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# **Time-varying betas and cross-sectional return-risk relation: evidence from the UK**

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# **Time-varying betas and cross-sectional return-risk relation: evidence from the UK**

## **Abstract**

The seminal study by Fama and MacBeth (1973) initiated a stream of papers testing for the cross-sectional relation between return and risk. The debate whether beta is a valid measure of risk has been reanimated by Fama and French (1992) and subsequent studies. Rather than focusing on exogenous variables that have a larger explanatory power than an asset's beta in cross sectional tests, we assume the matrix of variances-covariances to follow a time varying ARCH process. Using monthly data from the UK market from February 1975 to December 1996, we compare the cross sectional return-risk relations obtained with an unconditional specification for assets' betas to those obtained when the estimated betas are based on an ARCH model. We also investigate the Pettengill, Sundaram and Mathur (1995) approach, which allows a negative cross sectional return-risk relation in periods in which the market portfolio yields a negative return relative to the risk free rate. These tests are also carried out on samples pertaining to a specific month and on samples from which a particular month is removed. Our results suggest that CAPM holds better in downward moving markets than in upward moving markets hence beta is a more appropriate measure of risk in bear markets.

**Keywords:** CAPM, QTARCH, return-risk relation, UK market

# **Time-varying betas and cross-sectional return-risk relation: evidence from the UK**

## **1. Introduction**

This paper is concerned with assessing the adequacy of the Capital Asset Pricing Model (CAPM) in explaining the cross-sectional behaviour of returns on portfolios of shares traded on the London International Stock Exchange between February 1975 and December 1996. In particular, our aim is to evaluate beta as a measure of risk by focusing on alternative multivariate specifications of portfolios' first and second moments. While early empirical tests concluded in favour of the CAPM [Fama and MacBeth (1973)], subsequent studies provide evidence that is less than conclusive. Recent examples using US data are Fama and French (1992), Davis (1994) and He and Ng (1994) who find no statistically significant linear relationship between realised returns and beta. Similarly, Groenewold and Fraser (1997) using Australian data conclude that the empirical version of the APT clearly outperforms the CAPM in terms of within-sample explanatory power, but both models perform poorly out-of-sample. Clare, Priestly and Thomas (1996), however, argue that the rejection of the CAPM may be the result of failing to take into account possible correlations between idiosyncratic returns. The authors present results using UK data and non-linear SUR (seemingly unrelated regressions) and suggest that “ $\beta$  is not dead” (p. 23).

The limited empirical support found for the CAPM is interpreted in the literature either as evidence against the CAPM itself [Fama and French (1992) conclude that “beta is dead”], or as evidence that the testing methodology is inappropriate [Calvet and Lefoll (1989) and Roll and Ross (1994)]. The Fama and MacBeth (1973) testing methodology suffers from known deficiencies, however, in absence of any better approach it is still used in most empirical studies. This study seeks to contribute to the specification of the CAPM itself by proposing time-varying beta specifications; and to the testing methodology by investigating the return-risk relation in a variety of situations.

As far as the CAPM is concerned, the unrealistic assumptions of most empirical CAPM frameworks, namely, that the first and second moments of asset returns are constant over time and residual risk is uncorrelated are well known. One of the most popular methods to overcome this problem has been to model risk by assuming it is driven by an ARCH (autoregressive conditional heteroscedastic) process or some generalisation such as GARCH. Overall, the ARCH-based empirical models, without being really convincing, yield a stronger return-risk relationship than the early unconditional models [see, for example, Bollerslev, Engle and Wooldridge (1988), Bollerslev, Chou and Kroner (1992), Gallant, Rossi and Tauchen (1992),

Engle and Rodrigues (1989), Giovannini and Jorion (1989), Thomas and Wickens (1993) and Clare *et al.* (1997)].

The testing methodology developed by Fama and MacBeth has been criticised for numerous reasons. Roll (1977), for example, argued that although the CAPM *must* hold because it is a mathematical derivation from Markowitz's Modern Portfolio Theory, it cannot be tested because the composition of the real market portfolio is unknown. Recently, Pettengill, Sundaram and Mathur (1995) have stressed the fact that the CAPM is stated in *expectational* terms, whereas the model is tested using *realised* returns. The authors suggest that proxying expected returns by realised returns biases against finding a positive relationship between beta and realised returns. They report an inverse relationship between realised returns and beta when excess market returns (the market portfolio less the risk-free rate) are negative. When this stylised fact is accounted for, the authors, using US data, find considerable support for the CAPM.

In general, the inconclusive nature of the empirical literature has serious consequences for the practical use of CAPM's predictions, particularly the use of betas to predict an asset's return<sup>1</sup>. If riskier stocks or portfolios in terms of beta do not earn a higher return, only the least risky stocks should be held by investors. This study adds to the empirical literature on the CAPM, further extending the modelling developments discussed above by analysing cross-sectional return-risk relationships in all sectors of the UK stock market over a period of time. First, we apply the Qualitative Threshold ARCH (QTARCH) model of Gouriéroux and Monfort (1992) to a multivariate time series framework, where, for different regimes, we compute first and second moments conditional on past values of returns. The model, therefore, allows for asymmetries in the first and second moments of returns, which are driven by the "mood" of the market-place reflecting investors perceptions of the "state-of-the-world" of the market-place. As far as the authors are aware such a modelling procedure has not been applied to UK share price data. Second, we compare the out-of-sample predictions of the QTARCH model to that of the multivariate GARCH-M model and to the multivariate version of the traditional unconditional Fama-MacBeth (1973) model. Third, for all specifications, we follow Pettengill, Sundaram and Mathur (1995) and allow for possible negative return-risk relationships when excess returns on the market are negative. By doing this, we take account of possible bias against finding a positive relationship between beta and realised returns. This has been done recently by Fletcher (1997) for the UK but only for the case where assets' betas do not vary over time. Fourth, using the different model specifications, we investigate the existence of monthly anomalies and their effect on return-risk relationships.

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<sup>1</sup> Betas are widely used in practice. It is common for stock-exchange research services to sell "beta books" to market participants who in turn use them as an input into the process of share valuation. As Fama (1991) points out "market professionals (and academics) still think of risk in terms of market  $\beta$ " (p. 1593).

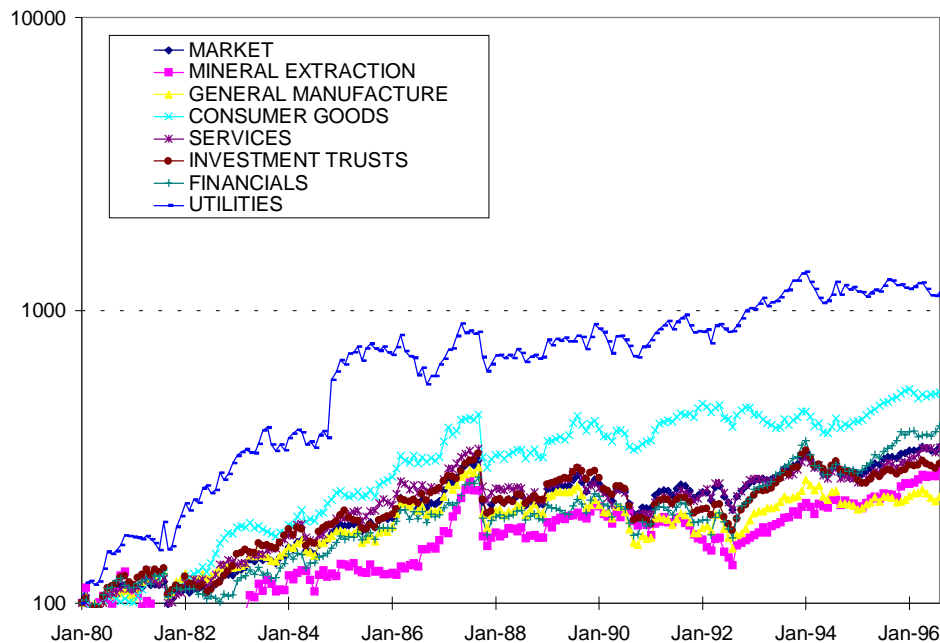
The remainder of the paper is organised as follows. In section 2 we describe the data base. The methodology and econometric framework are given in section 3. Section 4 reports the empirical results while section 5 provides some concluding remarks.

## 2. Data and summary statistics

### 2.1 Data

The data used in this study are the seven market sectors return indices in excess of the risk free rate on the London Stock Exchange as defined by Datastream. Appendix 1 provides the summary of the composition of each sector. A return is defined as the difference of the logarithm of two consecutive index levels, which are fully adjusted for dividends, splits and other operations. The same source also provided the market values series. The period under consideration is from February 1975 to December 1996. While pre-1975 data is available we excluded it from our analysis because of the special circumstances prevailing in the UK stock market in the early 1970's<sup>2</sup>.

*Figure 1 : Evolution of £100 invested in each of the indices and in the market index (logarithmic scale), starting January 1980, date of the first cross-sectional regression (for details of the portfolios, see Appendix 1).*



<sup>2</sup> In the early 1970's the UK stock market was under particular pressure due to a number of special factors such as: oil price rises, miners' strikes and a three day working week. Such events caused unprecedented price swings over this period which, given they were "one off" occurrences, may bias results.

## *2.2 Preliminary Statistics*

The evolution of a £100 (in excess of the risk free rate) investment in each of the sectors in January 1980 (which is the date of the first cross-sectional regression) as well as in the market portfolio is provided in Figure 1. The Utilities portfolio is clearly the one that has not only performed best in absolute terms, it is also the one that has suffered the least from the October 1987 crash. The worst portfolios were Mineral Extraction and General Manufacture.

Appendix 2 provides the mean, the standard deviation and the MSD ratios (mean over standard deviation) of the portfolios over the full period, as well as month by month. The Utilities portfolio has the highest average return, but also the highest standard deviation. In terms of MSD, General Manufacture has performed best.

## **3. Methodology**

The general in-sample / out-of-sample approach is the one developed by Fama and MacBeth (1973) and widely used since. The out-of-sample data are used for the cross-sectional regressions of returns on betas estimated using data from the in-sample period. The conditional ARCH specifications include the well known GARCH-M model and the QTARCH of Gouriéroux and Monfort (1992), previously used for modeling conditional betas in a similar framework by Hamelink (1997) for the Swiss market.

A common framework is used to test the unconditional specification and the three conditional models. For each model, we use the first five years of monthly returns (60 observations) to estimate the betas for each of the seven market sectors. The sector returns during the 61<sup>st</sup> period are regressed on these estimated betas. The estimation period is then moved one period “forward”, that is, betas are estimated from the 2<sup>nd</sup> through the 61<sup>st</sup> observation, and the returns during the 62<sup>nd</sup> period are regressed on these beta estimates.

### *3.1 The unconditional Fama and MacBeth approach*

Most empirical validations of the CAPM are based on the study by Fama and MacBeth (1973). Their method has the advantage of being a true “out-of-sample” test of the model, as beta coefficients are first estimated using “in-sample” data, then cross sectional regressions based on “out-of-sample” data test whether these beta estimates can be considered as a valid measure of risk.

Fama and MacBeth (1973) split the methodology into three parts. The purpose of the first one is to form portfolios from the universe of shares. A first period of 60 months is used to calculate a traditional beta coefficient for each share. The shares are then grouped together into portfolios

according to the individual beta estimate for each share. The beta for each portfolio  $p$  is then estimated over the second period of 60 months as well, using realised returns for both the portfolios and the market proxy:

$$R_{p,\tau} - R_{f,\tau} = \alpha_p + \beta_p (R_{m,\tau} - R_{f,\tau}) + \varepsilon_{p,\tau}, \text{ for } \tau \in [t-60, \dots, t-1]. \quad [1]$$

These beta estimates are then used to perform a cross sectional regression at time  $t$  of realised returns on these beta estimates, in other words, the risk-return trade off is tested :

$$R_{p,t} - R_{f,t} = \gamma_{0,t} + \gamma_{1,t} \hat{\beta}_p + u_{p,t}. \quad [2]$$

The coefficient  $\gamma_{0,t}$  is interpreted as the expected return of a zero beta portfolio and  $\gamma_{1,t}$  is the market price of risk, which, in the Fama and MacBeth approach, has to be significantly positive to support the validity of the CAPM.

This procedure is repeated by rolling the two periods of 60 months of observations and the cross sectional observations, one period (1 month) ahead. This way, time series for  $\gamma_{0,t}$  and  $\gamma_{1,t}$  are generated. Assuming the independence of the consecutive estimations, a simple  $t$  test can be applied to these coefficients. Fama and MacBeth find that, over the time period from 1935 to 1968, a positive relationship exists between beta and monthly returns.

The CAPM states a relation between assets' *expected* returns and risks. In order to test the model, Fama and MacBeth proxy the expected returns by realised returns. In expectations, the return on any risky asset must be higher than the risk free rate, otherwise no investor would hold the risky asset. However, investors perceive the possibility of the risky assets' return being below the risk free rate. This is the approach of Pettengill, Sundaram and Mathur (1995), which we use for our analysis. They conclude that "the existence of a large number of negative market excess return periods suggests that previous studies that test for unconditional positive correlation between beta and realised returns are biased against finding a positive relationship".

### 3.2 The Qualitative Threshold ARCH

The Qualitative Threshold ARCH (QTARCH) suggested by Gouriéroux and Monfort (1992) is a model in which the conditional mean and the conditional variance are endogenous stepwise functions. The space of past innovations (considered over the past  $\tau$  periods) is divided into partitions using thresholds<sup>3</sup>. In a multivariate framework, let  $Y_t$  be the  $n$ -dimensional vector of

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<sup>3</sup> One of the reasons why threshold models in general have seen so few applications is because the choice of the values of the thresholds is not straightforward. Tsay (1986, 1989) for example suggests some techniques for deriving them.



returns at time  $t$ , which may also be referred to as the “innovation process” at time  $t$  if  $Y_t$  has zero conditional mean, and  $J$  the number of partitions of  $\mathbb{R}^n$ . Each of these partitions is denoted  $A_j$  and is associated with a binary characteristic function  $I_{A_j}$ , which takes the value 1 if the matrix of the  $\tau$  past innovations  $[Y_{t-\tau} \dots Y_{t-1}]'$  belongs to  $A_j$  and 0 otherwise.

It is obvious that the number  $J$  of partitions increases rapidly with the dimension  $n$  of the multivariate time series and in order to limit this number, only one time lag is considered<sup>4</sup> in most cases, i.e.  $\tau$  is set to 1. The characteristic function  $I_{A_j}$  thus only depends on the vector  $Y_{t-1}$ . Each partition is referred to as regime.

In terms of conditional mean and conditional covariances, the expression of the QTARCH with single lag is :

$$\begin{aligned} E(Y_t | Y_{t-1}) &= \sum_{j=1}^J \alpha_j I_{Y_{t-1} \in A_j} \\ V(Y_t | Y_{t-1}) &= \sum_{j=1}^J \beta_j I_{Y_{t-1} \in A_j}, \end{aligned} \quad [3]$$

or :

$$Y_t = \sum_{j=1}^J \alpha_j I_{Y_{t-1} \in A_j} + \sum_{j=1}^J \beta_j I_{Y_{t-1} \in A_j} u_t, \quad [4]$$

where  $\alpha_j$  ( $j=1 \dots J$ ) and the strong white noise  $u_t$  are  $n$ -dimensional vectors and  $\beta_j$  ( $j=1 \dots J$ ) are symmetric positive definite matrices.

The parameters of the model may be estimated using pseudo-maximum likelihood (PML) methods based on the normal distribution. As shown by Gouriéroux and Monfort (1992), the PML estimators are the solutions of :

$$\max_{\alpha, \beta} L_t = \sum_{j=1}^J \sum_{t \in B_j} \left( -\frac{n}{2} \log 2\pi - \log |\beta_j| - \frac{1}{2} (Y_t - \alpha_j)' \beta_j^{-2} (Y_t - \alpha_j) \right) \quad [5]$$

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<sup>4</sup> Empirical studies of most ARCH class of models have shown that increasing the number of time lags does not necessarily lead to significantly better results. Most of the information for predicting time  $t$  is found in time  $t-1$ . In order to limit the number of parameters to estimate, there is a trade off between including higher order lags and imposing as few restrictions as possible on the basic specification. Especially in a multivariate case, which is our case, the number of parameters has to be restricted in order to keep the model tractable.

where  $B_j$  is the set defined by  $\{t : 1 \leq t \leq T, Y_{t-1} \in A_j\}$ . By this method, the least square estimators for  $\hat{\alpha}_j$ , the conditional mean vector of regime  $j$ , and  $\hat{\beta}_j^2$ , the conditional variance-covariance matrix of regime  $j$ , are given by :

$$\hat{\alpha}_j = \bar{Y}_j = \frac{1}{T_j} \sum_{t \in B_j} Y_t, \quad [6]$$

$$\hat{\beta}_j^2 = \frac{1}{T_j} \sum_{t \in B_j} (Y_t - \bar{Y}_j)(Y_t - \bar{Y}_j)', \quad [7]$$

where  $T_j$  is the number of observations belonging to regime  $j$ . The conditional two first order moments are thus simply the empirical mean and variance of each regime.

The asymptotic covariance matrix of  $\sqrt{T}(\hat{\alpha}_j - \alpha_j)$  is written as :

$$V_{as} \left( \sqrt{T} [\hat{\alpha}_j - \alpha_j] \right) = \beta_j^2 \times T_j/T, \quad [8]$$

where  $T = \sum_{j=1}^J T_j$ .

The QTARCH specification offers several important advantages over traditional ARCH specifications. First, the asymmetry of variance is easily accounted for. By choosing the thresholds in order to partition the space of past innovations into regimes where the return during the previous period was strongly negative, around zero, or strongly positive, we get three regimes ( $J=3$ ), each of them yielding a specific conditional return and a specific conditional variance. Negative shocks can influence the conditional means and the conditional variance-covariances differently from positive shocks. Furthermore, extreme shocks only influence the estimated parameters of a single regime.

Second, contrary to a GARCH-M model for instance, where the conditional mean at time  $t$  is a function of the conditional variance at that time, the mean is estimated independently from the variances-covariances (although, by construction, the calculation of the variances and covariances involves the means) and different thresholds based on different information sets may be used to calculate the conditional means and variance-covariances.

Third, once the difficult procedure of setting the appropriate thresholds prior to estimation has been accomplished, estimating even higher dimensional multivariate processes is not the same technical problem that it is with traditional GARCH specifications : only the number of available observations has to be sufficient.

This study considers two partitioning methods, that is, a set of rules to partition the 60 historical observations into “states-of-the-world”. The first one defines partitions according to the previous month’s market return. It is close to the initial study by Gouriéroux and Monfort, who first consider zero as “natural” threshold for a time series, arguing that investors may be differently influenced by positive or negative returns during the period. In our case, setting the threshold to zero would yield far more observations in the “positive return” class than in the “negative return” one, because of the strong drift found in the series. Furthermore, the number of observations we have allows us to have more than two partitions. Gouriéroux and Monfort also came up with the idea of partitioning a univariate series into ten deciles. We choose to construct four partitions. Each of the 60 observations is in the first, second, third or fourth partition according to whether the previous month’s market return was in the first, second, third or fourth quartile of the monthly market returns during the previous 60 months. The four partitions thus all have the same number of observations.

The second approach considers Principal Component Analysis (PCA), which is a statistical method which extracts the most important factors from the past 60 monthly returns on all seven sectors. The value of each of our two extracted factors is compared to zero, which yields again four partitions (both factors were negative, first factor was negative and second positive, first factor was positive and second negative, both factors were positive). It makes sense here to compare the values of the factors to zero, as by construction, each factor has zero mean.

### 3.3 The GARCH-M Model

The traditional methodology used by Bollerslev, Engle and Wooldridge (1988) is used here. Let  $Y_t$  be the vector of asset returns in excess of the risk free rate and  $\omega_t$  the vector of value weights, a general multivariate GARCH-M (p, q) model of dimension n takes the form :

$$\begin{aligned}
 Y_t &= b + \delta H_t \omega_{t-1} + \varepsilon_t, \\
 \text{vech}(H_t) &= C + \sum_{i=1}^q A_i \text{vech}(\varepsilon_{t-i} \varepsilon_{t-i}') + \sum_{j=1}^p B_j \text{vech}(H_{t-j}), \\
 \varepsilon_t \mid \phi_{t-1} &\sim N(0, H_t),
 \end{aligned} \tag{9}$$

where vech is the operator transforming an original n by n matrix into an  $n \times (n+1)/2$  dimensional vector containing the columns of the lower triangular matrix issued from the original matrix. b is a n dimensional vector representing the unconditional part in the conditional mean equation,  $\varepsilon_t$  is the innovation process and C,  $A_i$  and  $B_j$  are  $n(n+1)/2$  dimensional matrices.

In order to reduce the number of parameters and to keep the model tractable, we limit the number of time lags to one (that is,  $p=q=1$ ) and make the “traditional” assumption of diagonality of the A and B matrices.

The parameters are estimated by optimising the likelihood function, which is :

$$\sum_{t=1}^T -\frac{N}{2} \log 2\pi - 0.5 \log |H_t(\theta)| - 0.5 \varepsilon_t(\theta)' H_t^{-1}(\theta) \varepsilon_t(\theta), \quad [10]$$

where all the parameters have been combined into  $\theta' = (b', \delta, C', \text{vech}'(A), \text{vech}'(B))$ .

### 3.4 The Pettengill, Sundaram and Mathur (1995) Approach for Up and Down Markets

The models for unconditional and conditional betas use the 60 observations from  $t-60$  to  $t-1$  to estimate a beta coefficient for each of the portfolios at time  $t$ . These estimates are denoted  $\hat{\beta}_{p,t}$ . The next period  $t$  is then used to regress cross sectionally :

$$R_{p,t} - R_{f,t} = \gamma_{0,t} + \gamma_{1,t} \hat{\beta}_{p,t} + u_{p,t} \quad [11]$$

for the traditional (Fama and MacBeth) regression, and

$$R_{p,t} - R_{f,t} = \gamma_{0,t} + \gamma_{2,t} \delta \hat{\beta}_{p,t} + \gamma_{3,t} (1-\delta) \hat{\beta}_{p,t} + u_{p,t}, \quad [12]$$

for the Pettengill *et al.* approach, where  $\delta=1$  if  $(R_{mt} - R_{ft}) \geq 0$  (i.e. when market excess returns are positive) and  $\delta=0$  if  $(R_{mt} - R_{ft}) < 0$  (i.e. when market excess returns are negative). The coefficient  $\gamma_{2,t}$ , which is estimated in periods in which the excess market return is positive, is expected to be positive, whereas  $\gamma_{3,t}$ , dealing with periods in which the market excess return is negative, is expected to be negative. The two sets of hypotheses ( $H_0 : \gamma_2 = 0, H_a : \gamma_2 > 0$ ) and ( $H_0 : \gamma_3 = 0, H_a : \gamma_3 < 0$ ) can be tested using a simple  $t$ -statistic. If both null hypotheses are rejected, we conclude in favour of a systematic relationship between beta and *realised* returns.

## 4. Results

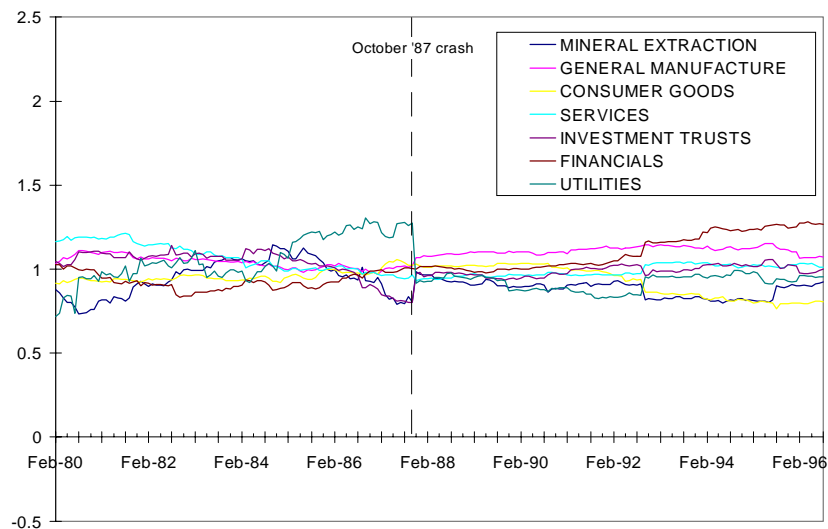
Figure 2 contains plots of the time-varying betas for the unconditional model, both of the QTARCH specifications and for the GARCH-M model. Each graph contains seven lines for the seven portfolios that are considered. Each point on the graph represents, at a given time  $t$ , the forecast of the beta for that period, according to each of the four models. As could be expected, the figure shows that conditional models yield more volatile betas than the unconditional model. The forecasts for beta according to the unconditional model are also time varying because each forecast is based on the past 60 observations. The QTARCH models, in particular the one based on the principal component analysis, are more volatile than the GARCH-M model which is fairly stable except for some peaks. GARCH-M beta estimates change from one period to another only because observation  $t+1$  is added to the estimation sample and observation  $t-60$  is removed from this sample. Thus, from one period to another, 58 observations are the same and 2 are different. The beta parameter estimate does not, therefore, change very much. For QTARCH, in addition to the two observations that are different, there is also the possibility of a regime switch, which makes the QTARCH more volatile. This volatility can be seen in Figure 3 which contains a close-up of Figure 2 for the QTARCH model based on market returns for 1992-1996.

Panel A of Figure 2 dealing with the unconditional model shows that prior to the October 1987 stock market crash the betas estimated for each of the portfolios ranged from a low of 0.80 for Investment Trusts to a high of 1.28 for Utilities. Immediately after the crash, the estimates ranged between 0.92 for Utilities to 1.07 for General Manufacture. Thus, adding the one single October 1987 set of observations to the estimation sample completely changed the forecast of the beta coefficients.

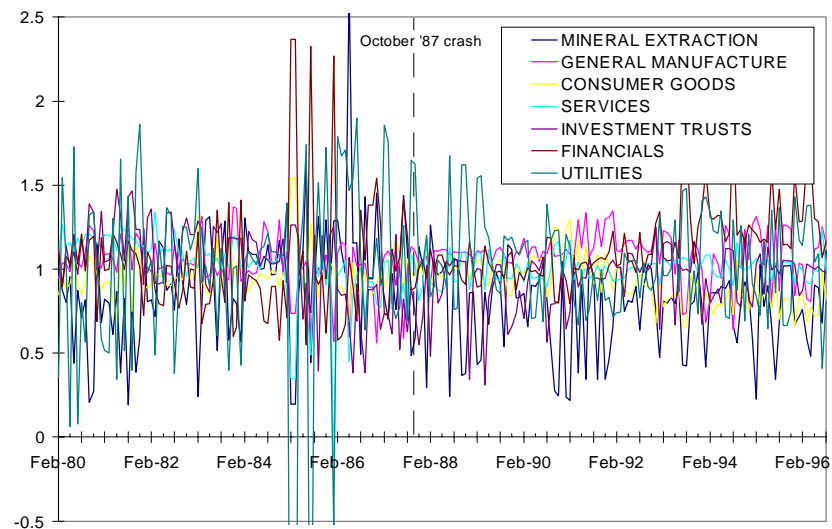
The GARCH-M model (Panel D) also shows how the estimates for beta are dramatically changed by adding the October 1987 observation. The two QTARCH based models, however, do not show a very different pattern after the crash. The reason is that the regimes that are yielded after the crash do not often contain the “extreme” October 1987 observations. However, the QTARCH model based on market returns (Panel B) shows that the partition containing the extreme observations appears from time to time, yielding higher volatility of the graph of estimated betas. This clearly shows the advantage of a non-parametric approach such as the QTARCH. Whereas the forecasts done by means of the parametric GARCH-M models remain influenced for a long time by the crash, the QTARCH models do not.

Figure 2 : Time-varying betas for unconditional and conditional models.

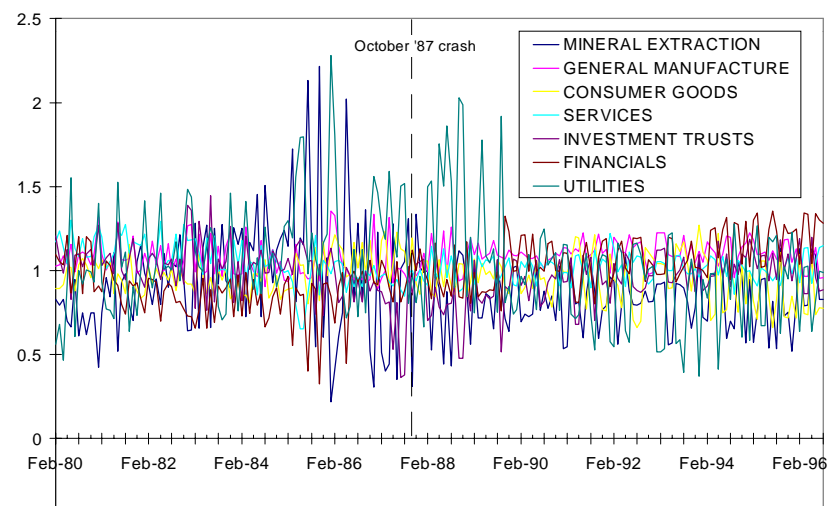
A. Unconditional model



C. QTARCH with PCA analysis



B. QTARCH on market returns



D. GARCH-M

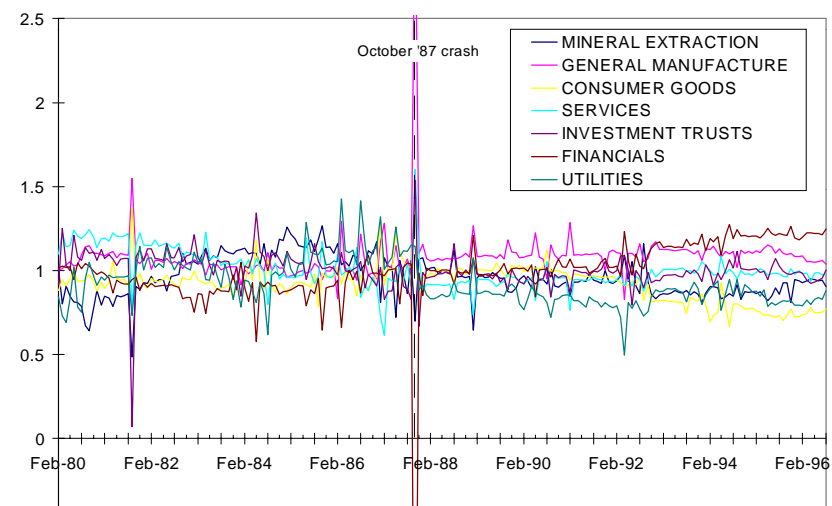


Table 1 contains under the heading “all markets” the results of the Fama and MacBeth procedures for the four models under investigation. All averages of the  $\gamma_{0,t}$  coefficients, denoted  $\gamma_0$ , are positive but not significantly different from zero. This is what could be expected, as the returns are calculated in excess of the risk free rate. All but one  $\gamma_1$  coefficient (average of all  $\gamma_{1,t}$  coefficients) are negative, but show low levels of significance. Only the QTARCH based on PC analysis has a coefficient of the expected sign but its p-value is high (0.28) indicating this coefficient is not significantly positive. Overall, the Fama and MacBeth results suggest that the CAPM should be rejected.

Figure 3 : Time-varying betas for QTARCH on market returns (close-up of Figure 2 for 1992- 1996).

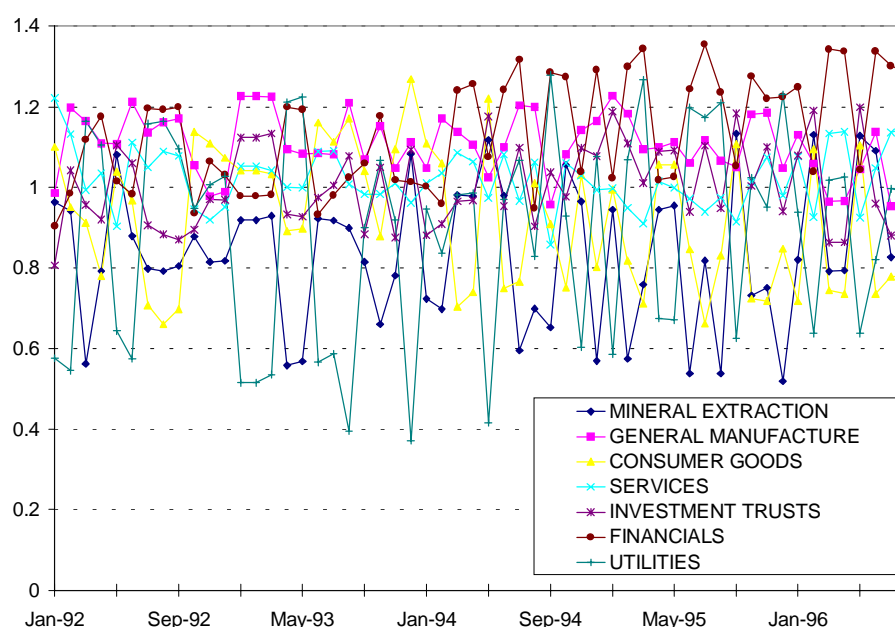


Table 1 Average estimated coefficients and their p-values for the regressions :

$$R_{p,t} - R_{f,t} = \gamma_{0,t} + \gamma_{1,t} \hat{\beta}_{p,t} + u_{pt} \text{ and } R_{p,t} - R_{f,t} = \gamma_{0,t} + \gamma_{2,t} \delta \hat{\beta}_{p,t} + \gamma_{3,t} (1-\delta) \hat{\beta}_{pt} + u_{pt};$$

(standard Fama and MacBeth methodology and Pettengill *et al.* approach).

Method	all markets		up and down markets		
	$\gamma_0$	$\gamma_1$	$\gamma_0$	$\gamma_2$	$\gamma_3$
Unconditional	0.86%	-0.17%	0.86%	1.05%	-1.22%
	<i>0.43</i>	<i>0.87</i>	<i>0.43</i>	<i>0.20</i>	<i>0.07</i> *
QTARCH	1.17%	-0.50%	1.17%	-0.03%	-0.47%
on market returns	<i>0.20</i>	<i>0.56</i>	<i>0.20</i>	<i>0.97</i>	<i>0.25</i>
QTARCH	0.08%	0.59%	0.08%	0.31%	0.28%
PC analysis	<i>0.91</i>	<i>0.28</i>	<i>0.91</i>	<i>0.45</i>	<i>0.42</i>
Garch-M	1.35%	-0.65%	1.35%	0.62%	-1.26%
	<i>0.16</i>	<i>0.50</i>	<i>0.16</i>	<i>0.41</i>	<i>0.03</i> **

Notes to Table 1: The figures in italics are p-values. \* (respectively \*\*) indicates the result is significantly different from zero at the 90% (respectively 95%) level.

When the conjecture by Pettengill *et al.* is examined (denominated “up and down markets” in Table 1), the results are slightly more encouraging. As expected, the  $\gamma_0$  coefficients are the same as for the Fama and MacBeth procedure. All  $\gamma_2$  coefficients (which are expected to be positive) are indeed positive in all but one case but not significant at usual levels. The  $\gamma_3$  coefficients are negative in all but one case, significantly negative in two instances (unconditional and GARCH-M). These results suggest that the CAPM holds better in downward moving markets than in upward moving markets, in other words, beta is a valid measure of an assets’ risk in bear markets only, not in bull markets. A possible explanation for this fact is that assets in general are more correlated in negative markets. This correlation can be expected to be even stronger when sector returns are used rather than individual asset returns. Fletcher (1997), using 1975-94 data from the UK market and 10 portfolios formed on size, reports similar figures, especially the stronger effect in down moving markets.

Given the above results, it can be expected that the overall results are highly influenced by some large market movements. In the UK and over the period under considerations, the largest negative returns on the market (still in excess of the risk free rate) were -27.9% in October 1987, -16.3% in September 1981, -9.8% in November 1987. On the other hand, the largest positive monthly market returns were 13.3% in January 1989, 11.0% in September 1992 and 10.7% in April 1992 and February 1991. This asymmetry could be the reason why the return-risk relation is apparently stronger in downward moving markets than in up moving markets.

In order to obtain a better insight, Table 3 contains the results for the same conducted above, but without particular months, and Table 4 contains these tests for particular months only. A



summary of these two tables is given in Table 2, which provides an idea of how often the results are satisfactory.

*Table 2 : How often are the  $\gamma$  coefficients of the correct sign and significant?*

Model	All months except...						Only month of...					
	$\gamma_1$			$\gamma_2$			$\gamma_3$			$\gamma_1$		
Unconditional	4	0	0	12	1	1	12	6	3	6	0	0
QTARCH on market returns	0	0	0	5	0	0	12	0	0	5	2	0
QTARCH with PCA	12	1	0	12	0	0	0	0	0	7	3	0
GARCH-M	0	0	0	12	0	0	12	12	6	5	2	0

Note to Table 2: For the analysis carried out by removing a given month (“All months except...”) and for the one dealing with a particular month (“Only month of...”) the table reports respectively for each of the three  $\gamma$  coefficients the number of times (out of 12) that, a) the coefficient is of the expected sign, b) the coefficient is significant at the 90% level, and c) the coefficient is significant at the 95% level.

When a particular month is removed from the sample and for the unconditional model, 4 out of the 12 cases report a positive  $\gamma_1$ , that is, a positive relation between return and beta. This is interesting, as when no month is removed, the relation is negative. In other words, 4 out of 12 months have such an important influence that each of them makes the global return-risk relation negative. For this unconditional model, however, the return-risk relation never becomes significant, neither at the 90% level, nor at the 95% level. When the partitioning is done using QTARCH in combination with PCA, removing October makes  $\gamma_1$  significantly positive at the 90% level. This is an important result, in favour of this particular conditional specification.

When we account for up and down moving markets, the results show that for the unconditional model  $\gamma_2$  and  $\gamma_3$  not only become of the correct sign but even significant at the 95% level when some particular months are removed from the sample. For QTARCH with PCA, the cross sectional tests are also heavily influenced by a few months in which unusual events happened.

Table 2 also reports the results for individual months, which are reported in detail in Table 4. For the unconditional model without consideration for up and down moving markets, the return-risk relation is positive for 6 out of 12 months only, but never significant neither at the 90% level, nor at the 95% level. The results are, however, marginally better for the conditional specifications in general.

We now turn to Table 3. It could be expected that the results without January would be different if a January effect occurs in the UK. Investigations into US stock market overreaction have shown that statistically significantly higher returns are earned in January

than in any other month of the year [for example, Zarowin (1989, 1990)]. Fraser and Power (1992), using UK Investment Trust data, also report evidence which suggests a strong seasonal effect in January. As the end of the UK tax year is on 5 April, we might also expect an April/May effect. We detect no such effect: the unconditional specifications without January, April or May are very similar to the results when all months are considered. When October is excluded from the sample, the unconditional model reports significant  $\gamma_2$  and  $\gamma_3$  coefficients at the 95% level, whereas QTARCH results based on principal component analysis report a positive  $\gamma_1$  at the 90% level. In other words, when October is excluded from the sample, support for the CAPM is found either when betas are unconditional and up and down moving markets are accounted for, or when betas are specified according to QTARCH with PCA in any kind of market.

As shown above, the detailed results obtained when one particular month is removed from the sample clearly show the heavy influence of October. When this month is dropped from the sample, the unconditional beta can be considered as a valid measure of risk. This is, however, somewhat unsatisfactory, as it shows the sensitivity of beta in general to a few extreme observations. Furthermore, the global results reported by Pettengill *et al.* also contain some extreme observations such as the October 1987 crash of the US market.

Table 4 contains the results for particular months. A common pattern that emerges from all unconditional and conditional specifications is that the return-risk relation is positive for the month of January, without consideration of the market movement. Only the conditional betas, however, yield a significant positive relation. When up and down moving markets are accounted for, all conditional beta specifications suggest that the  $\gamma_2$  coefficients (for up moving markets) are significant at the 90% level, which is not the case for the unconditional betas. None of the  $\gamma_3$  coefficients, however, is here significant. An explanation for this can easily be found in Appendix 2, which shows that the average monthly return during the month of January was 2.75%, much higher than any other average monthly return. Overall, the January results suggest that for this month only a conditional beta is a significant measure of an asset's risk.

The above findings are somewhat disappointing with respect to the Pettengill *et al.* methodology. Their 1995 study carried out on US data reports very strong evidence in favour of the CAPM when up and down moving markets are taken into account. For Switzerland, Hamelink (1997) also reports similar strong results with this methodology. The results we report here are much weaker as, when beta is unconditional, the  $\gamma_2$  and  $\gamma_3$  coefficients have p-values of only 0.20 and 0.07, respectively. The coefficient for down moving markets ( $\gamma_3$ ) can be considered significant at usual levels.

Table 3 : Average of the estimated coefficients and the p-values for the regressions

$$R_{p,t} - R_{f,t} = \gamma_{0,t} + \gamma_{1,t} \hat{\beta}_{p,t} + u_{p,t} \text{ and } R_{p,t} - R_{f,t} = \gamma_{0,t} + \gamma_{2,t} \delta \hat{\beta}_{p,t} + \gamma_{3,t} (1-\delta) \hat{\beta}_{p,t} + u_{p,t}$$

when a particular month is removed from the sample.

Panel A: Unconditional model						Panel C: QTARCH with PCA					
	all markets		up and down markets				all markets		up and down markets		
	$\gamma_0$	$\gamma_1$	$\gamma_0$	$\gamma_2$	$\gamma_3$		$\gamma_0$	$\gamma_1$	$\gamma_0$	$\gamma_2$	$\gamma_3$
full sample	0.86%	-0.17%	0.86%	1.05%	-1.22%	full sample	0.08%	0.59%	0.08%	0.31%	0.28%
	0.43	0.87	0.43	0.20	0.07 *		0.91	0.28	0.91	0.45	0.42
All months except...						All months except...					
January	1.01%	-0.52%	1.01%	0.77%	-1.28%	January	0.08%	0.40%	0.08%	0.09%	0.30%
	0.36	0.63	0.36	0.33	0.08 *		0.91	0.49	0.91	0.83	0.43
February	1.26%	-0.59%	1.26%	0.89%	-1.48%	February	-0.01%	0.67%	-0.01%	0.35%	0.32%
	0.27	0.60	0.27	0.30	0.04 **		0.98	0.25	0.98	0.43	0.40
March	1.04%	-0.31%	1.04%	0.89%	-1.20%	March	0.16%	0.55%	0.16%	0.36%	0.19%
	0.36	0.78	0.36	0.30	0.09 *		0.82	0.33	0.82	0.42	0.59
April	0.56%	0.04%	0.56%	1.20%	-1.16%	April	0.03%	0.55%	0.03%	0.41%	0.14%
	0.62	0.97	0.62	0.17	0.09 *		0.96	0.31	0.96	0.32	0.70
May	0.80%	-0.03%	0.80%	1.11%	-1.14%	May	0.05%	0.70%	0.05%	0.42%	0.27%
	0.49	0.98	0.49	0.21	0.11		0.94	0.24	0.94	0.34	0.48
June	0.78%	-0.01%	0.78%	1.15%	-1.16%	June	0.24%	0.51%	0.24%	0.25%	0.26%
	0.50	0.99	0.50	0.18	0.11		0.73	0.37	0.73	0.57	0.47
July	0.68%	0.00%	0.68%	0.96%	-0.95%	July	-0.24%	0.90%	-0.24%	0.43%	0.48%
	0.56	1.00	0.56	0.28	0.17		0.74	0.12	0.74	0.33	0.20
August	0.88%	-0.25%	0.88%	0.78%	-1.03%	August	0.21%	0.40%	0.21%	0.09%	0.31%
	0.44	0.82	0.44	0.37	0.13		0.77	0.49	0.77	0.83	0.42
September	1.21%	-0.30%	1.21%	0.84%	-1.14%	September	0.10%	0.78%	0.10%	0.49%	0.30%
	0.29	0.79	0.29	0.33	0.10		0.88	0.17	0.88	0.26	0.43
October	0.51%	0.28%	0.51%	1.76%	-1.48%	October	-0.11%	0.88%	-0.11%	0.58%	0.30%
	0.63	0.79	0.63	0.03 **	0.04 **		0.86	0.10 *	0.86	0.16	0.38
November	0.55%	0.10%	0.55%	1.25%	-1.15%	November	0.20%	0.44%	0.20%	0.15%	0.29%
	0.63	0.93	0.63	0.15	0.11		0.78	0.45	0.78	0.73	0.45
December	1.07%	-0.46%	1.07%	0.96%	-1.42%	December	0.23%	0.36%	0.23%	0.11%	0.25%
	0.35	0.68	0.35	0.25	0.05 **		0.74	0.52	0.74	0.80	0.48
Panel B: QTARCH on market returns						Panel D: GARCH-M					
	all markets		up and down markets				all markets		up and down markets		
	$\gamma_0$	$\gamma_1$	$\gamma_0$	$\gamma_2$	$\gamma_3$		$\gamma_0$	$\gamma_1$	$\gamma_0$	$\gamma_2$	$\gamma_3$
full sample	1.17%	-0.50%	1.17%	-0.03%	-0.47%	full sample	1.35%	-0.65%	1.35%	0.62%	-1.26%
	0.20	0.56	0.20	0.97	0.25		0.16	0.50	0.16	0.41	0.03 **
All months except...						All months except...					
January	1.32%	-0.84%	1.32%	-0.34%	-0.50%	January	1.74%	-1.23%	1.74%	0.12%	-1.35%
	0.18	0.35	0.18	0.66	0.26		0.09 *	0.21	0.09 *	0.87	0.04 **
February	1.37%	-0.72%	1.37%	-0.04%	-0.68%	February	1.23%	-0.55%	1.23%	0.84%	-1.39%
	0.16	0.43	0.16	0.96	0.12		0.23	0.58	0.23	0.29	0.03 **
March	0.96%	-0.25%	0.96%	0.06%	-0.31%	March	1.98%	-1.25%	1.98%	0.43%	-1.67%
	0.33	0.78	0.33	0.94	0.47		0.05 **	0.21	0.05 **	0.58	0.01 **
April	1.18%	-0.60%	1.18%	-0.23%	-0.37%	April	1.38%	-0.77%	1.38%	0.35%	-1.12%
	0.23	0.51	0.23	0.77	0.39		0.15	0.41	0.15	0.64	0.04 **
May	1.51%	-0.77%	1.51%	-0.15%	-0.62%	May	1.00%	-0.23%	1.00%	0.78%	-1.01%
	0.12	0.40	0.12	0.85	0.13		0.32	0.82	0.32	0.32	0.09 *
June	0.92%	-0.17%	0.92%	-0.04%	-0.13%	June	1.17%	-0.39%	1.17%	0.79%	-1.18%
	0.35	0.85	0.35	0.96	0.74		0.24	0.69	0.24	0.29	0.06 *
July	1.07%	-0.40%	1.07%	0.01%	-0.41%	July	1.36%	-0.67%	1.36%	0.50%	-1.17%
	0.28	0.66	0.28	0.99	0.32		0.19	0.51	0.19	0.53	0.07 *
August	1.67%	-1.06%	1.67%	-0.48%	-0.59%	August	1.37%	-0.73%	1.37%	0.48%	-1.21%
	0.08 *	0.22	0.08 *	0.53	0.19		0.19	0.47	0.19	0.54	0.05 *
September	1.36%	-0.47%	1.36%	-0.06%	-0.41%	September	1.54%	-0.63%	1.54%	0.47%	-1.10%
	0.16	0.60	0.16	0.94	0.33		0.13	0.53	0.13	0.54	0.08 *
October	1.13%	-0.34%	1.13%	0.21%	-0.56%	October	1.00%	-0.19%	1.00%	1.12%	-1.30%
	0.23	0.70	0.23	0.78	0.20		0.31	0.85	0.31	0.14	0.04 **
November	0.71%	-0.08%	0.71%	0.47%	-0.55%	November	1.09%	-0.42%	1.09%	0.81%	-1.23%
	0.36	0.91	0.36	0.40	0.21		0.29	0.68	0.29	0.30	0.06 *
December	0.86%	-0.27%	0.86%	0.23%	-0.50%	December	1.34%	-0.71%	1.34%	0.70%	-1.41%
	0.38	0.77	0.38	0.77	0.24		0.19	0.48	0.19	0.37	0.03 **

Table 4: Average of the estimated coefficients and the p-values for the regressions

$$R_{p,t} - R_{f,t} = \gamma_{0,t} + \gamma_{1,t} \hat{\beta}_{p,t} + u_{p,t} \text{ and } R_{p,t} - R_{f,t} = \gamma_{0,t} + \gamma_{2,t} \delta \hat{\beta}_{p,t} + \gamma_{3,t} (1-\delta) \hat{\beta}_{p,t} + u_{p,t}$$

month by month.

Panel A: Unconditional model

	all markets		up and down markets		
	$\gamma_0$	$\gamma_1$	$\gamma_0$	$\gamma_2$	$\gamma_3$
full sample	0.86% 0.43	-0.17% 0.87	0.86% 0.43	1.05% 0.20	-1.22% 0.07 *
<b>Only month of...</b>					
January	-0.84% 0.87	3.78% 0.46	-0.84% 0.87	4.24% 0.40	-0.45% 0.50
February	-3.36% 0.32	4.28% 0.23	-3.36% 0.32	2.68% 0.36	1.60% 0.45
March	-1.06% 0.78	1.31% 0.73	-1.06% 0.78	2.72% 0.32	-1.41% 0.57
April	4.12% 0.28	-2.42% 0.52	4.12% 0.28	-0.62% 0.77	-1.80% 0.56
May	1.54% 0.58	-1.63% 0.55	1.54% 0.58	0.37% 0.76	-1.99% 0.42
June	1.79% 0.56	-1.85% 0.53	1.79% 0.56	-0.03% 0.99	-1.82% 0.22
July	2.81% 0.28	-2.00% 0.52	2.81% 0.28	2.03% 0.14	-4.03% 0.13
August	0.65% 0.85	0.71% 0.86	0.65% 0.85	3.90% 0.09 *	-3.19% 0.35
September	-3.10% 0.41	1.26% 0.75	-3.10% 0.41	3.41% 0.21	-2.15% 0.46
October	4.84% 0.45	-5.34% 0.27	4.84% 0.45	-7.09% 0.08 *	1.75% 0.48
November	4.44% 0.22	-3.30% 0.31	4.44% 0.22	-1.29% 0.63	-2.01% 0.28
December	-1.52% 0.70	3.17% 0.43	-1.52% 0.70	2.03% 0.56	1.15% 0.58

Panel B: QTARCH on market returns

	all markets		up and down markets		
	$\gamma_0$	$\gamma_1$	$\gamma_0$	$\gamma_2$	$\gamma_3$
full sample	1.17% 0.20	-0.50% 0.56	1.17% 0.20	-0.03% 0.97	-0.47% 0.25
<b>Only month of...</b>					
January	-0.48% 0.84	3.41% 0.10	-0.48% 0.84	3.56% 0.08 *	-0.15% 0.67
February	-0.95% 0.68	1.86% 0.34	-0.95% 0.68	0.08% 0.97	1.79% 0.05 *
March	3.45% 0.15	-3.17% 0.17	3.45% 0.15	-1.02% 0.58	-2.15% 0.13
April	1.08% 0.59	0.58% 0.81	1.08% 0.59	2.14% 0.22	-1.56% 0.34
May	-2.44% 0.36	2.39% 0.30	-2.44% 0.36	1.27% 0.11	1.11% 0.61
June	3.82% 0.05 *	-3.96% 0.07 *	3.82% 0.05 *	0.10% 0.91	-4.06% 0.04 **
July	2.29% 0.23	-1.53% 0.45	2.29% 0.23	-0.48% 0.61	-1.05% 0.55
August	-4.16% 0.19	5.57% 0.09 *	-4.16% 0.19	4.77% 0.15	0.80% 0.19
September	-1.04% 0.74	-0.81% 0.77	-1.04% 0.74	0.32% 0.88	-1.13% 0.53
October	1.65% 0.66	-2.26% 0.37	1.65% 0.66	-2.81% 0.21	0.55% 0.62
November	6.47% 0.40	-5.26% 0.45	6.47% 0.40	-5.70% 0.41	0.44% 0.50
December	4.67% 0.06 *	-3.10% 0.15	4.67% 0.06 *	-3.01% 0.07 *	-0.09% 0.95

Panel C: QTARCH with PCA

	all markets		up and down markets		
	$\gamma_0$	$\gamma_1$	$\gamma_0$	$\gamma_2$	$\gamma_3$
full sample	0.08% 0.91	0.59% 0.28	0.08% 0.91	0.31% 0.45	0.28% 0.42
<b>Only month of...</b>					
January	0.08% 0.96	2.87% 0.05 *	0.08% 0.96	2.79% 0.06 *	0.08% 0.60
February	1.07% 0.57	-0.20% 0.89	1.07% 0.57	-0.12% 0.91	-0.08% 0.94
March	-0.79% 0.76	1.06% 0.58	-0.79% 0.76	-0.17% 0.89	1.24% 0.41
April	0.57% 0.84	1.10% 0.69	0.57% 0.84	-0.74% 0.74	1.84% 0.25
May	0.37% 0.84	-0.50% 0.65	0.37% 0.84	-0.90% 0.32	0.41% 0.49
June	-1.69% 0.45	1.49% 0.44	-1.69% 0.45	0.96% 0.39	0.53% 0.74
July	3.50% 0.03 **	-2.68% 0.10	3.50% 0.03 **	-0.91% 0.45	-1.77% 0.13
August	-1.35% 0.41	2.66% 0.09 *	-1.35% 0.41	2.66% 0.06 *	0.00% 1.00
September	-0.20% 0.94	-1.57% 0.36	-0.20% 0.94	-1.69% 0.28	0.12% 0.86
October	2.28% 0.60	-2.70% 0.36	2.28% 0.60	-2.80% 0.19	0.10% 0.96
November	-1.29% 0.41	2.36% 0.05 *	-1.29% 0.41	2.15% 0.06 *	0.21% 0.62
December	-1.66% 0.45	3.30% 0.19	-1.66% 0.45	2.60% 0.10 *	0.71% 0.72

Panel D: GARCH-M

	all markets		up and down markets		
	$\gamma_0$	$\gamma_1$	$\gamma_0$	$\gamma_2$	$\gamma_3$
full sample	1.35% 0.16	-0.65% 0.50	1.35% 0.16	0.62% 0.41	-1.26% 0.03 **
<b>Only month of...</b>					
January	-3.07% 0.37	6.03% 0.08 *	-3.07% 0.37	6.25% 0.07 *	-0.23% 0.71
February	2.59% 0.40	-1.65% 0.56	2.59% 0.40	-1.75% 0.40	0.10% 0.96
March	-5.41% 0.14	5.78% 0.10	-5.41% 0.14	2.65% 0.35	3.13% 0.16
April	1.00% 0.83	0.67% 0.89	1.00% 0.83	3.43% 0.25	-2.75% 0.48
May	5.05% 0.15	-5.15% 0.15	5.05% 0.15	-1.13% 0.57	-4.01% 0.19
June	3.25% 0.41	-3.36% 0.38	3.25% 0.41	-1.25% 0.71	-2.11% 0.26
July	1.29% 0.57	-0.43% 0.86	1.29% 0.57	1.88% 0.31	-2.31% 0.12
August	1.17% 0.64	0.24% 0.93	1.17% 0.64	2.03% 0.25	-1.79% 0.41
September	-0.84% 0.79	-0.89% 0.79	-0.84% 0.79	2.23% 0.37	-3.12% 0.14
October	5.34% 0.19	-5.91% 0.06 *	5.34% 0.19	-5.11% 0.08 *	-0.79% 0.57
November	4.37% 0.14	-3.26% 0.23	4.37% 0.14	-1.61% 0.51	-1.64% 0.17
December	1.50% 0.62	0.11% 0.97	1.50% 0.62	-0.31% 0.91	0.42% 0.71

## 5. Conclusion

Most empirical studies seeking to validate the CAPM fail to report convincing results, and so does ours. The literature has documented well the possible reasons why so little support in favour of the model can be found and the causes either refer to the model itself, or to the way it is tested. In this study, we address both possible sources by examining some time-varying specifications for the assets' variances-covariances (and therefore betas) on the one hand, and various "anomalies" on the other. In particular, following Pettengill *et al.* (1995), we allow for a negative return-risk relation in periods in which the market excess return is negative. The conditional time-varying betas were estimated according to a non parametric threshold based ARCH specification (QTARCH), which was shown to have a good ability to explain the out-of-sample, cross-sectional return-risk relation. These new conditional specifications were compared to the more traditional, but also conditional, GARCH-M model, and to the unconditional case. Additionally, the Pettengill *et al.* approach, which argues that the return-risk relation should be negative in periods in which excess market returns are negative, was tested.

The global results were shown to be much influenced by the month October. Removing this month from the sample dramatically changes the results, both for the unconditional and conditional specifications for beta and whether or not upward and downward moving markets are taken into account. On the full sample, an unconditional beta seems to be a valid measure of risk in down moving markets only. When October is dropped, two interesting points arise. First, the CAPM is supported with unconditional betas only when up and down moving markets are accounted for. This shows the limitation of the traditional Fama and MacBeth approach. The CAPM is formulated in expectational terms, but the model is tested (cross-sectional) using realised returns. Although in our sample the mean excess return on the market portfolio has been significantly positive, there are numerous periods (that is, months) in which the excess market return was negative. In these periods, a negative relation between return and beta can be expected and will influence the significance level of the overall return-risk relation. Second, there is a conditional specification for beta, the QTARCH with PCA, which yields a significant beta without having to account for up and down moving markets. From a practical point of view, this is what a portfolio manager, who is unable to anticipate the direction of the next period's market move, is interested in. A long term overall positive relation between return and some measure of risk (such as a QTARCH-beta) is more important than an even stronger relation that depends on a rather unpredictable variable: the sign of the excess market return.

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*Appendix 1 : Composition of the seven sector portfolios.*

**MINERAL EXTRACTION**

**P1**

<b>Classification</b>	<b>Datastream Mnemonic</b>
Other Mining	MINESUK
Mining Finance	MIFINUK
Oil, Integrated	OILINUK
Oil Exploration & Production	OILEPUK
Oil Services	OILSVUK

**GENERAL MANUFACTURE**

**P2**

<b>Classification</b>	<b>Datastream Mnemonic</b>
Building & Construction	BUILDUK
Cement & Concrete	CMENUK
Other Building Materials	OTHBMUK
Builders Merchants	BMERCUK
Chemicals, Commodity	CHEMSUK
Chemicals, Speciality	CHMSPUK
Chemicals, Materials Technology	CHMSTUK
Diversified Industrials	DIVINUK
Electrical Equipment	ELEQPUK
Electronic Equipment	ELETRUK
Office Equipment	OFFEQUK
Metallurgy	METALUK
Steel	STEELUK
Engineering, Fabrication	ENGFAUK
Engineering, Specialities	ENGSPUK
Engineering, Contractors	ENGCOUK
Engineering, Aerospace	AERSPUK
Instruments & Tools	INSTRUK
Vehicle Components	VCOMPUK
Vehicle Assemblers	VASSAMUK
Paper & Packaging	PAPERUK
Printing	PRINTUK
Clothing Manufacturers	CLTHGUK
Wool	WOOLSUK
Footwear & Leather	FTWERUK

**CONSUMER GOODS**

**P3**

<b>Classification</b>	<b>Datastream Mnemonic</b>
Breweries	BREWSUK
Spirits, Wines & Ciders	WINESUK
Food Manufacturers	FDMFGUK
Furniture & Furnishings	FURBSUK
Household Requisites	HREQSUK
Health Care	HLTHCUK
Pharmaceuticals	PHARMUK
Tobacco	TOBACUK

**SERVICES**

**P4**

<b>Classification</b>	<b>Datastream Mnemonic</b>
Distributors of Industrial Components & Equipment	DCOMPUK
Vehicle Distributors	DSVHLUK
Leisure	LEISRUUK
Hotels & Caterers	HOTELUK
Media Agencies	MEDAGUK
Broadcasting	BRCASUK
Publishing	PUBLSUK
Retailers, Food	FDENTUK
Retailers, Multi-Department	MULTIUK
Retailers, Chain Stores	CHAINUK
Business Support	BUSUPUK
Security & Alarms	SECALUK
Laundries & Cleaners	LAUNDUK
Computer Services	COMPSUK
Shipping	SHPNGUK
Bus & Coach	COACHUK
Railways	RAILSUK
Airlines	AIRLNUK
Other Transport	OTHTRUK
Pollution Control	POLUTUK
Other Business	OTHBUUK

**INVESTMENT TRUSTS**

**P5**

<b>Classification</b>	<b>Datastream Mnemonic</b>
Investment Trusts	INVTSTUK

**FINANCIALS**

**P6**

<b>Classification</b>	<b>Datastream Mnemonic</b>
Insurance Brokers	INSBRUK
Insurance, Composite	INSCMUK
Insurance, Lloyds Funds	INSLLUK
Life Assurance	LIFEAUK
Merchant Banks	BNKMRUK
Miscellaneous Financial	MISFIUK
Property	PROPSUK
Property Agencies	PRPAGUK

**UTILITIES**

**P7**

<b>Classification</b>	<b>Datastream Mnemonic</b>
Electricity	ELECTUK
Gas Distribution	GASDSUK
Telecommunications	TELCMUK
Water	WATERUK



Appendix 2 : Summary statistics of the market portfolio and the seven sectors.

MONTH		MARKET	MINERAL EXTRACTION	GENERAL MANUFACTURE	CONSUMER GOODS	SERVICES	INVESTMENT TRUSTS	FINANCIALS	UTILITIES
Full sampl	MEAN	0.62%	0.53%	0.42%	0.83%	0.62%	0.55%	0.70%	1.23%
	S.D.	4.99%	6.24%	5.69%	5.23%	5.23%	5.45%	5.67%	7.19%
	MSD	0.124 *	0.085	0.074	0.160 **	0.118 *	0.101	0.124 *	0.171 **
January	MEAN	2.75%	1.78%	3.47%	2.82%	2.17%	3.31%	3.54%	3.35%
	S.D.	4.00%	5.98%	4.57%	5.32%	4.42%	5.24%	4.18%	5.46%
	MSD	0.687 **	0.297 **	0.761 **	0.531 **	0.491 **	0.631 **	0.847 **	0.614 **
February	MEAN	1.01%	-0.90%	2.04%	0.84%	1.54%	0.97%	0.57%	1.42%
	S.D.	4.39%	5.20%	5.36%	4.56%	4.28%	5.45%	4.94%	5.73%
	MSD	0.230 **	-0.173 **	0.381 **	0.185 **	0.360 **	0.177 **	0.116	0.249 **
March	MEAN	0.16%	1.46%	0.21%	-0.10%	1.05%	-0.78%	-0.70%	1.14%
	S.D.	4.37%	7.12%	3.99%	5.23%	4.93%	4.66%	4.95%	5.62%
	MSD	0.037	0.205 **	0.054	-0.019	0.214 **	-0.167 **	-0.142 **	0.203 **
April	MEAN	1.86%	3.28%	2.52%	1.95%	2.30%	1.03%	0.75%	-0.64%
	S.D.	3.61%	4.66%	3.89%	3.38%	4.12%	2.88%	4.48%	6.50%
	MSD	0.517 **	0.704 **	0.647 **	0.575 **	0.557 **	0.359 **	0.166 **	-0.098
May	MEAN	-0.20%	0.89%	-0.71%	-0.36%	-0.89%	-0.39%	-0.34%	1.49%
	S.D.	4.55%	4.11%	5.20%	4.89%	5.41%	4.50%	5.18%	6.08%
	MSD	-0.045	0.217 **	-0.136 *	-0.073	-0.165 **	-0.086	-0.066	0.245 **
June	MEAN	-0.19%	-1.23%	-0.80%	0.90%	-0.83%	0.52%	1.38%	-0.62%
	S.D.	4.27%	6.19%	5.17%	4.59%	4.19%	5.36%	5.15%	5.59%
	MSD	-0.045	-0.200 **	-0.154 **	0.197 **	-0.199 **	0.097	0.268 **	-0.111
July	MEAN	0.79%	0.41%	0.55%	0.89%	1.05%	0.76%	1.30%	0.86%
	S.D.	3.60%	4.29%	4.36%	3.93%	4.55%	3.53%	4.03%	6.83%
	MSD	0.218 **	0.095	0.127 *	0.225 **	0.230 **	0.217 **	0.324 **	0.126 *
August	MEAN	1.19%	1.85%	0.28%	1.46%	1.17%	1.42%	0.63%	3.54%
	S.D.	4.92%	6.65%	5.81%	5.31%	5.35%	5.34%	4.86%	6.49%
	MSD	0.241 **	0.278 **	0.047	0.275 **	0.219 **	0.266 **	0.129 *	0.545 **
Septembe	MEAN	-1.76%	-1.18%	-2.45%	-1.46%	-1.56%	-1.69%	-1.67%	-2.45%
	S.D.	6.52%	7.09%	6.97%	6.44%	7.20%	6.90%	7.73%	8.69%
	MSD	-0.270 **	-0.167 **	-0.352 **	-0.227 **	-0.217 **	-0.246 **	-0.215 **	-0.282 **
October	MEAN	-0.48%	-0.41%	-2.01%	0.09%	-0.92%	-0.44%	0.52%	-1.45%
	S.D.	9.26%	11.34%	10.07%	9.67%	8.48%	10.26%	9.94%	7.50%
	MSD	-0.052	-0.036	-0.200 **	0.010	-0.108	-0.043	0.052	-0.193 **
November	MEAN	0.79%	0.04%	0.31%	0.88%	0.99%	0.28%	0.70%	5.18%
	S.D.	4.65%	4.52%	5.28%	3.89%	4.83%	4.49%	6.33%	12.49%
	MSD	0.169 **	0.008	0.059	0.225 **	0.204 **	0.063	0.111	0.415 **
December	MEAN	1.49%	0.22%	1.56%	2.12%	1.27%	1.61%	1.81%	2.98%
	S.D.	3.05%	4.85%	3.60%	2.80%	2.74%	3.34%	3.92%	4.78%
	MSD	0.489 **	0.046	0.434 **	0.756 **	0.465 **	0.482 **	0.463 **	0.623 **

Notes: MSD is the mean over the standard deviation. \* (respectively \*\*) indicates the result is significantly different from zero at the 90% (respectively 95%) level.