Michel Kervaire : 1927-2007

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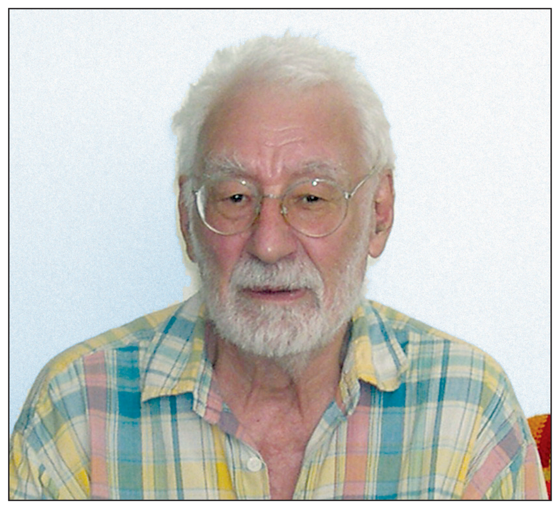
Shalom Eliahou, Pierre de la Harpe, Jean-Claude Hausmann, and Claude Weber

Michel Kervaire died on November 19, 2007, in Geneva. He was born in Poland on April 26, 1927. His secondary school education was in France and he studied mathematics at Eigdenössisches Technische Hochschule Zurich, defending his thesis in 1955 under the supervision of Heinz Hopf. He published a second thesis in France in 1965. He was a professor in New York (1959–1971) and in Geneva (1972–2007), with several long-term academic visits to Princeton, Paris, Chicago, Massachusetts Institute of Technology, Cambridge (UK) and Bombay. He was very active as editor of *Commentarii Mathematici Helvetici* (1980–2001) and chief editor of *L'Enseignement Mathématique* (1978–2007). He had an h.c. doctorate from Neuchâtel (1986).

Kervaire was an inspiring example that a mathematician can be both a generalist and a specialist at the highest level. As a generalist he knew how to appreciate a large range of subjects and encourage others to progress in their work. Among other things, he organized uncountably many meetings in the small mountain village of Les Plans-sur-Bex, mixing younger students with the best international specialists in all kinds of domains: Brauer groups, foliations, arithmetic, von Neumann algebras, knot theory, representations of Lie groups, the theory of codes, ergodic theory and finite groups (to name but a few). As a specialist, he made his mark in several subjects: he strongly contributed to changing differential topology and homotopy theory from the mid-1950s onwards; he created the subject of high-dimensional knots; he formulated a famous conjecture (still open) in abstract group theory and he made deep contributions on problems mixing algebra, number theory, and combinatorics.



Photograph by Teresa Dib.

Michel Kervaire, 2005.

The Kervaire Manifold and the Kervaire-Milnor Results

In his 1960 *Commentarii* paper, Kervaire constructed the first example in history of a closed topological manifold (indeed a PL manifold) that does not admit any differentiable structure, not

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The first author was one of the many Ph.D. students of Kervaire. The other three authors were colleagues of Kervaire in Geneva. The text of this article previously appeared in the European Mathematical Society Newsletter, *March 2008. A* longer version appeared in French in *La Gazette des Mathématiciens,* April 2008.

even up to homotopy type. This 10-dimensional manifold is now known as the Kervaire manifold. The main tools include an invariant of quadratic forms defined over the field of two elements (the Arf invariant) and deep results on stable homotopy groups of spheres; the closely related Kervaire invariant is now important in this subject.

In the mid-1950s Milnor had discovered that, on the sphere of dimension 7, there exists a differentiable structure that is not diffeomorphic to the standard one. In their 1963 *Annals* paper Kervaire and Milnor showed that the set of differentiable structures on S^n, which for $n \neq 3,4$ is also the set of h-cobordism classes of smooth homotopy n-spheres, is a finite abelian group. In particular, S^7 has exactly twenty-eight differentiable structures.

Thus a topological or a PL manifold can have zero or more than one differentiable structures. This was a revolution in our understanding of regularity conditions in topology. The work of Kervaire and Milnor (first independently and then jointly) is a gem from this flourishing time of the topology of manifolds. It announces most of the capital discoveries of the next period: Browder-Novikov, Wall, Sullivan, Kirby-Siebenmann and so on.

From High-dimensional Knots to the Kervaire Conjecture and to Later Work

In his French thesis (*Bull. Soc. Math. France*, 1965), Kervaire created the theory of high-dimensional knots, namely that of smooth embeddings of homotopy n-spheres in the $(n+2)$-sphere, for $n \geq 2$. His first result is a characterization by three purely group-theoretic conditions of those finitely presented groups that can be fundamental groups of knot complements $S^{n+2} \setminus S^n$, for any $n \geq 3$ (stating results for $n = 2$ would require too much space here). The proof involves completely original arguments, using the new technique of surgery. The same paper was also the origin of knot modules and of the cobordism of knots.

A by-product of this was the following conjecture, first stated by Kervaire in 1963–4 during conversations with G. Baumslag: let G be a group presented by generators and relations; if adding one generator and one relation gives rise to the trivial group, then G itself is (conjecturally) trivial. This has been solved in the torsion-free case by Klyachko (1993) but the full conjecture is still open.

Kervaire knew much more than the topics he published about; he was an expert on many subjects of arithmetic, algebra, and combinatorics: class field theory, quadratic forms, algebraic K-theory, etc. In the last twenty years he published nearly thirty papers, most of them on the borderline between algebra and combinatorics, covering subjects such as commutative algebra (the so-called Eliahou-Kervaire resolution for stable monomial ideals in polynomial rings), the Hadamard conjecture (on square matrices having ± 1 entries that are orthogonal up to a factor), the possible lengths of Golay complementary pairs of sequences of ± 1 (the original proof used properties of cyclotomic integers) and vast generalizations of the Cauchy-Davenport theorem from additive number theory (given integers $r, s \geq 1$ and a group G, say abelian here, the question is to compute the minimal size of all sets of sums $a + b$ where a, b range in subsets A, B of cardinality r, s respectively).

Michel Kervaire had a special gift for turning moments of life into celebration: at the blackboard; enjoying a coffee, offering great wines or memorable meals, sometimes in restaurants and sometimes at home with his wife, painter Aimée Moreau.