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Anisotropic String Cosmology at Large Curvatures

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We study the effect of the antisymmetric tensor field $B_{\mu\nu}$ on the large curvature phase of string cosmology. It is well-known that a non-vanishing value of $H = dB$ leads to an anisotropic expansion of the spatial dimensions. Correspondingly, in the string phase of the model, including α' corrections, we find anisotropic fixed points of the evolution, which act as regularizing attractors of the lowest order solutions. The attraction basin can also include isotropic initial conditions for the scale factors. We present explicit examples at order α' for different values of the number of spatial dimensions and for different ansätze for H .

1 Introduction

Cosmological models derived from the low-energy action of string theory [1-5] can address fundamental problems of cosmology, like the initial singularity, and at the same time are a testing ground for our present knowledge of string theory. The pre-big-bang model developed in [1-4] has attractive physical features. It has a scale factor duality symmetry [1, 6, 7, 8] which suggests the existence of a dual, “pre-big-bang” phase and a possible solution of the initial singularity problem, and has a built-in mechanism for obtaining an inflationary phase driven by the kinetic energy of the dilaton (see also [9]). Whether this mechanism is sufficient to solve the flatness/horizon problems has been a matter of some discussion recently [10].

Another aspect of the model, which distinguishes it from previous attempts, is that it allows us to perform quite explicit computations. This is due to the fact that the evolution starts in a regime of low curvatures and weak couplings, where both sigma-model corrections (controlled by the string constant $\alpha' = \lambda_s^2$) and string loops (controlled by the value of the dilaton) are fully under control. The existence of this perturbative regime allows to perform detailed computation of the spectra of particles produced by amplification of vacuum fluctuations [11]. Important features of these spectra, such as the $\sim \omega^3$ dependence of the graviton spectrum at low frequencies, are only sensitive to this phase and therefore are under good theoretical control.

Starting at the initial time in this perturbative regime, the model evolves toward a large curvature regime. In this phase α' corrections certainly become important. In fact, at lowest order α' only enters as an overall constant in the effective action. Therefore it drops from the equations of motion and there is no scale at which we can stop the growth of the curvature. The inclusion of α' corrections provides such a scale, and changes qualitatively the form of the solution in the large curvature regime. Computations in this phase are not under such a good theoretical control. What has been done to date is the following:

(i) one can study perturbative α' corrections, working at first order in α' , i.e. including terms $\sim R_{\mu\nu\rho\sigma}^2$ in the low energy action [12]. Working at this order we can already see the regularizing effect of α' corrections and we can therefore hope to have a glimpse of the structure which could be obtained at all orders in α' . In fact, it was found in [12] that a De Sitter solution with linearly growing dilaton exists at all orders in α' if a set of algebraic (rather than differential) equations admit a real solution. Such indications, of course, cannot

be obtained at zero order in α' , since in this case, as we said, α' drops from the equations and there is no scale at which a new regime can set in.

Working at any finite order in α' , one must however keep in mind the limitations of the computation, which are due not only to the fact that, in the large curvature regime, higher order corrections are potentially of the same order as the lowest order terms, but also to the fact that results obtained at finite order are very sensitive to the scheme used for renormalization of the sigma model, or equivalently are sensitive to field redefinitions [12, 13].

(ii): non-perturbative effects in α' are also very important. In particular the production of massive string modes is non-perturbative in α' and provides a mechanism which stops the growth of the curvature [13].

So, even if the large curvature phase is certainly not as much under theoretical control as the previous dilaton-dominated phase, still we can perform a number of computations which gives us some indications on the behavior of the cosmological model.

Finally, the large curvature phase of the model should be matched with the standard radiation dominated era. This is the so called graceful exit problem [14].

In this paper we consider the effect of the antisymmetric tensor field $B_{\mu\nu}$ on the large curvature phase of the model. It has been recognized for some time that a non vanishing value of its field strength $H = dB$ leads to an anisotropic expansion; the fact that H is a three-form opens in principle the possibility that in the presence of a non zero H three spatial dimensions expand while six (or, for bosonic string, twenty-two) contract, realizing a scenario of dynamical “ten into four” compactification [15]. These ideas can be tested in the context of the model of refs. [1-4]. The evolution with a non-vanishing H , in the low curvature dilaton-dominated phase of the model, has been studied in ref. [16], where it has been shown that a three-form H with a component $H_{012} \neq 0$ inevitably leads to an anisotropic cosmology. The effect of form fields in string cosmology and in M-theory has also been considered in [17, 18, 19]. Here we extend the analysis to the large curvature region of the model, considering the effect of perturbative α' corrections. Following the approach of ref. [12], we will find the fixed points of the evolution which can be obtained after inclusion of perturbative α' corrections, and we will discuss the resulting scenario for dynamical compactification of the extra dimensions.

The paper is organized as follows. In sect. 2 we discuss the model, we recall some of the

relevant results in the literature and we present possible ansätze for the three-form H . In sect. 3 we discuss the corresponding solutions of the equations of motion.

2 The model

The starting point of our investigation is the effective action, up to $\mathcal{O}(\alpha')$, of the bosonic sector of a string theory with nontrivial antisymmetric tensor field strength $H_{\rho\mu\nu} = \partial_{[\rho}B_{\mu\nu]} = \partial_\rho B_{\mu\nu} + \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu}$; in the string frame it has the following form [20]:

$$S_0 = -\frac{1}{2\lambda_s^{d-1}} \int d^{d+1}x \sqrt{|g|} e^{-\phi} \left\{ R + (\nabla\phi)^2 - \frac{H^2}{12} - \frac{k\alpha'}{4} [R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} + \right. \\ \left. - \frac{1}{2} R^{\mu\nu\rho\sigma} H_{\mu\nu\alpha} H_{\rho\sigma}{}^\alpha + \frac{1}{24} H_{\mu\nu\lambda} H^\nu{}_{\rho\alpha} H^{\rho\sigma\lambda} H_\sigma{}^{\mu\alpha} - \frac{1}{8} (H^2_{\mu\nu})^2 \right\}, \quad (2.1)$$

where $k = 1, \frac{1}{2}, 0$ for bosonic, heterotic and type II strings respectively. We restrict in the following to the bosonic and heterotic string case. For the type II string the first corrections are due to terms $\sim R^4_{\mu\nu\rho\sigma}$ [21].

Our conventions are $\eta_{\mu\nu} = (+, -, -, -, \dots)$ and $R^\mu{}_{\nu\rho\sigma} = (\partial_\rho \Gamma^\mu_{\nu\sigma} - \dots)$. In eq. (2.1) we have defined $H^2_{\mu\nu} = H_{\mu\alpha\beta} H_\nu{}^{\alpha\beta}$ and $H^2 = H_{\mu\nu\rho} H^{\mu\nu\rho}$. The $\mathcal{O}(\alpha')$ part of (2.1) is not uniquely fixed: actually, by means of the redefinitions of the fields $g_{\mu\nu} \rightarrow g_{\mu\nu} + \delta g_{\mu\nu}$, $\phi \rightarrow \phi + \delta\phi$, $B_{\mu\nu} \rightarrow B_{\mu\nu} + \delta B_{\mu\nu}$ with

$$\delta g_{\mu\nu} = k\alpha' \left\{ b_1 R_{\mu\nu} + b_2 \partial_\mu \phi \partial_\nu \phi + g_{\mu\nu} [b_2 (\nabla\phi)^2 + b_4 R + b_5 \square\phi + b_6 H^2] + b_7 H^2_{\mu\nu} \right\}, \\ \delta\phi = k\alpha' [c_1 R + c_2 (\nabla\phi)^2 + c_3 \square\phi + c_4 H^2], \\ \delta B_{\mu\nu} = k\alpha' [(d_1 \nabla^\rho + d_2 \partial^\rho \phi) H_{\rho\mu\nu}],$$

one can generate an action of the type

$$S = S_0 + \frac{1}{2\lambda_s^{d-1}} \left(\frac{k\alpha'}{4} \right) \int d^{d+1}x \sqrt{-g} e^{-\phi} \left[a_1 R_{\mu\nu} R^{\mu\nu} + a_2 R^2 + a_3 (\nabla\phi)^4 + a_4 R^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \right. \\ \left. + a_5 R (\nabla\phi)^2 + a_6 R \square\phi + a_7 \square\phi (\nabla\phi)^2 + a_8 (\square\phi)^2 + a_9 R H^2 + a_{10} R^{\mu\nu} H^2_{\mu\nu} + \right. \\ \left. + a_{11} H^2_{\mu\nu} \partial^\mu \phi \partial^\nu \phi + a_{12} (\nabla^\mu \partial^\nu \phi) H^2_{\mu\nu} + a_{13} H^2 (\nabla\phi)^2 + a_{14} H^2 \square\phi + a_{15} (H^2)^2 + \right. \\ \left. + a_{16} (H^2_{\mu\nu})^2 + a_{17} (\nabla_\rho H^{\rho\mu\nu}) (\nabla^\sigma H_{\sigma\mu\nu}) + a_{18} (\nabla_\rho H^{\rho\mu\nu}) \partial^\sigma \phi H_{\sigma\mu\nu} \right], \quad (2.2)$$

where S_0 is given by eq. (2.1) and we have eliminated terms that can be reduced to those displayed, by means of integration by parts or use of Bianchi identity. The parameters a_i

are functions of b_i, c_i, d_i and satisfy the following relations:

$$\begin{aligned}
a_2 + a_3 - a_5 - a_6 + a_7 + a_8 &= 0 , \\
a_1 + 4a_{10} - 4a_{12} &= 0 , \\
a_{12} + 4a_{16} &= 0 , \\
a_1 - a_4 - 4a_{11} - 4a_{17} - a_{18} &= 0 , \\
5a_1 + 9a_2 + 4a_3 - 2a_4 + 6a_5 + 36a_9 + 24a_{13} + 144a_{15} &= 0 , \\
5a_1 + 25a_3 - 5a_4 + 15a_7 + 9a_8 + 60a_{13} + 36a_{14} + 144a_{15} &= 0 .
\end{aligned} \tag{2.3}$$

Within these constraints the a_i can be chosen at will with the appropriate field redefinitions: the different actions one obtains all reproduce the correct string theory S -matrix elements on-shell and so they are equally all good candidates for a description of the low energy regime (up to $\mathcal{O}(\alpha')$) of the corresponding string theory. The problem is that, since we work at a finite order in α' , different schemes (*i.e.*: different choices of the a_i) can lead to different physical results as, for instance, the existence or not of a fixed point of the evolution. We will return to this question later, but from the above consideration it is clear that whatever result is obtained within a particular field redefinition should be taken only as an illustrative example rather than a definite prediction.

After this remark, let us make our choice for the a_i , in order to have a concrete model to deal with. First of all, we require the action (2.2) not to produce higher than second order derivatives in the equations of motion. This can be done by allowing only the presence of the structures reported in eq.(4.4) of [7]; this request reduces the 18-parameter family of eq. (2.2) to the following 2-parameter one:

$$\begin{aligned}
S = & -\frac{1}{2\lambda_s^{d-1}} \int d^{d+1}x \sqrt{|g|} e^{-\phi} \left\{ R + (\nabla\phi)^2 - \frac{H^2}{12} - \frac{k\alpha'}{4} [R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} + \right. \\
& - (1 + \lambda_1 - \lambda_2) (\nabla\phi)^4 + \lambda_1 \square\phi (\nabla\phi)^2 + \lambda_2 (g^{\mu\nu} R - 2R^{\mu\nu}) \partial_\mu\phi \partial_\nu\phi + \\
& - \frac{1}{2} \left(R^{\mu\nu\rho\sigma} H_{\mu\nu\alpha} H_{\rho\sigma}^\alpha - 2R^{\mu\nu} H_{\mu\nu}^2 + \frac{1}{3} R H^2 \right) + \\
& + \frac{1}{24} H_{\mu\nu\lambda} H^\nu{}_{\rho\alpha} H^{\rho\sigma\lambda} H_{\sigma}{}^{\mu\alpha} + \frac{5}{144} (H^2)^2 - \frac{1}{8} (H_{\mu\nu}^2)^2 \\
& \left. - \left(1 - \frac{\lambda_2}{2} \right) H_{\mu\nu}^2 \partial^\mu\phi \partial^\nu\phi + \frac{2}{3} \left(1 + \frac{\lambda_1}{4} - \frac{7\lambda_2}{8} \right) H^2 (\nabla\phi)^2 \right\} . \tag{2.4}
\end{aligned}$$

Moreover, we set $\lambda_1 = \lambda_2 = 0$; with this prescription our model is the straightforward generalization of the one studied in ref. [12] to the case of nonvanishing antisymmetric

tensor field strenght.

Since the basic ingredients of our model are:

- (i) the presence of α' corrections,
- (ii) a nontrivial antisymmetric tensor field strenght $H_{\mu\nu\rho}$,

we first briefly recall what are the principal effects that these ingredients produce when they act separately.

Concerning the role of α' corrections alone, we refer to the treatment in ref. [12], since in the next section we will apply quite the same procedure to the case of nonvanishing $H_{\mu\nu\rho}$. The model considered in [12] corresponds to the ours without antisymmetric tensor; by means of the following ansatz for the metric

$$g_{00} = N^2(t), \quad g_{ij} = -\delta_{ij}e^{2\beta(t)}, \quad i, j = 1, \dots, d, \quad (2.5)$$

and after integration by parts, the actions becomes (discarding an irrelevant overall multiplicative constant)

$$S = \int dt e^{d\beta - \phi} \left[\frac{1}{N} (\dot{\phi}^2 + d(d-1)\dot{\beta}^2 - 2d\dot{\beta}\dot{\phi}) - \frac{k\alpha'}{4N^3} (c_1\dot{\beta}^4 + c_2\dot{\phi}\dot{\beta}^3 - \dot{\phi}^4) \right], \quad (2.6)$$

where

$$c_1 = -\frac{d}{3}(d-1)(d-2)(d-3), \quad c_2 = \frac{4d}{3}(d-1)(d-2). \quad (2.7)$$

Varying the action with respect to β , ϕ and N , one gets two dynamical equations of motions plus a constraint on the initial data. To look for a solution one makes the ansatz $\dot{\phi} = x = \text{const}$, $\dot{\beta} = y = \text{const}$; then, in the gauge $N = 1$, the differential equations reduce to three algebraic equations in two unknowns. One should not be worried about this fact, because, due to reparametrization invariance, these equations are not independent, *i.e.* once the constraint and one of the ‘‘dynamical’’ equation is satisfied, the other is automatically satisfied. Therefore the problem of finding a solution in the string phase is reduced to solving the following system of two nonlinear equations in two unknowns:

$$\begin{aligned} x^2 + d(d-1)y^2 - 2dxy - \frac{k\alpha'}{4} (c_1y^4 + c_2xy^3 - x^4) + \\ - (dy - x) \left[-2x + 2dy + \frac{k\alpha'}{4} (c_2y^3 - 4x^3) \right] = 0, \\ x^2 + d(d-1)y^2 - 2dxy - \frac{3k\alpha'}{4} (c_1y^4 + c_2xy^3 - x^4) = 0. \end{aligned} \quad (2.8)$$

In [12] it was verified that the system (2.8) has a real solution for any d from 1 to 9. Moreover, by integrating numerically the full differential equations for β and ϕ and imposing the constraint on the initial data, it was found that this solution acts as late-time attractor for the evolution of the system, i.e. is a fixed point of the evolution, and its basin of attraction includes initial conditions corresponding to a state of pre-big bang evolution from the vacuum. The idea that emerges from the above example is that large curvature corrections to the lowest order string effective action can regularize the otherwise singular pre-big bang solution. As previously mentioned, it must be remembered that the last statement depends on the choice of the field redefinition: it has been verified ([12, 13]) that there are other choices of the a_i for which the above fixed point are not smoothly connected to the perturbative vacuum or do not exist at all. Nevertheless, it can still be useful to study models with corrections $\mathcal{O}(\alpha')$, at least to obtain some indicative examples of what can be the role of large curvature corrections in pre-big bang cosmology.

If perturbative α' corrections, computed at all orders in α' , would not regularize the lowest order solution, then the regularizing mechanism is the production of massive string modes, which is non perturbative in α' , and has been discussed in ref. [13].

Concerning the action of the antisymmetric tensor on the low curvature part of our model (*i.e.* without α' corrections), it has been studied in detail by Copeland et al. (see ref. [16] and references therein): they present the explicit solutions of the lowest-order equations of motion in $D = 4$ and find that a nonvanishing $H_{\mu\nu\rho}$ noticeably affects the dynamics of the system. In particular they find that a homogeneous B_{ij} , which corresponds to a nonvanishing H_{0ij} (latin indices indicate spatial components), produces anisotropic evolution even in the presence of isotropic initial conditions. They also studied the case $H_{0\mu\nu} = 0, H_{ijk} \neq 0$ in $4 + n$ dimensions: it turns out that the presence of the antisymmetric tensor accelerates the expansion of three spatial directions with respect to the other n , leading to an anisotropic $3 + n$ cosmology. However, without α' corrections, the evolution eventually runs into a singularity.

The fact that nonvanishing form fields produce an anisotropic expansion is quite general: the presence of the field strength F_r of an r -form in a string effective action makes the system evolve in an anisotropic way and (see also [15]) the number of spatial dimensions that are separated from the other is equal to the number of spatial components of the field

strenght; so, given an r -form, two simple situations are possible:

a) $F_{i_1 \dots i_{r+1}} \neq 0, F_{0i_1 \dots i_r} = 0$. In this situation $r + 1$ spatial dimensions separate from the other;

b) $F_{0i_1 \dots i_r} \neq 0, F_{i_1 \dots i_{r+1}} = 0$. In this case the splitting is r vs. $D - r - 1$.

We shall refer to a) and b) as the *solitonic* and *elementary* ansätze, respectively: the nomenclature comes from the relation with elementary and solitonic p -branes in M-theory explored in ref. [17].

Different types of string effective action have a different content of form fields. However, the three form $H = dB$ is common to all of them (bosonic, heterotic, type II) and we focus on it in the following.

3 The evolution with a non-vanishing H

Having recalled what are the basic features of models characterized by torsion and large curvature corrections separately, we want to study what happens when both are present. We will show that in this case the distinctive features of each factor are not lost, but rather merge in the way one expects: the presence of anisotropic fixed points.

We are going to apply the procedure of ref. [12] to our model in order to find the fixed points of the evolution; obviously we must release the request of isotropy, but we still want to work with a homogeneous metric, so we assume that all fields depend only on time.

Let us work first in $D = 4$ and consider the elementary ansatz: with a spatial rotation we can always reduce ourselves to the situation in which only one component H_{0ij} of the field strenght, say H_{012} (and its permutations), is nonzero and we call it simply h_e . The equations that h_e must obey are the integrability condition $dH = 0$ and the $(1, 2)$ -component of the equation of motion for $B_{\mu\nu}$ (the other components are trivially satisfied by our ansatz and by the request of homogeneity):

$$\frac{\delta S}{\delta B_{\mu\nu}} = -\nabla^\rho \frac{\delta S}{\delta \partial_\rho B_{\mu\nu}} = 0. \quad (2.9)$$

The integrability condition reduces to

$$\partial_3 h_e = 0,$$

which is obviously satisfied by an homogeneous field. With our choice for h_e we expect directions 1, 2 to evolve differently from direction 3, and so we describe the metric tensor in

the following way:

$$g_{00} = N^2(t), \quad g_{11} = g_{22} = -e^{2\beta(t)}, \quad g_{33} = -e^{2\gamma(t)}, \quad g_{\mu\nu} = 0 \text{ if } \mu \neq \nu.$$

We can now compute all the terms contained in action (2.4), and we get

$$S = \int dt e^{2\beta+\gamma-\phi} \left\{ \frac{1}{N} \left[\dot{\phi}^2 + 2\dot{\beta}(\dot{\beta} + 2\dot{\gamma}) - 2\dot{\phi}(2\dot{\beta} + \dot{\gamma}) - \frac{1}{2}e^{-4\beta}h_e^2 \right] + \right. \\ \left. - \frac{k\alpha'}{4N^3} \left[8\dot{\phi}\dot{\beta}^2\dot{\gamma} - \dot{\phi}^4 + 2e^{-4\beta}\dot{\phi}^2h_e^2 \right] \right\}.$$

It is now straightforward to write down the equations of motion; we report them introducing a new variable $z \equiv e^{-2\beta}h_e$:

$$\begin{aligned} & -\dot{\phi}^2 - 2\dot{\beta}(\dot{\beta} + 2\dot{\gamma}) + 2\dot{\phi}(2\dot{\beta} + \dot{\gamma}) + \frac{3k\alpha'}{4}(8\dot{\phi}\dot{\beta}^2\dot{\gamma} - \dot{\phi}^4) + \frac{z^2}{2}(1 + 3k\alpha'\dot{\phi}^2) = 0, \\ & -2\ddot{\phi} + 2(2\ddot{\beta} + \ddot{\gamma}) + \dot{\phi}^2 + 2(3\dot{\beta}^2 + \dot{\gamma}^2 + 2\dot{\beta}\dot{\gamma}) - 2\dot{\phi}(2\dot{\beta} + \dot{\gamma}) + \\ & + \frac{k\alpha'}{4} [3\dot{\phi}^4 - 4\dot{\phi}^3(2\dot{\beta} + \dot{\gamma}) + 8\dot{\beta}(2\dot{\beta}\dot{\gamma} + \ddot{\gamma}\dot{\beta}) - 12\ddot{\phi}\dot{\phi}^2 + 8\dot{\beta}^2\dot{\gamma}(2\dot{\beta} + \dot{\gamma})] + \\ & + \frac{z^2}{2} [1 + k\alpha'(-\dot{\phi}^2 + 2\dot{\phi}(2\dot{\beta} + \dot{\gamma}) + 2\ddot{\phi})] + k\alpha'z\dot{z}\dot{\phi} = 0, \\ & 2\ddot{\phi} - 2(\ddot{\beta} + \ddot{\gamma}) - \dot{\phi}^2 - 2(\dot{\beta}^2 + \dot{\beta}\dot{\gamma} + \dot{\gamma}^2) + 2\dot{\phi}(\dot{\beta} + \dot{\gamma}) + \\ & + \frac{k\alpha'}{4} [\dot{\phi}^4 + 8\dot{\phi}\dot{\beta}\dot{\gamma}(\dot{\beta} + \dot{\gamma}) + 8\dot{\phi}(\dot{\beta}\dot{\gamma} + \ddot{\gamma}\dot{\beta}) + 8(\ddot{\phi} - \dot{\phi}^2)\dot{\beta}\dot{\gamma}] + \frac{z^2}{2}(1 + k\alpha'\dot{\phi}^2) = 0, \\ & 2\ddot{\phi} - 4\ddot{\beta} - \dot{\phi}^2 - 6\dot{\beta}^2 + 4\dot{\phi}\dot{\beta} + \frac{k\alpha'}{4} [\dot{\phi}^4 + 16\dot{\phi}\dot{\beta}^3 + 16\dot{\phi}\dot{\beta}\dot{\beta}\ddot{\beta} + 8(\ddot{\phi} - \dot{\phi}^2)\dot{\beta}^2] + \\ & - \frac{z^2}{2}(1 + k\alpha'\dot{\phi}^2) = 0, \end{aligned} \tag{2.10}$$

while eq.(2.9) becomes

$$\dot{z}(1 + k\alpha'\dot{\phi}^2) + z[(\dot{\beta} - \dot{\phi})(1 + k\alpha'\dot{\phi}^2) + 2k\alpha'\dot{\phi}\ddot{\phi}] = 0. \tag{2.11}$$

By setting $\dot{\phi} = x = \text{const}$, $\dot{\beta} = y = \text{const}$, $\dot{\gamma} = w = \text{const}$, $z = \text{const}$ (note that this does not mean $h_e = \text{constant}$), we obtain from (2.10), (2.11) an algebraic system of five equations in four unknowns; as previously stated, the equations are not independent and we have numerically solved the system, finding the following fixed points (we have set $k\alpha' = 1$):

	x	y	w	z
A	1.40439	0.616596	0.616596	0
B	1.41421	0.707107	0.471405	0
C	1.66003	0.378784	1.66003	0.442278

We see that A is the same (isotropic) fixed point found in [12]. Its existence follows from the fact that for $z = 0$ the above equations reduce to the equations of ref. [12]. Studying its domain of attraction by direct numerical integration of the equations of motion we found that A can be a late-time attractor only if we impose $z = 0$ as initial condition (this condition is conserved by eq. (2.11)).

Quite the same statement holds for the anisotropic fixed point B , i.e. it doesn't attract solutions of the equations of motion whose initial conditions are such that $z \neq 0$, while it is the late time attractor of a certain region of the (x, y, w, z) -space characterized, as well as by $z = 0$, also by $y \neq w$: we conclude that in both cases the antisymmetric tensor plays little role and that the anisotropy of B is uniquely due to the choice of anisotropic initial conditions.

Up to now we are only elaborating on the results of ref. [12], in the sense that we have found that even non-isotropic fixed points are possible, and that the presence of torsion seems to compromise the stability of the system, i.e., in the enlarged parameter space which includes z these fixed point have an attraction basin of zero measure. On the contrary, C is the first real novelty of our analysis: since $z_C \neq 0$, it is clear that its existence is due to the presence of the antisymmetric tensor; moreover, it turns out to be a late-time attractor of a region of the phase space with non-zero measure. Unfortunately, we have found by numerical integration that this region does not contain isotropic ($y = w$) initial conditions. One can hope that working in a different number of dimensions could improve the situation: we will see that this can indeed be the case, but we prefer to develop this idea by studying the solitonic ansatz that, on account of what stated at the end of the last section, should provide a more appealing $3 + n$ dimensional separation of spatial dimensions, rather than the $2 + (n + 1)$ separation provided by the elementary ansatz.

We then choose $H_{123} \equiv h_s$ as the only non-zero component of $H_{\mu\nu\rho}$, modulo permutations of the indices; the integrability condition now reads

$$\partial_\mu h_s = 0,$$

where the index μ runs from 0 to $n + 3$ and $\mu \neq 1, 2, 3$. The equations with $\mu = 4, \dots, n + 3$ are trivially satisfied on account of homogeneity, while the equation with $\mu = 0$ implies that h_s must be a constant; moreover, our ansatz automatically satisfies every component of the equation of motion (2.9).

After having parametrized the metric tensor in the following way

$$g_{00} = N^2, \quad g_{ij} = -\delta_{ij}e^{2\beta}, \quad i, j = 1, \dots, 3, \quad g_{ab} = -\delta_{ab}e^{2\gamma}, \quad a, b = 4, \dots, n+3,$$

it is not difficult to write down the new form of action (2.4):

$$\begin{aligned} S = & \int dt e^{3\beta+n\gamma-\phi} \left\{ \frac{1}{N} \left[\dot{\phi}^2 + 6\dot{\beta}^2 + 6n\dot{\beta}\dot{\gamma} + n(n-1)\dot{\gamma}^2 - 2\dot{\phi}(3\dot{\beta} + n\dot{\gamma}) + \frac{N^2}{2}e^{-6\beta}h_s^2 \right] + \right. \\ & - \frac{k\alpha'}{4N^3} \left[-8n\dot{\beta}^3\dot{\gamma} - 12n(n-1)\dot{\beta}^2\dot{\gamma}^2 - 4n(n-1)(n-2)\dot{\beta}\dot{\gamma}^3 - \frac{n}{3}(n-1)(n-2)(n-3)\dot{\gamma}^4 + \right. \\ & \quad \left. + \dot{\phi}^3 \left(8\dot{\beta}^3 + 24n\dot{\gamma}\dot{\beta}^2 + 12n(n-1)\dot{\gamma}^2\dot{\beta} + \frac{4n}{3}(n-1)(n-2)\dot{\gamma}^3 - \dot{\phi}^3 \right) + \right. \\ & \quad \left. \left. + N^2e^{-6\beta}h_s^2 \left(n(n-1)\dot{\gamma}^2 - 6n\dot{\gamma}\dot{\beta} - 2n\dot{\gamma}\dot{\phi} - 4\dot{\phi}^2 \right) \right] \right\}. \end{aligned}$$

We are now ready to repeat the usual procedure of writing down the equations of motion (this time we will not report them), finding the fixed points, and integrating the full numerical system, for generic values of n .

The case $n = 0$, *i.e.* four dimensional space-time, doesn't tell anything interesting: we know that, since we have only three spatial dimension, h_s cannot induce any anisotropy on the system: actually the only fixed point we found is the fixed point A of the previous discussion, with the difference that this time A is a good attractor even if we start with $z \neq 0$ (having defined a new $z \equiv e^{-3\beta}h_s$).

Trying different values for n we have found, as well as an isotropic fixed point for each n , also many anisotropic ones. Unfortunately, they result to be all (with reference to the previous discussion) B -like, *i.e.* not torsion-induced fixed point, as well as unstable; but there is one exception, for $n = 2$: a fixed point F of coordinates

$$x = 1.38259, \quad y = 0.238441, \quad w = 0.438904, \quad z = 0.$$

In fact the stability analysis reveals not only that F is a good attractor, but also that the attraction basin includes even an isotropic region, characterized by $h_s \neq 0$; in other words, the nonvanishing component of the antisymmetric field strenght drives an initially isotropic region towards an anisotropic fixed point. The behaviour of the system is presented in fig. 1. As we already discussed, our model has only an illustrative value. Therefore, we are not too much disappointed by the fact that the compactification showed in F acts in the “wrong” sense (three dimensions expanding *slower* than the others): we think in fact that

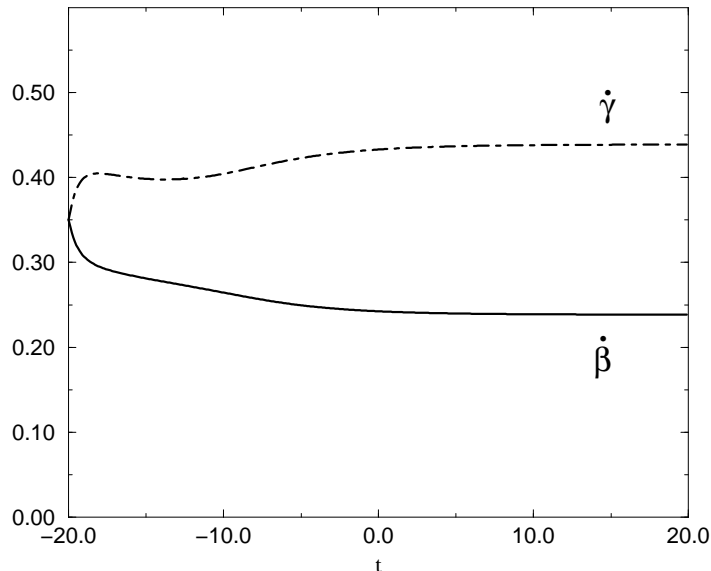


Figure 1: Evolution of the system towards anisotropic fixed point F , starting from isotropic initial conditions.

this behaviour is not a general rule. Our feeling is enforced by some trials we have made with various values of n : although we have not found significant fixed points, we have seen that in many cases, starting from isotropic initial conditions, three dimensions expand driven by the torsion while the others contract (see fig. 2 for an example in $D = 10$). Parenthetically, we note that “contraction” and “expansion”, *i.e.* the signs of $\dot{\beta}$ and $\dot{\gamma}$, are frame-dependent concepts: in fact, since the metric tensor and time in the Einstein frame (here denoted by a tilde) are related to those of the string frame (which is by definition the frame in which the action has the form (2.1)) by

$$\tilde{g}_{\mu\nu} = \exp\left(-\frac{2\phi}{D-2}\right) g_{\mu\nu}, \quad \frac{d\tilde{t}}{dt} = \exp\left(-\frac{\phi}{D-2}\right), \quad (2.12)$$

then the Hubble parameter in the E -frame can be written as

$$\dot{\tilde{\beta}} = e^{\frac{\phi}{D-2}} \left(\dot{\beta} - \frac{1}{D-2} \dot{\phi} \right). \quad (2.13)$$

Equations (2.12), (2.13) show that what is really frame-independent is the ratio of the scale factors; it follows that the compactification process, that is the growth of some dimensions *with respect* to others, is not altered by the choice of the frame. As discussed in detail in

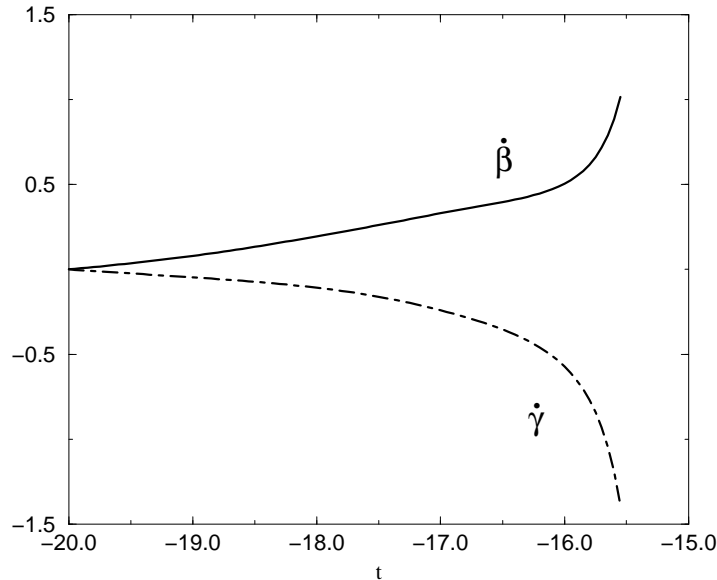


Figure 2: Behaviour of the system in $D = 10$, starting from isotropic initial conditions.

ref. [3], statements concerning the scale factors are in general frame-dependent. A scale factor growing in the string frame can be decreasing in the Einstein frame, but physical properties like the number of e-folds of an inflationary period, or the spectrum of metric perturbations amplified in the course of the background evolution, are independent of the frame.

For the same reason previously discussed, we are not especially worried about the fact that we have not found appealing attractors in a more significant number of dimensions, say 10 or 26, (even if we would had liked it!); it is possible that the system exhibits the expected behaviour for another choice of the a_i (we have unsuccessfully tried some of it, but we explored only a small fraction of the whole space of parameters). In other words, it is clear that a model in which α' corrections are truncated to any finite order cannot be used to *prove* anything; what we rather could hope was to find (as we did it) some concrete example of the action of $\mathcal{O}(\alpha')$ corrections and antisymmetric tensor combined together, *i.e.* non singular, anisotropic cosmological solutions.

Finally, another possibility for obtaining regular anisotropic solutions, is that in the low curvature regime three spatial dimension expand and the remaining contract, as can be ob-

tained easily with the solitonic ansatz, and then the regularization mechanism is provided by massive string modes production, as discussed in ref. [13] (see ref. [22] for particle production in anisotropic space-times). In this case we do not expect a fixed point describing a DeSitter phase but rather a bounce in the scale factor, as it happens in the isotropic case.

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