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Trade-in-goods and trade-in-tasks: An integrating framework

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ABSTRACT

We introduce a simple but flexible analytical framework in which both trade in goods and trade in tasks arise. We use this framework to provide versions of the gains-from-trade and the famous four HO theorems (Heckscher-Ohlin, factor-price-equalisation, Stolper-Samuelson, and Rybczynski) that apply to this environment. We extend our framework to accommodate monopolistic competition and two-way offshoring and to integrate theoretical results of the early offshoring literature.

1. Introduction

A growing list of economists argues that the nature of international trade is changing in important ways.¹ Instead of simply creating more trade in goods, global integration is increasingly marked by trade of intermediate goods and services, also known as ‘fragmentation’, ‘offshoring’ or ‘task trade’. The importance of this trade has been clarified with new data sets that remove the double counting in customs statistics that arises when intermediates cross borders on their own and then are embodied in further processed goods (Johnson and Noguera 2012, Koopman, Powers, Wang, and Wei 2013). The new trade numbers are called ‘value added’ trade to distinguish them from the ‘gross’ trade flows that are traditionally measured.

In this paper, we introduce a simple but flexible analytical framework in which both trade in goods and trade in tasks arise endogenously in response to exogenous changes in the cost of moving goods and services. We then use this framework to provide versions of the traditional four theorems (Heckscher-Ohlin-Vanek, factor-price-equalisation, Stolper-Samuelson, and Rybczynski) that are valid when both trade in goods and trade in tasks occur. We also revisit the gains-from-trade theorem. Finally, we show how specific assumptions in the offshoring literature simplify the analysis and lead to specific and strong results in our framework.

Integrating task-trade into the trade-in-goods literature is important for at least three reasons. First, a substantial fraction of world output remains traded across international boundaries and trade in components

¹ Inter alia, Blinder (2006, 2009), Grossman and Rossi-Hansberg (2006, 2008, 2012), Hanson, Mataloni and Slaughter (2005), Hummels, Ishii and Yi (2001), and Jones and Kierzkowski (1990). The ‘task trade’ terminology was introduced by Grossman and Rossi-Hansberg (2008).

and intermediate services represents a growing fraction of this trade (WTO 2008). It is thus important to study these facts jointly.

Second, this trend has elicited a substantial number of theoretical contributions, reviewed in some detail below, that is marked by a wide range of cases where outcomes seemingly contradict standard trade theory results. We show that this arises because task trade and trade in goods display interesting similarities but task trade differs in that it typically involves some kind of technology transfer that is akin to *product-augmenting technical change* of the type identified by Dixit and Norman, (1980, chapter 5).

Third, this technological transfer interacts with traditional sources of comparative advantage and these interactions require that the traditional four theorems be amended. Specifically, trade in tasks: i) becomes a source of comparative advantage in final goods which means that the Heckscher-Ohlin-Vanek and the factor-price-equalisation theorems break down in their standard formulation. ii) has important wage effects beyond and above those predicted by the standard Stolper-Samuelson theorem, and iv) implies ‘shadow migration’ that exhibits Rybczynski-like effects on production and trade patterns for final goods.

Formally, we extend the traditional Heckscher-Ohlin-Vanek (henceforth HO) framework to allow for trade in tasks in the wake of Grossman and Rossi-Hansberg (2008), henceforth GRH. In our model, there are two countries, Home and Foreign, producing final goods by combining tasks, each of which involves primary factors of production. Importantly, Home has a Hicks-neutral technological superiority. Thus before task-trade becomes possible, Home wages are higher even though there is conditional factor price equalisation. This wage gap drives the offshoring when trade in some, but not all, tasks becomes feasible.

Task trade has five noteworthy consequences in this environment.

First, the reduction in Home firm production costs due to offshoring (i.e. task trade) is analogous to a technical change (the ‘productivity effect’ in GRH), so at least some – and possibly all – Home factor prices must rise. This analogy and this finding are central to the analysis of GRH.

Second, offshoring is akin to ‘shadow migration’ – i.e. it is as if foreign factors migrated to the offshoring nation and we show that the quantity effects on production and trade in final goods follow the logic of the Rybczynski theorem. This analogy, to the best of our knowledge, is novel.

Third, the combination of heterogeneous factor intensities at the task level implies heterogeneous task intensities at the final good level so that Home’s effective technological superiority is no longer Hicks-neutral. Put differently, trade patterns in final goods are now governed by a combination of Ricardian and HO forces. The four traditional HO theorems break down in this environment. A key contribution of our

paper is to transform the HO equations using shadow endowments instead of actual ones and to establish analogues to the traditional HO theorems in this modified version.

Fourth, we show that task-trade creates intraindustry trade in a Walrasian economy.

Finally, starting from an equilibrium with trade in goods but no task trade, allowing for offshoring has ambiguous welfare effects because it affects the terms-of-trade. In this respect, increasing access to trade at the extensive margin is similar to increasing it at the intensive margin; more trade unambiguously raises the welfare of a country only starting from autarky.²

Our framework departs from the offshoring literature of the 1990s and early 2000s (which we review below) by making two specific assumptions – arbitrage opportunities arising from Hicks-neutral technology differences (an assumption we maintain throughout) and Leontief technologies (an assumption we sometimes relax). This enables us to obtain strong analytical results and precise conditions to sign the factor price, output, and trade effects of offshoring, another key novelty of our paper. Several of these effects are comparable to results uncovered by this early literature and we also show under which conditions they also arise in our setting. In this sense, our framework enables us to integrate this literature within the standard HO toolkit.

We conclude the paper with an extension that departs further from the early offshoring literature by assuming that task trade implies technology diffusion. In this environment, offshoring has novel price and quantity effects on the host country, Foreign. Finally, we extend the basic framework to accommodate monopolistic competition and two-way offshoring/task-trade.

1.1. Existing literature

Our paper integrates and extends a large body of theoretical literature in trade theory. The early HO theory incorporates trade in intermediate goods (Batra and Casas 1973, Woodland 1977, Dixit and Grossman 1982, and Helpman 1984) and the 1990s saw a number of informal analyses of fragmentation as well as some formal modelling (Deardorff 1998a, b, and Venables 1999). Task-trade issues, however, were more recently crystallised by Kohler (2004a), Markusen (2006), Antràs et al. (2006), and Grossman and Rossi-Hansberg (2006, 2008).

The most commonly cited reference in the early offshoring/fragmentation literature is the diagrammatic analysis of Jones and Kierzkowski (1990), which seems to be the first to leverage the insight that

² The gains-from-trade results in our paper were simultaneously and independently developed by Markusen (2013). Both papers use the methodology introduced by Dixit (1985). Markusen then applies the analysis to the positive models developed by Markusen and Venables (2007) and GRH.

fragmentation acts as technological progress and should therefore be expected to have complex wage effects. The ensuing line of modelling typically works with small open economies where fragmentation occurs in one sector and in one direction.³ The focus of the analysis is firmly on wage effects. Jones and Kierzkowski (1990), for instance, argue that workers whose jobs are ‘lost’ to offshoring may, somewhat paradoxically, see their wages rise in some special cases.

Among the mathematical formalisations of fragmentation, Deardorff (1998a,b) studies fragmentation in a multi-cone HO model where cost-saving offshoring is driven by non-factor price equalisation. The focus is on factor prices and showing that task-trade need not foster wage convergence. Venables (1999) works with a 2x2x2 HO model where offshoring is cost saving due to non-factor price equalisation arising from a factor-intensity reversal. Fragmentation occurs in one industry and in one direction. He uses numerical simulations and Lerner-Pearce diagrammatic analysis to study examples where task-trade produces wage convergence and divergence. Kohler (2004a) works with a small-open-economy specific-factor model where fragmentation occurs in one sector. The focus is on the reward to the specific capital that moves offshore when fragmentation occurs and on the overall welfare effects on the home nation. Markusen (2006) works with a multi-cone HO model that he simulates numerically assuming that fragmentation occurs in the skill-intensive sector and the fragment is of middle skill-intensity. He typically finds that skilled workers gain. Kohler (2004b) works with a small open economy where fragmentation/offshoring can only happen in one sector, using the Dixit and Grossman (1982) model with a continuum of intermediate goods; he shows that cheaper offshoring raises or lowers factor prices according to the relative factor intensity of the two sectors and the fragments offshored. We add production and trade effects to this literature. By imposing additional structure on the model, we are also able to repeat some of its simulation findings by analytic means.

More recently, Grossman and Rossi-Hansberg (2006, 2008) present a perfect competition model where two final-goods are produced using two continuums of tasks, each employing only one type of labour. Offshoring arises endogenously and the range of tasks offshored varies continuously with the cost of offshoring. The paper formalises the analogy between offshoring and technological change (the ‘productivity effect’) showing that task-trade, unlike trade-in-goods, can generate gains for all factors in the offshoring nation. The paper establishes necessary and sufficient conditions for wage-changes in the small open economy case with two factors and two goods. It also explores the novel labour supply effect that influences wages when there are more factors than goods.

³ See Jones and Marjit (1992), Arndt (1997, 1999), Jones and Findlay (2000, 2001), Jones and Kierzkowski (1998, 2000), Jones, Kierzkowski and Leonard (2002), and Francois (1990a, b, c).

There also exists a string of recent offshoring papers that are not encapsulated by our integrating approach. Rodriguez-Clare (2010) embodies the GRH approach in a Ricardian model à la Eaton and Kortum (2002). He studies the impact of task-trade on the gains from trade for the home and host nations. Global welfare rises due to offshoring's productivity effect, but terms-of-trade effect can mean that the home nation loses despite this. Antràs, Garicano and Rossi-Hansberg (2006, 2008) propose a model in which all tasks are potentially offshorable. The focus is on the formation, composition and size of (cross-border) teams when workers have different abilities (skills), and countries have different skill endowments. Among other results, they show that improved communication technology yields larger teams and larger wage inequalities. Their model also provides a trade-induced explanation for the rise in returns to skills. Fujita and Thisse (2006) and Robert-Nicoud (2008) study how offshoring interacts with agglomeration forces in 'new economic geography' settings while Antràs and Staiger (2012) analyse the optimal design of trade agreements in the presence of offshore outsourcing.

The next section introduces notation and a slightly modified HO model of trade in final goods. Section 3 adds offshoring/task-trade and Section 4 solves for the equilibrium. Section 5 discusses some measurement issues that arise with task trade, Section 6 uses our framework to integrate previous results of the fragmentation/offshoring/task-trade literature, Section 7 extends the framework in various directions, and Section 8 concludes.

2. The standard HO results

To fix notation, this section presents the standard HO model, slightly modified – à la Trefler (1993) – to create an incentive for task-trade when the possibility arises in Section 3. As the model is familiar, we move quickly through the analysis.

There are two countries, Home and Foreign, with homothetic and identical tastes, F factors of production, and I perfectly competitive industries. Foreign variables are distinguished by asterisks and the indices for factors and industries are, respectively, $f = 1, \dots, F$ and $i = 1, \dots, I$. The Home factor price, goods price, factor endowment, production, consumption and import vectors are denoted by $\mathbf{w} \equiv \{w_f\}$, $\mathbf{p} \equiv \{p_i\}$, $\mathbf{V} \equiv \{V_f\}$, $\mathbf{X} \equiv \{X_i\}$, $\mathbf{C} \equiv \{C_i\}$ and $\mathbf{M} \equiv \{M_i\}$, respectively.⁴ The $I \times F$ matrix $\mathbf{A}(\mathbf{w}) \equiv \{a_{fi}(\mathbf{w})\}$ and its transpose $\mathbf{A}^T$ summarise Home's constant returns technology with typical element a_{fi} giving the cost-minimizing input requirement of factor f in industry i as a function of $\mathbf{w}$, namely $a_{fi} = \partial c_i(\mathbf{w}) / \partial w_f$, where

⁴ Vectors and matrices are denoted by bold letters; variables and parameters by italics, and $\mathbf{Z} > \mathbf{N}$ means that *each element* of $\mathbf{Z}$ exceeds the corresponding element of $\mathbf{N}$.

c_i denotes the unit cost function in industry I . We define the I -dimensional vector $\mathbf{c}(\mathbf{w}) \equiv \{c_i(\mathbf{w})\}$. Throughout, well-known regularity conditions ensure that production is fully diversified in Home and Foreign.⁵

Our slight modification is to assume:

Assumption 1 (homothetic technologies). Foreign unit cost functions are $\gamma > 1$ times higher than Home's, i.e. $\mathbf{c}^*(\cdot) = \gamma \mathbf{c}(\cdot)$.

Such Hicks-neutral technology differences create no Ricardian motives for trade. Indeed, the model can be mechanically transformed into a standard HO model by defining Foreign factor supplies in 'effective units', i.e. dividing V_f^* by the technology gap γ . We denote effective units of factors by " $\sim$ ", so the world factor endowment in effective units is $\tilde{\mathbf{V}}^w \equiv \mathbf{V} + \mathbf{V}^*/\gamma$.

The autarky equilibria are characterised by market-clearing conditions $\mathbf{M}^* = \mathbf{0}$ and $\mathbf{M} = \mathbf{0}$ as well as I pricing conditions and F employment conditions in each nation. Using familiar notation these are:

$$\mathbf{p} = \mathbf{c} = \mathbf{A}\mathbf{w}, \quad \mathbf{p}^* = \gamma \mathbf{c}^* = \gamma \mathbf{A}^* \mathbf{w}^*; \quad \mathbf{V} = \mathbf{A}^T \mathbf{X}, \quad \mathbf{V}^* = \gamma \mathbf{A}^{*T} \mathbf{X}^* \quad (1)$$

where the arguments are suppressed for simplicity, so $\mathbf{c}(\mathbf{w})$ is $\mathbf{c}$, etc.

With free trade, goods prices are equalised (law of one price), goods-markets clear globally ($\mathbf{M}^* + \mathbf{M} = \mathbf{0}$), and equation (1) characterises the equilibrium with $\mathbf{p} = \mathbf{p}^*$. Regularity conditions ensure production and price vectors are strictly positive.⁶ We refer to this as the goods-trade equilibrium.

The famous four HO results follow from well-known demonstrations. When $\mathbf{A}$ is invertible (an assumption we maintain throughout), the law-of-one-price can, and, assuming no factor intensity reversals, must imply **effective factor price equalisation (FPE)**, i.e. $\mathbf{w} = \gamma \mathbf{w}^*$.⁷ This is established using $\mathbf{p} = \mathbf{p}^*$ with (1) and the

⁵ The necessary condition is that the $\mathbf{V}$'s lie in the Deardoff (1994) lens, i.e. the space spanned by the columns of $\mathbf{A}^T$ evaluated at equilibrium factor prices. Loosely speaking, this requires relative factor endowments of countries to differ by less than factor intensities of industries.

⁶ See the appendix of our working paper for necessary and sufficient conditions for existence (http://www.dagliano.unimi.it/media/wp2008_250.pdf).

⁷ Effective FPE can be obtained under milder conditions than under Assumption 1 but Assumption 1 ensures that the model retains the elegance and analytical tractability of the standard HOV model. We thank Pol Antràs for pointing this out to us.

fact that $\mathbf{c} = \mathbf{c}^*$ if and only if $\mathbf{w} = \gamma \mathbf{w}^*$.⁸ With homothetic preferences, $\mathbf{p} = \mathbf{p}^*$ and $\mathbf{A} = \mathbf{A}^*$ (since $\mathbf{w} = \gamma \mathbf{w}^*$), the effective-factor-content of $\mathbf{C}$ must be equal to $s\tilde{\mathbf{V}}^w$, where s is Home's share of world income. The factor content of Home production is $\mathbf{V}$, so the pattern of trade must respect the **HOV** and **HO theorems**:

$$\mathbf{A}^T \mathbf{M} = s\tilde{\mathbf{V}}^w - \mathbf{V} \quad \mathbf{M} = (\mathbf{A}^T)^{-1}(s\tilde{\mathbf{V}}^w - \mathbf{V}) \quad (2)$$

The third and fourth famous theorems consider the impact on $\mathbf{w}$ of an exogenous variation in $\mathbf{p}$ (**Stolper-Samuelson theorem**), and the impact on $\mathbf{X}$ of an exogenous variation of $\mathbf{V}$ (**Rybczynski theorem**). These follow from manipulations of (1) given that $\mathbf{A} = \mathbf{A}^*$.

The standard **gains-from-trade (GFT) theorem** (Ohyama 1972, Smith 1982, Dixit 1985) states that (i) some trade is better than none, regardless of what happens to terms of trade, and (ii) more trade is desirable for a nation if its terms of trade do not worsen. As it turns out, the analogue for part (i) of the GFT theorem fails for tasks-trade, but part (ii) goes through. To set the stage for that analysis, we prove both parts for the goods-trade equilibrium.

By revealed preference arguments (Samuelson 1939, 1962, Kemp 1962), one equilibrium is preferred to another if the inferior equilibrium's consumption vector is affordable at the preferred equilibrium's prices. Thus, the trade equilibrium is preferred by Home if $\mathbf{p}(\mathbf{C} - \mathbf{C}_a) \geq 0$, where $\mathbf{C}$ and $\mathbf{C}_a$ are Home's autarky and trading consumption vectors respectively. By definition of $\mathbf{M}$, we may rewrite this expression as $\mathbf{p}(\mathbf{M} - \mathbf{M}_a) + \mathbf{p}(\mathbf{X} - \mathbf{X}_a) \geq 0$. The GFT are proved by noting that the inequality holds since the first term is zero (due to balanced trade $\mathbf{pM} = 0$ and to $\mathbf{M}_a = \mathbf{0}$ in autarky) while the second term is positive (due to profit maximisation). This holds regardless of what happens to the terms of trade. A symmetric result holds for Foreign, so all gain from any move away from the no-goods-trade situation. Dixit (1985) applies this logic to show that a trading nation gains from any partial liberalisation as long as its terms of trade do not deteriorate, i.e. $\mathbf{p}(\mathbf{X} - \mathbf{X}_a) \geq 0$.

3. Trade in tasks

In the goods-trade model, all 'tasks' involved in producing a good are implicitly bundled together so that production could be conceptualised as the transformation of primary factors into final goods; see (1). This

⁸ To see why effective FPE easily holds when $I \geq F$, we write $\mathcal{C}(\mathbf{w}^*) = \mathbf{c}(\gamma \mathbf{w}^*) = \mathbf{c}(\mathbf{w})$. The former equality follows from the fact that $\mathbf{c}(\cdot)$ is homogenous of degree 1 in factor prices and the latter follows from the law of one price, $\mathbf{p}^* = \mathbf{p}$. The implications of relaxing the assumption $I \geq F$ are not innocuous but well understood (Ethier 1984).

implicitly assumed that all tasks in a given production process had to be performed in a single nation. Here we consider an exogenous change that allows the production process to function even when tasks are spatially unbundled – thus opening the door to offshoring.

We use ‘offshoring’ to refer to production organised abroad and task-trade as the attendant cross border flows that this engenders. Tasks are either segments of the physical production process (so the task’s output is an intermediate good) or a slice of the necessary factor inputs (so the task’s output is a productive services).

3.1. Framework

Production in industry I involves N_i tasks indexed by $t = 1, \dots, N_i$, $N_i \geq 2$; c_{it} denotes the cost of producing the amount of task t necessary to produce one unit of good i . In keeping with the literature (e.g. GRH), we assume that all tasks are assembled using a Leontief technology so that $c_i(\cdot) = \sum_{t=1}^{N_i} c_{it}(\cdot)$ and we assume each task involves a non-negative quantity of each factor. With constant returns, a_{fi} , can be written as the sum of task-level input coefficients:

$$a_{fi}(\mathbf{w}) \equiv \sum_{t=1}^{N_i} a_{fit}(\mathbf{w}); \quad \forall f = 1, \dots, F; i = 1, \dots, I \quad (3)$$

where a_{fit} is the unit input-requirement of factor f for task t in sector i . This allows substitutability across factors for a given task, but rules out substitutability across tasks (e.g. each car requires four wheels; extra wheels cannot be substituted for the engine). A key additional assumption is:

Assumption 3 (offshore-able, firm-specific technologies). Firms that offshore a task can do so using the technology of their own nation.⁹

This makes offshoring economical despite effective FPE. Offshoring reduces cost for Home firms since they can combine their superior technology with lower Foreign factor prices. Foreign-to-Home offshoring, however, is never economic since Home wages are higher and Foreign technology is worse. One interpretation of this assumption is that Foreign workers are themselves as productive or as well educated as

⁹ The concept of what constitutes a firm does not seat well with our otherwise Walrasian model and we use it as a shortcut for the common idea in the FDI literature that technologies may be implemented in countries different from those in which they have originated, as in e.g. Ramondo and Rodriguez-Clare (2013). Section 7 shows that our results all got through in a monopolistic competition trade model where firms are well-defined; here we stick with the HO setting to improve comparison with the four theorems.

Home workers but that Foreign technology or management practices are inferior to Home's (Bloom and Van Reenen 2007).¹⁰

To be concrete, the exogenous change that allows task-trade concerns "coordination costs". These comprise the cost of organising tasks in different nations, e.g. the cost of exchanging coordination information. The coordination costs are task-specific parameters that can be thought of as reflecting how easily codified a particular task is. For example, they may be lower for routine task than complex tasks that require frequent face-to-face interactions. The change may be brought about by the IT revolution, which allows exchanging information with arm-length companies or affiliates via the optic fibre cable.

Working with Leontief technologies enables us to obtain sharper results in many instances, so we sometimes impose:

Assumption 4 (Leontief technologies). Input-output coefficients of all factors, tasks, and goods are invariant in factor prices.

3.2. Trade in tasks

To integrate task-trade results with goods-trade theory that typically focuses on autarky to free trade changes, we focus on similarly extreme changes in coordination costs. Without further loss of generality, we set $N_i = 2$. Task $t = 1$ is the set of tasks that are offshore-able at zero coordination cost; task $t = 2$ is the set of tasks for which coordination costs are prohibitive. It may be convenient to think of the former as routine tasks and the latter as complex tasks. By the usual cost-savings logic, all Home production of type-1 tasks is offshored to Foreign (assuming standard regularity conditions that ensure diversified production in both economies).¹¹ Formally:

Lemma 1 (task-trade occurs). Under regularity conditions that assure diversified production, all type-1 tasks are offshored from Home to Foreign in the task-trade equilibrium.

Proof. Suppose first that trade in type-1 tasks was possible but none occurred in equilibrium. As this prospective equilibrium is identical to the goods-trade equilibrium, $\mathbf{w}$ would equal $\gamma \mathbf{w}^*$, so by Assumption 2

¹⁰ Assuming that offshoring involves a convex combination of domestic and foreign technologies would make the model more realistic but would not change our qualitative results provided that this combination is the same for all tasks. More interesting and far-reaching is the idea that offshoring also implies coordinating production over several, distant locations and possibly among different firms; both involve important costs. If such costs vary by task then the implications on factor prices and on quantities produced and traded are much more complex.

¹¹ The appendix of our working paper provides exact necessary and sufficient conditions for diversification in the 2x2x2 version of the model (http://www.dagliano.unimi.it/media/wp2008_250.pdf).

an atomistic firm deviating from the prospective equilibrium would reduce costs by offshoring its type-1 tasks; the per-unit-output cost reduction would be equal to $c_{il}(\mathbf{w}) - c_{il}(\mathbf{w}^*) = c_{il}(\gamma\mathbf{w}^*) - c_{il}(\mathbf{w}^*) > 0$. The resulting pure profit contradicts the definition of a competitive equilibrium, so some task-trade occurs; factor prices adjust so that:

$$c_{il}(\mathbf{w}_O) - c_{il}(\mathbf{w}_O^*) \geq 0; \quad \forall i=1, \dots, I \quad (4)$$

in the trade-in-task equilibrium (the subscript ‘O’ stands for ‘offshoring’). Suppose next that some Home firms do not offshore all type-1 tasks. Such firms are foregoing the cost-saving opportunity and thus earn negative profits when competing with firms that do, except in the knife-edge case with $c_{il}(\mathbf{w}_O) = c_{il}(\mathbf{w}_O^*)$, in which case they earn zero profit and are indifferent. To see this, note that

$$\begin{aligned} 0 = \mathbf{p}_O^* - \mathbf{p}_O &= \gamma \mathbf{c}(\mathbf{w}_O^*) - [\mathbf{c}_1(\mathbf{w}_O^*) + \mathbf{c}_2(\mathbf{w}_O)] \\ &\leq \gamma \mathbf{c}(\mathbf{w}_O^*) - [\mathbf{c}_1(\mathbf{w}_O) + \mathbf{c}_2(\mathbf{w}_O)] = \gamma \mathbf{c}(\mathbf{w}_O^*) - \mathbf{c}(\mathbf{w}_O), \end{aligned}$$

where the first equality follows by the law of one price and where the second one follows from diversification as well as from the Home and Foreign zero profit conditions. The inequality follows from (4), which applies to Home firms that do not offshore all of task 1. Thus $\mathbf{c}(\mathbf{w}_O) \geq \mathbf{p}_O$ and these firms cannot compete with either Foreign firms or Home firms that do offshore. *QED*.

Given Lemma 1, Home’s pricing and employment equations reflect Foreign-factor usage (for type-1 tasks), while Foreign’s pricing condition is unaffected (Foreign firms continue to use Foreign technology for all goods and tasks and to pay Foreign wages). Foreign’s employment condition, however, reflects offshoring employment. Here we follow GRH in assuming that all offshore task production is re-imported to Home, so Foreign employment in the offshore sector is proportional to Home’s production vector. Thus the pricing and employment equations for the task-trade equilibrium are:

$$\begin{aligned} \mathbf{p}_O &= (\mathbf{A}_O - \mathbf{A}_1)\mathbf{w}_O + \mathbf{A}_1^*\mathbf{w}_O^*, \quad \mathbf{p}_O = \gamma \mathbf{c}(\mathbf{w}_O^*) = \gamma \mathbf{A}_O^*\mathbf{w}_O^* \\ \mathbf{V} &= (\mathbf{A}_O^T - \mathbf{A}_1^T)\mathbf{X}_O, \quad \mathbf{V}^* = \gamma \mathbf{A}_O^{*T}\mathbf{X}_O^* + \mathbf{A}_1^{*T}\mathbf{X}_O, \end{aligned} \quad (5)$$

where the technology-matrix subscripts ‘O’ indicates the offshoring equilibrium values for the whole $I \times F$ technology matrix, namely: $\mathbf{A}_O \equiv \mathbf{A}(\mathbf{w}_O)$ and $\mathbf{A}_O^* \equiv \mathbf{A}(\mathbf{w}_O^*)$. We define the technology matrix pertaining to

the traded tasks as $\mathbf{A}_1 \equiv \{a_{f1}(\mathbf{w}_O)\}$ and $\mathbf{A}_1^* \equiv \{a_{f1}(\mathbf{w}_O^*)\}$ (we do not add a subscript ‘O’ in order to lighten notation).¹²

Importantly, applied technologies are no long homothetic in (5). Home factors use Home technology, but some Foreign factors use Home technology while others use Foreign technology. As we shall see below, this creates Ricardian motives for trade in final goods as well as blurring the HO logic and its analytic elegance.

4. Task-trade Equilibrium and the shadow migration approach

Henceforth, we use ‘goods-trade’ to mean free trade in final goods without trade in tasks.

4.1. Task-trade and the GFT, FPE and HO theorems

We start by showing that the familiar GFT theorem breaks down with task-trade. Specifically, the move from free trade in goods to free trade in goods and some tasks does not always improve welfare. The proof is straightforward even for the general technology case.

Under our Walrasian assumptions, the cost of combining the output of type-1 and type-2 sets of tasks into a final good is zero, so we can readily apply the well-known results in Dixit (1985). In particular, we think of there being $2I$ goods (the two sets of tasks for each of the I goods) whose ‘shadow prices’ are the actual marginal production costs (i.e. including offshoring in the task-trade equilibrium).¹³ The relevant GFT condition is therefore $\bar{\mathbf{p}}_O(\bar{\mathbf{C}}_O - \bar{\mathbf{C}}) \geq 0$ where bars indicate the artificially extended vectors, and the price vector consists of the shadow prices (marginal costs). Recalling that $\bar{\mathbf{p}}\bar{\mathbf{M}} = \bar{\mathbf{p}}_O\bar{\mathbf{M}}_O = 0$, this implies $(\bar{\mathbf{p}} - \bar{\mathbf{p}}_O)\bar{\mathbf{M}} + \bar{\mathbf{p}}_O(\bar{\mathbf{X}}_O - \bar{\mathbf{X}}) \geq 0$. Profit maximisation assures that $\bar{\mathbf{p}}_O(\bar{\mathbf{X}}_O - \bar{\mathbf{X}})$ is positive, but the term $(\bar{\mathbf{p}} - \bar{\mathbf{p}}_O)\bar{\mathbf{M}}$ can be positive or negative. This is the Laspeyres index of Home’s terms-of-trade loss when task-trade is allowed, as in Dixit’s (1985) extension of the GFT theorem to the case of already trading nations.

Offshoring could, for example, boost global production of Home exports more than Home’s imports, engendering a terms-of-trade loss. Relative output, however, could fall in the opposite direction, so the terms-of-trade impact is ambiguous. Isomorphic reasoning implies Foreign GFT is also not assured. At least

¹² In writing (5) we maintain the assumption of diversified production, though offshoring brings additional motives for specialisation. See the appendix of our 2008 working paper for details (http://www.dagliano.unimi.it/media/wp2008_250.pdf).

¹³ In his ‘fragmentation’ paper, Kohler (2003) uses the term ‘effective price’ instead of ‘shadow price’ but both terms recover the same concept.

one nation must gain from task-trade, however, since global output rises (some Foreign factors are made more productive with Home technology). If goods prices are unaffected by task-trade, Home gains and Foreign is unaffected (Foreign is also strictly better off in the model of Section 7.1). Formally we have proved:

Proposition 1 (ambiguous GFT from task-trade). Starting from free trade in goods, allowing some task-trade when there was none: (i) is Pareto improving if terms-of-trade are unaffected (i.e. if $\bar{\mathbf{p}}_0 = \bar{\mathbf{p}}$); (ii) always improves global welfare as terms-of-trade effects disappear at the global level; and (iii) a necessary condition for a nation to lose is that it experiences a terms-of-trade loss.

We turn now to showing that the HO and FPE theorems break down for task-trade in the general technology case. Starting with the FPE theorem:

Proposition 2 (effective factor price divergence). Unless there exists a real number ϕ in the unit interval such that $\mathbf{c}_1(\cdot) = \phi \mathbf{c}(\cdot)$ (and thus $\mathbf{A}_1 = \phi \mathbf{A}$), task-trade forces a divergence of (effective) factor prices.

Proof. The proof is by contradiction. Assume that effective FPE holds, that is, $\alpha \mathbf{w}_0 = \mathbf{w}_0^*$ for some $\alpha \in (0, 1/\gamma)$. Then the law of one price and (5) yield:

$$\begin{aligned} 0 &= \gamma [\mathbf{c}_1(\alpha \mathbf{w}_0) + \mathbf{c}_2(\mathbf{w}_0)] - [\mathbf{c}_1(\alpha \mathbf{w}_0) + \mathbf{c}_2(\mathbf{w}_0)] \\ &= \alpha(\gamma - 1) \mathbf{c}_1(\mathbf{w}_0) - (1 - \alpha\gamma) \mathbf{c}_2(\mathbf{w}_0) = \alpha(\gamma - 1) \mathbf{c}(\mathbf{w}_0) - (1 + \alpha) \mathbf{c}_2(\mathbf{w}_0), \end{aligned}$$

where the first equality follows by the law of one price and by effective FPE, the second one by the homogeneity of degree 1 in factor prices of the unit cost functions, and the third one by the definition $\mathbf{c}(\cdot) \equiv \mathbf{c}_1(\cdot) + \mathbf{c}_2(\cdot)$. The final equality holds in either of the following cases: (i) no task-trade occurs, which violates Lemma 1; (ii) all tasks are offshorable (i.e. $\mathbf{c}_1 = \mathbf{c}$), which is ruled out by assumption; (iii) all factor prices are zero, which violates the zero profit condition for positive goods prices; or (iv) the factor intensities of type-1 and type-2 tasks are equal (and thus also equal to aggregate factor intensity) industry by industry, i.e. $\mathbf{c}_1(\cdot) = \phi \mathbf{c}(\cdot)$ for $\phi = (\gamma - 1)\alpha / (1 - \alpha)$. Thus the supposition that effective FPE occurs under trade in task must be false in general. *QED.*

Intuition for this result is simple. As authors from Jones and Kierzkowski (1990) to GRH have argued, task-trade is akin to technological progress for the offshoring nation. As the new trade involves a subset of tasks and offshoring is unidirectional, the technological change is non-homothetic, so the basis of effective FPE is destroyed. In other words, task-trade introduces Ricardian elements endogenously.

Perhaps the most robust theoretical finding in the model is the Heckscher-Ohlin-Vanek theorem, yet this fails for task-trade. Given homothetic preferences, Home's consumption vector of final goods is proportional to world output, i.e. $\mathbf{C}_o = s\mathbf{X}_o^w$, however solving for $\mathbf{X}_o$ and $\mathbf{X}_o^*$ from (5):

$$\begin{aligned}\mathbf{M}_o &= s\mathbf{X}_o^w - \mathbf{X}_o \\ &= s\left\{\left[\mathbf{I} - (\gamma\mathbf{A}_o^{*T})^{-1}\mathbf{A}_1^{*T}\right](\mathbf{A}_o^T - \mathbf{A}_1^T)^{-1}\mathbf{V} + (\gamma\mathbf{A}_o^{*T})^{-1}\mathbf{V}^*\right\} - (\mathbf{A}_o^T - \mathbf{A}_1^T)^{-1}\mathbf{V}\end{aligned}\quad (6)$$

The only circumstance in which this reduces to the standard HO expression in (2) is when the offshoring matrices $\mathbf{A}_1$ and $\mathbf{A}_1^*$ are both zero – i.e. when no offshoring occurs.¹⁴ In short, given Lemma 1, we have proved:

Proposition 3 (HO fails). If task-trade occurs, the HO theorem fails to hold in its standard formulation.

4.2. The shadow migration approach

Allowing trade in task ruins much the elegance and tractability of the HO model algebra as is clear from Proposition 3 and expressions (5) and (6). The inelegance with task-trade stems from the non-proportionality of the technology matrices due to both non-FPE and the use of Home technology by some Foreign factors. Moreover, Home final good exports contain some Foreign factors. A key contribution of our paper is to suggest a transformation of the model, which we call the ‘shadow migration approach. This approach restores much of the HO elegance and does so in a way that enables us to integrate task-trade into the received body of goods-trade theory. It also allows the integration of a wide range of special cases considered in the fragmentation/offshoring literature.

The essential elements to this ‘shadow migration’ transformation are twofold. First we continue to write the pricing and employment conditions in terms of final goods only instead of following the more common practice of expanding them to include newly traded intermediate goods. Second, we write the conditions as if Foreign factors employed in offshored activities had migrated to Home but were paid Foreign factor prices. This transformation involves the introduction of two new vectors that are potentially observable given modern datasets (more on this in Section 5).

Starting with the pricing equations in (1), the task-trade Home and Foreign equivalents to these expressions are rewritten as:

¹⁴ Note that we have written (6) for imports in final goods only. Adding task imports would only reinforce the point.

$$\mathbf{p}_O + \tilde{\mathbf{S}} = \mathbf{c}(\mathbf{w}_O), \quad \mathbf{p}_O = \mathbf{c}(\gamma \mathbf{w}_O^*); \quad \tilde{\mathbf{S}} \equiv \mathbf{c}_1(\mathbf{w}_O) - \mathbf{c}_1(\mathbf{w}_O^*) \geq \mathbf{0}, \quad (7)$$

where $\tilde{\mathbf{S}}$ is the difference between the cost of performing the offshored tasks in Home and Foreign; the inequality follows from $\mathbf{w}_O \geq \mathbf{w}_O^*$ in equilibrium. $\tilde{\mathbf{S}}$ is identical to the vector Jones (1965) uses to illustrate the impact of technical change, but it arises from Home technology being deployed in Foreign rather than changes in national technologies. Using (7), we can formulate the

Proposition 4 (task-trade analogue to FPE theorem). Starting from free trade in goods, the opening of task-trade produces a divergence in effective factor prices. In equilibrium the divergence is:

$$\mathbf{w}_O - \gamma \mathbf{w}_O^* = \mathbf{c}(\mathbf{p}_O + \tilde{\mathbf{S}})^{-1} - \mathbf{c}(\mathbf{p}_O)^{-1}$$

where the vector $\mathbf{c}^{-1}(\cdot)$ is the element-by-element inverse of $\mathbf{c}(\cdot)$.

Proof. The proof is by inspection of (7). *QED.*

The link between the effective FPE gap, $\mathbf{w}_O - \gamma \mathbf{w}_O^*$, and $\tilde{\mathbf{S}}$ is intuitive. Offshoring saves Home firms money because they pay the shadow migrants the going (lower) Foreign wage, so departures from effective FPE tends to be increasing in the cost-savings. It also follows by inspection of (7) that we may use Jones algebra, specifically that developed in Dixit and Norman (1980, Chapter 5), and that the effect of trade-in-tasks on Home factor prices is akin to what they call ‘product augmenting technical change’. This is a central finding of GRH, who call this the ‘productivity effect’ of task-trade.

A further implication, whose proof is by inspection of (7), is:

Corollary 4.1 (shadow migration not necessarily a substitute for real migration). From Proposition 4, the shadow migration caused by task-trade can widen or narrow the international wage gap for each type of labour, so task-trade has an ambiguous effect on the pressure for real migration.

4.3. Analytic results for the Leontief case

To obtain sharp, closed-form results, this subsection embraces Assumption 4 in which case the price and employment conditions simplify to:

$$\begin{aligned} \mathbf{p}_O + \mathbf{S} &= \mathbf{A} \mathbf{w}_O, & \mathbf{p}_O &= \gamma \mathbf{A} \mathbf{w}_O^* \\ \mathbf{V}_O &= \mathbf{A}^T \mathbf{X}_O, & \mathbf{V}_O^* &= \gamma \mathbf{A}^T \mathbf{X}_O^*. \end{aligned} \quad (8)$$

where $\mathbf{S} = \mathbf{A}_1(\mathbf{w}_0 - \mathbf{w}_0^*)$, $\mathbf{V}_0 \equiv \mathbf{V} + \Delta\mathbf{V}$, and $\mathbf{V}_0^* \equiv \mathbf{V}_0 - \Delta\mathbf{V}$ where $\Delta\mathbf{V}$ is the shadow migration vector, namely $\Delta\mathbf{V} \equiv \mathbf{A}_1^T \mathbf{X}_0 > \mathbf{0}$. In words, the shadow migration vector, $\Delta\mathbf{V}$, is the vector of Foreign factors employed in performing the offshored tasks with Home technology.

Using shadow-migration-adjusted employment conditions from (8), Home's import of final goods under task-trade, $\mathbf{M}_0$, is related to endowments by:

$$\mathbf{M}_0 = s\mathbf{X}_0^w - \mathbf{X}_0 = (\mathbf{A}^T)^{-1} (s\tilde{\mathbf{V}}_0^w - \mathbf{V}_0) \quad (9)$$

where $\tilde{\mathbf{V}}_0^w \equiv \tilde{\mathbf{V}}^w + (1 - \gamma^{-1})\Delta\mathbf{V}$. Note that $\tilde{\mathbf{V}}_0^w > \tilde{\mathbf{V}}^w$, namely, the effective world endowment rises because a subset of Foreign workers use Home's superior technology. Inspection of this expression yields:

Proposition 5 (task-trade analogue to HO theorem). The pattern of trade in goods in the task-trade equilibrium is explained by the HO theorem where actual endowments are replaced by shadow-migration-adjusted endowments.

This is subject to the usual provisos that apply to higher-dimension versions of the HOV and HO theorems – see Ethier (1974, 1984) or Dixit and Norman (1980). Note that such adjustments are symmetric to those introduced in Davis and Weinstein (2001) to account for *non-tradable goods*; here, the adjustment accounts for *tradable tasks*.¹⁵

By comparing the goods-trade expression for Home imports, (2), to the task-trade version, (9), we have:

Corollary 5.3 (offshoring as a source of comparative advantage): Offshoring is a source of comparative advantage in the sense that it creates trade in final goods that would not occur otherwise.

The intuition can be highlighted with the special case where no goods-trade would occur without offshoring that arises when Home and Foreign endowment vectors are proportional, i.e. $\mathbf{V} = b\mathbf{V}^*$, $b > 0$. Allowing task-trade creates Ricardian comparative advantage and thus trade in goods (except in the knife-edge case where $\mathbf{A}_1 = \phi\mathbf{A}$).

An additional implication of offshoring is the creation of intra-industry trade. The point is that task-trade may be recorded as intraindustry trade because some final goods exports embody imports of components

¹⁵ Relaxing the assumption for Leontief technologies would require to include an additional adjustment to account for the fact that the technology matrix of Foreign, $\mathbf{A}_0^*$, is different from Home's, $\mathbf{A}_0$, in the offshoring equilibrium. Without access to country-specific input-output matrices, the only adjustment in (12) accounts for task imports.

within the same industry classification at standard levels of aggregation.¹⁶ For instance, one of Home's final good exports was motor vehicles (NAICS 3361) and it offshored some tasks (parts), the resulting task-trade, i.e. import of motor vehicle parts (NAICS 3363), would show up as intraindustry trade at the three-digit level of aggregation (NAISC 336 for both). Such trade can be more precisely characterised as 'vertical specialisation of trade' (Hummels, Ishii and Yi, 2001). To see this for industry i , define $\mathbf{a}_{i1}^T$ as the row vector of offshored tasks and $-m_{<0,i}$ as the exports of final goods; Hummels *et al.*'s index of vertical specialisation of industry i is thus:

$$VS_i = \frac{\mathbf{a}_{i1}^T \mathbf{w}_0^*}{-p_{0,i} m_{<0,i}}$$

if $-m_{<0,i}$ is positive and zero otherwise. This would be recorded as intraindustry trade whenever the task/part import transaction was recorded in the same industry i as final good exports. Formally:

Corollary 5.4 (intraindustry trade and vertical specialisation): If the offshored tasks produce intermediate goods then intraindustry trade of the vertical specialisation type can arise in a perfect competition, constant returns setting.

We next turn to showing that going from no-task-trade to some tasks-trade is equivalent to particular changes in prices and endowments, whose impact can be studied using the (existing) Rybczynski and Stolper-Samuelson theorems. As is well known, the Stolper-Samuelson and Rybczynski results are cleanest when there the model has the same number of goods and factors, so we specialise to this case in developing task-trade analogues for the Stolper-Samuelson and Rybczynski theorems.

Inverting the Home pricing equation in (7), Home's equilibrium factor prices are equal to $\mathbf{w}_0 = \mathbf{A}^{-1}(\mathbf{p}_0 + \mathbf{S})$ at the task-trade equilibrium while it is equal to $\mathbf{w} = \mathbf{A}^{-1}\mathbf{p}$ at the goods-trade equilibrium (without task trade); Foreign wages are only affected by terms-of-trade changes. Home's shadow endowment equals $\mathbf{V}_0 = \mathbf{A}^T \mathbf{X}_0$, so Home production is $\mathbf{X}_0 = (\mathbf{A}^T)^{-1}(\mathbf{V} + \Delta\mathbf{V})$ when task-trade occurs but $\mathbf{X} = (\mathbf{A}^T)^{-1}\mathbf{V}$ without task-trade. Analogous expressions hold for $\mathbf{X}_0^*$ and $\mathbf{X}^*$. Thus with $\Delta\mathbf{p} \equiv \mathbf{p}_0 - \mathbf{p}$, the equations of change are:

¹⁶ Grubel-Lloyd and other indices quantifying intraindustry trade also suffer from a well-known aggregation bias: for instance, exports of Automobiles (NAICS 33611) and imports of Heavy trucks (NAICS 33612) would account as interindustry trade at the five-digit level of aggregation but as intraindustry trade at the four-digit level (NAISC 3361 for both).

$$\begin{aligned} \mathbf{X}_0 - \mathbf{X} &= (\mathbf{A}^T)^{-1} \Delta \mathbf{V}, & \mathbf{X}_0^* - \mathbf{X}^* &= -(\gamma \mathbf{A}^T)^{-1} \Delta \mathbf{V} \\ \mathbf{w}_0 - \mathbf{w} &= \mathbf{A}^{-1}(\Delta \mathbf{p} + \mathbf{S}), & \mathbf{w}_0^* - \mathbf{w}^* &= (\gamma \mathbf{A})^{-1} \Delta \mathbf{p} \end{aligned} \quad (10)$$

Using these in the standard ways (see Dixit and Norman 1985):

Proposition 6 (task-trade analogue to Stolper-Samuelson theorem). Starting from free trade in goods, allowing task-trade affects Home factor prices in exactly the way predicted by the standard Stolper-Samuelson theorem with the offshoring cost-savings vector $\mathbf{S}$ added to any exogenous variation in prices.

Proposition 7 (task-trade analogue to Rybczynski theorem). Starting from free trade-in-goods, allowing task-trade affects Home production in exactly the way predicted by the standard Rybczynski theorem with the implied ‘shadow migration’ vector replacing the usual exogenous variation of factor endowments.

Standard Jonesian magnification effects occur for both theorems.

The proofs are by application of standard HO techniques to (10). Note that the conditions under which the Stolper-Samuelson magnification effect arises are fairly general and, in particular, do not hinge on Assumption 4 (Leontief technologies). Generalising the conditions under which Proposition 6 applies only requires the absence of joint production and that each sector combine two primary factors or more to produce only one output (Jones and Scheinkman 1977). By contrast, generalizing Proposition 7 is problematic when there are more factors than goods. In the ‘even case’ $I = F$, each factor has at least one ‘natural enemy’, that is, the price of f falls as the price of i rises while all other prices remain constant. The dual of this proposition is that for any f , there exists an i such that the output of i falls as the endowment of f rises, keeping good prices constant because $\partial w_f / \partial p_i = \partial x_i / \partial V_f$ holds for all i and f in this case (Jones and Scheinkman 1977, Dixit and Norman 1980).

Standard trade theory rarely addresses the impact of free trade on global output. With task-trade, however, there are important and systematic global changes in output since shadow migration expands the effective world endowments, i.e. $\tilde{\mathbf{V}}_0^w > \tilde{\mathbf{V}}^w$. From (10) and the definition of $\mathbf{X}^w$ we get:

$$\mathbf{X}_0^w - \mathbf{X}^w = (1 - \gamma^{-1})(\mathbf{A}^T)^{-1} \Delta \mathbf{V}. \quad (11)$$

Each factor having a natural enemy, it follows that shadow migration of a single factor reduces the production of at least one good if all factors are employed in equilibrium, that is shadow migration is like actual migration in this respect:

Proposition 8 (global production effects). If task-trade produces shadow migration in only one factor, then global production of at least one good must rise and that of at least one other good must fall.

The proof is by the usual Ethier (1984) approach to the $I \times F$ version of the Rybczynski theorem.

As a minor corollary, we note that the expansion of the shadow-migration-adjusted world endowment vector is proportional to the augmentation of Home's shadow-migration-adjusted endowment, thus the global production effects tend to be proportional to Home's production effects as shown by comparison of (10) and (11).

5. Measuring shadow migration

The offshoring nation's shadow-migration vector, ΔV , is the factor content of imported tasks used in producing final goods. To ease comparison between goods-trade – where only final goods are traded – and task-trade – where 'tasks' are also traded – our theory formulates (8) in terms of the production and consumption vectors of final goods only. This content depends upon the Home technology matrix for the imported tasks, A_1 , and the Home final-good production vector X_0 . Task production and task trade – be it in the form of intermediate goods, producer services, etc. – are compressed into the shadow migration vector ΔV .

One must address several empirical challenges in order to measure ΔV . First, some imported tasks, especially service tasks, are not well captured by conventional trade statistics. Second, while statistics on imported goods is readily available, standard trade databases do not distinguish between final and intermediate usage of goods. For example an automobile wheel imported should be consider a final good when sold as a replacement part, but as an intermediate good when used in a new auto. There is nothing to be done about the first problem, but recently developed databases can be used to address the second. The World Input-Output Database (at www.WIOD.org) provide a breakdown of the final/intermediate usage of imports and exports for all major trading nations.¹⁷ With the trade flows identified, standard factor intensity coefficients – elements of the A matrices – would identify the factor content of the goods trade.

For example, focusing only on trade-task trade in the form of intermediate goods, Home's import of offshore-produced intermediates will be proportional to Home's final-good production vector, X_0 . To be specific, we can introduce the matrix ' C ' that gives the type-1 tasks needed to produce any level of final

¹⁷ The Trade in Value Added database at (available from oe.cd/tiva) provides the same breakdown for a slightly different set of nations and reports the matrix manipulations that allow one to determine the nationality of the value added in every nation's production vector.

goods, so $\mathbf{CX}_0$, is Home's vector of imported intermediate goods. To get the factor content of these, we use the Home technology matrix for these intermediate goods, call this $\mathbf{B}$. Thus $\mathbf{BCX}_0$ is equal to the shadow-migration vector, $\Delta\mathbf{V}$. To put it differently, the matrix $\mathbf{C}$ transforms $\mathbf{X}_0$ into the space of intermediate goods and $\mathbf{B}$ transforms these intermediates into factor-content space. The WIOD database provides data on nation's intermediate good imports, i.e. $\mathbf{CX}_0$.

A third challenge is the choice input-output tables, the $\mathbf{B}$ matrices, to be used in crafting the technology matrices. Our model assumes that all offshored parts are produced with the offshoring nation's technology, so the literal interpretation would use the technology matrix of the task-importing nation. However, some imported intermediates are produced by unrelated firms who presumably use their national technology. One way forward would be to calculate the shadow migration vectors for a weighted average of the importing and exporting nations' technology matrices, trying a variety of weights.

With such measures of shadow migration, it should be possible to calculate $(s\tilde{\mathbf{V}}_0^w - \mathbf{V}_0)$ and thus test how well (10) fits the final goods trade pattern, where final exports and imports are again identified using WIOD or TiVA data. Specifically, one could combine the $\Delta\mathbf{V}$ calculated this way with factor intensity figures from the literature (e.g. from national IO tables). Finally, the usual HO tests that have hereto been performed using unadjusted endowment vectors (e.g. Trefler 1993, Davis and Weinstein 2001, Trefler and Zhu 2010; see Bernhofen 2010 for a survey) could be re-run with shadow-migration-adjusted endowments instead.

Some more refined tests are also possible. Using the definition of $\tilde{\mathbf{V}}_0^w$ from above:

$$(s\tilde{\mathbf{V}}^w - \mathbf{V}) - \mathbf{A}^T \mathbf{M}_0 = [1 - s(1 - \gamma^{-1})] \Delta\mathbf{V} \quad (12)$$

Without task-trade, the HOV theorem asserts that the factor-content of Home's net exports of final goods, i.e. the left-hand-side of equation (12) above, should be zero as per equation (2). With task-trade, Home's net exports of final goods must pay for its imports of tasks from Foreign, i.e. the right-hand-side of equation (12). This leads to:

Corollary 5.1: The difference between the factor-content predicted by the HOV theorem and the measured factor-content of Home's import vector, $\mathbf{A}^T \mathbf{M}_0$, is (i) proportional to and smaller than the shadow migration vector $\Delta\mathbf{V}$; (ii) this difference is increasing in the size of the country (ceteris paribus) and is exactly equal to $\Delta\mathbf{V}$ in the limiting case of $s = 0$ (a country so small in the world that its size is negligible); and (iii) this difference is decreasing in the technology gap (ceteris paribus) and is exactly equal to $\Delta\mathbf{V}$ in the limiting case of $\gamma = 1$ (technological differences are zero).

The proof is by inspection of (12). In the two-factor example (skilled and unskilled workers) where Home is skill-abundant and coordination costs are such that the offshored tasks are particularly unskilled intensive, Home's shadow-migration-adjusted endowment is skewed towards unskilled labour, so, as per Proposition 4, it may import the skill-intensive good for reasons that are conceptually different from the exogenous Ricardian differences suggested by Leontief (1953) and confirmed by Trebler (1993). From (12), $(s\tilde{\mathbf{V}}^w - \mathbf{V}) - (\mathbf{A}^T \mathbf{M}_O + \Delta \mathbf{V}) = -s(1 - \gamma^{-1})\Delta \mathbf{V}$ and combining this with Corollary 5.1 we have:

Corollary 5.2 (bounded HOV errors): In the presence of task-trade, the standard HOV factor-content prediction, $s\tilde{\mathbf{V}}^w - \mathbf{V}$, should overstate the factor-content of final-goods trade but understate the factor-content of final-goods trade plus that of task-trade. More precisely, the factor-content of final-goods and traded tasks are $\mathbf{A}^T \mathbf{M}_O$ and $\Delta \mathbf{V}$ respectively, so the following bounds should hold:

$$\mathbf{A}^T \mathbf{M}_O < s\tilde{\mathbf{V}}^w - \mathbf{V} < \mathbf{A}^T \mathbf{M}_O + \Delta \mathbf{V}.$$

As an illustrative example of this, consider Vietnam, an unskilled-abundant country that has recently become a net exporter of high-technology engine parts to skill-abundant Japan. This violates the HOV logic but is consistent with our integrating framework where task-trade arises from the combination of Japanese technology with Vietnamese labour.

6. Integrating previous work in the literature

This section uses the shadow migration approach to revisit previous work and well-known cases. As most the literature works with what are effectively 2x2x2 models, we follow suit. Moreover, to illustrate how the various cases fit together we maintain Assumption 4 (Leontief technologies) in this section.

6.1. The 2x2x2 case

The two factors (skilled labour K and unskilled labour L) are paid r and w , respectively. The sectors are X and Y where Y is capital intensive (i.e. $\kappa_Y > \kappa_X$ where $\kappa_i \equiv a_{Ki}/a_{Li}$ for $i = X, Y$) and X is numeraire. Foreign is abundantly endowed with unskilled labour (i.e. $k^* < k$ where $k \equiv K/L$ and $k^* \equiv K^*/L^*$). To ensure diversified production with goods-trade, we impose the regularity condition $\kappa_Y > k > k^* > \kappa_X$. Preferences are identical and homothetic.

Inverting expressions in (1) yields the good-trade equilibrium values:

$$\mathbf{w} = \mathbf{A}^{-1} \mathbf{p}, \quad \mathbf{w}^* = \gamma^{-1} \mathbf{A}^{-1} \mathbf{p}, \quad \mathbf{X} = (\mathbf{A}^T)^{-1} \mathbf{V}, \quad \mathbf{X}^* = \gamma^{-1} (\mathbf{A}^T)^{-1} \mathbf{V}^*, \quad (13)$$

where

$$\mathbf{w} \equiv \begin{bmatrix} w \\ r \end{bmatrix}, \quad \mathbf{p} \equiv \begin{bmatrix} 1 \\ p \end{bmatrix}, \quad \mathbf{V} \equiv \begin{bmatrix} L \\ K \end{bmatrix}, \quad \mathbf{X} \equiv \begin{bmatrix} X \\ Y \end{bmatrix}, \quad \mathbf{A} \equiv \begin{bmatrix} a_{LX} & a_{KX} \\ a_{LY} & a_{KY} \end{bmatrix}, \quad \mathbf{A}_1 \equiv \begin{bmatrix} a_{LX1} & a_{KX1} \\ a_{LY1} & a_{KY1} \end{bmatrix}.$$

Global market-clearing yields

$$p = \frac{\alpha}{1-\alpha} \frac{a_{LY}}{a_{LX}} \frac{\kappa_Y - \tilde{k}^w}{\tilde{k}^w - \kappa_X},$$

where $\alpha \in (0,1)$ denotes the equilibrium expenditure share on Y and

$$\tilde{k}^w \equiv \frac{K + K^*/\gamma}{L + L^*/\gamma}$$

is the world's capital-to-labour relative endowment in effective units.

The literature typically investigates task-trade by comparing the outcomes with and without offshoring/task-trade. Following suit, we solve (8) for the case at hand and subtract the goods-trade values to get:

$$\begin{aligned} X_o - X &= \frac{1}{a_{LY}} \frac{\kappa_Y \Delta L - \Delta K}{\kappa_Y - \kappa_X}, & Y_o - Y &= \frac{1}{a_{LY}} \frac{\Delta K - \kappa_X \Delta L}{\kappa_Y - \kappa_X} \\ w_o - w &= \frac{a_{KY} S_X - a_{KX} S_Y}{a_{LX} a_{LY} (\kappa_Y - \kappa_X)} + \mathbf{A}^{-1} \begin{bmatrix} 0 \\ \Delta p \end{bmatrix}, & r_o - r &= \frac{a_{LX} S_Y - a_{LY} S_X}{a_{LX} a_{LY} (\kappa_Y - \kappa_X)} + \mathbf{A}^{-1} \begin{bmatrix} 0 \\ \Delta p \end{bmatrix} \end{aligned} \quad (14)$$

where variables with and without the subscripts 'o' indicate the goods-trade and task-trade equilibrium values respectively. Additionally, $\Delta p \equiv p_o - p$ is the terms-of-trade variation;

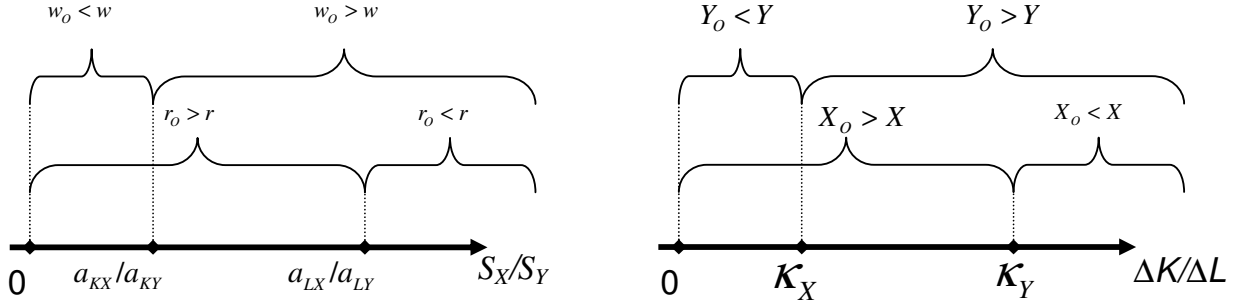
$S_X = a_{LX1}(w_o - w_o^*) + a_{KX1}(r_o - r_o^*)$ and $S_Y = a_{LY1}(w_o - w_o^*) + a_{KY1}(r_o - r_o^*)$ are the cost savings in sectors X and Y ; and $\Delta K = a_{KX1}X_o + a_{KY1}Y_o$ and $\Delta L = a_{LX1}X_o + a_{LY1}Y_o$ are the shadow migrations of K and L .¹⁸

Expression (14) implicitly defines the necessary and sufficient conditions for signing production and wage effects of offshoring/task-trade.¹⁹ To elucidate these conditions, we depict the full range of wage and production outcomes in Figure 1 (left panel for wages and right panel for quantities).

Figure 1: Necessary and sufficient conditions for wage and production effects.

¹⁸ For example, $\Delta X/X = [(\Delta L/L)/(1 - k/\kappa_Y) - (\Delta K/K)/(\kappa_Y/k - 1)]$ and $k/\kappa_Y < 1$.

¹⁹ The appendix of our working paper provides exact necessary and sufficient conditions for diversification in the 2x2x2 version of the model (http://www.dagliano.unimi.it/media/wp2008_250.pdf).



The quality effects depend upon the nature of the implied shadow migration. If it is heavily skewed towards L (specifically, $\Delta K/\Delta L$ is lower than the capital intensity of the L -intensive good X , κ_X) then X rises and Y falls. If shadow migration has an intermediate factor ratio, i.e. $\kappa_X < \Delta K/\Delta L < \kappa_Y$, then both X and Y rise, and if it is heavily skewed towards K ($\Delta K/\Delta L > \kappa_Y$), then X falls and Y rises. Foreign production effects are characterised in an isomorphic manner.

Turning to the wage effects, we see from (14) that the wage of Home L -workers rises (controlling for terms-of-trade effects) if and only if the cost-saving is sufficiently greater in the L -intensive sector than in the K -intensive sector, namely, $w_o > w \Leftrightarrow S_X / S_Y > a_{KX} / a_{KY}$; in this case, r may actually fall. The necessary and sufficient condition for r to fall (controlling for terms-of-trade effects) is that the ratio of cost-savings exceeds the ratio of L -input coefficients, namely, $S_X / S_Y > a_{LX} / a_{LY}$. If the cost-savings ratio lays between the skilled-unskilled endowment ratios, then both w and r rise by an effect analogous to product augmenting technical change. Controlling for the terms-of-trade effects, task-trade has no effect on foreign wages because Foreign firms do not benefit from this technical change.

6.2. Integrating specific cases in the literature

Most of the literature ignores terms-of-trade effects by imposing the small open economy assumption. In this case, the impact of offshoring on w and r are given by the bottom row of (14) taking $\Delta p = 0$.

Offshoring as sector-specific technical progress. Many papers assume that offshoring occurs in a single sector only. This includes Jones and Kierzkowski (1990) and follow-on papers.²⁰ In such papers, either $S_X = 0$ or $S_Y = 0$, so offshoring acts like sector-specific technical progress. The wage effects thus depend on the

²⁰ For example, Jones and Marjit (1992), Arndt (1997, 1999), Jones and Kierzkowski (1998, 2000), Deardorff (1998a), Venables (1999), Jones, Kierzkowski and Leonard (2002), Kohler (2004a, b), and Markusen (2006). Francois (1990a, b, c) explicitly considers the impact of offshoring on the factor price equalization set.

factor intensity of the progressing sector; offshoring in the L -intensive sector only, namely X , raises w and lowers r ; while offshoring in Y only does the opposite, as (14) shows. That is to say, offshoring in unskilled labour intensive sectors *raises* unskilled wages – even if the tasks being offshored are themselves labour intensive.

Offshoring as *factor-specific technical progress*. Other papers consider offshoring involving only one factor but in both sectors, so offshoring is like factor-specific technical progress, specifically $S_X = a_{LX1}(w_O - w_O^*)$ and $S_Y = a_{LY1}(w_O - w_O^*)$. Factor-specific technical progress has ambiguous effects on w and r (Jones 1965); what matters is the relative size of the cost savings by sector (the necessary and sufficient conditions are summarised by the left-panel in Figure 1). That is to say, offshoring of unskilled tasks may raise unskilled wages because such trade in tasks tends to reduce production costs in unskilled labour intensive sectors more than in skilled labour intensive sectors.

This result was famously popularized in the main body of analysis of GRH. Unskilled labour unambiguously gains from the offshoring of unskilled-intensive tasks while the other factor's wage effect is exactly zero. How does this fit in with the ambiguity apparent in (14)? This sharp result is driven by the concatenation of three assumptions. Working in what could be boiled down to a $2 \times 2 \times 2$ model, GRH describe the production process of each sector as involving two continuums of tasks – one that uses only L , the other only K . The four continuums are normalised to the unit interval, and within each continuum, tasks are assumed to use the same amount of the relevant factor. After ordering the tasks by increasing offshoring costs, GRH normalise the offshoring costs across sectors. In the famous special case, only L -tasks are offshorable, but the two normalisations and the assumption of identical task-intensity imply that exactly the same fraction λ of L is offshored in X and Y at equilibrium. In our notations $S_X = \lambda a_{LX}(w_O - w_O^*)$ and $S_Y = \lambda a_{LY}(w_O - w_O^*)$. Plugging these into (14) yields $r_O - r = 0$ and $w_O - w = \lambda(w_O - w_O^*)$. Ambiguity of the wage effects is restored in subsequent analysis in GRH when they drop the constant proportion assumption or allow for offshoring of H -tasks.

7. Extending the basic model

We extend the basic task-trade model to allow technology diffusion, Ricardian differences among nations that result in two-way offshoring – a common phenomenon among OECD nations (Amiti and Wei 2005) – and to incorporate monopolistic competition following Helpman and Krugman (1985). To focus on the new economic elements, we maintain Assumption 4 (Leontief technologies).

7.1. Offshoring with technology diffusion

In the previous section, all output of the offshored sector was ‘sold’ to Home. Here we allow local sales of offshored tasks; put differently, we assume that Home technology diffuses to Foreign for the offshored tasks in the sense that Foreign final good producers can procure their type-1 tasks at the same price as Home producers. This version of the model captures well long-run technology spillovers brought about by vertical FDI (Javorcik 2004) and it is also consistent with, e.g., Apple’s offshore outsourcing activities.

Home firms have an incentive to sell type-1 tasks to Foreign producers as their superior technology gives them a cost-edge over local producers. The purchase of type-1 tasks by Foreign final-good makers alters the price and employment conditions. With local task-1 sales, (8) becomes:

$$\mathbf{p}_O + \mathbf{S} = \mathbf{A}\mathbf{w}_O, \quad \mathbf{p}_O + \mathbf{S}^* = \gamma\mathbf{A}\mathbf{w}_O^*; \quad \mathbf{V} + \Delta\mathbf{V} = \mathbf{A}^T\mathbf{X}_O, \quad \mathbf{V}^* + \Delta\mathbf{V}^* = \gamma\mathbf{A}^T\mathbf{X}_O^*. \quad (15)$$

Solve for wages:

$$\mathbf{w}_O - \mathbf{w} = \mathbf{A}^{-1}(\mathbf{S} + \Delta\mathbf{p}), \quad \mathbf{w}_O^* - \mathbf{w}^* = \mathbf{A}^{-1}(\mathbf{S}^* + \Delta\mathbf{p}), \quad (16)$$

where $\Delta\mathbf{p} \equiv \mathbf{p}_O - \mathbf{p}$ and $\mathbf{S} \equiv \mathbf{A}_1(\mathbf{w}_O - \mathbf{w}_O^*)$. In addition, Foreigners now benefit directly from offshoring’s cost saving, whereas they were only indirectly affected via terms-of-trade effects in the case of Sections 3 to 6.

There is a crucial difference, though, between the factor price effects on Home versus Foreign labour. For Home labour, it is *rents* that generate the cost-savings (i.e. the fact that Home firms use their superior technology but pay Foreign wages); for Foreign labour it is *technology diffusion* that generates the cost-savings and this cost-savings are proportional to the technology gap: $\mathbf{S}^* = (\gamma - 1)\mathbf{A}_1\mathbf{w}_O^* > \mathbf{0}$. Despite this qualitative difference, the Foreign wage changes in (16) are isomorphic to those of Home. Consequently, all the detailed analysis in the previous sections relating the cost-savings to the wage effects is now applicable to the impact of offshoring on Foreign wages when technology diffusion (via local sales) is allowed.

Solving (15) for task-trade production in the extended model and comparing it to goods-trade values yields:

$$\mathbf{X}_O - \mathbf{X} = (\mathbf{A}^T)^{-1}\Delta\mathbf{V}, \quad \mathbf{X}_O^* - \mathbf{X}^* = \gamma^{-1}(\mathbf{A}^T)^{-1}\Delta\mathbf{V}^*.$$

Three aspects of this expression are noteworthy. First, $\Delta\mathbf{V} \equiv \mathbf{A}_1^T\mathbf{X}_O$ is as in Section 4. By contrast, the impact on Foreign production is qualitatively different because $\Delta\mathbf{V}^* \equiv -\Delta\mathbf{V} + (\gamma - 1)\mathbf{A}^T\mathbf{X}_O^*$. Second, the shadow migration interpretation is less clear-cut than in Section 4. The first term in the right-hand side of the definition of $\Delta\mathbf{V}^*$ does relate to the shadow migration of Foreign workers conveying Home tasks and this effect tends to reduce Foreign production of final goods. However, the second term, $(\gamma - 1)\mathbf{A}^T\mathbf{X}_O^*$, relates to

the *technology diffusion* that now benefit all Foreign workers conveying offshoreable tasks. This increase in effective factor endowments is proportional to the technology gap and it tends to expand Foreign production. The net effect on Foreign production of final goods is thus ambiguous.

Finally, effective world endowments are unambiguously larger with offshoring, i.e. $\tilde{\mathbf{V}}_0^w > \tilde{\mathbf{V}}^w$. In the case without technology diffusion (Sections 3 to 6), Home offshores technology that is used only for Home production, so the Foreign labour employed in the offshoring sector is diverted from Foreign production and this means that the Foreign production change is proportional to the Home production effect but of the opposite sign. Here the tech-transfer embodied in offshoring diffuses to and stimulates Foreign production, so simple proportionality breaks down. Nevertheless, the basic analysis of production effects for Foreign follows the reasoning of Proposition 4 and Figure 1 with $\Delta\mathbf{X}^*$ substituted for $\Delta\mathbf{X}$.

Since the trade effects follow from the production and factor price changes, it is clear that offshoring in the technology-diffusion case at hand will also be a source of comparative advantage and intra-industry trade.

To summarise, the main difference between the two cases is that offshoring with technology diffusion spreads some of the benefit of the implicit technology transfers to Foreign factors whereas in the case studied in Sections 3 to 6 all the benefits accrued to Home factors (modulo terms of trade effects).

7.2. Ricardian differences and intra-industry two-way offshoring

Davis (1995) shows that intraindustry trade arises in a HO-like model due to minor technological differences among nations when there are more goods than factors. As many production patterns are consistent with the system of equations in (1) when $I > F$, even minor technological advantages can shift global production of individual goods to a single nation. We apply this insight to generate two-way task-trade that arises from task-level technology differences across nations (e.g. Italy may be especially excellent at making brakes for small cars, while France may be especially excellent at making air bags for small cars, even though France and Italy are roughly at parity when it comes to small car technology).

To implement this idea cleanly, we eliminate all macro differences between Home and Foreign by assuming $\gamma = 1$, and $\mathbf{V} = \mathbf{V}^*$ (we do not impose Assumption 4). The trade-in-goods equilibrium is thus marked by absolute FPE and zero trade. There are, however, task-level technology differences in the sense that Foreign's task technology is as in (3), but Home's task technology is now:

$$a_{fi}(\mathbf{w}) \equiv \sum_{t=1}^{N_i} \varepsilon_{it} a_{fit}(\mathbf{w}); \quad \forall f = 1, \dots, F; i = 1, \dots, I$$

where $\varepsilon \in [1-\mu, 1+\mu]$ is a random variable that is iid across sectors and tasks, symmetrically distributed around $E\{\varepsilon\} = 1$ and with $\mu > 0$.²¹

We assume that all tasks are potentially offshorable and firms can supply tasks to one another. We also assume that N_i is sufficiently large for all industries and that the coefficients a_{ji} are symmetrically distributed around a_{ii} so that the realization of $a_{ji}(\cdot)$ in Home is arbitrarily close to $a_{ji}^*(\cdot)$ in Foreign. Thus, factor prices are equalised and Home is competitive in all tasks where $\varepsilon_{ji} < 1$ while Foreign has the edge in all other tasks. The law of large numbers implies that Home has the edge in half the tasks sector-by-sector. Moreover, the tasks in which Home has the Ricardian comparative advantage will be a random sample of all tasks, so the Home employment condition will be:

$$\mathbf{V} = \frac{1}{2} \mathbf{A}^T \mathbf{X} + \frac{1}{2} \mathbf{A}^T \mathbf{X}^*.$$

As Home and Foreign are symmetric at the macro level, it is clear that task-trade will have no impact on the $\mathbf{w}$'s or $\mathbf{X}$'s, but intraindustry offshoring and intraindustry task-trade will arise. There are no terms-of-trade effects, so gains from task-trade are assured. To see this, note that $\mathbf{A}_0^T = \frac{1}{2} [1 + F_\varepsilon(1)] \mathbf{A}^T = \frac{3}{4} \mathbf{A}^T < \mathbf{A}^T$ holds (by the law of large numbers), where $F_\varepsilon(\cdot)$ is the CDF of ε . Further, all factor owners are better off if preferences are homothetic; to see this, note that $\mathbf{w}_0 > \mathbf{w}$ follows by unit cost pricing ($\mathbf{w}_0 = \mathbf{A}_0^{-1} \mathbf{p}_0$ and $\mathbf{w} = \mathbf{A}^{-1} \mathbf{p}$) and homothetic preferences imply $\mathbf{p}_0 = \mathbf{p}$.

7.3. Offshoring in a Helpman-Krugman trade model

A fact that has been well appreciated in trade theory since Helpman and Krugman (1985) is that the basic HO results carry through unaltered in a Dixit-Stiglitz monopolistic competition setting provided that the increasing returns technology is homothetic, i.e. the cost function is equal to $(mx + f) \sum_f a_{if} w_f$, where m and f are parameters that govern marginal and fixed costs, respectively, and x is firm-level output. Here we use this insight to show that the above analysis could easily be conducted in a monopolistic competition trade model setting.

The key to the Section-3 analysis lies in the pricing and employment equations and their restatement using the shadow migration insight. As is well known, the free-entry output of a typical variety under monopolistic competition with homothetic technologies depends only on cost and taste parameters and so does not vary

²¹ Here we treat the realizations of ε as parameters. These are the endogenous result of external scale economies in the recent work of Grossman and Rossi-Hansberg (2012).

across the equilibria we consider. This implies that monopolistic competition sectors display constant returns at the sector level (doubling all inputs results in doubling sectoral output at equilibrium), specifically, $X = n\bar{x}$ is the sector's total output where $\bar{x}$ is the invariant firm-level output and n is the free-entry equilibrium number of firms. The sector's employment of factor f is thus $n\bar{x}(m + f/\bar{x})a_{if}$ where i is the Dixit-Stiglitz sector. Likewise the price of the Dixit-Stiglitz sector equals average cost, namely $(m + f/\bar{x})\sum_f a_{if}w_f$. Choosing units such that $m + f/\bar{x}$ is equal to unity, the employment and pricing equations for this model are identical to those of the HO model of Section 3, as are the Foreign pricing and employment conditions. With this, we have reduced the problem to the one solved in Sections 3, so can conclude that the relevant Propositions also in a model that allows monopolistic competition.

8. Summary and concluding remarks

Recent theoretical contributions have renewed interest in characterising the effects of offshoring and the result has been a wide range of cases that generate unexpected results – many which seem to contradict intuition based on standard trade theory. This paper is an attempt to integrate the theoretical task-trade literature into standard trade-in-goods theory. We present a simple modification of the HO model that allows us to consider trade-in-goods in the traditional sense (i.e., the exogenous shift from no-trade to free-trade in goods) as well as task-trade (i.e. the exogenous shift from no-trade to free-trade in a range of routine tasks).

The expressions for the trade and production patterns, and goods and factor prices are highly complex in the task-trade equilibrium and clearly violate the standard HO predictions. However, if one views offshoring as ‘shadow migration’, and uses shadow-migration adjusted endowments instead of actual endowments, the HO trade and production predictions work perfectly. As such, we can use the elegant HO theorems to establish necessary and sufficient conditions for the trade and production effects of offshoring. As the quantity of factors employed in offshore production is potentially observable with firm-level datasets, these task-trade analogues of the HO and Rybczynski theorems are testable in principle. We also show that offshoring creates intra-industry trade when the various tasks are considered as being in the same sector. On the price side, we show how using the vector of the cost-savings that ‘shadow migration’ produces can be used to develop task-trade analogues of the FPE and Stolper-Samuelson theorems.

Our integrating framework does not encompass the many important contributions in the literature that focus on issues of corporate governance, e.g. Antràs and Helpman (2004) and Antràs and Chor (2013). These papers typically focus on the division of rents and how they depend upon the corporate structure chosen. As the division of rents will affect the division of the benefits from offshoring, we conjecture that it could have

significant general equilibrium effects as well as the more direct effects on ownership. Incorporating such issues would seem to be an important topic for future theoretical research.

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