rule Performance and the Bootstrap, *Journal of Finance* 54, 1647-1691.
- Tang, Yongqiang, Subhashis Ghosal, and Anindya Roy, 2007, Nonparametric Bayesian Estimation of Positive False Discovery Rates, *Biometrics* 63, 1126-1134.
- Wermers, Russ, 1999, Mutual Fund Herding and the Impact on Stock Prices, *Journal of Finance* 55, 581-622.
- Wermers, Russ, 2000, Mutual Fund Performance: An Empirical Decomposition into Stock-Picking Talent, Style, Transaction Costs, and Expenses, *Journal of Finance* 55, 1655-1695.

Notes

¹From an investor perspective, "skill" is manager talent in selecting stocks sufficient to generate a positive alpha, net of trading costs and fund expenses.

²This multiple test should not be confused with the joint test of the null hypothesis that all fund alphas are equal to zero in a sample (e.g., Grinblatt and Titman (1989)) or to the KTWW test of single-fund performance. The first test addresses only whether at least one fund has a non-zero alpha among several funds, but is silent on the prevalence of these non-zero alpha funds. The second tests the skills of a single fund that is chosen from the universe of alpha-ranked funds. In contrast, our approach simultaneously estimates the prevalence and location of multiple outperforming funds in a group. As such, our approach examines fund performance from a more general perspective, with a richer set of information about active fund manager skills.

³However, perhaps a manager exhibits skill sufficient to more than compensate for trading costs, but the fund management company overcharges fees or inefficiently generates other services (such as administrative services, e.g., record-keeping)—costs that the manager usually has little control over. In a later section (III.D.1), we redefine stockpicking skill in an absolute sense (net of trading costs only) and revisit some of our basic tests to be described.

⁴Individual funds within a given skill group are assumed to have identical true alphas in this illustration. In our empirical section, our approach makes no such assumption.

⁵The actual t -statistic distributions for individual funds are non-normal for most U.S. domestic equity funds (KTWW). Accordingly, in our empirical section, we use a bootstrap approach to more accurately estimate the distribution of t -statistics for each fund (and their associated p -values).

⁶From Panel A, the probability that the observed t -statistic is greater than $t_\gamma^+ = 1.65$ equals 5% for a zero-alpha fund and 91% for a skilled fund. Multiplying these two probabilities by the respective proportions represented by their categories (π_0 and π_A^+) gives 5.6%.

⁷From Figure 1 (Panel A), the probability of including a zero-alpha fund (skilled fund) in the portfolio equals 2.5% (85%) in population A . This gives $E(T_\gamma^+) = \pi_A^+ \cdot 85\% = 1.7\%$, $E(F_\gamma^+) = \pi_0 \cdot 2.5\% = 1.8\%$, $E(S_\gamma^+) = 3.5\%$, and an expected alpha of $(E(T_\gamma^+)/E(S_\gamma^+)) \cdot 3.8\% = 1.8\%$ per year.

⁸To see this, we denote by T_i and P_i the t -statistic and p -value of the zero-alpha fund, \hat{t}_i and \hat{p}_i their estimated values, and $T_i(P_i)$ the t -statistic associated with the p -value, P_i . We have $\hat{p}_i = 1 - F(|\hat{t}_i|)$, where $F(|\hat{t}_i|) = \text{prob}(|T_i| < |\hat{t}_i| \mid \alpha_i = 0)$. The p -value P_i is uniformly distributed over $[0, 1]$ since its cdf, $\text{prob}(P_i < \hat{p}_i) = \text{prob}(1 - F(|T_i(P_i)|) < \hat{p}_i) = \text{prob}(|T_i(P_i)| > F^{-1}(1 - \hat{p}_i)) = 1 - F(F^{-1}(1 - \hat{p}_i)) = \hat{p}_i$.

⁹This estimation procedure cannot be used in a one-sided multiple test, since the null hypothesis is tested under the least favorable configuration (LFC). For instance, consider the following null hypothesis $H_{0,i} : \alpha_i \leq 0$. Under the LFC, it is replaced with $H_{0,i} : \alpha_i = 0$. Therefore, all funds with $\alpha_i \leq 0$ (i.e., drawn from the null) have inflated p -values which are not uniformly distributed over $[0, 1]$.

¹⁰The MSE is the expected squared difference between $\hat{\pi}_0(\lambda)$ and the true value, π_0 : $MSE(\hat{\pi}_0(\lambda)) = E(\hat{\pi}_0(\lambda) - \pi_0)^2$. Since π_0 is unknown, it is replaced with $\min_\lambda \hat{\pi}_0(\lambda)$ to compute the estimated MSE (see Storey (2002)).

¹¹Jensen (1968) summarizes the ‘‘full luck’’ approach as follows: ‘‘...if all the funds had a true α equal to zero, we would expect (merely by random chance) to find 5% of them having t values ‘significant’ at the 5% level.’’

¹²We choose $\gamma = 0.20$ to examine a large portion of the tails of the cross-sectional t -distribution. As shown in Appendix D online, the results using $\gamma = 0.10$ are similar.

¹³Specifically, $\hat{\pi}_0 = (1 - \lambda^*)^{-1} \cdot 1/M \sum_{i=1}^M x_i$, where x_i follows a binomial distribution with probability of success $p_{\lambda^*} = \text{prob}(P_i > \lambda^*) = 0.30$, where P_i denotes the fund p -value (p_{λ^*} equals the rectangle area delimited by the horizontal black line and the vertical line at $\lambda^* = 0.6$ in Figure 2). Therefore, from the standard deviation of a binomial random variable, $\sigma_x = (p_{\lambda^*}(1 - p_{\lambda^*}))^{\frac{1}{2}} = 0.46$, and $\sigma_{\pi_0} = (1 - \lambda^*)^{-1} \cdot \sigma_x / \sqrt{M} = 2.5\%$.

¹⁴It is well known that the sample average, $\bar{x} = 1/M \sum x_i$, is a consistent estimator under many forms of dependence (i.e., \bar{x} converges to the true mean value when M is large; see Hamilton (1994), p. 47). Since our FDR estimators can be written as sample averages (see endnote 13), it is not surprising that they are also consistent under cross-sectional dependence among funds (for further discussion, see Storey, Taylor, and Siegmund (2004)).

¹⁵To determine whether assuming homoscedasticity and temporal independence in individual fund residuals is appropriate, we have checked for heteroscedasticity (White test), autocorrelation (Ljung-Box test), and Arch effects (Engle test). We

have found that only a few funds present such regularities. We have also implemented a block bootstrap methodology with a block length equal to $T^{\frac{1}{5}}$ (proposed by Hall, Horowitz, and Jing (1995)), where T denotes the length of the fund return time-series. All of our results to be presented remain unchanged.

¹⁶This minority of funds is the driving force explaining the negative average estimated alpha that is widely documented in the literature (e.g., Jensen (1968), Carhart (1997), Elton et al. (1993), and Pastor and Stambaugh (2002a)).

¹⁷The dynamic proportion estimators, $\hat{\pi}_0$, $\hat{\pi}_A^-$, and $\hat{\pi}_A^+$, measured at the end of each year treat the universe of existing funds as a new fund population (to be included, a fund must have at least 60 return observations, ending with that year). For these estimators to be accurate (in terms of bias and variability), it is necessary that the cross-sectional fund dependence at each point in time remains sufficiently low (see Section I.C).

¹⁸Under a structural change, the long-term alpha is a time-weighted average of the two subperiod alphas. A zero or negative performance after 1996 progressively drives the long-term alphas of the skilled funds towards zero. This explains why our estimate of the proportion of skilled funds at the end of 2006 is close to zero ($\hat{\pi}_A^+ = 0.6\%$). We have verified this pattern using the Monte-Carlo setting described in Appendix B online. Assuming that all skilled funds become zero-alpha (unskilled) after 1996, we find that the average value of $\hat{\pi}_A^+$ (1,000 iterations) over the entire period equals 2.9% (0.3%).

¹⁹Time-varying betas may also affect the inference on the estimated alpha. As mentioned earlier, we have measured performance using the conditional version of the four-factor model (Equation (10)), and find that the results remained qualitatively unchanged.

²⁰Our new measure, FDR_γ^+ , is an extension of the traditional FDR introduced in the statistical literature (e.g., Benjamini and Hochberg (1995), Storey (2002)), since the latter does not distinguish between bad and good luck. The traditional measure is $FDR_\gamma = E(F_\gamma/S_\gamma)$, where $F_\gamma = F_\gamma^+ + F_\gamma^-$, $S_\gamma = S_\gamma^+ + S_\gamma^-$.

²¹Besides its financial interpretation, the FDR has also a natural statistical meaning, as it is the extension of the Type I error (i.e., rejecting the null H_0 , while it is correct) from single- to multiple-hypothesis testing. In the single case, the Type I error is controlled by using the significance level γ (i.e., the size of the test). In the multiple case, we replace γ with the FDR , which is a *compound* Type I error measure. In both cases, we face a similar trade-off: in order to increase

power, we have to increase γ or the FDR , respectively (see the survey of Romano, Shaikh, and Wolf (2008)).

²²For instance, the minimum achievable FDR at the end of 2003 and 2004 is equal to 47.0% and 39.1%, respectively. If we look at the $\widehat{FDR}_{\gamma(z^+)}^+$ distribution for the portfolio $FDR10\%$ in Panel A, we observe that in 6 years out of 27, the $\widehat{FDR}_{\gamma(z^+)}^+$ is higher than 70%.

²³We use the t -statistic to be consistent with the rest of our paper, but the results are qualitatively similar when we rank on the estimated alpha.

²⁴We discard funds which do not have at least 60 pre-expense return observations over the period 1975-2006. This leads to a small reduction in our sample from 2,076 to 1,836 funds.

²⁵The average expense ratio across funds with $|\widehat{\alpha}_i| < 1\%$ is approximately 10 bp per month. Adding back these expenses to a fund with zero net-expense alpha only increases its t -statistic mean from 0 to 0.9 (based on $T^{\frac{1}{2}}\alpha_A/\sigma_\varepsilon$, with $T = 384$, and $\sigma_\varepsilon = 0.021$). It implies that the null and alternative t -statistic distributions are extremely difficult to distinguish (i.e., for a hypothetical fund with a (pre-expense) t -statistic mean of 0.9, the probability of observing a negative (pre-expense) t -statistic is equal to 18%).

²⁶Our demonstration follows from the arguments used by Efron and Tibshirani (2002) and Storey (2003) for the traditional FDR , defined as $FDR_\gamma = E(F_\gamma/S_\gamma)$, where $F_\gamma = F_\gamma^+ + F_\gamma^-$, $S_\gamma = S_\gamma^+ + S_\gamma^-$.

²⁷A full Bayesian estimation of fdr_γ^+ requires to posit prior distributions for the proportions π_0 , π_A^- , and π_A^+ , and for the distribution parameters of T_i for each skill group. This method, based on additional assumptions (including independent p -values) as well as intensive numerical methods, is applied by Tang, Ghosal, and Roy (2007) to estimate the traditional FDR in a genomics study.

²⁸See Bell (1982) and Loomes and Sugden (1982) for a presentation of Regret Theory which includes in the investor's utility function the cost of regret about foregone investment alternatives.

Table I
Performance of the Equally-Weighted Portfolio of Funds

Results for the unconditional and conditional four-factor models are shown in Panels A and B for the entire fund population (All funds), as well as for Growth, Aggressive Growth, and Growth & Income funds. The regressions are based on monthly data between January 1975 and December 2006. Each panel contains the estimated annualized alpha ($\hat{\alpha}$), the estimated exposures to the market (\hat{b}_m), size (\hat{b}_{smb}), book-to-market (\hat{b}_{hml}), and momentum factors (\hat{b}_{mom}), as well as the adjusted R^2 of an equally-weighted portfolio that includes all funds that exist at the beginning of each month. Figures in parentheses denote the Newey-West (1987) heteroscedasticity and autocorrelation consistent estimates of p -values, under the null hypothesis that the regression parameters are equal to zero.

Panel A Unconditional Four-Factor Model						
	$\hat{\alpha}$	\hat{b}_m	\hat{b}_{smb}	\hat{b}_{hml}	\hat{b}_{mom}	R^2
All (2,076)	-0.48% (0.12)	0.95 (0.00)	0.17 (0.00)	-0.01 (0.38)	0.02 (0.09)	98.0%
Growth (1,304)	-0.45% (0.16)	0.95 (0.00)	0.16 (0.00)	-0.03 (0.15)	0.02 (0.07)	98.0%
Aggressive Growth (388)	-0.53% (0.22)	1.04 (0.00)	0.43 (0.00)	-0.17 (0.00)	0.09 (0.00)	95.8%
Growth & Income (384)	-0.47% (0.09)	0.87 (0.00)	-0.04 (0.02)	0.17 (0.00)	-0.03 (0.01)	98.2%
Panel B Conditional Four-Factor Model						
	$\hat{\alpha}$	\hat{b}_m	\hat{b}_{smb}	\hat{b}_{hml}	\hat{b}_{mom}	R^2
All (2,076)	-0.60% (0.09)	0.96 (0.00)	0.17 (0.00)	-0.02 (0.23)	0.02 (0.08)	98.2%
Growth (1,304)	-0.59% (0.10)	0.96 (0.00)	0.16 (0.00)	-0.03 (0.08)	0.03 (0.05)	98.2%
Aggressive Growth (388)	-0.49% (0.24)	1.05 (0.00)	0.43 (0.00)	-0.19 (0.00)	0.08 (0.00)	96.2%
Growth & Income (384)	-0.58% (0.05)	0.87 (0.00)	-0.04 (0.02)	0.16 (0.00)	-0.03 (0.02)	98.3%

Table II
Impact of Luck on Long-Term Performance

Long-term performance is measured with the unconditional four-factor model over the entire period 1975-2006. Panel A displays the estimated proportions of zero-alpha, unskilled, and skilled funds ($\hat{\pi}_0$, $\hat{\pi}_A^-$, and $\hat{\pi}_A^+$) in the entire fund population (2,076 funds). Panel B counts the proportions of significant funds in the left and right tails of the cross-sectional t -statistic distribution (\hat{S}_γ^- , \hat{S}_γ^+) at four significance levels ($\gamma=0.05, 0.10, 0.15, 0.20$). In the leftmost columns, the significant group in the left tail, \hat{S}_γ^- , is decomposed into unlucky and unskilled funds (\hat{F}_γ^- , \hat{T}_γ^-). In the rightmost columns, the significant group in the right tail, \hat{S}_γ^+ , is decomposed into lucky and skilled funds (\hat{F}_γ^+ , \hat{T}_γ^+). The bottom of Panel B also presents the characteristics of each significant group (\hat{S}_γ^- , \hat{S}_γ^+): the average estimated alpha (% per year), expense ratio (% per year), and turnover (% per year). Figures in parentheses denote the standard deviation of the different estimators.

Panel A Proportion of Unskilled and Skilled Funds									
	Zero alpha ($\hat{\pi}_0$)	Non-zero alpha	Unskilled ($\hat{\pi}_A^-$)	Skilled ($\hat{\pi}_A^+$)					
Proportion	75.4 (2.5)	24.6	24.0 (2.3)	0.6 (0.8)					
Number	1,565	511	499	12					

Panel B Impact of Luck in the Left and Right Tails									
Signif. level (γ)	Left Tail				Right Tail				Signif. level (γ)
	0.05	0.10	0.15	0.20	0.20	0.15	0.10	0.05	
Signif. \hat{S}_γ^- (%)	11.6 (0.7)	17.2 (0.8)	21.5 (0.9)	25.4 (0.9)	8.2 (0.6)	6.0 (0.5)	4.2 (0.4)	2.2 (0.3)	Signif. \hat{S}_γ^+ (%)
Unlucky \hat{F}_γ^- (%)	1.9 (0.0)	3.8 (0.1)	5.6 (0.2)	7.6 (0.3)	7.6 (0.3)	5.6 (0.2)	3.8 (0.1)	1.9 (0.0)	Lucky \hat{F}_γ^+ (%)
Unskilled \hat{T}_γ^- (%)	9.8 (0.7)	13.6 (0.9)	16.1 (1.0)	18.2 (1.1)	0.6 (0.7)	0.4 (0.6)	0.4 (0.5)	0.3 (0.3)	Skilled \hat{T}_γ^+ (%)
Alpha(% year)	-5.5 (0.2)	-5.0 (0.2)	-4.7 (0.1)	-4.6 (0.1)	4.8 (0.3)	5.2 (0.4)	5.6 (0.5)	6.5 (0.7)	Alpha(% year)
Exp.(% year)	1.4	1.4	1.4	1.4	1.3	1.2	1.2	1.2	Exp.(% year)
Turn.(% year)	100	97	95	95	94	95	95	104	Turn.(% year)

Table III
Impact of Luck on Short-Term Performance

Short-term performance is measured with the unconditional four-factor model over non-overlapping 5-year periods between 1977-2006. The different estimates shown in the table are computed from the pooled alpha p -values across all 5-year periods. Panel A displays the estimated proportions of zero-alpha, unskilled, and skilled funds ($\hat{\pi}_0$, $\hat{\pi}_A^-$, and $\hat{\pi}_A^+$) in the population (3,311 funds). Panel B counts the proportions of significant funds in the left and right tails of the cross-sectional t -statistic distribution (\hat{S}_γ^- , \hat{S}_γ^+) at four significance levels ($\gamma=0.05, 0.10, 0.15, 0.20$). In the leftmost columns, the significant group in the left tail, \hat{S}_γ^- , is decomposed into unlucky and unskilled funds (\hat{F}_γ^- , \hat{T}_γ^-). In the rightmost columns, the significant group in the right tail, \hat{S}_γ^+ , is decomposed into lucky and skilled funds (\hat{F}_γ^+ , \hat{T}_γ^+). The bottom of Panel B also presents the characteristics of each significant group (\hat{S}_γ^- , \hat{S}_γ^+): the average estimated alpha (% per year), expense ratio (% per year), and turnover (% per year). Figures in parentheses denote the standard deviation of the different estimators.

Panel A Proportion of Unskilled and Skilled Funds									
	Zero alpha ($\hat{\pi}_0$)	Non-zero alpha		Unskilled ($\hat{\pi}_A^-$)	Skilled ($\hat{\pi}_A^+$)				
Proportion	72.2 (2.0)	27.8	25.4 (1.7)	2.4 (0.7)					
Number	2,390	921	841	80					

Panel B Impact of Luck in the Left and Right Tails									
Signif. level (γ)	Left Tail				Right Tail				Signif. level (γ)
	0.05	0.10	0.15	0.20	0.20	0.15	0.10	0.05	
Signif. \hat{S}_γ^- (%)	11.2 (0.5)	16.8 (0.6)	21.4 (0.7)	24.9 (0.8)	9.6 (0.5)	7.8 (0.5)	5.9 (0.4)	3.5 (0.3)	Signif. \hat{S}_γ^+ (%)
Unlucky \hat{F}_γ^- (%)	1.8 (0.0)	3.6 (0.0)	5.4 (0.1)	7.2 (0.2)	7.2 (0.2)	5.4 (0.1)	3.6 (0.0)	1.8 (0.0)	Lucky \hat{F}_γ^+ (%)
Unskilled \hat{T}_γ^- (%)	9.4 (0.6)	13.2 (0.7)	16.0 (0.8)	17.7 (0.8)	2.4 (0.6)	2.4 (0.5)	2.3 (0.4)	1.7 (0.3)	Skilled \hat{T}_γ^+ (%)
Alpha(% year)	-6.5 (0.2)	-5.9 (0.2)	-5.5 (0.1)	-5.3 (0.1)	6.7 (0.3)	7.0 (0.4)	7.2 (0.4)	7.5 (0.6)	Alpha(% year)
Exp.(% year)	1.4	1.3	1.3	1.3	1.2	1.2	1.2	1.2	Exp.(% year)
Turn.(% year)	98	95	94	93	80	80	81	78	Turn.(% year)

Table IV

Fund Characteristics and Performance Dynamics

We examine the relation between short-term performance and fund size (Panel A), age (Panel B), and annual flows (Panel C and D). At the beginning of each non-overlapping 5-year period between 1977-2006, funds are ranked according to each characteristic, and grouped into quintiles (Low, 2, 3, 4, High). Short-term performance is measured with the unconditional four-factor model over the next 5 years, except for Panel C (Annual Flow-Past Performance), where we use the previous 5 years. For each quintile, we pool the fund alpha p -values, characteristic levels, and estimated alphas across all 5-year periods to compute the estimated proportions of zero-alpha, unskilled, and skilled funds ($\hat{\pi}_0$, $\hat{\pi}_A^-$, and $\hat{\pi}_A^+$), average characteristic levels, and estimated alphas ($\hat{\alpha}$). Median Size denotes the median quintile total net asset under management (million USD), while Avg. Age and Flow denote the average quintile age (years), and annual flow (%). Figures in parentheses denote the standard deviation of the different estimators.

Panel A Size (TNA)						
Quintile	Low	2	3	4	High	High-Low
Zero-alpha ($\hat{\pi}_0$)	81.0 (3.5)	72.2 (4.0)	77.7 (3.8)	64.2 (4.2)	62.1 (4.2)	-18.9
Unskilled ($\hat{\pi}_A^-$)	16.4 (3.1)	23.1 (3.7)	22.3 (3.5)	33.5 (3.9)	34.3 (3.9)	+17.9
Skilled ($\hat{\pi}_A^+$)	2.6 (1.6)	4.6 (1.7)	0.0 (1.5)	2.3 (1.5)	3.6 (1.6)	+1.0
Median Size (million \$)	9.8	52.9	166.0	453.1	1,651.7	+1,641.9
Avg. $\hat{\alpha}$ (% year)	-0.5 (0.1)	-0.6 (0.1)	-1.1 (0.1)	-1.1 (0.1)	-0.9 (0.1)	-0.4

Panel B Age						
Quintile	Low	2	3	4	High	High-Low
Zero-alpha ($\hat{\pi}_0$)	79.6 (3.5)	65.0 (4.2)	72.5 (3.7)	70.2 (4.0)	70.1 (4.2)	-9.5
Unskilled ($\hat{\pi}_A^-$)	16.5 (3.0)	29.8 (3.9)	25.5 (3.4)	26.7 (3.6)	29.9 (4.0)	+13.4
Skilled ($\hat{\pi}_A^+$)	3.9 (1.7)	5.2 (1.6)	2.0 (1.5)	3.1 (1.5)	0.0 (1.3)	-3.9
Avg. Age (year)	2.1	5.2	8.6	15.5	37.8	+35.7
Avg. $\hat{\alpha}$ (% year)	-0.3 (0.1)	-0.8 (0.1)	-0.9 (0.1)	-0.7 (0.1)	-1.4 (0.1)	-1.1

Panel C Annual Flow–Past Performance						
Quintile	Low	2	3	4	High	High-Low
Zero-alpha ($\hat{\pi}_0$)	52.9 (4.0)	73.5 (3.8)	84.0 (2.7)	71.0 (3.8)	78.6 (3.5)	+25.7
Unskilled ($\hat{\pi}_A^-$)	47.1 (3.8)	26.5 (3.5)	16.0 (2.4)	22.5 (3.5)	3.4 (1.6)	-43.7
Skilled ($\hat{\pi}_A^+$)	0.0 (1.2)	0.0 (1.2)	0.0 (1.3)	6.5 (1.8)	18.0 (3.0)	+18.0
Avg. Flow (% year)	-26.8	-11.0	-3.2	7.5	67.5	+94.3
Avg. $\hat{\alpha}$ (% year)	-2.8 (0.1)	-1.7 (0.1)	-0.9 (0.1)	0.1 (0.1)	1.2 (0.1)	+4.0

Panel D Annual Flow–Future Performance						
Quintile	Low	2	3	4	High	High-Low
Zero-alpha ($\hat{\pi}_0$)	69.9 (4.6)	59.7 (4.4)	70.6 (3.6)	73.8 (4.3)	80.6 (2.9)	+10.7
Unskilled ($\hat{\pi}_A^-$)	27.0 (4.2)	37.5 (4.0)	26.8 (3.3)	25.7 (3.5)	17.0 (2.5)	-10.0
Skilled ($\hat{\pi}_A^+$)	3.1 (1.7)	2.7 (1.6)	2.6 (1.6)	0.5 (1.5)	2.4 (1.7)	-0.7
Avg. Flow (% year)	-23.2	-7.1	3.0	24.0	205.3	+228.5
Avg. $\hat{\alpha}$ (% year)	-0.9 (0.1)	-1.4 (0.1)	-1.0 (0.1)	-1.0 (0.1)	-0.7 (0.1)	+0.2

Table V

Performance Persistence Based on the False Discovery Rate

For each of the five *FDR* targets ($z^+=10\%$, 30% , 50% , 70% , and 90%), Panel A contains descriptive statistics on the *FDR* level ($\widehat{FDR}_{\gamma(z^+)}^+$) achieved by each portfolio, as well as the proportion of funds in the population that it includes ($\widehat{S}_{\gamma(z^+)}^+$). The panel shows the average values of $\widehat{FDR}_{\gamma(z^+)}^+$ and $\widehat{S}_{\gamma(z^+)}^+$ over the 27 annual formation dates (from December 1979 to 2005), as well as their respective distributions. Panel B displays the performance of each portfolio over the period 1980-2006. We estimate the annual four-factor alpha ($\widehat{\alpha}$) with its bootstrap *p*-value, its annual residual standard deviation ($\widehat{\sigma}_\varepsilon$), its annual information ratio ($IR=\widehat{\alpha}/\widehat{\sigma}_\varepsilon$), its loadings on the market (\widehat{b}_m), size (\widehat{b}_{smb}), book-to-market (\widehat{b}_{hml}), and momentum factors (\widehat{b}_{mom}), and its annual excess mean, and standard deviation. In Panel C, we examine the turnover of each portfolio. We compute the proportion of funds that are still included in the portfolio 1, 2, 3, 4, and 5 years after their initial selection.

Panel A Portfolio Statistics

Target (z^+)	Achieved False Discovery Rate ($\widehat{FDR}_{\gamma(z^+)}^+$)					Included proportion of funds ($\widehat{S}_{\gamma(z^+)}^+$)				
	Mean	10-30	30-50	50-70	>70%	Mean	0-6	6-12	12-24	>24%
<i>FDR</i> 10%	41.5%	14	6	1	6	3.0%	25	2	0	0
<i>FDR</i> 30%	47.5%	8	12	1	6	8.2%	15	7	3	2
<i>FDR</i> 50%	60.4%	0	14	7	6	20.9%	5	7	4	11
<i>FDR</i> 70%	71.3%	0	4	12	11	29.7%	1	5	5	16
<i>FDR</i> 90%	75.0%	0	4	9	14	33.7%	0	3	4	20

Panel B Performance Analysis

Target (z^+)	$\widehat{\alpha}$ (<i>p</i> -value)	$\widehat{\sigma}_\varepsilon$	IR	\widehat{b}_m	\widehat{b}_{smb}	\widehat{b}_{hml}	\widehat{b}_{mom}	Mean	Std dev
<i>FDR</i> 10%	1.45%(0.04)	4.0%	0.36	0.93	0.16	-0.04	-0.02	8.3%	15.4%
<i>FDR</i> 30%	1.15%(0.05)	3.3%	0.35	0.94	0.17	-0.02	-0.03	8.1%	15.4%
<i>FDR</i> 50%	0.95%(0.10)	2.9%	0.33	0.96	0.20	-0.06	-0.01	8.1%	16.1%
<i>FDR</i> 70%	0.68%(0.15)	2.7%	0.25	0.97	0.19	-0.06	-0.01	7.9%	16.1%
<i>FDR</i> 90%	0.39%(0.30)	2.7%	0.14	0.97	0.19	-0.05	-0.00	7.8%	16.0%

Panel C Portfolio Turnover

Target (z^+)	Proportion of funds remaining in the portfolio...				
	After 1 year	After 2 years	After 3 years	After 4 years	After 5 years
<i>FDR</i> 10%	36.7	12.8	3.4	0.8	0.0
<i>FDR</i> 30%	40.0	14.7	5.1	1.7	1.3
<i>FDR</i> 50%	48.8	23.5	12.3	4.7	2.6
<i>FDR</i> 70%	52.2	29.0	17.4	9.5	6.3
<i>FDR</i> 90%	55.9	33.8	20.4	13.0	8.5

Table VI

Impact of Luck on Long-Term Pre-Expense Performance

We add the monthly expenses to net return of each fund, and measure long-term performance with the unconditional four-factor model over the entire period 1975-2006. Panel A displays the estimated proportions of zero-alpha, unskilled, and skilled funds ($\hat{\pi}_0$, $\hat{\pi}_A^-$, and $\hat{\pi}_A^+$) in the entire fund population on a pre-expense basis (1,836 funds). Panel B counts the proportions of significant funds in the left and right tails of the cross-sectional t -statistic distribution (\hat{S}_γ^- , \hat{S}_γ^+) at four significance levels ($\gamma=0.05, 0.10, 0.15, 0.20$). In the leftmost columns, the significant group in the left tail, \hat{S}_γ^- , is decomposed into unlucky and unskilled funds (\hat{F}_γ^- , \hat{T}_γ^-). In the rightmost columns, the significant group in the right tail, \hat{S}_γ^+ , is decomposed into lucky and skilled funds (\hat{F}_γ^+ , \hat{T}_γ^+). The bottom of Panel B also presents the characteristics of each significant group (\hat{S}_γ^- , \hat{S}_γ^+): the average estimated alpha prior to expenses (in % per year), expense ratio (in % per year), and turnover (in % per year). Figures in parentheses denote the standard deviation of the different estimators.

Panel A Proportion of Unskilled and Skilled Funds									
	Zero alpha ($\hat{\pi}_0$)	Non-zero alpha		Unskilled ($\hat{\pi}_A^-$)	Skilled ($\hat{\pi}_A^+$)				
Proportion	85.9 (2.7)	14.1		4.5 (1.0)	9.6 (1.5)				
Number	1,577	259		176	83				

Panel B Impact of Luck in the Left and Right Tails									
Signif. level (γ)	Left Tail				Right Tail				Signif. level (γ)
	0.05	0.10	0.15	0.20	0.20	0.15	0.10	0.05	
Signif. \hat{S}_γ^- (%)	4.3 (0.5)	7.5 (0.6)	10.2 (0.7)	12.8 (0.8)	17.3 (0.9)	13.1 (0.8)	9.3 (0.7)	5.8 (0.5)	Signif. \hat{S}_γ^+ (%)
Unlucky \hat{F}_γ^- (%)	2.1 (0.0)	4.3 (0.1)	6.4 (0.1)	8.6 (0.2)	8.6 (0.2)	6.4 (0.1)	4.3 (0.1)	2.1 (0.0)	Lucky \hat{F}_γ^+ (%)
Unskilled \hat{T}_γ^- (%)	2.2 (0.5)	3.2 (0.6)	3.8 (0.8)	4.2 (0.9)	8.7 (1.0)	6.6 (0.9)	5.0 (0.7)	3.6 (0.5)	Skilled \hat{T}_γ^+ (%)
Pre Expense Alpha(% year)	-5.9 (0.5)	-5.2 (0.3)	-4.8 (0.2)	-4.5 (0.2)	4.4 (0.2)	4.8 (0.2)	5.0 (0.3)	5.3 (0.4)	Pre Expense Alpha(% year)
Exp.(% year)	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.2	Exp.(% year)
Turn.(% year)	105	107	108	108	90	89	91	84	Turn.(% year)

Table VII
Loadings on Omitted Factors

We determine the proportions of significant funds in the left and right tails (\widehat{S}_γ^- , \widehat{S}_γ^+) at four significance levels ($\gamma=0.05, 0.10, 0.15, 0.20$) according to each asset pricing model over the period 1975-2006. For each of these significant groups, we compute their average loadings on the omitted factors from the four-factor model: size (\widehat{b}_{smb}), book-to-market (\widehat{b}_{hml}), and momentum (\widehat{b}_{mom}). Panel A shows the results obtained with the unconditional CAPM, while Panel B repeats the same procedure with the unconditional Fama-French model.

Panel A Unconditional CAPM									
	Left Tail				Right Tail				
Signif. level (γ)	0.05	0.10	0.15	0.20	0.20	0.15	0.10	0.05	Signif. level (γ)
Size(\widehat{b}_{smb})	0.06	0.07	0.09	0.09	0.27	0.28	0.28	0.36	Size(\widehat{b}_{smb})
Book(\widehat{b}_{hml})	-0.14	-0.14	-0.13	-0.14	0.34	0.35	0.36	0.37	Book(\widehat{b}_{hml})
Mom.(\widehat{b}_{mom})	0.00	0.00	0.00	0.01	-0.01	-0.01	-0.02	-0.01	Mom.(\widehat{b}_{mom})

Panel B Unconditional Fama-French model									
	Left Tail				Right Tail				
Signif. level (γ)	0.05	0.10	0.15	0.20	0.20	0.15	0.10	0.05	Signif. level (γ)
Mom.(\widehat{b}_{mom})	-0.02	-0.03	-0.02	-0.03	0.09	0.10	0.11	0.12	Mom.(\widehat{b}_{mom})

Figure 1
Outcome of the Multiple Performance Test

Panel A shows the distribution of the fund t -statistic across the three skill groups (zero-alpha, unskilled, and skilled funds). We set the true four-factor alpha equal to -3.2% and $+3.8\%$ per year for the unskilled and skilled funds (implying that the t -statistic distributions are centered at -2.5 and $+3$). Panel B displays the cross-sectional t -statistic distribution. It is a mixture of the three distributions in Panel A, where the weight on each distribution depends on the proportion of zero-alpha, unskilled, and skilled funds in the population (π_0 , π_A^- , and π_A^+). In this example, we set $\pi_0 = 75\%$, $\pi_A^- = 23\%$, and $\pi_A^+ = 2\%$ to match our average estimated values over the final 5 years of our sample.

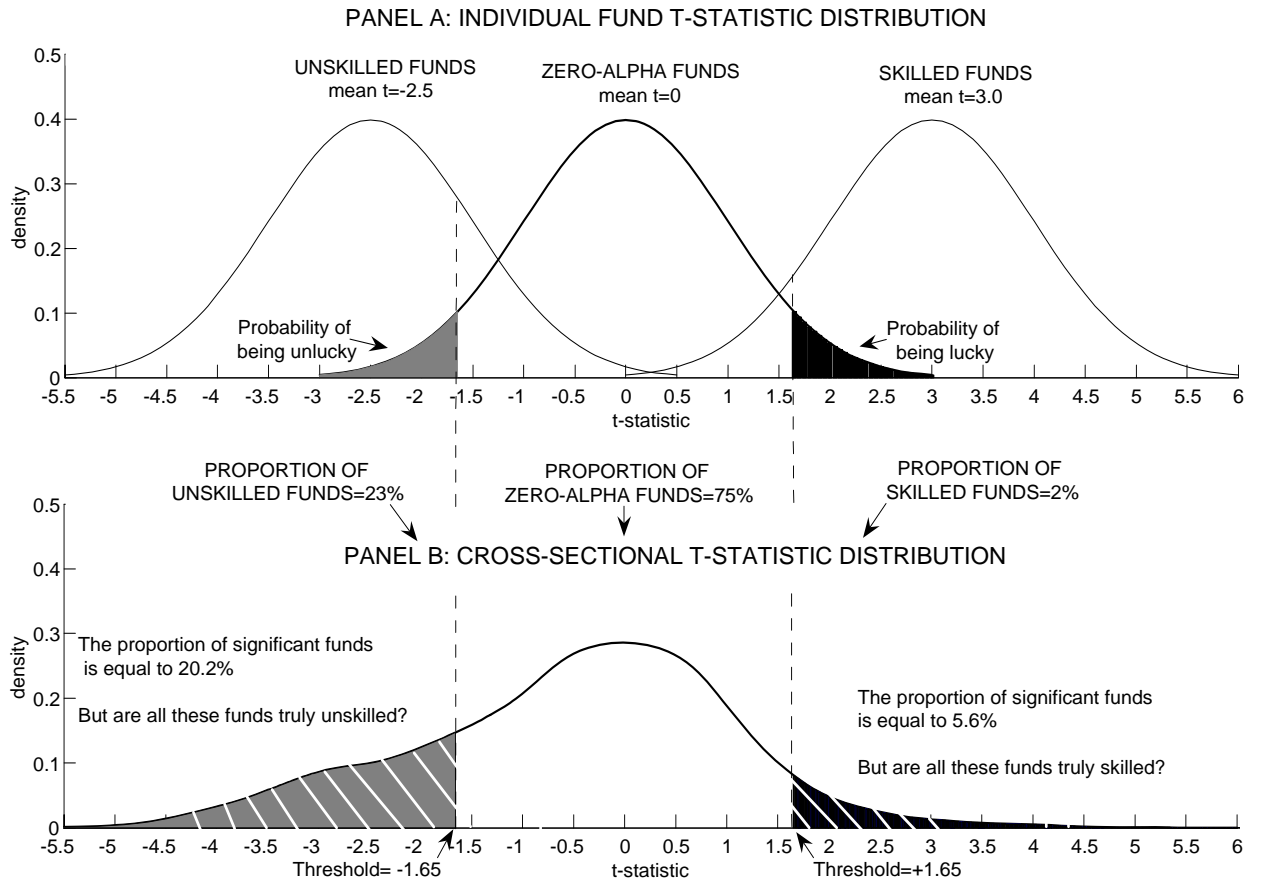


Figure 2
Histogram of Fund p -values

This figure represents the p -value histogram of $M=2,076$ funds (as in our database). For each fund, we draw its t -statistic from one of the distributions in Figure 1 (Panel A) according to the proportion of zero-alpha, unskilled, and skilled funds in the population (π_0 , π_A^- , and π_A^+). In this example, we set $\pi_0 = 75\%$, $\pi_A^- = 23\%$, and $\pi_A^+ = 2\%$ to match our average estimated values over the final 5 years of our sample. Then, we compute the two-sided p -values of each fund from its respective sampled t -statistic, and plot them in the histogram.

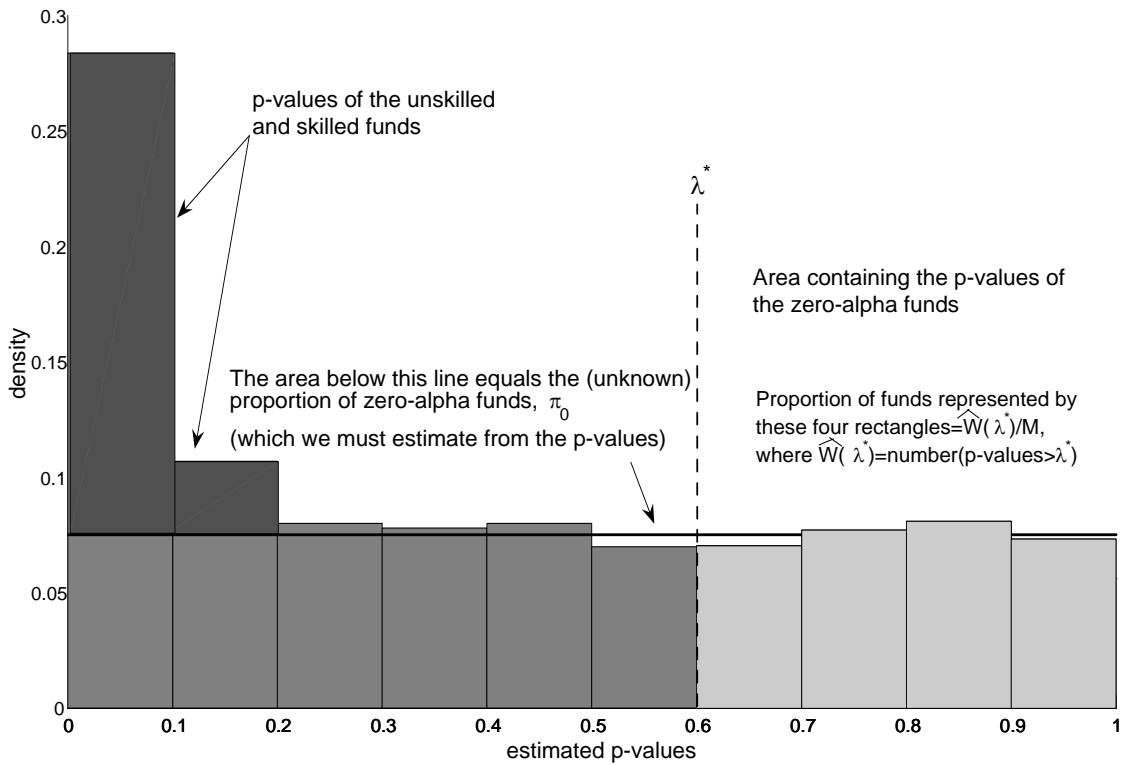
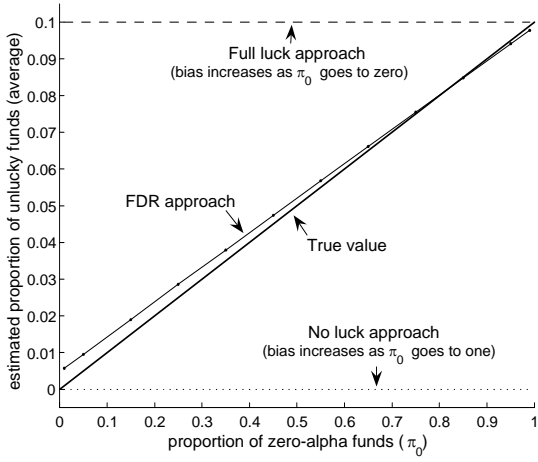


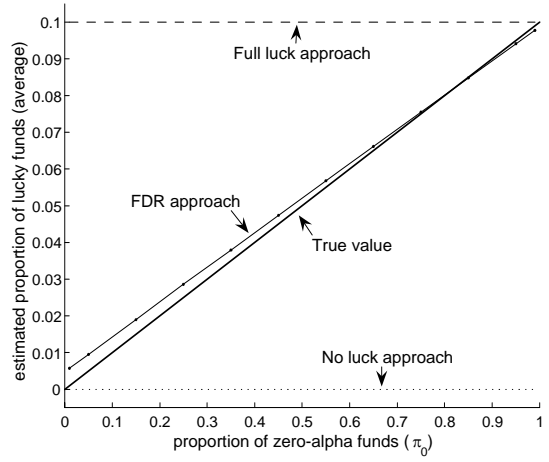
Figure 3

Measuring Luck: Comparison with Existing Approaches

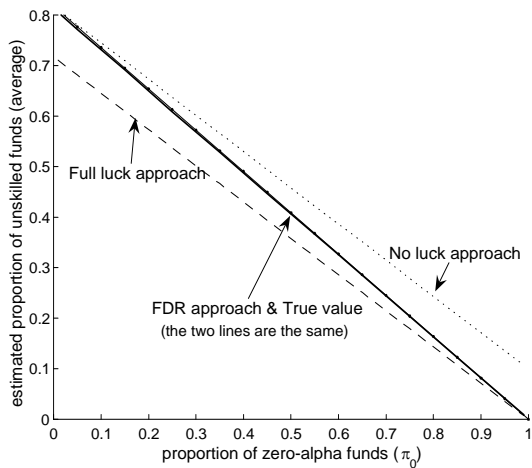
This figure examines the bias of different estimators produced by the three approaches ("no luck", "full luck", and "FDR approach") as a function of the proportion of zero-alpha funds, π_0 . We examine the estimators of the proportions of unlucky, lucky, unskilled, and skilled funds in Panel A, B, C, and D, respectively. The "no luck" approach assumes that $\pi_0=0$, the "full luck" approach assumes that $\pi_0=1$, while the "FDR approach" estimates π_0 directly from the data. For each approach, we compare the average estimator value (over 1,000 replications) with the true population value. For each replication, we draw the t -statistic for each fund i ($i=1, \dots, 2,076$) from one of the distributions in Figure 1 (Panel A) according to the weights π_0 , π_A^- , and π_A^+ , and compute the different estimators at the significance level $\gamma = 0.20$. For each π_0 , the ratio π_A^- over π_A^+ is held fixed to 11.5 (0.23/0.02) as in Figure 1.



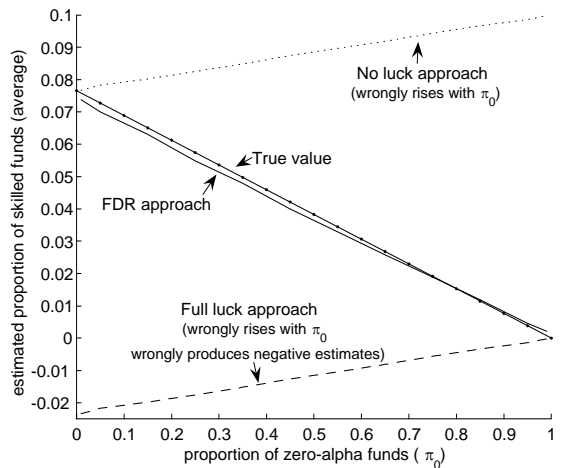
(A) Unlucky funds (left tail)



(B) Lucky funds (right tail)



(C) Unskilled funds (left tail)



(D) Skilled funds (right tail)

Figure 4

Evolution of Mutual Fund Performance over Time

Panel A plots the evolution of the estimated proportions of unskilled and skilled funds ($\hat{\pi}_A^-$ and $\hat{\pi}_A^+$) between 1989 and 2006. At the end of each year, we measure $\hat{\pi}_A^-$ and $\hat{\pi}_A^+$ using the entire fund return history up to that point. The initial estimates at the end of 1989 cover the period 1975-1989, while the last ones in 2006 use the period 1975-2006. The performance of each fund is measured with the unconditional four-factor model. Panel B displays the growth in the mutual fund industry (proxied by the total number of funds used to compute $\hat{\pi}_A^-$ and $\hat{\pi}_A^+$ over time), as well as its average alpha (in % per year).

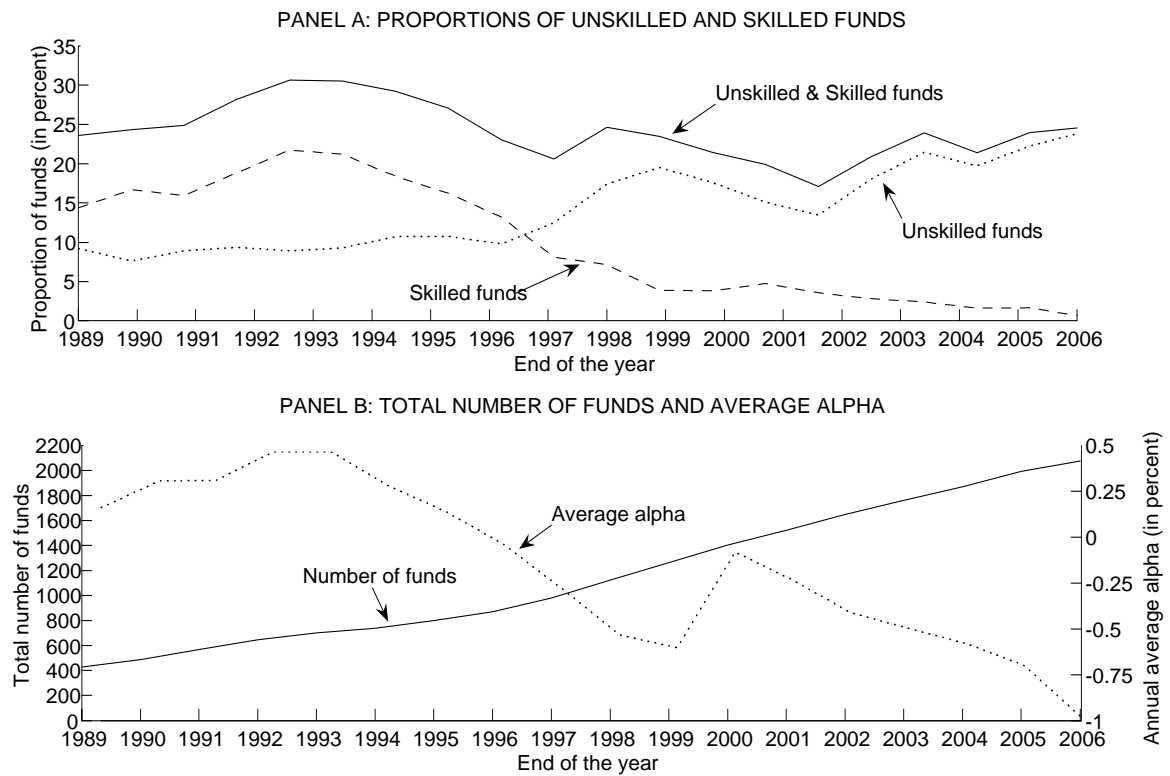
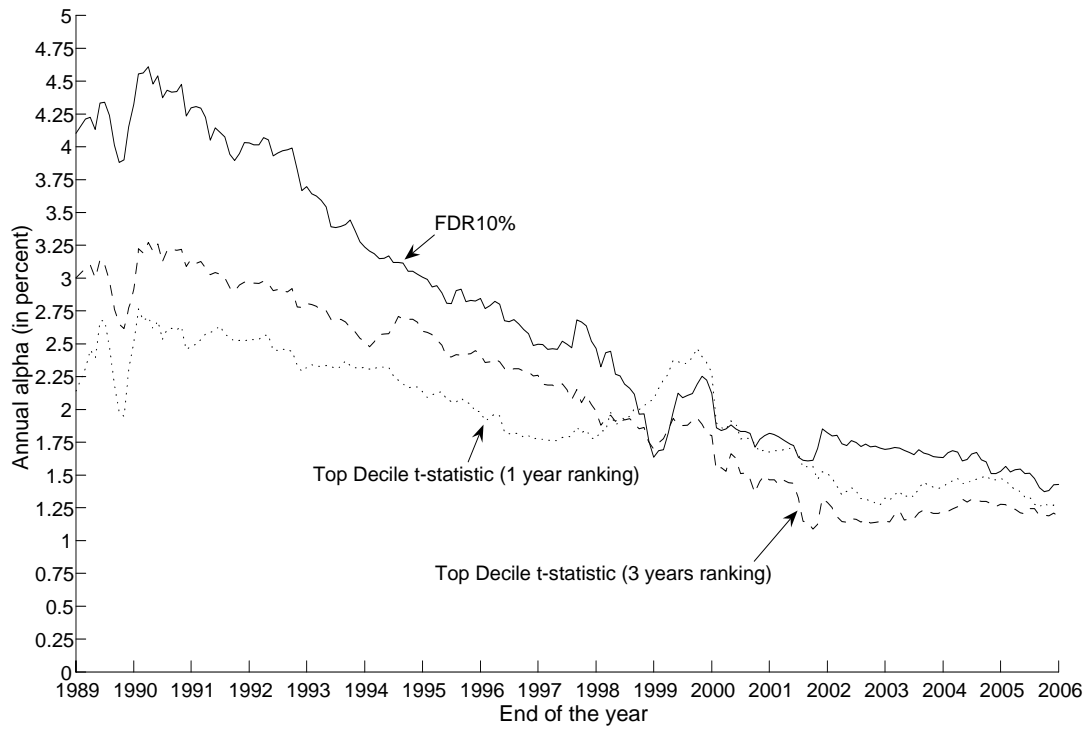


Figure 5

Performance of the Portfolio FDR10% over Time

The graph plots the evolution of the estimated annual four-factor alpha of the portfolio $FDR10\%$. To construct this portfolio, we estimate the (alpha) p -values of each existing fund at the end of each year using the previous five-year period. After determining the significance level, $\gamma(z^+)$, such that the estimated FDR , $\widehat{FDR}_{\gamma(z^+)}^+$, is closest to 10%, we include all funds in the right tail of the cross-sectional t -statistic distribution with p -values lower than $\gamma(z^+)$ in an equally-weighted portfolio. At the end of each year from 1989 to 2006, the portfolio alpha is estimated using the portfolio return history up to that point. The initial estimates cover the period 1980-1989 (the first five years are used for the initial portfolio formation on December 31, 1979), while the last ones use the entire portfolio history from 1980 up to 2006. For comparison purposes, we also show the performance of top decile portfolios formed according to a t -statistic ranking, where the t -statistic is estimated over the prior one and three years, respectively.



Appendix

False Discoveries in Mutual Fund Performance: Measuring Luck in Estimated Alphas

Laurent Barras, Olivier Scaillet, and Russ Wermers

A Estimation Procedure

A.1 Determining the Value for λ^* from the Data

We use the bootstrap procedure proposed by Storey (2002) and Storey, Taylor, and Siegmund (2004) to estimate the proportion of zero-alpha funds in the population, π_0 . This resampling approach chooses λ from the data such that an estimate of the Mean-Squared Error (MSE) of $\hat{\pi}_0(\lambda)$, defined as $E(\hat{\pi}_0(\lambda) - \pi_0)^2$, is minimized. First, we compute $\hat{\pi}_0(\lambda)$ using Equation (5) of the paper across a range of λ values ($\lambda = 0.30, 0.35, \dots, 0.70$). Second, for each possible value of λ , we form 1,000 bootstrap replications of $\hat{\pi}_0(\lambda)$ by drawing with replacement from the $M \times 1$ vector of fund p -values. These are denoted by $\hat{\pi}_0^b(\lambda)$, for $b = 1, \dots, 1,000$. Third, we compute the estimated MSE for each possible value of λ :

$$\widehat{MSE}(\lambda) = \frac{1}{1,000} \sum_{b=1}^{1,000} \left[\hat{\pi}_0^b(\lambda) - \min_{\lambda} \hat{\pi}_0(\lambda) \right]^2. \quad (\text{A1})$$

We choose λ^* such that $\lambda^* = \arg \min_{\lambda} \widehat{MSE}(\lambda)$.

A.2 Determining the Value for γ^* from the Data

To estimate the proportions of unskilled and skilled funds in the population, π_A^- and π_A^+ , we use a bootstrap procedure which minimizes the estimated MSE of $\hat{\pi}_A^-(\gamma)$ and $\hat{\pi}_A^+(\gamma)$. First, we compute $\hat{\pi}_A^-(\gamma)$ using Equation (8) of the paper across a range of γ values ($\gamma = 0.30, 0.35, \dots, 0.50$). Second, we form 1,000 bootstrap replications of $\hat{\pi}_A^-(\gamma)$ for each possible value of γ . These are denoted by $\hat{\pi}_A^{b-}(\gamma)$, for $b = 1, \dots, 1,000$. Third, we compute the estimated MSE for each possible value of γ :

$$\widehat{MSE}^-(\gamma) = \frac{1}{1,000} \sum_{b=1}^{1,000} \left[\hat{\pi}_A^{b-}(\gamma) - \max_{\gamma} \hat{\pi}_A^-(\gamma) \right]^2. \quad (\text{A2})$$

We choose γ^- such that $\gamma^- = \arg \min_{\gamma} \widehat{MSE}^-(\gamma)$. We use the same data-driven procedure for $\widehat{\pi}_A^+(\gamma)$ to determine $\gamma^+ = \arg \min_{\gamma} \widehat{MSE}^+(\gamma)$. If $\min_{\gamma} \widehat{MSE}^-(\gamma) < \min_{\gamma} \widehat{MSE}^+(\gamma)$, we set $\widehat{\pi}_A^-(\gamma^*) = \widehat{\pi}_A^-(\gamma^-)$. To preserve the equality $1 = \pi_0 + \pi_A^+ + \pi_A^-$, we set $\widehat{\pi}_A^+(\gamma^*) = 1 - \widehat{\pi}_0 - \widehat{\pi}_A^-(\gamma^*)$. Otherwise, we set $\widehat{\pi}_A^+(\gamma^*) = \widehat{\pi}_A^+(\gamma^+)$ and $\widehat{\pi}_A^-(\gamma^*) = 1 - \widehat{\pi}_0 - \widehat{\pi}_A^+(\gamma^*)$.

A.3 Computing the fund (alpha) p -value

We use a bootstrap procedure (instead of asymptotic theory) to compute the fund (alpha) p -value, \widehat{p}_i , for the two-sided test, $H_{0,i} : \alpha_i = 0$ ($i = 1, \dots, M$), with equal-tail significance level, $\gamma/2$. We follow the approach proposed by Davidson and MacKinnon (2004, p. 187) which allows the distribution of the fund t -statistic to be asymmetric:

$$\widehat{p}_i = 2 \cdot \min \left(\frac{1}{Q} \sum_{q=1}^Q I\{\widehat{t}_i^{*q} > \widehat{t}_i\}, \frac{1}{Q} \sum_{q=1}^Q I\{\widehat{t}_i^{*q} < \widehat{t}_i\} \right), \quad (\text{A3})$$

where Q is the number of bootstrap iterations ($Q = 1,000$), $I\{\widehat{t}_i^{*q} > \widehat{t}_i\}$ is an indicator function that takes the value 1 if the bootstrap t -statistic, \widehat{t}_i^{*q} , is higher than the estimated t -statistic, \widehat{t}_i . If the fund t -statistic distribution is symmetric, Equation (A3) is equivalent to the more familiar p -value computation: $\widehat{p}_i = \left(\frac{1}{Q} \sum_{q=1}^Q I\{|\widehat{t}_i^{*q}| > |\widehat{t}_i|\} \right)$.

A.4 Determining the Standard Deviation of the Estimators

We rely on the large-sample theory proposed by Genovese and Wasserman (2004) to determine the standard deviation of the estimators used in the paper.¹ The essential idea is to recognize that these estimators are all stochastic processes indexed by λ or γ which converge to a Gaussian process when the number of funds, M , goes to infinity. Proposition 3.2 of Genovese and Wasserman (2004) shows that $\widehat{\pi}_0(\lambda^*)$ is asymptotically normally distributed when $M \rightarrow \infty$, with standard deviation $\widehat{\sigma}_{\widehat{\pi}_0} = \left(\frac{\widehat{W}(\lambda^*)(M - \widehat{W}(\lambda^*))}{M^3(1 - \lambda^*)^2} \right)^{\frac{1}{2}}$, where $\widehat{W}(\lambda^*)$ denotes the number of funds having p -values exceeding λ^* . Similarly, we have $\widehat{\sigma}_{\widehat{F}_\gamma^+} = (\gamma/2) \widehat{\sigma}_{\widehat{\pi}_0}$, $\widehat{\sigma}_{\widehat{S}_\gamma^+} = \left(\frac{\widehat{S}_\gamma^+(1 - \widehat{S}_\gamma^+)}{M} \right)^{\frac{1}{2}}$, and $\widehat{\sigma}_{\widehat{T}_\gamma^+} = \left(\widehat{\sigma}_{\widehat{S}_\gamma^+}^2 + (\gamma/2)^2 \widehat{\sigma}_{\widehat{\pi}_0}^2 + 2 \frac{(\gamma/2) \widehat{S}_\gamma^+ \widehat{W}(\lambda^*)}{1 - \lambda^* M^2} \right)^{\frac{1}{2}}$ (using the equality $\widehat{S}_\gamma^+ = \widehat{F}_\gamma^+ + \widehat{T}_\gamma^+$). Standard

¹These are the estimated proportions of zero-alpha, unskilled, and skilled funds in the population, $\widehat{\pi}_0, \widehat{\pi}_A^-, \widehat{\pi}_A^+$; the estimated proportion of significant funds with positive (negative) estimated alpha at the significance level γ , $\widehat{S}_\gamma^+ (\widehat{S}_\gamma^-)$; the estimated proportion of lucky (unlucky) funds (at level γ), $\widehat{F}_\gamma^+ (\widehat{F}_\gamma^-)$; and the estimated proportion of skilled (unskilled) funds (at level γ), $\widehat{T}_\gamma^+ (\widehat{T}_\gamma^-)$.

deviations for the estimators in the left tail ($\widehat{S}_\gamma^-, \widehat{F}_\gamma^-, \widehat{T}_\gamma^-$) are obtained by simply replacing \widehat{S}_γ^+ with \widehat{S}_γ^- in the above formulas.

Finally, if $\gamma^* = \gamma^+$, the standard deviations of $\widehat{\pi}_A^+$ and $\widehat{\pi}_A^-$ are respectively given by $\widehat{\sigma}_{\widehat{\pi}_A^+} = \widehat{\sigma}_{\widehat{T}_{\gamma^*}^+}$, and $\widehat{\sigma}_{\widehat{\pi}_A^-} = \left(\widehat{\sigma}_{\widehat{\pi}_A^+}^2 + \widehat{\sigma}_{\widehat{\pi}_0(\lambda^*)}^2 - 2 \left(\frac{1}{1-\lambda^*} \right) \widehat{S}_{\gamma^*}^+ \frac{\widehat{W}(\lambda^*)}{M^2} - 2(\gamma^*/2) \widehat{\sigma}_{\widehat{\pi}_0}^2 \right)^{\frac{1}{2}}$ (using the equality $\widehat{\pi}_A^+ = 1 - \widehat{\pi}_0^+ - \widehat{\pi}_A^-$). Otherwise if $\gamma^* = \gamma^-$, we just reverse the superscripts $+/-$ in the two formulas above.

B Monte-Carlo Analysis

B.1 Under Cross-Sectional Independence

We use Monte-Carlo simulations to examine the performance of all estimators used in the paper: $\widehat{\pi}_0, \widehat{\pi}_A^-, \widehat{\pi}_A^+, \widehat{S}_\gamma^-, \widehat{F}_\gamma^-, \widehat{T}_\gamma^-$, and $\widehat{S}_\gamma^+, \widehat{F}_\gamma^+, \widehat{T}_\gamma^+$. We generate the $M \times 1$ vector of fund monthly excess returns, r_t , according to the four-factor model (market, size, book-to-market, and momentum factors):

$$\begin{aligned} r_t &= \alpha + \beta F_t + \varepsilon_t, & t = 1, \dots, T, \\ F_t &\sim N(0, \Sigma_F), & \varepsilon_t \sim N(0, \sigma_\varepsilon^2 I), \end{aligned} \quad (\text{A4})$$

where α denotes the $M \times 1$ vector of fund alphas, and β is the $M \times 4$ matrix of factor loadings. The 4×1 vector of factor excess returns, F_t , is normally distributed with covariance matrix Σ_F . ε_t is the $M \times 1$ vector of normally distributed residuals. We initially assume that the residuals are cross-sectionally independent and have the same variance σ_ε^2 , so that the covariance matrix of ε_t can simply be written as $\sigma_\varepsilon^2 I$, where I is the $M \times M$ identity matrix.

Our estimators are compared with their respective true population values defined as follows. The parameters π_0, π_A^- , and π_A^+ denote the true proportions of zero-alpha, unskilled, and skilled funds. The expected proportions of unlucky and lucky funds, $E(F_\gamma^-)$ and $E(F_\gamma^+)$, are both equal to $\pi_0 \cdot \gamma/2$. To determine the expected proportions of unskilled and skilled funds, $E(T_\gamma^-)$ and $E(T_\gamma^+)$, we use the fact that, under the alternative hypothesis $\alpha_i \neq 0$, the fund t -statistic follows a non-central student distribution with $T - 5$ degrees of freedom and a noncentrality parameter equal to $T^{\frac{1}{2}} \alpha_A / \sigma_\varepsilon$ (Davidson and MacKinnon (2004), p. 169):

$$\begin{aligned} E(T_\gamma^-) &= \pi_A^- \cdot \text{prob}(t < t_{T-5, \gamma/2} | H_A, \alpha_A < 0), \\ E(T_\gamma^+) &= \pi_A^+ \cdot \text{prob}(t > t_{T-5, 1-\gamma/2} | H_A, \alpha_A > 0), \end{aligned} \quad (\text{A5})$$

where $t_{T-5,\gamma/2}$ and $t_{T-5,1-\gamma/2}$ denote the quantiles of probability level $\gamma/2$ and $1 - \gamma/2$, respectively (these quantiles correspond to the thresholds t_γ^- and t_γ^+ used in the text). Finally, we have $E(S_\gamma^-) = E(F_\gamma^-) + E(T_\gamma^-)$, and $E(S_\gamma^+) = E(F_\gamma^+) + E(T_\gamma^+)$.

To compute these population values, we need to set values for the (true) proportions π_0 , π_A^- , π_A^+ , as well as for the means of the non-central student distributions (required to compute Equation (A5)). In order to set realistic values, we estimate π_0 , π_A^- , and π_A^+ at the end of each of the final five years of our sample (2002-2006) using the entire return history for each fund up to that point in time. These estimates are then averaged to produce values that reflect the recent trend observed in Figure 4 of the paper: $\pi_0 = 75\%$, $\pi_A^- = 23\%$, and $\pi_A^+ = 2\%$. To determine the means of the t -statistic distributions of the unskilled and skilled funds, we use a simple calibration method. First, we compute \widehat{T}_γ^- and \widehat{T}_γ^+ (at $\gamma = 0.20$) at the end of each of the final five years of our sample (2002-2006) using the entire return history for each fund up to that point in time. These estimates are then averaged and inserted along with $\pi_A^- = 23\%$ and $\pi_A^+ = 2\%$ in Equation (A5) in order to determine the means of the distributions which satisfy both equalities. The resulting values are -2.5 and 3, and correspond to an annual four-factor alpha of -3.2% and 3.8%, respectively (using the equality $t_A = T^{\frac{1}{2}}\alpha_A/\sigma_\varepsilon$).

The total number of funds, M , used in the simulation is equal to 1,400.² The input for β is equal to the empirical loadings of a random draw of 1,400 funds (among the total population of 2,076 funds). Consistent with our database, we set $T = 384$ (months), $\sigma_\varepsilon = 0.021$ (equal to the empirical average across the 1,400 funds), and proxy Σ_F by its empirical counterpart. To build the vector of fund alphas, α , we need to determine the identity of the unskilled and skilled funds. This is done by randomly choosing 322 funds (i.e., 23% of the entire population) to which we assign a negative alpha (-3.2% per year), and 28 funds (2% of the population), to which we assign a positive alpha (3.8% per year).

After randomly drawing F_t and ε_t ($t = 1, \dots, 384$), we construct the fund return time-series according to Equation (A4), and compute their t -statistics by regressing the fund returns on the four-factor model. To determine the alpha p -values, we use the fact that the fund t -statistic follows a Student distribution with $T - 5$ degrees of freedom under the null hypothesis $\alpha_i = 0$. Then, we compute $\widehat{\pi}_0$, $\widehat{\pi}_A^-$, and $\widehat{\pi}_A^+$ using Equations (5) and (8) of the paper. \widehat{S}_γ^- and \widehat{S}_γ^+ correspond to the observed number of significant funds with negative and positive alphas, respectively. \widehat{F}_γ^- and \widehat{F}_γ^+ are computed with

²We use this sample size to allow for comparison with the dependence case (described hereafter), which uses a sample of 1,400 correlated fund returns. Since our original sample of funds is larger than 1,400 ($M = 2,076$), our assessment of the precision of the estimators in this section is conservative.

Equation (6) of the paper. \widehat{T}_γ^- and \widehat{T}_γ^+ are given in Equation (7) of the paper. We repeat this procedure 1,000 times.

In Table AI, we compare the average value of each estimator (over the 1,000 replications) with the true values. The figures in parentheses denote the lower and upper bounds of the estimator 90%-confidence interval. We set γ equal to 0.05 and 0.20. In all cases, the simulation results reveal that the average values of our estimators closely match the true values, and that their 90%-confidence intervals are narrow. This result is not surprising in light of the large cross-section of funds available in our sample.

Please insert Table AI here

B.2 Under Cross-Sectional Dependence

The return-generating process is the same as the one shown in Equation (A4), except that the fund residuals are cross-correlated:

$$\varepsilon_t \sim N(0, \Sigma), \quad (\text{A6})$$

where Σ denotes the $M \times M$ residual covariance matrix. The main constraint imposed on Σ is that it must be positive semi-definite. To achieve this, we select all funds with 60 valid return observations over the final five years (2002-2006), which is the period over which we have the largest possible cross-section of funds existing simultaneously—898 funds, whose covariance matrix, Σ_1 , is directly estimated from the data.³ To assess the precision of our estimators, we also need to account for the non-overlapping returns observed in the long-term fund data due to funds that do not exist at the same time. To address this issue, we introduce 502 uncorrelated funds, and write the covariance matrix for the resulting 1,400 funds as follows:⁴

$$\Sigma = \begin{pmatrix} \Sigma_1 & 0 \\ 0 & \sigma_\varepsilon^2 I \end{pmatrix}. \quad (\text{A7})$$

An an input for β , we use the empirical factor loadings of the 898 funds, along with the loadings of a random draw of the 502 remaining funds. The vector of fund alphas,

³The 25%, 50%, and 75% pairwise correlation quantiles are -0.09, 0.05, and 0.19, respectively.

⁴The total number of fund pairs, P , is given by $M(M - 1)/2$, where $M = 1,400$. If there are X uncorrelated funds in the population, the total number of uncorrelated fund pairs, I , equals $X \cdot (M - X) + X(X - 1)/2$. In our data, 15% of the funds pairs do not have any return observations in common, and 55% of the observations are common to the remaining pairs (85%). Therefore, we estimate that the proportion of uncorrelated pairs is equal to 53% (15% + 85% · 45%). With 502 uncorrelated funds, I/P amounts to 58%, and is very close to the ratio observed in the data.

α , is built by randomly choosing the identity of the unskilled and skilled funds, as in the independence case. The results in Table AII indicate that all estimators remain nearly unbiased ($\hat{\pi}_0$, $\hat{\pi}_A^-$, and $\hat{\pi}_A^+$ exhibit small biases). Looking at the 90% confidence intervals, we logically observe that the dispersion of the estimators widens under cross-sectional dependence. However, the performance of the estimators is still very good.

Please insert Table AII here

Apart from this baseline dependence scenario, we also examine three other cases (the results are available upon request). First, we introduce correlation by block among each skill group (zero-alpha, unskilled, and skilled funds) to account for their possible similar bets. Inside each block (representing 10% of each skill group), we set the pairwise correlation equal to 0.15 or 0.30. Second, we use the residual factor specification proposed by Jones and Shanken (2005) in order to capture the role of non-priced factors. We assume that all fund residuals depend on a common residual factor, and that the unskilled and skilled funds are affected by specific residual factors. In the two cases, the results show that the precision of the estimators remain very close to those obtained under the independence case. Finally, we consider the extreme dependence case where the fund population only consists of the 898 correlated funds. We find that all estimators remain unbiased as shown in Tables AI and AII. But unsurprisingly, the confidence intervals widen slightly compared to those shown in Table AII (on average 2% are added on each side of the interval).

C Performance Analysis across Investment Categories

C.1 Long-Term Performance

Similar to our tests for the overall mutual fund sample, we conduct long-term tests for individual investment-objective subgroups—Growth, Aggressive-Growth, and Growth & Income categories. Panel A of Table AIII shows the estimated proportions of zero-alpha, unskilled, and skilled funds ($\hat{\pi}_0$, $\hat{\pi}_A^-$, and $\hat{\pi}_A^+$) in the population of Growth funds (1,304 funds), as defined in Section I.A.1 of the paper, with standard deviations of estimates in parentheses. These point estimates are computed using the procedure described in Section I.A.3, while standard deviations are computed using the method of Genovese and Wasserman (2004)—which is described in Appendix A. In the leftmost columns, we also compute the proportion of significant alpha funds in the left tail, \hat{S}_γ^- , at four different significance levels ($\gamma = 0.05, 0.10, 0.15, 0.20$) along with its decomposition into lucky and unskilled funds (\hat{F}_γ^- and \hat{T}_γ^-). The rightmost columns repeat the analysis for the

significant funds in the right tail, \widehat{S}_γ^+ , and decompose them into lucky and skilled funds (\widehat{F}_γ^+ and \widehat{T}_γ^+). We find that Growth funds show similar results to the overall universe of funds, as 76.5% of the funds are zero-alpha funds. The rest of the population (23.5%) is comprised of unskilled funds, which are unable to pick stocks well enough to recover their trading costs and other expenses.

Please insert Table AIII here

Panel B repeats these estimates for Aggressive-Growth funds (388 funds). While the vast majority of these funds produce zero alphas ($\widehat{\pi}_0 = 75.5\%$), a small proportion of funds have long-term skills ($\widehat{\pi}_A^+ = 3.9\%$). Despite the high level of turnover observed for right-tail funds (between 119% and 134% per year), some of their managers are sufficiently skilled to more than compensate for these additional trading costs. We also find that 20.6% of the funds are unskilled, partly because of their high expense ratios (1.6% per year, on, average for the left-tail funds).

Finally, Panel C shows results for the Growth & Income funds (384 funds). This category produces the lowest performance: not only is the proportion of skilled funds equal to zero, but this category also contains the highest proportion of unskilled funds ($\widehat{\pi}_A^- = 30.7\%$). Despite a low level of expenses and trading costs (compared to the other categories), our results reveal that these managers do not have sufficient stockpicking skills to produce a positive performance in the long-run.

C.2 Short-Term Performance

Table AIV repeats the short-term tests conducted for the overall mutual fund sample on the same investment-objective subgroups—Growth, Aggressive-Growth, and Growth & Income categories. That is, for each category, we partition our data into six non-overlapping subperiods of five years, beginning with 1977-1981 and ending with 2002-2006. For each subperiod, we include all funds having 60 monthly return observations, then compute their respective alpha p -values—in other words, we treat each fund during each five-year period as a separate “fund.” We pool these five-year records together across all time periods to represent the average experience of an investor in a randomly chosen fund during a randomly chosen five-year period.

Please insert Table AIV here

In Panel A, the results for Growth funds are similar to the short-term performance

of the overall universe of funds. First, a small fraction of funds (2.4% of the population) exhibit skill over the short-run. These skilled funds are located in the extreme right tail of the cross-sectional t -distribution. For instance, at a significance level, γ , of 5%, approximately 50% of the significant Growth funds are skilled ($\widehat{T}_\gamma^+/\widehat{S}_\gamma^+ = 1.7/3.5 = 48.6\%$). Proceeding toward the center of the distribution (by increasing γ to 0.05 and 0.20) produces almost no additional skilled funds and almost entirely additional zero-alpha funds that are lucky (i.e., $\widehat{T}_\gamma^+/\widehat{S}_\gamma^+$ decreases from 48.6% to 26.2%). Second, we still observe a large proportion of unskilled funds ($\widehat{\pi}_A^- = 24.6\%$), which are dispersed throughout the left tail.

The short-term performance of the Aggressive-Growth funds in Panel B (i.e., $\widehat{\pi}_A^- = 24.0\%$ and $\widehat{\pi}_A^+ = 4.2\%$) is similar to their long-term performance shown in Table AIII. Similar to Growth funds, we observe that skilled Aggressive-Growth funds are concentrated in the extreme right tail of the distribution. For instance, at $\gamma = 0.05$, more than 60% of the significant funds are skilled ($\widehat{T}_\gamma^+/\widehat{S}_\gamma^+ = 3.1/4.9 = 63.3\%$).

Finally, Panel C shows that Growth & Income funds contain about the same proportion of unskilled funds as Growth and Aggressive-Growth funds ($\widehat{\pi}_A^+ = 25.9\%$). But, contrary to these two categories, no Growth & Income funds are able to generate positive short-term alphas.

D Additional Results

D.1 The Proportion of Zero-Alpha Funds and the p -value Histogram

We examine in detail how the histogram of fund p -values is modified when the proportion of zero-alpha funds in the population, π_0 , changes. We consider two different fund populations of $M = 2,076$ funds (as in our database), whose t -statistics are drawn randomly from one of the three t -statistic distributions in Figure 1 of the paper (Panel A). While the first population contains only zero-alpha funds ($\pi_0 = 100\%$, $\pi_A^- = 0\%$, and $\pi_A^+ = 0\%$), the second population contains 75% of zero-alpha funds and 25% of skilled funds ($\pi_0 = 75\%$, $\pi_A^- = 0\%$, $\pi_A^+ = 25\%$). After computing the two-sided p -values for each of the 2,076 funds for each population, we compare their respective p -value histograms in Figure A1.⁵

Please insert Figure A1 here

⁵We have purposely separated each histogram bar, so that the two histograms can be easily compared.

The histogram of the first population (only zero-alpha funds) is displayed with black bars. Since all funds satisfy the null hypothesis $\alpha = 0$, their p -values are drawn from the uniform distribution over the interval $[0,1]$. As a result, the histogram closely approximates the uniform distribution shown by the horizontal dark line at 0.10 (some black bars are slightly below or above 0.10 because of sampling variation).

The histogram of the second population (75% of zero-alpha funds, 25% of skilled funds) is displayed with grey bars. It is formed with: 1) a set of light grey bars with constant height over the interval $[0,1]$, corresponding to the 75% of zero-alpha funds; 2) an additional bar (dark grey) corresponding to the p -values of the 25% of skilled funds in the population. Note that the height of each grey bar is close to the horizontal grey line at 0.075 (once again, the difference comes from sampling variability). Summing the area covered by these light grey bars over the entire interval, we get the correct proportion of zero-alpha funds, π_0 (i.e., $0.075 \cdot 10 = 75\%$).

Comparing the second histogram with the first one, we observe an important increase in the proportion of extremely small p -values due to the existence of 25% of skilled funds. But since the area covered by the histogram bars must sum to one, this increase is offset by a decline in all light grey bars over the interval $[0,1]$ (compared to the black bars). The reason for this decline over $[0,1]$ is straightforward: while we draw all p -values of the first population from the uniform distribution (i.e., $\pi_0 = 100\%$), the p -values of the second population comes from this uniform distribution only 75% of the time ($\pi_0 = 75\%$).

D.2 Comparison Between the Bootstrap and Fixed-Value Procedures

The threshold λ^* used to estimate the proportion of zero-alpha funds, π_0 , determines the number of funds, $W(\lambda^*)$, with p -values higher than λ^* . If λ^* is too low, the estimator, $\hat{\pi}_0(\lambda^*)$, overestimates π_0 (i.e., $\hat{\pi}_0$ is biased upward), since $W(\lambda^*)$ includes the p -values of many unskilled (skilled) funds (generating Type II errors). On the contrary, if λ^* is too high, we estimate π_0 using only the few p -values at the extreme right of the histogram—thus, making the estimator $\hat{\pi}_0(\lambda^*)$ extremely volatile. In the paper, we propose a simple procedure which chooses λ^* such that the estimated Mean-Squared Error (MSE) of $\hat{\pi}_0$ is minimized (see Equation (A1)).

While the main advantage of this procedure is that it is entirely data-driven, it turns out that the estimate of π_0 is not overly sensitive to the choice of λ^* . So, we believe that a researcher studying the performance of a large population of mutual funds can simply use a value for λ^* of 0.5 or 0.6. At these levels, we avoid including the p -values of the unskilled (skilled) funds. In addition, the estimator is not too volatile, since there

are still many p -values located at the right of λ^* .⁶ To illustrate, consider the baseline example used in the Monte-Carlo analysis and illustrated in Figure 1 of the paper. The number of funds, M , is equal to 2,076 (as in our database). For each fund, we draw its t -statistic from one of the three t -statistic distributions shown in Panel A of Figure 1 according to the following weights: $\pi_0 = 75\%$, $\pi_A^- = 23\%$, and $\pi_A^+ = 2\%$. Then, we compute the two-sided p -value for each of the 2,076 funds, and estimate π_0 using two procedures. The first one is our bootstrap procedure, while the second one (called the fixed-value procedure) sets λ^* equal to 0.5 and 0.6, respectively. The results for 10 different simulations shown in Table AV (Panel A) indicate that the estimated values, $\hat{\pi}_0$, are very close to one another.

Please insert Table AV here

In order to estimate the proportions of unskilled and skilled funds in the population, π_A^- and π_A^+ , we also need to determine the significance level γ^* . If γ^* is too low, the estimators $\hat{\pi}_A^-(\gamma^*)$ and $\hat{\pi}_A^+(\gamma^*)$ underestimate π_A^- and π_A^+ (i.e., $\hat{\pi}_A^-$ and $\hat{\pi}_A^+$ are biased downward), because the power of the test (i.e., the probability of detecting unskilled (skilled) funds) is not sufficiently high (especially if the unskilled (skilled) funds are dispersed in the tails). On the other hand, if γ^* is too high, we inflate the variance of $\hat{\pi}_A^-(\gamma^*)$ and $\hat{\pi}_A^+(\gamma^*)$ by investigating a very large portion of the tails of the cross-sectional t -distribution. In the paper, we use a bootstrap procedure which minimizes the estimated MSE of $\hat{\pi}_A^-$ and $\hat{\pi}_A^+$ (see Equation (A2)).

As for λ^* , we find that the estimates of π_A^- and π_A^+ are not overly sensitive to the choice of γ^* . To illustrate it, we compare our bootstrap procedure with a fixed-value approach, where γ^* is set to 0.35 and 0.4, respectively. In Panel B of Table AV, we compare the estimated values over different 10 simulations. The results show that they are very close to one another. To sum up, while we use a bootstrap approach, there is still some flexibility in the way the proportions of unskilled and skilled funds are estimated.

D.3 Comparison of the FDR Approach with Existing Methods

Figure 3 of the paper compares the different approaches using a significance level γ equal to 0.20. To assess the result sensitivity to changes in γ , we plot the different

⁶One may wonder why $\hat{\pi}_0$ remains almost unchanged at $\lambda^* = 0.5$ and 0.6 , although there are less p -values at the right of λ^* when $\lambda^* = 0.6$. The reason is that the area $\widehat{W}(\lambda^*)/M$ (where M is the number of funds) has to be stretched over the entire interval $[0, 1]$: when λ^* rises from 0.5 to 0.6, $\widehat{W}(\lambda^*)/M$ gets smaller, but you stretch it more, so these effects offset each other.

relations using a significance level γ equal to 0.10 (i.e., we measure the proportions of unlucky (lucky), unskilled (skilled) funds further into the extreme left (right) tails of the cross-sectional t -distribution). The results are displayed in Figure A2.

Please insert Figure A2 here

In Panel A, we compare the estimators of the expected proportion of unlucky funds under the “no luck”, “full luck”, and FDR approaches for different values for the proportion of zero-alpha funds in the population, π_0 . This graph is similar to the one shown in Figure 3 of the paper. While the average value of our FDR estimator closely tracks $E(F_\gamma^-)$, the “no luck” approach (which assumes that $\pi_0 = 0$) consistently underestimates $E(F_\gamma^-)$, and the “full luck” approach (which assumes that $\pi_0 = 1$) overestimates $E(F_\gamma^-)$ when $\pi_0 < 1$. The only difference is that the bias of these two approaches is lower when γ declines from 0.20 to 0.10 (i.e., the scale of the vertical axis is lower in Figure A2 than in Figure 3 of the paper). To understand this result, we can write the bias in the expected proportion of unlucky funds under the two approaches as:

$$\begin{aligned} \text{No luck:} \quad & E\left(\widehat{F}_\gamma^- - E(F_\gamma^-)\right) = \pi_0 \cdot \text{prob}(t < t_\gamma^- | H_0) = \pi_0 \cdot \gamma/2, \\ \text{Full luck:} \quad & E\left(\widehat{F}_\gamma^- - E(F_\gamma^-)\right) = (1 - \pi_0) \cdot \text{prob}(t < t_\gamma^- | H_0) = (1 - \pi_0) \gamma/2. \end{aligned} \quad (\text{A8})$$

In both cases, the bias depends on the probability, $\gamma/2$, of finding an unlucky fund, which declines as γ falls from 0.20 to 0.10. Stated differently, the errors in the luck measurement made by the “no luck” and “full luck” approaches increase as we investigate larger portions of the tails.⁷ All these comments also apply to the expected proportion of lucky funds, $E(F_\gamma^+)$, in Panel B, since $E(F_\gamma^+) = E(F_\gamma^-)$.

Panel C displays the estimates of the expected proportion of unskilled funds, $E(T_\gamma^-)$, at $\gamma = 0.10$. Here again, the graph is very close to the one shown in Figure 3 of the paper. Our approach closely captures the negative relation between $E(T_\gamma^-)$ and π_0 . On the contrary, the “no luck” approach overestimates the expected proportion of unskilled funds (since it does not adjust for luck), while the “full luck” approach underestimates $E(T_\gamma^-)$ (because it overadjusts for luck).

Finally, Panel D shows that our FDR approach provides a nearly unbiased estimator of the expected proportion of skilled funds, $E(T_\gamma^+)$, at $\gamma = 0.10$, as opposed to the other approaches. The main difference with Figure 3 of the paper (Panel D) lies in the

⁷Note that we need to examine such large portions when estimating the proportions of unskilled and skilled funds in the population (π_A^- and π_A^+), (see Appendix D.2). As a result, the “no luck” and “full luck” approaches can produce very poor estimates of π_A^- and π_A^+ .

slope of the relation between π_0 and the estimators under the “no luck” and “full luck” approaches. While in Figure 3 of the paper, we observe a nonsensical positive slope, it is negative in Figure A2 (as it should be). To understand the reason for this change, we can write the expected proportion of significant funds with positive estimated alphas, $E(S_\gamma^+)$, as:

$$E(S_\gamma^+) = E(F_\gamma^+) + E(T_\gamma^+) = \gamma/2 \cdot \pi_0 + \pi_A^+ \cdot \text{prob}(t > t_\gamma^+ | H_A, \alpha_A > 0). \quad (\text{A9})$$

Using the following equalities $(1 - \pi_0) = \pi_A^- + \pi_A^+$ and $\pi_A^-/\pi_A^+ = 11.5$ (0.23/0.02),⁸ we can write $\pi_A^+ = (1 - \pi_0)/12.5$. Replacing π_A^+ in Equation (A9), we have:

$$\begin{aligned} E(S_\gamma^+) &= \gamma/2 \cdot \pi_0 + (1 - \pi_0)/12.5 \cdot \text{prob}(t > t_\gamma^+ | H_A, \alpha_A > 0) \\ &= c + \pi_0(\gamma/2 - \text{prob}(t > t_\gamma^+ | H_A, \alpha_A > 0)/12.5) \\ &= c + \pi_0(\gamma/2 - b), \end{aligned} \quad (\text{A10})$$

where $c = (1/12.5) \cdot \text{prob}(t > t_\gamma^+ | H_A, \alpha_A > 0)$, and $b = \text{prob}(t > t_\gamma^+ | H_A, \alpha_A > 0)/12.5$. Using this result, the average value of the estimators of skilled funds under both approaches can be written as a function of π_0 :

$$\begin{aligned} \text{No luck} &: E(\widehat{T}_\gamma^+) = E(S_\gamma^+) = c + \pi_0(\gamma/2 - b), \\ \text{Full luck} &: E(\widehat{T}_\gamma^+) = E(S_\gamma^+) - \gamma/2 = d + \pi_0(\gamma/2 - b), \end{aligned} \quad (\text{A11})$$

where d is a constant: $d = c - \gamma/2$. Equation (A11) reveals that an increase in π_0 has two contradictory effects on $E(\widehat{T}_\gamma^+)$. On the one hand, it increases the expected proportion of lucky funds which are wrongly included in \widehat{T}_γ^+ (through $\gamma/2$). On the other hand, it decreases the proportion of skilled funds in the population (through b) (i.e., a rise in π_0 leads to a decline in π_A^+).

If $\gamma/2 > b$, the “no luck” and “full luck” approaches produce a nonsensical positive relation between $E(T_\gamma^+)$ and π_0 , as it is the case in Figure 3 of the paper at $\gamma = 0.20$.⁹ On the contrary, when $\gamma/2 < b$, the relation is negative, as in Figure A2.¹⁰ Therefore, the slope is positive when the proportion of skilled funds in the population is low (as

⁸The ratio, π_A^-/π_A^+ , is held fixed at 11.5 both in Figure 3 of the paper and in Figure A2 to guarantee that, as π_0 varies, the proportion of skilled funds remains low compared to the unskilled funds.

⁹Under $H_A, \alpha_A > 0$, the fund t -statistic follows a non-central student distribution (see Appendix B.1) Using $t_\gamma^+ = 1.28$, $T = 384$ (the number of observations), and a t -mean equal to 3 (the noncentrality parameter), we find that $b = 0.96/12.5 = 0.076$, implying that $b < \gamma/2 = 0.10$.

¹⁰Under $H_A, \alpha_A > 0$, the fund t -statistic follows a non-central student distribution (see Appendix B.1) Using $t_\gamma^+ = 1.65$, $T = 384$ (the number of observations), and a t -mean equal to 3 (the noncentrality parameter), we find that $b = 0.89/12.5 = 0.071$, implying that $b > \gamma/2 = 0.05$.

we empirically find), and when the probability of finding a lucky fund, $\gamma/2$ is high (as in Figure 3 of the paper, where γ is set equal to 0.20).

Finally, note that our FDR approach is immune to this problem, as the average value of its estimator is always negatively related to π_0 :

$$\text{FDR approach: } E(\widehat{T}_\gamma^+) = E(S_\gamma^+) - \pi_0 \cdot \gamma/2 = c - \pi_0 b. \quad (\text{A12})$$

The reason is that the luck adjustment, $\pi_0 \cdot \gamma/2$, depends on π_0 , and correctly captures the increase in $E(S_\gamma^+)$ due to the inclusion of the additional lucky funds.

D.4 Fund Selection From a Bayesian Perspective

Instead of controlling the False Discovery Rate (FDR^+) as in the paper, the Bayesian approach to fund selection consists in minimizing the investor's loss function. We denote by G_i , a random variable which takes the value of -1 if fund i is unskilled, 0 if it has zero alpha, and +1 if it is skilled. The prior probabilities for the three possible values (-1, 0, +1) are given by the proportion of each skill group in the population, π_A^- , π_0 , and π_A^+ .

In her attempt to determine whether to include fund i ($i = 1, \dots, M$) in her portfolio, the Bayesian investor is subject to two sorts of misclassification. First, she may wrongly include a zero-alpha fund in the portfolio (i.e., rejecting H_0 , while it is true). Second, she may fail to include a skilled fund in the portfolio (i.e., accepting H_0 , while it is wrong). Following Storey (2003), we can model the investor's loss function, BE , as a weighted average of each misclassification type:

$$BE(\Gamma^+) = (1 - \psi) \text{prob}(T_i \in \Gamma^+) \cdot \text{fdr}_\gamma^+(\Gamma^+) + \psi \cdot \text{prob}(T_i \notin \Gamma^+) \cdot \text{fnr}_\gamma^+(\Gamma^+), \quad (\text{A13})$$

where T_i is the t -statistic of fund i , $\text{fdr}^+(\Gamma^+) = \text{prob}(G_i = 0 | T_i \in \Gamma^+)$ is the False Discovery Rate (i.e., the probability of falsely including zero-alpha funds), $\text{fnr}^+(\Gamma^+) = \text{prob}(G_i = +1 | T_i \notin \Gamma^+)$ is the False Nondiscovery Rate (i.e., the probability of failing to detect skilled funds), and ψ is a cost parameter which can be interpreted as the investor's regret after failing to detect skilled funds.

The decision problem faced by the Bayesian investor is to choose the significance threshold, $t^+(\psi)$, such that $\Gamma^+(\psi) = (t^+(\psi), +\infty)$ minimizes Equation (A13). After updating her prior belief using fund i observed t -statistic, \widehat{t}_i , she will include fund i in her portfolio only if the posterior loss incurred when wrongly considering a skilled fund as a zero-alpha fund is larger than the posterior loss incurred when wrongly considering

a zero-alpha fund as skilled:

$$\psi \cdot \text{prob}(G_i = +1 | T_i = \hat{t}_i) > (1 - \psi) \text{prob}(G_i = 0 | T_i = \hat{t}_i), \quad (\text{A14})$$

or equivalently, if \hat{t}_i belongs to the significance region, Γ^+ (see Storey (2003)):

$$\Gamma^+ = \left\{ \hat{t}_i : \frac{\pi_0 \cdot f_0(\hat{t}_i)}{\pi_0 \cdot f_0(\hat{t}_i) + \pi_A^+ \cdot f_A(\hat{t}_i)} \leq \psi \right\}, \quad (\text{A15})$$

where $f_0(\hat{t}_i) = \text{prob}(T_i = \hat{t}_i | G_i = 0)$ and $f_A(\hat{t}_i) = \text{prob}(T_i = \hat{t}_i | G_i = +1)$. The optimal significance threshold, $t^+(\psi)$, is therefore defined as

$$t^+(\psi) = t^+ : \frac{\pi_0 \cdot f_0(t^+)}{\pi_0 \cdot f_0(t^+) + \pi_A^+ \cdot f_A(t^+)} = \psi. \quad (\text{A16})$$

Equation (A15) reveals that a Bayesian approach requires an extensive parameterization, contrary to the frequentist approach used in the paper. This includes the exact specification of the null and alternative distributions, $f_0(\hat{t}_i)$ and $f_A(\hat{t}_i)$, the cost parameter, ψ , as well as the assumptions that the t -statistics are IID and homogeneous across the population (i.e., $f_0(\hat{t}_i)$ and $f_A(\hat{t}_i)$ must be similar across the individual test statistics).¹¹ In addition, a full Bayesian analysis requires to posit prior distributions for the proportions π_0 , π_A^- , and π_A^+ , and for the distribution parameters of $f_0(\hat{t}_i)$ and $f_A(\hat{t}_i)$.

Our frequentist approach to fund selection (Section III.C of the paper) consists in controlling the FDR^+ of the portfolio at some specific target z^+ ($z^+ = 10\%$, 30% , 50% , 70% , and 90%). If we agree to make the additional parameterization mentioned above, we can use Equation (A16) to determine the optimal Bayesian decision implied by each FDR^+ target. To illustrate, let us consider the hypothetical example presented in Figure 1 of the paper, where the individual fund t -statistic distributions for the three skill groups are normal, and centered at -2.5, 0, and 3.0, respectively (with a unit variance). The proportions of zero-alpha, unskilled, and skilled funds in the population (π_0 , π_A^- , and π_A^+) are equal to 75%, 23%, and 2%, respectively. Since these values are directly estimated from the data (see Appendix B.1), this example should provide a realistic analysis of the relation between the frequentist and Bayesian approaches.

First, we determine the significance threshold, $t^+(z^+)$, such that the fdr^+ is equal

¹¹See Efron et al. (2001) and Storey (2003) for further discussion in the context of genomics.

to the chosen target z^+ :

$$t^+(z^+) = t^+ : fdr^+ = \frac{\pi_0(1 - \Phi(t^+; 0, 1))}{\pi_0(1 - \Phi(t^+; 0, 1)) + \pi_A^+(1 - \Phi(t^+; 3, 1))} = z^+, \quad (\text{A17})$$

where $\Phi(x; \mu, \sigma^2) = \text{prob}(X < x; \mu, \sigma^2)$ is the cumulative distribution function of a normal random variable X with mean μ and variance σ^2 . In Section III.C of the paper, we use the significance level, γ , (related to p -values) as opposed to the significance threshold, t^+ , (related to t -statistics). Using the definition of a p -value, we can easily determine its value from $t^+(z^+) : \gamma(z^+) = 2 \cdot (1 - \Phi(t^+(z^+); 0, 1))$. Second, we use Equation (A16) to determine the implied cost parameter, $\psi(z^+) :$

$$\psi(z^+) = \frac{\pi_0 \cdot \phi(t^+(z^+); 0, 1)}{\pi_0 \cdot \phi(t^+(z^+); 0, 1) + \pi_A^+ \cdot \phi(t^+(z^+); 3, 1)}, \quad (\text{A18})$$

where $\phi(x; \mu, \sigma^2)$ is the density of the normal distribution with mean μ and variance σ^2 (at the point $X = x$). Finally, using $t^+(z^+)$ and $\psi(z^+)$, we can easily determine the implied False Nondiscovery rate, $fnr^+(z^+)$, and the Bayesian loss function, $BE(z^+)$.

In Table AVI we display the significance threshold, $t^+(z^+)$, the significance level $\gamma(z^+)$, the cost parameter, $\psi(z^+)$, the $fnr^+(z^+)$, and the loss function, $BE(z^+)$ implied by the five FDR^+ targets, z^+ , chosen in the paper ($z^+ = 10\%$, 30% , 50% , 70% , and 90%). We observe that fixing a high FDR^+ target (such as 90%) is consistent with the behavior of a Bayesian investor with a high cost of regret, $\psi(90\%) = 0.997$. Therefore, she chooses a very high significance level, $\gamma(90\%) = 0.477$, in order to include the vast majority of the skilled funds in the portfolio ($fnr^+(90\%)$ is essentially equal to zero). On the contrary, a low FDR^+ target (such as 10%) implies a low cost parameter, $\psi(10\%) = 0.318$. In this case, the Bayesian investor sets a very high significance threshold, $t_\gamma^+(10\%) = 2.96$ (a low significance level, $\gamma(10\%) = 0.003$), in order to avoid including a large proportion of zero-alpha funds in the portfolio.

Please insert Table AVI here

References

- [1] Davidson, Russell, and James G. MacKinnon, 2004, *Econometric Theory and Methods*, Oxford University Press, New-York.
- [2] Efron, Bradley, Robert Tibshirani, John D. Storey, and Virginia Tusher, 2001, Empirical Bayes Analysis of a Microarray Experiment, *Journal of the American Statistical Association* 96, 1151-1160.
- [3] Genovese, Christopher, and Larry Wasserman, 2004, A Stochastic Process Approach to False Discovery Control, *Annals of Statistics*, 32, 1035-1061.
- [4] Jones, Christopher S., and Jay Shanken, 2005, Mutual Fund Performance with Learning across Funds, *Journal of Financial Economics* 78, 507-552.
- [5] Storey, John D., 2002, A Direct Approach to False Discovery Rates, *Journal of the Royal Statistical Society* 64, 479-498.
- [6] Storey, John D., 2003, The Positive False Discovery Rate: A Bayesian Interpretation and the q-Value, *Annals of Statistics* 31, 2013-2035.
- [7] Storey, John D., Jonathan E. Taylor, and David Siegmund, 2004, Strong Control, Conservative Point Estimation and Simultaneous Conservative Consistency of False Discovery Rates: A Unified Approach, *Journal of the Royal Statistical Society* 66, 187-205.

Table AI
Monte-Carlo Analysis under Cross-Sectional Independence

We examine the average value and the 90%-confidence interval (in parentheses) of the different estimators based on 1,000 replications. For each replication, we generate monthly fund returns for 1,400 funds and 384 periods using the four-factor model (market, size, book-to-market, and momentum factors). Fund residuals are independent from one another. The true parameter values for the proportions of zero-alpha, unskilled, and skilled funds ($\pi_0, \pi_A^-,$ and π_A^+) are set to 75%, 23%, and 2%. We set the true four-factor annual alpha equal to -3.2% for the unskilled funds and +3.8% for the skilled ones. In each tail (left and right), we assess the precision of the different estimators at two significance levels ($\gamma=0.05$ and 0.20).

Fund Proportion	True	Estimator (90% interval)		
Zero-alpha funds (π_0)	75.0	75.1 (71.7,78.6)		
Unskilled funds (π_A^-)	23.0	22.9 (19.7,25.9)		
Skilled funds (π_A^+)	2.0	2.0 (0.3,3.8)		
	Significance level $\gamma = 0.05$		Significance level $\gamma = 0.20$	
Left Tail	True	Estimator (90% interval)	True	Estimator (90% interval)
Significant funds $E(S_\gamma^-)$	18.1	18.1 (16.4,19.7)	27.9	27.9 (26.1,30.0)
Unlucky funds $E(F_\gamma^-)$	1.8	1.8 (1.8,1.9)	7.5	7.5 (7.1,7.9)
Unskilled funds $E(T_\gamma^-)$	16.2	16.2 (14.6,17.9)	20.4	20.4 (18.2,22.7)
	Significance level $\gamma = 0.05$		Significance level $\gamma = 0.20$	
Right Tail	True	Estimator (90% interval)	True	Estimator (90% interval)
Significant funds $E(S_\gamma^+)$	3.6	3.6 (2.8,4.4)	9.4	9.4 (8.2,10.8)
Lucky funds $E(F_\gamma^+)$	1.8	1.8 (1.8,1.9)	7.5	7.5 (7.1,7.9)
Skilled funds $E(T_\gamma^+)$	1.7	1.7 (0.9,2.5)	1.9	1.9 (0.5,3.3)

Table AII

Monte-Carlo Analysis under Cross-Sectional Dependence

We examine the average value and the 90%-confidence interval (in parentheses) of the different estimators based on 1,000 replications. For each replication, we generate monthly fund returns for 1,400 funds and 384 periods using the four-factor model (market, size, book-to-market, and momentum factors). We assume that funds are cross-sectionally correlated and use the empirical covariance matrix of the fund residuals as the true covariance matrix. The true parameter values for the proportions of zero-alpha, unskilled, and skilled funds ($\pi_0, \pi_A^-,$ and π_A^+) are set to 75%, 23%, and 2%. We set the true four-factor annual alpha equal to -3.2% for the unskilled funds and +3.8% for the skilled ones. In each tail (left and right), we assess the precision of the different estimators at two significance levels ($\gamma=0.05$ and 0.20).

Fund Proportion	True	Estimator (90% interval)		
Zero-alpha funds (π_0)	75.0	75.2 (69.5,80.8)		
Unskilled funds (π_A^-)	23.0	22.8 (17.0,28.9)		
Skilled funds (π_A^+)	2.0	1.9 (0.0,6.5)		
	Significance level $\gamma = 0.05$		Significance level $\gamma = 0.20$	
Left Tail	True	Estimator (90% interval)	True	Estimator (90% interval)
Significant funds $E(S_\gamma^-)$	18.1	18.1 (15.3,20.7)	27.9	27.9 (24.3,32.3)
Unlucky funds $E(F_\gamma^-)$	1.8	1.8 (1.6,2.1)	7.5	7.6 (6.6,8.3)
Unskilled funds $E(T_\gamma^-)$	16.2	16.2 (13.4,19.1)	20.4	20.4 (16.3,24.6)
	Significance level $\gamma = 0.05$		Significance level $\gamma = 0.20$	
Right Tail	True	Estimator (90% interval)	True	Estimator (90% interval)
Significant funds $E(S_\gamma^+)$	3.5	3.6 (2.4,5.3)	9.4	9.4 (6.6,12.6)
Lucky funds $E(F_\gamma^+)$	1.8	1.8 (1.6,2.1)	7.5	7.6 (6.6,8.3)
Skilled funds $E(T_\gamma^+)$	1.7	1.7 (0.5,3.8)	1.9	1.9 (0.1,5.6)

Table AIII
Impact of Luck on Long-Term Performance

We measure long-term performance with the unconditional four-factor model over the entire period 1975-2006 for three investment categories (Growth, Aggressive Growth, and Growth & Income funds) shown in Panels A, B, and C, respectively. In each panel, we display the estimated proportions of zero-alpha, unskilled, and skilled funds ($\hat{\pi}_0$, $\hat{\pi}_A^-$, and $\hat{\pi}_A^+$) in the entire investment category. We also count the proportions of significant funds in the left and right tails of the cross-sectional t -statistic distribution (\hat{S}_γ^- , \hat{S}_γ^+) at four significance levels ($\gamma=0.05, 0.10, 0.15, 0.20$). In the leftmost columns, the significant group in the left tail, \hat{S}_γ^- , is decomposed into unlucky and unskilled funds (\hat{F}_γ^- , \hat{T}_γ^-). In the rightmost columns, the significant group in the right tail, \hat{S}_γ^+ , is decomposed into lucky and skilled funds (\hat{F}_γ^+ , \hat{T}_γ^+). Finally, we present the characteristics of each significant group (\hat{S}_γ^- , \hat{S}_γ^+): the average estimated alpha (% per year), expense ratio (% per year), and turnover (% per year). Figures in parentheses denote the standard deviation of the different estimators.

Panel A Growth funds										
Proportion of Unskilled and Skilled Funds										
	Zero alpha ($\hat{\pi}_0$)		Non-zero alpha		Unskilled ($\hat{\pi}_A^-$)		Skilled ($\hat{\pi}_A^+$)			
Proportion	76.5 (3.2)		23.5		23.5 (2.9)		0.0 (1.0)			
Number	985		319		319		0			
Impact of Luck in the Left and Right Tails										
	Left Tail				Right Tail					
Signif. level (γ)	0.05	0.10	0.15	0.20	0.20	0.15	0.10	0.05	Signif. level (γ)	
Signif. \hat{S}_γ^- (%)	10.8 (0.8)	16.1 (1.0)	20.5 (1.1)	24.2 (1.2)	7.6 (0.7)	5.7 (0.6)	3.8 (0.5)	1.9 (0.4)	Signif. \hat{S}_γ^+ (%)	
Unlucky \hat{F}_γ^- (%)	1.9 (0.1)	3.8 (0.2)	5.7 (0.2)	7.6 (0.3)	7.6 (0.3)	5.7 (0.2)	3.8 (0.2)	1.9 (0.1)	Lucky \hat{F}_γ^+ (%)	
Unskilled \hat{T}_γ^- (%)	8.9 (0.8)	12.3 (1.1)	14.8 (1.2)	16.6 (1.3)	0.0 (0.8)	0.0 (0.7)	0.0 (0.6)	0.0 (0.4)	Skilled \hat{T}_γ^+ (%)	
Alpha(% year)	-5.5 (0.3)	-4.9 (0.2)	-4.6 (0.2)	-4.4 (0.1)	5.0 (0.4)	5.3 (0.5)	5.8 (0.6)	7.1 (0.9)	Alpha(% year)	
Exp.(% year)	1.4	1.4	1.4	1.4	1.3	1.2	1.2	1.2	Exp.(% year)	
Turn.(% year)	105	100	98	98	93	90	87	99	Turn.(% year)	

Table AIII
Impact of Luck on Long-Term Performance

Panel B Aggressive Growth funds									
Proportion of Unskilled and Skilled Funds									
	Zero alpha ($\hat{\pi}_0$)		Non-zero alpha		Unskilled ($\hat{\pi}_A^-$)		Skilled ($\hat{\pi}_A^+$)		
Proportion	75.5 (4.5)		24.5		20.6 (4.0)		3.9 (2.1)		
Number	293		95		80		15		
Impact of Luck in the Left and Right Tails									
	Left Tail				Right Tail				
Signif. level (γ)	0.05	0.10	0.15	0.20	0.20	0.15	0.10	0.05	Signif. level (γ)
Signif. \hat{S}_γ^- (%)	10.5 (1.5)	17.5 (1.9)	21.1 (2.0)	23.2 (2.1)	10.6 (1.5)	7.8 (1.3)	5.5 (1.0)	3.4 (0.9)	Signif. \hat{S}_γ^+ (%)
Unlucky \hat{F}_γ^- (%)	1.8 (0.1)	3.8 (0.2)	5.7 (0.3)	7.5 (0.5)	7.5 (0.5)	5.7 (0.3)	3.8 (0.2)	1.9 (0.1)	Lucky \hat{F}_γ^+ (%)
Unskilled \hat{T}_γ^- (%)	8.7 (1.6)	13.7 (2.0)	15.4 (2.2)	15.7 (2.4)	3.1 (1.7)	2.1 (1.4)	1.7 (1.1)	1.5 (0.9)	Skilled \hat{T}_γ^+ (%)
Alpha(% year)	-7.8 (0.4)	-6.8 (0.3)	-6.4 (0.3)	-6.2 (0.2)	5.4 (0.5)	6.0 (0.6)	6.7 (1.0)	7.0 (1.4)	Alpha(% year)
Exp.(% year)	1.6	1.6	1.6	1.6	1.3	1.2	1.2	1.3	Exp.(% year)
Turn.(% year)	121	120	118	119	119	116	130	134	Turn.(% year)

Panel C Growth & Income funds									
Proportion of Unskilled and Skilled Funds									
	Zero alpha ($\hat{\pi}_0$)		Non-zero alpha		Unskilled ($\hat{\pi}_A^-$)		Skilled ($\hat{\pi}_A^+$)		
Proportion	69.3 (4.8)		30.7		30.7 (4.4)		0.0 (1.7)		
Number	267		117		117		0		
Impact of Luck in the Left and Right Tails									
	Left Tail				Right Tail				
Signif. level (γ)	0.05	0.10	0.15	0.20	0.20	0.15	0.10	0.05	Signif. level (γ)
Signif. \hat{S}_γ^- (%)	15.4 (1.8)	20.2 (2.0)	25.2 (2.2)	31.5 (2.4)	7.0 (1.2)	5.2 (1.0)	3.3 (0.9)	1.9 (0.7)	Signif. \hat{S}_γ^+ (%)
Unlucky \hat{F}_γ^- (%)	1.9 (0.1)	3.3 (0.2)	5.2 (0.4)	7.0 (0.5)	7.0 (0.5)	5.2 (0.4)	3.3 (0.2)	1.9 (0.1)	Lucky \hat{F}_γ^+ (%)
Unskilled \hat{T}_γ^- (%)	13.5 (0.8)	16.9 (2.1)	20.0 (2.4)	24.5 (2.6)	0.0 (1.4)	0.0 (1.2)	0.0 (1.0)	0.0 (0.7)	Skilled \hat{T}_γ^+ (%)
Alpha(% year)	-3.8 (0.2)	-3.5 (0.2)	-3.3 (0.2)	-3.1 (0.1)	2.9 (0.4)	3.1 (0.5)	3.1 (0.8)	3.5 (1.1)	Alpha(% year)
Exp.(% year)	1.3	1.3	1.3	1.3	1.1	1.1	0.9	0.9	Exp.(% year)
Turn.(% year)	73	71	69	70	61	76	78	75	Turn.(% year)

Table AIV
Impact of Luck on Short-Term Performance

We measure short-term performance with the unconditional four-factor model with the unconditional four-factor model over non-overlapping 5-year periods between 1977-2006 for three investment categories (Growth, Aggressive Growth, and Growth & Income funds) shown in Panels A, B, and C, respectively. The different estimates shown in the table are computed from the pooled alpha p -values across all 5-year periods. In each panel, we display the estimated proportions of zero-alpha, unskilled, and skilled funds ($\hat{\pi}_0$, $\hat{\pi}_A^-$, and $\hat{\pi}_A^+$) in the entire investment category. We also count the proportions of significant funds in the left and right tails of the cross-sectional t -statistic distribution (\hat{S}_γ^- , \hat{S}_γ^+) at four significance levels ($\gamma=0.05, 0.10, 0.15, 0.20$). In the leftmost columns, the significant group in the left tail, \hat{S}_γ^- , is decomposed into unlucky and unskilled funds (\hat{F}_γ^- , \hat{T}_γ^-). In the rightmost columns, the significant group in the right tail, \hat{S}_γ^+ , is decomposed into lucky and skilled funds (\hat{F}_γ^+ , \hat{T}_γ^+). Finally, we present the characteristics of each significant group (\hat{S}_γ^- , \hat{S}_γ^+): the average estimated alpha (% per year), expense ratio (% per year), and turnover (% per year). Figures in parentheses denote the standard deviation of the different estimators.

Panel A Growth funds									
Proportion of Unskilled and Skilled Funds									
	Zero alpha ($\hat{\pi}_0$)	Non-zero alpha		Unskilled ($\hat{\pi}_A^-$)	Skilled ($\hat{\pi}_A^+$)				
Proportion	73.0 (2.3)	27.0		24.4(2.1)	2.6 (0.9)				
Number	1,442	534		483	51				
Impact of Luck in the Left and Right Tails									
	Left Tail				Right Tail				
Signif. level (γ)	0.05	0.10	0.15	0.20	0.20	0.15	0.10	0.05	Signif. level (γ)
Signif. \hat{S}_γ^- (%)	11.3 (0.7)	16.6 (0.8)	21.4 (0.9)	25.2 (1.0)	9.9 (0.7)	8.1 (0.6)	5.9 (0.5)	3.5 (0.4)	Signif. \hat{S}_γ^+ (%)
Unlucky \hat{F}_γ^- (%)	1.8 (0.0)	3.6 (0.1)	5.5 (0.2)	7.3 (0.2)	7.3 (0.2)	5.5 (0.2)	3.6 (0.1)	1.8 (0.0)	Lucky \hat{F}_γ^+ (%)
Unskilled \hat{T}_γ^- (%)	9.5 (0.7)	13.0 (0.9)	15.9 (1.0)	17.9 (1.1)	2.6 (0.8)	2.6 (0.7)	2.3 (0.6)	1.7 (0.4)	Skilled \hat{T}_γ^+ (%)
Alpha(% year)	-6.0 (0.3)	-5.6 (0.2)	-5.2 (0.2)	-5.1 (0.1)	6.8 (0.3)	6.8 (0.4)	6.8 (0.6)	7.3 (0.9)	Alpha(% year)
Exp.(% year)	1.4	1.3	1.3	1.3	1.2	1.2	1.2	1.2	Exp.(% year)
Turn.(% year)	98	96	96	97	79	79	78	79	Turn.(% year)

Table AIV
Impact of Luck on Short-Term Performance

Panel B Aggressive Growth funds									
Proportion of Unskilled and Skilled Funds									
	Zero alpha ($\hat{\pi}_0$)	Non-zero alpha		Unskilled ($\hat{\pi}_A^-$)	Skilled ($\hat{\pi}_A^+$)				
Proportion	71.8 (4.2)	28.2		24.0 (3.8)	4.2 (1.7)				
Number	436	171		145	26				
Impact of Luck in the Left and Right Tails									
	Left Tail				Right Tail				
Signif. level (γ)	0.05	0.10	0.15	0.20	0.20	0.15	0.10	0.05	Signif. level (γ)
Signif. \hat{S}_γ^- (%)	12.0 (1.1)	16.0 (1.4)	19.4 (1.6)	22.2 (1.8)	11.2 (1.3)	9.4 (1.1)	7.1 (1.0)	4.9 (0.8)	Signif. \hat{S}_γ^+ (%)
Unlucky \hat{F}_γ^- (%)	1.8 (0.1)	3.6 (0.2)	5.4 (0.3)	7.2 (0.4)	7.2 (0.4)	5.4 (0.3)	3.6 (0.2)	1.8 (0.1)	Lucky \hat{F}_γ^+ (%)
Unskilled \hat{T}_γ^- (%)	10.2 (1.3)	12.4 (1.6)	14.0 (1.7)	15.0 (1.9)	4.0 (1.4)	4.0 (1.2)	3.5 (1.1)	3.1 (0.9)	Skilled \hat{T}_γ^+ (%)
Alpha(% year)	-9.3 (0.6)	-8.6 (0.4)	-8.1 (0.4)	-7.6 (0.3)	8.5 (0.4)	8.8 (0.7)	9.7 (1.0)	9.7 (1.1)	Alpha(% year)
Exp.(% year)	1.5	1.5	1.5	1.5	1.4	1.3	1.3	1.3	Exp.(% year)
Turn.(% year)	116	113	111	109	105	104	107	104	Turn.(% year)

Panel C Growth & Income funds									
Proportion of Unskilled and Skilled Funds									
	Zero alpha ($\hat{\pi}_0$)	Non-zero alpha		Unskilled ($\hat{\pi}_A^-$)	Skilled ($\hat{\pi}_A^+$)				
Proportion	74.1 (3.8)	25.9		25.9 (3.5)	0.0 (1.4)				
Number	540	188		188	0				
Impact of Luck in the Left and Right Tails									
	Left Tail				Right Tail				
Signif. level (γ)	0.05	0.10	0.15	0.20	0.20	0.15	0.10	0.05	Signif. level (γ)
Signif. \hat{S}_γ^- (%)	11.6 (1.1)	17.4 (1.4)	22.5 (1.5)	26.8 (1.6)	7.3 (1.0)	5.5 (0.8)	3.7 (0.7)	1.8 (0.5)	Signif. \hat{S}_γ^+ (%)
Unlucky \hat{F}_γ^- (%)	1.8 (0.1)	3.7 (0.2)	5.5 (0.3)	7.3 (0.4)	7.3 (0.4)	5.5 (0.3)	3.7 (0.2)	1.8 (0.1)	Lucky \hat{F}_γ^+ (%)
Unskilled \hat{T}_γ^- (%)	9.8 (1.2)	13.7 (1.5)	17.0 (1.7)	19.5 (1.8)	0.0 (1.1)	0.0 (0.9)	0.0 (0.8)	0.0 (0.5)	Skilled \hat{T}_γ^+ (%)
Alpha(% year)	-4.9 (0.3)	-4.5 (0.2)	-4.2 (0.2)	-4.0 (0.1)	4.9 (0.5)	5.3 (0.6)	5.1 (0.8)	4.9 (1.3)	Alpha(% year)
Exp.(% year)	1.3	1.2	1.2	1.2	1.1	1.1	1.0	0.9	Exp.(% year)
Turn.(% year)	69	69	67	64	59	59	54	45	Turn.(% year)

Table AV

Comparison between the Bootstrap and the Fixed-Value Procedures

In a population of $M=2,076$ funds, we draw each fund t -statistic from one of the distributions in Figure 1 of the paper (Panel A) according to the proportion of zero-alpha, unskilled, and skilled funds in the population ($\pi_0 = 75\%$, $\pi_A^- = 23\%$, and $\pi_A^+ = 2\%$). Then, we compute the p -values of each fund, from which the different proportions are estimated. In Panel A, we compare the estimated proportion of zero-alpha funds, $\hat{\pi}_0$, computed with the bootstrap procedure (Bootstrap) and the fixed-value procedure (Fixed Value) where λ^* is set to 0.5 and 0.6, respectively. The last column shows the difference in $\hat{\pi}_0$ between the two approaches. In Panel B, we compare the estimated proportions of unskilled and skilled funds, $\hat{\pi}_A^-$ and $\hat{\pi}_A^+$, computed with the bootstrap procedure (Bootstrap) and the fixed-value procedure (Fixed Value) where γ^* is set to 0.35 and 0.45, respectively. The last column shows the difference in $\hat{\pi}_A^+$ between the two approaches (the difference in $\hat{\pi}_A^-$ is identical (but with opposite sign), because of the equality $1-\hat{\pi}_0=\hat{\pi}_A^-+\hat{\pi}_A^+$). To assess the estimator sample variability, we run 10 simulations. All figures are expressed in percent.

Panel A Proportion of Zero-Alpha Funds

Simulation	Bootstrap $\hat{\pi}_0$	Fixed Value ($\lambda = 0.5, 0.6$)		Difference in $\hat{\pi}_0$	
		$\hat{\pi}_0(0.5)$	$\hat{\pi}_0(0.6)$	0.5	0.6
1	74.6	74.5	74.7	0.1	-0.1
2	75.4	76.2	76.4	-0.8	-1.0
3	74.9	74.8	74.6	0.1	0.3
4	78.3	78.9	78.5	-0.6	-0.2
5	78.1	78.1	78.4	0.0	-0.3
6	76.7	77.3	76.8	-0.6	-0.1
7	79.3	79.6	79.5	-0.3	-0.2
8	73.2	74.0	73.2	-0.8	0.0
9	74.5	75.0	74.8	-0.5	-0.3
10	78.0	78.2	78.5	-0.2	-0.5

Panel B Proportions of Unskilled and Skilled Funds

Simulation	Bootstrap		Fixed Value ($\gamma = 0.35, 0.45$)				Difference in $\hat{\pi}_A^+$	
	$\hat{\pi}_A^-$	$\hat{\pi}_A^+$	$\hat{\pi}_A^-(0.35)$	$\hat{\pi}_A^+(0.35)$	$\hat{\pi}_A^-(0.45)$	$\hat{\pi}_A^+(0.45)$	0.35	0.45
1	22.6	2.9	21.5	4.1	23.4	2.1	-1.2	0.8
2	22.0	1.9	22.2	1.6	22.8	1.0	0.3	0.8
3	21.7	1.0	21.7	1.0	21.6	1.2	0.0	-0.2
4	23.2	2.1	23.2	2.1	23.3	1.9	0.0	0.2
5	23.2	0.6	23.3	0.5	22.4	1.4	0.1	-0.8
6	21.1	0.6	21.4	0.4	21.4	0.4	0.2	0.2
7	23.5	2.9	23.5	2.9	22.7	3.6	0.0	-0.7
8	21.7	2.7	22.5	2.0	21.8	2.7	0.7	0.0
9	20.6	1.9	20.7	1.7	20.6	1.9	0.2	0.0
10	21.8	0.6	21.4	0.9	21.5	0.8	-0.3	-0.3

Table AVI

Optimal Bayesian Decision implied by the FDR Target Levels

We examine the optimal Bayesian decision implied by five False Discovery Rate (FDR^+) targets ($z^+ = 10\%$, 30% , 50% , 70% , and 90%) in a fund population where the proportions of zero-alpha, unskilled, and skilled funds (π_0 , π_A^- , and π_A^+) are equal to 75%, 23%, and 2%, respectively. For each skill group, the fund estimated t -statistic is normally distributed and centered at -2.5, 0, and 3.0, respectively (with a unit variance). The significance threshold, $t^+(z^+)$ (related to the t -statistic), and significance level, $\gamma(z^+)$ (related to the p -value), are determined such that the FDR^+ of the portfolio is equal to the target z^+ . $\psi(z^+)$, $fnr^+(z^+)$ and $BE(z^+)$ denote the cost parameter, False Nondiscovery Rate, and loss function implied by the target value z^+ .

FDR^+ target z^+	Signif. $t^+(z^+)$	Signif. $\gamma(z^+)$	Cost $\psi(z^+)$	$fnr^+(z^+)$	Loss $BE(z^+)$
10%	2.96	0.003	0.318	0.98%	0.38
30%	2.39	0.017	0.719	0.55%	0.56
50%	2.00	0.045	0.891	0.33%	0.46
70%	1.57	0.116	0.967	0.16%	0.28
90%	0.71	0.477	0.997	0.02%	0.07

Figure A1
Comparison of Two p -value Histograms

This graph compares two p -value histograms of $M=2,076$ funds (as in our database). To plot these histograms, we draw each fund t -statistic from one of the distributions in Figure 1 of the paper (Panel A) according to the proportion of zero-alpha, unskilled, and skilled funds in the population (π_0 , π_A^- , and π_A^+). Then, we compute the two-sided p -values of each fund from its respective t -statistic, and plot them. For the first histogram (black bars), we assume that $\pi_0 = 100\%$, $\pi_A^- = 0\%$, and $\pi_A^+ = 0\%$ (i.e., there are only zero-alpha funds). For the second histogram (grey bars), we assume that $\pi_0 = 75\%$, $\pi_A^- = 0\%$, and $\pi_A^+ = 25\%$ (i.e., there are 75% of zero-alpha funds, and 25% of skilled funds).

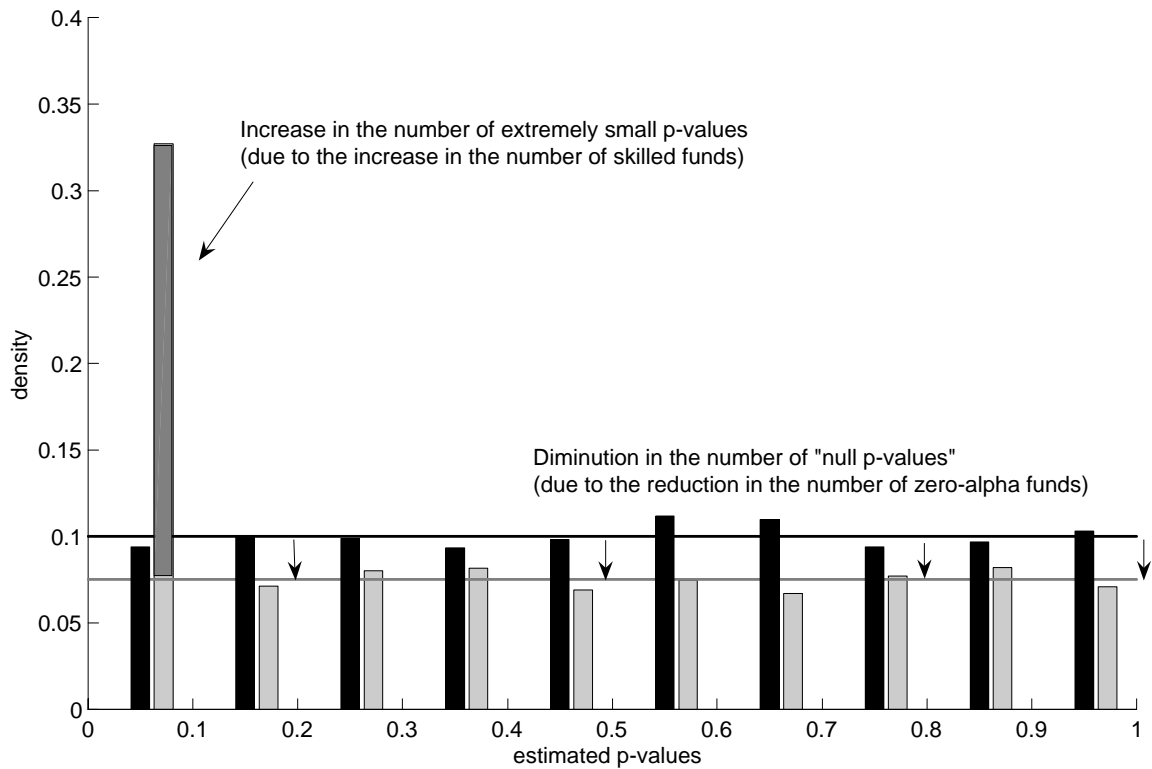
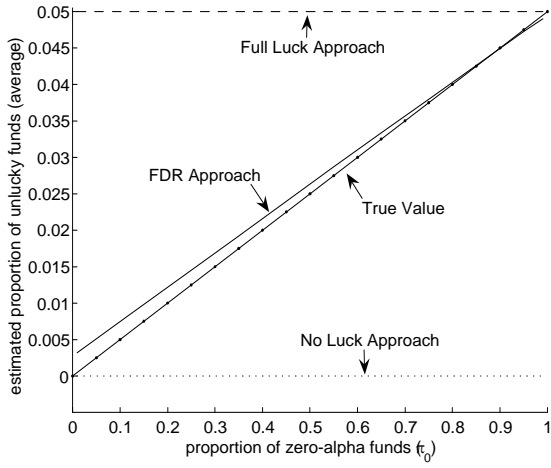


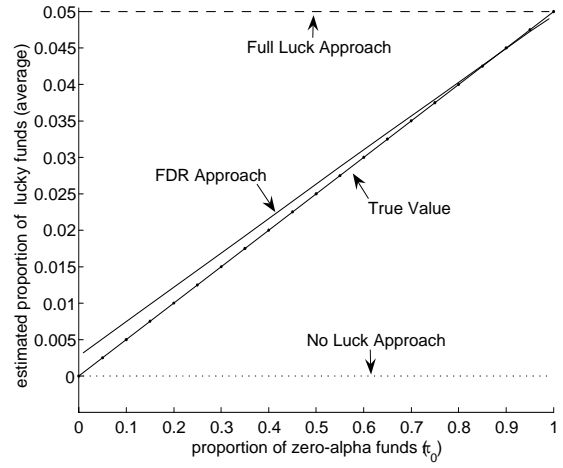
Figure A2

Measuring Luck: Comparison with Existing Approaches (at $\gamma = 0.10$)

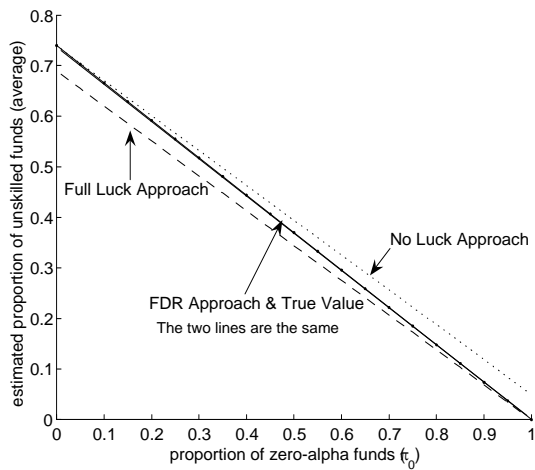
This figure examines the bias of different estimators produced by the three approaches ("no luck", "full luck", and "FDR approach") as a function of the proportion of zero-alpha funds, π_0 . We examine the estimators of the proportions of unlucky, lucky, unskilled, and skilled funds in Panel A, B, C, and D, respectively. The "no luck" approach assumes that $\pi_0=0$, the "full luck" approach assumes that $\pi_0=1$, while the "FDR approach" estimates π_0 directly from the data. For each approach, we compare the average estimator value (over 1,000 replications) with the true population value. For each replication, we draw the t -statistic for each fund i ($i=1, \dots, 2,076$) from one of the distributions in Figure 1 (Panel A) according to the weights π_0 , π_A^- , and π_A^+ , and compute the different estimators at the significance level $\gamma = 0.10$. For each π_0 , the ratio π_A^- over π_A^+ is held fixed to 11.5 (0.23/0.02) as in Figure 1 of the paper.



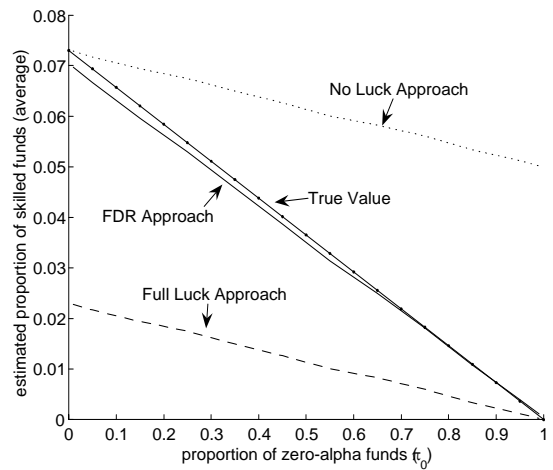
(A) Unlucky funds (left tail)



(B) Lucky funds (right tail)



(C) Unskilled funds (left tail)



(D) Skilled funds (right tail)