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Small self-contained quantum thermal machines function without external source of work or control but using only incoherent interactions with thermal baths. Here we investigate the role of entanglement in a small self-contained quantum refrigerator. We first show that entanglement is detrimental as far as efficiency is concerned—fridges operating at efficiencies close to the Carnot limit do not feature any entanglement. Moving away from the Carnot regime, we show that entanglement can enhance cooling and energy transport. Hence, a truly quantum refrigerator can outperform a classical one. Furthermore, the amount of entanglement alone quantifies the enhancement in cooling.

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I. INTRODUCTION

The study of quantum thermal machines has a long history, from the thermodynamic analysis of lasers [1–3] to considerable work on quantum cycles and the second law [4–17]. Recently, models of small self-contained quantum thermal machines [18–22] have attracted attention. The key feature of such machines is that they function without any external source of work or control. Only incoherent interaction with thermal baths are required. Interestingly, there exists no fundamental limit on the size of such machines [18] or on their efficiency [19]. Their main interest resides in their simplicity, which makes them an ideal test for exploring quantum thermodynamics.

Beyond the fundamental interest, these models may also turn out to be relevant from a more practical point of view. Proposals for experimental realizations were made considering various physical platforms. A scheme using superconducting qubits (three flux-biased qubits) driven by current noise was discussed in Ref. [23], while an electronic setup based on quantum dots was presented in Ref. [24] (see also Ref. [25]). Finally, Ref. [26] discussed the cooling of an optomechanical systems.

From a more fundamental point of view, an important question is whether quantum effects play any significant role in small self-contained thermal machines. Indeed, although these machines are described within the quantum formalism, it is not immediately clear to what extent their working is inherently quantum. One can give a heuristic account of the functioning of the machine in classical terms.

Here our aim is to establish the role of quantum effects in self-contained quantum thermal machines. Our main focus will be on the concept of entanglement, the paradigmatical quantum effect. Hence, if entanglement turns out to play a role in self-contained quantum thermal machines, this would make it clear that the working of such machines is truly quantum mechanical. Moreover, it would then raise the question of whether entanglement can enhance the performance of such machines.

Below, we address these questions focusing on the model of the smallest possible self-contained quantum refrigerator [18,19]. We first show that in the regime of high efficiency, that is, machines operating with efficiency close to Carnot limit, the machine does not feature any entanglement. Hence, entanglement appears to be *detrimental* as far as efficiency is concerned since an entangled state cannot get close to Carnot efficiency. Next, moving away from the high-efficiency regime, we show that there exist regimes featuring entanglement. In fact, a wide variety of types of entanglement can be found in our system—including genuine multipartite entanglement—depending upon the external conditions. Finally, we show that this entanglement is useful, as it enhances cooling and energy transport. Specifically, given an object to cool and a set of resources (for instance, fixing the temperatures of the heat baths), we show that a refrigerator featuring entanglement can outperform a "classical" refrigerator (i.e., featuring no entanglement), as it allows us to cool the object to lower temperatures. Moreover, we demonstrate that the improvement grows monotonically with entanglement measures, strongly suggesting a functional relationship.

We emphasize that the above results are not general but refer to the specific fridge model discussed in Refs. [18,19]. However, as this model represents the simplest possible refrigerator, it is in some sense the most fundamental one. Results obtained here are therefore bound to reflect some basic facts about quantum thermal machines; in one way or another, they are likely to have implications for all machines.

II. QUANTUM FRIDGE MODEL

We start by briefly reviewing the model of the smallest quantum refrigerator of Refs. [18,19] (see Fig. 1), which we will focus on in this work. Let us consider three qubits, which in the absence of interaction have vanishing ground-state energies and excited-state energies E_j ($j = 1,2,3$). The free Hamiltonian of the refrigerator is thus given by

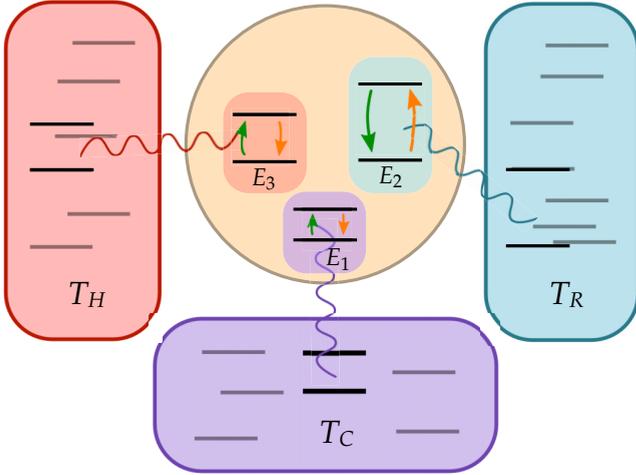$$H_0 = E_1 \Pi_1 + E_2 \Pi_2 + E_3 \Pi_3, \tag{1}$$

FIG. 1. (Color online) Schematic diagram of the quantum refrigerator. The fridge consists of three qubits (inside the yellow circle), each in weak thermal contact (wiggly lines) with a bath at a different temperature. The qubits interact via the weak interaction Hamiltonian H_{int}, which couples the two degenerate levels $|010\rangle$ and $|101\rangle$, depicted by the arrows. The lower qubit [purple (dark gray)] is the object to be cooled. At equilibrium, it reaches a temperature $T_S < T_C$. The other two qubits [red and blue (medium gray)] are the machine qubits, connected to heat baths at temperatures T_R and T_H.

where $|1\rangle_j$ is the exited state of qubit j (i.e., energy eigenstate at energy E_j) and $\Pi_j = |1\rangle_j\langle 1|$ the corresponding projector. We choose the energy levels such that $E_2 = E_1 + E_3$ and $E_1 \neq E_3$. Given this condition the Hamiltonian contains only two states degenerate in energy, $|010\rangle$ and $|101\rangle$. Next we consider weakly coupling the qubits by placing an interaction between them. In the weak-coupling regime, the main effect of a generic three-qubit Hamiltonian is to couple only states degenerate in energy—in this case $|010\rangle$ and $|101\rangle$ [27]; couplings between other states are suppressed by a factor inversely proportional to their energy difference. Thus, we focus here on this main effect, i.e., the interaction Hamiltonian,

$$H_{\text{int}} = g\left(|010\rangle\langle 101| + |101\rangle\langle 010|\right). \quad (2)$$

We impose $g \ll E_j$, which signifies the weak-coupling regime and also ensures that the eigenvalues and eigenstates of the fridge remain governed by H_0.

Finally, each qubit is taken to be in contact with a separate thermal reservoir. The temperatures of the reservoirs are denoted by T_C (cold), T_R (room), and T_H (hot) for qubits 1, 2, and 3, respectively. The thermal contact between each qubit and bath is governed by Linbladian dissipative dynamics, which we model here using a simple reset model, the justification of which we shall comment on briefly. In this model, with probability $p_i\delta t$ per time δt, qubit i is reset to the thermal state τ_i, at the temperature of its bath, while for all other times it evolves unitarily according to the combined Hamiltonian $H_0 + H_{\text{int}}$. That is, in this model thermalization events are taken to be rare but strong events. The equation of motion for the refrigerator using this model of dissipation [18]

is given by the master equation

$$\frac{\partial \rho}{\partial t} = -i[H_0 + H_{\text{int}}, \rho] + \sum_i p_i(\tau_i \otimes Tr_i(\rho) - \rho), \quad (3)$$

where $\tau_i = r_i|0\rangle_i\langle 0| + (1 - r_i)|1\rangle_i\langle 1|$ with $r_i = 1/(1 + e^{-E_i/T_i})$. In general, one would expect there to be additional terms in Eq. (3), corresponding to dissipative dynamics on qubit j originating from the combination of the interaction Hamiltonian and the dissipative dynamics on qubit $i \neq j$. In other words, one may expect each qubit to be effectively in contact with all three baths due to the interaction Hamiltonian [22,28]. However, in the regime where $p_i \approx g \ll E_i$, these additional effects, whose strength is approximately $gp_i \ll g$, can be safely neglected.

Here our focus is on the stationary state (i.e., long-term behavior) of the refrigerator, ρ_S, which satisfies $\dot\rho_S = 0$, i.e.,

$$i[H_0 + H_{\text{int}}, \rho_S] = \sum_i p_i(\tau_i \otimes Tr_i(\rho_S) - \rho_S). \quad (4)$$

As shown in Ref. [19], this equation can be solved analytically for all values of the parameters. The solution takes the form

$$\rho_S = \tau_1\tau_2\tau_3 + \gamma\sigma \quad (5)$$

where γ is a dimensionless parameter depending upon all parameters of the model (namely p_i, g, E_i, and temperatures $T_{C,R,H}$), and σ is a traceless matrix with a single off-diagonal term (see Ref. [19] for details). The important property of the solution is that it can be shown that the refrigerator cools qubit 1 whenever $\gamma > 0$. In this case, one finds that qubit 1 is in a stationary state that is diagonal, with corresponding temperature $T^S < T_C$. Moreover, the efficiency of the refrigerator tends to the Carnot limit in the limit $\gamma \to 0$.

III. AROUND THE CARNOT POINT

Let us first discuss the properties of ρ_S for those refrigerators which are operating close to the Carnot efficiency, which we hereafter refer to as refrigerators around the *Carnot point*. From inspection of Eq. (5), it is clear that for $\gamma = 0$, ρ_S is a fully separable state, as it is nothing other than the direct product of thermal state for each qubit. Hence, no entanglement is present at the Carnot point. More interestingly, this statement remains true for a small region within the set of all ρ_S in the vicinity of the Carnot point. Thus all refrigerators which are highly efficient function without entanglement. To see this let us first rewrite ρ_S in the following form:

$$\rho_S = w|\text{GHZ}\rangle\langle\text{GHZ}| + (1 - w)\sigma_{\text{diag}}, \quad (6)$$

where $|\text{GHZ}\rangle = (|010\rangle + i|101\rangle)/\sqrt{2}$ is tripartite entangled state (of the Greenberger-Horne-Zeilinger form) and σ_{diag} is a diagonal density matrix, hence, corresponding to a fully separable state. While there is no unique notion of entanglement in multipartite systems, it turns out that the entanglement of states of the form (6) can be conveniently characterized.

In the vicinity of any Carnot point, the state ρ_S has full rank and off-diagonal terms which are small compared to diagonal ones. Hence, in this regime, the state can be decomposed as

TABLE I. Regimes of the quantum refrigerator featuring entanglement. Here we give the full list of parameters required to exhibit the different entanglement regimes present in the quantum refrigerator. The first row gives a point where there is genuine multipartite entanglement between all three qubits, while the second and third rows are points where there is only bipartite entanglement along a single bipartition. The last row shows a regime with bipartite entanglement across every bipartition but without genuine tripartite entanglement.

$\mathcal{C}_{C|RH}$	$\mathcal{C}_{R|CH}$	$\mathcal{C}_{H|CR}$	$\mathcal{C}_{CRH}$	T_C	T_R	T_H	E_1	E_3	g	p_1	p_2	p_3
0.003	0.004	0.004	0.003	1.0	1.1	1.0×10^4	2.0	300	1.0×10^{-4}	1.0×10^{-5}	1.0×10^{-3}	1.0×10^{-5}
0	0	0.00002	0	1.0	2.0	1.0×10^4	4.4	379	3.4×10^{-4}	1.3×10^{-5}	1.6×10^{-4}	1.3×10^{-5}
0	0.00005	0	0	1.0	42.6	1.0×10^4	4.4	421	2.8×10^{-4}	1.3×10^{-5}	8.3×10^{-4}	1.0×10^{-5}
0.0008	0.005	0.004	0	1.0	1.1	1.0×10^4	2.0	300	1.0×10^{-4}	1.0×10^{-5}	2.0×10^{-4}	1.0×10^{-5}

$\rho_S = (1 - \epsilon)\sigma'_{\text{diag}} + \epsilon\rho(p)$, where σ'_{diag} is a diagonal separable state and $\rho(p) = p|\text{GHZ}\rangle\langle\text{GHZ}| + (1 - p)\frac{1}{8}\mathbb{1}$. Since $\rho(p)$ is fully separable for $p \leqslant \frac{3}{11}$ [29], it follows that ρ_S is fully separable in the vicinity of any Carnot point. In fact, it can even be shown that any ρ_S at the Carnot point has a small ball of fully separable state in the full Hilbert space around it—see Appendix for more details.

IV. ENTANGLED REGIMES

Next we ask whether there exist regimes in which entanglement is present in ρ_S. At this point it is useful to recall that entanglement can appear under several forms in a state of 3 qubits. Indeed, there can be bipartite entanglement along a given bipartition (e.g., qubit 1 versus qubits 2 and 3) or genuine tripartite entanglement. Here our main tool will be a class of entanglement witnesses developed in Refs. [30,31] which allow one to fully characterize the entanglement of states of the form ρ_S. Moreover, these witnesses also provide a meaningful entropy based measure of multipartite entanglement [32] and necessary and sufficient conditions for biseparability for our system [33]. Formally, these witnesses are given by inequalities of the form

$$W_S(\rho) = 2\left(|\rho_{3,6}| - \sum_{k \in \mathcal{S}} \sqrt{\rho_{k,k}\rho_{9-k,9-k}}\right) \leqslant 0, \quad (7)$$

where $\rho_{i,j}$ denotes elements of the density matrix and the set $\mathcal{S}$ depends on the partition and type of entanglement one is interested in. When inequality (7) is violated, its left-hand side gives the concurrence [34] of $C|RH$ ($\mathcal{S} = \{2\}$), $R|CH$ ($\mathcal{S} = \{1\}$), $CR|H$ ($\mathcal{S} = \{3\}$) or the genuine multipartite concurrence (see Refs. [32,35]) for $\mathcal{S} = \{1,2,3\}$. When inequality (7) holds, no entanglement is present on the given bipartition.

Moving away from the Carnot point we find, by sweeping through the parameter space numerically, that there exist regimes where entanglement is present. In fact, most types of entanglement can be found. We find regimes where there is entanglement (i) along only a single bipartition of the system, (ii) on all three bipartitions at the same time, and (iii) genuine tripartite entanglement, the strongest form of multipartite entanglement. In Table I we characterize various entanglement regimes, giving the corresponding parameters of the refrigerator. Note that the only type of entanglement we did not observe is the following: entanglement on the $C|RH$ partition but no entanglement across the other two bipartitions.

V. ENTANGLEMENT ENHANCES COOLING

In the remainder of the paper, we investigate the usefulness of this entanglement that we have just uncovered in the fridge. We will see that entanglement can in fact enhance the performance of the refrigerator. For this we consider the task of cooling a qubit with given energy E_1, immersed in a bath at a given temperature T_C with fixed coupling p_1. As a source of free energy, we have at our disposal two heat baths, at temperatures T_R and T_H (again assuming that $T_C < T_R < T_H$). The challenge is then to adjust the remaining parameters in order to minimize the temperature of the qubit in its stationary state. The free parameters are the energy of the hot qubit E_3, the thermalization coefficients for the machine qubits p_2 and p_3, and the interaction strength g. Indeed, some of these parameters are constrained by the weak-coupling assumption. We require that $g \ll E_i$, $p_j \ll E_i$, $p_j g \ll g$, and $p_j g \ll p_j$, which can be enforced by choosing a cutoff for p_j and g. We observe that all of our conclusions below remain valid independent of the precise choice of this cutoff. The only change is that the strength of the effect becomes weaker as we make the constraints stronger, as is intuitively expected. We comment further on this at the end of this section.

First, considering all possible fridges, we look for the one achieving the best cooling, i.e., the smallest value of T_S. Next, we repeat this optimization but now adding the constraint that no entanglement is present in the fridge. More precisely, we find the optimal cooling (now denoted T_S^*), imposing that the stationary state ρ_S satisfies all the entanglement witness inequalities (7) (and their relevant symmetries), hence, ensuring separability across every bipartition. The results are presented in Fig. 2. We observe that the cold qubit can be cooled to lower temperatures when no restrictions are placed, compared to the case when the system is constrained to be separable. In the regime where $T_R \ll T_H$, entanglement provides an enhancement in cooling, which is quantified by the ratio

$$\zeta = \frac{T_C - T_S}{T_C - T_S^*}. \quad (8)$$

When restricting to optimal machines, we find that the entanglement is almost always preferentially between the room qubit and the other two. Only if $T_R \approx T_C$ and $T_R \ll T_H$ does entanglement between the other two bipartitions also appear. An intriguing aspect of this behavior is that the entanglement thus appears to be preferentially between the partitions *energy in vs energy out*, since in the stationary state it is the room qubit which is heated up and the cold and hot qubits which
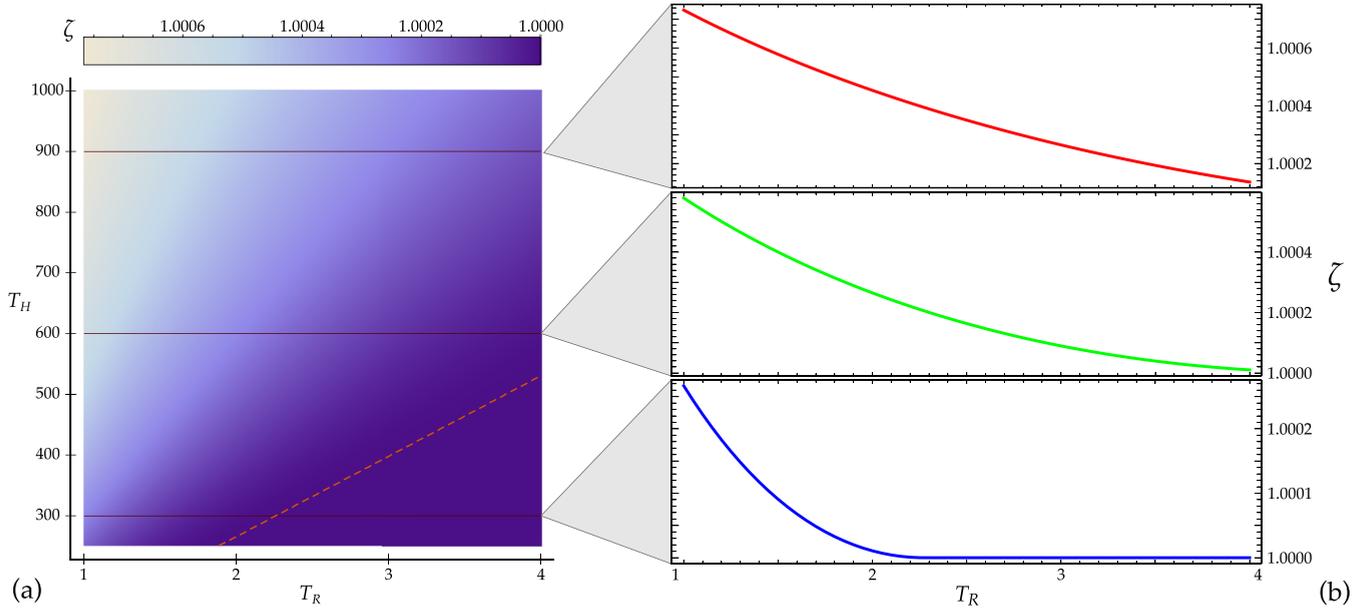
FIG. 2. (Color online) Entanglement can enhance cooling. (a) Density plot depicting the relative advantage ζ in cooling a qubit at $T_C = 1K$, as a function of the available bath temperatures (T_R and T_H) for optimal refrigerators. Entanglement provides an advantage ($\zeta > 1$) for all points above the red dashed line. In the dark blue region, below the red dashed line, the optimal fridge is separable on all bipartitions ($\zeta = 1$). For T_R close to T_C and sufficiently high T_H, entanglement is a widely present feature of the optimal quantum refrigerator. (b) Slices through the density plot for three different values of T_H, showing in more detail the relative advantage in cooling. The parameters used here are $p_C = 10^{-5}$, $E_1 = 1J$, and parameter bounds $p_R, p_H, g \leqslant 10^{-4}$.

are cooled down. Hence, it seems that quantum coherence (here entanglement) enhances energy transport, a phenomenon that has received considerable attention in the context of photosynthetic complexes [36].

It is important to note that, although the cooling enhancement observed here is relatively small, it is nevertheless much larger than the uncertainty in the result [which is $\mathcal{O}(gp)$] and, hence, represents a genuine effect and not a mere consequence

FIG. 3. (Color online) Cooling advantage is determined by the amount of entanglement. Plot of relative cooling enhancement ζ against amount of entanglement on the bipartition $R|CH$ (measured by the concurrence C), as evaluated from Fig. 2(a). Since all points lie on a single curve, it follows that ζ is determined solely by C and does not depend on the temperatures of the baths T_R and T_H. The fact that the behavior is monotonic strongly suggests a functional relationship. For convenience the data plotted correspond to the three horizontal slices of Fig. 2(a). Taking random sample points from Fig. 2(a) leads to a similar result.

of our approximate dynamics. Not unexpectedly, the optimal cooling is obtained by maximizing the interaction coupling as well as the thermalization rates. Here these parameters must be constrained in the optimization in order to remain within the regime of validity of the master equation (3). It would be interesting to consider more general master equations (see, e.g., Ref. [22]), accounting for more sophisticated thermalization processes. In this case, one could describe machines working at much higher rates, in which entanglement may become even more beneficial. We leave it for future research to explore this direction.

Finally, we investigate the link between the amount of entanglement on the bipartition $R|CH$ (as measured by the concurrence $\mathcal{C}$) and the relative cooling enhancement ζ. Remarkably, ζ appears to be solely determined by the concurrence (see Fig. 3) and, hence, does not depend on the temperatures of the heat baths T_R and T_H. This result strongly suggests a functional relationship between the relative cooling enhancement and concurrence. Indeed, the monotonic relationship observed shows that the more entanglement that is present, the larger the advantage one gains for cooling a system.
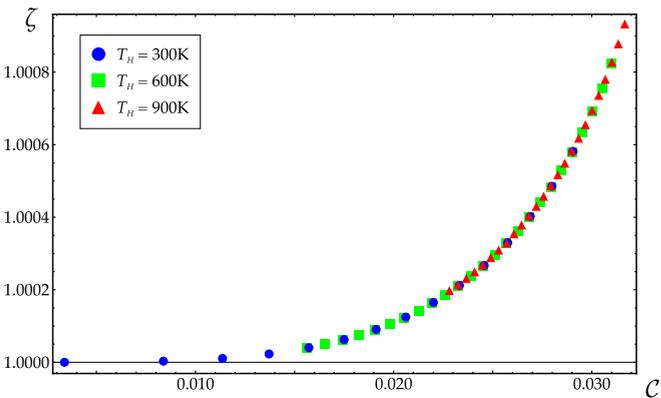
VI. CONCLUSIONS

We have discussed the role of entanglement in the smallest self-contained quantum refrigerator. Entanglement turns out to be a feature of a wide range of operation regimes, with the notable exception of the Carnot point and its vicinity. Crucially, this entanglement is not a mere by-product, but its presence is beneficial: The fridge is able to cool to lower temperatures when it becomes entangled. Moreover, our results show that

the extra quantum performance is directly related to measures of entanglement.

One important question is to what extent the results obtained here tell us about entanglement and its usefulness in general thermal machines. In Ref. [21] it was argued that any thermal machine which is operating close to the Carnot efficiency is necessarily functioning in the same manner as the machines studied here. This is precisely the regime where no entanglement is present, hence, we can conclude that close to the Carnot efficiency no thermal machine is entangled (at least regarding those degrees of freedom directly relevant to the cooling process). What would be more interesting, however, is to go in the other direction and make statements about the regime where entanglement is beneficial. At present such general results appear to be beyond reach, but any such statements in this direction would represent significant progress.

Another question is whether entanglement can enhance the performance of machines producing work. The model discussed here, when analyzed appropriately (in particular, by replacing the cooled qubit by a "weight" which can store work), can also function as a small work-producing heat engine [21]. Hence, an analysis of the presence and role of entanglement in this system will also be of particular interest, especially given the recent results [37–40] where the role of entanglement in work extraction from quantum systems is explored.

To conclude, we believe that the present results provide evidence that entanglement plays a significant role in thermodynamic processes. Clearly, turning these preliminary results into a general and quantitative understanding of the exact role played by entanglement in thermodynamics is highly desirable.

Note added in proof. We recently became aware of the work of Correa and colleagues [22] discussing the effect of quantum discord in a small self-contained quantum refrigerator.

APPENDIX

In this Appendix we show that around any Carnot point (i.e., any machine which is operating at the Carnot efficiency) there is a ball of states which are fully biseparable (i.e., separable on any bipartition). It is also shown that, in some circumstances, there is a ball of fully separable states.

More precisely, the first statement that we would like to prove is that there exists an $\epsilon > 0$, such that for all states σ which satisfy $\|\rho_S - \sigma\|_1 \leqslant \epsilon$ are biseparable, where ρ_S is a stationary state of the refrigerator corresponding to a machine at Carnot efficiency.

The first important point is that all Carnot points have the form $\rho_S = \tau_C \otimes \tau_R \otimes \tau_H$, where each τ_i is a thermal state at a strictly positive temperature $T_i > 0$ (since if we consider

the cold bath to be at absolute zero there is no cooling to be achieved). As such, it is immediately clear that ρ_S is a full rank fully separable state (in fact it is a direct product state). The fact that ρ_S is full rank guarantees that there is a ball of states around it (i.e., it is strictly in the interior of the set of quantum states). This ball, however, may contain entangled states, and so we must show that this ball contains within it a small ball of separable states.

Any states σ which are diagonal in the same basis as ρ_S are clearly also fully separable, and therefore we shall focus on states which are not diagonal. Let us consider first states σ which are still within the interior of the set of quantum states but contain only a single pair of off-diagonal elements, i.e., states of the form

$$\sigma = \rho_D + \sigma_{\mathbf{xy}}|\mathbf{x}\rangle\langle\mathbf{y}| + \sigma_{\mathbf{xy}}^*|\mathbf{y}\rangle\langle\mathbf{x}|, \tag{A1}$$

where ρ_D is diagonal in the same basis as ρ_S and $\mathbf{x} = (x_C, x_R, x_H)$ collectively specifies the state for the three qubits in the energy eigenbasis (and analogously for $\mathbf{y}$). States of this form are in the class of so-called X states. For such states, the three nonlinear witnesses W_i, for $i = 1,2,3$, given by

$$W_i(\rho) = 2(|\rho_{3,6}| - \sqrt{\rho_{i,i}\rho_{9-i,9-i}}) \leqslant 0, \tag{A2}$$

provide necessary and sufficient criteria for biseparability. This follows from the results of Refs. [33] and [35], using which one can show, respectively, that the W_i provide upper and lower bounds on the concurrence.

Since the witnesses W_i are convex and ρ_S is full rank, it is the case that the $W_i(\rho_S) < 0$, i.e., the witness is strictly negative on the Carnot point. It follows therefore that there is an η such that for all $|\sigma_{\mathbf{xy}}| < \eta$, the state σ is still separable. In other words, if we restrict to the set of states with only a single off diagonal element, then there exists an ϵ such that for all $\|\sigma - \rho_S\|_1 \leqslant \epsilon$ the state is biseparable.

To finish the proof note that we can simply take mixtures of the above states with only a single off-diagonal element to define a set of states with an arbitrary number of off-diagonal elements. That is, we write

$$\sigma = \sum_i p_i\big(\rho_D + \sigma_{\mathbf{x}_i\mathbf{y}_i}^i|\mathbf{x}_i\rangle\langle\mathbf{y}_i| + \sigma_{\mathbf{x}_i\mathbf{y}_i}^{i*}|\mathbf{y}_i\rangle\langle\mathbf{x}_i|\big), \tag{A3}$$

where again ρ_D is full rank and all $\sigma_{\mathbf{x}_i\mathbf{y}_i}^i$ are sufficiently small relative to elements of ρ_D. From the above it follows that there is an $\epsilon > 0$ such that each state in the decomposition is both biseparable and positive semidefinite. This thus demonstrates that there is a ball of fully biseparable states in the full Hilbert space around every Carnot point.

Moving on, the second claim we wish to prove is that around any Carnot point such that $\mathrm{Tr}(\rho_S^2) < \frac{19}{24}$, there is a ball of fully separable states. To do so we shall use the fact that there is a ball of fully separable states around the maximally mixed state [41]. In particular, in our case the size of this ball is $\epsilon_{\mathrm{sep}} = \sqrt{\frac{2}{3}}$ [42]. Hence, full separability of ρ_S is ensured when

$$\left\|\rho_S - \frac{\mathbb{1}}{8}\right\|_2 < \sqrt{\frac{2}{3}}, \tag{A4}$$

which implies the existence of a ball of separable states around ρ_S when $\mathrm{Tr}\rho_S^2 < \frac{19}{24}$.

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