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The Resounding Universe

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Abstract / Resumen / Résumé

In 1977, NASA launched two spacecrafts: Voyager 1 and 2. Officially designed to study the planetary systems of Jupiter and Saturn, the two probes still gather a large amount of data and on August, 25th 2012, Voyager 1 entered the unexplored region of space known as interstellar space. Both Voyager 1 and 2 carry with them a phonograph record, the Voyager Golden Record, containing images as well as musical examples designated for any extraterrestrial life form. The immensity of the sky and the sparkling stars have inspired mankind from time immemorial and many scientists were music lovers or even musicians themselves: Galileo, Herschel, Einstein... The aim of this article is to highlight the peculiar and intriguing connection between music and astronomy.

En 1977, la NASA lanzó dos cohetes: Voyager 1 y 2. Oficialmente diseñadas para estudiar los sistemas planetarios de Júpiter y Saturno, las dos sondas aún siguen recolectando una gran cantidad de datos y el 25 de Agosto 2012, Voyager 1 entró en la región inexplorada del espacio conocida como el espacio interestelar. Tanto la Voyager 1 como la 2 llevan consigo un disco fonográfico, el Disco de Oro de Voyager que contiene imágenes, así como ejemplos musicales designados para cualquier forma de vida extraterrestre. La inmensidad del cielo y las estrellas brillantes han inspirado a la humanidad desde tiempos inmemoriales y numerosos científicos fueron apasionados de música o incluso músicos ellos mismos: Galileo, Herschel, Einstein... El objetivo de este artículo es recalcar la conexión peculiar y fascinante entre música y astronomía.

En 1977, la NASA a lancé deux sondes : Voyager 1 et 2. Officiellement conçus pour étudier les systèmes planétaires de Jupiter et de Saturne, les deux sondes recueillent toujours une grande quantité de données et le 25 août 2012, Voyager 1 est entré dans la région inexplorée de l'espace connu comme espace interstellaire. Les deux sondes Voyager 1 et 2 portent avec eux un disque phonographique, le Disque d'Or de Voyager, contenant des images ainsi que des exemples musicaux conçus pour toute forme de vie extraterrestre. L'immensité du ciel et les étoiles scintillantes ont inspiré l'humanité depuis des temps immémoriaux et de nombreux scientifiques étaient amateurs de musique ou même musiciens eux-mêmes : Galilée, Herschel, Einstein... Le but de cet article est de mettre en évidence la connexion particulière et intrigante entre la musique et l'astronomie

Palabras clave / Mots-clé / Key Words

Astronomy, Music

Astronomía, Música

Astronomie, Musique

1. Pythagoras' hammers

The relationship that humankind probed between music and the sciences, in particular with astronomy, is very ancient and can be originally found in Pythagoras (c. 570 BC - c. 495 BC). In his investigation there is no epiphany from the Muses, indeed his research follows the scientific method. His starting point is a rational study of reality. Legend would have it that while listening to the noise caused by a blacksmith's hammer striking iron atop an anvil, Pythagoras noticed a relation between the weight of the hammer and the pitch of the sound produced. Using a monochord, he discovered that a different tension of the string corresponded to notes more or less consonant with respect to a reference note. This is what nowadays is referred to as interval of octave (obtained by halving the length of the string), fifth (taking two thirds of the length), fourth (with ratio four to three), and so on.

According to Pythagoreans, simple mathematical ratios preside over the cosmos: ratios such as those ones determining consonant chords. The Universe was an harmonious system: the distance between the Earth and the fixed stars, namely the stars that do not seem to move with respect to other stars in the night sky, is represented by an octave; the distance between the Earth and the Sun is a fifth; the

distance between Mercury and the Sun as well as between the Sun and the fixed stars is a fourth. The rotation and the revolution of Sun and planets also generate continuous sounds imperceptible to human beings: the combination of these sounds creates the *Harmony of the Spheres*.

Plato (c. 428 BC – c. 347 BC) in "Republic" supports this theory and describes astronomy and music as specular disciplines. Sight and hearing are complementary senses: eyes are made for looking at celestial bodies and ears to follow their harmonious motions. Aristotle (c. 384 BC – c. 322 BC) explains why mortals cannot hear these sounds. In fact, a sound or a noise can be perceived only when in contrast with its opposite, namely the absence of the sound itself. However, the sound of the planets is present since our birth and this explains why we cannot recognise it, not having the notion of its counterpart. Later, Aristotle retracts this enthralling theory affirming that if such a sound would exist, it would be so loud as to destroy life on Earth.

Another example that demonstrates the affinity between astronomy and music theory is given by Ptolemy (c. 90 AD - c. 168 AD). He is known as a mathematician, astrologer, poet, geographer, and especially an astronomer. In fact, everyone knows the Ptolemaic system described in the "Almagest" where he introduces the geocentric model with the Sun, the Moon and the other planets orbiting around the Earth. This theory influenced the western countries as well as the Arab world (e.g.: "Zīj al-Sindhind", "Astronomical Tables of Sind and Hind", written in the 8th century by the mathematician, astronomer, and geographer Al-Khwārizmī) until the formulation of the heliocentric model of the Universe by Nicolaus Copernicus (1473 - 1543) in "De Revolutionibus Orbium Coelestium" in 1543. However Ptolemy took an interest also in Pythagoras' ideas and he wrote the essay "Harmonics" where he explains how mathematics is the tool to describe the generation of sounds. He also revised the method for tuning instruments that was used until the 15th century.

2. The tripartition of music

In the Middle Ages the theoretical speculation on music thrived again both in philosophical and scientific disci-

plines. In medieval universities, the preparatory studies comprised rhetoric, grammar, and logic, the so-called Trivium (latin word standing for "the three ways"). This was followed by the *Quadrivium* ("the four ways") based on the knowledge of the number (arithmetic), its relation with space (geometry), with time (music), and both space and time (astronomy). These seven disciplines constituted the liberal arts (in that historical period, art and science, ars et scientia in latin, were synonyms) but music was the towering pillar since it encompassed the rules of metre (the rhythmic structure of a verse) in rhetoric, the science of numbers, and the star motions. One of the most known theorists of this period is Severinus Boëthius (480 AD - c. 525 AD) who wrote "De Institutione Musica" at the beginning of the 6th century, a treatise that was a benchmark until the 14th century. Music found a new, strong bond with theology: the Musica Divina, above the spatial and temporal dimensions, which can be audible with the knowledge of the own soul. This kind of music was put above a tripartite system: the *Musica Mundana* regulates the macrocosm namely the motion of planets and stars and the sequence of the seasons; the Musica Humana governs the microcosmic harmony related to the human body; the Musica Instrumentalis, that is the instrumental music or Musica *Naturalis*, related to the sounds present in Nature.

3. Musical theory during the Renaissance

At the beginning of the 16th century, the debate on the roles of music and science reinvigorates itself once again focusing on the same dilemmas as the Pythagoras' epoch. As in the past, philosophers and scientists who gave a solid impulse to (astro)physical sciences were the same ones who revisited the musical theory. Gioseffo Zarlino (1517 – 1590), besides developing the counterpoint theory as well as the musical tuning, integrates the fundamental Pythagorean notions with the novelties introduced by Christianity, being himself a priest. Following Boëthius, Zarlino separates the temporal, mundane act of composing and playing instruments from the divine rhythmical recurring of the events such as the season cycle or the periodical regular motion of the planets. He gives some peculiar rules for composition: a musical piece will be fair if written in imitation of Nature. He states that music is a display of mathematical laws as well as the movement of celestial



bodies, then astronomy and music are two different and complementary expressions of a common mathematical structure.

Another important contribution during this historical period was given by Vincenzo Galilei (1520 – 1591), disciple of Zarlino and father of Galileo. He was part of the Camerata Fiorentina de' Bardi (Florentine Camerata) a group of philosophers, musicians, and intellectuals arguing about the relation between drama and music. The Camerata holds an important place in the history of music since it led to the birth of the Recitar Cantando (acting while singing) or rather the Melodramma. Vincenzo Galilei reintroduces the significance of performing experiments against pure theoretical mathematical considerations recalling the school of thought of Aristoxenus of Tarentum (fl. 335 BC). Aristoxenus was a disciple of Aristotle, famous for having repudiated the rationalism of Pythagorean theories according to which arithmetic controls musical intervals. In fact, the approach of Aristoxenus is empirical: the magnitude of an interval can be judged just by hearing while modifying the tension of the string. It is interesting to stress that Galileo himself effectively partook in the musical dialectic with a strong belief that his discovery about the isochronous oscillations of the pendulum could be exploited in music. In fact, modern metronomes plainly resemble upside-down pendulums.

4. Kepler discovers a chorale in the solar system

Johannes Kepler (1571 – 1630) has a central role in the development of the scientific thought in the 17th century due to his laws of planetary motions that shortly afterwards gave a strong argument to the theory of universal gravitation by Isaac Newton. Nevertheless, besides his scientific discoveries, Kepler was even more interested in investigating the "harmony of the steady/eternal things" that is the Sun, the fixed stars, and the interstellar space in-between. While Pythagoreans stated that every planet contributes with a single sound to a musical scale, namely the harmony of the spheres is monophonic, Kepler discovers that the velocity of the revolution of a planet around the Sun is not constant. As he explains in the "Harmonice Mundi", this means that each celestial body emits a scale of sounds,

so that the harmony of the spheres becomes polyphonic, made up of chords played simultaneously by different planets. Looking at the length of the orbit's arc swept by a planet at the minimum and maximum distance from the Sun (the perihelion and the aphelion, respectively), he finds a correlation between musical intervals and vocal ranges. According to this concept Mercury "sings" such as a soprano, the Earth and Venus such as altos, Mars is a tenor, while Jupiter and Saturn are basses. Of course, at this time the musical talents of Uranus and Neptune remained undiscovered.

5. The astro-musical synthesis

Paul Hindemith (1895 - 1963) was not only one the most important composers of the 20th century, but also a renowned conductor, violinist, and violist. He transposes in music the scientific considerations of Kepler who asserted that the pleasure that one feels while listening to consonant intervals -given by particular combinations of couples of sounds—is a consequence of the archetypes, the primordial essences impressed in mankind by God. In 1939 Hindemith began to elaborate on a composition about Kepler's theories and in 1952 he completes the symphony "Die Harmonie der Welt" (the Harmony of the Universe) composed of three movements labelled following the tripartition by Boëthius: Musica Instrumentalis, Musica Humana, and Musica Mundana. Four years later, Hindemith gave birth to his magnum opus, the opera "Die Harmonie der Welt". In the last scene, characters are transfigured in allegoric characters among which the Sun (the Emperor Frederick II), Jupiter (the military leader Albrecht von Wallenstein), the Earth (Kepler), Venus (Susanna Reuttinger, the second wife of Kepler), the Moon (Katharina Guldenmann, mother of Kepler). Each planet-character sings using a scale determined by Kepler according to the velocity of the celestial body and the final chord is a unison, an E sung at different octaves by the different planets.

6. Musical Examples

In the last part of the article we give some of the most representative musical compositions where the synergistic union between music and astronomy found its maximum



expression, an intimate bond between two realms apparently very far away from each other.

"Three Irish Legends" – Henry Cowell

Henry Cowell (1897 – 1965) was one of the most innovative and original musicians who influenced many composers of the 20th century. He was among the firsts to make an extensive use of *clusters* on piano, namely chords composed of at least three adjacent tones in a scale. These were commonly very extended in Cowell's scores, having to be played using fists, palms, and forearms. Later, the Hungarian composer Béla Bartók (1881 – 1945) even asked for authorisation from Cowell to make use of clusters in his compositions (e.g. in the piano Sonata BB 88 and the piano Suite "Outdoors"). Cowell wrote the solo piano pieces "Three Irish Legends": "The Tides of Manaunaun" (1917), "The Voice of Lir" (1920), and "The Hero Sun" (1922). These three pieces, inspired by Irish mythology, include some references to astronomy. In fact, each movement of the score references lines by the Irish-born American poet, John Osborne Varian. "The Tides of Manaunaun", probably the most famous composition by Cowell, has a clear allusion to the origin of the Universe, the quotation reciting:

Manaunaun was the god of motion, and long before the creation, he sent forth tremendous tides, which swept to and fro through the universe, and rhythmically moved the particles and materials of which the gods were later to make the suns and the worlds.

"The Voice of Lir" contains an indirect reference to the philosophical concept of indeterminism that suggests the role of chance in the creation and the development of the Universe. The verses of Varian proclaim:

Lir of the half tongue was the father of the gods, and of the universe. When he gave the orders for creation, the gods who executed his commands understood but half of what he said, owing to his having only half a tongue; with the result that for everything that has been created there is an unexpressed and concealed counterpart, which is the other half of Lir's plan of creation. Finally, the introductive poem of "The Hero Sun" is a fascinating view on the origin of our solar system:

The gods created all the suns and sent them out into space. But these suns, instead of lighting the universe, congregated closely together, enjoying each others society, and the universe was in darkness. Then one of the gods told the suns of a place where people were living in misery on account for the lack of light, and a strong young sun rose and hurled himself out into the darkness, until he came to this place, which was our earth; and the Hero Sun who sacrificed the companionship of the other suns to light the earth is our Sun.

"As Três Marias" – Heitor Villa-Lobos

Heitor Villa-Lobos (1887 – 1959) was one of the most known and notable composers of Latin America to date, representative of Neoclassicism. "As Três Marias" (1939), namely The Three Maries, is one of his most popular piano works. It is a set of three short and brilliant pieces drawing inspiration from the popular Brazilian children's tale "The Three Maries of Earth" reproduced on the frontispiece of the score:

Once there were three little girls. "The Three Maries of the Earth", who romped and played in the countryside of Brazil. They were always gay and the best of friends. Smilingly they travelled all the paths of life together.

That this trinity might serve as a perpetual symbol of the union of humanity, Destiny has preserved them as eternal stars in the heavens to illuminate the path for the other children of Earth.

The three stars which lend the name their names to the three movements are in fact those constituting the asterism of the Orion's Belt: *Alnitah*, *Alnilam*, and *Mintika*.

"Ludus Tonalis" - Paul Hindemith

Paul Hindemith summarises the musical-scientific reflections by Kepler in the opera "Die Harmonie der Welt", but in "Ludus Tonalis" he materialises the equivalent to "The



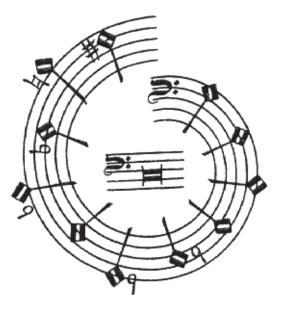


Fig. 1: The spiralling pentagram with the sequence of notes from the Series 1 by Paul Hindemith (from "The Craft of Musical Composition")

Well-Tempered Clavier" by Bach, implementing his own theory where chords and intervals are chosen according to the laws of acoustic physics. This masterpiece is composed of twelve fugues¹ interposed with eleven interludes framed by a prelude (Praeludium) and a postlude (Postludium). Each fugue is based on one of the twelve semitones, but while Bach uses the chromatic order and Chopin, Busoni, Scriabin, and Shostakovich a series of fifths, Hindemith creates a sequence (called *Series 1*) that makes use of twelve pitches that withdraw step by step from a central note, which acts as a gravitational centre. In this way, Hindemith builds a sort of solar system, a C being the sun at the centre and all the other notes/planets orbiting around. In this sense, it is remarkable the illustration that he gives in the theoretical essay "The Craft of Musical Composition" where the pentagram containing the twelve semitones of the Series 1 warps in a spiral shape around the first note, a C, the central star. The notes of this succession increase their degree of dissonance while distancing from the first note (the C). The same happens in a planetary system, where the gravitational bound with the central star decreases for farther and farther planets. Finally, Hindemith employs a brilliant expedient to circumscribe the alternation of fugues and interludes without altering their symmetry. In fact, the Postludium is exactly the retrograde inversion of the introductive *Praeludium*. This means that the *Postludium* score can be turned upside down and read backwards, finding a note-by-note correspondence with the *Praeludium*.

"Serenata per un Satellite" – Bruno Maderna

Bruno Maderna (1920 – 1973) was an Italian conductor and composer. In 1969 he wrote "Serenata per un Satellite" (Serenade for a Satellite) dedicated to Umberto Montalenti, director of the ESOC (European Space Operation Centre) in Darmstadt (Germany), who designed and coordinated the launch of ESTRO I, an European satellite for the study of phenomena related to northern lights. The "Serenata" can be played by various instruments, in groups or separately. The different musical sentences are organised in distinct pentagrams crossing the score in various directions: askew, along a straight line, from bottom to top. This allows the performers to follow different paths when playing and, concurrently, these pentagrams draw on the page a track that impressively calls to mind the orbit of a satellite traversing the space. This piece, whose duration is comprised between four and twelve minutes, represents one of the best outcomes of the inclusion of aleatoric theories in a musical score rigorously prescribed by the composer (see Section on John Cage for more details).

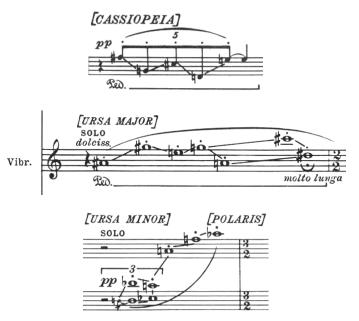
A fugue is a compositional technique in two or more voices, built on a *subject* (a main melody) that is introduced at the beginning of the piece and that recurs in the course of the composition at different pitches. Once all the voices have presented the subject, the part called Exposition ends, and a second part called Development begins. The subject of the fugue is then altered by means of the *inversion* (the subject is turned upside-down and played backwards such as in the Praeludium-Postludium of "Ludus Tonalis" by Paul Hindemith), the retrogradation (the subject is played from the last to the first note), the diminution and the augmentation (the rhythmic values of the subject are reduced or increased by a certain factor). The closing section of a fugue is the *Stretto and Coda*, whereby a voice answers with the subject before a previous voice has completed its entry, increasing the intensity of the music. Then a cadence closes the fugue with a series of chords that create a sense of resolution and conclusion.

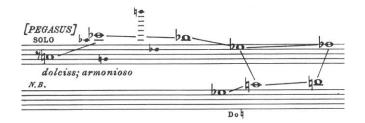
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"Sicut Umbra..." – Luigi Dallapiccola

EU-topías

Luigi Dallapiccola (1904 – 1975) was an Italian pianist and composer especially noted for having revisited the twelve-tone technique by Arnold Schönberg – also known as Serialism – by adding a deep, intense lyricism. "Sicut Umbra..." (1970), for soprano and fifteen instruments, represents one of his most innovative and original compositions. The title comes from the Book of Job in the Bible: Sicut umbra dies nostri sunt super terram (our days upon the Earth are like a shadow). The lyrics sung by the soprano come from three poems of the Spanish poet, Nobel prize, Juan Ramón Jiménez that celebrate the constellations and the cosmos: El Olvido (Forgetfulness), El Recuerdo (The Memory), Epitafio Ideal de un Marinero (Ideal Epitaph for a Sailor). In the fourth and last movement, Dallapiccola connects group of notes depicting in this way the shape of nine constellations: Volans, Cassiopeia, Columba, Ursa Major, Triangulum Australe, Ursa Minor, Pegasus, Andromeda, and Libra. In fact, as he explains in his essay "Appunti, Incontri, Meditazioni" (Annotations, Encounters, Meditations) when referring to the third poem by Jiménez, Dallapiccola attempts to render the picture of these constellations in the score by means of musical signs. Stars represent the place where a sailor searches for his own grave. Notes become one with stars and the tight, secular link between music and cosmos is restored.





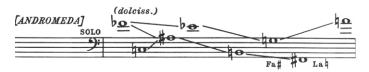


Fig. 2: Five out of the nine constellations evoked by Luigi Dallapiccola in the last movement of "Sicut Umbra..."

John Cage, he who placed notes in the sky

John Cage (1912 – 1992) was an American music theorist, composer, and writer. Disciple of Herny Cowell and Arnold Schönberg, he was the pioneer of the prepared piano, that is a piano whose sounds are altered by means of the placement of various kinds of objects and materials amongst the strings, the hammers, and the dampers. He had a strong hold on the musical aesthetics of the twentieth century, being the first musician to explore the possibility of introduce the *alea* in music, leaving to the chance some or all the elements of a composition: dynamics, pitch, timbre, order, rhythm. During his life, Cage was time after time inspired by a sky atlas compiled by the Czech astronomer Antonín Bečvář that he acquired in Prague, in 1964. To be more precise, it was a star chart of the southern sky, the "Atlas Eclipticalis". Cage put stripes of transparent paper on the pages of the atlas to isolate group of stars then, since the maps were depicted in different colours, each time he chose a colour and replaced the star with notes. The result is a series of compositions in which the listener experiences the same sensation that one has while looking at the night sky: single sounds correspond to isolated stars, cluster of notes to star clusters, and silence to darkness. This procedure was employed in "Atlas Eclipticalis" (1961 – 1962) for orchestra, "Études Australes" (1974 – 1975) for piano, "Études Boréales" (1978) for piano or cello, and "Freeman Études" (1977 – 1980, 1989 – 1990) for violin.



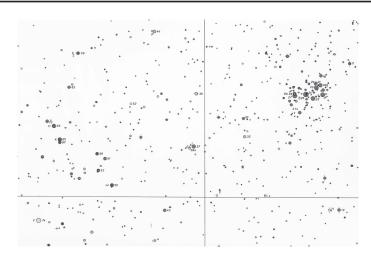


Fig. 3: Page from Bečvář's "Atlas Eclipticalis" (1958)

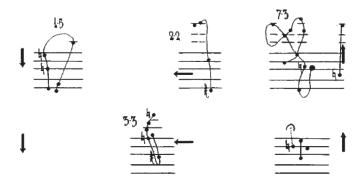


Fig. 4: Excerpt from the violin part of Cage's "Atlas Eclipticalis"



Fig. 5: Excerpt from Cage's "Étude Australe VII" for piano

"Tierkreis" – Karlheinz Stockhausen

Karlheinz Stockhausen (1928 – 2007) was one of the most influent composers of the 20th and 21st century, famous for his interest in serialism, electronic and aleatoric music. In 1974 he wrote a series of twelve melodies for six percussions and music boxes, "Musik im Bauch" (Music in the Belly), for one of his daughters. Each of these short pieces has a reference to an astrological sign and gravitates around its own *Zentralton* (central pitch). As Stockhausen wrote in an article for the German music journal "Die Reihe":

The first, Aquarius is attracted to E^b , and so forth according the ascending chromatic succession; Leo, to A, is located at the centre, and finally the D for the Capricorn.

With respect to the nature of the single melodies:

I began to think to friends and acquaintances born under the different zodiacal signs. Therefore each melody could be in harmony with traits and manners of the corresponding sign.

The following year Stockhausen arranged these melodies in "Tierkreis" (the Zodiac), for any melodic or chordal instrument solo, for a choir and keyboard (using lyrics written by the composer himself), for chamber orchestra, etc.

"Makrokosmos" and "Star Child" – George Crumb

George Crumb (1929) is an American composer. In his pieces, he explores timbres, notations, and peculiar playing techniques: the *prepared piano* such as John Cage; the *spoken flute*, according to which the flautist has to speak while insufflating the air into the instrument; the *seagull effect* for the cello; the *overpressure* for string instruments so that the pressure exerted by the arch is so high as to distort the sounds. Between 1972 and 1974 he wrote the four books constituting the "Makrokosmos". The first two books are for piano solo and are inspired by the constellations of the Zodiac. Here, Crumb experiments new forms of notation, bending the pentagrams to reproduce a spiral galaxy (as in "Aquarius", Book 1)

or two twin suns (as in "Gemini", Book 2). The third book, also called "Music for a Summer Evening", is for two pianos and percussions, whilst the last one, known as Celestial Mechanics: cosmic dances for amplified piano", is for four/six-hand piano. The homonym book written by the French mathematician and astronomer Pierre-Simon Laplace inspired this fourth book. Four movements dedicated to four stars with decreasing luminosity compose "Celestial Mechanics": Alpha Centauri, Beta Cygni, Gamma Draconis, and Delta Orionis. This last star is also known as Mintika (Mentaka or Mintaka), the star of the third movement of "As Três Marias" by Villa-Lobos.

In 1977 Crumb composed "Star-Child, a Parable for Soprano, Antiphonal Children's Voices, Male Speaking Choir and Bell Ringers, and Large Orchestra". It is the largest work by Crumb, requiring 47 instrumentalists plus a large number of choristers. The title derives from a section of the third book of Makrokosmos, "Hymn for the Advent of the Star-Child". This piece contains many biblical references, for example, the seven trumpets of the apocalypse, the lyrics of the *Dies Irae*, and John's New Testament. There are also references to the Musica Humana and Mundana by Boëthius, to which Crumb adds the Musica Apocalyptica, the only section of the score where every musician plays. In addition, the main musical idea is once again the Music of the Spheres that Crumb introduces by letting the strings play a sequence of chords containing fifths, one of the basic intervals of the Pythagorean (music-)cosmogony.

Suggested Listenings

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- H. VILLA-LOBOS: Piano Works (1974), N. Freire (piano), ed. Apex
- P. HINDEMITH: Ludus Tonalis, counterpoint, tonal and technical studies for the piano (2013, remastered), P. Roggenkamp (piano), ed. Essential Media
- B. MADERNA: Piano and Percussions in the 20th Century (1993), A. Orvieto, G. Facchin, R. Maioli and the Tammittam Percussion Ensemble, ed. Dynamic
- L. DALLAPICCOLA: Sicut Umbra... (1975), S. Michelow (soprano), G. Bertini (conductor), London Sinfonietta, ed. Argo
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