The normalization group in quantum theory

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Untersetzerstufen über den Divisionsfaktor m hinaus schlechter gewählt werden darf, falls der zusätzliche Zählverlust nur 1% bzw. 2% betragen soll.

Die Zählverluste lassen sich für $T_1 > 0$ nicht übersichtlich in einer graphischen Darstellung wiedergeben. Um wenigstens ein

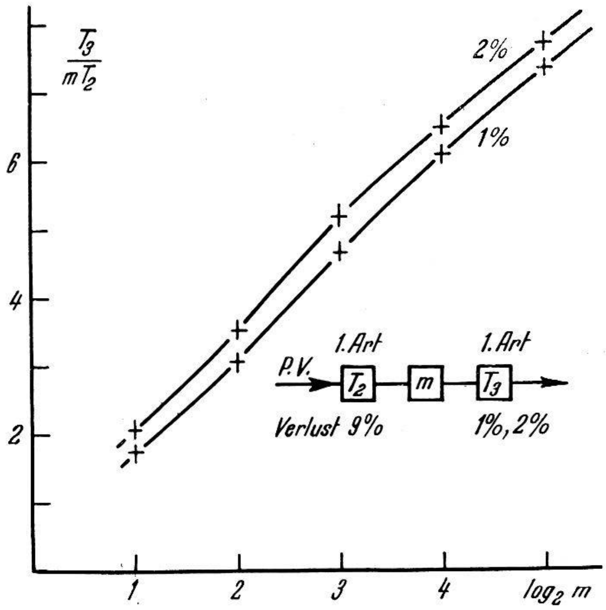

Fig. 2.
Zählverluste ohne Berücksichtigung des Verstärkers.

qualitatives Bild vom Verhalten der Anordnung zu bekommen, seien die Zahlen für einen Spezialfall angegeben: $n_E T_1 = n_E T_2 = 0{,}1$; m $= 4$; $T_3/mT_2 = 3{,}06$. Gesamter Zählverlust $= 10{,}4\%$. Ein höheres Auflösungsvermögen des Verstärkers würde keine wesentliche Verbesserung bedeuten, da selbst für $T_1 = 0$ der Zählverlust 10% beträgt.

The normalization group in quantum theory

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In order to discuss complex interactions, we generalize Dyson's method[1]) in the following way:

Consider a given n-th order contribution to the S-matrix

$$S_{nm}[V](u''\varphi''\ldots/u'\ldots)=\text{const.}\prod_i\Big(\int V dx_i\Big)u''^{\dagger}(x_1)\varphi''^{\dagger}(x_2)\sigma_{M_1}(x_1..x_i..)\cdot$$
$$\cdot\Delta(x_i x_k)\ldots u'(x_l)\ldots\sigma_{M_2}\ldots\sigma_{M_n}\,. \qquad (1)$$

corresponding to a definite Feynman graph of n specified points. It is given in terms of the wawe packets $\varphi, u, \ldots$ the causal func-

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tions $\Delta(xx')$, $D(xx')$,... and the distributions $\sigma_M(xx'...)$ specifying points MNL... They depend numerically on the *packet normalizations* Z,... their *rest mass* $\varkappa$... and the *coupling constants* ε_M in the following way:

$$(\partial/\partial \log Z)\,(u, \Delta) = (\tfrac{1}{2}\,u, \Delta); \quad (\partial/\partial \log \varkappa)\,\Delta = -\varkappa\,\Delta\,\Delta^{*})$$
$$(\partial/\partial \log \varepsilon_M)\,\sigma_M = \sigma_M; \text{ etc.} \tag{2}$$

The particular distributions $\sigma_0(x\,x') = i\,\varepsilon_0\,\varkappa\,\delta(x - x')$ and $\sigma_1(x\,x') = \varepsilon_1(\gamma\,\partial + \varkappa)\,\delta(x - x')$ contribute only to mass and packet renormalization**).

$$(\partial/\partial\,\varepsilon_0)\,S_n = (\partial/\partial \log \varkappa)\,S_n; \quad (\partial/\partial\,\varepsilon_1)\,S_n = (\partial/\partial \log z)\,S_n\,, \tag{3}$$

S_n is evaluated following the rule given in a previous paper[2]). Therefore it involves a great number of *arbitrary constants* arising from all proper self energy and vertex parts $\Sigma_\alpha(xx'...)$ contained in S_n. Let them be ordered according to their degree of complexity in such a way that Σ_β may need the definition of Σ_α only if $\beta > \alpha$. A change in thear bitrary constants can be expressed in terms of infinitesimal operators.

$$\boldsymbol{P}_{N\beta}\,\Sigma_\beta = \sigma_N; \quad \boldsymbol{P}_{N\beta}\,\Sigma_\alpha = 0 \qquad \text{if } \beta > \alpha\,. \tag{4}$$

Operating upon any S_n, they form a Lie Group:

$$[\boldsymbol{P}_{N\beta}, \boldsymbol{P}_{M\alpha}] = \sum_\gamma (\delta_{M,L}\,\boldsymbol{P}_{N\gamma} - \delta_{N,L}\,\boldsymbol{P}_{M\gamma}) \tag{5}$$

γ numbers the vertices Σ_γ arising from overlapping of Σ_α and Σ_β in a single point σ_L.

If $P_{M\alpha}$ operates on the sum $S = \Sigma\,S_n$, the sum being extended over all terms belonging to the same process, the identity

$$\boldsymbol{P}_{M\alpha}S = (\partial/\partial \log \varepsilon_M)\,S\,, \tag{6}$$

holds. Thus, the arbitrariness contained in the evaluation of the S_n's is equivalent to a renormalization of the interaction parameters ε_M.

This method shows that a chosen set of local interactions σ_M cannot be introduced without in general adding interactions up to an infinite order in the derivatives of $\delta(x)$ and involving actions between any number of quanta. We have shown[3]) that the only exceptions are: Actions of zero order involving three and four scalar

*) Matrix multiplication in x-space, for u the same equation holds with $\Delta\,u$.

**) To be shown by partial integration according to (3,3) and (5,17) in[2]) and omitting surface contributions.

fields. Actions of zero resp. first order between a vector field and two spinor resp. two scalar fields, if the corresponding charge satisfies continuity. Thus, a theory of the *Proca particle* or of a *Dirac particle with Pauli terms* introduces all derivatives and therefore describes non-local interactions. One can show, that non local interactions of the type $\sigma(xx')$ between the charge and the vector field are possible without contradicting *macroscopic causality*, if the FOURIER representation $\sigma(p^2)$ has only complex singularities. If this "finite extension" is developped in terms of p^2, we find all multipole actions corresponding to the infinite series formed of (4). This theory should be considered as a phenomenological approach to a true description of spin 1 particles, in which they appear as bound states of an even number of elementary particles of spin ½. If the pseudoscalar meson field should be unable to account simultaneously for the apparent magnetic moment of the nucleon and for the nuclear forces, the nucleon would also be a composite particle formed from an odd number of elementary particles of spin ½ and of particles of spin 0.

References:

1) DYSON, Phys. Rev. **78**, 1736 (1949).
2) STUECKELBERG and GREEN, HPA. **24**, 153 (1951).
3) STUECKELBERG and PETERMANN, Phys. Rev. **82**, 548 (1951).

Über die relativistische Invarianz der kanonischen Grundgleichung.

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Die Gleichung

$$\frac{dr}{dt} = \frac{\partial r}{\partial t} + [r,H] \tag{1}$$

welche bekanntlich die kanonischen Gleichungen der Dynamik zusammenfasst, findet auch in der kanonischen Formulierung der Relativitätsmechanik ihre Verwendung, freilich nur beim Problem des einzelnen Massenpunkts. Es frägt sich, ob die Gleichung in dieser Form als Lorentzvariant gelten kann. Zur Beantwortung dieser Frage wird der homogene kanonische Formalismus verwendet; das ist diejenige Formulierung der analytischen Dynamik, welche die Zeit t als zusätzliche generalisierte Koordinate und ein ihr kanonisch konjugiertes Moment p_t einführt.

Diese Methode drängt sich auf, weil die Lorentz-Transformation zwar als kanonische Transformation aufgefasst werden kann, jedoch, da sie die Zeit mittransformiert, als eine homogen-kanonische.