Essays in financial econometrics

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by

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Abstract

The recent events since the onset of the subprime crisis in 2007 have put forward the central role of the financial market in developed economies and its close linkages with other markets, such as the sovereign one. The extreme (co)variations in the financial market are difficult to explain by the value of the fundamentals, and factors like investor sentiment, liquidity and simultaneous comovement are privileged. This thesis contributes in better understanding the role of these factors in explaining highly debated topics of international financial markets such as (i) the predictability of their returns, (ii) the commonality in liquidity, and (iii) several overlooked aspects in the modeling of market comovement.

The common thread running through the three chapters of the thesis is the use and the further development of regime-switching models. This class of models is well adapted to capture potential impact variations in different phases of the business cycles. In two empirical studies we use Threshold and Smooth Transition Regression models in a panel framework, while in the last chapter we develop a flexible setting to model market comovement within a Smooth Transition Simultaneous Equation Model (ST-SEM).

In the first chapter we investigate the role of market-wide investor sentiment in predicting aggregate annual stock returns of financial companies - banks, insurances and real estate firms - in 20 developed countries, from January 1999 to August 2011. To the best of our knowledge, the sensitivity to investor sentiment of the prices of this class of assets has not been the subject of any study so far. From an international perspective, this paper appears to be the largest in this strand of literature, and includes the recent financial crisis during which the impact of investor sentiment has presumably changed. A consistent result is the negative impact of investor sentiment during normal times and a positive impact during crisis times. Although the positive sign of this latter coefficient is not common in the literature, the arguments advanced in a few papers might well explain this finding. This result is in favor of a differentiated impact of sentiment on stock returns of financial companies, compared to a wider horizon of stocks.

The second chapter examines the impact of commonality in liquidity in pricing the returns of U.S. real estate securities before and during the recent crisis. We consider the commonality (i) within the real estate market, (ii) with the stock market and (iii) with the underlying property market. Our main hypothesis is that commonality in liquidity should have a differentiated effect depending on the state of the economy. In good times, market liquidity is high, and thus commonality is a desired property of an asset. Investors are willing to accept lower returns for assets whose liquidity comoves strongly with the market, leading to a negative effect of commonality in their prices. On the contrary, during bad times – when market liquidity dries out – investors would require higher returns in compensation of commonality, leading to a positive effect. This hypothesis is validated by the data for the three types of commonality.

The financial crisis and the euro area sovereign debt issue that followed, brought market dependence to the forefront of a debate among economists, politicians, investors and portfolio managers to constrain its adverse effects. In the third chapter we formulate a flexible setting to model the dependence between two endogenous variables within a nonlinear simultaneous equation model. The framework considers comovement as a simultaneous event that can be disentangled from indirect dependence which intervenes through observed and unobserved common factors. The model can handle an endogenous identification of crisis periods rather than an ex-post and arbitrary definition of them. The transmission of shocks is smooth and potentially asymmetric, granting to the model a high degree of flexibility. We provide an accurate estimation method and a testing procedure for simultaneity, and treat the issue of potential multiple equilibria in the system of equations.

Résumé

Les événements récents depuis l'éclatement de la bulle des subprimes en 2007 ont mis en avant le rôle central du marché financier dans les pays développés et ses liens étroits avec d'autres marchés, comme le marché souverain. Les (co-)variations extrêmes de ce marché sont difficiles à expliquer par la valeur des fondamentaux, et des facteurs comme le sentiment des investisseurs, la liquidité et la interdépendance sont privilégiés. Cette thèse contribue à une meilleure compréhension du rôle de ces facteurs pour expliquer des sujets très débattus de la finance internationale comme (i) la prédictibilité des rendements, (ii) la co-variation (commonality) de la liquidité et (iii) certains aspects négligés dans la modélisation de la dépendance entre les marchés.

Le fil conducteur des trois chapitres de la thèse est l'utilisation et le développement de modèles à changement de régime avec seuil ou avec une transition graduelle entre les régimes. Ces modèles sont bien adaptés pour modéliser de potentielles variations d'impact dans les différentes phases du cycle conjoncturel. Dans deux études empiriques nous utilisons des modèles à changement de régime pour des données de panel, alors que dans le dernier chapitre nous mettons en place un cadre flexible pour la modélisation de la dépendance, plus précisément un modèle non-linéaire à équations simultanées avec une transition graduelle.

Dans le premier chapitre nous étudions le rôle d'une mesure générale du sentiment des investisseurs pour prédire les rendements annuels agrégés des actions des compagnies financières – banques, assurances, entreprises immobilières – dans 20 pays développés pour la période de janvier 1999 à août 2011. À notre connaissance, la réaction des prix de cette classe d'actifs par rapport au sentiment des investisseurs n'a fait le sujet d'aucune autre étude à ce jour. Dans une perspective internationale, cette étude semble être la plus étendue dans ce volet de littérature incluant également la récente crise, durant laquelle l'impact du sentiment des investisseurs a vraisemblablement changé. Un résultat constant est l'impact négatif du sentiment durant les périodes normales et l'impact positif durant les périodes de crise. Bien que le signe positif de ce dernier coefficient ne soit pas commun dans la littérature, les arguments avancés dans certains travaux de recherche pourraient très bien expliquer cette conclusion. Le résultat est en faveur d'un impact différencié du sentiment dans les rendements des actions des compagnies financières, comparé à un spectre plus large d'actions.

Le deuxième chapitre examine l'impact de la co-variation entre la liquidité d'un titre et la liquidité du marché dans l'évaluation de ce titre. En se focalisant sur le marché immobilier américain titrisé, nous étudions la co-variation (i) à l'intérieur de ce marché, (ii) avec le marché des actions et (iii) avec le marché immobilier privé. Notre hypothèse principale stipule que la co-variation de la liquidité a un impact qui dépend de la situation économique. Durant les périodes de croissance, la liquidité du marché est élevée et par conséquent la co-variation pour un actif est souhaitable. Les investisseurs sont disposés à recevoir des rendements plus faibles pour des actifs dont la liquidité co-varie fortement

avec celle du marché, conduisant à un impact négatif de cette co-variation dans les prix. À l'inverse, durant les périodes de crise – caractérisées par une liquidité du marché faible – les investisseurs devraient exiger des rendements plus élevés pour compenser la co-variation de la liquidité, conduisant à un impact positif de cette dernière. Notre hypothèse est validée par les données pour les trois types de co-variation de la liquidité mentionnés ci-dessus.

La crise financière et la crise de la dette dans la zone euro qui a suivi ont mis la dépendance entre les marchés au premier plan d'un débat entre économistes, politiciens, investisseurs et gestionnaires de portefeuille pour en limiter les effets défavorables. Dans le troisième chapitre nous formulons une structure économétrique flexible en modélisant la dépendance avec un modèle non-linéaire à équations simultanées. Le modèle traite la dépendance comme un événement simultané et qui peut être distingué de la dépendance indirecte qui intervient à travers des variables communes observées et inobservées. Le modèle est en mesure d'identifier de façon endogène les périodes de crise, ce qui est préférable à une définition arbitraire et ex-post de celles-ci. La dépendance entre les marchés est graduelle et potentiellement asymétrique, accordant au modèle un grand degré de flexibilité. Nous fournissons également une méthode d'estimation adaptée, une procédure de test de simultanéité, et traitons la question des potentiels équilibres multiples de ce système.

Contents

Për nënën time, për vëllezerit e mi, dhe për kujtimin e babait tim.

Introduction

When this thesis started the world economy was living one of its most severe crises, which originated in the U.S. financial market and was prolonged by the sovereign debt crisis in the euro area countries. From a research perspective, these conditions were an “ideal laboratory” to study the mechanisms through which the shocks spread across markets and countries, the extent to which these performance variations were predictable etc. Simultaneously, there was an increasing need for the development of advanced econometric tools to model these new phenomena in the financial markets. This thesis is situated at a crossing point between these empirical and methodological aspects of financial econometrics.

In the first chapter we investigate the role of investor sentiment in predicting long-term stock returns of financial companies – banks, insurances and real estate firms – in 20 developed countries from January 1999 to August 2011. Shefrin (2008) defines investor sentiment as excessive pessimism or optimism regarding stock prices in general. As a proxy for investor sentiment we use survey-based consumer sentiment indicators which are found to be accurate measures of investor sentiment. Financial institutions play a central role in developed economies but the investigation of their return predictability has not been the subject of any study so far. Many papers investigate the return predictability in the U.S. market and for a wide universe of stocks. The study which is closest to ours is Schmeling (2009) where the predictive power of investor sentiment is tested in the aggregate returns of a panel of 18 developed countries.

The impact of investor sentiment on future returns should be *negative* since high sentiment is associated with temporary overpricing and consequently with lower future returns, while low sentiment is associated with temporary underpricing and subsequent higher returns. This argument seems to be valid for aggregate returns, with a large empirical support. But, what about the impact of investor sentiment on different classes of assets? Baker and Wurgler (2007) argue that the predictive power of sentiment could turn to *positive* during distressed periods for easy-to-arbitrage and easy-to-value stocks, as opposed to speculative stocks. The reason is that during such periods, the sentiment is most likely low, and the demand variations due to the flight to quality increase the price of these types of stocks, leading to lower future returns. Stocks of financial companies included in our sample should exhibit such a predictability pattern since they contain many of the characteristics of easy-to-arbitrage and easy-to value stocks, and are traded in highly liquid international platforms.

Our estimation results are supportive of the above hypothesis, with a positive and robust impact of near lags of investor sentiment on future returns during crisis times, and a negative impact during normal times, but the result is less robust. Second, the returns of financial companies seem to contain a smaller predictability component, as compared to returns of broader indices. Finally, as suggested by the literature, future returns are less predictable in the short horizon.

The second chapter of this thesis studies the impact of commonality in liquidity on the returns of the securitized real estate market in the U.S. We consider the monthly disaggregated returns of a large panel of firms from January 1999 to December 2012. Commonality in liquidity is defined as the comovement of an asset's liquidity with market liquidity. Using linear models, the literature so far has found a small effect of commonality on the returns of various asset classes. We think that these weak results are not necessarily a proof that commonality has no impact on prices, but rather driven by the mismatch between the methods used and the underlying impact of commonality which presumably depends on the business cycles.

Our main hypothesis grants to commonality a positive impact during crisis times and a negative impact during normal times. Indeed, in distressed periods during which market liquidity dries out, commonality is not a desirable characteristic of an asset. Thus, an investor would require higher returns for assets with high commonality, leading to a positive impact. On the contrary, during normal times investors would be willing to receive lower returns for assets whose liquidity covaries strongly with market liquidity. Testing this hypothesis requires the use of methods with time-varying coefficients, such as regime-switching methods, instead of linear methods which average out the impact of commonality on prices.

Using a Panel Threshold Regression (PTR) model developed by Hansen (1999), we verify our hypothesis for three types of commonality: (i) within the securitized real estate market, (ii) with the stock market and (iii) with the underlying property market. Subsequently, we investigate the supply-side (i.e. funding constraints in times of crisis among financial intermediaries) and demand-side (i.e. correlated trading activity and investor sentiment) sources of commonality in liquidity, as in Kalolyi, Lee and van Dijk (2012). Our results support a demand-side explanation of the commonality in liquidity.

In the last chapter of the thesis we develop a flexible methodological setting to model the dependence between two endogenous variables. The approach is a generalization of the nonlinear simultaneous equation model of Pesaran and Pick (2007) which is a special case. We add flexibility in the dependence structure by modeling it as a smooth and potentially asymmetric function of the magnitude of the shock (i.e. the value of the endogenous variable). Following a large literature on Smooth Transition Regression models, the logistic function is used. The threshold, the smoothness and the asymmetric parameters are estimated jointly with the other parameters of the model using heuristic methods. The model controls also for indirect dependence which intervenes through common observed and unobserved factors.

The parameters are estimated using the nonlinear two-stage least squares method (Kelejian 1971, Amemiya 1974). Additionally, we derive a test for simultaneity in the presence of nuisance parameters, adapting a procedure developed in the framework of smooth transition regression models (Luukkonen, Saikkonen and Teräsvirta 1988). Finally, we investigate the properties of the model, and in particular the potential multiple equilibria that appear in Pesaran and Pick's model but could be avoided in our framework under some parameter constraints. The uniqueness of the reduced form is crucial for the efficiency of the estimators and for forecasting. We apply these ideas to the comovement between the sovereign and the banking sectors of a set of eight developed countries. We strongly believe that this project is the first of a research agenda concerned with further methodological extensions and with empirical studies in different economic and financial issues.

Chapter 1

Predictability of Stock Returns of Financial Companies and the Role of Investor Sentiment: A Multi-country Analysis*

Abstract

We investigate the role of investor sentiment in predicting annual stock returns of financial companies at the aggregate level and for a large panel of developed countries within two panel regime-switching models, with threshold and with smooth transition between regimes. We find a negative, but insignificant effect of sentiment on future returns during normal times, and a surprisingly positive and strongly significant effect during crisis times. This result could be explained by a differentiated impact of investor sentiment on specific types of stocks, as opposed to a wide horizon of stocks. We find less evidence of predictability for shorter-term financial stock returns. To our knowledge, this study is the first to examine the predictability of financial stock returns within a panel regime-switching framework.

JEL Classification: *C23, C58, G01, G15*

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1.1 Introduction

Many studies have found that stock returns contain a predictable component that is not only explained by economic and financial factors, but most importantly by the noise trading activity of irrational investors whose actions are persistent and correlated (i.e. herd behavior).[1] The literature on behavioral finance and investor sentiment that emerged in

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[1]See Keim and Stambaugh (1986), Fama and French (1988, 1989), Pesaran and Timmermann (1994, 1995, 2000), Cochrane (1999), Ang and Bekaert (2007), Bollerslev, Tauchen and Zhou (2009), Camponovo, Scaillet and Trojani (2012). Stock return predictability has been mainly investigated in the U.S. market, while the extension to other markets has been the research question of a few recent papers. To our knowledge, the only studies investigating the predictive power of investor sentiment in an international perspective are Schmeling (2009), Baker, Wurgler and Yuan (2012). See also Ang and Bekaert (2007), Bollerslev, Marrone, Xu and Zhou (2012), Cooper and Priestley (2013) for the impact of other predictors.

the 2000s reflects this change of perspective, compared with the efficient market paradigm initiated by Fama (1965, 1970): see Baker and Wurgler (2006, 2007), Shefrin (2008) and Barone-Adesi, Mancini and Shefrin (2012) among others. Indeed, Shefrin (2008) defines investor sentiment as excessive optimism or pessimism regarding stock prices in general. The impact of investor sentiment on future returns should be *negative* since high sentiment leads to temporary overpricing and consequently to lower future returns, while low sentiment leads to temporary underpricing and higher future returns.

Baker and Wurgler (2007) and Baker, Wurgler and Yuan (2012) argue that the above future return - sentiment relation could turn to *positive* for some types of stocks, especially for those which are easy to arbitrage and easy to value. The latter characteristics can be particularly desirable during distressed periods or low-sentiment episodes (e.g. flight to quality), leading to an increase of the demand for easy-to-arbitrage and easy-to-value stocks (higher prices and lower future returns), and to a decrease of the demand for speculative[2] stocks (lower prices and higher future returns).

In this study we investigate the role of market-wide investor sentiment in predicting aggregate stock returns of financial companies – banks, insurances and real estate firms – in 20 developed countries from January 1999 to August 2011. Focusing on stocks of financial companies is interesting for at least two reasons. First, financial stocks were severely hit by the market downturn which originated in the banking sector in August 2007. Although of different nature, the two other shocks covered by our time frame – the dot-com bubble and the euro area sovereign debt crisis – had also serious consequences on the performance of financial stocks. In this context of highly volatile market conditions, it is thus appealing to focus more particularly on this class of assets. Second, financial institutions play a central role in developed economies with strong links with many sectors and across countries. Nevertheless, following Fama and French (1992), the stocks of these companies were often excluded from empirical tests of return predictability because of the different role of their high leverage compared to other high leveraged firms.[3] Focusing on these stocks thus fills a gap in the literature and represents a robustness check for studies ignoring these stocks or considering a wider universe of stocks

Given the substantial evidence that the predictability patterns of stock returns are closely linked to variations in business cycles and to the arrivals of sudden shocks – such as the one that hit the financial markets in 2007 – several studies advise the use of models that allow potential regime changes (Pesaran and Timmermann 1995). We use two panel regime-switching models with threshold (Hansen 1999) and smooth (González, Teräsvirta and van Dijk 2005) transition between regimes and compare the robustness of the results through both specifications and with a standard panel linear model which serves as baseline. The use of a panel framework, instead of separate time-series at the country level, should result in efficiency gains from the econometric estimation due to a much larger sample, and should provide for a better understanding of financial stock predictability at the aggregate level. To the best of our knowledge, this study is the first

[2] Baker and Wurgler (2007) and Baker, Wurgler and Yuan (2012) classify as speculative, stocks which are hard to value and hard to arbitrage. Examples of speculative stocks are low capitalization, younger, unprofitable, high volatility, non-dividend paying, distressed, or extreme growth stocks. The authors write that these types of stocks are likely to be disproportionately sensitive to investor sentiment. On the contrary, relatively easy-to-value and easy-to-arbitrage stocks are less sensitive to investor sentiment, or even with a reversed relation.

[3] We thank a referee for noting that Barber and Lyon (1997) have argued that financial firms are not different from other firms on dimensions other than leverage and found that Fama and French (1992) 3-factor model works just as well with such firms.

to examine the predictability of financial stock returns within a panel regime-switching model.

Although we use a panel framework, we are able to uncover the country-specific predictability variations since in the regime-switching models that we employ, the regime changes are driven by an observable transition variable which tracks the business cycles. We show evidence that the transition variable that we use – the spread between 3-month and overnight interbank rates – is an early indicator of market fluctuations: it is positive and exhibits upward moves since the onset of distressed periods and is close to zero or even negative in normal times.

The study which is closest to ours is Schmeling (2009) in which the author investigates the role of investor sentiment in the predictability of aggregate stock market returns in 18 developed countries. Nevertheless, our study differs in several respects: (i) we focus on financial companies for reasons explained above, (ii) we use a different methodology, (iii) our data cover the recent financial crisis, and (iv) our set of predictors is larger, including measures of volatility, liquidity and market size.

We use as proxy of market-wide investor sentiment a Consumer Confidence Indicator which was found to be a highly adequate measure in recent studies such as Fisher and Statman (2003), Qiu and Welch (2006) and Schmeling (2009). Our main focus are returns at the one-year horizon, since medium to long-term returns have been found to be more predictable (e.g. Brown and Cliff 2005). We nevertheless test as a robustness check, the predictability of shorter-term returns at 1, 3, 6 and 9-month horizons.[4] It is important to notice that we conduct a time-series predictability analysis across the economic states where sentiment is as an independent variable, in contrast with cross-sectional studies where sentiment acts as a moderating variable across the sentiment states.

Keim and Stambaugh (1986) argue that theoretical models provide limited guidance about specific variables to be used as predictors. In the empirical literature, stock return predictability has been investigated by a wide range of variables that can be classified in three groups : (i) business-cycle indicators, (ii) financial variables, and (iii) investor sentiment.[5] Among the most used business-cycle indicators we find output and consumption growth (Lettau and Ludvigson 2001), inflation (Erb, Harvey and Viskanta 1995), interest rates (Pesaran and Timmermann 1994), credit and term spreads (Fama and French 1989, 1993). The most frequently used predictor among the financial variables is dividend yield which is found to have a predictive power on financial returns for different horizons (Lewellen 2004; Ang and Bekaert 2007; Camponovo, Scaillet and Trojani 2012). Other crucial variables are market volatility (Bollerslev, Tauchen and Zhou 2009), market liquidity (Amihud 2002) and indicators related to the profitability of financial firms.

Our empirical investigation uncovers that there is a substantial predictability component in the annual stock returns of financial companies. Investor sentiment is found to have a negative but insignificant impact in normal times, whereas in crisis times this impact is positive and strongly significant. A potential explanation of this result could be the mechanism provided in Baker and Wurgler (2007): easy-to-arbitrage and easy-to-value stocks are less sensitive to sentiment. The effect of sentiment might even be positive for this class of stocks. Such an effect need not be present at all times – indeed, our results

[4]Brown and Cliff (2005) argue that returns are predictable over longer horizons because (i) investor sentiment is a persistent variable that builds over time, (ii) arbitrage from informed investors is likely to eliminate mispricing in the short run, but in the long run informed investors face important limits to arbitrage.

[5]Cochrane (2006) states that “stock returns can be forecastable by other variables such as dividend yields, yet unforecastable by their own past.”

indicate that the positive impact of sentiment on future returns is only felt in times of economic crisis. Why this should be, is an interesting subject for future research.

The rather brusque turning point to a crisis regime is detected at the burst of the dot-com bubble by the beginning of year 2000, and at the inception of the global financial crisis by August 2007. The predictability power of some factors varies with the business cycle (inflation, short-term rate, market size), while other factors have a statistically constant impact (output growth, volatility, dividend yield, liquidity) or no impact (term spread).

We examined the robustness of our results in several directions. First we consider an alternative proxy of investor sentiment – an Economic Sentiment Indicator – and find that the predictability pattern of our main analysis is not entirely confirmed. The most plausible reason for this finding could be that Economic Sentiment Indicator, being a broader measure compared to Consumer Confidence Indicator, might be more contaminated by fundamentals. Second, there is much less evidence of predictability for shorter-term financial stock returns. Nevertheless, the future return - sentiment relation remains negative in normal times and positive in crisis times. Third, we examined further lags of the predictors and the results are much in favor of the same pattern of predictability. Fourth, for the U.S. market, we examined whether financial stock returns react differently to investor sentiment compared to the returns of a broad equity market index. In line with the literature, the results are in favor of a higher predictability of returns of broad indices. Finally, the results are robust to the bias correction due to the persistence of the predictors and to various measures of real growth and liquidity.

The outline of this paper is as follows. In Section 1.2 we discuss the set of the predictors used in this study and how they impact financial markets' performance. In Section 1.3 we introduce the Panel Threshold Regression (PTR) and the Panel Smooth Transition Regression (PSTR) models. Section 1.4 presents the data set and the construction of some variables. The estimation results and the robustness analysis are displayed in Section 1.5. In Section 1.6 we conclude on the main findings and suggest directions for future research.

1.2 Predictive factors of stock returns

In this section we briefly describe variables that are found by an important number of empirical studies to have a significant predictive power on stock returns. We classify them in three groups: (i) investor sentiment indicator (ii) macroeconomic variables related to the business cycles and (iii) variables specific to the financial markets. We do not claim that our set of predictors captures all aspects of financial return predictability, but we strongly believe that it captures the most persistent aspects of it.

1.2.1 Investor sentiment measure

Given the unavailability of a direct measure of investor sentiment for reasonably long periods and for a large number of countries, many studies have attempted to use accurate proxies of it.[6] Consumer Confidence Indices (CCI) are found to be highly adequate measures of investor sentiment (e.g. Fisher and Statman 2003; Qiu and Welch 2006;

[6] Several surveys are conducted to quantify investor sentiment, but they are mainly concerned with the U.S. market and generally involve quite short time spans (e.g. UBS/Gallup, Investor Intelligence, American Association of Individual Investors surveys). Baker and Wurgler (2006, 2007) construct a composite investor sentiment for the U.S. market, which is extended to (only) six developed countries in Baker, Wurgler and Yuan (2012).

Stambaugh, Yu and Yuan 2012). An advantage of this proxy is that it covers the sentiment of investors at the aggregate level and not exclusively of financial stock markets. For robustness tests we also use as a proxy for investor sentiment an Economic Sentiment Index (ESI) which is a composite index consisting of business and consumer surveys. In addition to capturing the sentiment of consumers, ESI captures also the role of retail investors which tend to buy and sell stocks in concert producing a systematic component that Kumar and Lee (2006) call *retail investor sentiment* with incremental predictive power for returns.

Lemmon and Portniaguina (2006) and Baker and Wurgler (2007) argue that indirect investor sentiment measures such as consumer confidence may also reflect expectations about future economic outlook. Therefore, in these studies it is proposed to regress these proxies on a set of macroeconomic variables such as output growth, inflation, unemployment, and consider the residuals as an investor sentiment unrelated to economic fundamentals.

1.2.2 Business-cycle variables

Very early studies relate stock return predictability with business-cycles indicators. In Clay (1925) one can find the following statement: “So it is that the stock market is a creature of every day economic forces.” Pesaran and Timmermann (1995) write that future stock returns not only vary with the business cycles, but they also depend on the magnitude of the real shocks. Variables having an important business-cycle component, and frequently used predictors of stock and bond returns, are real output growth or industrial production growth, inflation, short-term interest rates and term spread.

Real growth. The empirical evidence has found that the correlation between financial returns and different measures of real economic activity is positive, suggesting that stock returns are pro-cyclical (Fama 1981; Chen, Roll and Ross 1986; Fama and French 1989; Boyd, Levine and Smith 1997).

Inflation. Many empirical studies suggest that stock returns react negatively to inflation (Chen, Roll and Ross 1986; Amihud 1996). This finding appears to go against the classical view that higher future stock returns should compensate for higher inflation. Erb, Harvey and Viskanta (1995) confirm that even in the long horizon, stock returns do not appear to hedge against inflation. Indeed, higher inflation rates are a feature of economies with less developed financial markets. From a theoretical standpoint, Boyd, Levine and Smith (1997) write that high levels of inflation aggravate the inefficiency of financial systems through more intense market frictions, leading to lower returns.

Short-term interest rate. Ang and Bekaert (2007) and many other studies find that short rates are a robust predictor of stock returns. Variations in the short interest rates may reflect variability in the real rate of return (Pesaran and Timmermann 1994) or monetary policy decisions (Rosa 2011), both associated positively to future returns.

Term spread. There is extensive empirical evidence that the term spread between long- and short-term government bond yields is a leading indicator of business cycles, and as such it should have a significant power in predicting stock returns. Fama and French (1989, 1993), Fama (1990) and Amihud (2002) find a positive impact of the term spread on expected returns. In contrast with the previous results, the estimated impact of term spread in Chen, Roll and Ross (1986) is negative.

1.2.3 Factors related to the financial market

The variables described below are directly linked to the financial index that we use in this study, that is why they are classified in this group. The empirical literature shows strong evidence of the predictive power of these factors.

Volatility. Future stock returns are in general positively related to measures of volatility. Bollerslev, Tauchen and Zhou (2009) explain the association of higher variation in returns with higher future returns by the fact that high volatility includes a discount premium into prices. Higher levels of variance can be also related to increasing risk aversion (Bollerslev, Marrone, Xu and Zhou 2012). An accurate measure of stock market return variation is the realized volatility, as asserted by a large number of studies (see French, Schwert and Stambaugh 1987; Amihud 2002; Adrian and Rosenberg 2008).

Dividend yield. The dividend yield is undoubtedly the most frequently used predictor with vast empirical evidence of its predictive power on asset returns. There is nevertheless no unanimous conclusion about the sign of this relation (Ang and Bekaert 2007). Black and Scholes (1974) write that dividend-paying stocks may increase in value because of investors' preference to receive dividends rather than capital gains, and thus accept lower returns. This negative dividend-return linkage could hold if dividend yield is a characteristic of less risky assets, or dividend-paying assets are perceived as being more liquid (Amihud 2002). On the other hand, Brennan, Chordia and Subrahmanyam (1998) and Amihud (2002) suggest that the heavier taxation on dividends in comparison with capital gains may lead investors to require higher returns for dividend-paying stocks.

Liquidity. Seminal empirical studies (Amihud and Mendelson 1986; Amihud 2002; Pástor and Stambaugh 2003; Acharya and Pedersen 2005; Bekaert, Harvey and Lundblad 2007) have found that expected equity returns contain a premium for illiquidity: more illiquid stocks should offer larger returns. Chordia, Roll, Subrahmanyam (2001) and Li, Wang, Wu and He (2009) argue that higher trading volume is a feature of active markets and consequently an indicator of liquid markets. Thus, this variable could be used as an appropriate liquidity measure.[7]

Market size. Fama and French (1993) and Pesaran and Timmermann (2000) argue that market size reflects economic fundamentals and profitability. In our international perspective the average market capitalization per firm should capture the time variation in the average size of the financial firm within a country.

1.3 Predictive regressions

In this section we specify the Panel Threshold Regression (PTR) model of Hansen (1999) and the Panel Smooth Transition Regression (PSTR) model of González, Teräsvirta and van Dijk (2005), and show that the PTR is a special case of the PSTR when the smoothness parameter tends to infinity. We briefly describe the estimation method, some testing procedures to check the models' goodness of fit and the transition variable selection.

In our main analysis, we focus on the prediction of annual returns of financial companies by a set of variables lagged by one month. In the robustness analysis in Section

[7] We could not use the Amihud (2002) proxy for market-wide illiquidity because in our panel framework endogeneity issues could arise. Indeed, Fasnacht (2008) shows that not only the Amihud illiquidity measure explains stock market returns, but the inverse relation is also true. Concerning the Pástor and Stambaugh (2003) liquidity measure, in theory the coefficient associated with the signed volume can be interpreted as a liquidity cost and should be negative. In practice, there is no guarantee that its estimated value is negative and significantly different from zero. Our estimation results support this view.

1.5.2, we consider shorter-term returns of 1, 3, 6 and 9-month horizon explained by the first lag of the explanatory variables. Second, we explore the ability of further lags of the explanatory variables until 32 months in predicting the annual returns of financial companies. To keep the notation simple, we write the models only for the annual returns and for the 1-month lag. The same models apply to the other analyses.

1.3.1 PTR and PSTR specifications

The PTR model for the prediction of the annual stock returns of financial companies $r_{i,t}$ of a panel of developed countries can be written as follows:

$$r_{i,t} = \mu_i + \boldsymbol{\lambda}_1' \boldsymbol{x}_{i,t-1} \mathbb{1}_{(q_{i,t-1} \leq c)} + \boldsymbol{\lambda}_2' \boldsymbol{x}_{i,t-1} \mathbb{1}_{(q_{i,t-1} > c)} + \varepsilon_{i,t} \qquad i = 1, \ldots, N, \quad t = 1, \ldots, T \quad (1.1)$$

where $\mathbb{1}_{(\cdot)}$ is an indicator function, $\boldsymbol{x}_{i,t-1}$ is the 1-month lag of a set of k predictors described in Section 1.2, μ_i is an individual fixed effect and $\varepsilon_{i,t}$ is an error term which is assumed to be *i.i.d.* $\boldsymbol{\lambda}_1$ and $\boldsymbol{\lambda}_2$ are two sets of k coefficients indicating the marginal effect of $\boldsymbol{x}_{i,t-1}$ on expected financial returns during the first and the second regimes, respectively. The non-constancy of the marginal effect is the outstanding feature of the threshold models. The transition between regimes is driven by an observable variable $q_{i,t-1}$ with the parameter c designating their threshold. We follow the notation of Hansen (1999) and rewrite the two-regime[8] threshold model in Equation 1.1 in a compact representation.

$$r_{i,t} = \mu_i + \boldsymbol{\lambda}' \boldsymbol{x}_{i,t-1}(c) + \varepsilon_{i,t} \quad (1.2)$$

with $\boldsymbol{\lambda} = (\boldsymbol{\lambda}_1' \;\; \boldsymbol{\lambda}_2')'$ and $\boldsymbol{x}_{i,t-1}(c) = \left(\boldsymbol{x}_{i,t-1}' \mathbb{1}_{(q_{i,t-1} \leq c)} \;\; \boldsymbol{x}_{i,t-1}' \mathbb{1}_{(q_{i,t-1} > c)} \right)'$.

Hansen's (1999) specification constrains the transition between regimes to be abrupt, splitting the observations in two according to the value of $q_{i,t-1}$ with respect to c. The more flexible nonlinear specification of González, Teräsvirta and van Dijk (2005) allows for a smooth turning point, the parameter of smoothness being jointly estimated with the other coefficients of the model. The model can be written as follows:

$$r_{i,t} = \mu_i + \boldsymbol{\beta}_0' \boldsymbol{x}_{i,t-1} + \boldsymbol{\beta}_1' \boldsymbol{x}_{i,t-1} G(q_{i,t-1}; \gamma, c) + \varepsilon_{i,t}. \quad (1.3)$$

For the purposes of the estimation procedure described below, we write Equation 1.3 in a more compact form.

$$r_{i,t} = \mu_i + \boldsymbol{\beta}' \boldsymbol{x}_{i,t-1}(\gamma, c) + \varepsilon_{i,t} \quad (1.4)$$

with $\boldsymbol{\beta} = (\boldsymbol{\beta}_0' \;\; \boldsymbol{\beta}_1')'$ and $\boldsymbol{x}_{i,t-1}(\gamma, c) = \left(\boldsymbol{x}_{i,t-1}' \;\; \boldsymbol{x}_{i,t-1}' G(q_{i,t-1}; \gamma, c) \right)'$.

$G(q_{i,t-1}; \gamma, c)$ is the transition function, continuous and bounded between 0 and 1, given by Equation 1.5. We follow the literature[9] and choose as a transition function the

[8] The PTR model can be generalized in a straightforward manner to involve more than two regimes. For instance the three-regime model can be written as follows:

$$r_{i,t} = \mu_i + \boldsymbol{\lambda}_1' \boldsymbol{x}_{i,t-1} \mathbb{1}_{(q_{i,t-1} \leq c_1)} + \boldsymbol{\lambda}_2' \boldsymbol{x}_{i,t-1} \mathbb{1}_{(c_1 < q_{i,t-1} \leq c_2)} + \boldsymbol{\lambda}_1' \boldsymbol{x}_{i,t-1} \mathbb{1}_{(q_{i,t-1} > c_2)} + \varepsilon_{i,t}.$$

[9] See van Dijk, Teräsvirta and Franses (2002) and González, Teräsvirta and van Dijk (2005). In the first paper it is argued that regime-switching models using the logistic distribution are suitable to examine the business-cycle asymmetry relating the distinct regimes to the upward and downward moves.

logistic distribution (see Figure A.III in the Appendix).

$$G\left(q_{i,t-1};\gamma,c\right)=\left(1+\exp\left\{-\gamma\left(q_{i,t-1}-c\right)\right\}\right)^{-1},\quad \gamma>0. \tag{1.5}$$

The degree of smoothness of the transition from one regime to the other is determined by the scale parameter γ. Upon the value of this coefficient two limiting models can be distinguished. First, when γ tends to 0, the logistic function becomes constant (equal to 0.5) leading to the standard linear panel model with fixed effects. Given the positiveness constraint on γ, the second limiting case is when it converges to infinity. The transition function tends to an indicator function for $q_{i,t-1}$ greater than c and 0 otherwise. Consequently, the PSTR model converges to the PTR model.

As for the PTR, the principal advantage of the PSTR modeling upon the standard fixed-effects linear panel specification is that the marginal effect $(e_{i,t})$ of $\boldsymbol{x}_{i,t-1}$ on the dependent variable is time-varying and heterogeneous across individuals. It depends on the value of the transition variable $q_{i,t-1}$ pointing out its crucial role, and is given by the following formula:

$$e_{i,t}=\frac{\partial r_{i,t}}{\partial \boldsymbol{x}_{i,t-1}}=\boldsymbol{\beta}_0+\boldsymbol{\beta}_1 G\left(q_{i,t-1};\gamma,c\right). \tag{1.6}$$

With an increasing $q_{i,t-1}$, the marginal effect $e_{i,t}$ is increasing if $\boldsymbol{\beta}_1>0$ and decreasing if $\boldsymbol{\beta}_1<0$. Given that $G\left(q_{i,t-1};\gamma,c\right)$ is bounded between 0 and 1, the extreme regimes are associated with the values $\boldsymbol{\beta}_0$ and $\boldsymbol{\beta}_0+\boldsymbol{\beta}_1$.

In a generalized setting, the PSTR approach can account for more than two regimes. In Equation 1.7 there are L transition functions and therefore $L+1$ distinct regimes.

$$r_{i,t}=\mu_i+\boldsymbol{\beta}_0'\boldsymbol{x}_{i,t-1}+\sum_{l=1}^{L}\boldsymbol{\beta}_l'\boldsymbol{x}_{i,t-1}G_l\left(q_{i,t-1}^{(l)};\gamma_l,\boldsymbol{c}_l\right)+\varepsilon_{i,t} \tag{1.7}$$

The transition variable $q_{i,t-1}^{(l)}$ can be different, or the same among the L transition functions. For $q_{i,t-1}^{(l)}=q_{i,t-1}$ and $\gamma_l\to\infty$ with $l=1,\ldots,L$ the model in 1.7 converges to the PTR specification with $L+1$ distinct regimes.

1.3.2 Estimation

We estimate the parameters of the PTR and the PSTR specifications by nonlinear least squares method.[10] As in the standard panel linear model, Hansen (1999) and González, Teräsvirta and van Dijk (2005) suggest to demean the nonlinear models in order to eliminate the individual fixed effects. The specificity in this case is that the demeaning procedure takes the nonlinearity into account by subtracting away each regressor's regime-specific mean. Applying this transformation to Equations 1.2 (PTR) and 1.4 (PSTR) yields the following model:

$$r_{i,t}^*=\boldsymbol{\Psi}'\boldsymbol{x}_{i,t-1}^*(\boldsymbol{\theta})+\varepsilon_{i,t}^* \tag{1.8}$$

where $\boldsymbol{\theta}=c$ and $\boldsymbol{\Psi}=\boldsymbol{\lambda}$ in the PTR, or $\boldsymbol{\theta}=(\gamma\ \ c)'$ and $\boldsymbol{\Psi}=\boldsymbol{\beta}$ in the PSTR. $r_{i,t}^*=r_{i,t}-\bar{r}_{i,t}$ with $\bar{r}_{i,t}=\frac{1}{T}\sum_{t=1}^{T}r_{i,t}$, $\varepsilon_{i,t}^*=\varepsilon_{i,t}-\bar{\varepsilon}_{i,t}$ with $\bar{\varepsilon}_{i,t}=\frac{1}{T}\sum_{t=1}^{T}\varepsilon_{i,t}$, $\boldsymbol{x}_{i,t-1}^*(\boldsymbol{\theta})=\boldsymbol{x}_{i,t-1}(\boldsymbol{\theta})-\bar{\boldsymbol{x}}_{i,t-1}(\boldsymbol{\theta})$

[10]The coefficients of these models are estimated in Matlab using algorithms provided by C. Hurlin.

with $\boldsymbol{x}_{i,t-1}(\boldsymbol{\theta})$ defined above for both models and $\bar{\boldsymbol{x}}_{i,t-1}(\boldsymbol{\theta})$ given by Equations 1.9 for the PTR model and 1.10 for the PSTR model.

$$\bar{\boldsymbol{x}}_{i,t-1}(\boldsymbol{\theta}) = \begin{pmatrix} \frac{1}{T}\sum_{t=1}^{T} \boldsymbol{x}_{i,t-1}\mathbb{1}_{(q_{i,t-1}\leq c)} \\ \frac{1}{T}\sum_{t=1}^{T} \boldsymbol{x}_{i,t-1}\mathbb{1}_{(q_{i,t-1}>c)} \end{pmatrix} \tag{1.9}$$

$$\bar{\boldsymbol{x}}_{i,t-1}(\boldsymbol{\theta}) = \begin{pmatrix} \frac{1}{T}\sum_{t=1}^{T} \boldsymbol{x}_{i,t-1} \\ \frac{1}{T}\sum_{t=1}^{T} \boldsymbol{x}_{i,t-1}G(q_{i,t-1};\boldsymbol{\theta}) \end{pmatrix}. \tag{1.10}$$

Under the assumptions of exogeneity of the explanatory variables and of an *i.i.d.* error term, the estimation procedure is performed combining Equations 1.11 and 1.12. Conditioned upon the value of the vector $\boldsymbol{\theta}$, both PTR and PSTR specifications are linear on the slope parameters $\boldsymbol{\Psi}$ which can be estimated by an ordinary least squares method. The location parameter – and the smoothness parameter for the PSTR – are numerically estimated in a second step by the minimization of the residuals sum of squares.

$$\hat{\boldsymbol{\Psi}}(\boldsymbol{\theta}) = \left\{\sum_{i=1}^{N}\sum_{t=1}^{T} \boldsymbol{x}^{*}_{i,t-1}(\boldsymbol{\theta})\boldsymbol{x}^{*}_{i,t-1}(\boldsymbol{\theta})'\right\}^{-1} \left\{\sum_{i=1}^{N}\sum_{t=1}^{T} \boldsymbol{x}^{*}_{i,t-1}(\boldsymbol{\theta})r^{*}_{i,t}\right\} \tag{1.11}$$

$$\min_{\boldsymbol{\theta}}\ S(\boldsymbol{\theta}) = \sum_{i=1}^{N}\sum_{t=1}^{T}\left\{r^{*}_{i,t} - \hat{\boldsymbol{\Psi}}(\boldsymbol{\theta})'\boldsymbol{x}^{*}_{i,t-1}(\boldsymbol{\theta})\right\}^{2}. \tag{1.12}$$

The distribution of $\hat{\boldsymbol{\Psi}}(\hat{\boldsymbol{\theta}})$ depends on the estimates $\hat{\boldsymbol{\theta}}$. For the PTR model, Hansen (1999) argues that the asymptotic distribution of the slope parameters is normal since it is shown (see Chan 1993 and Hansen 2000) that the estimates of the threshold function which follow a nonstandard distribution, are not of *first-order asymptotic importance*. Under standard assumptions, the asymptotic distribution of all the parameters in the PSTR model is normal.

1.3.3 Hypothesis testing

The adequacy of the predictive regressions for financial stock returns with regime-switching patterns should be tested against the null hypothesis of a linear model. Nonetheless, the testing procedure in both models proposed above faces a nuisance parameter issue under the null hypothesis, rendering the distribution of the tests nonstandard (see Davies 1977, 1987; Hansen 1996).

For the PTR model in Equation 1.1 the null hypothesis of linearity is formulated as $H_0 : \boldsymbol{\lambda}_1 = \boldsymbol{\lambda}_2$ against the alternative that they are different. Under the null hypothesis the value of the threshold parameter c cannot be defined. The identification issue is overcome by means of a bootstrap procedure (see Hansen 1999) to simulate the asymptotic distribution of the likelihood ratio test given by the following formula:

$$F_1 = \left(S_0 - S_1\left(\hat{c}\right)\right)/\hat{\sigma}^2 \tag{1.13}$$

S_0 and $S_1(\hat{c})$ are the residuals sum of squares for the linear and the single threshold models, respectively. The residual variance $\hat{\sigma}^2$ is estimated under the alternative hypothesis. The null hypothesis is rejected if the actual value of the test is larger than the critical value at the required percentile.

In the PSTR model given by Equations 1.3 and 1.5, the linearity hypothesis against a two-regime smooth transition model can be set in two different manners: $H_0 : \boldsymbol{\beta}_1 = \mathbf{0}$, or $H_0' : \gamma = 0$. Under H_0 the parameters of the transition function γ and c are not identified, whereas under H_0' the value of the location parameter c and the vector $\boldsymbol{\beta}_1$ can take any value. In this case, the identification problem is sidestepped through a first-order Taylor series expansion of the logistic distribution around $\gamma = 0$, as initially proposed by Luukkonnen, Saikkonen and Teräsvira (1988). A chi-squared test and its F version are used for the testing.

For both regime-switching specifications, if the null hypothesis of linearity is rejected in favor of a threshold (or smooth transition) model, in a second step one should test for no remaining nonlinearity. In other words, one should test the null hypothesis of a two-regime model (the number of transition functions L is equal to 1) against the alternative of a model with three regimes ($L = 2$). The testing procedure should be applied until the null hypothesis is not rejected for the first time.

1.3.4 Transition variable selection

The selection of the transition variable is of crucial importance in our regime-switching modeling since it should exhibit variations which reflect the business cycles. We use as an early indicator of risk related to business cycles the spread between 3-month and overnight interbank rates and take the 1-month lag to remain in a predictive regression setting. For the euro area countries, the spread is identical since it is computed as the difference between the 3-month euribor and the eonia. A common transition variable for this group of countries should be suitable given that they are strongly interconnected through trade, financial system and unique monetary policy.

Figure A.I shows, for almost all the countries, a surge of this spread from the onset of the financial crisis by July-August 2007, with the highest level around the peak of the crisis by October 2008. The 3-month-overnight interbank spread also tracks the dot-com bubble with significant increases previous to its peak by March 2000. This evidence indicates that an increase of this variable is an early warning that market conditions will deteriorate. The empirical literature (e.g. Kuttner 2001) suggests that short-term rates respond more strongly to monetary policy actions than long-term rates. Moreover, tightening conditions in the interbank lending market and market liquidity constraints should apply pressure in favor of an increasing spread. Finally, this spread should also reflect increasing risk in the banking system and increasing risk aversion.[11]

1.4 Data

The data set involves 20 developed countries[12] and covers the period from January 1999 to August 2011. The monthly frequency of the data is constrained by the availability of some macroeconomic variables.

We construct annual returns from Datastream Financials Index (DFI) in local currency available in Datastream. For the robustness analysis we also use 1, 3, 6 and 9-month

[11] As a simple robustness test, we regressed this variable as a function of investor sentiment and four macro indicators: real GDP growth, CPI inflation, unemployment rate and term spread. In distressed periods this spread widens, while real growth and sentiment decrease. This expected negative impact from these two variables comes out from the data, while the three other variables do not have a significant impact.

[12] Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, the United Kingdom and the United States. The World Bank Group classifies Greece and Portugal as emerging countries.

returns. The DFI is a large index that involves mainly banks and insurance companies, and appropriately represents the financial stock markets of the countries considered in this study.[13] To compute the DFI return we consider the last price for each month and compute its percentage variation with respect to the last value of the same month during the previous year. This procedure creates an overlapping observation issue which can bias the covariance matrix of the estimated coefficients. We account for this using the Newey and West (1987) correction for heteroskedasticity and autocorrelation.

Similarly, for the other variables related with the DFI, such as the dividend yield or the market capitalization, the last value of each month is reported. The realized volatility is measured by the standard deviation computed from the daily data of the DFI annual returns within a month. To control for a different number of firms within the index of each country we divide the DFI market capitalization by the constituent number of firms. Following the line of Amihud and Mendelson (1986) and Gibson and Mougeot (2004), we define liquidity as the number of shares traded monthly standardized by the number of index constituent shares to account for size and changes in the composition of the index. An alternative measure is the ratio of the monthly volume in *value* over the index market capitalization to adjust for its size. These variables measure liquidity (rather than illiquidity) and accordingly they should negatively influence future financial stock returns.

The consumer price inflation for Australia and New Zealand, and the realized real GDP growth for all countries are available solely in a quarterly frequency. In order to have a monthly frequency we report the same value three times within a quarter. We alternatively employ the average real GDP growth forecasts with a 12-month horizon from Consensus Economics and industrial production growth which are available monthly. Table 1.1 displays summary statistics for the dependent and the explanatory variables. A description of the variables used in this study is given in Table A.I in the Appendix.

We proxy investor sentiment by a Consumer Confidence Indicator (CCI) stemming from different data sources. In order for these indices to be comparable from a scale viewpoint, we subtract from each series the individual sample mean and divide the result by the individual standard deviation. To examine the robustness of this variable, we use an alternative proxy for investor sentiment based on an Economic Sentiment Indicator (ESI).[14] For comparability reasons we apply the same transformation as for the CCI.

We follow the approach of Lemmon and Portniaguina (2006), Baker and Wurgler (2006, 2007) and Baker, Wurgler and Yuan (2012) and retrieve from the Consumer Confidence Indicator (CCI) the component related to economic fundamentals. To this aim, we regress the CCI on the 1-month lag of real GDP growth (seasonally adjusted), consumer price inflation, unemployment rate and the change in the 3-month interbank rate.[15] The

[13]To assess its adequacy, we compared the DFI annual returns to the Dow Jones Financials Index (DJFI) annual returns. The within country correlations between the two series span from 75.55% to 99.93% with an overall value of 97.80%. The graphical display shows that these two series almost overlap. Nevertheless, we could not use the Dow Jones Financials Index because variables related to this index, such as dividend yield, market capitalization and trading volume are not available.

[14]For the majority of the countries we have recourse to the Economic Sentiment Indicator (ESI) from the European Commission, a composite index of five sectoral business tendency indicators: industry, services, construction, retail trade and consumers, all constructed from surveys. Thus, this broad index includes the sentiment of consumers. For Switzerland we employ the KOF Economic Barometer which forecasts how the Swiss economy will perform in the next quarter or in the next two quarters. For the remaining countries (Australia, Canada, Japan, Ireland, New Zealand, Norway and U.S.) we employ a Composite Leading Indicator which is a six to nine months forward-looking measure of the economic activity.

[15]Within a panel framework, Baker, Wurgler and Yuan (2012) orthogonalize each investor sentiment

residuals from this model are assumed to reflect investor optimism or pessimism about the market dissociated to economic conditions. Given the stressful economic conditions since the inception of the global financial crisis, we consider that there has been a regime change in the consumer confidence indicator which should be appropriately modeled by a regime-switching approach. As for the predictability regression of financial stock returns, we employ as transition variable the 1-month lag of the spread between 3-month and overnight interbank rates. The estimation results are shown in Table A.II in the Appendix.[16]

Figure A.II in the Appendix (see also Table A.III) displays annual financial stock returns which have become negative since September 2007. For most of the countries the series were extremely low at the market bottom in February 2009 (from −75.3% for Belgium to −31.1% for New Zeland). Subsequently, financial market returns are generally positive but end up being quite low or negative by the end of the period under study. For the euro area countries, these negative returns should be related to the sovereign debt crisis. Additionally, financial markets experienced a severe drawdown (from −63.8% for Germany to −17.9% for Australia) contemporaneous with the dot-com burst that occurred over the period 2000-2003.

Before proceeding to the estimation, we test the stationarity of the explanatory and the dependent variables having course to five panel unit root tests (augmented with one lag of the dependent variable). The results are shown in Table A.IV in the Appendix. Overall, the tests strongly reject the null hypothesis of a unit root. The term spread and the average market capitalization per firm appear to be stationary when two lags and four lags are included, respectively. As regards the 3-month interbank rate, it should theoretically be stationary within a stable economic environment indicating for instance the ability of central banks to stabilize inflation. Nevertheless, to avoid any non-stationarity issue we consider the first difference of this variable which is found to be stationary.

component by a similar set of macroeconomic variables. More precisely they use (i) consumption growth, (ii) industrial production growth, (iii) inflation, (iv) employment growth, (v) short-term rate and (vi) term premium.

[16]To compare the estimation results we fit to the data, both the PSTR and the PTR specifications. The estimated coefficients are very similar through both specifications. For this reason, and in order to obtain estimation results for the predictive model of stock returns that are comparable through the smooth and the threshold regression specifications, we use only the residuals stemming from the PSTR on CCI as a proxy for investor sentiment. For the robustness tests we use the corresponding residuals from ESI.

Table 1.1

Summary statistics of the dependent and of the explanatory variables from Jan. 1999 to Aug. 2011

	Jan. 1999 - Aug. 2007					Sep. 2007 - Aug. 2011				
	Obs.	Mean	Std.	Min	Max	Obs.	Mean	Std.	Min	Max
<u>DEPENDENT VARIABLE</u>										
DFI annual return (%)	2,080	10.054	25.233	-63.839	268.126	960	-10.492	32.894	-75.302	169.226
DFI 9-month return (%)	2,080	7.415	19.878	-57.356	130.260	960	-7.954	30.811	-75.657	146.486
DFI 6-month return (%)	2,080	5.064	15.317	-51.496	100.807	960	-5.604	26.398	-68.159	112.601
DFI 3-month return (%)	2,080	2.367	10.123	-42.368	67.633	960	-3.449	17.088	-59.373	82.126
DFI 1-month return (%)	2,080	0.703	5.522	-27.564	31.083	960	-1.424	8.977	-38.935	32.188
<u>EXPLANATORY VARIABLES</u>										
Investor sentiment (CCI)	2,040	0.0244	0.669	-2.163	2.3903	960	-0.0518	0.788	-2.663	2.221
Investor sentiment (ESI)	2,040	-0.0256	0.633	-2.544	2.138	960	0.0544	0.806	-2.681	2.924
Real GDP growth rate (SADJ, %)	700	2.651	1.555	-1.869	6.564	320	0.220	3.165	-9.905	7.625
Real GDP forecasts (%)	1,872	2.319	0.865	-1.211	4.619	864	0.962	1.659	-4.675	3.95
Inds. prod. growth (%)	1,784	1.930	3.881	-19.59	22.93	912	-1.439	8.767	-34.70	29.11
CPI inflation (%)	2,080	1.977	1.138	-1.834	6.078	960	2.054	1.568	-2.524	5.912
3-month interbank rate (%)	2,080	3.443	1.639	0.046	8.65	960	2.443	2.036	-0.24	9.45
Term spread (%)	2,080	1.058	0.940	-2.711	3.957	960	1.527	2.129	-2.721	15.097
Volatility (%)	2,080	3.345	2.507	0.459	40.108	960	4.544	3.988	0.538	39.439
Dividend yield (%)	2,080	3.205	2.242	0.52	24.65	960	4.137	2.812	0	22.68
Liquidity (VO/NOSH)	2,080	0.0830	0.0586	0.0016	0.5641	960	0.1034	0.0955	0.00113	0.975
Liquidity (VA/MV)	2,080	0.0599	0.0368	0.0010	0.3414	960	0.0882	0.0692	0.000807	0.5647
Market capitalization (MV)	2,080	308,338.9	576,986.1	86.826	3,701,226	960	311,385.4	496,664.6	1,966.039	3,381,000
Number of firms	2,080	41.384	48.844	3	234	960	51.180	60.182	4	251
<u>TRANSITION VARIABLE</u>										
Spread: 3-month-overnight interb.	2,080	-0.00562	0.469	-2.849	2.96	960	0.453	0.374	-0.567	2.806

Data sources: Datastream, KOF, Bank of Denmark.

Notes: The data set contains a panel of 20 developed countries (see Table A.III in the Appendix) for the period from January 1999 to August 2011. In order to put in evidence the impact of the recent global financial crisis the data set is split in two subsamples, the threshold between the two being the outset of the downturn, August 2007. The financial variables are observed at the end of the month.

The market capitalization (MV) is expressed in millions of U.S. dollars. The real GDP growth, seasonally adjusted (SADJ), is available only quarterly, whereas the monthly forecasts of this variable are not available for Australia and New Zealand. The investor sentiment is obtained from the residuals of a model explaining Consumer Confidence Indicator (CCI) or Economic Sentiment Indicator (ESI) as a function of a set of macroeconomic variables (see Table A.II in the Appendix). The monthly volatility of financial returns is computed from daily observations of the annual return within a month. The liquidity measures defined as the ratios of the turnover by volume on the total number of shares in the index (VO/NOSH) and the turnover by value on the market size (VA/MV) are multiplied by 1000. The trading volume in value and in number of shares is the monthly sum of the daily observations. The term spread is the difference between the 10-year and the 1-year benchmark government bond yields. See also Table A.I in the Appendix for a description of the variables.

1.5 Empirical evidence

1.5.1 Predictability of financial stock returns

A first step in the specification of regime-switching models is testing the null hypothesis of a linear model against the alternative of a two-regime model. We perform two tests (Wald test and F test) for the PSTR specification and a bootstrap-based test for the PTR model (F_1 test). The results are shown in Table 1.2, Panel A. The null hypothesis is rejected for the two specifications at any significance level. A further step is to test the null hypothesis of a two-regime model against the alternative of a three-regime model. The PSTR model would suggest the presence of more than two regimes, whereas the PTR cannot reject the null of two regimes at any significance level.[17] Moreover, the two regimes have an economic interpretation since they are related with normal (Regime 1, low value of the transition variable) and crisis (Regime 2, high value of the transition variable) times. For these reasons, we proceed with a two-regime model.

Figure 1.1 presents the logistic distribution, over time and for each country, evaluated at the transition variable and at the estimated values of γ and c.[18] It points out that the financial stock returns switched to the crisis regime at the outset of the subprime crisis in July-August 2007. For several countries such as Canada and Switzerland the impact lasts solely a few months. Following two spikes by the onset of the subprime crisis, the U.S. financial stock market performance jumps from the normal to a long-lasting crisis regime from September 2008, after Lehman Brothers' bankruptcy. Subsequent to a short period of resurgence, the euro area financial stock markets seem to be absorbed by a crisis state which should reflect the European sovereign debt issue. The frequent oscillations for New Zealand and the crisis regime for Norway and Sweden by the end of the sample are less easy to be explained. Although of a different nature, the dot-com crash which started by the end of the 90s, has played an important role in shaping financial stock markets' expected returns. Bar Norway, all the countries have experienced a crisis regime during this period with a varying duration.

Figure 1.2 presents for each country, the time-series of the investor sentiment proxy, and the crisis periods estimated by the model and defined as $G(q_{i,t-1}; \hat{\gamma}, \hat{c}) > 0.5$. The aim is to analyze whether or not sentiment decreases during distressed periods. Sentiment has been low or decreasing since the onset of the financial crisis for all the countries, except for Italy where it starts decreasing by the end of 2009. Nevertheless, for several countries the recent crisis is not the only period to be associated with low levels of sentiment.

The estimation results for the linear, PSTR and PTR specifications are shown in Table 1.2, Panel B. Generally, both nonlinear models suggest very close values for the estimated parameters and the same direction for the marginal effect: whether it is significantly increasing or decreasing in the switch from one regime to the other. We also indicate whether the difference of impact between regimes is significant. Note that the impact of some variables is not constant over time, endorsing the choice of a nonlinear model. On the other hand, the estimation results from the linear model yield marginal effects from the predictors which are, in most cases, roughly an average of the coefficients across regimes.

[17] Even though the p-value of this test in the PSTR model is never too high, in the majority of the specifications that we tried it is above 5%. On the other hand, the PTR model never rejects the null hypothesis of two regimes.

[18] We reduce and standardize the transition variable at the country level to obtain a threshold level c which is homogenous across the countries.

Table 1.2

Linear, PSTR and PTR models for the predictability of financial stock returns

PANEL A	Testing the null hypothesis of linearity, and subsequently the null hypothesis of a two-regime specification		
	PSTR: LM test	PSTR: F test	PTR: F_1 test
$H_0 : L = 0$ vs $H_1 : L = 1$	204.399 (0.000)	24.146 (0.000)	310.206 (0.000)
$H_0 : L = 1$ vs $H_1 : L = 2$	22.046 (0.009)	2.429 (0.010)	48.549 (1.000)

PANEL B	Linear model	PSTR			PTR		
		Regime 1	Regime 2	Difference	Regime 1	Regime 2	Difference
	$\boldsymbol{\beta}$	$\boldsymbol{\beta}_0$	$\boldsymbol{\beta}_0 + \boldsymbol{\beta}_1$	$\boldsymbol{\beta}_1$	$\boldsymbol{\lambda}_1$	$\boldsymbol{\lambda}_2$	$\boldsymbol{\lambda}_2 - \boldsymbol{\lambda}_1$
Investor sentiment (CCI)	1.481	-1.174	5.921***	7.095***	-1.084	4.725***	5.809***
	(1.04)	(-0.685)	(6.420)	(3.094)	(-0.667)	(8.155)	(4.002)
Real GDP growth	3.197***	2.477***	2.165***	-0.312	2.439***	2.528***	0.089
	(10.29)	(4.495)	(4.089)	(-0.340)	(4.568)	(7.308)	(0.127)
CPI inflation	-5.730***	-3.660***	-7.091***	-3.430**	-4.059***	-6.475***	-2.416**
	(-4.91)	(-2.671)	(-6.741)	(-1.994)	(-3.133)	(-7.998)	(-1.993)
FDIFF.3-month interbank	9.925***	20.947***	5.041	-15.906**	19.069***	7.704***	-11.365*
	(4.13)	(4.282)	(1.521)	(-2.340)	(3.705)	(3.092)	(-1.862)
Term spread	0.118	-0.445	0.427	0.872	-0.437	0.244	0.681
	(0.19)	(-0.584)	(0.468)	(0.613)	(-0.588)	(0.349)	(0.597)
Volatility	2.885***	3.307***	2.261***	-1.046	3.128***	2.843***	-0.285
	(19.16)	(17.356)	(3.196)	(-1.260)	(15.330)	(4.881)	(-0.405)
Dividend yield	-2.817***	-2.403***	-2.101***	0.302	-2.393***	-2.438***	-0.046
	(-3.53)	(-3.006)	(-4.210)	(0.294)	(-3.041)	(-5.282)	(-0.055)
Liquidity (VO/NOSH)	-59.70***	-46.232**	-35.663***	10.569	-48.938***	-38.948***	9.991
	(-3.85)	(-2.514)	(-2.871)	(0.559)	(-2.793)	(-3.355)	(0.709)
Ln(Market value/Firms)	16.35***	17.010***	15.341***	-1.670**	17.063***	15.784***	-1.279***
	(5.53)	(5.173)	(4.481)	(-2.380)	(5.236)	(4.720)	(-2.675)
γ			3.700 [2.603, 5.742]				
c			1.000 [0.851, 1.169]			0.744 [0.659, 0.751]	
Observations	2,980		2,980			2,980	
Number of id	20		20			20	
R-squared	0.468		0.524			0.539	
AIC	6.069		5.968			5.982	
BIC	6.087		6.009			6.020	

Notes. PANEL A. We report the values of the test statistics testing the null hypothesis of linearity ($L = 0$) against a two-regime PSTR (PTR) model ($L = 1$) with L the number of transition functions. If the linearity hypothesis is rejected, one should test the null hypothesis of no remaining nonlinearity ($H_0 : L = 1$) against the alternative of three regimes ($L = 2$). For the PSTR specification we compute two tests. The LM-type test is given by $LM = NT\frac{RSS_0 - RSS_1}{RSS_0}$ and follows a χ^2_K with $K = 9$. The F-test statistic is given by $F = \frac{(RSS_0 - RSS_1)/K}{RSS_1/(NT-N-(L+1)K)}$ and follows $(F_{K,NT-N-(L+1)K})$. When we test for linearity, RSS_0 and RSS_1 represent respectively the residual sum of squares of the linear panel model with fixed effects and the residual sum of squares of a first-order Taylor series approximation of a two-regime PSTR model. When we test for no remaining nonlinearity RSS_0 and RSS_1 are respectively the residual sum of squares for a two-regime PSTR and the auxiliary regression of a first-order expansion of a three-regime PSTR model. The F_1 statistic for the PTR is obtained by a bootstrap procedure to overcome the nuisance parameter issue (see Hansen 1999). The p-values are reported in parentheses.

PANEL B. The estimated models are the following:

$$\text{Linear model:} \quad r_{i,t} = \mu_i + \boldsymbol{\beta}'\boldsymbol{x}_{i,t-1} + \varepsilon_{i,t}$$

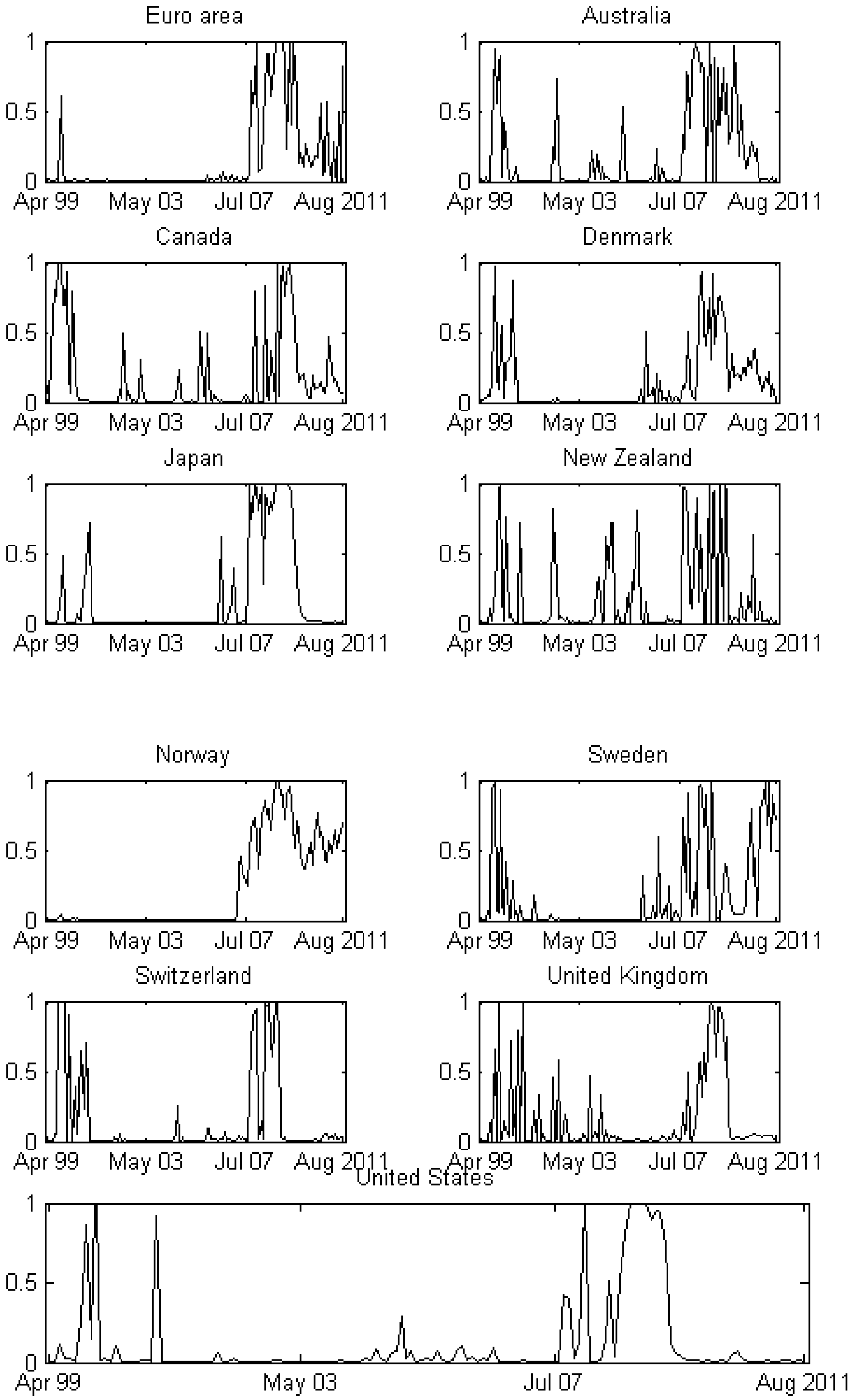
$$\text{PSTR model:} \quad r_{i,t} = \mu_i + \boldsymbol{\beta}_0'\boldsymbol{x}_{i,t-1} + \boldsymbol{\beta}_1'\boldsymbol{x}_{i,t-1}G(q_{i,t-1}, \gamma, c) + \varepsilon_{i,t}$$

$$\text{PTR model:} \quad r_{i,t} = \mu_i + \boldsymbol{\lambda}_1'\boldsymbol{x}_{i,t-1}\mathbb{1}_{(q_{i,t-1}<c)} + \boldsymbol{\lambda}_2'\boldsymbol{x}_{i,t-1}\mathbb{1}_{(q_{i,t-1}\geq c)} + \varepsilon_{i,t}.$$

The dependent variable $r_{i,t}$ is the annual return of Datastream Financials Index for country i at month t, and the transition variable ($q_{i,t-1}$) is the spread between 3-month and overnight interbank rates. We reduce and standardize this variable in order to have a comparable estimated threshold across the different countries. "FDIFF.3-month interbank" stands for the first difference of the 3-month interbank rate. In parentheses are reported t-statistics using the Newey-West correction for the variance and ***, ** and * denote significance at 0.01, 0.05 and 0.1 levels.

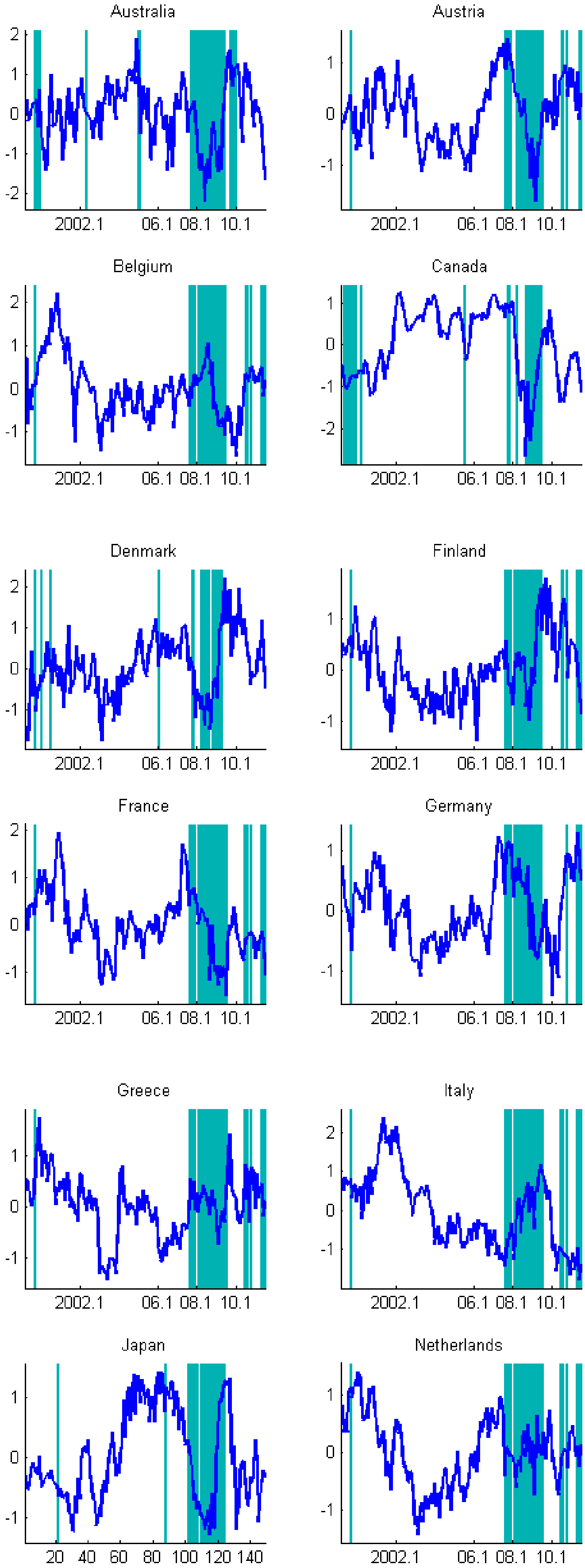
We obtain 95% confidence intervals for γ and c in the PSTR specification through a parametric bootstrap procedure by resampling in the time dimension with 1000 iterations. For the threshold parameter c in the PTR model, Hansen (1999) derives a non-standard likelihood ratio test and obtains the confidence interval from a no-rejection region. The confidence level is 95%.

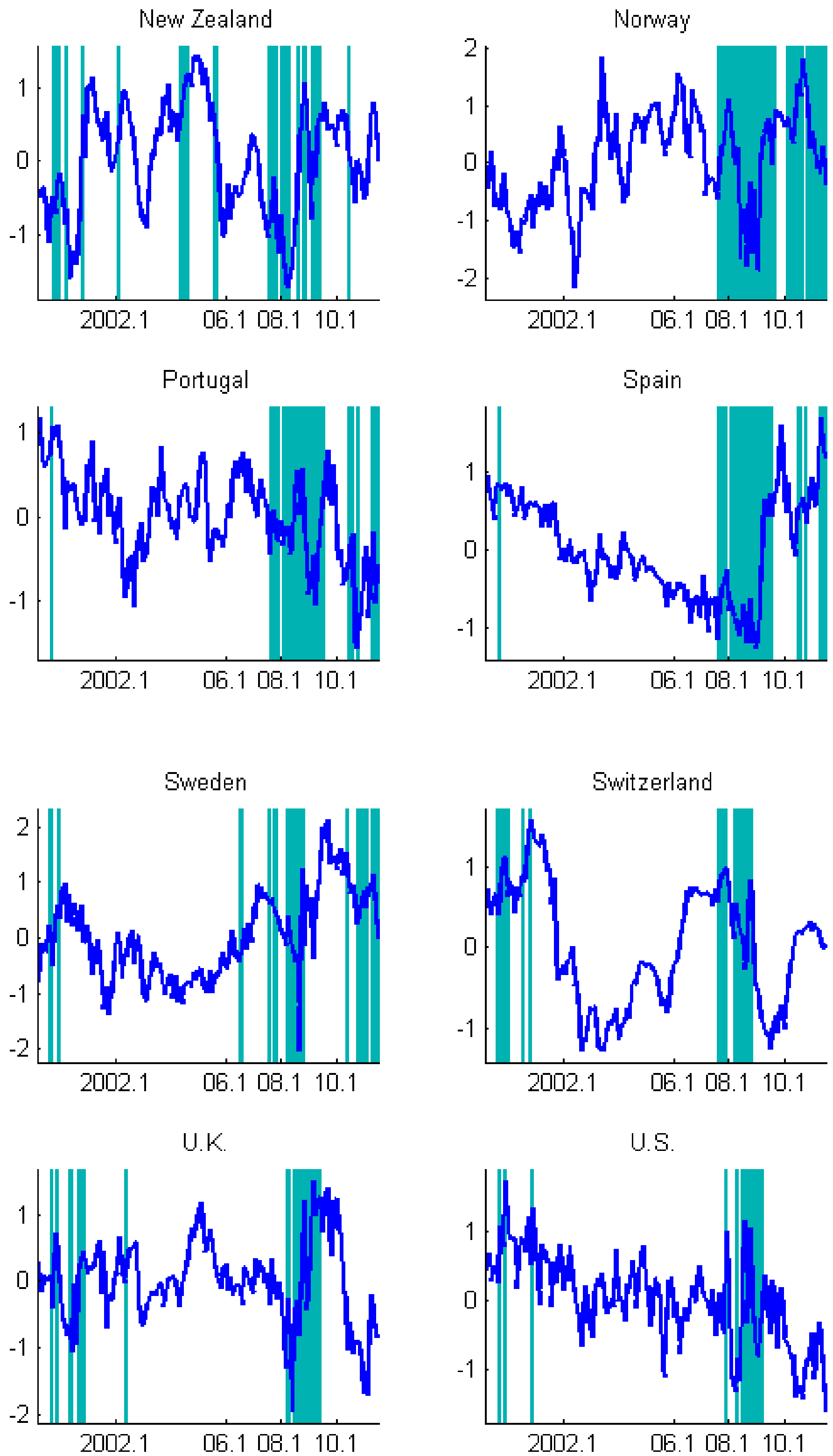
Figure 1.1
Estimated transition function for each country



Notes: The transition variable is the first lag of the spread between 3-month and overnight interbank rates, reduced and standardized to be comparable between different countries. The logistic distribution is given by the following formula: $G(q_{i,t-1}; \hat{\gamma}, \hat{c}) = \left(1 + e^{-\hat{\gamma}(q_{i,t-1} - \hat{c})}\right)^{-1}$ where $\hat{\gamma}$ and $\hat{c}$ correspond to the estimated values exhibited in Table 1.2. For the euro area countries the transition variable is identical and is given by the difference between the 3-month euribor and the eonia.

Figure 1.2
Investor sentiment (CCI) and crisis periods





Notes: This figure shows for each country the Consumer Confidence Indicator (CCI-first regressed on a set of macroeconomic variables) which is used as a proxy for investor sentiment (solid line) and the crisis period (shaded area) defined as $G(q_{i,t-1}; \hat{\gamma}, \hat{c}) > 0.5$, with $G(\cdot)$ the logistic distribution. The estimated values of γ and c can be found in Table 1.2.

In the linear model, investor sentiment yields a positive and insignificant coefficient.

In the nonlinear model, this impact is negative but insignificant in normal times (i.e. Regime 1) and positive and highly significant in crisis times (i.e. Regime 2). This result is surprising given the theoretical prediction that high levels of sentiment lead to temporary overpricing and thus low future returns, and vice-versa.[19] Baker and Wurgler (2007) provide a potential explanation for this finding, supported by empirical evidence in their study. They find that the sentiment - return relation depends on stock characteristics in the sense that relatively hard-to-arbitrage and hard-to-value stocks are highly sensitive to sentiment, while easy-to-arbitrage and easy-to-value stocks react less, or with an inverse effect. The latter class of stocks are perceived as “safer” by investors, engendering flight to quality during periods of distress. Stocks of financial companies composing our indices should be easy to arbitrage and easy to value because they are traded in highly liquid international platforms. These first results open the way for several directions in the exploration of the data.

First, although CCI is found to be a good measure of investor sentiment, its impact should be tested by using alternative proxies. Second, many studies show that investor sentiment and other predictors are characterized by a high degree of persistence which leads to biased estimated coefficients (e.g. Stambaugh 1999; Lewellen 2004). Third, as an exploratory analysis, it would be interesting to consider the predictability pattern of shorter-term financial stock returns. Fourth, given that the literature is not clear about the appropriate lag between business cycles and stock returns (Pesaran and Timmermann 1995), it would be worth examining the impact of further lags of the predictors on annual returns. Finally, it is possible that future stock prices of financial firms react differently to investor sentiment compared to broader equity indices which have been the subject of many studies so far. We explore in these directions in Section 1.5.2.

Among the business-cycle indicators, the impact of real GDP growth is positive and highly significant, but does not change magnitude when shifting from the normal to the crisis regime (the difference of impact is small and insignificant). A potential explanation of this result could be that real output growth is a headline indicator of the economic activity and as such, the same predictive power could be attributed to this factor whatever the state of the economy. On the other hand, inflation strongly affects future financial stock returns through a negative and significant premium which is increasing in absolute value during the crisis regime. This finding confirms the results from Pesaran and Timmermann (1995) and from other studies that inflation is a stronger predictor of stock returns during distressed periods, probably explained by the more intense market frictions. The change in the short-term interbank rate exhibits (as expected) a positive and significant (insignificant for Regime 2 in the PSTR) factor loading with a weaker effect during the crisis in both model specifications. The positive impact of this variable could plausibly be explained by the fact that investors require higher future returns during periods of monetary policy tightening. Despite the common finding in the literature that term spread explains future stock returns, we could not find evidence in this line of research among all the different specifications that we tried (Amihud 2002 shows results similar to ours).

In the group of the financial variables, volatility exhibits the expected positive coefficient with an insignificant change from one regime to the other. The predictive power of dividend yield and liquidity is negative, highly significant and appears to be constant across both specifications. Investors seem to perceive dividend-paying financial stocks as

[19] Our explanatory variable does not include dividends. We reran the regressions with returns including the dividends and the results remain unchanged.

being less risky or more liquid (Amihud 2002). The average market capitalization exhibits a positive factor loading with a weaker effect during the crisis in both nonlinear specifications.

As indicated by the estimated value of the smoothness parameter $\hat{\gamma} = 3.7$ in the PSTR specification, the turning point between the two regimes is quite brusque.[20] This finding could be a potential explanation of the striking similarity of the estimated parameters between the PSTR and the PTR specifications. In the PSTR model, the crisis occurs when the transition variable is one standard deviation above its mean. For the PTR model, the estimated location parameter is somewhat lower ($\hat{c} = 0.744$).

Regarding the goodness of fit, Akaike's Information Criterion (AIC) and Schwarz's Bayesian Information Criterion (BIC) show that the PSTR model fits the data the best, whereas the within R-squared is in favor of the PTR model (53.92%). On the other hand, the linear model performs the poorest although the difference is not large (within R-squared is 46.75%). This difference becomes larger when we consider regressions with further lags of the predictors. This gap could be partially explained by the higher flexibility of the nonlinear models. The remarkably high level of R-squared could be explained by the large number of observations and by the relevance of the predictive regressors. When investigating the predictability of aggregate stock returns at the one-year horizon for a panel of 18 developed countries using investor sentiment (and a smaller set of control variables), Schmeling (2009) finds an adjusted R-squared of 11%.

From the specification of the PTR model, country i is involved in a crisis (Regime 2) for month t if $q_{i,t-1} > c$. Based on the estimation of the location parameter c from the PTR, we investigate the number of countries classified in a crisis regime for each month and present some summary statistics for each year in Table 1.3. One can note that in October 1999 (bar Norway), November 2007 (bar U.K.), October 2008 (bar New Zealand) almost all the sampled countries are absorbed in a crisis. Over the 20 countries considered, there are on average 16 of them to be involved in a crisis during 2008. These findings are perfectly in line with the results from the PSTR discussed above and displayed in Figure 1.1.

The above analysis shows that aggregate financial stock returns at the 1-year horizon contain a substantial predictable component from economic and financial variables that varies with the business cycles. So far our variable of interest, the market-wide investor sentiment has a negative impact during normal times (but this coefficient is insignificant) and positive and strongly significant impact during crisis times. The latter positive impact is uncommon in the literature for at least three potential reasons: (i) based on the argument of Baker and Wurgler (2007), this class of stocks may react differently to investor sentiment (ii) the impact of sentiment may have changed during the recent crisis, for which there is no benchmark study, (iii) this paper is the first to use a regime-switching model. In the next section we explore in several directions to test the robustness of this finding.

[20]In Figure A.III is shown the logistic distribution for different values of the smoothness parameter. With $\gamma = 4$ the transition is already very steep and the functions are very close as γ increases. Additionally, this function is much closer to the function with $\gamma = 10$ or $\gamma = 100$ then to the function with $\gamma = 1$.

Table 1.3
Number of countries in the crisis regime based on the PTR model

Transition variable:		3-month-overnight interbank spread			
Year	Obs	Mean	Std. Dev.	Min	Max
1999	12	3.75	5.48	0	[a]17
2000	12	2.58	1.24	1	5
2001	12	0.08	0.29	0	1
2002	12	0.58	1.00	0	3
2003	12	0.17	0.39	0	1
2004	12	0.67	0.49	0	1
2005	12	0.58	0.51	0	1
2006	12	0.42	0.51	0	1
2007	12	7.08	8.28	0	[b]19
2008	12	15.58	5.81	3	[c]19
2009	12	10.75	7.02	1	[d]18
2010	12	4.50	4.98	1	13
2011	8	6.50	4.99	2	12

Notes. In this table we present summary statistics concerning the number of countries in the crisis regime for each month within a year, basing our analysis on the PTR estimation results. Our sample contains 20 developed countries with monthly data. We take the estimated location parameter from the PTR specification in Table 1.2 and compare it with the value of the transition variable (reduced and standardized) for each month and for each country. If $q_{i,t-1} > \hat{c}$, than for month t, country i is classified as being in a crisis regime.

"Obs" stands for the number of months within a year for which we have the data, "Mean" is the average number of countries for each month within a year estimated to be involved in a crisis. "Std. Dev", "Min", "Max" are respectively the standard deviation, the minimum and maximum number of countries to undergo a downturn within a month.

[a]October 1999, [b]November 2007, [c]May, July, October, December 2008, [d]May, June 2009.

1.5.2 Robustness tests

Alternative measures of investor sentiment, real activity and liquidity

We first test whether the estimation results are robust to alternative measures of investor sentiment, real growth and market liquidity.

In a novel strand of empirical literature, Consumer Confidence Indicators are documented to be a highly appropriate proxy for investor sentiment. Nevertheless, being a broad approximation of investors' expectations about the evolution of financial perspectives, its adequacy should be tested. For this reason, we use as an alternative measure a survey-based Economic Sentiment Indicator (ESI) consisting of business and consumer surveys, and so being a broader index than CCI.[21] The overall correlation between these two measures is large and equal to 46.3% with the highest value for France (78.6%) and lowest for the U.S. (-9%).[22] Additionally, ESI might capture the role of retail investors which tend to buy and sell stocks in concert producing a systematic component that Kumar and Lee (2006) call *retail investor sentiment* with incremental predictive power for returns.

The estimation results of the predictive regressions are shown in Table 1.4 and they are virtually the same as the results in our baseline specification in Table 1.2. Nevertheless, our variable of interest has now a positive and insignificant impact, except for the linear model where the coefficient is significant at 10% level. This result is surprising given the high correlation between these two variables. A potential explanation could be that ESI – being a larger index – is more contaminated by fundamentals already present in the model. Note that for the PSTR model there is now stronger evidence in favor of a two-regime specification with p-values in favor of this null hypothesis around 5%.

As mentioned in the description of the data, the real output growth is available only quarterly. To check the sensitivity of the estimation results due to the report of the same value of this variable three times within a quarter, we replace it by an average of 12-month fixed horizon forecasts provided by professionals such as banks and research institutes at a monthly frequency. As a second alternative variable we employ the industrial production growth.[23] The results presented in Table 1.5 remain qualitatively unchanged (real growth has a constant impact across the states of the economy), but the impact of the forecasts on returns is nearly two times stronger, and the impact of industrial production growth is cut by more than half. The impact of investor sentiment is robust across these three alternatives. Note that the negative impact of sentiment in Regime 1 is now significant, which mainly points out the special role of Australia and New Zealand, excluded from the sample because the alternative measures of growth are not available.

Finally, we assess the robustness of our liquidity measure replacing the number of transactions over the total number of shares in the index by the trading activity in value

[21] As for CCI, ESI is orthogonalized by a set of business-cycle variables (see Section 1.4 on the data construction and Table A.II in the Appendix).

[22] Only the U.S., Australia (3.7%) and Switzerland (15.5%) exhibit such low correlations; the other countries are above 30%.

[23] We tested for a potential colinearity between our investor sentiment measures (CCI and ESI) and GDP growth forecasts by computing the correlation for each country since forecasts of economic conditions are considered in several studies as a measure of investor sentiment (e.g. Lemmon and Portniaguana 2006). This analysis is in favor of independence with *Corr(CCI, real growth forecasts)* which ranges from -23.4% to 59.9% with an average of 14.9%, whereas the *Corr(ESI, real growth forecasts)* is in the range -48.6% to 74.8% with an average value of 12.8%. On the other hand, the average correlation of the realized GDP growth with the forecasts of this variable is 84.9% and with industrial production growth is 74.41%.

over the market capitalization (results not presented). Both variables are used in the literature as proxies of market liquidity. The impact of this variable on future returns appears significantly negative and constant across regimes, which is in line with the previous finding. The other results and conclusions are unaltered.

Bias correction for the persistence of the explanatory variables

Several studies such as Stambaugh (1999), Lewellen (2004), Campbell and Yogo (2006) show that there is a small-sample bias on the impact of the predictive variables on returns if the predictors are persistent. Note that the persistence of the explanatory variables is an issue only if the error term of the predictive model is correlated with the error term of the *stationary* AR(1) process of the explanatory variables.[24] We use the method developed by Lewellen (2004) (see also Maio 2013) and modify it for a multi-factor and panel regime-switching model. The bias-corrected impact of investor sentiment for the three specifications and the procedure for the construction of this test are presented in Table 1.6. The coefficients change slightly without modifying the general conclusions (magnitude and significance). The main reason of this robustness could be that our sample is large with almost 3,000 observations.

Predictability of short-term financial stock returns

In Table 1.7 we explore the shorter-term predictability of financial stock returns for 1, 3, 6, 9-month horizons, displaying the estimation results from the linear, PSTR and PTR models. The impact of investor sentiment is never significant in the linear model, while the R-squared increases rapidly with the return horizon. For the PSTR and PTR models, sentiment is associated, as previously, with a negative coefficient in normal times and a positive coefficient in crisis times. For the 1 and 3-month returns only the negative impact in Regime 1 is significant, while the results for the 6 and 9-month returns are very much in line with the yearly returns: only the positive coefficient in Regime 2 is significant.

In a snapshot, predictability improves as the horizon increases. This investigation thus supports the findings of many studies that stock returns are predictable at long horizons, whereas the predicable component at short horizons is small (e.g. Brown and Cliff 2005).

Higher lags for the explanatory variables

Table 1.8 shows the impact of investor sentiment on annual financial stock returns for the three specifications (linear, PSTR and PTR) where the transition and *all* the explanatory variables are taken in lags from 1 to 32 months. The estimation results reveal very interesting patterns. For the linear model, the impact of investor sentiment is positive until the 6th lag, but never significant. Then, it turns to negative and becomes significant from the 13th to the 26th lag. The lags 30, 31 and 32 have again a positive impact, but only the last is significant. When we consider Regime 1 of the PSTR and PTR models, the impact of investor sentiment is always negative, but significant from the 5th to the 29th lag. In crisis times (Regime 2) the story is somewhat different: the pattern uncovered above is kept, but now the negative coefficients which appear in the middle lags are not significant any more. On the other hand, the positive coefficients are significant only for very close or very far lags. The R-squared are about 50% for the first lag but rapidly

[24] The AR(1) assumption for the predictive variables is largely used in the literature. See for instance: Stambaugh (1999), Lewellen (2004), Cochrane (2005), Camponovo, Scaillet and Trojani (2012).

fall at a level around 25% for the two-regime models and 15% for the linear model. We perform the same exercise using investor sentiment based on ESI and the main findings hold.

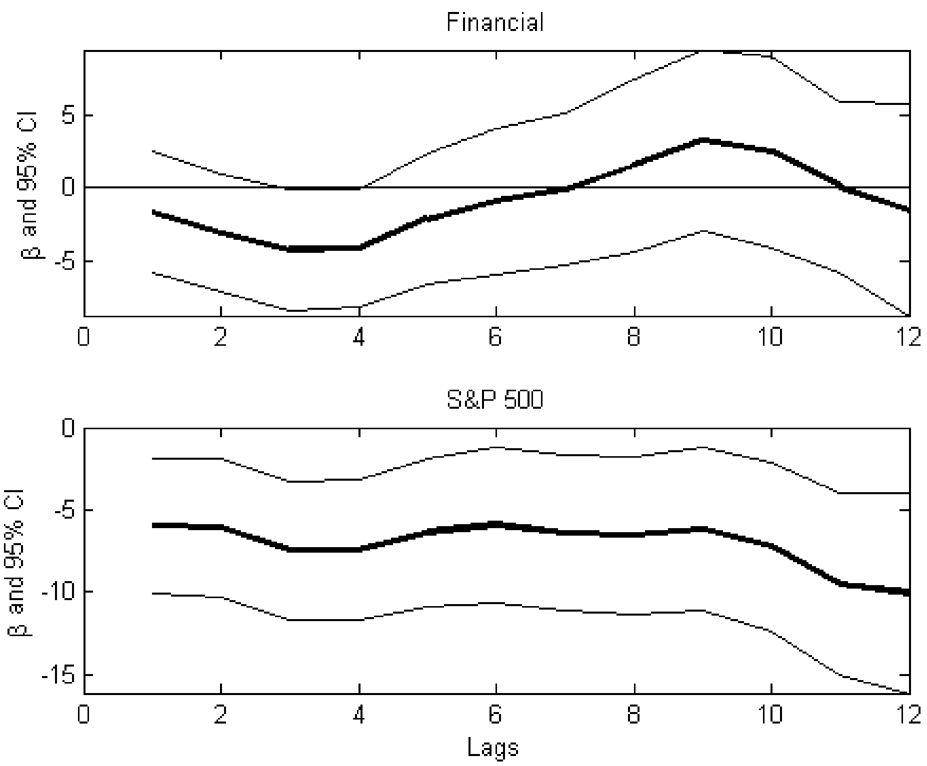
Predictability of U.S financial stock returns versus S&P 500 returns

We empirically investigate for the U.S. market whether annual financial stock returns react differently to investor sentiment compared to a broad equity index (here we consider the S&P 500). The linear model in Figure 1.3 shows that the lags of investor sentiment ranging from 1 to 12 months[25] have a negative and significant impact on the annual S&P 500. For the financial stock returns the impact oscillates between the negative and the positive signs and is almost never significant.

This simple exercise shows that stock returns of financial companies are less predictable from investor sentiment as compared to a wider universe of stocks, which are extensively studied so far in the literature with a central focus on the U.S. market.

Figure 1.3

Impact of investor sentiment on U.S. financial stocks and on *S&P500*



Notes: In this figure is displayed the impact of investor sentiment on annual returns of financial companies for the U.S. market (upper panel), and on S&P 500 annual returns (lower panel) using a linear model and considering the lags from 1 to 12 months for the transition and *all* the explanatory variables.

[25]We consider fewer lags since the time series is quite short with a maximum of 149 observations. We do not report the results for the nonlinear models because the number of observations is too small to precisely estimate a two-regime specification with 9 predictors. Moreover, there are too few observations in the crisis regime.

Table 1.4

Linear, PSTR and PTR estimation results for annual financial stock returns with an investor sentiment measure based on ESI

PANEL A	Testing the null hypothesis of linearity, and subsequently the null hypothesis of a two-regime specification		
	PSTR: LM test	PSTR: F test	PTR: F_1 test
$H_0 : L = 0$ vs $H_1 : L = 1$	194.653 (0.000)	22.914 (0.000)	282.545 (0.000)
$H_0 : L = 1$ vs $H_1 : L = 2$	16.962 (0.049)	1.866 (0.053)	42.791 (1.000)

PANEL B	Linear model	PSTR			PTR		
		Regime 1	Regime 2	Difference	Regime 1	Regime 2	Difference
	$\boldsymbol{\beta}$	$\boldsymbol{\beta}_0$	$\boldsymbol{\beta}_0 + \boldsymbol{\beta}_1$	$\boldsymbol{\beta}_1$	$\boldsymbol{\lambda}_1$	$\boldsymbol{\lambda}_2$	$\boldsymbol{\lambda}_2 - \boldsymbol{\lambda}_1$
Investor sentiment (ESI)	3.353*	2.539	1.231	-1.308	2.737	1.556	-1.181
	(1.84)	(1.266)	(0.683)	(-0.601)	(1.398)	(0.934)	(-0.693)
Real GDP growth	3.231***	2.509***	2.276***	-0.233	2.532***	2.551***	0.019
	(9.76)	(4.648)	(4.296)	(-0.266)	(4.642)	(7.256)	(0.028)
CPI inflation	-5.797***	-3.670***	-7.105***	-3.435**	-4.036***	-6.536***	-2.500**
	(-4.92)	(-2.695)	(-6.359)	(-1.980)	(-3.129)	(-7.751)	(-2.153)
FDIFF.3-month interbank	9.138***	19.577***	6.044*	-13.533**	18.319***	8.613***	-9.706
	(3.99)	(4.081)	(1.835)	(-1.993)	(3.599)	(3.707)	(-1.617)
Term spread	0.0338	-0.541	0.683	1.224	-0.534	0.595	1.130
	(0.05)	(-0.708)	(0.704)	(0.832)	(-0.706)	(0.844)	(0.992)
Volatility	2.889***	3.240***	2.348***	-0.892	3.042***	3.021***	-0.022
	(18.19)	(18.269)	(3.226)	(-1.088)	(15.194)	(5.697)	(-0.034)
Dividend yield	-2.682***	-2.187***	-2.159***	0.028	-2.174***	-2.607***	-0.433
	(-3.22)	(-2.761)	(-3.996)	(0.028)	(-2.775)	(-5.199)	(-0.547)
Liquidity (VO/NOSH)	-64.79***	-49.278***	-40.408***	8.870	-51.373***	-47.048***	4.324
	(-4.48)	(-2.864)	(-2.784)	(0.476)	(-3.065)	(-3.793)	(0.302)
Ln(Market value/Firms)	15.91***	16.103***	14.379***	-1.724**	15.992***	14.870***	-1.123***
	(5.62)	(5.140)	(4.438)	(-2.485)	(5.208)	(4.697)	(-2.559)
γ			3.794 [2.539, 6.290]				
c			1.045 [0.888, 1.189]			0.725 [0.654, 0.751]	
Observations	2,980		2,980			2,980	
Number of id	20		20			20	
R-squared	0.472		0.524			0.538	
AIC	6.060		5.968			5.984	
BIC	6.078		6.008			6.023	

Notes. We replace the investor sentiment based on the Consumer Confidence Indicator (CCI) by a proxy based on an Economic Sentiment Indicator (ESI). The transition variable is the spread between 3-month and overnight interbank rates (reduced and standardized). See also the notes under Table 1.2.

Table 1.5

Comparison of the impact of real GDP growth (realized and forecasts), and industrial production growth

	Impact of real GDP growth				Impact of real GDP growth forecasts				Impact of industrial prod. growth			
	Linear	PSTR			Linear	PSTR			Linear	PSTR		
		Regime 1	Regime 2	Difference		Regime 1	Regime 2	Difference		Regime 1	Regime 2	Difference
	β	β_0	$\beta_0 + \beta_1$	β_1	β	β_0	$\beta_0 + \beta_1$	β_1	β	β_0	$\beta_0 + \beta_1$	β_1
Investor sentiment (CCI)	1.366	-1.397**	6.479***	7.876***	0.425	-1.889***	5.602***	7.491***	1.464	-1.257*	6.396***	7.653***
	(0.937)	(-2.120)	(4.563)	(4.618)	(0.29)	(-2.972)	(5.505)	(5.778)	(0.999)	(-1.785)	(4.313)	(4.222)
Real growth	3.359***	2.753***	2.045***	-0.708	5.105***	3.772***	4.065***	0.293	1.143***	0.904***	0.665***	-0.239
	(12.83)	(10.814)	(5.717)	(-1.447)	(4.74)	(8.725)	(7.177)	(0.396)	(10.66)	(9.184)	(3.957)	(-1.118)

Notes: In this table we display the impact of the 1-month lag of investor sentiment on annual returns of financial companies using three different measures of real growth: real GDP growth, real GDP growth forecasts and industrial production growth. This sensitivity analysis is conducted because real GDP growth is available only quarterly (we repeat the same value three times within a quarter), while the two other measures are available monthly. The model includes the whole set of the control variables (real GDP growth, CPI inflation, 3-month interbank rate, term spread, realized volatility, dividend yield, liquidity, market value per firm), which are not shown here to save space. Given that the economic forecasts are not available for Australia and New Zealand, we remove these two countries from the data. We could not find any broad industrial index for New Zealand, that is why this country is dropped from the panel. For the following countries, the series of industrial production growth does not start from January 1999: Denmark, Greece, Portugal, Sweden (from January 2001), Switzerland (from January 2005). In order to obtain comparable results, we use the same sample across the three specifications. In parentheses are shown t-statistics based on Newey-West corrected variance. ***, ** and * denote significance at 0.01, 0.05 and 0.1 levels. See also the notes under Table 1.2.

Table 1.6

Linear, PSTR and PTR models: *bias correction* for the persistence of the predictive variables

Linear model	$\hat{\beta} = 1.48\,(1.04)$	$\hat{\beta}_{adj} = 0.640$	$t(\beta_{adj}) = 0.46$	$p(\beta_{adj}) = 0.648$	$\hat{\rho}_k = 0.836$	$t(\rho_k) = 43.07$	$\hat{\delta} = 3.67$	$t(\delta) = 2.69$
	Regime 1 ($\hat{b}_1$)	Regime 2 ($\hat{b}_2$)	Regime 1 ($\hat{b}_{1(adj)}$)	Regime 2 ($\hat{b}_{2(adj)}$)	Regime 1 ($\hat{\rho}_{k,1}$)	Regime 1 ($\hat{\rho}_{k,2}$)	Regime 1 ($\hat{\delta}_1$)	Regime 2 ($\hat{\delta}_2$)
PSTR	-1.174 (-0.69)	5.921 (6.42)	-1.232 (-0.82)	5.073 (5.43)	0.842 (36.78)	0.741 (20.86)	3.03 (1.80)	2.84 (1.75)
PTR	-1.084 (-0.67)	4.725 (8.16)	-1.301 (-0.86)	3.796 (3.57)	0.844 (33.03)	0.783 (26.30)	2.20 (1.37)	5.79 (3.36)

Notes: In this table we present the bias-corrected coefficients related with *investor sentiment* for the linear, PSTR and PTR models using a 1-month lag for the explanatory variables. The models include the whole set of the control variables (real GDP growth, CPI inflation, 3-month interbank rate, term spread, realized volatility, dividend yield, liquidity, market value per firm), whose impacts are not shown here to save space. The potential bias is due to the persistence of the explanatory variables. We base this correction on the procedure developed by Lewellen (2004). In the latter paper the procedure is explained only for the time series case and within a univariate linear model. The bias can be important for small samples, but it decreases as the sample size increases. In our case, the panel data contains around 3'000 observations, which might substantially reduce this bias. In Lewellen (2004), the univariate linear model is written as follows:

$$\text{Univariate principal model}: \quad r_t = \alpha + \beta x_{t-1} + \varepsilon_t, \qquad \text{Univariate auxiliary model}: \quad x_t = \phi + \rho x_{t-1} + \mu_t.$$

The persistence of the predictive (and stationary) variables engenders a bias if $corr(\varepsilon_t, \mu_t)$ is different from zero, in which case the errors would be linked by the following model: $\varepsilon_t = \gamma\mu_t + v_t$ with $v_t \sim i.i.d.(0, \sigma_v^2)$. If we replace this last equation in the principal model, the latter becomes (with v_t uncorrelated with r_t):

$$\text{Univariate principal model (bias adjusted)}: \quad r_t = \alpha + \beta x_{t-1} + \gamma\mu_t + v_t.$$

An estimate of μ_t can be obtained from the auxiliary model using the OLS estimation procedure.

We try to adapt this procedure to our multivariate and panel framework, but we do not check whether this method holds asymptotically. The linear model can be written as follows (the notations do not correspond to the univariate model):

$$\text{Multivariate principal model:} \quad r_{i,t} = \mu_i + \boldsymbol{\beta}'\boldsymbol{x}_{i,t-1} + \varepsilon_{i,t}, \qquad \text{Auxiliary model:} \quad x_{k,i,t} = \eta_{k,i} + \rho_k x_{k,i,t-1} + \nu_{k,i,t} \quad k = 1, \dots, K$$

with K the number of explanatory variables. We estimate the auxiliary equation by OLS (within method) for each explanatory variable, and compute the residuals $\hat{\nu}_{k,i,t}$. Then, we augment the multivariate principal model as follows and estimate the coefficients by OLS (within method) to obtain bias-corrected estimates:

$$\text{Multivariate principal model (bias-adjusted)}: \quad r_{i,t} = \mu_i + \boldsymbol{\beta}'\boldsymbol{x}_{i,t-1} + \boldsymbol{\delta}'\hat{\boldsymbol{\nu}}_{i,t} + \omega_{i,t}.$$

In the first row of the table, $\hat{\beta}$ is the estimated coefficient for investor sentiment in the principal model (Table 1.2) with no bias correction, $\hat{\beta}_{adj}$ is the bias-corrected estimate for β, $t(\beta_{adj})$ and $p(\beta_{adj})$ are the t-statistic and the p-value for testing the null hypothesis that $\beta_{adj} = 0$. $\hat{\rho}_k$ is the estimate of ρ_k in the auxiliary regression and its corresponding t-statistic $t(\rho_k)$. $\hat{\delta}$ and $t(\delta)$ is the estimate of δ and its corresponding t-statistic. $\boldsymbol{\delta}$ is the coefficient vector that links the residuals of the principal model with the residual of the auxiliary model: $\varepsilon_{i,t} = \boldsymbol{\delta}'\boldsymbol{\nu}_{i,t} + \omega_{i,t}$.
For the PSTR and PTR specifications we proceed as for the linear model, with the only difference that we run separate regressions for each regime (we denote the coefficients by b_1 and b_2). In the PSTR model the regimes are defined as follows: Regime 1: $G(q_{i,t-1}; \hat{\gamma}, \hat{c}) < 0.5$ and Regime 2: $G(q_{i,t-1}; \hat{\gamma}, \hat{c}) \geq 0.5$. In the PTR model the regimes are defined as follows: Regime 1: $q_{i,t-1} < \hat{c}$ and Regime 2: $q_{i,t-1} \geq \hat{c}$. $\hat{b}_1$, $\hat{b}_2$, $\hat{\gamma}$, $\hat{c}$ are sourced from Table 1.2. In parentheses are shown t-statistics with Newey-West corrected variance.

Table 1.7

Impact of investor sentiment on 1- to 12-month horizon stock returns of financial companies

	Linear model		PSTR				PTR			
			Regime 1	Regime 2	Difference		Regime 1	Regime 2	Difference	
	β	R^2	β_0	$\beta_0+\beta_1$	β_1	R^2	λ_1	λ_2	$\lambda_2-\lambda_1$	R^2
1-month horizon	-0.040	0.049	-0.520**	0.364	0.884	0.107	-0.533**	-0.524*	0.009	0.088
	(-0.186)		(-2.430)	(0.313)	(0.761)		(-2.469)	(-1.875)	(0.036)	
3-month horizon	0.200	0.165	-1.384*	2.655	4.039	0.243	-1.408**	0.563	1.971*	0.224
	(0.324)		(-1.939)	(1.085)	(1.560)		(-2.118)	(0.511)	(1.871)	
6-month horizon	1.090	0.272	-2.215	3.923***	6.138***	0.345	-1.794	2.799***	4.593***	0.342
	(1.036)		(-1.531)	(3.532)	(3.670)		(-1.373)	(2.674)	(3.971)	
9-month horizon	1.561	0.386	-1.943	5.852***	7.795***	0.439	-2.025	4.279***	6.304***	0.434
	(1.364)		(-1.147)	(6.225)	(3.444)		(-1.273)	(4.969)	(3.922)	
12-month horizon	1.481	0.468	-1.174	5.921***	7.095***	0.546	-1.084	4.725***	5.809***	0.539
	(1.037)		(-0.685)	(6.420)	(3.094)		(-0.667)	(8.155)	(4.002)	

Notes. This table presents the estimation results concerning the impact of investor sentiment (CCI) on future returns of financial companies using linear, PSTR and PTR models. The return horizons considered are 1, 3, 6, 9 and 12 months. In parentheses are presented Newey-West-corrected t-statistics and ***, ** and * denote significance at 0.01, 0.05 and 0.1 levels. The estimated specifications are the following:

$$\text{Linear model}: \quad r_{i,t:t+k} = \mu_i^{(k)} + \boldsymbol{\beta}^{(k)\prime}\boldsymbol{x}_{i,t} + \varepsilon_{i,t:t+k}^{(k)} \qquad i=1,\ldots,N, \quad t=1,\ldots,T, \quad k=1,3,6,9,12$$

$$\text{PSTR}: \quad r_{i,t:t+k} = \mu_i^{(k)} + \boldsymbol{\beta}_0^{(k)\prime}\boldsymbol{x}_{i,t} + \boldsymbol{\beta}_1^{(k)\prime}\boldsymbol{x}_{i,t} G\left(q_{i,t};\gamma^{(k)},c^{k}\right) + \varepsilon_{i,t:t+k}^{(k)}$$

$$\text{PTR}: \quad r_{i,t:t+k} = \mu_i^{(k)} + \boldsymbol{\lambda}_1^{(k)\prime}\boldsymbol{x}_{i,t}\mathbb{1}_{\left(q_{i,t}<c^{(k)}\right)} + \boldsymbol{\lambda}_2^{(k)\prime}\boldsymbol{x}_{i,t}\mathbb{1}_{\left(q_{i,t}\geq c^{(k)}\right)} + \varepsilon_{i,t:t+k}^{(k)}$$

where $r_{i,t:t+k}$ is the k-month return of financial companies and $\boldsymbol{x}_{i,t}$ contains in addition to the proxy for investor sentiment, macroeconomic variables (real GDP growth, CPI inflation, 3-month interbank rate, term spread) and financial variables (realized volatility, dividend yield, liquidity, market value per firm). $q_{i,t}$ is the transition variable between regimes given by the spread between the 3-month and the overnight interbank rates.

Table 1.8

Impact of different lags of investor sentiment on annual returns of financial companies

	Linear model		PSTR				PTR			
	β	R^2	β_0 (Reg.1)	$\beta_0+\beta_1$ (Reg.2)	β_1 (Diff.)	R^2	λ_1 (Reg.1)	λ_2 (Reg.2)	$\lambda_2-\lambda_1$ (Diff.)	R^2
Lag=1	1.481	0.468	-1.174	5.921***	7.095***	0.524	-1.084	4.725***	5.809***	0.539
	(1.040)		(-0.685)	(6.420)	(3.094)		(-0.667)	(8.155)	(4.002)	
Lag=2	1.502	0.414	-1.844	5.999***	7.843***	0.477	-1.858	4.713***	6.570***	0.496
	(1.050)		(-0.959)	(5.673)	(2.932)		(-1.051)	(6.125)	(3.866)	
Lag=3	1.242	0.366	-2.777	5.905***	8.683***	0.442	-2.417	4.681***	7.098***	0.464
	(0.820)		(-1.288)	(5.403)	(3.140)		(-1.258)	(5.821)	(3.918)	
Lag=4	0.973	0.325	-3.799	5.767***	9.566***	0.412	-3.086	4.605***	7.691***	0.435
	(0.600)		(-1.599)	(4.599)	(3.314)		(-1.477)	(4.484)	(3.860)	
Lag=5	1.008	0.276	-4.458*	5.856***	10.314***	0.372	-3.597	4.881***	8.478***	0.395
	(0.570)		(-1.714)	(3.790)	(3.409)		(-1.574)	(3.763)	(4.035)	
Lag=6	0.622	0.220	-4.960*	5.242***	10.202***	0.323	-4.483*	3.726**	8.209***	0.361
	(0.330)		(-1.873)	(2.788)	(3.366)		(-1.843)	(2.098)	(3.985)	
Lag=7	-0.069	0.181	-5.379*	3.988*	9.367***	0.289	-5.069*	2.591	7.661***	0.342
	(-0.030)		(-1.945)	(1.767)	(2.814)		(-1.952)	(1.249)	(3.269)	
Lag=8	-0.709	0.152	-5.566**	2.674	8.240**	0.260	-5.359**	1.268	6.626**	0.320
	(-0.340)		(-2.010)	(0.963)	(2.128)		(-2.037)	(0.535)	(2.457)	
Lag=9	-1.346	0.131	-5.598**	1.120	6.718	0.237	-5.648**	0.311	5.959*	0.304
	(-0.660)		(-2.139)	(0.339)	(1.562)		(-2.230)	(0.118)	(1.958)	
Lag=10	-2.31	0.128	-5.900**	-0.700	5.200	0.227	-6.131**	-0.708	5.424	0.295
	(-1.140)		(-2.256)	(-0.182)	(1.056)		(-2.449)	(-0.244)	(1.610)	
Lag=11	-3.492*	0.141	-6.215**	-2.597	3.617	0.238	-6.038***	-2.524	3.514	0.306
	(-1.730)		(-2.436)	(-0.629)	(0.690)		(-2.640)	(-0.735)	(0.891)	
Lag=12	-3.869*	0.165	-5.804**	-3.046	2.758	0.260	-6.308***	-1.698	4.610	0.328
	(-1.940)		(-2.294)	(-0.783)	(0.543)		(-2.571)	(-0.584)	(1.269)	
Lag=13	-4.443**	0.172	-5.646**	-3.370	2.275	0.264	-6.321***	-1.510	4.811	0.331
	(-2.250)		(-2.267)	(-0.854)	(0.435)		(-2.657)	(-0.525)	(1.287)	
Lag=14	-4.898**	0.172	-5.721**	-2.138	3.582	0.257	-6.169***	-1.253	4.916	0.326
	(-2.450)		(-2.335)	(-0.578)	(0.727)		(-2.720)	(-0.431)	(1.344)	
Lag=15	-5.498***	0.179	-5.548**	-2.355	3.193	0.258	-6.183***	-0.810	5.373	0.312
	(-2.720)		(-2.287)	(-0.583)	(0.599)		(-2.892)	(-0.263)	(1.425)	
Lag=16	-5.957***	0.182	-5.482**	-2.435	3.047	0.264	-6.073***	-0.673	5.400	0.305
	(-2.910)		(-2.048)	(-0.423)	(0.407)		(-2.860)	(-0.210)	(1.392)	
Lag=17	-6.476***	0.194	-6.034**	-1.949	4.085	0.277	-5.874***	-2.854	3.020	0.308
	(-3.120)		(-2.344)	(-0.386)	(0.625)		(-2.899)	(-0.985)	(0.961)	
Lag=18	-7.096***	0.203	-6.898***	-2.128	4.769	0.281	-7.144***	-0.979	6.165**	0.321
	(-3.360)		(-2.978)	(-0.649)	(1.149)		(-3.405)	(-0.369)	(2.047)	
Lag=19	-7.572***	0.208	-7.626***	-1.842	5.784*	0.292	-7.812***	-0.960	6.852**	0.327
	(-3.500)		(-3.467)	(-0.646)	(1.713)		(-3.667)	(-0.399)	(2.532)	
Lag=20	-7.462***	0.213	-8.045***	-1.149	6.895**	0.297	-8.168***	-0.028	8.140***	0.343
	(-3.430)		(-3.575)	(-0.397)	(1.979)		(-3.724)	(-0.012)	(2.903)	
Lag=21	-7.061***	0.211	-7.919***	-0.809	7.109*	0.298	-7.492***	-0.412	7.080**	0.353
	(-3.350)		(-3.490)	(-0.275)	(1.915)		(-3.570)	(-0.167)	(2.471)	
Lag=22	-6.271***	0.201	-7.578***	0.575	8.153**	0.297	-7.285***	1.076	8.360***	0.351
	(-3.090)		(-3.455)	(0.194)	(2.124)		(-3.524)	(0.424)	(2.594)	
Lag=23	-5.444***	0.195	-6.671***	1.931	8.603***	0.302	-6.707***	1.938	8.644**	0.342

	(-2.850)		(-3.296)	(0.748)	(2.434)		(-3.278)	(0.757)	(2.437)	
Lag=24	-4.557**	0.187	-6.000***	2.718	8.717**	0.302	-6.002***	2.658	8.660**	0.351
	(-2.510)		(-2.946)	(1.066)	(2.322)		(-2.949)	(1.047)	(2.316)	
Lag=25	-3.826**	0.176	-5.757***	4.036	9.792***	0.299	-5.736***	3.996	9.732***	0.341
	(-2.110)		(-2.825)	(1.629)	(2.628)		(-2.812)	(1.620)	(2.616)	
Lag=26	-3.281*	0.166	-5.408***	4.940**	10.348***	0.295	-5.502***	4.704*	10.206***	0.325
	(-1.890)		(-2.784)	(2.147)	(2.909)		(-2.837)	(1.927)	(2.804)	
Lag=27	-2.487	0.156	-4.891***	5.346**	10.237***	0.285	-4.728***	4.927**	9.655***	0.324
	(-1.470)		(-2.624)	(2.208)	(2.922)		(-2.566)	(2.188)	(2.952)	
Lag=28	-1.368	0.146	-4.398**	5.450**	9.848***	0.272	-4.472**	5.495**	9.967***	0.320
	(-0.820)		(-2.409)	(2.270)	(2.971)		(-2.468)	(2.303)	(3.049)	
Lag=29	-0.151	0.133	-3.532*	5.765**	9.297***	0.254	-3.525*	5.724**	9.249***	0.289
	(-0.090)		(-1.909)	(2.479)	(3.021)		(-1.905)	(2.460)	(3.005)	
Lag=30	0.897	0.130	-0.594	1.588	2.182	0.251	-0.680	1.622	2.302	0.261
	(0.510)		(-0.321)	(0.639)	(0.740)		(-0.369)	(0.668)	(0.813)	
Lag=31	2.58	0.134	-3.094	6.103**	9.197***	0.242	-2.780	5.904**	8.684***	0.255
	(1.430)		(-1.257)	(2.524)	(2.894)		(-1.167)	(2.470)	(2.847)	
Lag=32	4.103**	0.140	-1.75	6.739***	8.489***	0.238	-1.740	6.441***	8.181***	0.257
	(2.230)		(-0.660)	(3.048)	(2.906)		(-0.681)	(2.864)	(3.014)	

Notes: This table displays the impact of investor (CCI) sentiment on *annual* financial stocks returns when we consider the model with 1 to 32 lags for the transition and *all* the explanatory variables (the impact of the control variables is not shown to save space). We present the results for the linear model, the PSTR model and the PTR model: the coefficients, the t-statistics with the variance matrix corrected by the Newey-West method (in parentheses), and the within-R^2. More specifically we run the following models (see also the notes under Table 1.2):

$$\text{Linear model}: \quad r_{i,t}^{(j)} = \mu_i^{(j)} + \beta^{(j)\prime} \boldsymbol{x}_{i,t-j} + \varepsilon_{i,t}^{(j)} \qquad i=1,\dots,N, \quad t=1,\dots,T, \quad j=1,\dots 32$$

$$\text{PSTR}: \quad r_{i,t}^{(j)} = \mu_i^{(j)} + \beta_0^{(j)\prime} \boldsymbol{x}_{i,t-j} + \beta_1^{(j)\prime} \boldsymbol{x}_{i,t-j} G\left(q_{i,t-j}; \gamma^{(j)}, c^{(j)}\right) + \varepsilon_{i,t}^{(j)}$$

$$\text{PTR}: \quad r_{i,t}^{(j)} = \mu_i^{(j)} + \boldsymbol{\lambda}_1^{(j)\prime} \boldsymbol{x}_{i,t-j} \mathbb{1}_{\left(q_{i,t-j} \leq c^{(j)}\right)} + \boldsymbol{\lambda}_2^{(j)\prime} \boldsymbol{x}_{i,t-j} \mathbb{1}_{\left(q_{i,t-j} > c^{(j)}\right)} + \varepsilon_{i,t}^{(j)}$$

with $r_{i,t}^{(j)}$ the annual return at month t for country i, $\boldsymbol{x}_{i,t-j}$ the set of explanatory variables observed at month $t-j$.

1.6 Conclusion

A growing strand of literature explores the role of investor sentiment in predicting asset returns. Following this literature, we investigate the predictability power of market-wide investor sentiment on medium-term aggregate stock returns of financial companies for a large panel of developed countries, jointly with a broad set of business-cycle and financial variables. The return predictability of this asset category is not investigated separately in the literature, many studies being focused on a wider horizon of stocks. Moreover, with a few exceptions (Schmeling 2009; Baker, Wurgler and Yuan 2012), the impact of investor sentiment is investigated exclusively in the U.S. market.

Investor sentiment is defined as excessive optimism or pessimism from irrational investors. The predictive power of investor sentiment on returns is explained in the literature by the limits to arbitrage and the short-selling constraints that face rational investors, especially in the long run (Brown and Cliff 2005; Stambaugh, Yu and Yuan 2012).

As suggested by the results of several studies, the impact of investor sentiment might be time-varying across the business cycles. Following this literature, and the evidence from our data, we use panel regime-switching models with threshold (Hansen 1999) and smooth transition (González, Teräsvirta and van Dijk 2005) between regimes. The results show that the exclusive use of a linear model would hide very interesting patterns in predictability during normal and crisis times.

Investor sentiment is found to have a negative impact on medium-term aggregate stock returns of financial companies during normal times – but the evidence is weak – and (surprisingly) a positive and highly significant impact during crisis times. The recent financial crisis was characterized by remarkably low levels of sentiment. The latter fact, and the argument advanced by Baker and Wurgler (2007) that the future returns of easy-to-arbitrage and easy-to-value stocks can be inversely related to sentiment, is a tentative explanation of the strong and positive impact of investor sentiment in distressed periods. On the other hand, and consistent with the literature, there is weaker evidence of predictability for short-term financial stock returns. Nevertheless, the negative impact during normal times and the positive impact during crisis times of investor sentiment is maintained, although the coefficients are not always significant. The models detect the turning point to a crisis regime at the burst of the dot-com bubble by the beginning of year 2000, and at the inception of the global financial crisis by August 2007.

As a future line of research it would be interesting to explore more in depth to which extent the predictability of financial stock returns differs compared to a wider universe of stocks. Further, it would interesting to investigate whether this stock return predictability could be exploitable in an international context. Indeed, although stock returns can be predictable from behavioral and business-cycle measures, they are not necessarily profitable because of high transaction costs (Pesaran and Timmermann 1994, 1995). Moreover, knowing that stock returns are predictable, does not tell investors which factors have predictive power at the different phases of the business cycles.

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A Appendix

Table A.I
Variable description

Variable	Description
DEPENDENT VARIABLE	
Financial stock returns	1, 3, 6, 9 and 12-month returns of the Datastream Financial Index. It is a broad index for the financial stock market including banks, insurance and real estate companies. The index is not adjusted for dividend.
EXPLANATORY VARIABLES	
1. Investor sentiment proxies	
Investor sentiment (CCI)	We proxy investor sentiment by a Consumer Confidence Indicator (CCI) stemming from different data sources.
Investor sentiment (ESI)	Economic Sentiment Indicator is an alternative proxy for investor sentiment. It is a composite leading indicator based on surveys on consumers and on sectors such as industry, services, retail trade and construction.
2. Economic variables	
Real GDP growth	The frequency is quarterly. To obtain a monthly series we repeat the same value three times within a quarter.
Real GDP growth forecasts	Real output growth average forecasts from professionals such as research institutes and banks, in monthly frequency.
Industrial production growth	The industrial production is available monthly (unavailable for Australia and New Zealand) but for some countries the series start later than January 1999.
CPI inflation	Percentage change of the Consumer Price Index.
FDIFF. 3-month interbank rate	3-month interbank rate taken in first difference to obtain stationary series.
Term spread	Spread between 10-year and 1-year government bond yields.
3. Financial variables	
Volatility	Annual realized volatility of the financial stock returns.
Dividend yield	Dividend yield of financial stock returns.
Liquidity (VA/MV)	Ratio of the index monthly volume in value over the index market capitalization.
Liquidity (VO/NOSH)	Number of shares traded monthly standardized by the number of index constituent shares to account for size.
Ln(Market value/Firms)	Natural logarithm of the market value of the country-specific financial index over the number of firms included in this index.

Note: All the variables are in monthly frequency, except the real GDP growth which is reported only quarterly.

Table A.II

Auxiliary regression for CCI and ESI: PSTR and PTR specifications

PANEL A	Consumer Confidence Indicator (CCI)				Economic Sentiment Indicator (ESI)			
	PSTR		PTR		PSTR		PTR	
	Regime 1 $\boldsymbol{\delta}_0$	Regime 2 $\boldsymbol{\delta}_0 + \boldsymbol{\delta}_1$	Regime 1 $\boldsymbol{\theta}_1$	Regime 2 $\boldsymbol{\theta}_2$	Regime 1 $\boldsymbol{\delta}_0$	Regime 2 $\boldsymbol{\delta}_0 + \boldsymbol{\delta}_1$	Regime 1 $\boldsymbol{\theta}_1$	Regime 2 $\boldsymbol{\theta}_2$
GDP growth	0.208***	0.248***	0.233***	0.244***	0.223***	0.263***	0.226***	0.281***
	(17.852)	(12.882)	(22.433)	(24.880)	(25.258)	(9.348)	(25.831)	(17.297)
CPI inflation	-0.059***	-0.503***	-0.061***	-0.328***	-0.105***	-0.269***	-0.112***	-0.202***
	(-3.132)	(-15.535)	(-3.632)	(-17.659)	(-6.202)	(-5.847)	(-6.571)	(-7.803)
Unemployment rate	-0.117***	-0.071***	-0.120***	-0.069***	-0.032***	-0.102***	-0.038***	-0.064***
	(-10.489)	(-4.566)	(-10.906)	(-6.525)	(-3.689)	(-4.132)	(-4.307)	(-4.637)
FDIFF 3-month interbank	0.841***	0.647***	0.724***	0.781***	1.149***	0.424***	1.031***	0.816***
	(7.419)	(4.467)	(6.733)	(7.393)	(12.074)	(2.632)	(11.079)	(6.540)
Smoothness parameter γ	4.326 [2.707, 6.931]				6.029 [1.909, 13.293]			
Location parameter c	0.544 [0.422, 0.672]		0.228 [0.214, 0.266]		0.884 [0.749, 1.710]		0.605 [0.388, 0.630]	
Observations	3,000		3,000		3,000		3,000	
Number of id	20		20		20		20	
R-squared	0.493		0.489		0.518		0.514	
AIC	-0.681		-0.667		-0.727		-0.714	
BIC	-0.661		-0.649		-0.708		-0.696	

PANEL B — Testing the null hypothesis of linearity, and subsequently the null hypothesis of a two-regime specification

	Consumer Confidence Indicator			Economic Sentiment Indicator		
	PSTR: LM test	PSTR: F test	PTR: F_1 test	PSTR: LM test	PSTR: F test	PTR: F_1 test
$H_0 : L = 0$ vs $H_1 : L = 1$	126.407 (0.000)	32.728 (0.000)	203.650 (0.000)	58.110 (0.000)	14.696 (0.000)	80.550 (0.000)
$H_0 : L = 1$ vs $H_1 : L = 2$	10.779 (0.029)	2.676 (0.030)	64.214 (1.000)	28.631 (0.000)	7.150 (0.000)	62.191 (1.000)

Notes. In this table are displayed the estimation results (Panel A) of the PSTR and PTR models for Consumer Confidence Indicator (CCI) and Economic Sentiment Indicator (ESI) as a function of 1-month lags of real GDP growth (RGDP), CPI inflation (INFL), unemployment rate (URATE), first difference of 3-month interbank interest rate (D.IRATE). The transition variable is the 1-month lag of the spread between the 3-month and overnight interbank rates. We follow Lemmon and Portniaguina (2006) and Baker and Wurgler (2006, 2007) and consider the residuals from these estimations ($\hat{\upsilon}_{i,t}$, $\hat{\vartheta}_{i,t}$ below) as an investor sentiment unrelated to economic fundamentals. In parentheses are presented t-statistics with standard errors corrected for heteroskedasticity. ***, ** and * denote statistical significance at 0.01, 0.05 and 0.1 level.

The PSTR model can be written as: $I_{i,t} = \mu_i + \left[\delta_{0,1} RGDP_{i,t-1} + \delta_{0,2} INFL_{i,t-1} + \delta_{0,3} URATE_{i,t-1} + \delta_{0,4} D.IRATE_{i,t-1}\right] + \left[\delta_{1,1} RGDP_{i,t-1} + \delta_{1,2} INFL_{i,t-1} + \delta_{1,3} URATE_{i,t-1} + \delta_{1,4} D.IRATE_{i,t-1}\right] G(q_{i,t-1}, \gamma, c) + \upsilon_{i,t}$. $I_{i,t}$ stands for the level of either CCI, or ESI.

The PTR model is written as: $I_{i,t} = \mu_i + \left[\theta_{1,1} RGDP_{i,t-1} + \theta_{1,2} INFL_{i,t-1} + \theta_{1,3} URATE_{i,t-1} + \theta_{1,4} D.IRATE_{i,t-1}\right] \mathbb{1}_{(q_{i,t-1} \leq c)} + \left[\theta_{2,1} RGDP_{i,t-1} + \theta_{2,2} INFL_{i,t-1} + \theta_{2,3} URATE_{i,t-1} + \theta_{2,4} D.IRATE_{i,t-1}\right] \mathbb{1}_{(q_{i,t-1} > c)} + \vartheta_{i,t}$.

In Panel B are displayed some tests for the null hypothesis of linearity ($L = 0$) against the alternative hypothesis of a two-regime model ($L = 1$), with L the number of transition functions in the PSTR and the number of location parameters in the PTR. Subsequently, the null hypothesis of two regimes ($L = 1$) is tested against the alternative of a three-regime model ($L = 2$). For the PSTR we perform two tests: LM (χ^2 distribution), F (Fisher distribution), and for PTR we perform the F_1 test whose distribution is obtained by a bootstrapping procedure (see Hansen 1999). The p-values are reported in parentheses.

Table A.III

Summary statistics for the annual return of Datastream Financials Index from Jan. 1999 to Aug. 2011

	Overall sample					Jan. 1999 - Aug. 2007					Sep. 2007 - Aug. 2011				
	Obs.	Mean	Std.	Min	Max	Obs.	Mean	Std.	Min	Max	Obs.	Mean	Std.	Min	Max
Australia	152	4.82	18.55	-47.02	52.86	104	10.26	10.39	-17.88	27.64	48	-6.98	25.72	-47.02	52.86
Austria	152	11.05	31.02	-70.11	107.27	104	18.02	19.01	-25.81	52.16	48	-4.06	44.25	-70.11	107.27
Belgium	152	-1.14	29.72	-75.30	67.56	104	6.65	22.75	-42.52	52.93	48	-18.01	35.77	-75.30	67.56
Canada	152	8.21	19.31	-44.37	65.55	104	12.27	15.24	-19.38	52.79	48	-0.59	23.97	-44.37	65.55
Denmark	152	10.60	32.10	-66.21	94.08	104	19.09	24.61	-23.84	91.59	48	-7.82	38.43	-66.21	94.08
Finland	152	8.56	26.34	-46.70	78.30	104	11.40	24.69	-38.44	61.94	48	2.41	28.92	-46.70	78.30
France	152	5.32	26.98	-58.58	71.36	104	12.76	20.94	-35.77	57.58	48	-10.78	31.46	-58.58	71.36
Germany	152	0.37	28.26	-63.84	65.02	104	4.78	28.49	-63.84	65.02	48	-9.17	25.52	-52.92	45.94
Greece	152	5.26	57.25	-72.43	268.13	104	22.06	58.98	-50.81	268.13	48	-31.14	30.48	-72.43	37.20
Italy	152	-1.48	25.02	-61.14	43.22	104	6.75	21.18	-35.95	38.68	48	-19.29	23.57	-61.14	43.22
Japan	152	-1.80	32.89	-50.92	109.41	104	7.93	34.13	-36.51	109.41	48	-22.88	16.11	-50.92	9.98
Netherlands	152	-0.80	28.54	-72.45	74.96	104	4.19	21.86	-52.74	39.87	48	-11.62	37.36	-72.45	74.96
New Zealand	152	0.69	15.18	-31.08	32.73	104	6.19	13.60	-23.70	32.73	48	-11.22	11.09	-31.08	9.59
Norway	152	12.99	36.86	-70.86	169.20	104	14.12	22.22	-41.40	73.86	48	10.54	57.22	-70.86	169.22
Portugal	152	-3.64	27.54	-67.82	56.48	104	6.52	21.22	-35.74	56.48	48	-25.65	26.96	-67.82	32.34
Spain	152	2.24	24.09	-55.70	61.63	104	8.92	19.27	-33.38	46.45	48	-12.21	27.17	-55.70	61.63
Sweden	152	7.95	28.46	-51.41	85.79	104	11.19	24.90	-36.00	53.16	48	0.91	34.19	-51.41	85.79
Switzerland	152	-0.02	26.49	-54.03	61.52	104	6.13	24.45	-47.08	61.52	48	-13.34	26.08	-54.03	55.66
United Kingdom	152	0.42	21.72	-60.04	68.24	104	4.90	14.82	-34.25	33.17	48	-9.30	29.89	-60.04	68.24
United States	152	1.72	22.05	-63.83	76.63	104	6.97	12.84	-21.01	43.40	48	-9.67	31.75	-63.83	76.63

Data source: Datastream.

Notes: In this table are presented summary statistics by country for the annual rate of return (in %) of Datastream Financial Index. The data set contains a panel of 20 developed countries for the period starting from January 1999 to August 2011. In order to put in evidence the impact of the recent global financial crisis, the data set is split in two subsamples, the threshold between the two being the outset of the downturn, August 2007. The summary statistics for the overall sample are also displayed.

Table A.IV

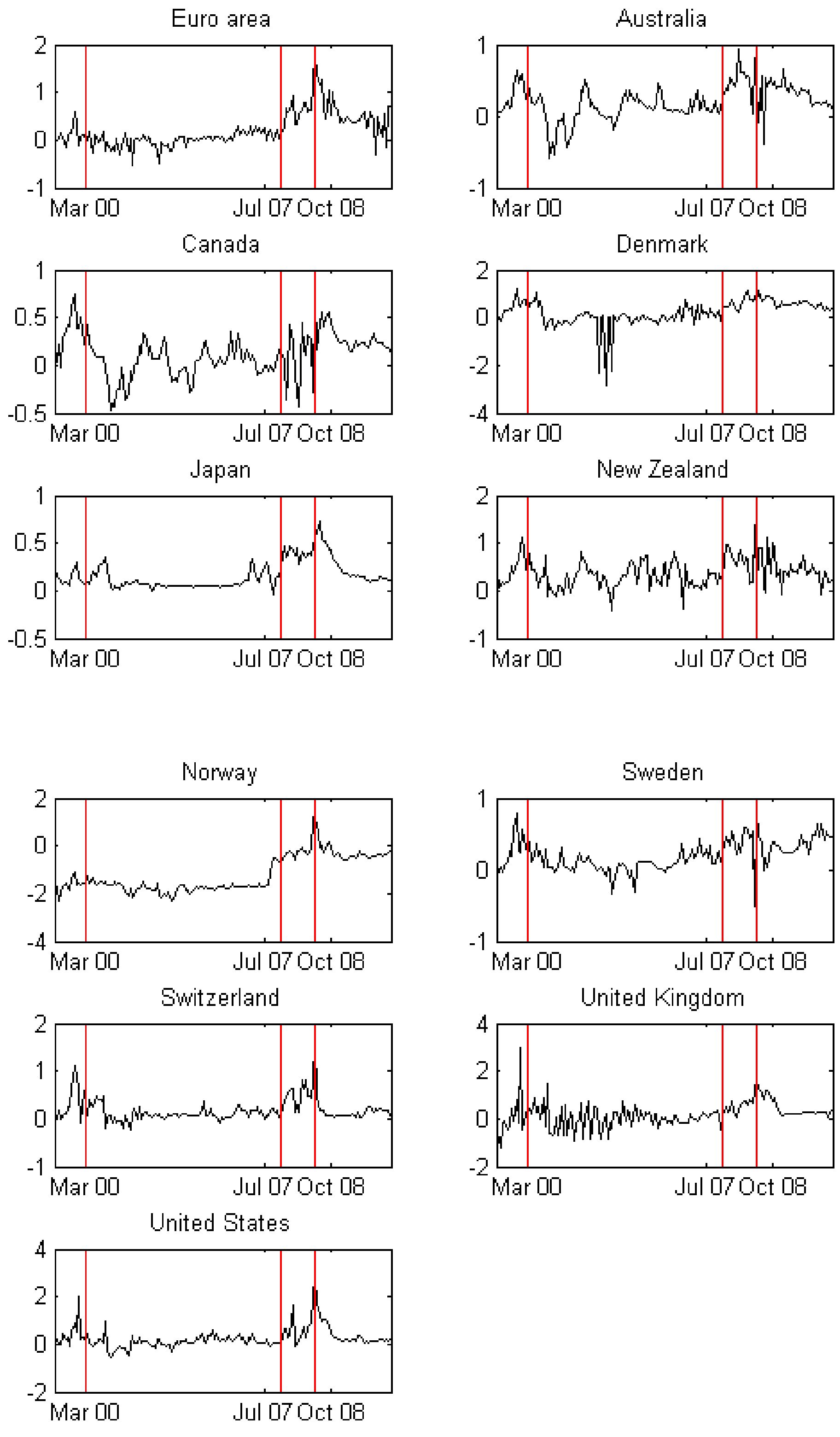
Stationarity tests, January 1999 to August 2011

Variable	Pesaran $\bar{t}$−statistic		Fisher D-Fuller		Fisher P-Perron		LLC t−statistic		IPS $\bar{t}$−statistic	
DEPENDENT VARIABLE										
Annual financial stock returns	-3.41	(0.000)***	124.58	(0.000)***	93.61	(0.000)***	-15.27	(0.000)***	-3.41	(0.000)***
9-month financial stock returns	-3.55	(0.000)***	163.68	(0.000)***	122.09	(0.000)***	-15.19	(0.000)***	-3.54	(0.000)***
6-month financial stock returns	-4.09	(0.000)***	262.38	(0.000)***	200.45	(0.000)***	-18.66	(0.000)***	-4.29	(0.000)***
3-month financial stock returns	-5.84	(0.000)***	533.19	(0.000)***	431.63	(0.000)***	-28.21	(0.000)***	-6.31	(0.000)***
1-month financial stock returns	-6.19	(0.000)***	1055.62	(0.000)***	1581.24	(0.000)***	-41.58	(0.000)***	-9.30	(0.000)***
EXPLANATORY VARIABLES										
Investor sentiment (CCI)	-3.01	(0.000)***	152.43	(0.000)***	220.32	(0.000)***	-12.23	(0.000)***	-2.99	(0.000)***
Investor sentiment (ESI)	-3.14	(0.000)***	144.16	(0.000)***	212.81	(0.000)***	-12.22	(0.000)***	-2.96	(0.000)***
Real GDP growth rate (SADJ)	-2.60	(0.000)***	159.30	(0.000)***	64.61	(0.008)***	-11.45	(0.000)***	-2.70	(0.000)***
Real GDP forecasts	-1.90	(0.306)	77.26	(0.000)***	23.71	(0.943)	-7.00	(0.5419)	-1.84	(0.062)*
CPI inflation	-2.60	(0.000)***	114.53	(0.000)***	87.90	(0.000)***	-10.84	(0.015)**	-2.58	(0.000)***
3-month interbank rate	-1.30	(0.992)	26.70	(0.947)	12.47	(1.000)	-6.88	(0.903)	-1.76	(0.119)
FDIFF.3-month interbank rate	-6.13	(0.000)***	588.28	(0.000)***	942.43	(0.000)***	-36.36	(0.000)***	-8.60	(0.000)***
Term spread (2 lags)	-2.22	(0.016)**	69.00	(0.003)***	221.83	(0.000)***	-8.09	(0.724)	-2.43	(0.000)***
Term spread	-0.273	(1.000)	47.259	(0.200)	37.775	(0.571)	2.043	(1.000)	0.151	(1.000)
FDIFF.Term spread	-6.186	(0.000)***	935.492	(0.000)***	1588.082	(0.000)***	-39.117	(0.000)***	-8.838	(0.000)***
Volatility	-5.86	(0.000)***	455.44	(0.000)***	840.46	(0.000)***	-25.92	(0.000)***	-5.97	(0.000)***
Dividend yield	-2.41	(0.001)***	87.43	(0.000)***	70.70	(0.002)***	-10.26	(0.001)***	-2.56	(0.000)***
Liquidity (VA/MV)	-4.31	(0.000)***	299.83	(0.000)***	531.27	(0.000)***	-15.42	(0.000)***	-4.00	(0.000)***
Liquidity (VO/NOSH)	-4.19	(0.000)***	326.46	(0.000)***	521.29	(0.000)***	-16.71	(0.000)***	-4.16	(0.000)***
Ln(Market size/No. firms)(4 lags)	-1.90	(0.285)	41.93	(0.387)	21.94	(0.991)	-9.36	(0.041)**	-2.36	(0.000)***
TRANSITION VARIABLE										
Spread: 3-month-overnight interb.	-5.54	(0.000)***	214.29	(0.000)***	468.29	(0.000)***	-19.91	(0.000)***	-5.70	(0.000)***

Data sources: Datastream, KOF, Bank of Denmark.
Notes: In this table are presented the results of five panel unit root tests (with 1 lag): the Pesaran test, the Fisher tests (augmented Dickey-Fuller and Phillips-Perron), the Levin, Lin and Chu (LLC) test, the Im, Pesaran and Shin (IPS) test. ***, **, * denote significance at 0.01, 0.05, 0.1 level, respectively and p-values are in parentheses. The tests assume under the null hypothesis that all the series composing the panel are non-stationary. The LLC test assumes that each individual unit in the panel shares the same AR(1) coefficient. Consequently, under the alternative hypothesis all cross-sections are stationary with the same parameter. In the other tests, not all individual series need to be stationary under the alternative hypothesis. For the real output growth the tests are performed from the quarterly data. The forecasts of the real output growth are not available for Australia and New Zealand. "FDIFF" is a prefix for the variables taken in first difference.

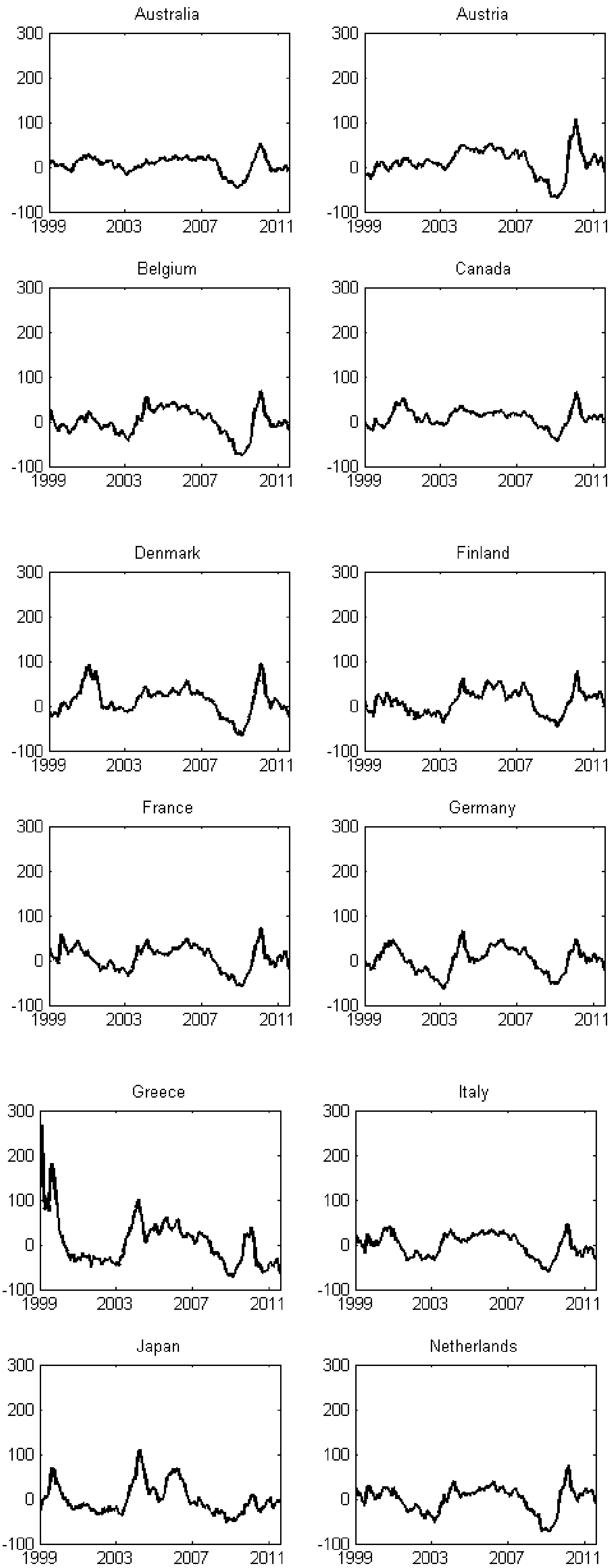
Figure A.I

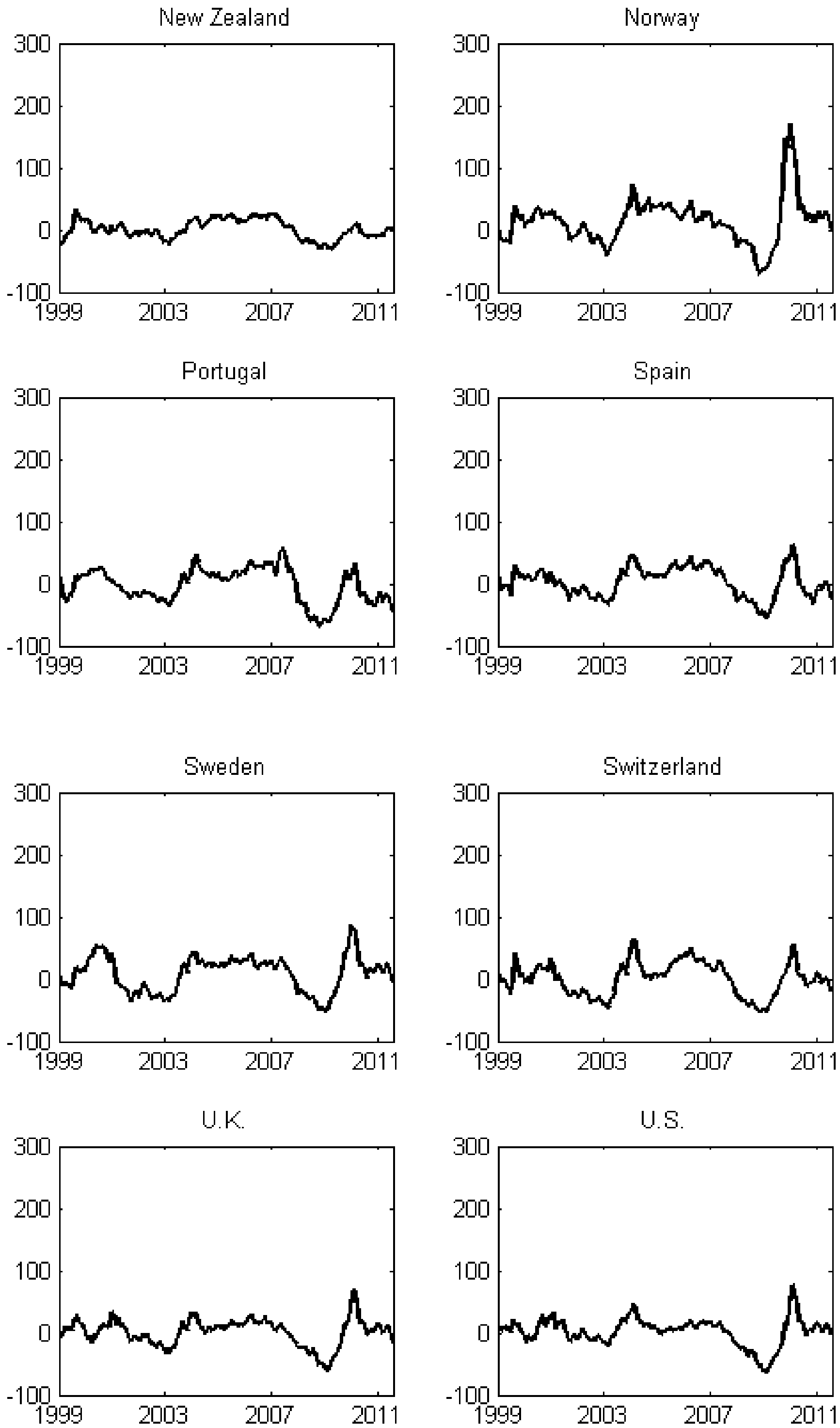
Transition variable: spread 3-month-overnight interbank rates



Notes: The transition variable is the spread between 3-month and overnight interbank rates. We draw vertical lines in March 2000, July 2007 and October 2008 which correspond to the peak of the dot-com bubble, onset and peak of the financial crisis, respectively. For the euro area countries the transition variable is identical and is given by the difference between the 3-month euribor and the eonia. For the predictive models we reduce and standardize this variable.

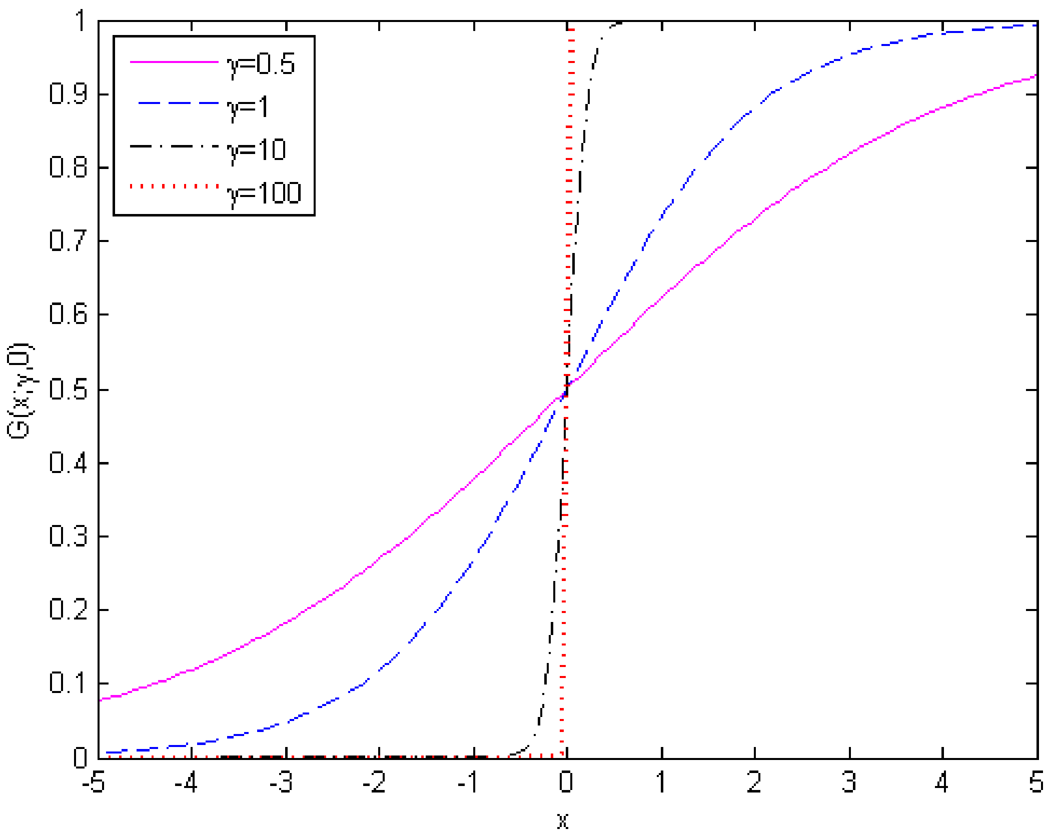
Figure A.II
Annual stock returns of financial companies





Notes: The annual return of Datastream Financials Index is computed as the year on year percentage change.

Figure A.III
Logistic distribution with different values of γ



Notes: In this figure is displayed the logistic distribution with different values of the smoothness parameter γ. The distribution is given by the following formula: $G(x;\gamma,c) = \left(1+e^{-\gamma(x-c)}\right)^{-1}$ with the location parameter c set equal to 0.

Chapter 2

Commonality in Liquidity and Real Estate Securities*

Abstract

We conduct an empirical investigation of the exposure of U.S. REIT returns to commonality in liquidity. Taking advantage of the specific characteristics of REITs, we study three types of commonality in liquidity: within-asset commonality, cross-asset commonality (with the stock market), and commonality with the underlying property market. We find evidence that the three types of commonality in liquidity represent significant risk factors for REIT returns but only during bad market conditions. We also find that using a linear approach, rather than a conditional, would have underestimated the role of commonality in liquidity risk. This could explain (at least partly) the small impact of commonality on asset prices documented in the extant literature. We also analyze the economic sources of commonality in liquidity and find that demand-side factors prevail over supply-side factors.

JEL Classification: *G12, G01, G02*

Keywords: *Real Estate Securities, REITs, Commonality in Liquidity, Liquidity Risk, Multi-Factor Model, Threshold Regression, Panel Data*

2.1 Introduction

Theoretical and empirical research has shown that liquidity risk is a priced factor for several assets. For instance, Acharya and Pedersen (2005) propose an asset pricing model in which liquidity risk has three dimensions: commonality in liquidity, the covariance of asset returns with market liquidity, and the covariance of asset liquidity with market returns. Their empirical analysis of the U.S. stock market provides support for their model. In this paper, we seek to expand our knowledge of the liquidity risk by focusing on one of its components, i.e., the commonality in liquidity, which is defined as the level of comovement between a security's liquidity and that of the overall market. This feature precludes investors from forming optimal portfolios where liquidity risk would be diversified away (i.e., systematic liquidity risk). Investors seek to hold assets that allow them to exit positions at a minimum cost when most needed, that is, during market downturns or liquidity dry-ups. Consequently, one would expect a compensation for

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holding an asset whose liquidity covaries with market liquidity. Despite this theoretical support, the empirical literature on the implications of liquidity commonality is quite scarce and shows that the impact of commonality on asset prices is small (Acharya and Pedersen 2005; Lee 2011). Our study aims to provide for a better understanding of this gap between theory and empirical evidence.

We investigate one potential explanation that is based on the idea that the exposure to liquidity risk could be time-varying. The extant empirical literature tests commonality in liquidity exclusively within an unconditional framework, while commonality in liquidity should be perceived differently by investors depending on the state of the economy. Indeed, in times of good economic conditions, when market liquidity tends to increase, investors would seek stronger commonality in liquidity (i.e., to hold an asset whose liquidity increases too), whereas they would want the opposite in adverse conditions, when market liquidity decreases (i.e., to hold an asset whose liquidity does not decrease). In other words, we expect the impact of commonality in liquidity risk on asset prices (i.e., *beta*) to increase, or to be only significant, in stressful times. Thus, we conjecture that previous studies find a small commonality effect on prices because this effect is smoothed out across the different states of the economy. Our empirical results support this hypothesis.

We use Real Estate Investment Trust (REIT) data as laboratory for our empirical investigation. Two main reasons motivate this choice. First, real estate companies own assets for which there are benchmarking and trading activity indicators, which is not the case for most other types of companies. Taking the commercial real estate market as proxy for the assets held by REITs, we are thus able to analyze the *commonality in liquidity with the underlying asset* and its impact on prices. We thus address the important question of whether liquid equity claims on relatively illiquid assets retain liquidity when that of the underlying asset is decreasing. Second, the hybrid nature of real estate securities allows us to look at yet another dimension of commonality in liquidity, i.e. *cross-asset commonality in liquidity*. Given that securitized real estate investments are listed on stock exchanges, it is reasonable to assume that the liquidity of the overall equity market could also affect the liquidity of real estate securities.[1] Thus, the specific characteristics of REITs allow us to look at two other types of commonality in liquidity whose pricing implications remain unexplored.

A focus on real estate securities is also important for investors as this type of asset is often preferred to direct real estate investments given the high unit value and illiquidity of real estate assets. Ciochetti, Craft and Shilling (2002), for instance, show that institutional investors take larger positions in REIT stocks, as compared with private real estate, because of liquidity considerations. However, little is known about the potential influence of liquidity risk on securitized real estate returns. Whereas Subrahmanyam (2007) shows that current negative liquidity shocks forecast higher future returns in the U.S. market, he does not investigate systematic liquidity risk per se. Although more liquid than a direct investment, real estate stocks are not necessarily immune to liquidity risk in the sense of Acharya and Pedersen (2005). Liquidity risk differs from the average level of liquidity of an asset. Indeed, liquidity risk pertains to the fact that liquidity varies over time and displays commonality across securities (Chordia, Roll and Subrahmanyam 2000), whereas the liquidity level is an asset-specific characteristic.

A multi-factor approach is adopted for testing the impact of commonality in liquidity on asset prices as we control for the effects of market return, value and size, momentum,

[1] Though quite scarce, the literature has shown some evidence of commonality in liquidity across different asset classes; see for instance Chordia, Sarkar and Subrahmanyam (2005).

credit conditions, investor sentiment, and market volatility. Consistent with Acharya and Pedersen (2005), both the sensitivity of an asset's liquidity to market returns and that of its returns to innovations in market liquidity[2] are also controlled for. We use U.S. REIT firm-specific data within a panel model with switching regimes (Panel Threshold Regression model) developed by Hansen (1999) for our empirical investigation. This nonlinear specification allows the impact of liquidity risk to be conditional on the state of the economy. With the exceptions of the papers by Watanabe and Watanabe (2008), and Acharya, Amihud and Bharath (2013), studies usually adopt an unconditional approach for analyzing the liquidity patterns. A linear model is also estimated in order to gauge the relevance of a regime-switching modeling.

We further seek to identify the sources of the within- and cross-asset commonality in liquidity. We employ a time-series Threshold Regression (TR) model (Tsay 1989) for the analysis of the determinants of liquidity commonality (as averaged at the market level) over time, which should better reflect market dynamics. We test both supply-side and demand-side determinants of liquidity as in Karolyi, Lee and van Dijk (2012). The theoretical underpinning for the supply-side explanation of liquidity is based on the relation between market participants' funding constraints (i.e., funding liquidity) and asset liquidity as suggested by Brunnermeier and Pedersen (2009). Their theory predicts that commonality in liquidity results from the fact that the ability to obtain funding for leveraged market participants (e.g. financial intermediaries) holding various securities is impaired in times of large market declines or high volatility (funding liquidity shock). This forces them to liquidate positions across several securities, which increases the commonality in liquidity. Brunnermeier and Pedersen also show that a decrease of market liquidity further tightens funding liquidity creating a liquidity spiral that increases the commonality in liquidity even further. Thus, financial intermediaries fail to provide liquidity to the market in bad environments. In sum, the supply-side hypothesis stipulates that commonality in liquidity is negatively linked to credit conditions and is higher when markets decline.[3]

As regards the demand-side explanation, we rely on two potential sources: correlated trading activity and investor sentiment. The first source argues that if investors tend to trade in concert, this leads to common buying or selling pressures (i.e., trade imbalances) that reinforce the degree of comovement between securities' liquidity. This argument stems from the idea that investors with similar trading patterns should face the same shocks in liquidity or changes in the information available and would therefore trade in the same way in response to those shocks (Chordia, Roll and Subrahmanyam 2000). This hypothesis is also consistent with Barberis, Shleifer and Wurgler's (2005) category-based model of comovement, which states that investors form asset categories and trade based on such categories, leading to increases in the level of comovement within these categories.[4] We expect therefore a positive impact of correlated trading activity (as proxied by the commonality in turnover) on commonality in liquidity.

The second source is in the wake of the growing behavioral finance literature that

[2]This dimension of liquidity risk is the same as the one suggested by Pástor and Stambaugh (2003).

[3]Other important models that demonstrate the effects of financial intermediaries' financing constraints on market liquidity include, among others, Kyle and Xiong (2001), and Gromb and Vayanos (2002).

[4]More generally, Barberis, Shleifer and Wurgler (2005) propose a theory that explains the comovements between asset prices in economies with frictions or with irrational investors (i.e., "friction-based" or "sentiment-based" theory of comovement). This theory includes three different views that could explain the level of comovement between asset prices over what is justified by fundamentals: the category view, the habitat view and the information diffusion view. For more details, we refer the reader to this reference.

stresses the role of noise traders and investor sentiment in the price formation and return comovements (Baker and Wurgler 2006; Kumar and Lee 2006). The sentiment hypothesis is in the spirit of the habitat-based model of comovement of Barberis, Shleifer and Wurgler (2005) that suggests a higher comovement between securities held by a subset of investors, such as individual investors, that choose to trade only some securities due to, for instance, lack of information. Thus, changes in preferences or sentiment in those investor groups influence the linkages between securities' returns. Similar effects on liquidity commonality are expected as shown by Huberman and Halka (2001). The investor sentiment hypothesis does not offer clear theoretical predictions regarding the sign of the relationship between investor sentiment and commonality in liquidity (Karolyi, Lee and van Dijk 2012). However, we conjecture that within our two-regime framework a higher optimism should impact commonality in the normal regime, whereas a higher pessimism should impact it in the crisis regime.

Our main results are as follows. First, we find that the exposure (i.e., beta) of REIT returns to the within-asset commonality in liquidity varies between two distinct regimes: a low-volatility regime characterized by a negative beta and a high-volatility regime characterized by a positive beta. These results imply that expected REIT returns increase following a rise in commonality in liquidity only during bad market conditions, consistent with the existence of time-varying liquidity risk. We also find that both the cross-asset commonality in liquidity and the liquidity correlation between REITs and the underlying property market represent significant risk factors but again only during market downturns. The comparison of these results with those of the linear model shows that commonality in liquidity would not have been considered as a risk factor (in most cases) if such an approach had been adopted. Thus, the use of an unconditional approach could be misleading with respect to the impact of commonality in liquidity on REIT prices. In addition, we find that these results are economically significant. We further uncover that REIT prices are sensitive to shocks in REIT and stock market liquidity but that they are immune to those in the private real estate market liquidity. Taken together, these findings suggest that the liquidity advantages of REITs should be nuanced. Finally, our results favor a demand-side explanation of commonality in liquidity.

Our paper contributes to the existing literature in several ways. First, we provide new insights to the literature on liquidity risk by showing that commonality in liquidity risk is time-varying, which contrasts with the extant literature that has only analyzed commonality within an unconditional framework. This finding is important because it could explain (at least partly) the gap between the theory on commonality in liquidity and the empirical evidence. In addition, the asset pricing literature has mainly studied the sensitivity of asset returns to market liquidity shocks, whereas commonality in liquidity has received much less attention. We also contribute to the literature by showing that different types of commonality in liquidity could have an impact on asset prices. Finally, we contribute to the real estate literature by examining an asset whose higher liquidity represents a key characteristic in real estate investment decision-making but whose liquidity risk is not fully comprehended.

The paper is structured as follows. We first review the extant empirical literature. Then, we develop our empirical strategy, before describing the data. We then turn to a discussion of our results. Given the nature of private real estate data, the analysis of the commonality in liquidity between REITs and direct real estate is contained in a separate section. A final section concludes.

2.2 Empirical literature

The bulk of the literature on liquidity risk has focused on the stock market. Pástor and Stambaugh (2003), by means of a four-factor model with liquidity risk, show that U.S. stocks with higher sensitivity to innovations in market liquidity exhibit higher expected returns. Acharya and Pedersen (2005) develop a liquidity-adjusted capital asset pricing model (LCAPM), where liquidity risk has three distinct dimensions (commonality in liquidity, the covariance of a security's return with market liquidity, and the covariance of a security's liquidity with market returns). In an analysis of the U.S. stock market, their empirical findings strongly support the LCAPM. Lee (2011) extends the previous work by applying the LCAPM in an international context; his findings are in line with Acharya and Pedersen's theory. However, those authors find that commonality in liquidity carries a relatively small premium within an unconditional framework. Focusing on the pricing of commonality in liquidity, Anderson et al. (2013) argue that this small premium is due to the correlation between illiquidity and commonality risk and the portfolio-sorting strategy in asset pricing tests. Instead of sorting portfolios only on liquidity level, they apply a double-sorting procedure that considers commonality in liquidity and thus eliminate any overlap between the two premia. Their empirical results show that commonality commands an economically significant premium in the U.S. stock market.

The evidence of a priced liquidity risk has been extended recently to other types of assets. Li et al. (2009) and Acharya, Amihud and Bharath (2013) study the exposure of U.S. bond returns to liquidity shocks, whereas Mancini, Ranaldo and Wrampelmeyer (2013) analyze liquidity risk in the foreign exchange markets. Hedge funds (Sadka 2010; Gibson Brandon and Wang 2013), credit derivatives (Longstaff, Mithal and Neis 2005; Bongaerts, de Jong and Driessen 2010) and private equity (Franzoni, Nowak and Phalippou 2012) have also been investigated. All of these studies show that liquidity risk is a significant risk factor. However, these studies only consider one dimension of liquidity risk, i.e., the sensitivity of asset returns to market liquidity. Overall, the literature on the implications of commonality in liquidity is thus quite limited.

Although the relationship between real estate stock prices and liquidity risk remain unexplored, several studies have however analyzed the liquidity dynamics within the securitized real estate market. Clayton and MacKinnon (2000) use U.S. trade-by-trade data to investigate the changes in REIT liquidity with an emphasis on the period following the 1993 boom. They find strong evidence of increased REIT liquidity between 1993 and 1996. Marcato and Ward (2007) compare these results with those obtained from daily data. Having shown that their results are consistent with those of Clayton and MacKinnon (2000), they extend their work to an international setting. Brounen, Eichholtz and Ling (2009) also analyze the liquidity dynamics of property company shares in an international context but they further investigate their determinants. They find that the liquidity of real estate firms varies importantly across countries and that nonproperty firms of similar size are on average more liquid. Moreover, they find that liquidity is positively related to the firm's market capitalization and negatively to the percentage of shares held by nonretail investors.

A branch of the literature has looked at the relationship between the liquidity of a firm's security and the liquidity of the assets owned by that firm. Gopalan, Kadan and Pevzner (2012) find that asset liquidity significantly impacts stock liquidity. They also document that an increase in asset liquidity leads to a higher firm value. In a REIT context, Benveniste, Capozza and Seguin (2001) show that creating liquid claims on illiquid real estate assets increases the value of these claims. In a setting closely related to this topic, Bond and Chang (2012) look at the cross-asset liquidity between REITs and the private real estate market and also document some relationship. Thus, the underlying assets' liquidity can influence both the liquidity and the price of a security, which further motivates our hypothesis that the commonality in liquidity with the underlying real estate market could be a significant risk factor in the REIT market.

Finally, several studies have sought to disentangle the dynamics of commonality in liquidity. Consistent with Brunnermeier and Pedersen's (2009) theory, Hameed, Kang and Viswanathan (2010) show that market liquidity dry-ups and commonality in liquidity in the U.S. stock market are more likely during market declines and times of tightness in the funding market. Although these results strongly support the supply-side explanation, the authors do acknowledge that factors related to the demand for liquidity may also influence their findings. In this spirit, Karolyi, Lee and van Dijk (2012) examine the sources of commonality in liquidity across 40 developed and emerging countries. Their results favor the demand-side determinants of liquidity and challenge therefore the ability of Brunnermeier and Pedersen's theory to fully explain commonality in liquidity. In a study of the impact of mutual funds' correlated trading on commonality in liquidity, Koch, Ruenzi and Starks (2010) also suggest an important role for demand-side factors.

2.3 Research methodology

2.3.1 Liquidity variables

In this section, we discuss the liquidity-related variables used for the within- and cross-asset commonality in liquidity analyses. We first specify the commonality in liquidity variable and the variable proxying for the sensitivity of a firm's liquidity to market returns. Then, we discuss the computation of the market liquidity risk factor.

Commonality in liquidity variable

Liquidity is an elusive concept and no perfect measure exists that captures the different aspects[5] of such a complex asset/market characteristic while having sufficient data to cover a large number of assets for a long time period, crucial for a reliable asset pricing analysis. We employ the Amihud's (2002) illiquidity measure which relies on daily data. This measure captures the market depth (price impact measure of liquidity) and is defined by:

$$ILLIQ^i_{t,d} = \frac{|r^i_{t,d}|}{VOLD^i_{t,d}} \tag{2.1}$$

with $r^i_{t,d}$ and $VOLD^i_{t,d}$, the daily return and volume on stock i on day d of month t, respectively. Many recent empirical studies have used this proxy (e.g. Karolyi, Lee and van Dijk 2012; Acharya, Amihud and Bharath 2013) which is, as shown by Goyenko, Holden and Trzcinka (2009), highly correlated with liquidity measures based on microstructure data (i.e., bid-ask spread). We apply a detrending factor to the above measure to control

[5]Kyle (1985) suggests three transactional characteristics that can be used to describe the liquidity of a financial asset: (1) the cost of liquidating an asset over a short period of time (tightness); (2) the ability to trade a large quantity of securities with the minimal price impact (depth); and (3) the propensity of prices to recover quickly after a shock.

for the effects of inflation by multiplying $ILLIQ$ by the ratio between the total REIT market capitalization at time $t-1$ and that at the beginning of the sample period (Pástor and Stambaugh 2003; Acharya and Pedersen 2005). To obtain a measure of liquidity (LIQ), we simply take the inverse of $ILLIQ$.

Karolyi, Lee and van Dijk (2012) utilize the R^2 of regressions as a measure of commonality in liquidity. More specifically, the R^2 of a regression of a security's liquidity on market liquidity (both contemporaneous and lagged) is estimated for each firm. Importantly, the market liquidity excludes the security analyzed to avoid any endogeneity issue and it is computed as the value-weighted average across the remaining securities' liquidity. We follow those authors and adopt the same approach for our measure of commonality in liquidity:

$$LIQ^i_{t,d} = \alpha^i_t + \beta^i_{1,t} LIQ^M_{t,d} + \beta^i_{2,t} LIQ^M_{t,d-1} + \upsilon^i_{t,d} \tag{2.2}$$

where $LIQ^i_{t,d}$ denotes the liquidity level of security i on day d within month t, and $LIQ^M_{t,d}$ the aggregate market liquidity. The market liquidity can be either that of securitized real estate (within-asset commonality in liquidity) or that of stocks (cross-asset commonality in liquidity). This regression is estimated each month t for each firm i which yields a time-series of commonality in liquidity ($R^2_{i,t}$) for all individuals that make up our sample. We require a minimum of 10 daily observations within a month for a given firm to estimate the R^2. We proceed identically for constructing the correlated trading activity variable which is proxied by the commonality in turnover ($Cturn_{i,t}$). The turnover, i.e., the ratio between the trading volume and the number of shares outstanding, is modeled instead of liquidity in Equation 2.2.

So far, we have discussed the computation of the liquidity and commonality in liquidity levels. We turn now to the construction of the commonality in liquidity risk factor to be used in the asset pricing model ($CLIQ$). We construct a commonality in liquidity factor at the aggregate level and return-based (traded liquidity factor). Hence, the construction of this risk factor follows the same rationale as, for example, the size factor in the Fama and French three-factor model (i.e., mimicking portfolios). Based on the levels of commonality in liquidity calculated previously, securities are ranked into P portfolios $p = 1, 2, \ldots, P$ and for each portfolio the value-weighted monthly return (r^p_t) is calculated. As commonality in liquidity risk factor, we take the return differential (that is, the risk premium) between the portfolio which exhibits the strongest commonality and the portfolio with the weakest commonality:

$$CLIQ_t = r^P_t - r^1_t. \tag{2.3}$$

The portfolios are formed according to the level of liquidity commonality during the previous month; the portfolios are therefore rebalanced every month. This return-based liquidity factor can be interpreted as the additional return required by investors for holding a real estate security whose liquidity comoves strongly with that of the overall securitized real estate market (or that of the stock market). For commonality in liquidity to be potentially priced, a positive coefficient on this variable is expected, meaning that the required rate of return is higher for securities whose liquidity covaries with market liquidity. In other words, security prices should therefore fall when commonality in liquidity rises, all else being equal.

The same approach is adopted for constructing the risk factor associated with the comovement between a firm's liquidity and market returns (either the securitized real estate market or the stock market). First, each REIT's liquidity is regressed on market returns (instead of the market liquidity $LIQ^M_{t,d}$ in Equation 2.2) and then, the associated traded risk factor ($CLIQRET$) is computed by means of mimicking portfolios (Equation 2.3). We consider this liquidity variable in order to be conservative in our conclusions with respect to the importance of commonality in liquidity in asset pricing.

Market liquidity risk factor

The market liquidity risk factor is used as an additional liquidity-related risk factor in this study. This liquidity risk factor, initially suggested by Pástor and Stambaugh (2003), captures the sensitivity of asset returns to the aggregate market liquidity. Indeed, an investor could demand a premium when an asset's returns comove with market liquidity. Again, we construct two market liquidity risk factors: one for the public real estate market and one for the stock market. In the spirit of our liquidity commonality factor, the security being tested is not included in the estimation of the market liquidity, which is calculated as the value-weighted average across the remaining securities' liquidity levels.

Given the persistence of market liquidity,[6] only innovations (news) should drive returns. We obtain innovations in market liquidity by means of an autoregressive process of order one. This method of constructing innovations of liquidity is similar to that used by Pástor and Stambaugh (2003) and Acharya and Pedersen (2005). This non-traded liquidity risk factor (labeled $MLIQ$) is therefore integrated in the asset pricing model along with $CLIQ$ and $CLIQRET$ (as in Acharya and Pedersen's model). Realized returns are affected by liquidity shocks because expected returns are affected by expected liquidity (Amihud and Mendelson 1986). Thus, a negative shock to liquidity reduces future expected liquidity and raises the expected return, which in turn lowers prices today. This usually gives a positive liquidity beta. Therefore, we also expect a positive beta for the market liquidity risk factor.

2.3.2 Model specification

In this section, we first specify the Panel Threshold Regression (PTR) model of Hansen (1999) as utilized for testing the sensitivity of asset prices to several factors and briefly discuss its characteristics. Then, we present the time-series Threshold Regression (TR) model that depicts the sources of commonality in liquidity within the public real estate market. Finally, the estimation method and linearity testing procedure of the PTR and TR models are discussed. Note that, in addition to the nonlinear models presented next, we also estimate their respective linear counterparts for comparison purposes.

Factor model

In our specific setting[7], the PTR model with two regimes which explains REIT excess returns over the 3-month T-bill rate can be written as:

$$\begin{aligned} r_{i,t} =& \mu_i + [\alpha_1 CLIQ_t + \beta_1 CLIQRET_t + \gamma_1 MLIQ_{i,t} + \delta_1' Z_t]\, \mathbb{1}_{(q_{i,t-1} \leq c)} \\ &+ [\alpha_2 CLIQ_t + \beta_2 CLIQRET_t + \gamma_2 MLIQ_{i,t} + \delta_2' Z_t]\, \mathbb{1}_{(q_{i,t-1} > c)} + \varepsilon_{i,t} \end{aligned} \quad (2.4)$$

[6]The serial correlation of the public real estate and stock market liquidity is 0.93 and 0.97, respectively.
[7]The parameters of the model were estimated using the Matlab code written by C. Hurlin.

where $r_{i,t}$ is the monthly excess return on the securitized real estate asset i for month t and $CLIQ_t$ the commonality in liquidity factor as estimated in Equation 2.3. $CLIQRET_t$ represents the risk factor associated with the covariation of a security's liquidity with market returns and $MLIQ_{i,t}$ is the asset-specific market liquidity risk factor.[8] Z_t is a vector of factors such as credit conditions, market return, Fama and French's (1993) factors, momentum, market volatility (Ang et al. 2006) and investor sentiment (Baker and Wurgler 2006), which reflect the state of the economic and financial outlook at the aggregate level and are consequently only time-varying. Finally, μ_i is an individual fixed effect and $\varepsilon_{i,t}$ is the remaining error term. When we examine cross-asset commonality in liquidity, $CLIQ_t$, $CLIQRET_t$ and $MLIQ_{i,t}$ will stem from the analysis in which the securitized real estate market is replaced by the stock market.

Within this setup, the impact of a given factor on returns is not constant over time. Notably, we expect to find a differentiated impact of the various factors in a crisis regime compared to a normal one. The transition from one regime to the other is conducted by the observable transition variable $q_{i,t-1}$ (lagged by one month) through the indicator function $\mathbb{1}_{(\cdot)}$ which satisfies the condition in parentheses.[9] The threshold between the two regimes is defined by the parameter c: if $q_{i,t-1} \leq c$ we are in the first regime and if $q_{i,t-1} > c$ we are in the second regime. This threshold parameter is unknown and will be jointly estimated with the other parameters of the model. The marginal impact of a given variable, say $CLIQ_t$, is α_1 conditional on being in the first regime and α_2 in the second regime.

The choice of the observable transition variable is essential in threshold modeling. In general, this variable should reflect the business cycle variations. Economic and financial intuition could be used to choose an appropriate transition variable. In our case, REITs' realized volatility should be a convenient transition variable since it properly reflects fluctuations in the REIT market. The monthly realized volatility is measured by the standard deviation of monthly returns (calculated daily) within a month. The data show that periods of high REIT volatility correspond to bad general conditions in the economy such as during the 2008-2009 crisis or during the more recent debt crisis. Moreover, the realized volatility for each firm should reflect the firm's specific conditions which does not constrain the transition from a normal to a crisis regime to be common to all firms.

Sources of commonality in liquidity

Let us now set up the time-series regime-switching model which examines the economic sources of commonality in liquidity within the REIT market and the cross-asset commonality in liquidity.[10] Specifically, the R^2 measure of commonality in liquidity (Equation 2.2) averaged across firms, is regressed on supply-side and demand-side determinants along with control variables that proxy for general market conditions. Given that the range of R^2 falls within the $[0; 1]$ interval, we apply the logit link function in order to obtain a dependent variable which potentially takes values in the total support of real

[8]The market liquidity risk factor is asset-specific because for each asset i tested, we do not consider its liquidity in the computation of the market liquidity.

[9]Given that the transition variable $q_{i,t}$ is defined at the firm level, the impact of a given factor is also not constant across firms.

[10]The potential drivers of the commonality in liquidity with the underlying asset are still not well established in the theoretical literature. Hence, we leave this point for future research and focus on the factors explaining the within- and cross-asset commonality.

numbers. The model is as follows:

$$\log\left\{\bar{R^2}_t / \left(1-\bar{R^2}_t\right)\right\} = \theta_1' H_t\, \mathbb{1}_{(q_{t-1}\leq c)} + \theta_2' H_t\, \mathbb{1}_{(q_{t-1}>c)} + \omega_t \tag{2.5}$$

where $\bar{R^2}_t$ is the average of the commonality in liquidity measure across firms. The vector H_t includes the determinants of commonality in liquidity and the control variables. The average commonality in turnover within the public real estate market,[11] the investor sentiment, the market volatility and various proxies of credit availability are chosen as potential factors explaining the commonality in liquidity as discussed in the introduction. The control variables include the market return, liquidity and turnover for both the REIT and stock markets. $\mathbb{1}_{(.)}$ is the indicator function conducting the transition between regimes and the procedure for choosing the transition variable is similar to that of Equation 2.4. Given our market approach in studying the sources of commonality in liquidity, we use the VIX index as a measure of market volatility. As it measures the market's expectations about future stock market volatility, the VIX is a good indicator of market conditions. Indeed, the VIX exhibits spikes during periods of market distress and is low during normal times.

Estimation method and linearity testing procedure

The parameters of Equations 2.4 and 2.5 are estimated by nonlinear least squares (NLS) which also accounts for a White correction for heteroskedasticity in the error term. Conditional on the value of the parameter c, both specifications are linear on the other coefficients which can be estimated by a standard least squares method. c is estimated using a grid search procedure which minimizes the residual sum of squares. The linear models are estimated by OLS with White-correction for heteroskedasticity.

A test to evaluate the accuracy of a nonlinear modeling, both in the time-series and in the panel framework, is also performed. Indeed, one should test the null hypothesis of a linear model against the alternative hypothesis of a two-regime specification. We use a likelihood ratio test whose distribution is non-standard because under the null hypothesis of a linear model the threshold parameter c is a nuisance parameter. This issue is solved by a bootstrap procedure that simulates the distribution of the test as suggested by Hansen (1999). A detailed review of the estimation and testing procedures can be found in Tsay (1989), Chan (1993) and Hansen (1999).

2.4 Data

We focus on REIT data from the Center for Research in Security Prices/Ziman Real Estate Data Series. Prices, trading volume and shares outstanding (daily frequency) at the individual REIT level are collected to compute the returns, the liquidity-related variables and the commonality in turnover (monthly frequency). The S&P 500 index is utilized as proxy for the U.S. stock market in the analysis of the cross-asset commonality in liquidity. Daily prices, trading volume and shares outstanding are collected from Thomson Reuters Datastream for each constituent of the index to construct the aggregate stock market liquidity.[12] The market return, size, book-to-market and momentum factors, used in the

[11]When the cross-asset commonality in liquidity is analyzed, the commonality in turnover with the stock market is used.

[12]Since several REITs are included in the S&P 500 index, these firms are removed from this index in order to avoid any endogeneity issue and any confusion with the within-asset commonality in liquidity.

asset pricing model as controls, are obtained from Kenneth French's website.[13] In the model, we further include the credit spread (difference between Moody's Baa corporate bond and 10-year U.S. government bond yields) and the term spread (difference between 10-year and 1-year U.S. government bond yields) as business cycle proxies. The VIX index is chosen as proxy for the stock market volatility. These data are obtained from Bloomberg.

We use the University of Michigan consumer confidence index (sourced from Bloomberg) as the investor sentiment variable. This index is based on surveys that poll U.S. households on their current financial situation and their expectations about the future of the U.S. economy. However, this raw investor sentiment indicator may reflect to some extent economic fundamentals (Lemmon and Portniaguina 2006). Therefore, we orthogonalize it with respect to various macroeconomic variables in order to obtain a 'pure' sentiment index in the spirit of noise trader theories. These macroeconomic variables include: growth in industrial production, growth in durable, nondurable and services consumption, growth in employment, inflation, and an NBER recession indicator. This filtered sentiment variable is used both in the asset pricing model and in the analysis of the economic sources of commonality in liquidity (i.e, the demand-side determinants).

As funding liquidity variables in the investigation of the determinants of liquidity commonality (i.e, the supply-side determinants), we use the TED spread, the difference between the 3-month commercial paper rate and the 3-month Treasury bill rate (commercial paper spread), and the spread between the 30-year conventional mortgage rate and the 3-month Treasury bill rate (mortgage spread). Increased spreads imply higher borrowing costs, that is a narrowing of the funding liquidity. The extant literature has shown that those indirect measures of aggregate supply of funding are relevant. The data needed to construct those variables are also collected from Bloomberg. The time period for our analyses goes from January 1, 1999 through December 31, 2012. For this period, CRSP reports data for 366 REITs. However, we restrict our sample to firms having a minimum of two years of observations, yielding a sample of 295 REITs.

2.5 Estimation results

We present in this section the empirical results from the analyses of the within- and cross-asset commonality in liquidity. First, we discuss their patterns and present some related statistics. Descriptive statistics for the main variables used in this study are also presented. Second, the results of the asset pricing model estimation which aims to test whether or not commonality in liquidity drives REIT returns are discussed. Finally, the sources of commonality in liquidity within the REIT market and with the stock market are investigated.

2.5.1 Commonality in liquidity: statistics & characteristics

This section aims to provide a detailed picture of the commonality in liquidity involving the U.S. public real estate market. Summary statistics are reported and various features of the liquidity commonality are examined, such as its evolution over time as well as its level with respect to firm size and liquidity level. Table 2.1 provides some summary

[13]http://mba.tuck.dartmouth.edu/pages/faculty/ken.french

statistics for the commonality in liquidity variables estimated (see Equations 2.2 and 2.3) as well as for the main variables utilized in the various analyses.

The average levels of the within- and cross-asset commonality in liquidity (i.e., R^2) during the 1999-2012 period are 12.2% and 12.9%, respectively, with a standard deviation of about 11%. These figures are calculated from over 28,000 monthly firm-level observations stemming from an unbalanced panel of 295 REITs. These levels of commonality are quite high and broadly in line with the levels found for the U.S. stock market by Karolyi, Lee and van Dijk 2012.[14] These values tell us that the phenomenon of commonality in liquidity exists within the U.S. REIT market, which warrants for further analysis of this phenomenon.

Figures 2.1 and 2.2 show the evolution, over the period 1999-2012, of the average levels across firms of the within- and cross-asset commonality in liquidity, respectively. It appears from Figure 2.1 that the level of liquidity commonality slightly decreased from 1999 to 2008, even though some spikes emerged in early 2000 and in 2006, and then remained quite constant throughout the remaining period. Interestingly, the commonality in liquidity did not increase during the recent financial crisis. In contrast, the cross-asset commonality in liquidity, although it behaves similarly to the within-asset commonality during the first part of the period, sharply increases during the 2007-2009 financial crisis. Thus, one can observe an increasing connection between REIT and stock markets' liquidity, which is particularly pronounced in stressful periods. This finding provides tentative support for Brunnermeier and Pedersen's (2009) theory.

It is commonly recognized that liquidity is positively correlated with firm size. It seems therefore reasonable to expect also a relationship between commonality in liquidity and firm size. Figure 2.3 depicts bar graphs of the levels of liquidity commonality sorted according to firms' average market capitalization. As can be seen, there is only a slightly negative tendency between commonality in liquidity and firm size (for both types of commonality in liquidity), meaning that small firms have in general a higher level of commonality. We also analyze how commonality in liquidity varies cross-sectionally according to firms' liquidity level (Figure 2.4). Sorting firms from low to high liquidity shows that illiquid REITs tend to comove more with the overall REIT and stock market liquidity. But again, this relationship is quite weak. Therefore, the explanatory power of commonality in liquidity in the cross-section of REIT returns should be unrelated to firm size and liquidity level.

Turning now to our commonality in liquidity factors ($CLIQ$) to be tested in the factor model, we can see from Figures 2.5 (within-asset commonality) and 2.6 (cross-asset commonality) their evolution over our sample period. For computing these return-based risk factors, we group the firms into five portfolios according to their level of commonality in liquidity and take the value-weighted return differential between the portfolio with the strongest commonality and the portfolio with the weakest commonality (i.e., "5-1" spread). Both risk factors show some high levels during the recent financial crisis, emphasizing their importance in crisis periods. It is important to note that the cross-asset commonality in liquidity risk factor reaches however a very low level at the peak of the crisis. Also, although the levels of the within-asset commonality in liquidity did not

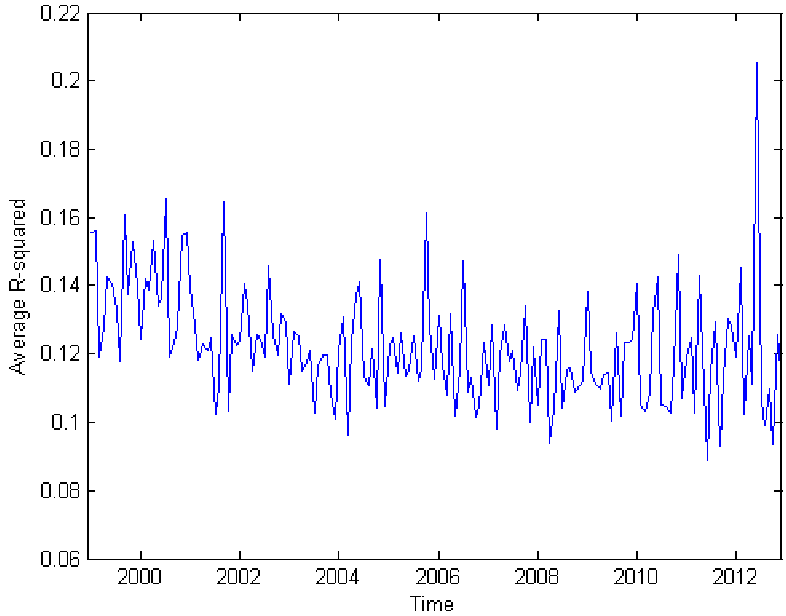
[14]These authors report an average R^2 of about 23%, but they include in their regressions for the commonality in liquidity (see Equation 2.2) the market liquidity in $t+1$. We do not include this variable because it seems doubtful that a variable in the future would influence another variable in the past (i.e., look-ahead bias). However, we ran our regressions with this additional variable for comparison purposes and found an average commonality in liquidity level of about 18%.

Table 2.1
Summary statistics

	Obs.	Mean	Std. Dev.	Minimum	Maximum
DEPENDENT VARIABLE					
REIT return	29,717	0.359	8.273	-57.75	52.34
COMMONALITY IN LIQUIDITY VARIABLES					
Commonality in liquidity	28,665	0.122	0.114	$2.6{\cdot}10^{-6}$	0.909
Cross-asset comm. in liq.	28,665	0.129	0.114	$5.6{\cdot}10^{-6}$	0.891
Comm. in liq. with DRE	13,585	0.056	0.190	-0.836	0.825
CLIQ	167	0.038	0.634	-1.774	2.625
Cross-CLIQ	167	-0.031	0.956	-5.684	3.529
DRECLIQ	142	-0.093	0.587	-1.695	1.709
OTHER VARIABLES					
CLIQRET	167	-0.083	0.644	-2.319	3.457
Cross-CLIQRET	167	-0.069	0.647	- 2.337	1.970
DRECLIQRET	142	0.200	0.976	-3.678	5.767
MLIQ (REIT)	167	-0.000	19.74	-56.49	82.00
MLIQ (Stock)	167	0.000	875.26	-2,320.5	2,722.2
MLIQ (Private RE)	142	0.726	498.90	-1,726.1	1,966.5
$R_M - R_f$	167	0.200	4.737	-17.23	11.34
SMB	167	0.038	3.766	-22.00	7.73
HML	167	0.627	3.546	-9.78	13.84
Momentum	167	0.289	6.149	-34.74	18.39
Credit spread	167	0.002	0.224	-1.000	1.450
Term spread	167	0.008	0.210	-0.620	0.770
VIX	167	22.04	8.289	10.42	59.89
Sentiment	167	-0.066	11.30	-28.09	26.58
Commonality in turnover	167	0.207	0.098	0.086	0.543
Cross-asset comm. in turn.	167	0.179	0.084	0.078	0.543

Note: The 'Commonality in liquidity' and 'Cross-asset comm. in liq.' variables are the R^2 of a regression in which the liquidity of each asset within a month is explained by the liquidity of the REIT market and of the stock market, respectively (see Equation 2.2). 'Comm. in liq. with DRE' is the liquidity correlation between REITs and the direct real estate market (see Equation 2.7). 'CLIQ', 'Cross-CLIQ' and 'DRECLIQ' are the return-based commonality in liquidity risk factors computed from the REIT market, the stock market and the direct real estate market, respectively, as given in Equation 2.3. 'CLIQRET', 'Cross-CLIQRET' and 'DRECLIQRET' are the return-based liquidity risk factors related to the covariation between REITs' liquidity and market returns from the REIT market, the stock market and the direct real estate market, respectively. 'MLIQ (REIT)', 'MLIQ (Stock)' and 'MLIQ (Private RE)' are the market liquidity risk factors. '$R_M - R_f$' is the spread between the market return and the 3-month T-bill rate. 'SMB', 'HML' and 'Momentum' are the Fama and French factors controlling for size, book-to-market and momentum. The 'Credit spread' is computed as the difference between Moody's Baa corporate bond and 10-year U.S. government bond yields. The 'Term spread' is the difference between the 10-year and 1-year U.S. government bond yields. 'VIX' stands for the Chicago Board of Options Exchange implied volatility index. As a proxy for investor sentiment, we use the University of Michigan consumer confidence index. 'Commonality in turnover' and 'Cross-asset comm. in turn.' are computed in the same manner as 'Commonality in liquidity' and 'Cross-asset comm. in liq.' from the REIT and stock market turnover (turnover denotes the ratio between the trading volume and the number of shares outstanding), and then averaged across firms. All return-based variables are expressed as percentages.

Figure 2.1
Cross-sectional average R^2 (commonality in liquidity)



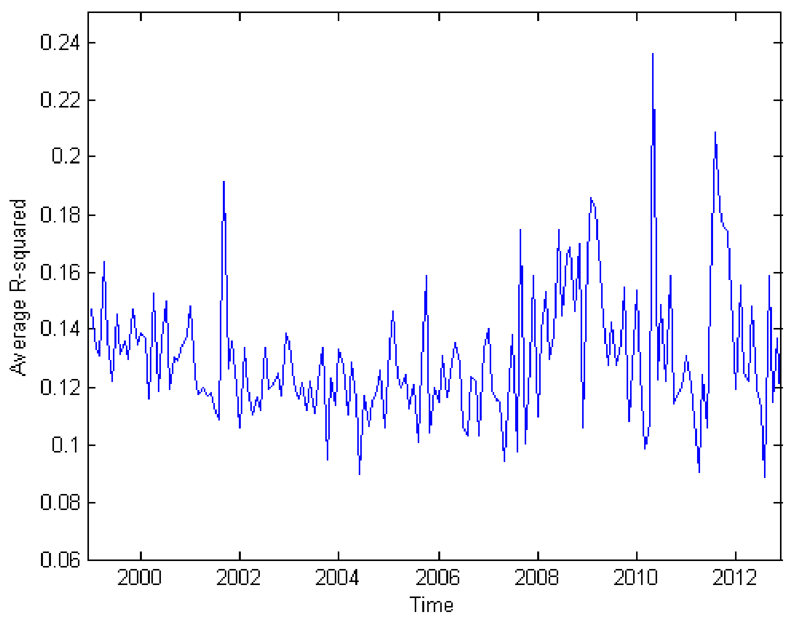
Note: This figure displays the cross-sectional average R^2 (commonality in liquidity) over time. The sample includes 295 firms.

increase during the 2007-2009 financial crisis, we observe here that $CLIQ$ did nevertheless increase. We explain this by the fact that in crisis periods, investors ask for a higher premium for the same level of commonality in liquidity. Over our sample period, $CLIQ$ exhibits an average return of 0.04% with a standard deviation of 0.63%, whereas *cross-asset* $CLIQ$ has an average return of -0.03% with a standard deviation of 0.96%.

Table 2.1 also reports the summary statistics of the other variables used in this study. To reduce the impact of outliers, the REIT returns are winsorized at the 1% and 99% quantiles. The average monthly return is about 0.36% with a standard deviation of 8.27% based on nearly 30,000 firm-level observations. Interestingly, the levels of the within- and cross-asset commonality in turnover (i.e., R^2) are on average higher than their liquidity counterparts. The commonality in turnover is on average 20.7% within the REIT market and 17.9% for the stock market. In line with the results reported previously for the commonality in liquidity, a REIT's turnover comoves quite strongly with that of the overall market and with that of the stock market. In general, the remaining variables have characteristics that are consistent with the extant literature. Finally, all series are stationary[15] (test results not reported) except the credit and term spreads. To make them stationary, we take the first difference of these variables.

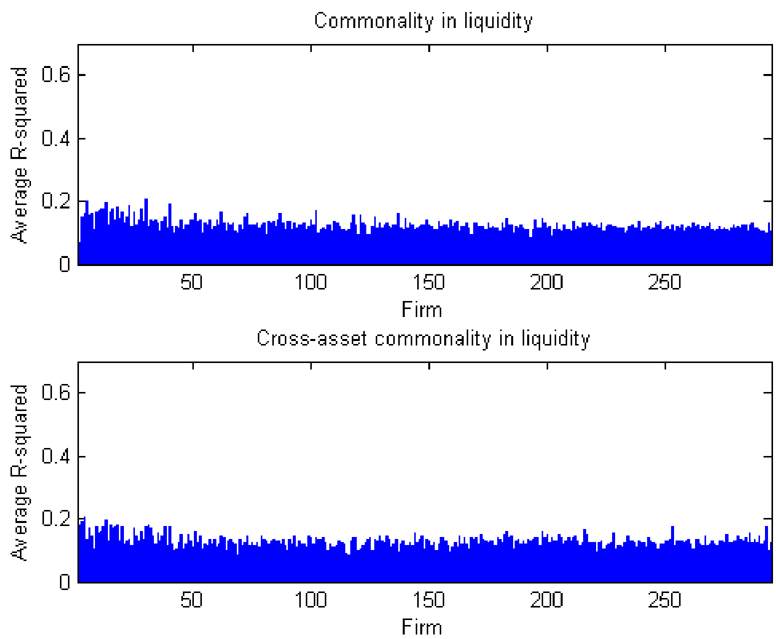
[15]Several unit root tests are used for testing stationarity: the Phillips-Perron and Dickey-Fuller tests are used for the time series variables, while the Levin-Lin-Chu and Pesaran panel unit root tests are used for the variables in panel data.

Figure 2.2
Cross-sectional average R^2 (cross-asset commonality in liquidity)



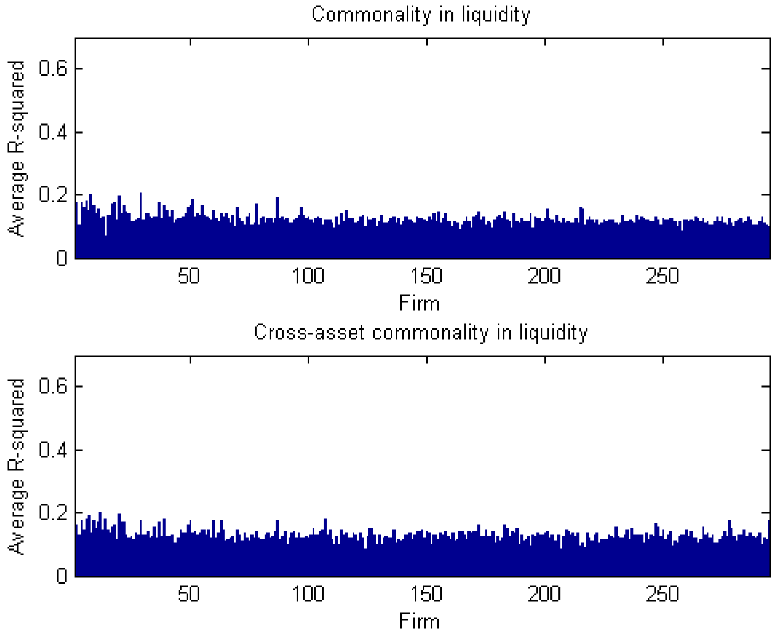
Note: This figure displays the cross-sectional average R^2 (cross-asset commonality in liquidity) over time. The sample includes 295 firms.

Figure 2.3
Average R^2 per firm ordered according to the average market capitalization



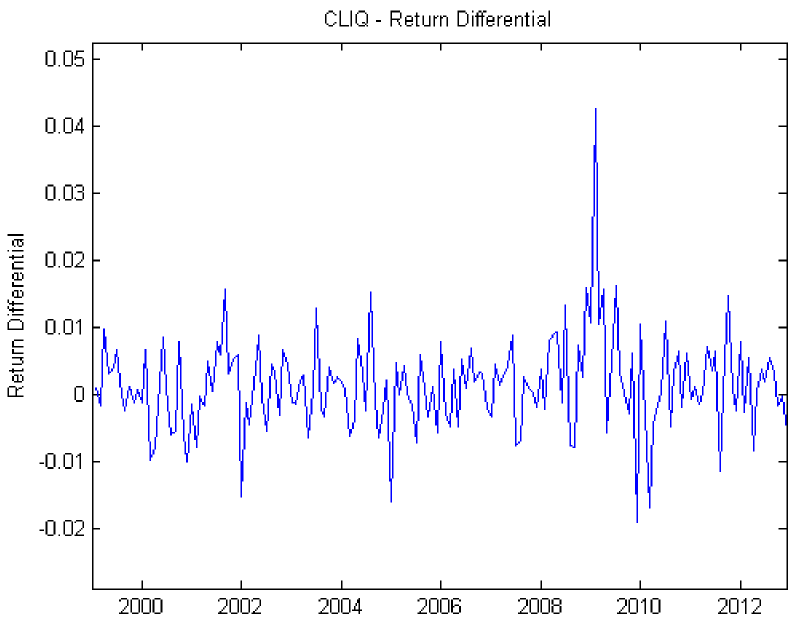
Note: These figures display the average within- and cross-asset commonality in liquidity by firm ordered by market capitalization. The sample includes 295 firms.

Figure 2.4
Average R^2 per firm ordered according to the average liquidity



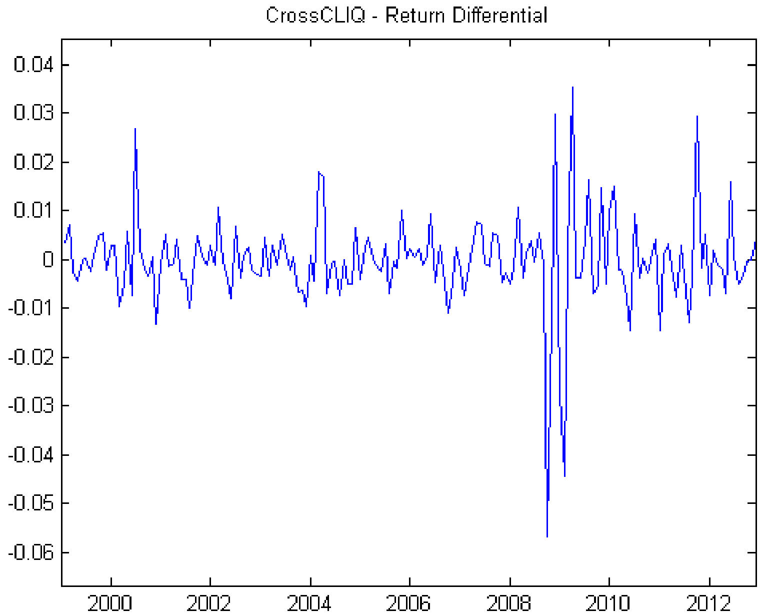
Note: These figures display the average within- and cross-asset commonality in liquidity by firm ordered by liquidity level. The sample includes 295 firms.

Figure 2.5
Commonality in liquidity - return differential



Note: This figure displays the commonality in liquidity risk factor given by the difference in returns between a portfolio with high R^2 and a portfolio with low R^2 (“5-1” spread).

Figure 2.6
Cross-asset commonality in liquidity - return differential



Note: This figure displays the cross-asset commonality in liquidity risk factor given by the difference in returns between a portfolio with high R^2 and a portfolio with low R^2 ("5-1" spread).

2.5.2 Factor model

In order to disentangle the exposure of REITs to commonality in liquidity, we estimate a linear model as well as a nonlinear model where REIT excess returns are regressed on our return-based commonality in liquidity factor (Equation 2.3) and a set of control variables. The estimation results of the multi-factor model which includes the within-asset commonality in liquidity factor are shown in Table 2.2. The table reports the estimation results of the linear model at the security level (i.e., the linear panel data model) as well as those of the PTR model.

The estimation results for the linear model show that most of the factor loadings (i.e., λ) are highly significant based on t-statistics calculated from robust standard errors. Of primary interest to this paper, the coefficient on $CLIQ$ is significantly negative, indicating that prices increase (and expected returns decrease) when commonality in liquidity rises. Investors should therefore not be compensated for bearing such a risk; they are even willing to pay for commonality in liquidity. This result is the opposite of what is predicted by theory. We explain this result as follows: the negative beta may be driven by periods during which markets are dominated by high liquidity and investors are looking for such a characteristic. Indeed, when market liquidity is high, one would like to buy a security whose liquidity is also high. This finding is consistent with our hypothesis that the impact of commonality may disappear within a linear framework. We also find a significantly negative coefficient for the sensitivity of assets' liquidity to market returns ($CLIQRET$).[16]

[16]Theoretically, $CLIQ$ and $CLIQRET$ could be correlated since periods of low (high) market liquidity generally correspond to periods of low (high) market returns, which would lead to some multicollinearity issues. However, given that the actual correlation between these two variables equals 0.3% (see Table B.II in the Appendix), our results should therefore not be biased by multicollinearity.

As for commonality in liquidity, investors are thus willing to pay for holding a security whose liquidity is correlated with market returns. On the other hand, we find that a negative shock to market liquidity lowers REIT prices (and increases expected returns). Indeed, a significantly positive beta is associated with the innovations in REIT market liquidity. In short, REIT market liquidity risk is the only liquidity risk factor consistent with theory when a linear approach is adopted. Finally, the other factor loadings exhibit relatively consistent patterns with the extant empirical asset pricing literature.

Turning now to the estimation results for the PTR model, we first examine the adequacy of using a regime-switching model by testing the null hypothesis of a linear model against the alternative hypothesis of a two-regime specification. We use firms' realized volatility for detecting the regimes. The F-statistic strongly rejects the hypothesis of linearity, which supports the use of a two-regime modeling for examining REIT pricing. The factor loadings in the low-volatility regime (i.e., λ_1 in Table 2.2) are relatively close to those of the linear model. In particular, we find that the coefficients on our three liquidity risk variables are very similar. We also find that several factor betas significantly vary between the normal regime and the crisis regime (i.e., $\lambda_2 - \lambda_1$), which shows the inability of a linear model to correctly capture the various impacts on REIT prices in stressful times. Importantly, we find that the commonality in liquidity beta significantly increases in the second regime (i.e., λ_2) and becomes positive, suggesting that expected returns increase when commonality in liquidity rises. This finding suggests that commonality in liquidity is a significant risk factor but only during stressful times. Hence, the gap between the theory and the extant empirical evidence on the pricing implications of commonality in liquidity may be explained by the use of a linear model when testing this risk factor. The impact of the REIT market liquidity risk on prices also increases in the second regime (but not significantly), whereas $CLIQRET$ becomes insignificant. In general, our liquidity risk factors become therefore more important in bad market conditions.

The coefficients on the control variables also give us some interesting insights. The market factor as well as the size, book-to-market and momentum factors are highly significant. Those factors impact REIT returns in both regimes, but systematically more in the high-volatility regime. On the other hand, we find that the credit and term spreads have nearly no impact on REIT returns. Consistent with Ang et al. (2006), the market volatility is negatively associated with returns in both regimes, implying that investors are willing to pay to hedge themselves against increases in market volatility. Finally, we find that investor sentiment do influence REIT returns. While insignificant in the linear model, the impact of sentiment switches from positive in the low-risk state to negative in the high-risk state within the nonlinear setting. Thus, bullish (bearish) investor sentiment leads to an asset overvaluation (undervaluation) in the first regime, whereas the opposite happens in the second regime.

Based on the estimation of the threshold parameter c, we further investigate the number of firms in the high-volatility regime (i.e., if $q_{i,t-1} > c$) for each month and present summary statistics for each year in Table 2.3. We find the highest percentages of firms in the high-volatility regime in 2008 and 2009 (approximatively 60% of the sample). Furthermore, we find that almost all REITs (i.e., 97%) included in the sample in November 2008 were in the high-volatility regime, consistent with what happened in the financial markets during that time. Thus, our transition variable does a good job in characterizing the different states of the economy.

The estimation results of the multi-factor model which includes the cross-asset commonality in liquidity factor are shown in Table 2.4. The estimates of the linear model

Table 2.2
Factor model estimation results

Commonality within the REIT Market - Realized Volatility

	Linear model	PTR model		
	λ	λ_1	λ_2	$\lambda_2 - \lambda_1$
CLIQ	-0.387***	-0.951***	1.114***	2.064***
	(-2.973)	(-9.439)	(2.996)	(5.361)
CLIQRET	-1.197***	-1.671***	-0.029	1.643***
	(-8.640)	(-15.53)	(-0.075)	(4.161)
MLIQ (REIT)	$3{\cdot}10^{-4}$***	$3{\cdot}10^{-4}$***	$4{\cdot}10^{-4}$***	$1{\cdot}10^{-4}$
	(7.815)	(10.49)	(2.878)	(1.039)
$R_M - R_f$	0.575***	0.435***	0.808***	0.373***
	(19.96)	(27.34)	(16.54)	(7.438)
SMB	0.464***	0.359***	0.670***	0.311***
	(19.87)	(22.52)	(9.325)	(4.213)
HML	0.702***	0.514***	1.017***	0.503***
	(21.28)	(27.22)	(13.32)	(6.435)
Momentum	-0.168***	-0.096***	-0.236***	-0.140***
	(-11.20)	(-8.956)	(-5.944)	(-3.414)
Credit spread	-0.668	0.581	-1.308	-1.889
	(-1.098)	(1.303)	(-1.057)	(-1.403)
Term spread	-1.944***	-1.557***	0.568	2.125
	(-4.551)	(-5.024)	(0.365)	(1.333)
VIX	$-1{\cdot}10^{-4}$	$-3{\cdot}10^{-4}$**	$-4{\cdot}10^{-4}$***	$-1{\cdot}10^{-4}$
	(-1.333)	(-2.508)	(-2.905)	(-1.178)
Sentiment	$8{\cdot}10^{-5}$	$3{\cdot}10^{-4}$***	$-4{\cdot}10^{-4}$*	-0.001***
	(1.028)	(4.816)	(-1.661)	(-2.794)
c		0.0622 [0.0515, 0.0625]		
F-test		1,397.5 (0.000)		
R-squared	0.145	0.159		
Observations	29,717	29,510		

Note: This table contains the estimation results for the multi-factor model (PTR) given by the following equation: $r_{i,t} = \mu_i + \left[\alpha_1 CLIQ_t + \beta_1 CLIQRET_t + \gamma_1 MLIQ_{i,t} + \delta_1' Z_t\right] \mathbb{1}_{(q_{i,t-1} \leq c)} + [\alpha_2 CLIQ_t + \beta_2 CLIQRET_t + \gamma_2 MLIQ_{i,t} + \delta_2' Z_t] \mathbb{1}_{(q_{i,t-1} > c)} + \varepsilon_{i,t}$, where $r_{i,t}$ is the monthly excess return on the securitized real estate asset i for month t. $CLIQ_t$ is the commonality in liquidity risk factor, $CLIQRET_t$ is the risk factor associated with the covariance between firms' liquidity and market returns, and $MLIQ_{i,t}$ is the market liquidity risk factor. Z_t includes all the market-wide factors considered in this study. The transition variable $q_{i,t-1}$ is the one-month lagged realized volatility of each firm. We define $\lambda_k = [\alpha_k, \beta_k, \delta_k']'$, $k = 1, 2$. The estimation method is Nonlinear Least Squares with the covariance matrix corrected for White heteroskedasticity (t-statistics in parentheses); the estimation results are presented in the last three columns. The estimation results of a standard panel linear model with fixed effects are displayed in the first column. The estimation method is OLS with White-corrected t-statistics presented in parentheses. ***, **, and * denote significance at the 1%, 5% and 10% confidence levels, respectively. The brackets contain the 95%-confidence interval of the threshold parameter c. The F-test (p-value in parentheses) tests the null hypothesis of a linear model against the alternative of a two-regime specification. The credit spread and the term spread are taken in first difference in order to obtain stationary series.

Table 2.3
Percentage of firms in the crisis regime based on the PTR model

Year	Mean	Std. dev.	Minimum	Maximum
1999	17.85	11.52	8.72	48.97
2000	17.92	4.92	12.00	26.13
2001	17.72	4.60	12.50	27.69
2002	15.82	10.84	6.49	40.54
2003	9.74	2.55	5.46	13.26
2004	10.80	9.50	3.65	34.07
2005	6.97	2.82	4.02	13.57
2006	7.17	2.27	2.60	11.89
2007	24.24	18.89	6.40	63.69
2008	59.25	29.55	16.35	[a]97.37
2009	60.27	25.57	18.42	[b]93.79
2010	27.35	14.40	9.32	52.53
2011	29.31	25.83	6.33	69.43
2012	8.29	5.15	2.60	19.87

Note: This table reports summary statistics concerning the percentage of firms in the crisis regime for each month within a year, based on the PTR estimation results. We take the estimated threshold parameter from the PTR specification in Table 2.2 and compare it with the value of the transition variable for each month: if $\hat{c} > q_{i,t-1}$, then for month t, firm i is classified in a crisis regime. "Mean" is the average proportion of firms for each month within a year estimated to be in the crisis regime. "Std. dev", "Minimum", "Maximum" are respectively the standard deviation, the minimum and maximum percentage of firms to be classified in a crisis regime within a month. [a]November 2008, [b]February 2009.

reveal that cross-asset commonality in liquidity is a significant risk factor (i.e., positive beta) in explaining REIT excess returns, which stresses the importance of also taking a cross-asset perspective when investigating liquidity risk.[17] On the other hand, the betas associated with the stock market liquidity risk and the covariation between REITs' liquidity and stock market returns are not consistent with the hypothesis of priced risk factors. Again, the linearity hypothesis is strongly rejected in favor of a two-regime specification. The estimation of the nonlinear model[18] provides values for the parameters in the first regime very close to those reported for the linear model. However, the coefficients on *cross*-$CLIQ$ and $MLIQ$ change from significantly positive to insignificant and from insignificant to significantly negative, respectively. In sum, REIT exposure to any type of liquidity risk does not lead to lower asset prices in the first regime. The dynamics significantly change when the economy switches to the high-risk state. The effects of *cross*-$CLIQ$ and $MLIQ$ become both significantly positive, whereas the effect of *cross*-$CLIQRET$ becomes insignificant. Thus, cross-asset commonality and stock market liquidity turn to be significant liquidity risk factors in crisis periods, consistent with the within-asset commonality in liquidity findings. The parameters associated with the control variables (both in the linear and nonlinear models) are very close to those of the first specification (Table 2.2).

We also measure the economic significance of our liquidity risk factors by means of the following equation: $\hat{\phi}\ \bar{\sigma}_{factor}/\bar{\sigma}_{return}$, where $\hat{\phi}$ is one of the estimated parameters λ, $\lambda 1$ or $\lambda 2$ in Tables 2.2 and 2.4. $\bar{\sigma}_{factor}$ and $\bar{\sigma}_{return}$ are the standard deviation (averaged across firms) of one of the factors of interest and of REIT returns, respectively. For the nonlinear model, these standard deviations are calculated separately for each regime. We report in the Appendix (Table B.III) the average effects across firms. For instance, a 1% change in CLIQ has an average impact of 0.06% on REIT returns in the second regime which represents an increase of 0.15% as compared to the first regime. The effect of *cross*-$CLIQ$ on REIT returns is as important as $CLIQ$ in the crisis regime and goes from no impact to an impact also of 0.06%. For comparison, the effect of the market risk factor ($R_M - R_f$) rises from 0.30% to 0.33% (results not reported), showing a moderate increase compared to commonality in liquidity. These results suggest therefore that commonality in liquidity is an important risk factor especially in inopportune moments. In addition, we find that commonality in liquidity is the most important liquidity risk factor in the high-risk state in both cases.

To summarize, both within- and cross-asset commonality in liquidity are relevant for REIT pricing and their impacts are independent from other types of liquidity risk, noise trader sentiment and standard systematic risks. We also find that in general the betas of our liquidity variables increase in a high-volatility regime. These findings contribute thus to the debate on the pricing implications of commonality in liquidity by showing that the impact of commonality in liquidity risk is time-varying and multi-faceted. Given these results, an additional analysis, critical to fully understanding the commonality in liquidity phenomenon, is to examine its determinants. This is the purpose of the following section.

[17] This impact is distinct from that of the within-asset commonality in liquidity, since the correlation between these risk factors is -2% (see Table B.II in the Appendix).

[18] Counting the number of firms in the high-volatility regime leads to very similar results as those reported in Table 2.3. This is explained by the fact that the estimated threshold parameters c of both specifications are close.

Table 2.4
Factor model estimation results

Commonality with the Stock Market - Realized Volatility

	Linear model	PTR model		
	λ	λ_1	λ_2	$\lambda_2 - \lambda_1$
Cross-CLIQ	0.683***	-0.078	0.679***	0.757***
	(4.157)	(-0.661)	(2.865)	(2.839)
Cross-CLIQRET	-0.387***	-0.775***	-0.051	0.725**
	(-2.606)	(-8.403)	(-0.144)	(1.986)
MLIQ (Stock)	$3{\cdot}10^{-7}$	$-3{\cdot}10^{-6}$***	$8{\cdot}10^{-6}$***	$1{\cdot}10^{-5}$***
	(0.344)	(-3.737)	(4.419)	(5.429)
$R_M - R_f$	0.575***	0.445***	0.675***	0.230***
	(20.79)	(25.98)	(16.28)	(5.361)
SMB	0.478***	0.415***	0.561***	0.146***
	(20.92)	(24.42)	(11.34)	(2.794)
HML	0.710***	0.539***	0.853***	0.314***
	(21.66)	(27.75)	(14.66)	(5.173)
Momentum	-0.175***	-0.117***	-0.194***	-0.077***
	(-11.77)	(-10.96)	(-7.103)	(-2.623)
Credit spread	-0.463	1.660***	-0.639	-2.298**
	(-0.750)	(3.150)	(-0.689)	(-2.112)
Term spread	-1.926***	-1.720***	-0.238	1.483*
	(-4.918)	(-4.947)	(-0.289)	(1.672)
VIX	$-2{\cdot}10^{-4}$**	$-4{\cdot}10^{-4}$***	$-4{\cdot}10^{-4}$***	$3{\cdot}10^{-5}$
	(-2.224)	(-3.166)	(-2.830)	(0.539)
Sentiment	$6{\cdot}10^{-5}$	$3{\cdot}10^{-4}$***	$-3{\cdot}10^{-4}$*	-0.001***
	(0.868)	(6.267)	(-1.954)	(-4.029)
c		0.0468 [0.0467, 0.0468]		
F-test		1,376.2 (0.010)		
R-squared	0.142	0.154		
Observations	29,717	29,510		

Note: This table contains the estimation results for the multi-factor model given by the following equation: $r_{i,t} = \mu_i + \left[\alpha_1 \text{Cross-CLIQ}_t + \beta_1 \text{Cross-CLIQRET}_t + \gamma_1 \text{MLIQ}_{i,t} + \delta_1' Z_t\right] \mathbb{1}_{(q_{i,t-1} \leq c)} + [\alpha_2 \text{Cross-CLIQ}_t + \beta_2 \text{Cross-CLIQRET}_t + \gamma_2 \text{MLIQ}_{i,t} + \delta_2' Z_t] \mathbb{1}_{(q_{i,t-1} > c)} + \varepsilon_{i,t}$, where $r_{i,t}$ is the monthly excess return on the securitized real estate asset i for month t. $Cross\text{-}CLIQ_t$ is the commonality in liquidity risk factor (with the stock market), $Cross\text{-}CLIQRET_t$ is the risk factor associated with the covariance between firms' liquidity and stock market returns, and $MLIQ_{i,t}$ is the stock market liquidity risk factor. Z_t includes all the market-wide factors considered in this study. The transition variable $q_{i,t-1}$ is the one-month lagged realized volatility of each firm. We define $\lambda_k = [\alpha_k, \beta_k, \delta_k']'$, $k = 1, 2$. The estimation method is Nonlinear Least Squares with the covariance matrix corrected for White heteroskedasticity (t-statistics in parentheses); the estimation results are presented in the last three columns. The estimation results of a standard panel linear model with fixed effects are displayed in the first column. The estimation method is OLS with White-corrected t-statistics presented in parentheses. ***, **, and * denote significance at the 1%, 5% and 10% confidence levels, respectively. The brackets contain the 95%-confidence interval of the threshold parameter c. The F-test (p-value in parentheses) tests the null hypothesis of a linear model against the alternative of a two-regime specification. The credit spread and the term spread are taken in first difference in order to obtain stationary series.

2.5.3 Sources of commonality in liquidity

As determinants of commonality in liquidity, we consider two supply-side variables, credit availability and market volatility, and two demand-side variables, commonality in turnover and investor sentiment. In addition to these factors, a set of control variables is also included in the analysis. Again, we estimate a linear as well as a nonlinear model since the impact of the above variables could be regime-dependent. Three models are estimated, each time with a different proxy for funding liquidity (i.e., Models 1-3). These proxies are the TED spread, the commercial paper spread (CP spread) and the mortgage spread.

The F-statistics[19] show that the null hypothesis of linearity is rejected only in two cases, i.e., when the mortgage spread is used as funding variable in both analyses. Except for these cases, our discussion focuses thus on the results of the linear models. Table 2.5 displays the results of the within-asset commonality in liquidity analysis. We find that commonality in turnover (i.e., $Cturn$), our proxy for correlated trading activity, represents a significant economic source of commonality in liquidity. The coefficient on this variable being positive, it is therefore in line with the theory. This finding is robust to the funding liquidity proxy used. Thus, investors who tend to trade in concert the same assets influence positively their level of comovement in liquidity (Koch, Ruenzi and Starks 2010). We also find that investor sentiment has a significantly positive impact on the within-asset commonality in liquidity. This indicates that more optimistic investor sentiment increases the commonality in liquidity level within the REIT market, and vice versa.

In contrast to the recent literature stressing the importance of funding liquidity in explaining commonality in liquidity, our empirical results strongly reject this hypothesis. Indeed, the impact of funding liquidity is either not significantly different from zero or negative, and this finding is robust across various proxies of credit availability. The commonality in liquidity within the public real estate market is also not driven by market volatility (i.e., VIX), consistent with the fact that this commonality did not increase during the recent financial crisis (see Figure 2.1). When we switch to a nonlinear framework (Table 2.6), we find that the impact of the mortgage spread is insignificant in the first state (i.e., θ_1) and significantly negative in the second state (i.e., θ_2). Although the risk that liquidity suppliers face funding constraints is much higher during pervasive market declines, the funding hypothesis is however rejected again. We further find that the market volatility is insignificant in any states of the economy. As regards the demand-side factors, we find $Cturn$ and investor sentiment to be significant (and positive) only in the high-volatility regime. The positive coefficient on sentiment is rather counter-intuitive, but, as noted by Karolyi, Lee and van Dijk (2012), the sentiment hypothesis does not offer clear theoretical predictions regarding the sign of the relationship between sentiment and commonality in liquidity. In sum, our empirical evidence favors a demand-side explanation of commonality in liquidity within the REIT market.

Table 2.7 displays the results of the linear model that analyzes the cross-asset commonality in liquidity. Globally, we find very little support for our various hypotheses. Indeed, only $Cturn$, when the TED spread is used as funding liquidity proxy, is in line with the theory (i.e., significantly positive coefficient). On the other hand, the estimation of a Threshold Regression model that includes the mortgage spread yields a number of interesting results (Table 2.8). Notably, several factors become important in the second regime. However, the TR model does not provide more support to the funding hypothesis

[19]The results of these tests are not reported but can be obtained upon request

since the mortgage spread is either negative or insignificant. On the other hand, market volatility appears to become a key variable (i.e., significantly positive) in explaining the level of comovement between REIT liquidity and stock market liquidity when the economy switches to the high-risk state. In addition, the increased impact is highly significant (i.e., $\theta_2 - \theta_1$). We find that sentiment is not an important source in the normal regime but becomes a major determinant with a negative coefficient in the crisis regime. This finding is consistent with the extant literature that stresses the increased behavioral biases during bad market conditions. This is also consistent with our conjecture that a higher pessimism should increase commonality in liquidity in stressful times. We also find that investors' correlated trading behavior increases liquidity commonality but only in the low-volatility regime. The demand-side determinants remain therefore relatively important within a two-regime framework, even though we observe an increasing role played by supply-side factors.

Overall, our findings are more in favor of demand-side determinants of liquidity and are thus in line with recent empirical works such as those by Karolyi, Lee and van Dijk (2012), and Koch, Ruenzi and Starks (2010). Our empirical setup thus shows the limited scope of Brunnermeier and Pedersen's theory in explaining commonality in liquidity. Furthermore, we uncover that most of the effects on liquidity commonality do not vary according to the state of the economy.

2.6 Commonality in liquidity with the underlying asset

In this section, we extend our work by analyzing the commonality in liquidity between REITs and the underlying real estate market. More specifically, we test whether the covariation between REITs' liquidity and the private market liquidity affects REIT returns. Several papers have analyzed the relationship between REIT prices, and more broadly closed-end fund share prices, and the value of the assets owned by these funds. Deviations from Net Asset Value (NAV) have been attributed to investor irrationality (Lee, Shleifer and Thaler 1991), but also to the differences between the liquidity of the assets held by the funds and the liquidity of their shares (Cherkes, Sagi and Stanton 2009). This latter theory conjectures thus some linkages between REIT liquidity and the direct real estate market liquidity. Furthermore, Benveniste, Capozza and Seguin (2001) show that the liquidity of the underlying real estate market has a significant influence on REIT liquidity and prices. Thereby, the commonality in liquidity between REITs and the private real estate market may also influence REIT returns since real estate investors generally shift their holdings to the public market for liquidity purposes.

Assessing the liquidity of the private real estate market is no trivial task. Due to the nature of the available data, we need to implement an alternative strategy as regards the liquidity proxy and the construction of the commonality in liquidity variable to that adopted for the other two analyses of commonality. We use the number of properties sold as our measure of liquidity in the private real estate market,[20] as Amihud's (2002) measure is inapplicable for the private market. Transaction frequency represents a reliable liquidity indicator especially in a highly illiquid market (Ling, Naranjo and Scheick 2012). The data are sourced from Real Capital Analytics and are available at a monthly frequency for the period 2001-2012. As regards the measure of commonality in liquidity, our previous

[20] We apply a detrending factor to this raw measure of liquidity to remove any time trend.

Table 2.5

Sources of commonality in liquidity: within REIT market - linear model

	Model 1	Model 2	Model 3
TED spread	-4.889*		
	(-1.811)		
CP spread		-4.609	
		(-1.469)	
Mortgage spread			-0.777
			(0.742)
Cturn	0.403**	0.407**	0.357**
	(2.539)	(2.524)	(2.266)
Sentiment	0.003**	0.003**	0.003**
	(2.130)	(1.998)	(1.965)
VIX	0.001	0.001	0.002
	(0.305)	(0.224)	(0.619)
REIT market return	0.070	0.101	0.156
	(0.298)	(0.435)	(0.672)
Stock market return	-0.062	-0.072	-0.018
	(-0.197)	(-0.229)	(-0.056)
REIT market liquidity	$4{\cdot}10^{-5}$	$7{\cdot}10^{-6}$	$-6{\cdot}10^{-5}$
	(0.077)	(0.013)	(-0.108)
Stock market liquidity	$-5{\cdot}10^{-6}$	$-5{\cdot}10^{-6}$	$-2{\cdot}10^{-6}$
	(-0.700)	(-0.611)	(-0.316)
REIT market turnover	-0.009**	-0.010***	-0.011***
	(-2.431)	(-2.792)	(-3.091)
Stock market turnover	18.82***	18.246***	13.87**
	(2.671)	(2.576)	(1.933)
Constant	-2.134***	-2.125***	-2.094***
	(-21.85)	(-21.74)	(-20.11)
R-squared	0.2317	0.2263	0.2183
Observations	167	167	167

Note: This table contains the estimation results for the sources of commonality in liquidity within the REIT market given by the equation: $\log\left\{\bar{R^2}_t/\left(1-\bar{R^2}_t\right)\right\} = \theta' H_t + \omega_t$ where $\bar{R^2}_t$ is the average of the commonality in liquidity measure across firms. The vector H_t involves supply- and demand-side determinants of commonality in liquidity, and a set of control variables. We use three alternative funding liquidity variables: 'Model 1' uses the TED spread, 'Model 2' uses the spread between the 3-month commercial paper rate and the 3-month Treasury bill rate, and 'Model 3' uses the spread between the 30-year conventional mortgage rate and the 3-month Treasury bill rate. The estimation method is OLS with White-corrected t-statistics presented in parentheses. ***, **, and * denote significance at the 1%, 5% and 10% confidence levels, respectively.

Table 2.6

Sources of commonality in liquidity: within the REIT market - TR model

	θ_1	θ_2	$\theta_2 - \theta_1$
Mortgage spread	0.066	-3.604**	-3.671*
	(0.057)	(-2.021)	(-1.856)
Cturn	0.256	0.783***	0.527
	(1.342)	(2.773)	(1.523)
Sentiment	$2{\cdot}10^{-4}$	0.011***	0.011***
	(0.120)	(4.747)	(3.515)
VIX	-0.001	0.005	0.006
	(-0.235)	(1.495)	(1.161)
REIT market return	-0.351	0.142	0.493
	(-1.121)	(0.507)	(1.163)
Stock market return	0.279	0.085	-0.194
	(0.679)	(0.302)	(-0.402)
REIT market liquidity	$4{\cdot}10^{-5}$	0.003***	0.003**
	(0.056)	(3.096)	(2.545)
Stock market liquidity	$-3\cdot10^{-6}$	$-2{\cdot}10^{-5}$*	$-2\cdot10^{-5}$
	(-0.300)	(-1.678)	(-1.087)
REIT market turnover	-0.020***	-0.007**	0.013
	(-2.264)	(-2.234)	(1.545)
Stock market turnover	25.93**	-5.908	-31.84***
	(2.472)	(-0.746)	(-2.645)
c	23.540 [23.540, 24.060]		
F-test	31.404 (0.03)		
R-squared	0.343		
Observations	167		

Note: This table contains the estimation results for the sources of commonality in liquidity within the REIT market given by the equation: $\log\left\{\bar{R^2}_t/\left(1-\bar{R^2}_t\right)\right\} = \theta_1' H_t \mathbb{1}_{(q_{t-1}\leq c)} + \theta_2' H_t \mathbb{1}_{(q_{t-1}>c)} + \omega_t$ where $\bar{R^2}_t$ is the average of the commonality in liquidity measure across firms. The vector H_t involves supply- and demand-side determinants of commonality in liquidity, and a set of control variables. In this regression the spread between the 30-year conventional mortgage rate and the 3-month Treasury bill rate is used as funding liquidity variable. The VIX index is used as transition variable. The estimation method is Nonlinear Least Squared with the covariance matrix corrected for White heteroskedasticity, t-statistics presented in parentheses. ***, **, and * denote significance at the 1%, 5% and 10% confidence levels, respectively. The brackets contain the 95%-confidence interval of the threshold parameter c. The F-test (p-value in parentheses) tests the null hypothesis of a linear model against the alternative of two-regime specification.

Table 2.7

Sources of commonality in liquidity: with the stock market - linear model

	Model 1	Model 2	Model 3
TED spread	-5.373* (-1.700)		
CP spread		-1.186 (-0.323)	
Mortgage spread			-1.476 (-1.228)
Cturn	0.400** (1.969)	0.333 (1.614)	0.308 (1.558)
Sentiment	-0.002 (-1.333)	-0.002 (-1.329)	-0.003 (-1.528)
VIX	0.000 (0.105)	0.001 (0.215)	0.002 (0.642)
REIT market return	-0.342 (-1.267)	-0.262 (-0.970)	-0.221 (-0.832)
Stock market return	-0.256 (-0.705)	-0.249 (-0.679)	-0.208 (-0.569)
REIT market liquidity	-0.001 (-0.853)	-0.001 (-0.910)	-0.001 (-1.035)
Stock market liquidity	$-3{\cdot}10^{-6}$ (-0.326)	$-5{\cdot}10^{-7}$ (-0.054)	$8{\cdot}10^{-7}$ (0.092)
REIT market turnover	0.002 (0.418)	$-4{\cdot}10^{-4}$ (-0.116)	$-3{\cdot}10^{-4}$ (-0.087)
Stock market turnover	33.14*** (4.076)	30.22*** (3.680)	26.54*** (3.220)
Constant	-2.220*** (-19.92)	-2.209*** (-19.68)	- 2.159*** (-18.15)
R-squared	0.3569	0.3454	0.3512
Observations	167	167	167

Note: This table contains the estimation results for the sources of cross-asset commonality in liquidity given by the equation: $\log\left\{\bar{R^2}_t/\left(1-\bar{R^2}_t\right)\right\} = \theta' H_t + \omega_t$ where $\bar{R^2}_t$ is the average of the commonality in liquidity measure across firms. The vector H_t involves supply- and demand-side determinants of commonality in liquidity, and a set of control variables. We use three alternative funding liquidity variables: 'Model 1' uses the TED spread, 'Model 2' uses the spread between the 3-month commercial paper rate and the 3-month Treasury bill rate, and 'Model 3' uses the spread between the 30-year conventional mortgage rate and the 3-month Treasury bill rate. The estimation method is OLS with White-corrected t-statistics presented in parentheses. ***, **, and * denote significance at the 1%, 5% and 10% confidence levels, respectively.

Table 2.8
Sources of commonality in liquidity: with the stock market - TR model

	θ_1	θ_2	$\theta_2 - \theta_1$
Mortgage spread	-0.079	-7.972***	-7.893***
	(-0.061)	(-2.805)	(-2.578)
Cturn	0.609**	-0.474	-1.083**
	(2.211)	(-1.298)	(-2.364)
Sentiment	-0.001	-0.005**	-0.004
	(-0.384)	(-2.163)	(-1.353)
VIX	-0.001	0.015***	0.016**
	(-0.124)	(3.559)	(2.508)
REIT market return	-0.721**	-0.342	0.379
	(-2.169)	(-1.178)	(0.845)
Stock market return	-0.842*	0.995*	1.837***
	(-1.903)	(1.915)	(2.669)
REIT market liquidity	$-4\cdot10^{-4}$	0.003*	0.003*
	(-0.506)	(1.891)	(1.935)
Stock market liquidity	$2\cdot10^{-6}$	$9\cdot10^{-6}$	$7\cdot10^{-6}$
	(0.146)	(0.709)	(0.400)
REIT market turnover	-0.011	-0.001	0.010
	(-0.925)	(-0.280)	(0.882)
Stock market turnover	38.61***	15.09*	-23.52*
	(3.408)	(1.778)	(-1.810)
c		25.400 [24.950, 31.170]	
F-test		34.048 (0.03)	
R-squared		0.462	
Observations		167	

Note: This table contains the estimation results for the sources of cross-asset commonality in liquidity given by the equation: $\log\left\{\bar{R^2}_t/\left(1-\bar{R^2}_t\right)\right\} = \theta_1' H_t \mathbb{1}_{(q_{t-1}\leq c)} + \theta_2' H_t \mathbb{1}_{(q_{t-1}>c)} + \omega_t$ where $\bar{R^2}_t$ is the average of the commonality in liquidity measure across firms. The vector H_t involves supply- and demand-side determinants of commonality in liquidity, and a set of control variables. In this regression the spread between the 30-year conventional mortgage rate and the 3-month Treasury bill rate is used as funding liquidity variable. The VIX index is used as transition variable. The estimation method is Nonlinear Least Squared with the covariance matrix corrected for White heteroskedasticity, t-statistics presented in parentheses. ***, **, and * denote significance at the 1%, 5% and 10% confidence levels, respectively. The brackets contain the 95%-confidence interval of the threshold parameter c. The F-test (p-value in parentheses) tests the null hypothesis of a linear model against the alternative of two-regime specification.

approach (i.e., R^2) is no longer adequate since it requires daily data. We choose to use a copula modeling to overcome this issue. A copula is a function that joins or couples two or more marginal distribution functions and describes their dependence structure. We use a normal copula, whose unique parameter is the correlation level, to estimate the degree of comovement between a REIT's liquidity (i.e., the inverse of Amihud's measure) and that of the private real estate market. The bivariate normal copula $C_N(v_1, v_2)$ takes the following form in a static case:

$$C_N(v_1, v_2; \rho) = \int_{-\infty}^{\Phi^{-1}(v_1)} \int_{-\infty}^{\Phi^{-1}(v_2)} \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left[\frac{-(r^2 - 2\rho rs + s^2)}{2(1-\rho^2)}\right] drds \tag{2.6}$$

where ρ is the correlation parameter and Φ^{-1} the inverse of the standard normal cumulative distribution function (cdf). v_1 and v_2 are the marginal distribution functions of our two liquidity series defined on a unit rectangle (i.e., $0 \leq v_k \leq 1$, $k = 1, 2$). We follow Patton (2006) and allow the correlation parameter ρ to vary over time by specifying a conditional normal copula. Formally, the following model (akin to an ARMA(1,10)) is estimated:[21]

$$\rho_t = \tilde{\Lambda}\left(\omega_\rho + \beta_\rho \rho_{t-1} + \alpha_\rho \frac{1}{10}\sum_{j=1}^{10} \Phi^{-1}(v_{1,t-j})\Phi^{-1}(v_{2,t-j})\right) \tag{2.7}$$

where $\tilde{\Lambda}(x) = (1 - e^{-x})(1 + e^{-x})^{-1}$ is the modified logistic transformation, which ensures that the correlation parameter ρ_t remains bounded between -1 and 1. ρ_{t-1} is included in the equation to capture any persistence in the correlation parameter, whereas the product of the transformed variables $\Phi^{-1}(v_{1,t-j})$ and $\Phi^{-1}(v_{2,t-j})$, to capture any variation in dependence.

Relying on a conditional copula allows us to have a monthly commonality in liquidity measure from monthly data. However, this approach has the disadvantage of restricting our analysis to firms with no missing values over the sample period. Thus, our tests are conducted on a sample of 95 REITs (instead of 295).[22] Equation 2.7 is estimated for each firm i included in our sample, which yields a monthly time-series of commonality in liquidity for each REIT (i.e., $\rho_{i,t}$). The commonality in liquidity risk factor that we use in the asset pricing model is constructed in the same way as for the within-asset commonality in liquidity (i.e., "5-1" spread). The risk factor associated with the correlation between REITs' liquidity and direct real estate market returns is computed following the same procedure. We also construct a real estate liquidity risk factor. Again, we use the residuals from an AR(1) applied to the detrended real estate market liquidity as indicator of shocks to market liquidity. The three types of liquidity risk are included in the asset pricing model together with a set of control variables. This model is estimated both within a linear and a regime-switching framework (Equation 2.4).

Figure 2.7 shows the evolution of the average commonality in liquidity with the private market over the 2001-2012 period. Although quite low (i.e., 6% on average; see Table 2.1),

[21] In a first stage, we filter our series (i.e., liquidity measures) by an AR(1)-GARCH(1,1) and estimate the marginal distributions by means of an empirical *cdf* based on the standardized residuals coming from the filtering process. In a second stage, the conditional copula is estimated by maximum likelihood.

[22] We estimated our asset pricing model with the within- and cross-asset commonality in liquidity using the same restricted sample and the results are economically similar, suggesting that our findings with respect to the commonality in liquidity with the private real estate market could also be generalized to the REITs that are not included in the sample.

the correlations are always positive and show some interesting features. The correlation sharply increases during the subprime crisis period, then declines during the 2008-2009 period, and increases again afterwards. It seems therefore that REITs offer some liquidity diversification but not consistently, given the strong fluctuations in the correlation levels. Thus, commonality in liquidity could affect REIT returns, especially in a context of high market volatility. Consistent with this intuition, the highest excess returns due to commonality in liquidity (i.e., the return-based risk factor) appear in 2008-2009 (Figure 2.8). We further analyze the characteristics of the commonality in liquidity between REITs and the underlying real estate market by sorting the levels of liquidity correlation according to firm size and liquidity (Figure 2.9). We observe that larger and more liquid REITs tend to comove less with the private market as shown by, inter alia, the higher number of negative correlations on the right-hand side of the graphs. These findings suggest that large and liquid REITs provide better diversification in terms of liquidity (with respect to direct investments) than small and less liquid REITs. This is consistent with Benveniste, Capozza and Seguin (2001) who show that the liquidity gains of creating equity claims on illiquid property assets are significant only above a certain firm size.

Table 2.9 reports the results of the multi-factor model estimation. In the linear model, the commonality in liquidity with the direct market ($DRECLIQ$) is not a risk factor that increases expected returns since its coefficient is negative. Investors are even willing to receive a lower return if a REIT exhibits this feature. Also, negative shocks to the private market liquidity ($MLIQ$) are found to increase REIT prices (negative coefficient). This finding suggests that investors place a greater value on the liquidity of REITs (i.e., they are willing to pay more) subsequent to a decrease in the underlying real estate market liquidity, and vice versa. As for the sensitivity of REITs' liquidity to direct real estate market returns, we find a positive beta suggesting this risk factor is potentially priced. Overall, the liquidity advantages of REITs seem to be valued.

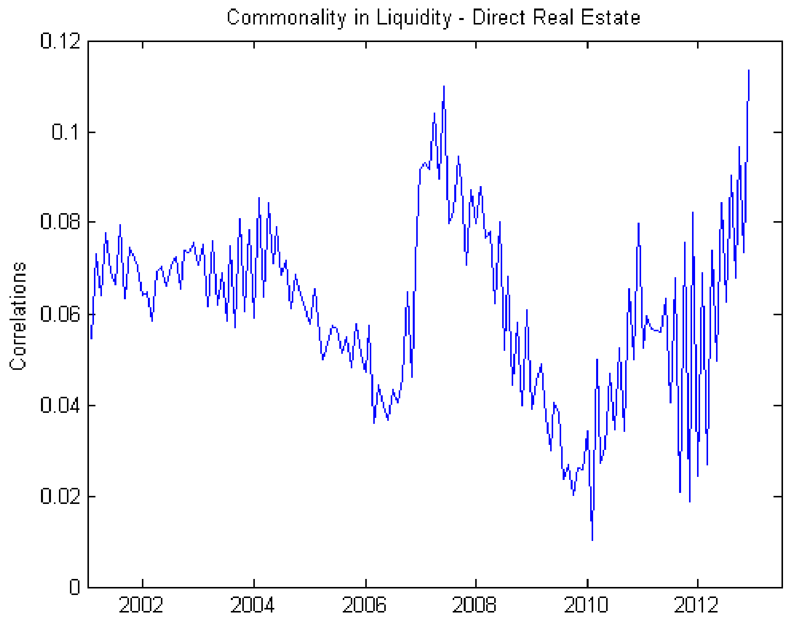
The null hypothesis of linearity being rejected (i.e., F-test in Table 2.9), we turn now to the discussion of the regime-switching model estimation results. Consistent with our hypothesis, the coefficient on $DRECLIQ$ switches from negative in the first regime to positive in the second regime (significant at the 10% confidence level). Moreover, this change is highly significant, suggesting that the dynamics between the commonality in liquidity with the underlying asset and REIT returns are time-varying. Although REITs are a liquid alternative to a direct investment, the extent to which their liquidity is correlated with the private market liquidity during high return volatility periods leads to lower their prices. This finding constitues further evidence in favor of our explanation of the mismatch between theory and empirical evidence on the impact of commonality on asset prices. Furthermore, we find that the economic significance of $DRECLIQ$ is important, with an average impact on REIT returns of 0.05% (for a 1% change in $DRECLIQ$) in the high-risk state (Table B.III in the Appendix). As for the two other types of commonality, this effect is the most important in the high-volatility regime amongst those of the different liquidity risks and is characterized by a sharp increase (0.16%) when the state of the economy changes. Based on our findings concerning the pricing implications of the three types of commonality in liquidity, we conclude thus that, in most cases, a linear approach would have led to underestimate the role of commonality in liquidity risk. It is also important to note that the three types of commonality in liquidity capture different risks given that their respective return-based factors exhibit a relatively low level of correlations (Table B.II in the Appendix).[23]

[23]The correlation between $CLIQ$ and $Cross\text{-}CLIQ$ is -23%, that between $CLIQ$ and $DRECLIQ$ is

We further find that the coefficient on $MLIQ$ switches from significantly negative in the low-risk state to insignificant in the high-risk state. Thus, when the private market liquidity decreases, investors do not pay more for owning a liquid REIT in the high-volatility regime as opposed to in the low-volatility regime. This is likely due to the fact that the REIT's liquidity also declines (see the above discussion on the commonality in liquidity) and does not constitute a diversifying investment in terms of liquidity anymore. This is consistent with the general liquidity dry-up which was observed during the recent financial crisis. Finally, the correlation between REITs' liquidity and market returns, although significantly positive in the low-risk state, is insignificant in the high-risk state.[24] Globally, our results indicate that the liquidity advantages of REITs should be nuanced.

Figure 2.7

Cross-sectional average correlation (commonality in liquidity with the private real estate market)



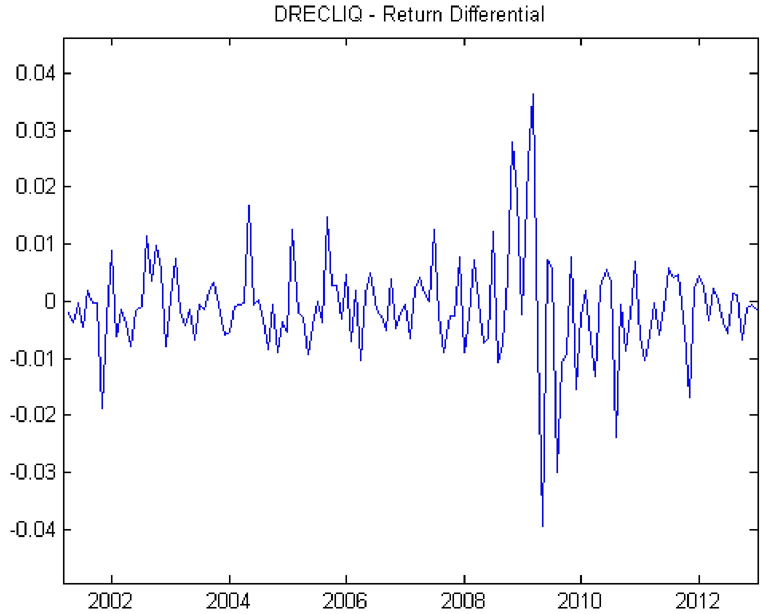
Note: This figure displays the cross-sectional average correlation (commonality in liquidity with the private real estate market) over time. The sample includes 95 firms which have no missing values over the entire sample period.

25%, and that between $Cross\text{-}CLIQ$ and $DRECLIQ$ is -28%.

[24]This insignificant impact is not driven by multicollinearity, since the correlation between $DRECLIQ$ and $DRECLIQRET$ is only 10% (see Table B.II in the Appendix).

Figure 2.8

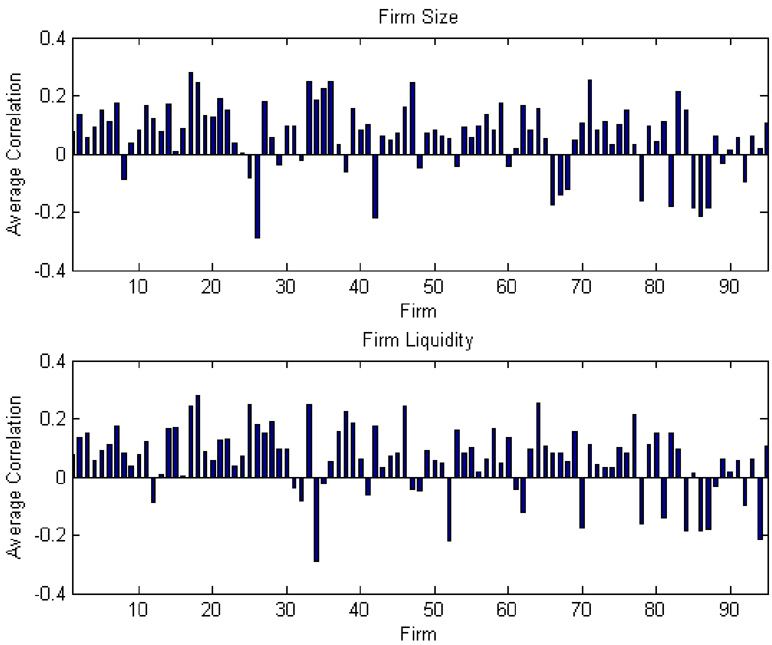
Commonality in liquidity with the private real estate market - return differential



Note: This figure displays the commonality in liquidity risk factor given by the difference in returns between a portfolio with high liquidity *correlation* and a portfolio with low liquidity *correlation* ("5-1" spread).

Figure 2.9

Average liquidity correlation per firm ordered according to the average size and liquidity



Note: These figures display the average commonality in liquidity with the private real estate market by firm ordered by size and liquidity level. The sample includes 95 firms.

Table 2.9
Factor model estimation results

Commonality with the Private Real Estate Market - Realized Volatility

	Linear model	PTR model		
	λ	λ_1	λ_2	$\lambda_2 - \lambda_1$
DRECLIQ	-2.296***	-1.263***	0.751*	2.013***
	(-12.96)	(-8.364)	(1.805)	(4.553)
DRECLIQRET	0.373***	1.094***	-0.003	-1.096***
	(3.596)	(8.709)	(-0.012)	(-4.477)
MLIQ (Private RE)	$-9\cdot10^{-6}$***	$-1\cdot10^{-5}$***	$-7\cdot10^{-6}$	$5\cdot10^{-6}$
	(-5.741)	(-10.53)	(-1.453)	(1.087)
$R_M - R_f$	0.470***	0.439***	0.908***	0.470***
	(11.26)	(17.42)	(17.00)	(8.341)
SMB	0.423***	0.407***	0.650***	0.243***
	(13.41)	(15.18)	(7.754)	(2.762)
HML	0.634***	0.423***	1.156***	0.734***
	(10.77)	(13.00)	(13.73)	(8.143)
Momentum	-0.055***	-0.010	-0.080*	-0.069
	(-2.986)	(-0.527)	(-1.881)	(-1.510)
Credit spread	-0.763*	0.797	-0.486	-1.283
	(-1.840)	(1.230)	(-0.623)	(-1.260)
Term spread	-2.161***	-2.880***	-3.007***	-0.127
	(-5.078)	(-7.039)	(-3.287)	(-0.130)
VIX	$-3\cdot10^{-4}$**	$-5\cdot10^{-4}$***	$-2\cdot10^{-4}$	$2\cdot10^{-4}$**
	(-2.472)	(-3.275)	(-1.245)	(2.552)
Sentiment	$-2\cdot10^{-4}$**	$3\cdot10^{-4}$***	$5\cdot10^{-5}$	$-2\cdot10^{-4}$
	(2.377)	(5.120)	(0.224)	(-1.135)
c		0.0495 [0.0491, 0.0495]		
F-test		591.58 (0.000)		
R-squared	0.3005	0.3133		
Observations	13,490	13,490		

Note: This table contains the estimation results for the multi-factor model (PTR) given by the following equation: $r_{i,t} = \mu_i + \left[\alpha_1 DRECLIQ_t + \beta_1 DRECLIQRET_t + \gamma_1 MLIQ_{i,t} + \delta_1' Z_t\right] \mathbb{1}_{(q_{i,t-1} \leq c)} + [\alpha_2 DRECLIQ_t + \beta_2 DRECLIQRET_t + \gamma_2 MLIQ_{i,t} + \delta_2' Z_t] \mathbb{1}_{(q_{i,t-1} > c)} + \varepsilon_{i,t}$, where $r_{i,t}$ is the monthly excess return on the securitized real estate asset i for month t. $DRECLIQ_t$ is the commonality in liquidity risk factor (with the private real estate market), $DRECLIQRET_t$ is the risk factor associated with the covariance between firms' liquidity and real estate market returns, and $MLIQ_{i,t}$ is the real estate market liquidity risk factor. Z_t includes all the market-wide factors considered in this study. The transition variable $q_{i,t-1}$ is the one-month lagged realized volatility of each firm. We define $\lambda_k = [\alpha_k, \beta_k, \delta_k']'$, $k = 1, 2$. The estimation method is Nonlinear Least Squares with the covariance matrix corrected for White heteroskedasticity (t-statistics in parentheses); the estimation results are presented in the last three columns. The estimation results of a standard panel linear model with fixed effects are displayed in the first column. The estimation method is OLS with White-corrected t-statistics presented in parentheses. ***, **, and * denote significance at the 1%, 5% and 10% confidence levels, respectively. The brackets contain the 95%-confidence interval of the threshold parameter c. The F-test (p-value in parentheses) tests the null hypothesis of a linear model against the alternative of a two-regime specification. The credit spread and the term spread are taken in first difference in order to obtain stationary series.

2.7 Conclusion

Liquidity is a key element in investment decision-making. The theoretical and empirical research is almost unanimous concerning the impact of liquidity risk on asset prices. In particular, Acharya and Pedersen (2005) show that liquidity risk has several components. In this paper, we focus on one of these components, i.e., the commonality in liquidity. Although the existence of commonality in liquidity has been extensively documented in many markets, studies on its pricing implications are scarce. In addition, those studies find that commonality in liquidity has a negligible impact on asset prices. However, this literature analyzes commonality in liquidity exclusively within an unconditional framework, while commonality in liquidity should be perceived differently by investors regarding the state of the economy. Thus, we adopt a conditional approach for investigating commonality in liquidity and test whether its impact on asset returns varies according to the state of the economy. For comparison purposes, we also estimate a linear model. Thus, we contribute to the literature by providing a potential explanation for the gap between the theory and the empirical evidence on the price effects of commonality in liquidity.

We use U.S. REIT data for our empirical investigation. Taking advantage of the hybrid nature of REITs, we are thus able to further examine the pricing implications of two other dimensions of commonality in liquidity: the commonality with the stock market liquidity and the commonality with the underlying asset liquidity. To the best of our knowledge, these aspects have to date remained unexplored. Securitized real estate is also of a certain importance for investors because it represents a liquid alternative to direct real estate investments. Despite its potential importance, liquidity risk remains insufficiently studied within this market. Indeed, although a relatively liquid asset, securitized real estate is not necessarily immune to liquidity risk.

Within a conditional framework, we find that the betas of the three types of commonality in liquidity increase in the second regime, showing that REITs are more exposed to these risks during bad market conditions (i.e., time-varying liquidity risk). We also find that these results are economically significant. The comparison of these results with those of the linear model shows that commonality in liquidity would not have been considered as a risk factor (in most cases) if such an approach had been adopted. Thus, the documented small effect of commonality on asset prices could be explained (at least partly) by the use of an unconditional approach. We also uncover that REIT prices are sensitive to shocks in REIT and stock market liquidity, but that they are relatively immune to those in the private real estate market.

In the analysis of the economic sources of commonality in liquidity, we find that agents' correlated trading activity and investor sentiment play a major role in explaining within- and cross-asset commonality in liquidity. Our evidence is therefore in favor of a demand-side explanation of commonality in liquidity. These results, in line with Karolyi, Lee and van Dijk (2012), and Koch, Ruenzi and Starks (2010), thus challenge the popular funding hypothesis of Brunnermeier and Pedersen (2009).

Our empirical findings offer interesting insights to real estate investors. Although more liquid than direct investments, real estate securities embed a particular risk as materialized by commonality in liquidity. Furthermore, we show that commonality in liquidity has several dimensions which also influence REIT prices. Our paper is the first one highlighting the importance of such risks. Investors should be aware of such risks and avoid making decisions based solely on the liquidity level of the assets; they would benefit from holding assets whose liquidities are not correlated, in particular during stressful periods. Bearing this in mind, an avenue for future research would be to assess the role of commonality in liquidity in portfolio construction.

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B Appendix

Table B.I
Variable description

Variable	Description
<u>ASSET PRICING VARIABLES</u>	
REIT return ($r_{i,t}$)	Monthly return for each REIT. Dependent variable.
R-squared of Eq. 2.2	Commonality in liquidity measure of each REIT with the overall REIT market (LIQ (REIT)) and the stock market (LIQ (Stock)).
CLIQ	Return-based commonality in liquidity risk factor computed from the REIT market as in Eq. 2.3.
CLIQRET	Return-based liquidity risk factor related to the covariation between REITs' liquidity and market REIT returns.
MLIQ (REIT)	REIT market liquidity risk: residuals from an AR(1) process of the value-weighted average of REITs' liquidity.
Cross-CLIQ	Return-based commonality in liquidity risk factor of each REIT firm with the stock market as in Eq. 2.3.
Cross-CLIQRET	Return-based liquidity risk factor related to the covariation between REITs' liquidity and stock market returns.
MLIQ (Stock)	Stock market liquidity risk: residuals from an AR(1) process of the value-weighted average of the liquidity of the stocks in S&P500.
$\rho_{i,t}$	Commonality in liquidity between a REIT and the underlying property market computed from a copula as in Eq. 2.6 and 2.7.
DRELIQ	Return-based commonality in liquidity risk factor of REITs with the underlying property market as in Eq. 2.3.
DRECLIQRET	Return-based liquidity risk factor related to the covariation between REITs' liquidity and private real estate market returns.
MLIQ (Private RE)	Liquidity risk from the private real estate market: residuals from an AR(1) process on the number of properties sold within a month.
$R_M - R_f$	Spread between the market return and the 3-month Treasury bill rate.
SMB	Fama and French factor controlling for size.
HML	Fama and French factor controlling for book-to-market value.

Momentum	Factor controlling for momentum.
Credit spread	Difference between Moody's Baa corporate bond and 10-year U.S. government bond yields.
Term spread	Difference between the 10-year and the 1-year U.S. government bond yields.
VIX	Implied volatility of the S&P 500.
Sentiment	University of Michigan Consumer Confidence Index.
SOURCES OF COMMONALITY IN LIQUIDITY	
$\bar{R}^2{}_t$	Equally-weighted average across firms of the commonality in liquidity (within the REIT market and with the stock market, alternatively). Dependent variable.
TED spread	Difference between the 3-month Eurodollar rate and the 3-month Treasury bill rate.
CP spread	Commercial Paper spread: difference between 3-month commercial paper rate and the 3-month Treasury bill rate.
Mortgage spread	Difference between 30-year conventional mortgage rate and the 3-month Treasury bill rate.
Cturn	Commonality in turnover: proxy for correlated trading activity.
REIT market return	Value-weighted monthly return of the REIT market.
Stock market return	Value-weighted monthly return of the S&P 500 index.
REIT market liquidity	Value-weighted average of the liquidity of the REITs.
Stock market liquidity	Value-weighted average of the liquidity of the stocks in S&P 500.
REIT market turnover	Ratio between the trading volume and the number of shares outstanding in the REIT market.
Stock market turnover	Ratio between the trading volume and the number of shares outstanding in the S&P 500 index.

Table B.II

Correlations - asset pricing variables

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
(1) CLIQ	1.00	$3 \cdot 10^{-3}$	-0.02	-0.23	-0.02	0.03	0.25	0.23	-0.04	-0.21	-0.06	-0.25	0.11	0.15	-0.04	0.31	-0.07
(2) CLIQRET		1.00	-0.09	-0.23	0.37	0.04	0.16	0.10	0.03	-0.25	-0.13	-0.17	0.26	0.03	0.06	0.09	-0.11
(3) MLIQ (REIT)			1.00	0.05	-0.18	0.37	-0.07	-0.16	-0.10	0.23	0.12	-0.02	-0.11	-0.23	-0.02	-0.23	-0.04
(4) Cross-CLIQ				1.00	-0.06	0.04	-0.28	-0.40	0.13	0.49	0.22	0.28	-0.31	-0.22	-0.09	-0.28	0.02
(5) Cross-CLIQRET					1.00	-0.02	-0.02	0.01	-0.06	-0.12	-0.19	-0.05	0.04	0.04	0.04	0.10	0.02
(6) MLIQ (Stock)						1.00	-0.04	-0.03	0.03	0.36	0.24	$-4 \cdot 10^{-3}$	-0.13	-0.39	0.05	-0.19	0.04
(7) DRECLIQ							1.00	0.10	0.04	-0.43	-0.18	-0.16	0.26	-0.07	0.01	0.16	-0.03
(8) DRECLIQRET								1.00	-0.06	0.09	-0.02	-0.22	0.26	0.19	0.23	0.23	0.01
(9) MLIQ (Private RE)									1.00	-0.01	-0.01	0.15	0.14	0.04	-0.03	-0.26	0.14
(10) $R_M - R_f$										1.00	0.37	0.09	-0.51	-0.42	0.08	-0.41	-0.02
(11) SMB											1.00	0.07	-0.15	-0.25	0.06	-0.12	0.06
(12) HML												1.00	-0.02	-0.10	-0.10	-0.29	0.20
(13) Momentum													1.00	0.23	-0.11	-0.05	0.05
(14) Credit spread														1.00	-0.12	0.36	-0.05
(15) Term spread															1.00	0.17	0.09
(16) VIX																1.00	-0.26
(17) Sentiment																	1.00

Note: This table displays the correlations between the variables used in the asset pricing model for the period March, 2001 - December 2012. 'CLIQ', 'Cross-CLIQ' and 'DRECLIQ' are the return-based commonality in liquidity risk factors computed from the REIT market, the stock market and the direct real estate market, respectively, as given in Equation 2.3. 'CLIQRET', 'Cross-CLIQRET' and 'DRECLIQRET' are the return-based liquidity risk factors related to the covariation between REITs' liquidity and market returns from the REIT market, the stock market and the direct real estate market, respectively. 'MLIQ (REIT)', 'MLIQ (Stock)' and 'MLIQ (Private RE)' are the market liquidity risk factors. '$R_M - R_f$' is the spread between the market return and the 3-month T-bill rate. 'SMB', 'HML' and 'Momentum' are the Fama and French factors controlling for size, book-to-market and momentum. The 'Credit spread' is computed as the difference between Moody's Baa corporate bond and 10-year U.S. government bond yields. The 'Term spread' is the difference between the 10-year and 1-year U.S. government bond yields. 'VIX' stands for the Chicago Board of Options Exchange implied volatility index. As a proxy for investor sentiment, we use the University of Michigan consumer confidence index.

Table B.III

Economic significance of liquidity variables

	REIT Market			Stock Market			Real Estate Market		
	Linear	Regime 1	Regime 2	Linear	Regime 1	Regime 2	Linear	Regime 1	Regime 2
CLIQ	-0.027$^\sharp$	-0.083$^\sharp$	0.062$^\sharp$	0.063$^\sharp$	-0.009	0.061$^\sharp$	-0.166$^\sharp$	-0.103$^\sharp$	0.052$^\sharp$
CLIQRET	-0.086$^\sharp$	-0.156$^\sharp$	-0.001	-0.029$^\sharp$	-0.083$^\sharp$	-0.003	0.045$^\sharp$	0.123$^\sharp$	$-4{\cdot}10^{-4}$
MLIQ	0.054$^\sharp$	0.075$^\sharp$	0.045$^\sharp$	0.003	-0.037$^\sharp$	0.059$^\sharp$	-0.053$^\sharp$	-0.083$^\sharp$	-0.027

Note: 'CLIQ' and 'CLIQRET' are return-based commonality factors associated with the commonality in liquidity and the covariance between firms' liquidity and market returns, respectively. 'MLIQ' is a market liquidity risk factor. The three factors are computed within the REIT market (Panel: REIT Market), with the stock market (Panel: Stock Market) and with the underlying real estate market (Panel: Real Estate Market). For each case, the economic significance of the three variables is measured for the linear model (Linear), and for the Panel Threshold Regression model in the normal regime (Regime 1) and in the crisis regime (Regime 2). For the computation of the economic significance we use the following equation: $\hat{\phi}\ \bar{\sigma}_{factor}/\bar{\sigma}_{return}$, where $\hat{\phi}$ is one of the estimated parameters α, β or γ from Equation 2.4 (and its linear counterpart). The estimation results can be found in Tables 2.2, 2.4 and 2.9, respectively. $\bar{\sigma}_{factor}$ and $\bar{\sigma}_{return}$ are the average across firms of the standard deviation of one of the factors of interest and of REIT returns, respectively. For the nonlinear model, these standard deviations are calculated separately for each regime. $^\sharp$ denotes that the corresponding $\hat{\phi}$ coefficient is significant at the 10% significance level or lower.

Chapter 3

Measuring Comovement by a Smooth Transition Simultaneous Equation Model*

Abstract

This article develops a flexible econometric framework to investigate the comovement between two endogenous variables within a nonlinear simultaneous equation model. The model controls also for indirect dependence which intervenes through common observed and unobserved factors. The comovement is modeled as a smooth and potentially asymmetric function of the magnitude of the endogenous variable. The threshold at which a shock is transmitted is estimated with the other parameters of the model. We investigate the properties of an accurate estimation method which takes into account endogeneity, and a testing procedure for simultaneity in the presence of nuisance parameters under the null hypothesis. In a two-equation setup, we study the conditions on the parameters which ensure the uniqueness of the implicit reduced form of the model. Multiple equilibria have important drawbacks such as the unsuitability of the model for forecasting. We illustrate the methodology by studying the impact of the recent crises on the comovement between the sovereign and banking sectors for eight developed countries.

JEL Classification: *C51, C58, G01.*

Keywords: *Market dependence; Smooth transition SEM; Equation-specific factors; Coherency conditions.*

3.1 Introduction

This article proposes an econometric framework to measure the dependence between two possibly endogenous variables. The approach is a generalization of the nonlinear simultaneous equation model of Pesaran and Pick (2007) which is a special case. The parameters are estimated using the nonlinear two-stage least squares method (Kelejian 1971, Amemiya 1974). Additionally, we derive a test for simultaneity in the presence of nuisance parameters, adapting a procedure developed in the framework of smooth transition regression (STR) models (Luukkonen, Saikkonen and Teräsvirta 1988). Finally, we discuss the conditions under which the system of equations has a unique solution.

*This paper is coauthored with Jaya Krishnakumar.

A large body of literature has investigated the dependence among various asset classes, with a renewed interest since the onset of the financial crisis during which comovement became a topical issue. Different approaches are adopted to quantify market comovement.[1] The most used approach is the cross-market correlation analysis (Forbes and Rigobon 2002; Bekaert, Hodrick and Zhang 2009). Other methods are VAR models (Eun and Shim 1989; Dees, di Mauro, Pesaran and Smith 2007; Longstaff 2010; Ehrmann, Fratzscher and Rigobon 2011), latent factor models (Bekaert, Harvey and Ng 2005; Dungey and Martin 2007), probability analysis (Eichengreen, Rose and Wyplosz 1996), extreme value analysis (Longin and Solnik 2001), quantile regressions (Cappiello, Gérard, Kadareja and Manganelli 2014), jump-diffusion models (Aït-Sahalia, Laeven and Pelizzon 2014; Aït-Sahalia, Cacho-Diaz and Laeven 2015).

Various approaches fail to capture important features of market comovement such as (i) the potentially simultaneous occurrence across different markets (i.e. endogeneity), (ii) the nonlinear and potentially asymmetric transmission of shocks, (iii) the level at which a shock is transmitted and its smooth or abrupt nature, (iv) the indirect dependence implied by common observed and unobserved factors which can be of local or global nature. Our econometric framework allows to adequately treat all these issues, thus allowing for a high degree of flexibility in many directions.

The rising cross- and within-market dependence during the last episode of turmoil, brought to the forefront the concept of contagion which is commonly defined as an increase of the financial market comovements during crisis times (e.g. Forbes and Rigobon 2002; Longstaff 2010). Contagion should not be confused with spillovers which are the lagged transmission of shocks, while the former is simultaneous in nature. Financial contagion can be a potential application of our methodology, without being limited to it. This framework can be used to study the asset comovement for portfolio allocation purposes, the transmission of macroeconomic and monetary shocks and many other topics.

In Pesaran and Pick's (2007) framework the dependence between the two endogenous variables is modeled by a dummy variable. This variable takes the value 1 if the endogenous variable present in each equation exceeds an unknown threshold. This modeling presents a limitation since the dependence structure is insensitive tof the magnitude of the shock above the threshold. Moreover, the model assumes that there is a jump in the transmission of shocks. The aim of this article is to add flexibility in the dependence structure by modeling it as a smooth and potentially asymmetric function of the magnitude of the shock (i.e. the value of the endogenous variable). Following a broad literature on STR models, we define the endogenous variable by the logistic function.[2] The threshold, the smoothness and the asymmetric parameters are estimated jointly with the other parameters of the model using heuristic methods.[3]

[1]For a recent review of these methods, their advantages and limitations see Forbes (2012).

[2]If we were to make a parallel, our framework that we name smooth transition SEM (ST-SEM) is the SEM equivalent of the smooth transition regression (STR) models in time series, as Pesaran and Pick's model is the SEM version of the threshold regression (TR) models. The literature on (S)TR models is a great success with many theoretical extensions and empirical applications (see Chan and Tong 1986; Teräsvirta 1994; van Dijk, Teräsvirta and Franses 2002; Teräsvirta, Tjøstheim and Granger 2010), but to our knowledge Pesaran and Pick's (2007) article is the first to deal with simultaneity.

[3]The difficulty of a precise estimation of the smoothness and of the threshold parameters is a well known issue in the STR literature (e.g. González and Teräsvirta 2006; Maringer and Meyer 2008; Schleer 2015). In our framework another difficulty is added since at the first step of the estimation procedure, the logistic function is approximated by a polynomial. To assess the accuracy of the parameter estimation for each equation, we proceed in three steps: (i) fix both the above-mentioned coefficients, (ii) fix only the smoothness parameter and (iii) estimate both of them with the other parameters of each equation.

The nonlinearity of the model in the parameters and in the endogenous variable has a number of consequences concerning the estimation method, the testing procedure for simultaneity and the equilibrium of the system. To perform the nonlinear two-stage least squares, we use low-order polynomials of the exogenous variables as instruments for the logistic function, implying that each equation must be identified (Kelejian 1971). The test for simultaneity contains unidentified parameters under the null hypothesis, in which case the usual tests have non-standard distributions. We adapt a testing procedure developed by Luukkonen, Saikkonen and Teräsvirta (1988) in the framework of STR models. To avoid the nuisance parameter issue, a polynomial approximation is used around a fixed value of the smoothness parameter.

The presence of dummy variables in Pesaran and Pick's model introduces multiple solutions in the system. This is known in the literature as an incoherency issue with important drawbacks such as the unsuitability of the model for prediction and the unknown efficiency of the parameters (e.g. Gourieroux, Laffont and Monfort 1980; Blundell and Smith 1994). In our framework, conditions on the parameters can be set to ensure a unique equilibrium of the system, and thus a unique reduced form. The nonlinearity of the system does not allow an explicit reduced form, but an implicit one can be obtained using numerical methods.

We conduct a series of Monte Carlo simulations to assess the properties of this novel setting, the estimation method and the tests for simultaneity. The model is able to produce excessive skewness and strong dependence between the endogenous variables, although the error terms are i.i.d. Gaussian and not correlated between the equations. Conditioning on the smoothness and on the threshold parameters, the estimation method provides highly accurate results for the dependence coefficient and the parameters related to the common and to the equation-specific factors. When those parameters are unknown, the results are less precise, as expected, but generally improve as the sample size increases. The tests for simultaneity are easy to implement and have good size and power properties which improve with the sample size.

We use our methodology to study the comovement between the sovereign and the banking sectors in the main euro area countries, Switzerland, the U.K. and the U.S. Our results show that comovement strengthens in distressed times. For most of the countries the transmission of shocks from the banking to the sovereign sector dominates. The inverse channel is present in Greece where the deep debt crisis paralyzed the banking sector. Using a linear model and failing to correct for endogeneity leads to the conclusion that both effects are important for the euro area countries. The data reveals that the transmission of shocks is rather abrupt and occurs for negative shocks of moderate to high magnitude.

The outline of the paper is as follows. In Section 3.2 we present the smooth transition simultaneous equation model and its main characteristics. The estimation method and the tests for simultaneity are described in Sections 3.3 and 3.4 respectively. The empirical investigation of the comovement between the sovereign and the banking sectors is displayed in Section 3.5. In Section 3.6 we give several directions for further methodological research and in Section 3.7 we conclude with a brief discussion.

3.2 The model

We first describe the main features of Pesaran and Pick's (2007) model. Next, we introduce our more flexible setting, discuss the reduced form of the model and set the data generating process.

3.2.1 The two-equation threshold SEM

The two-equation nonlinear threshold simultaneous equation model (T-SEM) developed in Pesaran and Pick (2007) can be written as follows:

$$y_{1,t} = \boldsymbol{\delta}_1' \boldsymbol{z}_t + \boldsymbol{\alpha}_1' \boldsymbol{x}_{1,t} + \beta_1 \mathbb{1}(y_{2,t} > c_2) + u_{1,t} \quad (3.1)$$
$$y_{2,t} = \boldsymbol{\delta}_2' \boldsymbol{z}_t + \boldsymbol{\alpha}_2' \boldsymbol{x}_{2,t} + \beta_2 \mathbb{1}(y_{1,t} > c_1) + u_{2,t} \quad (3.2)$$

where $y_{i,t}$ for $i = 1, 2$ and $t = 1, \ldots, T$, can be a return, an interest rate, a spread, $\boldsymbol{z}_t$ and $\boldsymbol{x}_{i,t}$ are p- and k_i-vectors of exogenous / predetermined common and equation-specific variables, respectively. $\boldsymbol{z}_t$ contains a column of ones associated with the constant term along with other common exogenous factors. $\boldsymbol{x}_{i,t}$ can contain lags of the dependent variable, leading to a dynamic system. $\boldsymbol{\delta}_i$, $\boldsymbol{\alpha}_i$, β_i and c_j are a set of unknown parameters. $u_{i,t}$ is $i.i.d(0, \sigma^2_{u_i})$ and the correlation between the error terms – $corr(u_{1,t}, u_{2,t})$ – is denoted by ρ. Even though $corr(u_{1,t}, u_{2,t})$ is assumed to be constant, it can be shown that $corr(y_{1,t}, y_{2,t})$ is time-varying.

$\mathbb{1}(y_{j,t} > c_j)$ introduces the direct dependence from market j to market i, when market j is hit by a negative[4] shock. As shown by a large literature, negative shocks are transmitted to a much larger extent compared to positive shocks, that is why we focus on the former. The variable takes the value 1 if $y_{j,t} > c_j$ and 0 if $y_{j,t}$ is below this threshold c_j. The latter parameter can be interpreted as the level of $y_{j,t}$ which disentangles a normal regime ($y_{j,t} < c_j$) from a crisis regime ($y_{j,t} > c_j$). This interpretation is very often given in threshold regression (TR) models. The magnitude of the comovement is measured by β_i, which is our parameter of interest that we name dependence parameter.

It is important to note that the model controls also for indirect dependence which intervenes through the set of the observed variables $\boldsymbol{z}_t$, and through ρ which controls for unobserved variables. Note that the presence of $\boldsymbol{z}_t$ and a correlation between the error terms different from 0 are not necessary to have direct comovement between the endogenous variables. On the contrary, the equation-specific variables are necessary to identify these two channels since functions of $\boldsymbol{x}_{j,t}$ serve as instruments for the endogenous variable $\mathbb{1}(\cdot)$.

3.2.2 The two-equation smooth transition SEM

Modeling the dependence structure as a dummy variable constraints its value to 1 whatever the magnitude of the shock beyond the threshold level. Similarly, a shock just below the threshold has no impact on the other market. In this article, our aim is to develop a more flexible model in which the variable modeling the dependence is a continuous function of the magnitude of the shock. More specifically, we replace the indicator function by a continuous and bounded function between 0 and 1, which allows a smooth transmission of shocks. The dependence variable tends to 0 for $y_{j,t}$ far below the threshold parameter and to 1 for $y_{j,t}$ far above the threshold parameter. Further, the impact of a shock hitting one market, on another market does not only depend on its magnitude, but

[4]For simplicity, we assume that a negative shock is associated with high values of y. If a negative shock is associated with very low values of y, then taking $-y$ leads to the same model.

also on whether it is transmitted smoothly or abruptly. For this reason, to the unknown threshold parameter we add a smoothness parameter, both to be estimated from the data. We name this setting by smooth transition simultaneous equation model (ST-SEM) and write below a two-equation system:

$$y_{1,t} = \boldsymbol{\delta}_1' \boldsymbol{z}_t + \boldsymbol{\alpha}_1' \boldsymbol{x}_{1,t} + \beta_1 G(y_{2,t}; \gamma_1, c_2) + u_{1,t} \tag{3.3}$$

$$y_{2,t} = \boldsymbol{\delta}_2' \boldsymbol{z}_t + \boldsymbol{\alpha}_2' \boldsymbol{x}_{2,t} + \beta_2 G(y_{1,t}; \gamma_2, c_1) + u_{2,t} \tag{3.4}$$

where $G(y_{j,t}; \gamma_i, c_j)$ is the dependence variable, γ_i is the smoothness parameter and c_j the location parameter, which plays the same role as before.

Many functional forms could be envisaged for $G(y_{j,t}; \gamma_i, c_j)$. In a large literature concerned with smooth transition regression (STR) models, the most commonly employed function is the logistic function (e.g. Teräsvirta 1994; van Dijk, Teräsvirta and Franses 2002; Teräsvirta, Tjøstheim and Granger 2010), which is given by:

$$G(y_{j,t}; \gamma_i, c_j) = \left\{1 + e^{-\gamma_i(y_{j,t} - c_j)}\right\}^{-1} \quad \gamma_i > 0, \quad i, j = 1, 2, \quad i \neq j. \tag{3.5}$$

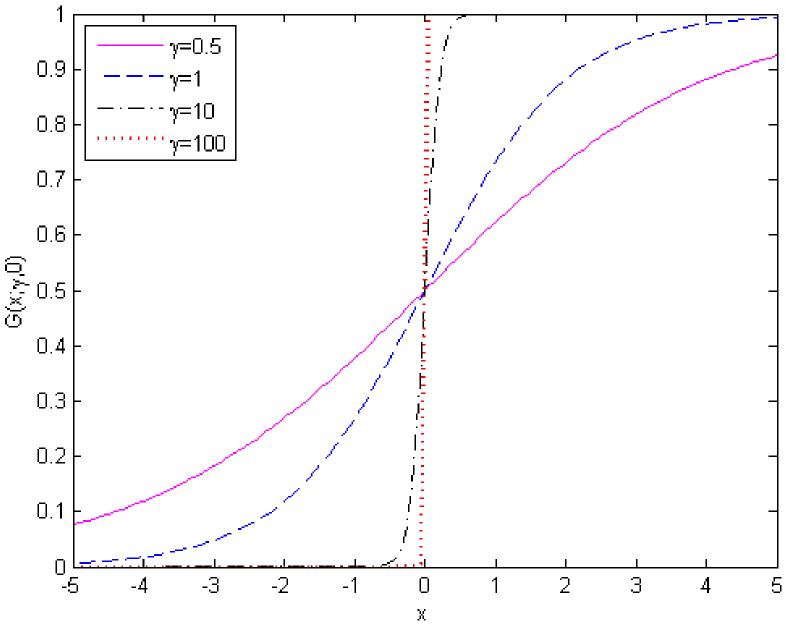
The logistic function has very nice properties such as continuity and monotonicity. It also possesses continuous and bounded first and second derivatives with respect to $y_{j,t}$ for $\gamma_i < +\infty$. Moreover, the logistic function is an appropriate alternative of more complicated and computationally-heavy functions, such as the normal distribution (Chan and Tong 1986).

Theoretically, the ST-SEM nests the T-SEM model of Pesaran and Pick (2007) as a special case when $\gamma_i \to +\infty$, with the logistic function converging to a heavyside function. As we will discuss in Section 3.2.3, for increasing values of the smoothness parameter, the system is more likely to contain multiple solutions. Moreover, the logistic function is almost a step function for moderate values of γ_i. On these grounds we focus on finite values of this parameter. If γ_i happens to be large, then we should consider a step function instead.

Figure 3.1 shows how the steepness of the logistic function changes with the values of γ_i. For values of γ_i not far from 0, the function is flat and it converges to a constant as γ_i tends to 0 ($G(y_{j,t}; 0, c_j) = 0.5$). For $\gamma_i > 0$, the function rapidly converges to a step function for quite low values of this parameter, with a very small distance between two functions. This property presents an important issue for the precise estimation of γ_i, as it will be discussed below. As in Pesaran and Pick (2007), the parameter c_j can be interpreted as the threshold between a normal and a crisis regime (if $y_{j,t} = c_j$ then $G(\cdot) = 0.5$).

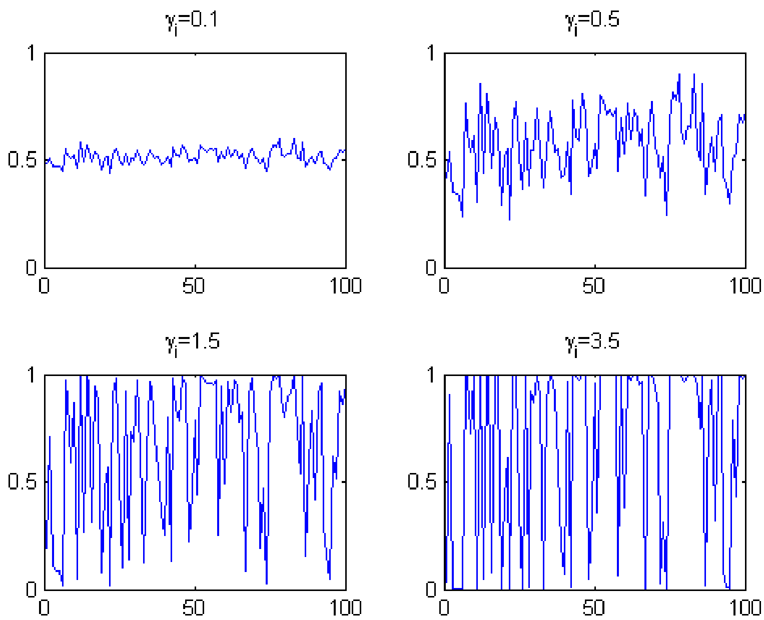
Figure 3.2 shows 100 realizations of the logistic function from simulations using the ST-SEM for different values of $\gamma_i = [0.1, 0.5, 1.5, 3.5]$, $\delta_i = \alpha_i = \beta_i = c_j = 1$, $u_{i,t} \sim i.i.d.\mathcal{N}(0, 1)$ and $corr(u_{1,t}, u_{2,t}) = 0.5$. Keeping in mind that the function is in the range [0, 1], its variation for $\gamma_i = 0.1$ and even for $\gamma_i = 0.5$ is very low. This brings about colinearity issues with the constant term. Thus, for the simulation of the model and for the estimation, the minimum value of γ_i should not be very close to 0. For smoothness parameters as low as 1.5, the series of the logistic function exhibits a much higher variation and for $\gamma_i = 3.5$ the function roughly switches back and forth between 0 and 1.

Figure 3.1
Logistic distribution with different values of γ



Notes: In this figure is displayed the logistic distribution for different values of γ and $c = 0$: $G(x; \gamma, c) = \left(1 + e^{\gamma(x-c)}\right)^{-1}$.

Figure 3.2
Realizations of the logistic function with different values of γ



Notes: This figure displays realizations of the logistic function using our DGP.

3.2.3 Reduced form of the model

For the simulation of the $y_{i,t}$ series, the reduced form of the system of equations is needed. Given the nonlinearity of the system, an explicit reduced form cannot be obtained. Nevertheless, in order to use this model, we need to make sure that a unique value of $y_{i,t}$ is associated with any admissible value of $\boldsymbol{\zeta}_i = [\boldsymbol{\delta}_i', \boldsymbol{\alpha}_i', \beta_i, \gamma_i, c_j]'$, $\mathbf{h}_{i,t} = [\boldsymbol{z}_t', \boldsymbol{x}_{i,t}', G(y_{j,t}; \gamma_i, c_j)]'$ and $u_{i,t}$, i.e. an implicit reduced form is well defined. The conditions on the parameters $\boldsymbol{\zeta}_i$ that ensure the uniqueness of the solution are called *coherency conditions*. An incoherent system can be thought of as not fully specified and a major consequence is that it cannot be used to predict $y_{i,t}$. Next, if the system is incoherent, the efficiency of the estimator of the dependence parameter β_i is unknown. Further, very often the parameters of incoherent models are rather set than point identified. Examples of articles where these conditions are discussed for different specifications of SEM are Gourieroux, Laffont and Monfort (1980), Blundell and Smith (1994), Lewbel (2007) and Pick (2007).

The incoherency issue appears also in Pesaran and Pick's model which corresponds to the limiting case of our framework with $\gamma \to +\infty$. The authors discuss the solution of this binary system and the possibility of multiple equilibria. For the values of $\boldsymbol{z}_t$, $\boldsymbol{x}_{i,t}$ and $u_{i,t}$ for which two solutions are possible, an index d_t which follows a Bernoulli distribution is introduced. The latter takes the value 1 if the favorable solution occurs (non-crisis times) and the value 0 for the unfavorable solution (crisis times). The reduced form depends on the value of d_t.

In our framework, the existence of a unique solution is related to the spectral radius[5] of the Jacobian matrix of the system which has to be smaller than one. The Jacobian matrix $\nabla H(\boldsymbol{y}_t)$ of the two-equation ST-SEM can be written as follows:

$$\nabla H(\boldsymbol{y}_t) = \begin{pmatrix} 0 & G(y_{2,t}; \gamma_1, c_2)[1 - G(y_{2,t}; \gamma_1, c_2)]\beta_1\gamma_1 \\ G(y_{1,t}; \gamma_2, c_1)[1 - G(y_{1,t}; \gamma_2, c_1)]\beta_2\gamma_2 & 0 \end{pmatrix}$$

The spectral radius is given by the following formula:

$$\rho(\nabla H(\boldsymbol{y}_t)) = \left| \sqrt{G(y_{1,t}; \gamma_1, c_2)\,[1 - G(y_{1,t}; \gamma_1, c_2)]\, G(y_{2,t}; \gamma_2, c_1)\,[1 - G(y_{2,t}; \gamma_2, c_1)]\,\gamma_1\beta_1\gamma_2\beta_2} \right|$$

The condition highly depends on the combinations of γ_i and β_i values. The product of the first four terms is bounded since the maximum value it can take is $1/2^4$ which corresponds to $G(y_{j,t}; \gamma_i, c_j) = 1/2$. In this case the product $|\gamma_1\beta_1\gamma_2\beta_2|$ has to be smaller than 16 such that $\rho(\nabla H(\boldsymbol{y}_t)) < 1$. This is a sufficient but a non-necessary condition because if $G(y_{j,t}; \gamma_i, c_j)$ is close to 0 or close to 1 for either of the equations, the product of the first four elements approaches 0 such that $|\gamma_1\beta_1\gamma_2\beta_2|$ can be higher than 16 as far as the spectral radius remains smaller than 1. Surely, the product of the first four elements highly depends on γ_i and β_i, both directly and indirectly through $G(y_{j,t}; \gamma_i, c_j)$ and $y_{j,t}$.

Figures 3.3, 3.4 and 3.5 show examples of the graphical solution of our two-equation system for $\beta_i = 1$ and for different values of the parameter γ_i. In Figure 3.3 with $\gamma_i = 2.5$, the system has a unique solution. In Figure 3.4 which corresponds to $\gamma_i = 10$, the system has three solutions. Putting a negative β_1 in the equation of y_1 would lead to a unique

[5] $\rho(A)$ denotes the spectral radius of a matrix $A \in \mathbb{R}^{n\times n}$ and is defined by $\rho(A) = \max_{1\le i\le n} |\lambda_i|$ where $\lambda_1, \ldots, \lambda_n$ are all the eigenvalues of A. If $\rho(A) < 1$ this is equivalent to stating that the matrix A is convergent or that $\lim_{k\to\infty} A^k x = 0$, for $\forall x \in \mathbb{R}^n$. A square matrix $A \in \mathbb{R}^{n\times n}$ is convergent if $||A^k|| \to 0$ as $k \to \infty$, which is equivalent to $(A^k)_{i,j} \to 0$ as $k \to \infty$ for all i, j.

solution again. In Figure 3.5 with $\gamma_1 = 6$ and $\gamma_2 = 1,000$ the solution is also unique. Increasing γ_2 will not have any effect, but a slight increase of γ_1 brings us back to three solutions. In summary, the number of solutions depends on the value of γ_i, β_i, $G(y_{j,t};\gamma_i,c_j)$ and $y_{j,t}$. Our system is coherent most likely for (but not limited to) low values of the smoothness parameter, that is why we focus on them for our Monte Carlo simulations. We solve the system numerically using Fixed Point and Newton's methods whose description and pseudocodes are given in Appendix C.2 and C.2, respectively.

3.2.4 Data Generating Process

For the Monte Carlo simulations, we generate the data using symmetric parameters for both equations of the system. This will allow us to only focus on the first equation, with symmetric results for the second equation. We use the following data generating process:

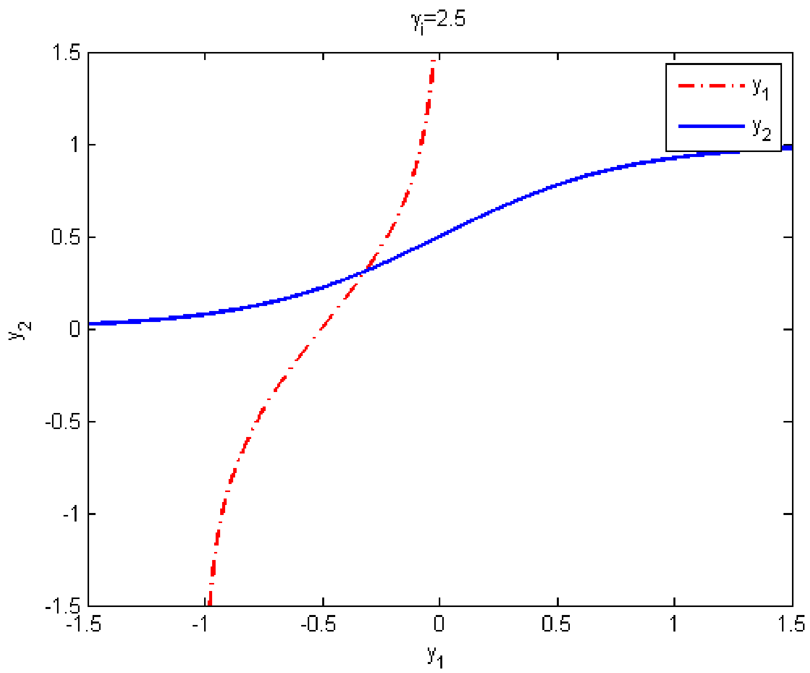
$$y_{i,t}^{(r)} = \delta_{0,i} + \delta_{1,i} z_t^{(r)} + \alpha_i x_{i,t}^{(r)} + \beta_i G(y_{j,t}^{(r)};\gamma_i,c_j) + u_{i,t}^{(r)} \quad i,j = 1,2 \quad i \neq j \quad t = 1,\ldots,T$$

$$G(y_{j,t}^{(r)};\gamma_i,c_j) = \left\{1 + e^{\gamma_i(y_{j,t}^{(r)} - c_j)}\right\}^{-1} \quad \gamma_i > 0$$

with $r = 1,\ldots,R$ the number of replications $R = 5,000$, and the number of observations $T = [100,\ 500]$. Given the high frequency of the financial data, series with 500 observations are easily available. On the other hand, a sample size of 100 is relatively small if we take into account the complexity of the estimation method. The error term follows a normal distribution $u_{i,t}^{(r)} \sim i.i.d.\mathcal{N}(0,1)$ and the correlation between the error terms of the two equations takes three different values $\rho = corr(u_{1,t},u_{2,t}) = [0,\ 0.5,\ 0.8]$.

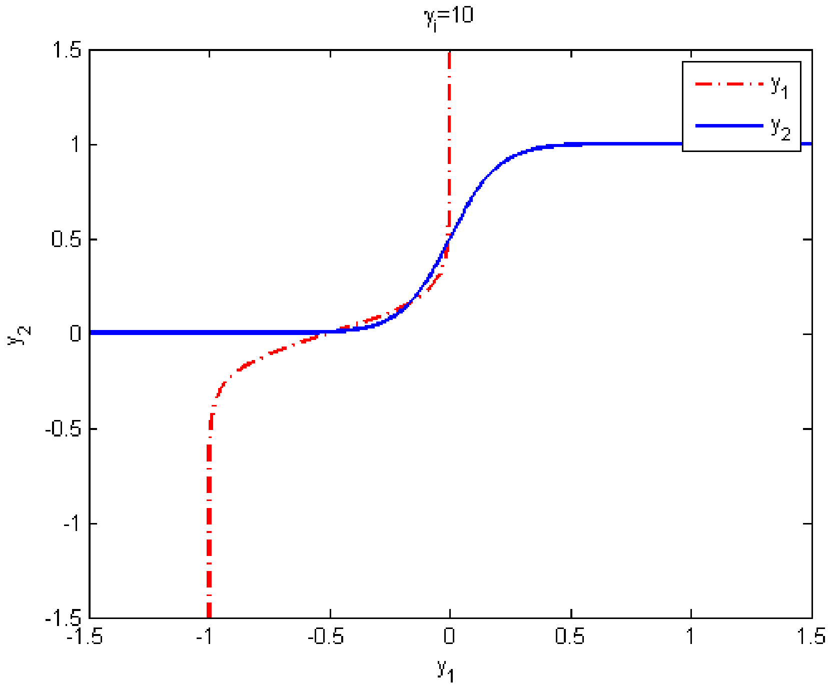
Figure 3.3

Graphical solution of a two-equation system, $\gamma_i = 2.5$



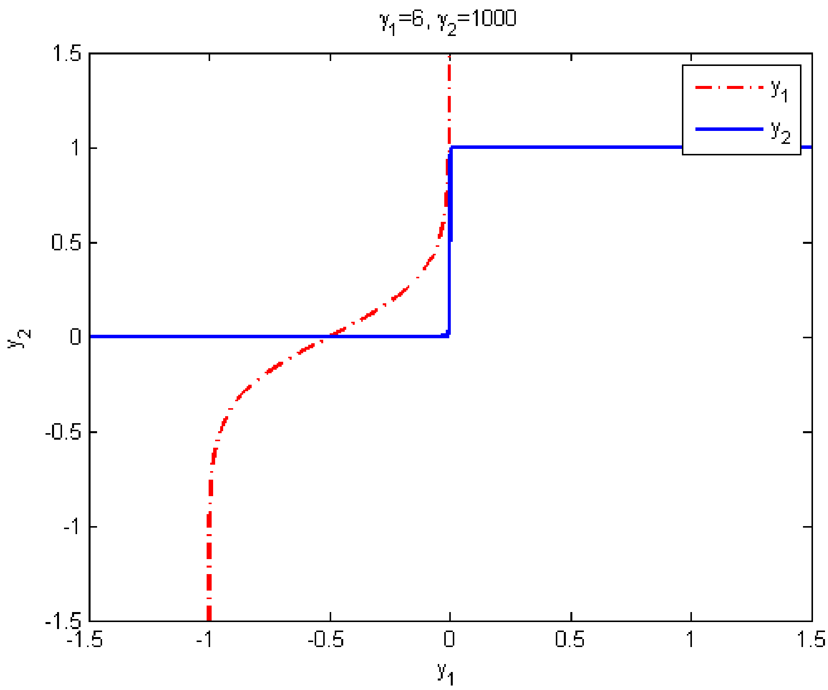
Notes: $y_1 = -1 + \{1 + e^{-2.5y_2}\}^{-1}$ and $y_2 = \{1 + e^{-2.5y_1}\}^{-1}$.

Figure 3.4
Graphical solution of a two-equation system, $\gamma_i = 10$



Notes: $y_1 = -1 + \{1 + e^{-10y_2}\}^{-1}$ and $y_2 = \{1 + e^{-10y_1}\}^{-1}$.

Figure 3.5
Graphical solution of a two-equation system, $\gamma_1 = 6$, $\gamma_2 = 1,000$



Notes: $y_1 = -1 + \{1 + e^{-6y_2}\}^{-1}$ and $y_2 = \{1 + e^{-1000y_1}\}^{-1}$. A zoom-in shows that there is only one solution.

We set $\delta_{0,i} = \delta_{1,i} = \alpha_i = 1$, while we try several values for the dependence parameter $\beta_i = [0,\ 0.3,\ 0.5,\ 1,\ 1.5]$ in order to carefully check the properties of the model. Given the sensitivity of the model with respect to the smoothness parameter, we test $\gamma_i = [0.5,\ 1.5,\ 2.5]$. The smallest value is close to the lower bound of 0, and given the low variation of the logistic function in this case, colinearity with the constant term may occur. On the other hand, the highest value is set such that the coherency of the model and numerical convergence of the system are guaranteed. Nevertheless, this should not be restrictive since a logistic function with $\gamma_i = 2.5$ is much closer to a step function than to a flat line.

The location parameter is set at $c_j = 1$ and in the estimation procedure the values it can take are constrained to a range corresponding to the 20$^{\text{th}}$ and 80$^{\text{th}}$ quantiles of the distribution of $y_{j,t}$. c should not be far in the tails of the distribution because the dependence variable will have a disproportionately high number of values close to 0 or close to 1. In both cases, identification issues will arise. Given that the generation of the values of $y_{j,t}$ depends on the value of c_j, we cannot set in advance the latter as a precise quantile of the former. Ex-post computations show that $c_j = 1$ corresponds to middle quantiles. We also performed analyses with $c_j = 1.5$ without changing the results.

The explanatory variables for each equation follow the autoregressive processes written below:

$$z_t^{(r)} = \xi z_{t-1}^{(r)} + \nu_t, \quad x_{i,t}^{(r)} = \phi_i x_{i,t-1}^{(r)} + (1-\phi_i^2)^{1/2} \varepsilon_{i,t}^{(r)}$$

with the error term written as:

$$\varepsilon_{1,t}^{(r)} = \theta \varepsilon_{2,t}^{(r)} + \eta_t^{(r)}$$

and $v_t^{(r)}, \varepsilon_{2,t}^{(r)}, \eta_t^{(r)} \sim i.i.d.\mathcal{N}(0,1)$, $\xi = \phi_i = [0,\ 0.5]$ and $\theta = corr(\varepsilon_{1,t}, \varepsilon_{2,t}) = [0,\ 0.3]$. If θ is different from 0, then the equation-specific variables are correlated, violating the identification condition. In real data it happens very frequently that the equation-specific variables are not independent of each other, and hence we are interested in checking the properties of the estimators and of the test statistics in this case.

Following this data generating process, in Table 3.1 and Figure 3.6 we show characteristics of the distribution of $y_{i,t}$, performing similar exercises to Pesaran and Pick (2007), Table 1, page 1252, Figures 2, 3 and 4, pages 1253-1254. We find very similar patterns compared to the above article, notably (i) a non-zero correlation between $y_{1,t}$ and $y_{2,t}$ when the error terms are not correlated, and (ii) bimodal distributions for $y_{i,t}$ associated with high values of the dependence and of the smoothness parameters.

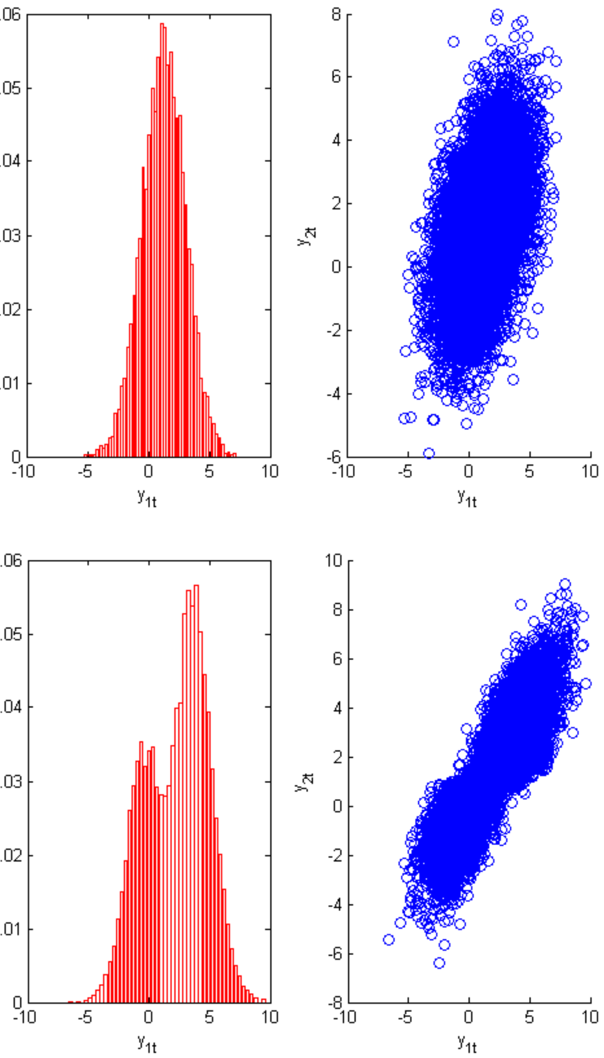
Figure 3.6 shows the distribution of $y_{1,t}$ and a scatter plot between $y_{1,t}$ and $y_{2,t}$ for two combinations of β_i and γ_i: upper panel $\beta_i = 0.5$, $\gamma_i = 1$, lower panel $\beta_1 = 2$, $\gamma_1 = 6$, $\beta_2 = 0.5$ and $\gamma_2 = 1$. In the upper panel, the distribution of $y_{1,t}$ has a regular shape and so does the cloud of points in the scatter plot. In the lower panel the distribution of $y_{1,t}$ is bimodal. A further analysis showed that the bimodality is visible as γ_i becomes larger, while for low values of γ_i and large β_i the distribution is well-behaved. If bimodality is present, it will strengthen as the correlation between the error terms ρ becomes larger.

Table 3.1 shows summary statistics for $y_{i,t}$ using different parameter combinations. It is important to note that even when $\rho = 0$, the correlation between $y_{1,t}$ and $y_{2,t}$ is almost always above 0.5 with a maximum value of 0.75. Thus, the analyses of comovement based on the correlation between the error terms might be inaccurate since absence of correlation in the error term does not imply no linkages between markets. Naturally, $corr(y_{1,t}, y_{2,t})$

increases as β_i and ρ increase with a value as high as 0.87 for $\beta_i = 1.5$ and $\rho = 0.8$. The model is also capable of producing skewness, which is a desirable property given that many empirical studies find that financial series have asymmetric distributions.

Figure 3.6

Distribution of $y_{1,t}$ and scatterplot of $y_{1,t}$, $y_{2,t}$



$$y_{i,t} = \delta_{0,i} + \delta_{1,i} z_t + \alpha_i x_{i,t} + \beta_i G(y_{j,t}; \gamma_i, c_j) + u_{i,t} \quad i,j = 1,2 \quad i \neq j$$

$$G(y_{j,t}; \gamma_i, c_j) = \left\{1 + e^{\gamma_i (y_{j,t} - c_j)}\right\}^{-1} \qquad \gamma_i > 0$$

$$z_t = \xi z_{t-1} + \nu_t, \quad x_{i,t} = \phi_i x_{i,t-1} + (1 - \phi_i^2)^{1/2} \varepsilon_{i,t}, \quad \varepsilon_{1,t} = \theta \varepsilon_{2,t} + \eta_t$$

$$v_t, u_{i,t}, \varepsilon_{2,t}, \eta_t \sim i.i.d.\mathcal{N}(0,1),\ \delta_{0,i} = \delta_{1,i} = \alpha_i = 1,\ c_j = 1,\ T = 10,000$$

Upper panel: $\beta_i = 0.5$, $\gamma_i = 1$, $\rho = 0$, $\phi_i = \xi = \theta = 0$. Lower panel: $\beta_1 = 2$, $\beta_2 = 0.5$, $\gamma_1 = 6$, $\gamma_2 = 1$, $\rho = 0.8$, $\phi_i = \xi = 0.5$, $\theta = 0.3$.

Table 3.1

Moments of the distribution of y_1

$\boldsymbol{\beta_1}$	$\boldsymbol{\xi=0,\ \phi_i=0,\ \theta=0}$										$\boldsymbol{\xi=0.5,\ \phi_i=0.5,\ \theta=0.3}$									
	$\gamma_1=1$					$\gamma_1=2$					$\gamma_1=1$					$\gamma_1=2$				
	$\bar{y}_1$	$\sigma(y_1)$	Kurt	Skew	Corr	$\bar{y}_1$	$\sigma(y_1)$	Kurt	Skew	Corr	$\bar{y}_1$	$\sigma(y_1)$	Kurt	Skew	Corr	$\bar{y}_1$	$\sigma(y_1)$	Kurt	Skew	Corr
$\boldsymbol{\rho=0}$																				
0.5	1.266	1.799	2.990	-0.060	0.470	1.255	1.825	2.875	-0.012	0.491	1.314	1.917	2.935	0.032	0.589	1.264	1.976	2.880	-0.052	0.624
1	1.566	1.922	2.879	-0.068	0.588	1.593	1.939	2.860	-0.026	0.610	1.552	2.047	2.867	-0.007	0.672	1.576	2.095	2.780	-0.034	0.696
1.5	1.965	1.993	2.827	-0.122	0.647	2.031	2.072	2.725	-0.160	0.667	1.931	2.149	2.755	-0.092	0.737	1.966	2.273	2.699	-0.168	0.749
$\boldsymbol{\rho=0.5}$																				
0.5	1.263	1.805	2.897	-0.036	0.601	1.256	1.861	2.851	0.010	0.628	1.247	1.939	2.851	-0.026	0.718	1.237	1.989	2.848	-0.005	0.729
1	1.604	1.915	2.883	-0.083	0.689	1.608	1.985	2.783	-0.079	0.709	1.555	2.079	2.836	-0.021	0.772	1.616	2.134	2.756	-0.054	0.785
1.5	1.975	2.032	2.778	-0.148	0.745	1.989	2.160	2.636	-0.216	0.767	2.024	2.213	2.707	-0.134	0.820	1.951	2.300	2.559	-0.190	0.829
$\boldsymbol{\rho=0.8}$																				
0.5	1.284	1.827	2.920	-0.032	0.692	1.253	1.858	2.947	-0.012	0.705	1.237	1.954	2.999	-0.036	0.779	1.220	1.980	2.789	-0.031	0.787
1	1.597	1.952	2.860	-0.049	0.760	1.617	2.029	2.751	-0.078	0.777	1.595	2.080	2.840	-0.069	0.832	1.597	2.125	2.725	-0.100	0.835
1.5	1.897	2.057	2.821	-0.144	0.801	2.004	2.148	2.625	-0.196	0.814	1.943	2.242	2.735	-0.134	0.864	2.007	2.346	2.524	-0.205	0.866

Note: This table presents summary statistics for y_1: $\bar{y}_1$ the sample average, $\sigma(y_1)$ the standard error, 'Kurt' the kurtosis of y_1, 'Skew' the skewness and 'Corr' the correlation between y_1 and y_2. Given that the system is symmetric, we present the results only for y_1. The data are obtained from simulations according to the data generating process below:

$$y_{it} = \delta_{0i} + \delta_{1i} z_t + \alpha_i x_{it} + \beta_i G(y_{jt}; \gamma_i, c_j) + u_{it} \quad i, j = 1, 2 \quad i \neq j$$

$$G(y_{jt}; \gamma_i, c_j) = \left\{1 + e^{\gamma_i (y_{jt} - c_j)}\right\}^{-1} \quad \gamma_i > 0$$

$$z_t = \xi z_{t-1} + \nu_t, \quad x_{it} = \phi_i x_{it-1} + (1 - \phi_i^2)^{1/2} \varepsilon_{it}, \quad \varepsilon_{1t} = \theta \varepsilon_{2t} + \eta_t, \quad v_t, u_{it}, \varepsilon_{2t}, \eta_t \sim i.i.d.\mathcal{N}(0,1)$$

$$\delta_{0i} = \delta_{1i} = \alpha_i = 1,\ \beta_i = [0.5\ 1\ 1.5],\ \gamma_i = [1\ 2],\ c_j = 1,\ \rho = corr(u_{1t}, u_{2t}) = [0\ 0.5\ 0.8], \theta = corr(\varepsilon_1, \varepsilon_2) = [0\ 0.3], \xi = \phi_i = [0\ 0.5], T = 10{,}000.$$

3.3 Estimation method and simulations

3.3.1 Estimation method

The dependence variable $G(y_{j,t};\gamma_i,c_j)$ is a nonlinear function of the endogenous variable $y_{j,t}$ and of the unknown parameters γ_i and c_j. Kelejian (1971) and Amemiya (1974) show that the parameters of nonlinear SEM models can be consistently estimated by a Nonlinear Two-Stage Least Squares (NL2SLS) procedure for each equation of the system. Kelejian (1971) suggests to regress at the first stage of the estimation procedure $G(y_{j,t};\gamma_i,c_j)$ on a polynomial of the instruments. The accuracy of the estimator should improve as the order d of the polynomial increases. As mentioned earlier, the presence of exogenous equation-specific variables is necessary to ensure identification. Kelejian (1971) and Amemiya (1974) show the consistency of the estimates for the overall set of parameters and their asymptotic normality under usual regularity conditions. Appendix C.1 gives the limiting distribution of the ST-SEM.

Below we describe the NL2SLS estimation procedure for each of the equations of our two-equation system. Let us denote by $\boldsymbol{\psi}_{i,t}=[\boldsymbol{\delta}_i',\ \boldsymbol{\alpha}_i',\ \beta_i]'$ the set of parameters which appear linearly in Equation i, by $\mathbf{h}_{i,t}=[\boldsymbol{z}_t',\ \boldsymbol{x}_{i,t}',\ G(y_{j,t};\gamma_i,c_j)]'$ the set of explanatory variables in this same equation for $i=1,2,\ i\neq j$, and by $\mathbf{w}_t=[\boldsymbol{z}_t',\ \boldsymbol{x}_{1,t}',\ \boldsymbol{x}_{2,t}']'$ the overall set of the exogenous variables of the system.

Step 1: Take initial values of γ_i and c_j denoted by $\hat{\gamma}_i$ and $\hat{c}_j$ and compute $\tilde{G}_{i,t}=G(y_{j,t};\hat{\gamma}_i,\hat{c}_j)$.

Step 2: For the first stage of the NL2SLS, regress $\tilde{G}_{i,t}$ on a d-order polynomial of $\mathbf{w}_t$ using OLS method, and denote the fitted value by $\hat{\tilde{G}}_{i,t}$.

Step 3: For the second stage of the NL2SLS, regress using OLS $y_{i,t}$ on $\hat{\mathbf{h}}_{i,t}=[\boldsymbol{z}_t',\ \boldsymbol{x}_{i,t}',\ \hat{\tilde{G}}_{i,t}]'$ to obtain an estimate of $\boldsymbol{\psi}_{i,t}$ denoted by $\hat{\boldsymbol{\psi}}_{i,t}$, then compute the sum of squared residuals $SSR_i=\sum_{t=1}^{T}\hat{u}_{i,t}^2$.

Step 4: Repeat Steps 1 to 3, using an updating procedure for γ_i and c_j, until SSR_i is minimized.

The literature concerned with STAR models points out the difficulty of precisely estimating the smoothness parameter γ_i for sufficiently large values. Indeed, as γ_i increases the logistic function rapidly converges to a step function, and large variations of this parameter lead to slight changes in the shape of the function (see Figure 3.1). To improve the estimator for γ_i, a large number of observations is required around c_j (see van Dijk, Teräsvirta and Franses 2002). An accurate estimation of γ_i is even more difficult in our case because of the approximation of $G(y_{j,t};\gamma_i,c_j)$ by a polynomial. Nevertheless, Teräsvirta (1994) and Teräsvirta, Tjøstheim and Granger (2010) argue that (i) a lack of significance of γ_i does not suggest that the model is linear, and (ii) a precise estimation of this parameter is not necessary for an accurate estimation of the other parameters of the model. In practice, γ_i is often bounded to a range of values not very close to zero and not too large; for instance, not higher than 100 (e.g. Schleer 2015).

In TR models the distribution of c_j is non-standard (see Hansen 2000). In STR models (with known γ_i), Andrews and Cheng (2013) show that the distribution of c_j depends on the value of $|\beta_i|$ and is also non-standard under no identification or weak identification.

The parameter c_j is very often winsorized in the quantile range 15^{th} to 85^{th} (e.g. Andrews 1993; González and Teräsvirta 2006), or $20^{\text{th}}$to 80^{th} (e.g. Hansen 1996). Indeed, for extreme values of c_j the logistic function only takes values that are either close 0 or close to 1, which raises a colinearity issue with the constant term, and ill-behavior of the covariance matrix and of the test statistics.

3.3.2 Optimization with heuristic methods

We use the Differential Evolution (DE) method to minimize the objective function with respect to γ_i and c_j, following a recent article by Maringer and Meyer (2008) who employ heuristic methods in STAR models.[6] The DE heuristic approach was first developed by Storn and Price (1997) for the minimization of possibly nonlinear and non-differentiable functions. Many studies show that the DE method outperforms other single-agent or multi-agent methods (e.g. Maringer and Meyer 2008; Schleer 2015). The DE is a population based method, and as such it simultaneously considers several solutions. The algorithm mimics the evolutionary process in nature where the offspring with better features produced through mutation and through the combination of parents' properties have better chances to survive.

For each individual solution $\mathbf{x}_p$ of the current population, an offspring solution $\boldsymbol{o}_p$ is generated in three steps. First, three distinct members of the current population are randomly picked m_1, m_2 and m_3, and $p \neq m_1 \neq m_2 \neq m_3$. Second, an interim solution $\tilde{\boldsymbol{o}}_p$ is computed from $\tilde{\boldsymbol{o}}_p = \mathbf{x}_{m_1} + F \cdot (\mathbf{x}_{m_2} - \mathbf{x}_{m_3})$. Third, the offspring $\boldsymbol{o}_p$ is a combination of $\tilde{\boldsymbol{o}}_p$ with cross-over probability π and of $\mathbf{x}_p$ with probability $1 - \pi$. The offspring survives if it has a better fit than the corresponding current solution.

The DE method has several main advantages compared to gradient-based methods: (i) it is able to escape local minima through the presence of random components, (ii) it does not require a sophisticated initial solution. Compared to other heuristic methods, DE needs very little tuning. Crucial parameters are (i) the population size, which should be large enough to reflect diversity, (ii) the number of simulations to allow the algorithm to converge, (iii) the scaling factor F and (iv) the cross-over probability π. Based on the study by Maringer and Meyer (2008), we set $F = 0.5$, $\pi = 0.8$, the population size to $10D$, with D the dimension of the optimization problem and the number of iterations to 500. For more details about the algorithm, see the pseudocode in the Appendix, Section C.2.

3.3.3 Simulation results

We now study the properties of the estimation method presented above, proceeding in three steps. First, we fix γ_i and c_j, then only γ_i is fixed and finally all the parameters of each equation are estimated jointly. For ease of interpretation, the distributions are centered around 0.

Known smoothness and location parameters

Tables 3.2 and 3.3 show the bias and the root mean squared error (RMSE) for the dependence parameter β_1 with $d = 1$ and $d = 3$, respectively. The first column of each of the

[6] We have also used gradient-based methods, grid search, threshold acceptance and simulated annealing. We found out that the DE method performs better. A comparison of these methods in our framework will be the subject of a future study. We use the notations of Maringer and Meyer (2008).

Table 3.2
Bias and root mean squared error for β_1 with *known* γ and c, $d = 1$

	$\xi = 0$, $\phi_i = 0$, $\theta = 0$						$\xi = 0.5$, $\phi_i = 0.5$, $\theta = 0.3$					
β_1	T=100			T=500			T=100			T=500		
γ	0.5	1.5	2.5	0.5	1.5	2.5	0.5	1.5	2.5	0.5	1.5	2.5
$\rho = 0$												
Bias												
0	0.017	0.006	0.004	-0.005	0.006	0.000	0.002	0.005	0.003	-0.009	0.004	-0.001
0.3	0.021	-0.009	-0.017	0.005	-0.005	-0.001	0.023	-0.009	-0.021	-0.002	-0.005	-0.001
0.5	0.008	-0.011	0.007	-0.001	-0.005	0.003	0.007	-0.015	0.008	-0.004	-0.002	0.000
1	-0.015	-0.014	0.011	-0.004	0.004	-0.005	-0.008	-0.021	0.011	-0.007	0.009	-0.008
1.5	-0.012	-0.030	-0.009	0.003	-0.014	0.000	0.007	-0.031	-0.033	0.008	-0.012	-0.004
RMSE												
0	0.972	0.540	0.483	0.418	0.233	0.210	1.050	0.595	0.542	0.446	0.257	0.233
0.3	0.968	0.545	0.498	0.421	0.238	0.215	1.057	0.620	0.569	0.452	0.258	0.236
0.5	0.966	0.569	0.506	0.427	0.242	0.220	1.055	0.633	0.583	0.455	0.264	0.243
1	0.977	0.593	0.550	0.425	0.250	0.236	1.080	0.676	0.640	0.456	0.277	0.266
1.5	0.999	0.648	0.621	0.426	0.275	0.268	1.101	0.757	0.722	0.458	0.309	0.303
$\rho = 0.5$												
Bias												
0	-0.030	-0.016	-0.014	-0.014	0.002	-0.004	-0.053	-0.022	-0.020	-0.018	-0.001	-0.005
0.3	-0.026	-0.032	-0.037	-0.004	-0.010	-0.004	-0.033	-0.039	-0.048	-0.012	-0.010	-0.005
0.5	-0.039	-0.036	-0.014	-0.010	-0.009	-0.001	-0.048	-0.046	-0.021	-0.015	-0.008	-0.004
1	-0.062	-0.042	-0.013	-0.013	0.000	-0.010	-0.066	-0.055	-0.021	-0.017	0.003	-0.014
1.5	-0.061	-0.062	-0.040	-0.005	-0.020	-0.006	-0.052	-0.074	-0.078	-0.002	-0.019	-0.011
RMSE												
0	0.979	0.541	0.486	0.419	0.233	0.211	1.061	0.594	0.546	0.447	0.257	0.234
0.3	0.977	0.556	0.510	0.422	0.240	0.218	1.069	0.634	0.586	0.454	0.261	0.240
0.5	0.975	0.583	0.521	0.428	0.246	0.224	1.071	0.651	0.603	0.457	0.268	0.248
1	0.994	0.618	0.574	0.428	0.256	0.245	1.103	0.711	0.671	0.459	0.284	0.276
1.5	1.021	0.684	0.666	0.430	0.285	0.281	1.128	0.807	0.792	0.462	0.320	0.319
$\rho = 0.8$												
Bias												
0	-0.059	-0.028	-0.025	-0.019	-0.001	-0.006	-0.086	-0.038	-0.035	-0.024	-0.004	-0.008
0.3	-0.054	-0.046	-0.051	-0.009	-0.012	-0.006	-0.066	-0.057	-0.066	-0.017	-0.014	-0.008
0.5	-0.067	-0.051	-0.027	-0.015	-0.012	-0.003	-0.081	-0.066	-0.039	-0.021	-0.011	-0.007
1	-0.091	-0.059	-0.030	-0.018	-0.003	-0.013	-0.101	-0.078	-0.045	-0.023	0.000	-0.018
1.5	-0.092	-0.084	-0.061	-0.011	-0.024	-0.010	-0.089	-0.103	-0.110	-0.008	-0.024	-0.016
RMSE												
0	0.990	0.546	0.491	0.420	0.234	0.211	1.077	0.600	0.555	0.448	0.258	0.234
0.3	0.992	0.567	0.521	0.423	0.242	0.220	1.087	0.649	0.603	0.455	0.263	0.243
0.5	0.988	0.596	0.534	0.430	0.248	0.227	1.089	0.669	0.623	0.459	0.271	0.253
1	1.012	0.639	0.598	0.430	0.261	0.251	1.127	0.741	0.707	0.462	0.289	0.285
1.5	1.042	0.715	0.702	0.433	0.292	0.291	1.157	0.850	0.849	0.464	0.328	0.332

Note: This table presents the *bias* and the *root mean squared error* (RMSE) of the dependence coefficient β_1 for a model with *known* γ_i and c_j. The system is symmetric, that is why we present the results only for the first equation. The data generating process with $5,000$ simulations is given below:

$$y_{i,t} = \delta_{0,i} + \delta_{1,i} z_t + \alpha_i x_{i,t} + \beta_i G(y_{j,t}; \gamma_i, c_j) + u_{i,t} \quad i, j = 1, 2 \quad i \neq j$$

$$G(y_{j,t}; \gamma_i, c_j) = \left\{1 + e^{\gamma_i (y_{j,t} - c_j)}\right\}^{-1} \quad \gamma_i > 0$$

$$z_t = \xi z_{t-1} + \nu_t, \quad x_{i,t} = \phi_i x_{i,t-1} + (1 - \phi_i^2)^{1/2} \varepsilon_{i,t}, \quad \varepsilon_{1,t} = \theta \varepsilon_{2,t} + \eta_t, \quad \nu_t, u_{i,t}, \varepsilon_{2,t}, \eta_t \sim i.i.d.\mathcal{N}(0, 1)$$

$$\delta_{0,i} = \delta_{1,i} = \alpha_i = 1,\ \beta_i = [0\ 0.3\ 0.5\ 1\ 1.5],\ \gamma_i = [0.5\ 1.5\ 2.5],\ c_j = 1$$

$$\rho = corr(u_{1,t}, u_{2,t}) = [0\ 0.5\ 0.8],\ \theta = corr(\varepsilon_{1,t}, \varepsilon_{2,t}) = [0\ 0.3],\ \xi = \phi_i = [0\ 0.5],\ T = [100,\ 500]$$

The $y_{i,t}$ series is obtained through an implicit reduced form using numerical procedures. We use nonlinear 2SLS as estimation method, approximating in the first stage of the procedure the logistic function by a polynomial of degree d of all the exogenous variables of the system. In this table $d = 1$.

tables shows the different values taken by $\beta_i = [0,\ 0.3,\ 0.5,\ 1,\ 1.5]$. In the left panel there is no structure in the regressors, while in the right panel correlation and autocorrelation

Table 3.3

Bias and root mean squared error for β_1 with *known* γ and c, $d = 3$

	$\xi = 0$, $\phi_i = 0$, $\theta = 0$						$\xi = 0.5$, $\phi_i = 0.5$, $\theta = 0.3$					
β_1	T=100			T=500			T=100			T=500		
γ	0.5	1.5	2.5	0.5	1.5	2.5	0.5	1.5	2.5	0.5	1.5	2.5
$\rho = 0$												
Bias												
0	0.006	-0.004	-0.003	0.001	-0.002	0.000	-0.006	-0.005	0.003	0.004	-0.002	0.001
0.3	0.008	-0.004	0.012	0.003	-0.002	-0.004	0.002	-0.001	0.012	0.002	-0.002	-0.002
0.5	0.022	0.010	0.005	0.014	0.002	0.005	0.030	0.007	0.011	0.017	0.001	0.004
1	0.045	0.051	0.030	0.010	0.008	0.001	0.045	0.061	0.049	0.012	0.011	0.004
1.5	0.060	0.040	0.049	-0.004	0.007	0.006	0.051	0.055	0.056	-0.007	0.009	0.011
RMSE												
0	0.942	0.525	0.455	0.419	0.235	0.206	1.025	0.575	0.514	0.442	0.253	0.223
0.3	0.936	0.530	0.472	0.423	0.235	0.214	1.023	0.582	0.522	0.444	0.255	0.235
0.5	0.951	0.537	0.474	0.420	0.235	0.214	1.026	0.585	0.527	0.447	0.260	0.236
1	0.949	0.554	0.512	0.414	0.247	0.226	1.025	0.612	0.565	0.444	0.270	0.248
1.5	0.991	0.582	0.532	0.428	0.261	0.245	1.055	0.634	0.579	0.455	0.282	0.267
$\rho = 0.5$												
Bias												
0	0.221	0.099	0.082	0.042	0.017	0.016	0.240	0.111	0.101	0.049	0.018	0.018
0.3	0.224	0.098	0.102	0.044	0.017	0.012	0.244	0.116	0.116	0.048	0.019	0.016
0.5	0.240	0.116	0.099	0.057	0.022	0.021	0.280	0.129	0.118	0.063	0.022	0.023
1	0.266	0.163	0.134	0.052	0.029	0.020	0.296	0.183	0.166	0.057	0.034	0.024
1.5	0.286	0.162	0.163	0.038	0.030	0.028	0.305	0.185	0.181	0.038	0.033	0.033
RMSE												
0	0.956	0.526	0.456	0.419	0.235	0.206	1.034	0.576	0.517	0.442	0.253	0.223
0.3	0.949	0.536	0.481	0.425	0.237	0.216	1.037	0.590	0.534	0.445	0.257	0.237
0.5	0.969	0.547	0.487	0.423	0.238	0.219	1.049	0.598	0.541	0.449	0.264	0.241
1	0.968	0.577	0.533	0.418	0.253	0.233	1.048	0.633	0.592	0.448	0.278	0.257
1.5	1.010	0.605	0.567	0.431	0.270	0.258	1.071	0.663	0.619	0.458	0.291	0.282
$\rho = 0.8$												
Bias												
0	0.349	0.160	0.134	0.067	0.028	0.025	0.389	0.180	0.159	0.076	0.030	0.029
0.3	0.354	0.162	0.157	0.069	0.028	0.022	0.390	0.188	0.180	0.075	0.031	0.027
0.5	0.371	0.183	0.159	0.082	0.034	0.032	0.430	0.205	0.187	0.090	0.035	0.035
1	0.399	0.233	0.204	0.077	0.043	0.033	0.448	0.260	0.243	0.084	0.049	0.038
1.5	0.423	0.242	0.240	0.063	0.045	0.043	0.459	0.272	0.266	0.066	0.048	0.049
RMSE												
0	0.974	0.530	0.461	0.420	0.235	0.206	1.050	0.580	0.522	0.443	0.254	0.224
0.3	0.966	0.542	0.491	0.427	0.238	0.218	1.060	0.600	0.546	0.447	0.258	0.239
0.5	0.989	0.558	0.499	0.426	0.241	0.222	1.071	0.611	0.556	0.452	0.266	0.245
1	0.993	0.596	0.552	0.421	0.258	0.239	1.075	0.651	0.614	0.451	0.283	0.264
1.5	1.032	0.625	0.597	0.434	0.276	0.267	1.096	0.687	0.651	0.460	0.298	0.293

Note: This table presents the *bias* and the *root mean squared error* (RMSE) of the dependence coefficient β_1 for a model with *known* γ_i and c_j. The system is symmetric, that is why we present the results only for the first equation. The data generating process with 5,000 simulations is given below:

$$y_{i,t} = \delta_{0,i} + \delta_{1,i} z_t + \alpha_i x_{i,t} + \beta_i G(y_{j,t}; \gamma_i, c_j) + u_{i,t} \quad i,j = 1,2 \quad i \neq j$$

$$G(y_{j,t}; \gamma_i, c_j) = \left\{1 + e^{\gamma_i (y_{j,t} - c_j)}\right\}^{-1} \quad \gamma_i > 0$$

$$z_t = \xi z_{t-1} + \nu_t, \quad x_{i,t} = \phi_i x_{i,t-1} + (1 - \phi_i^2)^{1/2} \varepsilon_{i,t}, \quad \varepsilon_{1,t} = \theta \varepsilon_{2,t} + \eta_t, \quad \nu_t, u_{i,t}, \varepsilon_{2,t}, \eta_t \sim i.i.d.\mathcal{N}(0,1)$$

$$\delta_{0,i} = \delta_{1,i} = \alpha_i = 1, \; \beta_i = [0 \; 0.3 \; 0.5 \; 1 \; 1.5], \; \gamma_i = [0.5 \; 1.5 \; 2.5], \; c_j = 1$$

$$\rho = corr(u_{1,t}, u_{2,t}) = [0 \; 0.5 \; 0.8], \; \theta = corr(\varepsilon_{1,t}, \varepsilon_{2,t}) = [0 \; 0.3], \; \xi = \phi_i = [0 \; 0.5], \; T = [100, \; 500]$$

The $y_{i,t}$ series is obtained through an implicit reduced form using numerical procedures. We use nonlinear 2SLS as estimation method, approximating in the first stage of the procedure the logistic function by a polynomial of degree d of all the exogenous variables of the system. In this table $d = 3$.

is introduced. The simulations are performed for $T = 100$ and $T = 500$.

In Table 3.2 we can see that the bias is small and takes negative and positive values. It becomes smaller as the correlation between the error terms decreases, and as the sample size or the smoothness parameter increases. A similar pattern can be observed for the RMSE which is low for $T = 500$ and $\gamma_i > 0.5$. For $d = 3$ the bias is positive, and sensibly larger as the correlation between the error terms strengthens, especially for the small sample. A larger bias in this case could be explained by the high number of parameters to be estimated as the order of the polynomial in the first stage of the estimation procedure increases. This result is confirmed for higher-order polynomials until $d = 6$, not displayed here to save space.

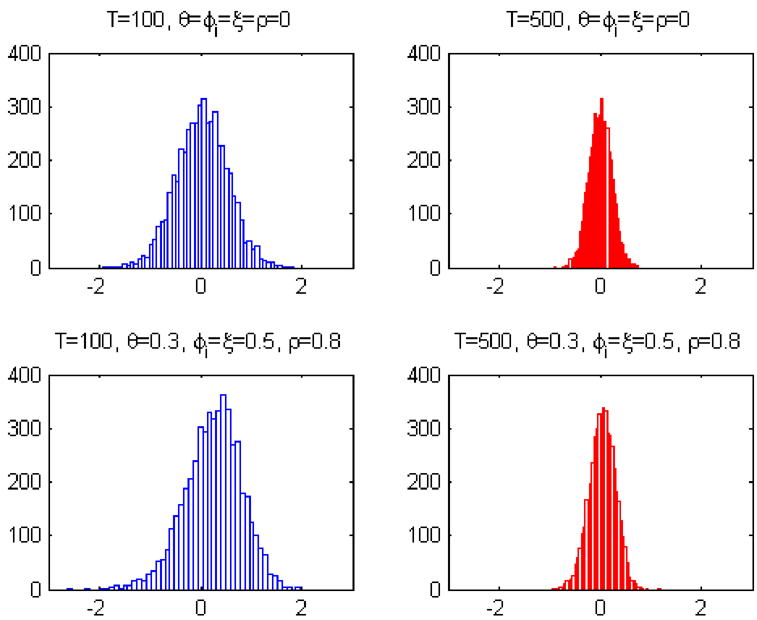
Figure 3.7 shows the distribution of the dependence parameter $\beta_1 = 1$ for $T = 100$ in the left panel and for $T = 500$ in the right panel. In the upper panel there is no structure in the regressors and no correlation between the error terms, while in the lower panel the explanatory variables have an AR(1) structure, are correlated between equations, and so are the error terms. The distributions are symmetric except for the lower left panel which has a longer left tail. The variance is considerably lower for the larger sample.

In our framework, it is not only important to accurately estimate the dependence parameter, but also the parameters related to the common and to the equation-specific variables. Their distributions are presented in Figure C.I for $d = 3$, $T = 500$, AR(1) structure and correlation in the regressors and the error term. The parameters related to z_t and $x_{i,t}$ have symmetric and narrow distributions; the distributions of the constant term and of the variance of the residuals are somewhat flatter, with a slightly longer right tail for the latter.

In general, there is evidence that the NL2SLS estimation procedure is able to provide accurate estimates for all the parameters of each equation of the ST-SEM. In empirical applications, γ_i could be set to reasonable values, while c_j could be fixed to a quantile in the tails of the distribution of y_j. Low order polynomials seems to be a good choice to approximate $G(y_{j,t}; \gamma_i, c_j)$, especially for small samples.

Figure 3.7

Distribution of β_1 for *known* γ and c



Notes: This figure shows the distribution of β_1 obtained from simulations for the following parameter combinations: $\beta_i = 1$, $\gamma_i = 2.5$, $d = 3$, and $5,000$ simulations. The parameter is centered around 0.

Known smoothness parameter and unknown location parameter

Table 3.4 displays the bias and the RMSE error for the dependence parameter β_1 when only γ_i is fixed. Given that the numerical procedure with $5,000$ is time-consuming, we only present the results for $\beta_1 = 1$. We have tried other values of β_i for a smaller number of simulations with similar conclusions. The structure of the table remains the same as above, except that the first column gives the order of the polynomial.[7] The bias is negligible for $d = 1$, but it rapidly increases from $d = 2$, especially for $T = 100$. Again, the bias is considerably lower as the sample size increases with $\gamma_i = 2.5$, and it deteriorates with ρ. Generally, the RMSE reaches lower values for higher-order polynomials. For $T = 100$ the RMSE is quite high but it does not explode.

Given the small bias for $d = 1$, we present the distribution of β_1 in this case and for $d = 3$. The corresponding Figures are 3.8 and 3.9 for the same parameter combinations as above. Compared to a case where c_j is known, the distributions are somewhat flatter but they still have a nice shape for $T = 500$. In the small sample case, the left tail is longer and has a bump. Figure C.II shows the distribution of the other parameters of the model, which are as good as the ones commented in Section 3.3.3.

Figure 3.10 shows the distribution of c_2 for different parameter combinations. As discussed previously, an accurate estimation of this parameter is difficult to be obtained. Although, the distribution does not seem to be normal, our results are not much different of those of Schleer (2015), page 79 and 83. In our framework, the optimization is more complex since it depends on the accuracy of the approximation of the logistic distribution by a polynomial in the first step of the NL2SLS. To the best of our knowledge, the endogeneity issue has not been treated before in the literature.

Unknown smoothness and location parameters

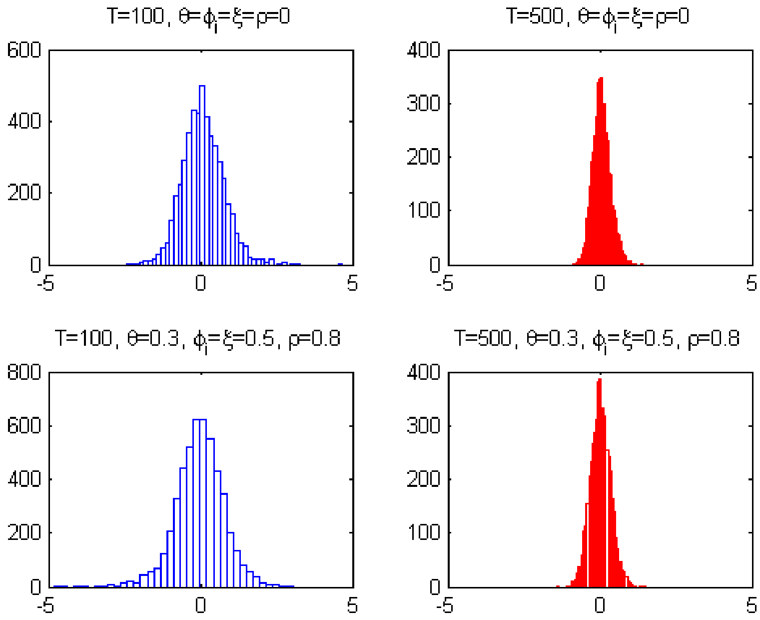
Table C.I displays the bias and the RMSE for β_1 in the case where γ_i is also unknown. Figures C.III, C.IV and C.V display the distribution of β_1, c_2 and $[\delta_{0,1}, \delta_{1,1}, \alpha_1, \sigma_1^2]$, respectively. The results are virtually the same as when γ_i is known, that is why we do not comment further on them. This finding is in line with the argument of Teräsvirta, Tjøstheim and Granger (2010) that a precise estimation of this parameter is not necessary for an accurate estimation of the other parameters of the model.

Note that if $d = 1$, the algorithm does not converge for several parameter combination. Despite that, for the cases in which it converges, the estimation procedure produces the most accurate distributions for γ_1 which are displayed in Figure 3.11. The values of γ_1 do not explode and the mean of the simulations is around 2.64 which is not far from the true value of 2.5. The only constraint we put on γ_i is that it has to be greater than 1 in order to avoid colinearity issues with the error term. For higher orders of the polynomial, the algorithm selects very frequently values close to the lower bound.

These results corroborate our above discussion that a precise estimation of the smoothness parameter is difficult due to a flat objective function with respect to this parameter. Studies in a time series framework such as Lundbergh and Teräsvirta (2006) and Andrews and Cheng (2013) fix the value of this parameter to 300 and 10, respectively. Schleer (2015) (page 79 and 83) does not seem to obtain better results that the ones presented here. We are not aware of any other study that shows the empirical distribution of γ.

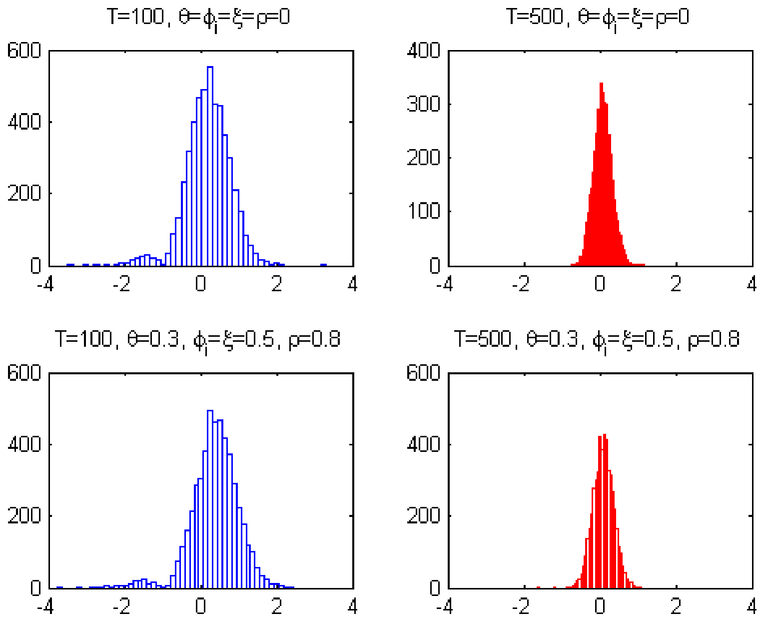
[7] Given the high RMSE seen previously, we do not consider the value of $\gamma_i = 0.5$.

Figure 3.8
Distribution of β_1 with *known* γ and *unknown* c, $d = 1$



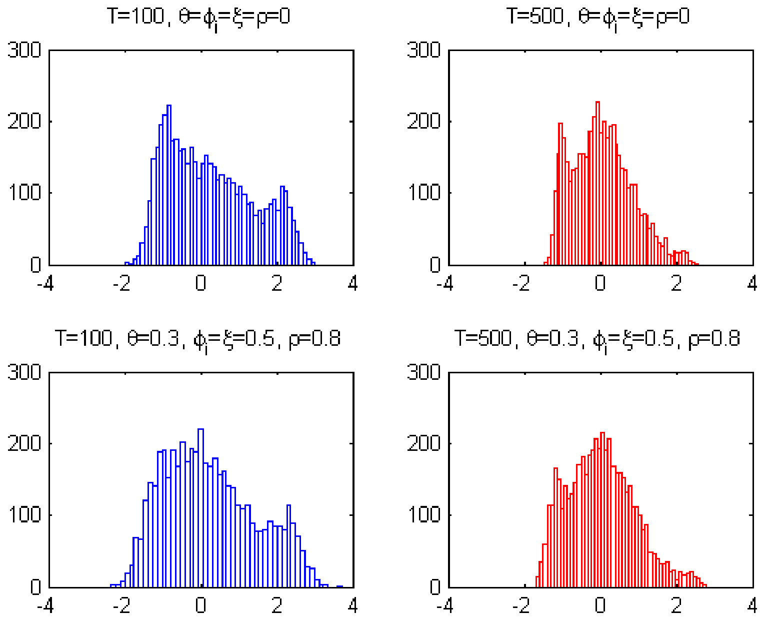
Notes: This figure shows the distribution of β_1 obtained from simulations for the following parameter combinations: $\beta_i = 1$, $d = 1$, $\gamma_i = 2.5$, and $5,000$ simulations. The parameter is centered around 0.

Figure 3.9
Distribution of β_1 with *known* γ and *unknown* c, $d = 3$



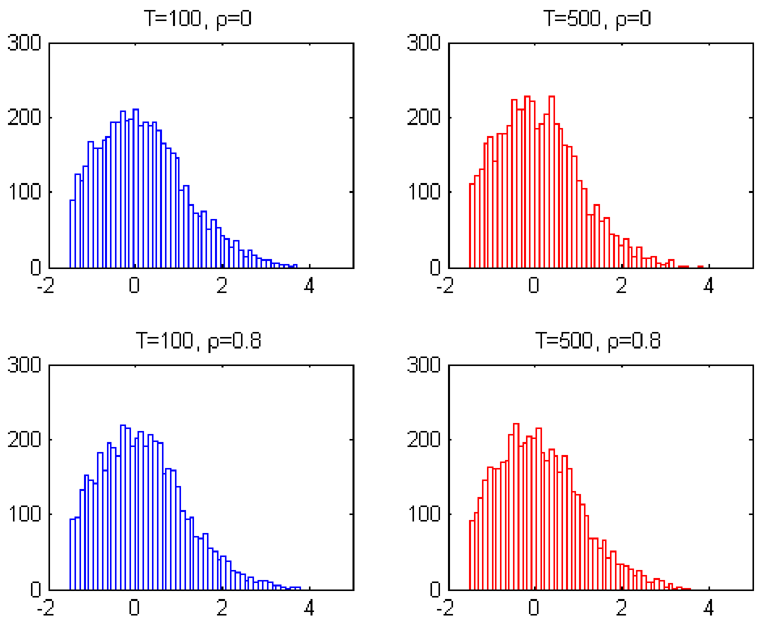
Notes: This figure shows the distribution of β_1 obtained from simulations for the following parameter combinations: $\beta_i = 1$, $d = 3$, $\gamma_i = 2.5$, and $5,000$ simulations. The parameter is centered around 0.

Figure 3.10
Distribution of c_2 with *known* γ, $d = 3$



Notes: This figure shows the distribution of c_2 obtained from simulations for the following parameter combinations: $c_j = 1$, $\beta_i = 1$, $\gamma_i = 2.5$, $d = 3$, and $5,000$ simulations. The parameter is centered around 0.

Figure 3.11
Distribution of γ_1, $d = 1$



Notes: This figure shows the distribution of γ_1 obtained from simulations for the following parameter combinations: $\gamma_i = 2.5$, $\beta_i = 1$, $c_j = 1$, $d = 1$, $\theta = \phi_i = \xi = 0$ and $5,000$ simulations. The parameter is centered around 0.

Table 3.4

Bias and root mean squared error for β_1 with *known* γ and *unknown* c

			$\xi = 0$, $\phi_i = 0$, $\theta = 0$				$\xi = 0.5$, $\phi_i = 0.5$, $\theta = 0.3$			
	$\beta_1 = 1$		T=100		T=500		T=100		T=500	
		γ	1.5	2.5	1.5	2.5	1.5	2.5	1.5	2.5
$\rho = 0$	*Bias*									
	$d = 1$		0.040	0.086	0.041	0.065	0.054	0.116	0.046	0.067
	$d = 2$		0.194	0.194	0.083	0.075	0.177	0.179	0.076	0.057
	$d = 3$		0.235	0.202	0.093	0.081	0.216	0.226	0.085	0.072
	$d = 4$		0.236	0.212	0.091	0.076	0.258	0.228	0.085	0.067
	$d = 5$		0.263	0.230	0.089	0.074	0.246	0.233	0.084	0.078
	$d = 6$		0.266	0.251	0.103	0.074	0.272	0.251	0.094	0.079
	RMSE									
	$d = 1$		0.660	0.685	0.288	0.302	0.758	0.796	0.326	0.337
	$d = 2$		0.683	0.637	0.277	0.260	0.774	0.695	0.297	0.276
	$d = 3$		0.666	0.614	0.284	0.259	0.732	0.677	0.293	0.270
	$d = 4$		0.650	0.596	0.273	0.252	0.727	0.651	0.290	0.266
	$d = 5$		0.643	0.582	0.278	0.248	0.699	0.622	0.283	0.265
	$d = 6$		0.619	0.568	0.279	0.254	0.681	0.610	0.288	0.263
$\rho = 0.5$	*Bias*									
	$d = 1$		0.014	0.034	0.045	0.045	0.017	0.028	0.040	0.041
	$d = 2$		0.249	0.241	0.093	0.086	0.246	0.224	0.083	0.069
	$d = 3$		0.341	0.034	0.114	0.096	0.361	0.314	0.104	0.092
	$d = 4$		0.439	0.403	0.128	0.103	0.455	0.405	0.125	0.105
	$d = 5$		0.521	0.453	0.145	0.122	0.543	0.473	0.137	0.117
	$d = 6$		0.584	0.510	0.164	0.138	0.611	0.532	0.158	0.129
	RMSE									
	$d = 1$		0.698	0.685	0.300	0.300	0.773	0.805	0.335	0.329
	$d = 2$		0.703	0.672	0.287	0.275	0.791	0.770	0.305	0.279
	$d = 3$		0.710	0.685	0.289	0.272	0.780	0.713	0.303	0.286
	$d = 4$		0.728	0.668	0.292	0.270	0.784	0.716	0.304	0.285
	$d = 5$		0.755	0.680	0.302	0.276	0.806	0.723	0.310	0.284
	$d = 6$		0.788	0.701	0.305	0.279	0.842	0.742	0.318	0.290
$\rho = 0.8$	*Bias*									
	$d = 1$		0.008	-0.003	0.026	0.026	-0.023	-0.025	0.022	0.021
	$d = 2$		0.266	0.238	0.101	0.084	0.268	0.258	0.094	0.074
	$d = 3$		0.429	0.381	0.125	0.110	0.447	0.377	0.118	0.092
	$d = 4$		0.538	0.495	0.148	0.130	0.568	0.514	0.146	0.118
	$d = 5$		0.660	0.586	0.178	0.150	0.685	0.629	0.167	0.137
	$d = 6$		0.753	0.672	0.201	0.172	0.809	0.694	0.191	0.160
	RMSE									
	$d = 1$		0.708	0.681	0.297	0.294	0.803	0.809	0.341	0.334
	$d = 2$		0.721	0.682	0.296	0.275	0.812	0.767	0.309	0.285
	$d = 3$		0.742	0.682	0.298	0.275	0.810	0.735	0.310	0.286
	$d = 4$		0.779	0.720	0.308	0.284	0.834	0.775	0.315	0.289
	$d = 5$		0.843	0.764	0.314	0.290	0.891	0.810	0.324	0.293
	$d = 6$		0.899	0.814	0.331	0.300	0.971	0.857	0.333	0.302

Note: This table contains the results of the *bias* and of the *root mean squared error* (RMSE) for the dependence parameter β_1 with *unknown* location parameter c_1 and *known* smoothness parameter γ_1. We focus on the first equation, but the second equation is symmetric. The true value of $\beta_1 = 1$. The data generating process with 5,000 simulations is as follows:

$$y_{i,t} = \delta_{0,i} + \delta_{1,i} z_t + \alpha_i x_{i,t} + \beta_i G(y_{j,t}; \gamma_i, c_j) + u_{i,t}, \quad G(y_{j,t}; \gamma_i, c_j) = \left\{1 + e^{\gamma_i (y_{j,t} - c_j)}\right\}^{-1}, \quad \gamma_i > 0, \quad i, j = 1, 2, \quad i \neq j$$

$$z_t = \xi z_{t-1} + \nu_t, \quad x_{i,t} = \phi_i x_{i,t-1} + (1 - \phi_i^2)^{1/2} \varepsilon_{i,t}, \quad \varepsilon_{1,t} = \theta \varepsilon_{2,t} + \eta_t, \quad v_t, u_{i,t}, \varepsilon_{2,t}, \eta_t \sim i.i.d.\mathcal{N}(0, 1).$$

3.4 Testing

3.4.1 Testing in the presence of nuisance parameters

In our framework, testing for dependence between the two endogenous variables translates into testing for simultaneity of a particular form. Let us focus on the first equation of the two-equation ST-SEM given by Equation 3.3. The test for dependence is equivalent to testing the null hypothesis that β_1 is equal to 0 ($H_0 : \beta_1 = 0$) against the alternative $H_a : \beta_1 \neq 0$. Under H_0, the smoothness parameter γ_1 and the threshold parameter c_2 are not identified and can take any value. The null hypothesis can alternatively be set as $H_0' : \gamma_1 = 0$, in which case the logistic function becomes a constant. In this case, neither β_1, nor c_2 are identified. This issue is known under the name of *nuisance parameter*, the presence of which renders the distributions of the conventional tests non-standard under the null hypothesis. The analytical expressions of these tests are often not available and simulation techniques must be used (van Dijk, Teräsvirta and Franses 2002).

Each of the above two null hypotheses leads to a different testing strategy, both being studied in a time-series context. The first class of tests based on $H_0 : \beta_1 = 0$ was initiated by Davies (1977, 1987) and taken over by Andrews (1993), Andrews and Ploberger (1994) and Hansen (1996, 2000). Davies (1977, 1987), Andrews (1993) and Andrews and Ploberger (1994) develop supremum, average and exponential average LM test statistics to solve the identification problem. Hansen (1996, 2000) derived the asymptotic distribution of the tests for TR models and suggested a sampling method to obtain critical values. The second class of tests based on $H_0' : \gamma_1 = 0$ versus $H_a' : \gamma_1 > 0$, first proposed by Luukkonnen, Saikkonen and Teräsvirta (1988) (LST), sidesteps the nuisance parameters through a Taylor-series expansion of first and third order[8] around $\gamma = 0$. The resulting test statistic has a standard χ^2 distribution and is widely used in STR models.

González and Teräsvirta (2006) examine both types of tests in a Monte Carlo study and come to the conclusion that their performance is comparable, although average tests have slightly better size and power properties compared to the LST test. Nevertheless the latter test performs as good as the most frequently used supremum test, while having the advantage of being easy to compute. In addition, Teräsvirta (1994) argues that the LST test can be seen as a general test of nonlinearity since it has power against other nonlinear additive models which yield the same auxiliary regression.

We adapt the LST test to our framework in the presence of an endogenous variable in the logistic function. Applying the first-order Taylor approximation to $G(y_{2t}; \gamma_1, c_2)$ around $\gamma_1 = 0$ yields the following auxiliary regression:[9]

$$y_{1,t} = \tilde{\boldsymbol{\delta}}_1' \boldsymbol{z}_t + \boldsymbol{\alpha}_1' \boldsymbol{x}_{1,t} + \tilde{\beta}_1 y_{2,t} + u_{1,t}^*. \tag{3.6}$$

where $\tilde{\boldsymbol{\delta}}_1 = \left[(\delta_{1,1} - \frac{1}{4}\beta_1\gamma_1 c_2) \; \boldsymbol{\delta}_1^{-\prime} \right]'$, with $\boldsymbol{\delta}_1^-$ equals to $\boldsymbol{\delta}_1$ minus the constant term $\delta_{1,1}$, $\tilde{\beta}_1 = \frac{1}{4}\beta_1\gamma_1$ and $u_{1,t}^* = \beta_1 R_{1,1} + u_{1,t}$. The above model is free of nuisance parameters and the standard asymptotic theory is applicable since under the null $u_{1,t}^* = u_{1,t}$. Note that this first-order Taylor approximation leads to a test for simultaneity within a linear SEM model. The null hypothesis of $\gamma_1 = 0$ can be equivalently set as $\tilde{\beta}_1 = 0$. The alternative hypothesis of $\tilde{\beta}_1 \neq 0$ necessarily implies that $\gamma_1 > 0$ and $\beta_1 \neq 0$. Similarly, the third-order

[8] The test based on the third-order Taylor series expansion is found to be more powerful in models where the intercept changes regime, which is the case in our model.

[9] The details of the Taylor approximation are given in Appendix C.3.

Taylor expansion can be written as:

$$y_{1,t} = \tilde{\boldsymbol{\delta}}_1' \boldsymbol{z}_t + \boldsymbol{\alpha}_1' \boldsymbol{x}_{1,t} + \tilde{\beta}_{1,1} y_{2,t} + \tilde{\beta}_{1,2} y_{2,t}^2 + \tilde{\beta}_{1,3} y_{2,t}^3 + u_{1,t}^* \tag{3.7}$$

where $\tilde{\boldsymbol{\delta}}_1 = [(\delta_{1,1} - \frac{1}{4}\gamma_1 c_2 + \frac{1}{48}\gamma_1^3 c_2^3)\ \boldsymbol{\delta}_1^{-\prime}]'$, with $\boldsymbol{\delta}_1^-$ defined as above, $\tilde{\beta}_{1,1} = \beta_1\gamma_1(\frac{1}{4} - \frac{1}{16}\gamma_1^2 c_2^2)$, $\tilde{\beta}_{1,2} = \frac{1}{16}\beta_1\gamma_1^3 c_2$, $\tilde{\beta}_{1,3} = -\frac{1}{48}\beta_1\gamma_1^3$ and $u_{1,t}^* = \beta_1 R_{3,1} + u_{1,t}$. The null hypothesis of no simultaneity can be written as $H_0 : \tilde{\beta}_{1,1} = \tilde{\beta}_{1,2} = \tilde{\beta}_{1,3} = 0$, while under the alternative at least one of these three parameters is different from zero. In this case, β_1 and γ_1 must be both non-zero since they are multiplicative.

Wooldridge (2002) pages 98-99, shows that the conventional LM statistic for 2SLS does not have a known limiting distribution and can even become negative. He proposes a modified version of this test that has a χ^2 square asymptotic distribution. Below we give the details of the implementation procedure of the LST test.

(i) Regress $y_{1,t}$ on $[\boldsymbol{z}_t'\ \boldsymbol{x}_{1,t}']'$ using the OLS[10] estimation procedure under the assumption of exogenous regressors. Take the residual $\hat{u}_{1,t}$, $t = 1, \ldots, T$ and compute the residual sum of squares $S\hat{S}E_0 = \sum_{t=1}^T \hat{u}_{1,t}^2$.

(ii) Regress $y_{1,t}$ on $[\boldsymbol{z}_t'\ \boldsymbol{x}_{1,t}'\ y_{2,t}]'$ (for the third-order expansion the set of regressors becomes $[\boldsymbol{z}_t'\ \boldsymbol{x}_{1,t}'\ y_{2,t}\ y_{2,t}^2\ y_{2,t}^3]'$) using the 2SLS method and $\mathbf{w}_t$ as set of instruments. Take the residual denoted by $\hat{u}_{1,t}^*$, and compute the residual sum of squares $SSE_1 = \sum_{t=1}^T \hat{u}_{1,t}^{*2}$.

(iii) Compute $S\hat{S}E_1$ which is the sum of squared residuals from the unrestricted second-stage regression of $y_{1,t}$ on $[\boldsymbol{z}_t'\ \boldsymbol{x}_{1,t}'\ \hat{y}_{2,t}]'$, with $\hat{y}_{2,t}$ the fitted value from the first stage regression of $y_{2,t}$ on $\mathbf{w}_t$ (third-order approximation: $[\boldsymbol{z}_t'\ \boldsymbol{x}_{1,t}'\ \hat{y}_{2,t}\ \widehat{y^2}_{2,t}\ \widehat{y^3}_{2,t}]'$).

(iv) Compute the test statistic as: $\text{LM} = T(S\hat{S}E_0 - S\hat{S}E_1)/SSE_1$.

The above LM test follows asymptotically a χ_k^2 with $k = 1$ for the first-order Taylor expansion and $k = 3$ for the third-order. In small samples, the F-test statistic has better size properties. In addition, Teräsvirta (1994) documents that the F-test has power against the limiting case where the dependence variable is a dummy. Note that the testing procedure does not require the estimation of the nonlinear ST-SEM, which can be computationally costly.

3.4.2 Testing simulation results

In this part we present simulation results related to the tests for simultaneity with null hypothesis $\beta_i = 0$ against the alternative $\beta_i \neq 0$, for different parameter combinations. For known smoothness and location parameters of the logistic function, standard test procedures can be applied. When these parameters are unknown, the LST test described above is used. We set completely symmetric parameters for both equations, that is why we only show the results for the first one.

[10] In the test suggested by Wooldridge (2002), $S\hat{S}E_0$ is the restricted sum of squared residuals from the *second* stage of 2SLS for $[\boldsymbol{z}_t'\ \boldsymbol{x}_{1,t}']'$ on a valid set of instruments (here $\mathbf{w}_t$). Given that $[\boldsymbol{z}_t'\ \boldsymbol{x}_{1,t}']'$ is assumed to be an exogenous set of variables, the fitted value of these regressors from the first-stage 2SLS is equal to their true value, and consequently OLS is equivalent to 2SLS.

Table 3.5

Size and power of the test for dependence (β_1) with *known* γ and c, $d = 1$

β_1		$\boldsymbol{\xi = 0, \phi_i = 0, \theta = 0}$						$\boldsymbol{\xi = 0.5, \phi_i = 0.5, \theta = 0.3}$					
		T=100			T=500			T=100			T=500		
	γ	0.5	1.5	2.5	0.5	1.5	2.5	0.5	1.5	2.5	0.5	1.5	2.5
$\boldsymbol{\rho = 0}$													
0		0.046	0.049	0.043	0.050	0.050	0.045	0.046	0.047	0.041	0.051	0.048	0.050
0.3		0.094	0.126	0.140	0.181	0.342	0.405	0.090	0.127	0.127	0.166	0.300	0.351
0.5		0.131	0.239	0.272	0.328	0.661	0.728	0.124	0.204	0.237	0.291	0.600	0.652
1		0.271	0.524	0.580	0.755	0.986	0.991	0.251	0.460	0.506	0.704	0.966	0.973
1.5		0.461	0.735	0.761	0.961	0.999	1.000	0.421	0.659	0.666	0.941	0.997	0.996
$\boldsymbol{\rho = 0.5}$													
0		0.062	0.063	0.058	0.054	0.057	0.054	0.060	0.062	0.058	0.056	0.055	0.055
0.3		0.117	0.150	0.162	0.195	0.347	0.405	0.109	0.151	0.147	0.177	0.308	0.355
0.5		0.154	0.257	0.287	0.335	0.642	0.704	0.150	0.230	0.257	0.300	0.587	0.631
1		0.293	0.509	0.553	0.734	0.973	0.976	0.276	0.453	0.495	0.688	0.946	0.951
1.5		0.460	0.690	0.711	0.944	0.998	0.997	0.430	0.625	0.617	0.920	0.991	0.989
$\boldsymbol{\rho = 0.8}$													
0		0.070	0.070	0.068	0.059	0.061	0.057	0.071	0.070	0.064	0.060	0.060	0.059
0.3		0.127	0.163	0.174	0.200	0.351	0.405	0.120	0.165	0.161	0.185	0.311	0.355
0.5		0.168	0.269	0.298	0.338	0.630	0.689	0.165	0.242	0.267	0.306	0.577	0.613
1		0.303	0.503	0.541	0.725	0.963	0.966	0.287	0.449	0.487	0.677	0.933	0.933
1.5		0.460	0.662	0.677	0.934	0.996	0.993	0.435	0.609	0.593	0.909	0.986	0.982

Note: This table shows the *rejection frequency* of the null hypothesis of no dependence ($H_0 : \beta_i = 0$) using standard testing procedures (we perform the F test). $d = 1$ denotes the degree of the polynomial in the first step of the NL2SLS method. We fix the value of γ_i and c_j and use the following data generating process:

$$y_{i,t} = \delta_{0,i} + \delta_{1,i} z_t + \alpha_i x_{i,t} + \beta_i G(y_{j,t}; \gamma_i, c_j) + u_{i,t}, \quad G(y_{j,t}; \gamma_i, c_j) = \left\{1 + e^{\gamma_i (y_{j,t} - c_j)}\right\}^{-1}, \quad \gamma_i > 0, \quad i = 1, 2 \quad i \neq j$$

$$d = 1,\ \delta_{0,i} = 1,\ \delta_{1,i} = 1,\ \alpha_i = 1,\ \beta_i = [0\ 0.3\ 0.5\ 1\ 1.5],\ \gamma_i = [0.5\ 1.5\ 2.5],\ c_j = 1,\ \rho = [0\ 0.5\ 0.8],\ \phi_i = [0\ 0.5],\ \theta = [0\ 0.3].$$

The results are presented only for Equation 1 since they are symmetric. We run 5,000 simulations.

Table 3.6

Size and power of the test for dependence (β_1) with *known* γ and c, $d = 3$

		$\xi = 0$, $\phi_i = 0$, $\theta = 0$						$\xi = 0.5$, $\phi_i = 0.5$, $\theta = 0.3$					
β_1		T=100			T=500			T=100			T=500		
	γ	0.5	1.5	2.5	0.5	1.5	2.5	0.5	1.5	2.5	0.5	1.5	2.5
$\boldsymbol{\rho = 0}$													
0		0.049	0.052	0.044	0.048	0.049	0.042	0.051	0.050	0.048	0.052	0.054	0.049
0.3		0.092	0.143	0.169	0.185	0.355	0.404	0.090	0.131	0.152	0.175	0.315	0.374
0.5		0.144	0.257	0.288	0.349	0.686	0.760	0.137	0.231	0.263	0.324	0.617	0.692
1		0.303	0.614	0.657	0.780	0.990	0.996	0.273	0.561	0.610	0.726	0.975	0.989
1.5		0.507	0.825	0.869	0.964	1.000	1.000	0.457	0.781	0.825	0.941	1.000	1.000
$\boldsymbol{\rho = 0.5}$													
0		0.102	0.094	0.083	0.068	0.067	0.061	0.108	0.097	0.094	0.069	0.072	0.067
0.3		0.165	0.223	0.249	0.228	0.394	0.438	0.162	0.219	0.235	0.218	0.355	0.407
0.5		0.239	0.352	0.375	0.401	0.704	0.763	0.239	0.329	0.360	0.379	0.641	0.701
1		0.426	0.672	0.695	0.793	0.984	0.990	0.410	0.632	0.665	0.746	0.967	0.981
1.5		0.597	0.837	0.870	0.955	1.000	0.999	0.570	0.801	0.838	0.935	0.999	0.999
$\boldsymbol{\rho = 0.8}$													
0		0.145	0.128	0.117	0.081	0.080	0.074	0.156	0.135	0.133	0.086	0.082	0.079
0.3		0.227	0.280	0.307	0.255	0.417	0.459	0.227	0.272	0.299	0.249	0.383	0.429
0.5		0.307	0.411	0.435	0.430	0.711	0.764	0.304	0.396	0.419	0.415	0.657	0.707
1		0.504	0.705	0.725	0.804	0.978	0.987	0.484	0.672	0.698	0.760	0.963	0.973
1.5		0.659	0.851	0.871	0.953	0.999	0.999	0.639	0.822	0.849	0.933	0.998	0.999

Note: This table shows the *rejection frequency* of the null hypothesis of no dependence ($H_0 : \beta_i = 0$) using standard testing procedures (we perform the F test). $d = 3$ denotes the degree of the polynomial in the first step of the NL2SLS method. We fix the value of γ_i and c_j and use the following data generating process:

$$y_{i,t} = \delta_{0,i} + \delta_{1,i} z_t + \alpha_i x_{i,t} + \beta_i G(y_{j,t}; \gamma_i, c_j) + u_{i,t}, \quad G(y_{j,t}; \gamma_i, c_j) = \left\{1 + e^{\gamma_i (y_{j,t} - c_j)}\right\}^{-1}, \quad \gamma_i > 0, \quad i = 1,2 \quad i \neq j$$

$$d = 3,\ \delta_{0,i} = 1,\ \delta_{1,i} = 1,\ \alpha_i = 1,\ \beta_i = [0\ 0.3\ 0.5\ 1\ 1.5],\ \gamma_i = [0.5\ 1.5\ 2.5],\ c_j = 1,\ \rho = [0\ 0.5\ 0.8],\ \phi_i = [0\ 0.5],\ \theta = [0\ 0.3].$$

The results are presented only for Equation 1 since they are symmetric. We run 5,000 simulations.

Table 3.7

Size and power of the test for simultaneity (β_1) with *unknown* γ and c – *first-order Taylor approximation*

	$\boldsymbol{\xi=0}$, $\boldsymbol{\phi_i=0}$, $\boldsymbol{\theta=0}$						$\boldsymbol{\xi=0.5}$, $\boldsymbol{\phi_i=0.5}$, $\boldsymbol{\theta=0.3}$					
β_1	T=100			T=500			T=100			T=500		
γ	0.5	1.5	2.5	0.5	1.5	2.5	0.5	1.5	2.5	0.5	1.5	2.5
$\boldsymbol{\rho=0}$												
0	0.050	0.051	0.041	0.048	0.052	0.046	0.050	0.045	0.041	0.047	0.046	0.056
0.3	0.068	0.095	0.124	0.112	0.305	0.394	0.056	0.087	0.115	0.096	0.279	0.374
0.5	0.087	0.192	0.251	0.260	0.679	0.780	0.099	0.172	0.205	0.227	0.614	0.714
1	0.190	0.527	0.594	0.686	0.994	0.997	0.184	0.489	0.518	0.655	0.988	0.997
1.5	0.384	0.743	0.759	0.944	1.000	1.000	0.334	0.695	0.689	0.927	1.000	1.000
$\boldsymbol{\rho=0.5}$												
0	0.050	0.051	0.045	0.054	0.052	0.041	0.050	0.048	0.046	0.062	0.044	0.053
0.3	0.074	0.115	0.126	0.128	0.311	0.394	0.073	0.115	0.137	0.125	0.284	0.349
0.5	0.108	0.212	0.258	0.236	0.644	0.740	0.109	0.193	0.230	0.243	0.600	0.702
1	0.224	0.487	0.549	0.671	0.985	0.994	0.218	0.458	0.491	0.630	0.974	0.986
1.5	0.400	0.677	0.698	0.928	1.000	1.000	0.365	0.633	0.646	0.919	0.998	0.999
$\boldsymbol{\rho=0.8}$												
0	0.045	0.050	0.047	0.041	0.049	0.046	0.052	0.052	0.046	0.055	0.054	0.054
0.3	0.080	0.120	0.146	0.141	0.321	0.358	0.092	0.105	0.148	0.126	0.315	0.361
0.5	0.105	0.220	0.268	0.258	0.644	0.718	0.123	0.214	0.238	0.261	0.598	0.679
1	0.242	0.472	0.536	0.670	0.979	0.983	0.228	0.448	0.465	0.614	0.956	0.970
1.5	0.382	0.670	0.670	0.906	0.999	0.999	0.376	0.618	0.579	0.896	0.998	0.996

Note: This table shows the *rejection frequency* of the null hypothesis of no simultaneity ($H_0 : \beta_i = 0$) using standard testing procedures (F tests) *after* approximating the logistic function by a *first-order* Taylor expansion around $\gamma_i = 0$. The auxiliary regression estimated by 2SLS is the following:

$$y_{i,t} = \delta_{0,i} + \delta_{1,i} z_t + \alpha_i x_{i,t} + \tilde{\beta}_i y_{j,t} + u^*_{i,t} \quad i = 1, 2 \quad i \neq j$$

The null hypothesis becomes: $H_0 : \tilde{\beta}_i = 0$. For the data simulation, we use the following parameter combinations:

$$\delta_{0,i} = 1,\ \delta_{1,i} = 1,\ \alpha_i = 1,\ \beta_i = [0\ 0.3\ 0.5\ 1\ 1.5],\ \gamma_i = [0.5\ 1.5\ 2.5],\ c_j = 1,\ \rho = [0\ 0.5\ 0.8],\ \phi_i = [0\ 0.5],\ \theta = [0\ 0.3].$$

The results are presented only for Equation 1 since they are symmetric. We run 2,000 simulations.

Table 3.8

Size and power of the test for s (β_1) with *unknown* γ and c – *third-order Taylor approximation*

		$\boldsymbol{\xi = 0}$, $\boldsymbol{\phi_i = 0}$, $\boldsymbol{\theta = 0}$						$\boldsymbol{\xi = 0.5}$, $\boldsymbol{\phi_i = 0.5}$, $\boldsymbol{\theta = 0.3}$					
β_1		T=100			T=500			T=100			T=500		
	γ	0.5	1.5	2.5	0.5	1.5	2.5	0.5	1.5	2.5	0.5	1.5	2.5
$\boldsymbol{\rho = 0}$													
0		0.016	0.015	0.015	0.036	0.046	0.035	0.014	0.010	0.016	0.038	0.034	0.035
0.3		0.026	0.038	0.038	0.088	0.182	0.238	0.024	0.026	0.040	0.066	0.175	0.225
0.5		0.023	0.074	0.074	0.148	0.500	0.633	0.024	0.065	0.075	0.142	0.464	0.580
1		0.074	0.291	0.291	0.519	0.978	0.992	0.079	0.255	0.341	0.492	0.958	0.983
1.50		0.185	0.545	0.545	0.879	1.000	0.999	0.178	0.509	0.550	0.859	0.999	1.000
$\boldsymbol{\rho = 0.5}$													
0		0.025	0.021	0.019	0.036	0.035	0.042	0.023	0.020	0.020	0.038	0.040	0.044
0.3		0.035	0.055	0.070	0.091	0.201	0.273	0.034	0.062	0.072	0.094	0.194	0.270
0.5		0.058	0.117	0.131	0.176	0.504	0.587	0.050	0.094	0.134	0.167	0.459	0.563
1		0.136	0.328	0.365	0.535	0.953	0.975	0.116	0.305	0.324	0.492	0.943	0.966
1.5		0.244	0.501	0.555	0.843	0.996	0.997	0.227	0.536	0.550	0.813	0.993	0.994
$\boldsymbol{\rho = 0.8}$													
0		0.026	0.032	0.030	0.030	0.035	0.041	0.032	0.030	0.023	0.034	0.038	0.031
0.3		0.050	0.083	0.083	0.083	0.217	0.286	0.051	0.074	0.088	0.096	0.219	0.257
0.5		0.069	0.149	0.160	0.160	0.492	0.577	0.081	0.140	0.161	0.178	0.467	0.530
1		0.165	0.331	0.386	0.386	0.929	0.958	0.138	0.327	0.366	0.494	0.924	0.942
1.5		0.289	0.515	0.540	0.540	0.993	0.992	0.264	0.499	0.536	0.804	0.990	0.987

Note: This table shows the *rejection frequency* of the null hypothesis of no simultaneity $(H_0 : \beta_i = 0)$ using standard testing procedures (F tests) *after* approximating the logistic function by a *third-order* Taylor expansion around $\gamma_i = 0$. The auxiliary regression estimated by NL2SLS is the following:

$$y_{i,t} = \delta_{0,i} + \delta_{1,i} z_t + \alpha_i x_{i,t} + \tilde{\beta}_{i,1} y_{j,t} + \tilde{\beta}_{i,2} y_{j,t}^2 + \tilde{\beta}_{i,3} y_{j,t}^3 + u_{i,t}^* \quad i = 1, 2 \quad i \neq j$$

The null hypothesis becomes: $H_0 : \tilde{\beta}_{i,1} = \tilde{\beta}_{i,2} = \tilde{\beta}_{i,3} = 0$. For the data simulation, we use the following parameter combinations:

$$\delta_{0,i} = 1,\ \delta_{1,i} = 1,\ \alpha_i = 1,\ \beta_i = [0\ 0.3\ 0.5\ 1\ 1.5],\ \gamma_i = [0.5\ 1.5\ 2.5],\ c_j = 1,\ \rho = [0\ 0.5\ 0.8],\ \phi_i = [0\ 0.5],\ \theta = [0\ 0.3].$$

The results are presented only for Equation 1 since they are symmetric. We run 2,000 simulations.

Known parameters of the logistic function

We comment on the simulation results of the tests for a system with known γ_i and c_j. The observed value of the test is compared to the 95% quantile of the t distribution with the corresponding degrees of freedom. In Table 3.5 the degree of the polynomial used to approximate the logistic function in the first stage of the NL2SLS is $d = 1$ and in Table 3.6, $d = 3$.

Focusing on Table 3.5, the size of the test is given by the first row of each horizontal block with different values of the parameter ρ. For $\rho = 0$ the empirical size is very close to the nominal size of 5%. The results appear to be marginally better for $T = 500$. As ρ becomes larger, the tests start being slightly oversized, especially for $T = 100$ and for low values of γ. On the contrary, the specifications with autocorrelated regressors and $corr(x_{1,t}; x_{2,t}) = 0.3$ do not produce sensibly different results. For $d = 3$, the size is still correct if $\rho = 0$, but as ρ increases, the upward bias in the size is more important, especially for $T = 100$.

The power of the test is shown from the second to the fifth row of each horizontal block with $\beta_i = [0.3,\ 0.5,\ 1,\ 1.5]$. The power is quite weak for $\beta_i = 0.3$, but it rapidly improves as β_i increases. It is also weak if $\gamma_i = 0.5$, but it is around the double already for $\gamma_i = 1.5$. The results are sensibly better for longer time series with figures which double or almost triple when switching from $T = 100$ to $T = 500$. The power is very close to 1 for $\beta_i \geq 1$, $T = 500$ and $\gamma_i > 0.5$. With $d = 3$ the power generally improves.

In summary, to obtain good size properties, a linear approximation of the logistic function is sufficient, while the power slightly improves as the degree of the polynomial increases. It is important to note the crucial role of the smoothness parameter. We also obtained the results for $d = 2, 4, 5, 6$, which we do not show here to save space. While the power of the test improves, the test is highly oversized, especially for $T = 100$. Given that the test performs well for low-order polynomials, we think that in practice it is not necessary to go further than a polynomial of degree 3.

Unknown parameters of the logistic function

We now discuss the testing results in the case where the smoothness and the location parameters are unknown. In Table 3.7, an auxiliary regression is constructed based on the first-order Taylor series approximation of the logistic function. The structure of the table is the same as above. We can see that the tests have very good size properties for any parameter combinations. Similarly as above, the power improves with the number of observations and with the smoothness parameter. Introducing a structure in the regressors seems to have a slight negative impact, although this is not always the case. The correlation in the error term does not seem to have an impact, neither on the size nor on the power. In Table 3.8 we perform the same exercise with the only difference that the dependence variable is approximated by a third-order expansion. The results deteriorate compared to a linear approximation: the test is undersized and the power weakens.

Given that the parameters of the two-equation system are completely symmetric, when testing $H_0 : \beta_1 = 0$, the value of β_2 is also equal to 0. We would like to make sure that the size of the test for $H_0 : \beta_1 = 0$ does not depend on the value of β_2. For this reason, in Table C.II we compute the size of the test for β_1 with $\beta_2 = [0,\ 0.3,\ 0.5,\ 1,\ 1.5]$ and find very good results for all the parameter combinations.

In a nutshell, the testing procedure proposed by Luukkonnen, Saikkonen and Teräsvirta (1988) works also very well in a model where endogeneity is present. In our framework, the simplest linear approximation seems to perform better than the heavier third-order approximation. In all cases, the test is simple and does not require the estimation of the ST-SEM which is computationally more demanding. This property clearly makes a difference when performing simulations.

3.5 Application to the comovement between the sovereign and the banking sectors

We use the ST-SEM to investigate the comovement between the sovereign and the banking sectors of eight developed countries, namely France, Greece, Italy, Portugal, Spain, Switzerland, the U.K. and the U.S. First, we briefly describe the content of each of the equations (Section 3.5.1). Second, in Section 3.5.2 we describe the data used and present summary statistics. Finally in Section 3.5.3 we summarize the empirical results.

3.5.1 Modeling sovereign and bank returns

During the recent financial and sovereign debt crises the within- and cross-country interactions between the sovereign and the banking sectors intensified as showed by recent studies (e.g. Kallestrup, Lando and Murgoci 2013). The nature of this comovement was rather simultaneous (i.e. contagion) than lagged in time (i.e. spillovers). Acharya, Drechsler and Schnabl (2014) design bank bailouts by the sovereign as the main mechanisms through which shocks were transmitted between these two markets. Government interventions to rescue distressed banks resulted in a dramatic increase of risk in the sovereign sector and considerable losses, liquidity and insolvency issues for banks which held a large part of sovereign bonds. This mechanism continued like a loop. Other channels of market comovement which do not require common factors are wake-up calls (e.g. Ahnert and Bertsch 2015), loss of confidence, general financial panic or herd behavior (e.g. Bae, Karolyi and Stulz 2003; Corsetti, Pericoli and Sbracia 2005).

We explain the monthly stock returns of banks ($Bret_t$) at the aggregate level by the dividend yield (DY_t), the implied volatility of the S&P500 (VIX_t) and the performance of the sovereign sector as shown in Equation 3.8. Referring to Equation 3.9, the sovereign sector monthly returns ($Sret_t$) are explained by the debt to GDP ratio ($d.debt_t$), the fiscal balance to GDP ratio ($d.fb_t$), the sovereign rating and the banking sector monthly returns ($Bret_t$). Following Favero (2013), the sovereign returns, the debt to GDP ratio and the fiscal balance to GDP ratio are taken in deviation with the respective variables for Germany.[11]

$$
\begin{aligned}
Bret_t &= \delta_1 + \alpha_{1,1} DY_t + \alpha_{1,2} VIX_t + \beta_1 G(Sret_t; \gamma_1, c_2) + u_{1,t} && (3.8)\\
Sret_t &= \delta_2 + \alpha_{2,1} d.debt_{t-1} + \alpha_{2,2} d.fb_{t-1} + \alpha_{2,3} rating_{t-1} && (3.9)\\
&\quad + \beta_2 G(Bret_t; \gamma_2, c_1) + u_{2,t}
\end{aligned}
$$

While we do not claim that the explanatory variables are strictly exogenous, we do believe that the weak exogeneity assumption is satisfied. Moreover, we tried several lags

[11] Given that Germany did not register any rating variation, this variable is not taken in deviation. The other countries with no changes in rating are Switzerland and the U.K., for which this variable is not included in Equation 3.9 given the perfect colinearity with the constant term.

for the explanatory variables. For banks the explanatory power is reduced, showing that the transmission lag is very short. On the other hand, the fiscal variables and the rating keep their impact on sovereign excess returns even for further lags, indicating that more time is needed for the structural reforms to be incorporated in sovereign bond prices. In Equation 3.9 these variables appear with a one-month lag.

3.5.2 Data description

Tables 3.9 and 3.10 show summary statistics of the dependent and of the explanatory variables for the period from January 1999 to December 2013, covering the financial and euro area sovereign debt crises. In Table 3.9 are computed summary statistics for each variable using the overall sample. In order to put in evidence the impact of the recent crises, the sample is split in two, taking as a threshold the onset of the subprime crisis in August 2007. The monthly stock returns of the banking sector averaged 0.6% during the pre-crisis period, while during the crisis the average was in negative territory at -1.1% with a much larger volatility. Similarly, the average and the volatility of the dividend yield were lower before the crisis. The VIX sensibly increased since 2007, indicating a higher risk in financial markets.

The conditions in the sovereign market deteriorated during the crises, as indicated by the decrease of the sovereign monthly excess returns and by their much higher volatility. The debt to GDP ratio further increased and the fiscal deficit widened. The rating companies proceeded to a series of downgrades for France, Greece, Italy, Portugal, Spain and the U.S. We convert the rating categories of Standard & Poor's into a numerical scale ranging from 25 (best rating) to 1 (worst rating) as shown in Table C.III in the Appendix.

Table 3.10 shows the average of all the variables for each country. The monthly returns of the banking sector are negative for Greece (-0.9%), followed by Portugal (-0.6%) and Italy (-0.3%), while the best performance is registered in France (0.6%). The monthly excess returns of the sovereign sector have been negative for all the countries (although close to 0), except for the U.S. Unsurprisingly, the worst performance can be found in Greece (-0.9%). This country has also the largest debt and deficit to GDP ratios, and the worst average rating. Switzerland, the U.K. and Spain have a lower debt compared to Germany, while Switzerland is the only country to have a better fiscal balance than the reference country.

3.5.3 Estimation results

In order to be able to compare the results with standard procedures, we first estimate a linear system frequently used in the literature using ordinary least squares (OLS) and two-stage least squares (2SLS). Then, we apply our nonlinear framework estimated by nonlinear 2SLS (NL2SLS). The estimation results are shown in Table 3.11. For the ST-SEM – as for the simulation procedure – we first fix γ_i and c_j respectively at 3 and the 30% quantile of the endogenous variable, then we only fix γ_i and finally we estimate both parameters using the differential evolution algorithm.[12] The order of the polynomial in the NL2SLS is equal to 2. We tried higher orders until 6, with comparable results for polynomials until the fourth order. For higher orders colinearity issues appear.

[12]We tried other low quantiles of $y_{j,t}$ to define c_j and alternative values for γ_i without sensibly affecting the estimation results. When either γ_i or c_j are not fixed the ratio $\hat{\beta}_i/\hat{\sigma}_i$ has an unknown asymptotic distribution. For comparison, we indicate the significance level as if the distribution were normal.

Table 3.9

Summary statistics of the dependent and of the explanatory variables from Jan. 1999 to Dec. 2013

	Jan. 1999 – Aug. 2007					Sep. 2007 – Aug. 2013				
	Obs.	Avg.	Std.	Min.	Max.	Obs.	Avg.	Std.	Min.	Max.
Banking variables										
Monthly return (%)	832	0.64	6.03	-28.12	24.06	608	-1.08	11.92	-39.98	71.94
Dividend yield (%)	832	3.01	0.95	0.35	7.91	608	3.70	2.81	0	15.31
VIX (%)	105	0.88	15.72	-29.14	47.98	75	1.69	21.76	-31.96	90.75
Sovereign variables										
Monthly excess return (%)	832	-0.02	0.44	-2.32	2.09	608	-0.32	3.19	-37.49	17.68
Diff.Debt/GDP (%)	832	2.85	22.11	-28.02	50.94	608	13.58	32.20	-34.30	100.50
Diff.Fisc. Bal./GDP (%)	832	-0.29	2.19	-7.53	4.50	608	-4.89	3.53	-12.25	4.42
S&P rating	832	23.69	1.77	19.00	25.00	608	21.58	4.68	3.00	25.00

Data sources: Datastream.
Notes: This table shows summary statistics for the dependent and for the explanatory variables. The data set contains eight developed countries, namely France, Greece, Italy, Portugal, Spain, Switzerland, the U.K. and the U.S. from January 1999 to December 2013. For the sovereign sector the dependent and the explanatory variables are taken in deviation with respect to the Germany series, indicated by the prefix "Diff" for the Debt/GDP and Fiscal Balance/GDP ratios. The rating categories of Standard & Poor's were translated into a numerical scale presented in Table C.III with the best rating taking the value 25 and the worst one taking the value 1. No changes were registered in the sovereign German rating during the period under study, that is why this variable is not taken in deviation with respect to this country. "Obs.", "Avg.", "Std.", "Min." and "Max." stand for the number of observations, the average, the standard error, the minimum and the maximum within the overall sample. The data is split in two in order to put in evidence the role of the recent crises.
Debt takes a positive value. This means that if a country is more indebted than Germany, Diff.Debt/GDP will be positive (e.g. Greece). On the other hand, if the country is less indebted, then the difference will be negative (e.g. Switzerland). The fiscal balance is positive when there is a fiscal surplus and negative when there is a fiscal deficit. As a result, a positive Diff.Fisc. Bal./GDP means that the country has a better fiscal position than Germany (e.g. Switzerland).

Table 3.10
Summary statistics for each country

	France	Greece	Italy	Portugal	Spain	Switzerland	UK	US
Banking variables								
Monthly return (%)	0.57	-0.86	-0.27	-0.59	0.21	0.06	0.01	0.17
Dividend yield (%)	3.76	2.19	3.89	3.17	4.52	2.14	4.02	2.74
VIX (%)	1.22	1.22	1.22	1.22	1.22	1.22	1.22	1.22
Sovereign variables								
Monthly excess return (%)	-0.007	-0.90	-0.01	-0.05	-0.02	-0.17	-0.06	0.03
Diff.Debt/GDP (%)	3.06	50.06	40.87	9.54	-13.91	-18.07	-15.19	2.70
Diff.Fisc. Bal./GDP (%)	-1.78	-5.07	-1.35	-3.58	-1.53	1.48	-2.14	-3.88
S&P rating	24.86	17.29	21.43	20.76	23.19	25.00	25.00	24.84

Data sources: Datastream.
Notes: This table shows the average for each country of the dependent and of the explanatory variables. The data set contains eight developed countries, namely France, Greece, Italy, Portugal, Spain, Switzerland, the U.K. and the U.S. from January 1999 to December 2013. For the sovereign sector the variables are taken in deviation with Germany, indicated by the prefix "Diff" for the Debt/GDP and Fiscal Balance/GDP ratios. The rating categories of Standard & Poor's were translated into a numerical scale presented in Table C.III with the best rating taking the value 25 and the worst one taking the value 1. No changes were registered in the sovereign German rating during the period under study, that is why this variable is not taken in deviation with respect to this country.

The OLS estimation results of the simple linear model show that for the euro area countries bank and sovereign sectors comove with a strong and positive mutual influence which is highly significant. The impact of the sovereign on banks is larger for France, while for Portugal the inverse effect is larger. For Switzerland and the U.K. only the transmission from banks towards the sovereign is significant, while for the U.S. none of the coefficients is significant. If we control for endogeneity several interesting patterns emerge for the euro area countries. For France, Italy and Portugal only the transmission channel from banks to the sovereign is now significant, while for Greece the impact of the sovereign on banks seems to play an important role. For Spain both channels remain significant with a larger effect than previously. For Switzerland and the U.K. the same conclusion as for OLS applies. For the U.S. the negative β_1 becomes significant at 10%.

The linear model captures the average comovement between sovereign and bank performances across different phases of the business cycle. On the other hand, the ST-SEM attributes a higher weight to negative shocks whose occurrence is more likely in crisis times during which market comovement strengthens. The model estimates a threshold value of the endogenous variables which indicates the turning points between normal and distressed times. Moreover, to adequately manage negative shocks and their potential dispersion in other markets, their speed of transmission estimated by the ST-SEM would be a valuable information. The direction of the comovement in the nonlinear system is broadly in line with the linear model that controls for endogeneity. Nevertheless, important differences emerge in terms of magnitude showing that comovement occurs mainly in distressed times: when significant, the impact is much larger within the nonlinear approach.

Greece is such an example with a considerably wider impact of the sovereign on the banking sector. This result is in line with the severe debt crisis that hit this country and subsequently paralyzed its banking sector. Another interesting case is the negative impact of sovereign excess returns on bank returns in the U.S. The mechanism behind this result might be driven by the government interventions through bank bailouts which improved banks' performance while simultaneously increasing government risk.

For the location parameter of the logistic function we find that the estimated value is in the majority of the cases higher than the predefined 30% quantile, suggesting that even moderate shocks can be transmitted. For Switzerland and the U.S. estimating c_j rather than fixing it, renders β_2 significant. In the bottom panel of Table 3.11 γ_i is also estimated. The only constraint that we put in the estimation procedure is that $\gamma_i > 1$ in order to avoid colinearity issues with the constant term. The estimated value of the smoothness parameter at around 20 suggests a brusque transmission of shocks. Exceptions are Italy, Portugal and Spain for which the transmission of shocks in banks seems to be smoother.

As shown in Figures C.VIII and C.IX in the Appendix, both markets, especially the sovereign one, were characterized by a higher volatility during the recent crisis. For this reason we filter them using a GARCH(1,1) process as shown in Figures C.X and C.XI. The estimation results displayed in Table 3.12 keep the main message. OLS concludes that for the euro area countries market dependence is bidirectional. Controlling for endogeneity within a linear or a nonlinear model shows that one of these effects dominates. Compared to Table 3.11 a few changes can be noticed: β_1 for the U.S. is now significant within the linear system while for Spain this parameter loses significance within the nonlinear system. For Greece, β_1 is less robust if γ_i and c_j are unknown. Finally, the impact of banks on the sovereign is also less robust in the U.K. and in the U.S.

The goodness of fit of the model is quite high as indicated by the R-squared. The equation on banks has a higher R-squared which is explained by the negative robust impact of the VIX and of the dividend yield. In the second equation the impacts of debt, fiscal balance and rating are not always significant and the coefficients do not always have the expected sign. For instance, in Switzerland, the U.K. and the U.S. the explicative power of the model for sovereign excess returns is low.

3.6 Further research

In this section we outline several directions for further methodological research. An immediate extension is the generalization of the model to a multivariate framework. Features such as asymmetry or a time-varying threshold parameter can be added to the dependence variable.

3.6.1 Multi-equation ST-SEM

The investigation of dependence in a multi-country or multi-market framework requires a system with more than two equations. Pesaran and Pick (2007) generalize the model defining the endogenous variable by two different functions:

$$y_{i,t} = \boldsymbol{\delta}_i'\boldsymbol{z}_t + \boldsymbol{\alpha}_i'\boldsymbol{x}_{i,t} + \beta_i\kappa_{i,t} + u_{i,t}, \qquad i = 1, \ldots, N \tag{3.10}$$

Alternative A1: Weighted average of dummy variables.

$$\kappa_{i,t} = \sum_{j=1,\, j\neq i}^{N} w_{i,j}\mathbb{1}(y_{j,t} > c_j)$$

Table 3.11

Estimation results for the banking and for the sovereign sectors

		France	Greece	Italy	Portugal	Spain	Switzerland	U.K.	U.S.
					Linear model				
OLS	β_1	5.05***	0.81***	2.13***	0.96***	2.21***	1.07	0.59	-0.64
	σ_1	(1.54)	(0.23)	(0.37)	(0.18)	(0.38)	(0.74)	(0.73)	(0.56)
	R_1^2	0.362	0.116	0.379	0.299	0.344	0.272	0.282	0.231
	β_2	0.012***	0.07***	0.08***	0.15***	0.08***	0.02**	0.01**	-0.01
	σ_2	(0.003)	(0.22)	(0.01)	(0.02)	(0.01)	(0.007)	(0.007)	(0.009)
	R_2^2	0.121	0.132	0.235	0.248	0.233	0.027	0.015	0.001
2SLS	β_1	1.61	1.84**	2.35	-0.29	6.48***	0.98	5.24	-10.1*
	σ_1	(7.08)	(0.82)	(2.80)	(0.96)	(2.51)	(6.45)	(7.55)	(5.69)
	R_1^2	0.359	0.030	0.388	0.129	0.231	0.260	0.131	0.252
	β_2	0.013***	0.03	0.10***	0.20***	0.11***	0.03**	0.03***	-0.02
	σ_2	(0.005)	(0.08)	(0.02)	(0.21)	(0.02)	(0.01)	(0.01)	(0.02)
	R_2^2	0.140	0.139	0.237	0.234	0.227	0.018	0.012	0.019
					Nonlinear ST-SEM model – *known* γ *and* c				
NL2SLS	β_1	-0.94	26.86***	8.44	-5.98	14.56*	2.97	9.93	-6.86
	σ_1	(13.14)	(10.15)	(11.06)	(11.06)	(8.03)	(7.68)	(11.49)	(7.88)
	γ_1	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00
	c_2	-0.10	-0.31	-0.16	-0.22	-0.12	-0.47	-0.32	-0.342
	R_1^2	0.338	0.103	0.273	0.206	0.240	0.279	0.295	0.242
	β_2	0.25**	-1.17	2.00***	3.67***	1.80***	0.30	0.38*	-0.33
	σ_2	(0.10)	(2.14)	(0.47)	(0.47)	(0.41)	(0.19)	(0.21)	(0.25)
	γ_2	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00
	c_1	-2.41	-6.94	-3.62	-3.45	-2.56	-4.11	-3.03	-2.19
	R_2^2	0.088	0.109	0.111	0.116	0.122	0.024	0.018	0.015
					Nonlinear ST-SEM model – *known* γ *and unknown* c				
	β_1	-1.04	46.73**	11.71	-8.55	14.66**	-9.08	16.80	-17.38***
	σ_1	(13.29)	(18.32)	(10.48)	(10.48)	(6.94)	(7.01)	(14.76)	(6.10)
	γ_1	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00
	c_2	-0.17	-0.4	0.19	-0.05	0.28	0.31	-0.56	0.66
	R_1^2	0.338	0.112	0.282	0.208	0.252	0.286	0.30	0.287
	β_2	0.28*	1.80	1.93***	3.73***	1.84***	1.00**	0.63**	-0.55
	σ_2	(0.16)	(2.84)	(0.54)	(0.54)	(0.52)	(0.43)	(0.27)	(0.50)
	γ_2	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00
	c_1	-4.13	5.34	-2.25	-2.68	-2.83	5.79	1.58	-4.91
	R_2^2	0.089	0.111	0.120	0.120	0.122	0.056	0.046	0.023
					Nonlinear ST-SEM model – *unknown* γ *and* c				
	β_1	1.80	51.89**	10.60	-8.62	20.81	9.99*	14.60	-14.04***
	σ_1	(10.22)	(23.75)	(10.94)	(10.26)	(14.79)	(5.97)	(21.09)	(4.90)
	γ_1	18.97	2.00	6.31	3.50	2.00	19.83	20.53	18.62
	c_2	0.054	-0.03	0.092	-0.05	0.28	0.31	-0.41	0.66
	R_1^2	0.339	0.113	0.282	0.208	0.258	0.287	0.304	0.292
	β_2	0.29**	1.97	1.95***	3.73***	1.83***	1.01***	0.63***	-0.56
	σ_2	(0.14)	(2.68)	(0.70)	(1.25)	(0.72)	(0.37)	(0.25)	(0.36)
	γ_2	18.79	18.45	18.70	20.32	19.01	18.06	20.95	19.90
	c_1	-4.13	5.51	-2.30	-2.62	-2.87	5.78	1.32	-4.90
	R_2^2	0.090	0.111	0.122	0.124	0.123	0.058	0.048	0.024

Note: This table shows the relation between the endogenous variables. The analysis is conducted for eight developed countries, using a linear model and the nonlinear ST-SEM model as shown in Equations 3.8 and 3.9. For each country, the monthly returns of the banking sector $Bret_t$ are regressed on the dividend yield DY_t, the implied volatility of the S&P index VIX_t and the monthly excess returns of a sovereign bond index with respect to German bonds $Sret_t$. The latter is regressed on $Bret_t$, on the difference of debt and fiscal balance with respect to Germany $d.debt_{t-1}$ and $d.fb_{t-1}$, and on the S&P sovereign rating $rating_{t-1}$ translated into a numerical scale as shown in Table C.III in the Appendix. In parentheses (σ_i) are shown estimated standard errors of the dependence parameter β_i. R_i^2 is the sum of squared residuals. In the nonlinear system, γ_i and c_j are first fixed respectively at 3 and the 30% quantile of y_{jt}, then only γ_i is fixed. Finally both of them are estimated using the differential evolution algorithm. ***, **, * represent significance at 1%, 5% and 10% respectively. For the cases where γ_i and/or c_j are estimated, the ratio $\hat{\beta}_i/\hat{\sigma}_i$ has an unknown distribution. We indicate the significance level as if the asymptotic distribution were normal.

Table 3.12

Estimation results for the banking and the sovereign sectors – GARCH(1,1)

		France	Greece	Italy	Portugal	Spain	Switzerland	U.K.	U.S.
					Linear model				
OLS	β_1	0.44**	0.05***	0.18***	0.07***	0.22***	0.11	0.07	-0.15**
	σ_1	(0.18)	(0.02)	(0.05)	(0.02)	(0.05)	(0.10)	(0.10)	(0.07)
	R_1^2	0.361	0.173	0.358	0.215	0.328	0.267	0.303	0.260
	β_2	0.23***	0.25***	0.45***	0.34***	0.42***	0.17**	0.15**	-0.15*
	σ_2	(0.08)	(0.06)	(0.07)	(0.08)	(0.07)	(0.07)	(0.08)	(0.08)
	R_2^2	0.07	0.137	0.183	0.131	0.222	0.020	0.018	0.015
2SLS	β_1	0.33	0.74**	-0.13	-0.10	0.05	0.08	0.11	-1.05*
	σ_1	(0.33)	(0.34)	(0.36)	(0.31)	(0.22)	(0.78)	(0.51)	(0.61)
	R_1^2	0.313	0.103	0.251	0.140	0.268	0.282	0.310	0.248
	β_2	0.40***	0.24	0.87***	0.88***	0.78***	0.38***	0.34**	-0.05
	σ_2	(0.13)	(0.15)	(6.08)	(0.21)	(0.14)	(0.14)	(0.14)	(0.15)
	R_2^2	0.061	0.157	0.052	0.127	0.132	0.030	0.070	0.019
					Nonlinear ST-SEM model – *known γ and c*				
Nonlinear 2SLS	β_1	0.35	3.30**	-0.02	-0.73	-0.10	1.05	0.40	-1.06
	σ_1	(0.74)	(1.38)	(0.91)	(1.35)	(0.62)	(1.31)	(1.11)	(0.95)
	γ_1	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00
	c_2	-0.41	-0.21	-0.37	-0.64	-0.49	-0.77	-0.54	-0.38
	R_1^2	0.355	0.166	0.312	0.187	0.251	0.280	0.313	0.260
	β_2	1.04***	0.62	2.41***	2.12***	1.82***	0.67*	0.75*	-0.46
	σ_2	(2.79)	(0.39)	(0.41)	(0.52)	(0.38)	(0.36)	(0.38)	(0.36)
	γ_2	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00
	c_1	-0.30	-0.54	-0.52	-0.51	-0.36	-0.54	-0.45	-0.40
	R_2^2	0.084	0.084	0.218	0.156	0.205	0.027	0.035	0.016
					Nonlinear ST-SEM model – *known γ and unknown c*				
	β_1	0.35	3.29	2.09	-0.89	0.30	-1.04	-0.99	-1.92**
	σ_1	(0.81)	(3.10)	(1.54)	(1.33)	(1.55)	(0.92)	(0.93)	(0.77)
	γ_1	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00
	c_2	-0.38	-0.21	0.76	-0.21	0.54	0.56	0.66	0.81
	R_1^2	0.355	0.166	0.324	0.188	0.252	0.284	0.318	0.288
	β_2	1.08	1.31	2.65***	2.93*	1.82***	1.52	1.44	-0.57
	σ_2	(0.81)	(1.30)	(0.77)	(1.62)	(0.39)	(1.72)	(1.80)	(0.61)
	γ_2	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00
	c_1	-0.57	0.64	-0.83	0.50	-0.15	0.84	0.80	-0.75
	R_2^2	0.084	0.093	0.222	0.160	0.206	0.046	0.046	0.052
					Nonlinear ST-SEM model – *unknown γ and c*				
	β_1	0.40	3.98	1.54	-0.97	-0.48	2.63*	-0.94	-1.51
	σ_1	(0.76)	(2.92)	(1.22)	(1.08)	(0.63)	(1.59)	(0.96)	(1.03)
	γ_1	16.74	2.00	18.73	18.37	20.75	19.75	19.35	19.17
	c_2	-0.15	-0.22	0.73	-0.27	-0.27	-0.62	0.66	0.81
	R_1^2	0.357	0.167	0.33	0.190	0.255	0.295	0.321	0.289
	β_2	0.91**	1.56	2.09***	1.73***	1.59***	1.69	0.98	-0.53
	σ_2	(0.43)	(0.98)	(0.38)	(0.60)	(0.38)	(1.58)	(0.64)	(0.48)
	γ_2	20.14	18.77	18.25	18.34	20.33	16.76	21.44	19.72
	c_1	-0.35	0.64	-0.47	-0.21	-0.25	0.84	0.28	-0.75
	R_2^2	0.088	0.097	0.225	0.163	0.214	0.053	0.048	0.021

Note: Before estimating the ST-SEM model, we filter the endogenous variables by a GARCH(1,1) process as follows:

$$R_{i,t} = \mu_i + \sigma_{i,t} X_{i,t}, \quad \sigma^2_{i,t} = \omega_i + \alpha_i (R_{i,t} - \mu_i)^2 + \beta_i \sigma^2_{i,t-1}, \quad X_{i,t} = \frac{R_{i,t} - \hat{\mu}_i}{\hat{\sigma}_{i,t}}, \; i = 1, 2, \; t = 1, 2, \ldots, T$$

where $\omega_i > 0$, $\alpha_i \geq 0$, $\beta_i \geq 0$ and $\alpha_i + \beta_i < 1$, with $R_{i,t}$ the non-filtered variable, $X_{i,t}$ the filtered series, μ_i and $\sigma^2_{i,t}$ the mean and the conditional variance of $R_{i,t}$. Refer also to the notes under Table 3.12.

Alternative A2: Dummy if any of the markets exceeds the threshold.

$$\kappa_{i,t} = I\left(\sum_{j=1,\, j\neq i}^{N} I(y_{j,t} > c_j)\right)$$

In *Alternative A1* the weights $w_{i,j}$ are defined such that $w_{i,j} \geq 0$ and $\sum_{j=1,\, j\neq i}^{N} w_{i,j} = 1$. Allen and Gale (2000) suggest three different weighting schemes, two of which assume an ordering of the countries/markets and the third one is an equal weights scheme. Other specifications can be thought of, for instance weights proportional to a country's fundamentals such as GDP or trade. *Alternative A2* is a simple way to model comovement, the outer indicator function taking the value 1 if the sum is positive. The drawback is that this variable takes the value 1 when either one, or more markets are hit by a shock ($y_{j,t}$ exceeds the threshold).

For the ST-SEM, we suggest three ways to define the dependence variable.

Alternative B1: Multivariate logistic function.

$$\kappa_{i,t} = \left\{1 + \sum_{j=1,\, j\neq i}^{N} e^{-\gamma_j^{(i)}(y_{j,t} - c_j^{(i)})}\right\}^{-1}$$

Alternative B2: Weighted average of univariate logistic functions.

$$\kappa_{i,t} = \sum_{j=1,\, j\neq i}^{N} w_{i,j} G(y_{j,t}; \gamma_j^{(i)}, c_j^{(i)})$$

Alternative B3: Univariate logistic function based on the weighted average of y.

$$\kappa_{i,t} = \left\{1 + e^{-\gamma_i(y_{i,t}^{*} - c_i^{*})}\right\}^{-1}$$

where $y_{i,t}^{*} = \sum_{j=1,\, j\neq i}^{N} w_{i,j} y_{j,t}$ and as previously $w_{i,j} \geq 0$, and $\sum_{j=1,\, j\neq i}^{N} w_{i,j} = 1$. The superscript (i) in *Alternatives B1* and *B2* indicates that the smoothness and threshold parameters are equation specific. For instance, if $N = 3$, a shock from Equation 2 is not necessarily transmitted at the same threshold value and with the same degree of smoothness to Equation 1 and to Equation 3.

Alternatives B1 and *B2* require the estimation of $N-1$ smoothness parameters and $N-1$ location parameters for each equation. If N is large, the numerical procedure for the estimation of these $2(N-1)$ parameters can become computationally heavy and imprecise. A way to simplify the model is to assign a data-based value to $c_j^{(i)}$ and estimate $\gamma_j^{(i)}$. A further simplification would be to impose that $\gamma_j^{(i)} = \gamma^{(i)}$ for all j which is equivalent to estimating of an average smoothness parameter for each equation.

3.6.2 Time-varying threshold parameter

In order to add more flexibility to the modeling of dependence, slight modifications can be thought of in the logistic function presented in Equation 3.5. First, as in Pesaran and Pick (2007), the threshold parameter can be time varying by introducing the conditional volatility $\sigma_{j,t} = var(y_{j,t}|\Omega_t)$ with Ω_t the information available until time t. The threshold becomes $c_j\sigma_{j,t}$ and c_j can be interpreted as the number of standard errors that a shock on $y_{j,t}$ has to exceed to switch from a normal to a crisis regime. $\sigma_{j,t}$ can for instance be specified as a GARCH model.

$$G(y_{j,t}; \gamma_i, c_j) = \left\{1 + e^{-\gamma_i(y_{j,t} - c_j\sigma_{j,t})}\right\}^{-1} \quad \gamma_i > 0, \quad i, j = 1, 2, \quad i \neq j.$$

3.6.3 Asymmetric transmission of contagion

There is empirical evidence that the transmission of shocks is asymmetric. A direct way to include asymmetry in the ST-SEM is to introduce a skewness parameter into the logistic function as in the Type I generalized logistic function given by the following formula (see Johnson, Kotz and Balakrishnan 1995):

$$G(y_{j,t}; \gamma_i, \theta_i, c_j) = \left\{1 + e^{-\gamma_i(y_{j,t} - c_j)}\right\}^{-\theta_i} \quad \gamma_i > 0, \quad \theta_i > 0, \quad i \neq j. \tag{3.11}$$

There is negative skewness for $0 < \theta_i < 1$, positive skewness for $\theta_i > 0$, and the standard logistic function for $\theta_i = 1$, as shown in Figure C.VI in the Appendix. Lundbergh and Teräsvirta (2006) use this skewed logistic function to build a flexible explicit or implicit target zone model to characterize the dynamic behavior of an exchange rate taking as illustrations the Norwegian and Swedish krona.

Crisis periods are extreme events during which comovement is more likely to occur and many financial and economic indicators reach extreme values. The extreme value theory (Poon, Rockinger and Tawn 2004) contains tools which are able to account not only for the magnitude but also for the intensity of the transmission of shocks. Figure C.VII in the Appendix shows the Gumbel distribution – a special case of extreme value distributions – for different values of the scale and location parameters, which can be a well-adapted candidate to model the dependence variable. A third highly flexible alternative would be a non-parametric specification of that variable.

3.7 Conclusion

Market comovement has become a highly topical issue since the burst of the financial crisis affecting the world economy. Politicians, economists, central bankers, investors, portfolio managers are concerned about limiting the harmful effects of comovement. The basis of successful policies is the understanding of the channels through which comovement spreads and the use of adapted tools to model this phenomenon.

In this study we develop a new econometric framework within a nonlinear simultaneous equation model to measure the comovement between two endogenous random variables. The model is a generalization of Pesaran and Pick's (2007) threshold SEM, which is included as a special case. Comovement intervenes through a smooth and potentially asymmetric function which is dependent on the magnitude of the endogenous variable. Following a large number of studies, market dependence is modeled to occur during bad events which are identified from the data through a threshold parameter. The framework also permits for an indirect channel of dependence controlling for constant links between markets and which is modeled through common observed and unobserved variables. The presence of market-specific factors is necessary to disentangle between direct and indirect market comovements.

We develop and adapt a valid estimation method and a testing procedure for simultaneity that we examine through Monte Carlo simulations. An advantage of our specification compared to Pesaran and Pick's model is that conditions can be set on the parameters in order to obtain a unique solution of the system. Models with multiple reduced forms

– explicit or implicit – are known in the literature as incoherent. They cannot be used for forecasting and have unknown properties of their estimators. We suggest numerical procedures to solve the system of equations.

The Monte Carlo simulations show that the dependence and other parameters of the model are correctly estimated when the smoothness and the threshold parameters of the dependence variable are known. When all the parameters are unknown, the results deteriorate for small samples, but highly improve for bigger samples. Although we obtain good results for some parameter combinations, an accurate estimation of the threshold parameter, and especially of the smoothness parameter proves difficult. This is a well known issue in the literature – maybe not enough explored – caused by a flat objective function. The tests for simultaneity which control for the presence of nuisance parameters are easy to implement and compute, and have good size and power properties.

As an illustration, we study the impact of the recent crises on the comovement between the sovereign and the banking sectors for the main euro area countries, Switzerland, the U.K. and the U.S. Our results document that market interdependence has significantly strengthened in such distressed times. For most of the countries the dominating channel is the transmission of shocks originating from the banking sectors. A major exception is Greece whose recent sovereign debt crisis had dramatic consequences for the banking sector.

As a future line of research, several extensions of this methodology are envisaged. First, for the investigation of comovement in a group of countries or markets, we plan to study the properties of a multi-equation system. Second, given that market dependence is likely to strengthen in extreme events, distributions from extreme value theory should be highly suitable. To further increase the flexibility, a non-parametric specification of the dependence structure could be developed. The methodology can be used to study the dependence structure in various markets such as stocks, bonds, interest rates, exchange rates and many others.

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C Appendix

C.1 Limiting distribution of the estimated parameters

We consider the ith equation of a two-equation ST-SEM which is written as follows:

$$y_{it} = f(\boldsymbol{h}_{it}, \boldsymbol{\zeta}_i) + u_{it}, \quad f(\boldsymbol{h}_{it}, \boldsymbol{\zeta}_i) = \boldsymbol{\delta}_i' \boldsymbol{z}_t + \boldsymbol{\alpha}_i' \boldsymbol{x}_{it} + \beta_i G(y_{jt}; \gamma_i, c_j)$$
$$G(y_{jt}; \gamma_i, c_j) = \left\{1 + e^{-\gamma_i (y_{jt} - c_j}\right\}^{-1} \quad \gamma_i > 0 \quad i \neq j \quad i = 1, 2 \quad t = 1, \dots T$$

with $\boldsymbol{\zeta}_i = [\boldsymbol{\delta}_i',\, \boldsymbol{\alpha}_i',\, \beta_i,\, \gamma_i, c_j]'$ the overall set of parameters, and $\mathbf{h}_{it} = [\boldsymbol{z}_t',\ \boldsymbol{x}_{it}',\ G(y_{jt}; \gamma_i, c_j)]'$ the explanatory variables. The NL2SLS estimator of $\boldsymbol{\zeta}_i$ denoted by $\hat{\boldsymbol{\zeta}}_i$ is the value of $\boldsymbol{\zeta}_i$ that minimizes the following equation:

$$\hat{\boldsymbol{\zeta}}_i = \text{argmin}\, (\boldsymbol{y}_i - f_i)'(Q(Q'Q)^{-1}Q')(\boldsymbol{y}_i - f_i)$$

with $\boldsymbol{y}_i = [y_{i1}\, y_{i2}\, \dots,\, y_{iT}]'$, $f_i = [f(\boldsymbol{h}_{i1}, \boldsymbol{\zeta}_i), \dots, f(\boldsymbol{h}_{iT}, \boldsymbol{\zeta}_i)]'$ and Q a $T \times K$ matrix containing low-order polynomials of the overall set of the exogenous parameters of the model.

Following Kelejian (1971) and Amemiya (1974), under usual regularity conditions, the limit in probability of $\hat{\boldsymbol{\zeta}}_i$ converges to its true value and the limiting distribution is normal.

$$\sqrt{T}(\hat{\boldsymbol{\zeta}}_i - \boldsymbol{\zeta}_i) \sim \mathcal{N}\left\{0,\, \sigma_i^2 \,\text{plim}\left[\frac{1}{T}\frac{\partial f_i'}{\partial \boldsymbol{\zeta}_i} Q(Q'Q)^{-1}Q' \frac{\partial f_i)}{\partial \boldsymbol{\zeta}_i}\right]^{-1}\right\}$$

with

$$\frac{\partial f(\boldsymbol{h}_{it}, \boldsymbol{\zeta}_i)}{\partial \boldsymbol{\zeta}_i} = \begin{pmatrix} \frac{\partial f(\boldsymbol{h}_{it}, \boldsymbol{\zeta}_i)}{\partial \boldsymbol{\delta}_i} \\ \frac{\partial f(\boldsymbol{h}_{it}, \boldsymbol{\zeta}_i)}{\partial \boldsymbol{\alpha}_i} \\ \frac{\partial f(\boldsymbol{h}_{it}, \boldsymbol{\zeta}_i)}{\partial \boldsymbol{\beta}_i} \\ \frac{\partial f(\boldsymbol{h}_{it}, \boldsymbol{\zeta}_i)}{\partial \gamma_i} \\ \frac{\partial f(\boldsymbol{h}_{it}, \boldsymbol{\zeta}_i)}{\partial c_j} \end{pmatrix} = \begin{pmatrix} \boldsymbol{z}_t \\ \boldsymbol{x}_{it} \\ G(y_{jt}; \gamma_i, c_j) \\ \beta_i\, G(y_{jt}; \gamma_i, c_j)\, [1 - G(y_{jt}; \gamma_i, c_j)]\, (y_{jt} - c_j) \\ -\beta_i\, G(y_{jt}; \gamma_i, c_j)\, [1 - G(y_{jt}; \gamma_i, c_j)]\, \gamma_i \end{pmatrix}$$

C.2 Algorithms of the numerical methods used

Fixed Point algorithm

We use the Fixed Point method to numerically solve the ST-SEM. We first present how the algorithm works for the solution of an n-equation system and then give the pseudocode for its implementation. Let us consider the system $F(\boldsymbol{y}) = \mathbf{0}$ and rewrite it as follows:

$$\boldsymbol{y} = H(\boldsymbol{y}) \quad \equiv \quad \begin{cases} y_1 = h_1(\boldsymbol{y}) \\ y_2 = h_2(\boldsymbol{y}) \\ \quad\vdots \\ y_n = h_n(\boldsymbol{y}) \end{cases}$$

where $\boldsymbol{y} = [y_1\, y_2 \dots y_n]' \in (a, b)^n$ and $H(\boldsymbol{y}) = [h_1(\boldsymbol{y})\, h_2(\boldsymbol{y}) \dots h_n(\boldsymbol{y})]' \in (a, b)^n$ continuous functions. We choose an initial value for $\boldsymbol{y}$ denoted by $\boldsymbol{y}^{(0)}$ and iterate according to the following equation:

$$\boldsymbol{y}^{(k+1)} = H\left(\boldsymbol{y}^{(k)}\right), \quad k = 0, 1, 2, \dots$$

The method converges if the spectral radius[13] of the Jacobian matrix of the system at the solution is smaller than one in absolute value. The Jacobian matrix $\nabla H(\boldsymbol{y}_t)$ of the two-equation ST-SEM can be written as follows:

$$\nabla H(\boldsymbol{y}_t) = \begin{pmatrix} 0 & G(y_{2t};\gamma_1,c_2)[1-G(y_{2t};\gamma_1,c_2)]\beta_1\gamma_1 \\ G(y_{1t};\gamma_2,c_1)[1-G(y_{1t};\gamma_2,c_1)]\beta_2\gamma_2 & 0 \end{pmatrix}$$

The spectral radius evaluated at the solution $\boldsymbol{y}_t^*$ is given by the following formula:

$$\rho(\nabla H(\boldsymbol{y}_t^*)) = \sqrt{G(y_{1t}^*;\gamma_1,c_2)\,[1-G(y_{1t}^*;\gamma_1,c_2)]\,G(y_{2t}^*;\gamma_2,c_1)\,[1-G(y_{2t}^*;\gamma_2,c_1)]\,\gamma_1\beta_1\gamma_2\beta_2}$$

The condition highly depends on the combinations of values for γ_i and β_i. The product of the first four terms is bounded since the maximum value it can take is 0.0625 which corresponds to $G(y_{jt};\gamma_i,c_j) = 0.5$. If $G(y_{jt};\gamma_i,c_j) = 0.5$ then the product $\gamma_1\beta_1\gamma_2\beta_2 < 16$ for the convergence condition to be satisfied. In practice, this condition cannot be verified since it depends on the solution (see Gilli, Maringer and Schumann 2011).

Pseudocode for the Fixed Point algorithm

1 Initialize $\boldsymbol{y}^{(0)}$, $\boldsymbol{y}^{(1)}$, the error tolerance ϵ, and the maximum number of iterations `itmax`
2 **while** convergence criterion not satisfied **do**
3 $\quad\boldsymbol{y}^{(0)} = \boldsymbol{y}^{(1)}$ (keep the previous iteration in $\boldsymbol{y}^{(0)}$)
4 $\quad$Evaluate $\boldsymbol{y}^{(1)}$
5 $\quad$Check the number of iterations
6 **end while**

Notes: $\boldsymbol{y}^{(k)} = [y_1^{(k)},\ y_2^{(k)}]'$; `itmax` = 100; Error tolerance: $\epsilon = 10^{-6}$; Convergence criterion: $\frac{|y_i^{(k+1)} - y_i^{(k)}|}{|y_i^{(k)}|+1} < \epsilon$, $i = 1, 2$. The algorithm stops when the convergence criterion is satisfied, or when the maximum number of iterations is reached.

Newton's algorithm

In Newton's method the system of equations is written as:

$$F(\boldsymbol{y}) = \mathbf{0} \quad \equiv \quad \begin{cases} f_1(\boldsymbol{y}) = 0 \\ f_2(\boldsymbol{y}) = 0 \\ \quad\vdots \\ f_n(\boldsymbol{y}) = 0 \end{cases}$$

where $F(\boldsymbol{y}) = [f_1(\boldsymbol{y}), f_2(\boldsymbol{y}), \ldots, f_n(\boldsymbol{y})]'$ are a system of continuous functions. Given the sequence $\{\boldsymbol{y}^{(k)}\}_{k=0,1,2\ldots}$, we approximate $F(\boldsymbol{y})$ around $\boldsymbol{y}^{(k)}$ using a first-order Taylor series expansion with $\nabla F(\boldsymbol{y}^{(k)})$ the Jacobian matrix.

$$F(\boldsymbol{y}) \approx F(\boldsymbol{y}^{(k)}) + \nabla F(\boldsymbol{y}^{(k)})(\boldsymbol{y} - \boldsymbol{y}^{(k)})$$

Given that $F(\boldsymbol{y}) = \mathbf{0}$, we can write a solution for $\boldsymbol{y}$ as:

[13] $\rho(A)$ denotes the spectral radius of a matrix $A \in \mathbb{R}^{n\times n}$ and is defined by $\rho(A) = \max_{1\le i\le n} |\lambda_i|$ where $\lambda_1, \ldots, \lambda_n$ are all the eigenvalues of A. If $\rho(A) < 1$ this is equivalent to stating that the matrix A is convergent or that $\lim_{k\to\infty} A^k x = 0$, for $\forall x \in \mathbb{R}^n$. A square matrix $A \in \mathbb{R}^{n\times n}$ is convergent if $||A^k|| \to 0$ as $k \to \infty$, which is equivalent to $(A^k)_{ij} \to 0$ as $k \to \infty$ for all i, j.

$$\boldsymbol{y} = \boldsymbol{y}^{(k)} - \left(\nabla F(\boldsymbol{y}^{(k)})\right)^{-1} F(\boldsymbol{y}^{(k)})$$

This approximation can be used to generate successive iterations of $\boldsymbol{y}$:

$$\boldsymbol{y}^{(k+1)} = \boldsymbol{y}^{(k)} - \left(\nabla F(\boldsymbol{y}^{(k)})\right)^{-1} F(\boldsymbol{y}^{(k)})$$

where $\boldsymbol{y}^{(k+1)}$ is the solution of the following linear system:

$$\underbrace{\nabla F(\boldsymbol{y}^{(k)})}_{J} \underbrace{(\boldsymbol{y}^{(k+1)} - \boldsymbol{y}^{(k)})}_{s} = \underbrace{-F(\boldsymbol{y}^{(k)})}_{b}$$

s can be computed as $s = J^{-1}b$ and $\boldsymbol{y}^{(k+1)} = s + \boldsymbol{y}^{(k)}$. The Jacobian of the two-equation ST-SEM is given by:

$$J = \begin{pmatrix} -1 & G(y_{2t};\gamma_1,c_2)[1-G(y_{2t};\gamma_1,c_2)]\beta_1\gamma_1 \\ G(y_{1t};\gamma_2,c_1)[1-G(y_{1t};\gamma_2,c_1)]\beta_2\gamma_2 & -1 \end{pmatrix}$$

In Gilli, Maringer and Schumann (2011) we find the following convergence criterion:

$$||\boldsymbol{y}_t^{(k+1)} - \boldsymbol{y}_t^*|| \leq \omega\tau||\boldsymbol{y}_t^{(k)} - \boldsymbol{y}_t^*||^2$$

with ω measuring the relative nonlinearity $||\nabla F(\boldsymbol{y}_t^*)^{-1}|| \leq \omega < 0$ and τ the Lipschitz constant. As for the Fixed Point method, this condition cannot be verified ex ante. Moreover, the convergence also depends on the choice a “good” starting value in the neighborhood (which has to be defined) of the solution $\boldsymbol{y}_t^*$.

Pseudocode for Newton's algorithm

Initialize $\boldsymbol{y}^{(0)}$, the error tolerance ϵ and the maximum number of iterations `itmax`
for $k = 0, 1, 2, \ldots$ until convergence criterion satisfied **do**
    Evaluate $b = -F(\boldsymbol{y}^{(k)})$ and $J = \nabla F(\boldsymbol{y}^{(k)})$
    Check whether J is invertible
    Solve $Js = b$
    $\boldsymbol{y}^{(k+1)} = \boldsymbol{y}^{(k)} + s$
    Check the number of iterations
end for

Notes: $\boldsymbol{y}^{(k)} = [y_1^{(k)},\, y_2^{(k)}]'$; `itmax` = 100; Error tolerance: $\epsilon = 10^{-6}$; Convergence criterion: $\frac{|y_i^{(k+1)} - y_i^{(k)}|}{|y_i^{(k)}|+1} < \epsilon$, $i = 1, 2$. The algorithm stops when the convergence criterion is satisfied, or when the maximum number of iterations is reached.

Differential Evolution algorithm

Pseudocode for the DE implementation

Initialize $\mathbf{x} = (\gamma, c)$: $\gamma \sim$ Uniform (1; 3.5), $c \sim$ Uniform in (0.2; 0.8) quantile range of y
Initialize real string $\mathscr{L}$ with random values
for $it = 1 : n_g$ **do**
    `% Generate new offspring solutions`
    for $p = 1 : n_p$ **do**

randomly pick m_1, m_2 and m_3 with $p \neq m_1 \neq m_2 \neq m_3$
$\tilde{\boldsymbol{o}}_c = \mathbf{x}_{m_1} + F \cdot (\mathbf{x}_{m_2} - \mathbf{x}_{m_3})$
$\tilde{\mathscr{O}}_c = \mathscr{L}_{m_1} + F \cdot (\mathscr{L}_{m_2} - \mathscr{L}_{m_3})$
for $i = 1 : 2$
  if Uniform $(0,1) < \pi$ **then**
    $o_{p,i} = \tilde{o}_{p,i}$
  else
    $o_{p,i} = x_{p,i}$
  end if
end for
for $i = 1 : length(\mathscr{L})$ **do**
  if Uniform $(0,1) < \pi$ **then**
    $\mathscr{O}_{p,i} = \tilde{\mathscr{O}}_{p,i}$
  else
    $\mathscr{O}_{p,i} = \mathscr{L}_{p,i}$
  end if
Compute the value of the objective function $f(\boldsymbol{o}_p, \mathscr{O}_p)$
end for
end for
`%` Replace pth current solution if offspring p has a better fit
for $p = 1 : n_p$ **do**
  if $f(\boldsymbol{o}_p, \mathscr{O}_p) < f(\mathbf{x}_p, \mathscr{L}_p)$ **then**
    Replace $\mathbf{x}_p$ with $\boldsymbol{o}_p$ and $\mathscr{L}_p$ with $\mathscr{O}_p$
    Check if offspring p is the best solution so far (new elitist)
  end if
end for
end for
Report elitist

Notes: Number of simulations: $n_g = 1,000$; Population size: $n_p = 10D$, with $D = 1$ if only c is estimated and $D = 2$ if both γ and c are estimated; Scaling factor: $F = 0.5$; Cross-over probability: $\pi = 0.8$. The only constraint put in γ is $\gamma > 1$. This algorithm is sourced from Maringer and Meyer (2008).

C.3 Derivation of the tests for contagion

Below we give the details of the derivation of the tests for contagion based on the procedure of Luukkonnen, Saikkonen and Teräsvira (1988) and Wooldridge (2002). Consider the following two-equation system:

$$y_{1t} = \boldsymbol{\delta}_1' \boldsymbol{z}_t + \boldsymbol{\alpha}_1' \boldsymbol{x}_{1t} + \beta_1 G(y_{2t}; \gamma_1, c_2) + u_{1t} \tag{C.I}$$

$$y_{2t} = \boldsymbol{\delta}_2' \boldsymbol{z}_t + \boldsymbol{\alpha}_2' \boldsymbol{x}_{2t} + \beta_2 G(y_{1t}; \gamma_2, c_1) + u_{2t} \tag{C.II}$$

$$G(y_{jt}; \gamma_i, c_j) = \left\{ 1 + e^{-\gamma_i (y_{jt} - c_j \sigma_{j,t-1})} \right\}^{-1} \quad \gamma_i > 0, \quad i,j = 1,2, \quad i \neq j. \tag{C.III}$$

First-order Taylor expansion

We compute the derivative with respect to γ_i of the logistic function in Equation C.III and evaluate it at $\gamma_i = 0$.

$$\frac{\partial G(y_{jt};\gamma_i,c_j)}{\partial\gamma_i} = \frac{e^{\gamma_i(y_{jt}-c_j)}}{\left(1+e^{\gamma_i(y_{jt}-c_j)}\right)^2}(y_{2t}-c_2) = G(y_{jt};\gamma_i,c_j)\left[1-G(y_{jt};\gamma_i,c_j)\right](y_{2t}-c_2)$$

$$\left.\frac{\partial G(y_{jt};\gamma_i,c_j)}{\partial\gamma_i}\right|_{\gamma_i=0} = \frac{1}{4}(y_{jt}-c_j)$$

We evaluate the logistic function at $\gamma_i = 0$ and then subtract the value obtained from the logistic function $G(y_{jt};\gamma_i,c_j)$. We denote this new function by $\tilde{G}(y_{jt};\gamma_i,c_j)$, whose value in $\gamma_i = 0$ is equal to 0. This reparametrization is one of the conditions set by Luukkonen, Saikkonen and Teräsvira (1988) and is used only for the derivation of the test.

$$G(y_{jt};\gamma_i,c_j)|_{\gamma_i=0} = \frac{1}{2}$$

$$\tilde{G}(y_{jt};\gamma_i,c_j) = G(y_{jt};\gamma_i,c_j) - G(y_{jt};\gamma_i,c_j)|_{\gamma_i=0}$$

The first-order Taylor expansion of $\tilde{G}(y_{jt};\gamma_i,c_j)$ is written as follows, with R_{1i} the remainder:

$$\tilde{G}(y_{jt};\gamma_i,c_j) = -\frac{1}{4}c_j\gamma_i + \frac{1}{4}\gamma_i y_{jt} + R_{1i}$$

Replacing the above approximation in the ith equation of the ST-SEM yields the following auxiliary regression:

$$y_{it} = \tilde{\delta}_i' z_t + \alpha_i' x_{it} + \tilde{\beta}_i y_{jt} + u_{it}^*$$

with $\tilde{\delta}_i = \left[(\delta_{i,1} - \frac{1}{4}\beta_i\gamma_i c_j)\ \delta_i^{-\prime}\right]'$, with δ_i^- equals to δ_i minus the constant term $\delta_{i,1}$, $\tilde{\beta}_i = \frac{1}{4}\beta_i\gamma_i$ and $u_{it}^* = \beta_i R_{1i} + u_{it}$.

Third-order Taylor expansion

We proceed in the same way for the construction of an auxiliary regression based on the the third-order expansion of the logistic function. We compute the second and the third derivative of $G(y_{jt};\gamma_i,c_j)$ and evaluate them at $\gamma_i = 0$. Given that the logistic function is odd, the second derivative is 0.

$$\frac{\partial^2 G(y_{jt};\gamma_i,c_j)}{\partial\gamma_i^2} = G(y_{jt};\gamma_i,c_j)\left[1-G(y_{jt};\gamma_i,c_j)\right]\left[1-2G(y_{jt};\gamma_i,c_j)\right](y_{2t}-c_2)^2$$

$$\left.\frac{\partial^2 G(y_{jt};\gamma_i,c_j)}{\partial\gamma_i^2}\right|_{\gamma_i=0} = 0$$

$$\frac{\partial^3 G(y_{jt};\gamma_i,c_j)}{\partial\gamma_i^3} = G(y_{jt};\gamma_i,c_j)\left[1-G(y_{jt};\gamma_i,c_j)\right]\left[6G(y_{jt};\gamma_i,c_j)^2 - 6G(y_{jt};\gamma_i,c_j)+1\right](y_{2t}-c_2)^3$$

$$\left.\frac{\partial^3 G(y_{jt};\gamma_i,c_j)}{\partial\gamma_i^3}\right|_{\gamma_i=0} = -\frac{1}{8}(y_{jt}-c_j)^3$$

The third-order Taylor expansion of $\tilde{G}(y_{jt};\gamma_i,c_j)$ – with R_{3i} the remainder – is given by the following equation.

$$\tilde{G}(y_{jt};\gamma_i,c_j) = -\frac{1}{4}\gamma_i c_j + \frac{1}{48}\gamma_i^3 c_j^3 + \left(\frac{1}{4}\gamma_i - \frac{1}{16}\gamma_i^3 c_j^2\right) y_{jt} + \frac{1}{16}\gamma_i^3 c_j y_{jt}^2 - \frac{1}{48}\gamma_i^3 y_{jt}^3 + R_{3i}$$

Finally, the auxiliary regression can be written as:

$$y_{it} = \tilde{\delta}_i' z_t + \alpha_i' x_{it} + \tilde{\beta}_{i,1} y_{jt} + \tilde{\beta}_{i,2} y_{jt}^2 + \tilde{\beta}_{i,3} y_{jt}^3 + u_{it}^*$$

with $\tilde{\delta}_i = [(\delta_{i,1} - \frac{1}{4}\gamma_i c_j + \frac{1}{48}\gamma_i^3 c_j^3)\ \delta_i^{-\prime}]'$, with δ_i^- defined as above, $\tilde{\beta}_{i,1} = \beta_i\gamma_i(\frac{1}{4} - \frac{1}{16}\gamma_i^2 c_j^2)$, $\tilde{\beta}_{i,2} = \frac{1}{16}\beta_i\gamma_i^3 c_j$, $\tilde{\beta}_{i,3} = -\frac{1}{48}\beta_i\gamma_i^3$ and $u_{it}^* = \beta_i R_{3i} + u_{it}$.

C.4 Additional results and further research

Table C.I
Bias and root mean squared error for β_1 with *unknown* γ and c

			$\xi=0$, $\phi_i=0$, $\theta=0$				$\xi=0.5$, $\phi_i=0.5$, $\theta=0.3$			
	$\beta_1=1$		T=100		T=500		T=100		T=500	
		γ	1.5	2.5	1.5	2.5	1.5	2.5	1.5	2.5
$\rho=0$	*Bias*									
	d=1		0.050	0.146	0.019	0.115	0.086	0.166	0.030	0.106
	d=2		0.152	0.238	0.056	0.139	0.159	0.233	0.053	0.126
	d=3		0.177	0.262	0.062	0.139	0.187	0.265	0.063	0.130
	d=4		0.193	0.280	0.070	0.147	0.207	0.288	0.068	0.135
	d=5		0.213	0.299	0.070	0.148	0.222	0.304	0.074	0.141
	d=6		0.226	0.312	0.073	0.146	0.244	0.324	0.072	0.137
	RMSE									
	d=1		0.686	0.725	0.304	0.335	0.836	0.876	0.334	0.356
	d=2		0.669	0.693	0.302	0.333	0.748	0.767	0.309	0.334
	d=3		0.655	0.679	0.304	0.329	0.735	0.756	0.314	0.334
	d=4		0.645	0.668	0.307	0.334	0.716	0.736	0.313	0.334
	d=5		0.636	0.661	0.305	0.332	0.703	0.723	0.318	0.337
	d=6		0.624	0.652	0.304	0.331	0.691	0.713	0.315	0.333
$\rho=0.5$	*Bias*									
	d=1		-	0.080	0.011	0.089	-	-	0.016	0.074
	d=2		0.202	0.278	0.073	0.147	0.218	0.279	0.070	0.132
	d=3		0.308	0.386	0.093	0.160	0.336	0.407	0.096	0.153
	d=4		0.404	0.477	0.125	0.193	0.435	0.504	0.120	0.177
	d=5		0.489	0.560	0.143	0.209	0.531	0.597	0.143	0.199
	d=6		0.569	0.635	0.163	0.223	0.617	0.676	0.165	0.219
	RMSE									
	d=1		-	0.717	0.305	0.330	-	-	0.337	0.352
	d=2		0.700	0.727	0.308	0.336	0.786	0.811	0.320	0.341
	d=3		0.705	0.740	0.317	0.343	0.784	0.811	0.326	0.347
	d=4		0.733	0.770	0.328	0.360	0.803	0.836	0.337	0.360
	d=5		0.776	0.819	0.336	0.369	0.837	0.873	0.350	0.374
	d=6		0.823	0.863	0.348	0.378	0.880	0.916	0.361	0.386
$\rho=0.8$	*Bias*									
	d=1		-0.025	0.038	0.004	0.070	-	-	0.004	0.046
	d=2		0.229	0.296	0.079	0.144	0.249	0.300	0.081	0.129
	d=3		0.389	0.456	0.111	0.174	0.426	0.485	0.114	0.163
	d=4		0.534	0.597	0.152	0.214	0.577	0.632	0.151	0.199
	d=5		0.661	0.723	0.184	0.244	0.715	0.769	0.185	0.227
	d=6		0.776	0.835	0.215	0.272	0.843	0.891	0.217	0.261
	RMSE									
	d=1		0.707	0.723	0.307	0.325	-	-	0.336	0.346
	d=2		0.713	0.739	0.310	0.337	0.806	0.829	0.328	0.345
	d=3		0.740	0.778	0.325	0.350	0.811	0.837	0.334	0.351
	d=4		0.805	0.843	0.341	0.373	0.868	0.897	0.352	0.373
	d=5		0.884	0.926	0.359	0.392	0.948	0.981	0.370	0.389
	d=6		0.968	1.011	0.377	0.412	1.039	1.071	0.390	0.412

Note: This table contains the results of the *bias* and of the *root mean squared error* (RMSE) for the contagion parameter β_1 with *unknown* location parameter c_1 and smoothness parameter γ_1. We focus on the first equation, but the second equation is symmetric. The true value of $\beta_1 = 1$. The DGP with $5,000$ simulations is as follows:

$$y_{it} = \delta_{0i} + \delta_{1i} z_t + \alpha_i x_{it} + \beta_i G(y_{jt}; \gamma_i, c_j) + u_{it}, \quad G(y_{jt}; \gamma_i, c_j) = \left\{1 + e^{\gamma_i (y_{jt} - c_j)}\right\}^{-1}, \quad \gamma_i > 0, \quad i, j = 1, 2, \quad i \neq j$$

$$z_t = \xi z_{t-1} + \nu_t, \quad x_{it} = \phi_i x_{it-1} + (1 - \phi_i^2)^{1/2} \varepsilon_{it}, \quad \varepsilon_{1t} = \theta \varepsilon_{2t} + \eta_t, \quad v_t, u_{it}, \varepsilon_{2t}, \eta_t \sim i.i.d. \mathcal{N}(0, 1).$$

Table C.II

Size of the test for simultaneity ($\beta_1 = 0$) with *unknown* γ and c – *first-order Taylor approximation*

		$\boldsymbol{\xi = 0}$, $\boldsymbol{\phi_i = 0}$, $\boldsymbol{\theta = 0}$						$\boldsymbol{\xi = 0.5}$, $\boldsymbol{\phi_i = 0.5}$, $\boldsymbol{\theta = 0.3}$					
β_2		T=100			T=500			T=100			T=500		
	γ	0.5	1.5	2.5	0.5	1.5	2.5	0.5	1.5	2.5	0.5	1.5	2.5
$\boldsymbol{\rho = 0}$													
0		0.039	0.045	0.051	0.046	0.046	0.053	0.050	0.048	0.043	0.049	0.040	0.050
0.3		0.047	0.045	0.049	0.052	0.051	0.043	0.045	0.045	0.042	0.045	0.050	0.054
0.5		0.047	0.044	0.041	0.058	0.051	0.046	0.040	0.045	0.048	0.048	0.045	0.051
1		0.051	0.047	0.048	0.055	0.052	0.051	0.042	0.046	0.043	0.046	0.039	0.055
1.5		0.048	0.046	0.053	0.051	0.047	0.055	0.040	0.049	0.058	0.050	0.052	0.041
$\boldsymbol{\rho = 0.5}$													
0		0.053	0.049	0.047	0.043	0.058	0.057	0.048	0.056	0.047	0.047	0.043	0.053
0.3		0.056	0.039	0.044	0.051	0.050	0.047	0.052	0.048	0.041	0.051	0.050	0.047
0.5		0.043	0.043	0.049	0.051	0.052	0.047	0.043	0.045	0.047	0.056	0.055	0.052
1		0.046	0.051	0.047	0.044	0.053	0.046	0.041	0.048	0.046	0.053	0.054	0.046
1.5		0.052	0.049	0.048	0.053	0.044	0.053	0.047	0.049	0.045	0.049	0.045	0.060
$\boldsymbol{\rho = 0.8}$													
0		0.054	0.049	0.047	0.053	0.053	0.049	0.047	0.044	0.038	0.053	0.055	0.046
0.3		0.036	0.057	0.048	0.054	0.049	0.049	0.045	0.051	0.046	0.052	0.056	0.042
0.5		0.053	0.060	0.056	0.037	0.055	0.041	0.046	0.050	0.051	0.048	0.046	0.043
1		0.045	0.047	0.043	0.049	0.052	0.056	0.050	0.056	0.047	0.050	0.048	0.045
1.5		0.053	0.045	0.061	0.043	0.052	0.055	0.049	0.056	0.058	0.047	0.052	0.048

Note: This table shows the *empirical size* of the test which tests the null hypothesis of no contagion ($H_0 : \beta_1 = 0$) for different values of β_2. We use standard testing procedures after approximating the logistic distribution by a *first-order* Taylor expansion around $\gamma_i = 0$. The auxiliary regression estimated by 2SLS is the following:

$$y_{i,t} = \delta_{0,i} + \delta_{1,i} z_t + \alpha_i x_{i,t} + \tilde{\beta}_i y_{j,t} + u^*_{i,t} \quad i = 1, 2 \quad i \neq j$$

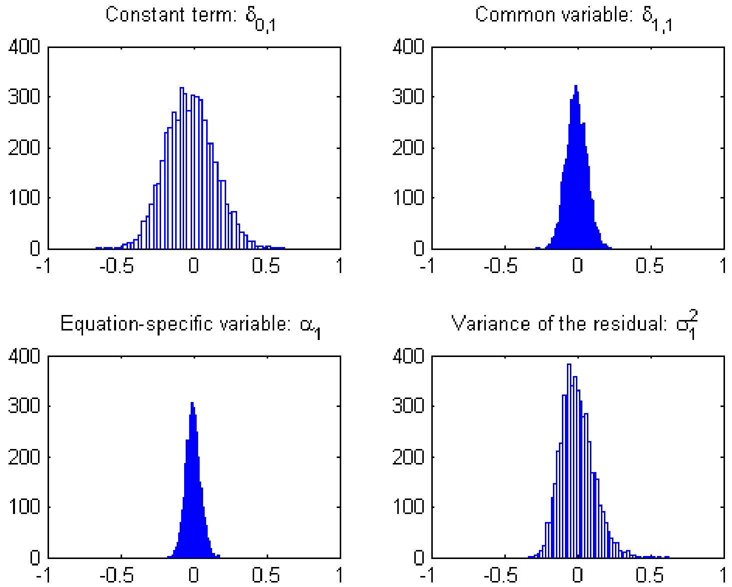
The null hypothesis becomes: $H_0 : \tilde{\beta}_i = 0$. For the data simulation, we use the following parameter combinations:

$$\delta_{0,i} = 1,\ \delta_{1,i} = 1,\ \alpha_i = 1,\ \beta_1 = 0,\ \beta_2 = [0\ 0.3\ 0.5\ 1\ 1.5],\ \gamma_i = [0.5\ 1.5\ 2.5],\ c_j = 1,\ \rho = [0\ 0.5\ 0.8],\ \phi_i = [0\ 0.5],\ \theta = [0\ 0.3].$$

We run 2,000 simulations.

Figure C.I

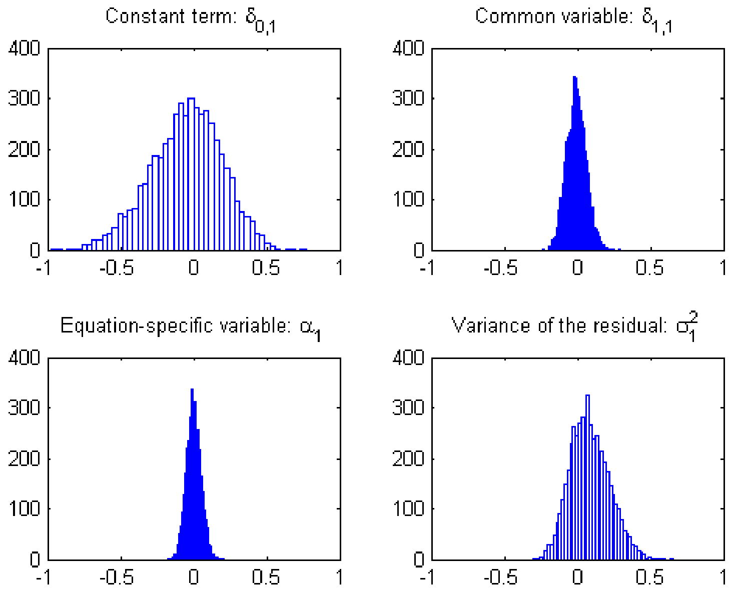
Distribution of the other parameters with *known* γ and c



Notes: This figure shows the distribution of the other parameters of the model, except for β_1 and c_2 obtained from simulations for the following parameter combinations: $\delta_{0,i} = \delta_{1,i} = \alpha_i = \beta_i = c_i = 1$, $\gamma_i = 2.5$, $\phi_i = \xi = 0.5$, $\theta = 0.3$, $\sigma_i^2 = 1$, $d = 3$, $T = 500$ and $5,000$ simulations. All the parameters are centered around 0.

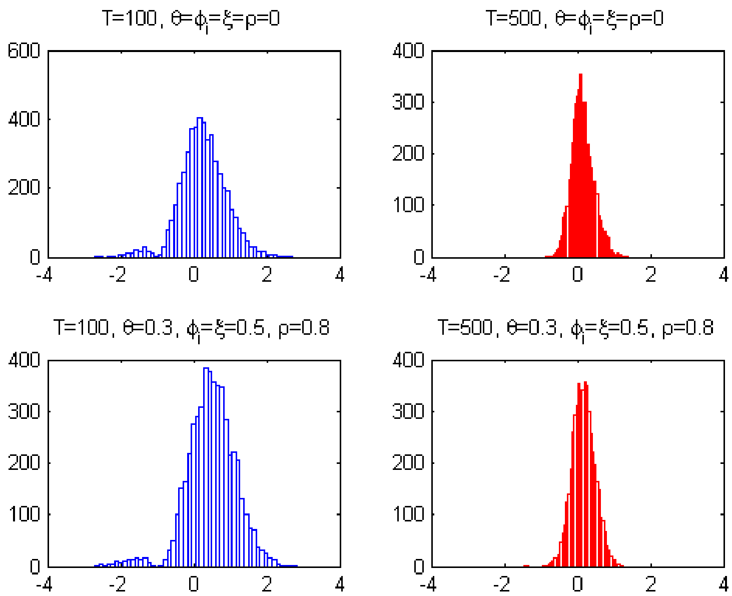
Figure C.II

Distribution of the other parameters with *known* γ and *unknown* c



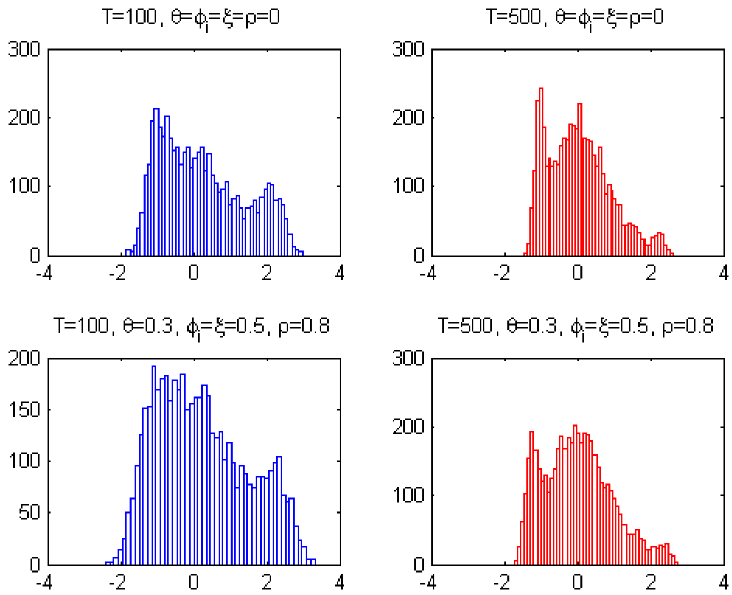
Notes: This figure shows the distribution of the other parameters of the model, except for β_1 and c_2 obtained from simulations for the following parameter combinations: $\delta_{0,i} = \delta_{1,i} = \alpha_i = \beta_i = c_i = 1$, $\gamma_i = 2.5$, $\phi_i = \xi = 0.5$, $\theta = 0.3$, $\sigma_i^2 = 1$, $d = 3$, $T = 500$ and $5,000$ simulations. All the parameters are centered around 0.

Figure C.III
Distribution of β_1 with *unknown* γ and c, $d = 3$



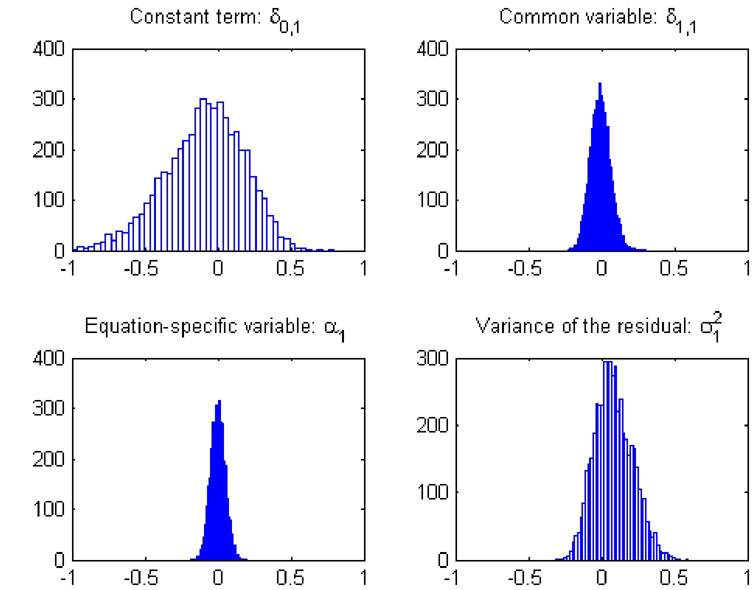
Notes: This figure shows the distribution of β_1 obtained from simulations for the following parameter combinations: $\beta_i = 1$, $d = 3$, $\gamma_i = 2.5$, and 5,000 simulations. The parameter is centered around 0.

Figure C.IV
Distribution of c_2 with *unknown* γ, $d = 3$



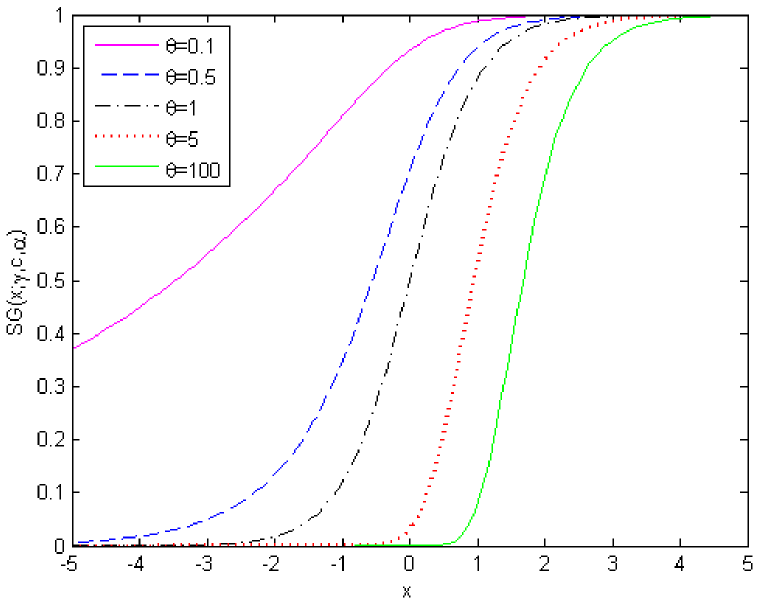
Notes: This figure shows the distribution of c_2 obtained from simulations for the following parameter combinations: $c_j = 1$, $\beta_i = 1$, $\gamma_i = 2.5$, $d = 3$, and 5,000 simulations. The parameter is centered around 0.

Figure C.V
Distribution of the other parameters with *unknown* γ and c



Notes: This figure shows the distribution of the other parameters of the model, except for β_1, γ_1 and c_2 obtained from simulations for the following parameter combinations: $\delta_{0,i} = \delta_{1,i} = \alpha_i = \beta_i = c_j = 1$, $\gamma_i = 2.5$, $\phi_i = \xi = 0.5$, $\theta = 0.3$, $\sigma_i^2 = 1$, $d = 3$, $T = 500$ and $5,000$ simulations. All the parameters are centered around 0.

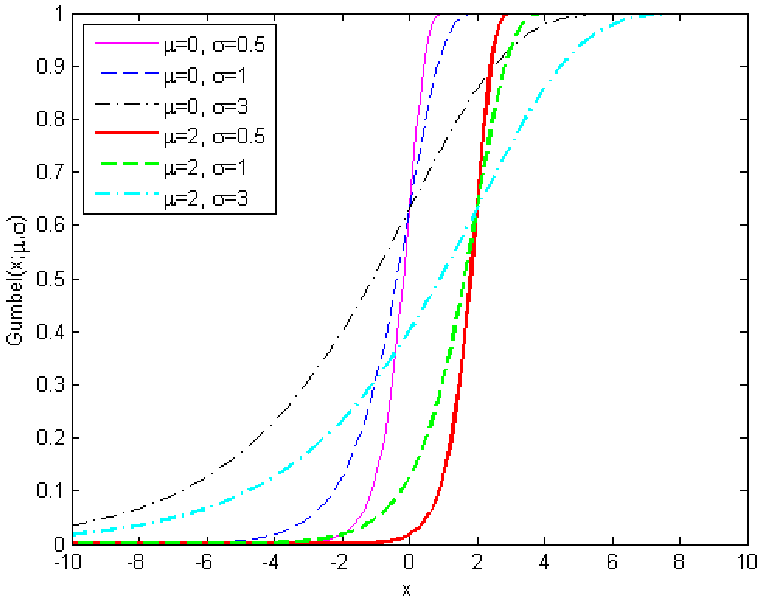
Figure C.VI
Type I generalized logistic distribution with different values of α



Notes: Skewed logistic c.d.f $\gamma = 2$, and $c = 0$: $G(x;\gamma,c) = \left(1 + e^{\gamma(x-c)}\right)^{-\theta}$.

Figure C.VII

Gumbel c.d.f. for different values of μ and σ



Notes: Gumbel distribution for different values of the scale and locations parameters: $G(x;\mu,\sigma) = \exp\left(-\exp(\frac{\mu-x}{\sigma})\right)$.

C.5 Empirical application

Table C.III

Numerical scale for the sovereign credit rating

Fitch ratings	S&P ratings	Numerical scale
AAA	AAA	25
AA+	AA+	24
AA	AA	23
AA-	AA-	22
A+	A+	21
A	A	20
A-	A-	19
BBB+	BBB+	18
BBB	BBB	17
BBB-	BBB-	16
BB+	BB+	15
BB	BB	14
BB-	BB-	13
B+	B+	12
B	B	11
B-	B-	10
CCC+	CCC+	9
CCC	CCC	8
CCC-	CCC-	7
CC	CC	6
C	-	5
RD	R	4
DDD	SD	3
DD	D	2
D		1

Notes: We convert to a numerical scale the rating categories of Fitch and Standard & Poor's. To the best rating AAA is attributed the value 25, while the worst rating D is attributed the value 1.

Figure C.VIII
Banking sector monthly returns for each country

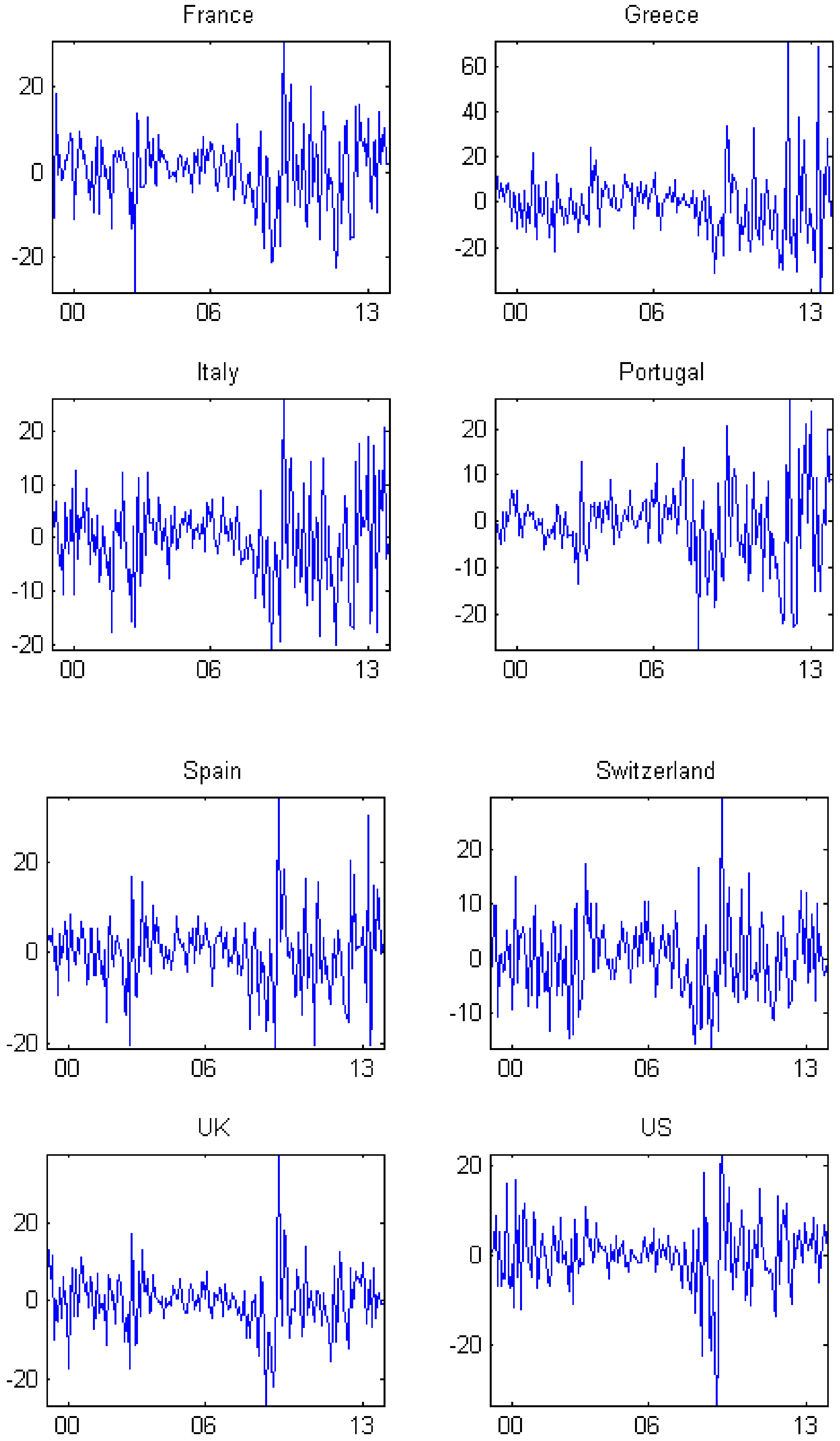
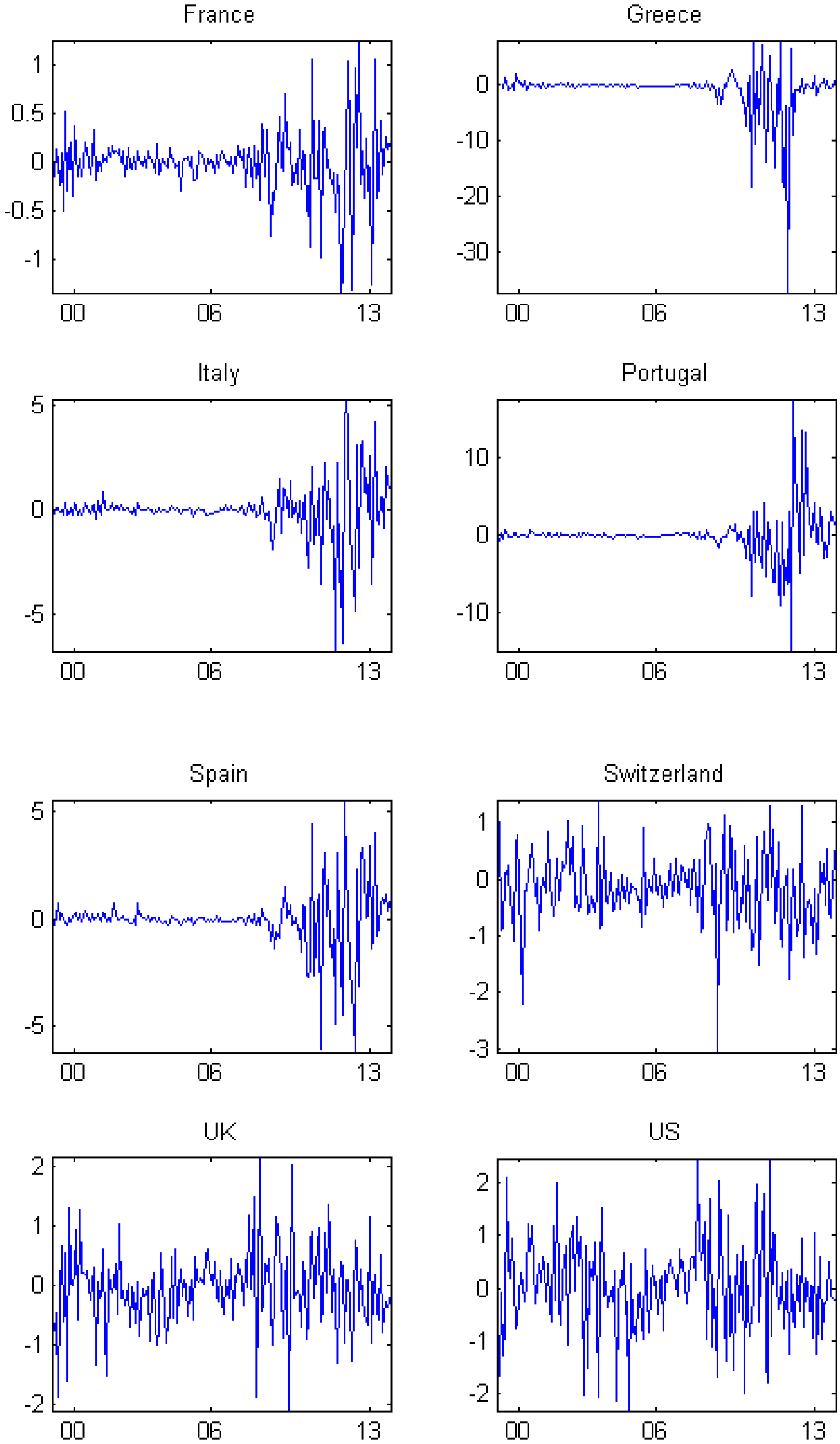
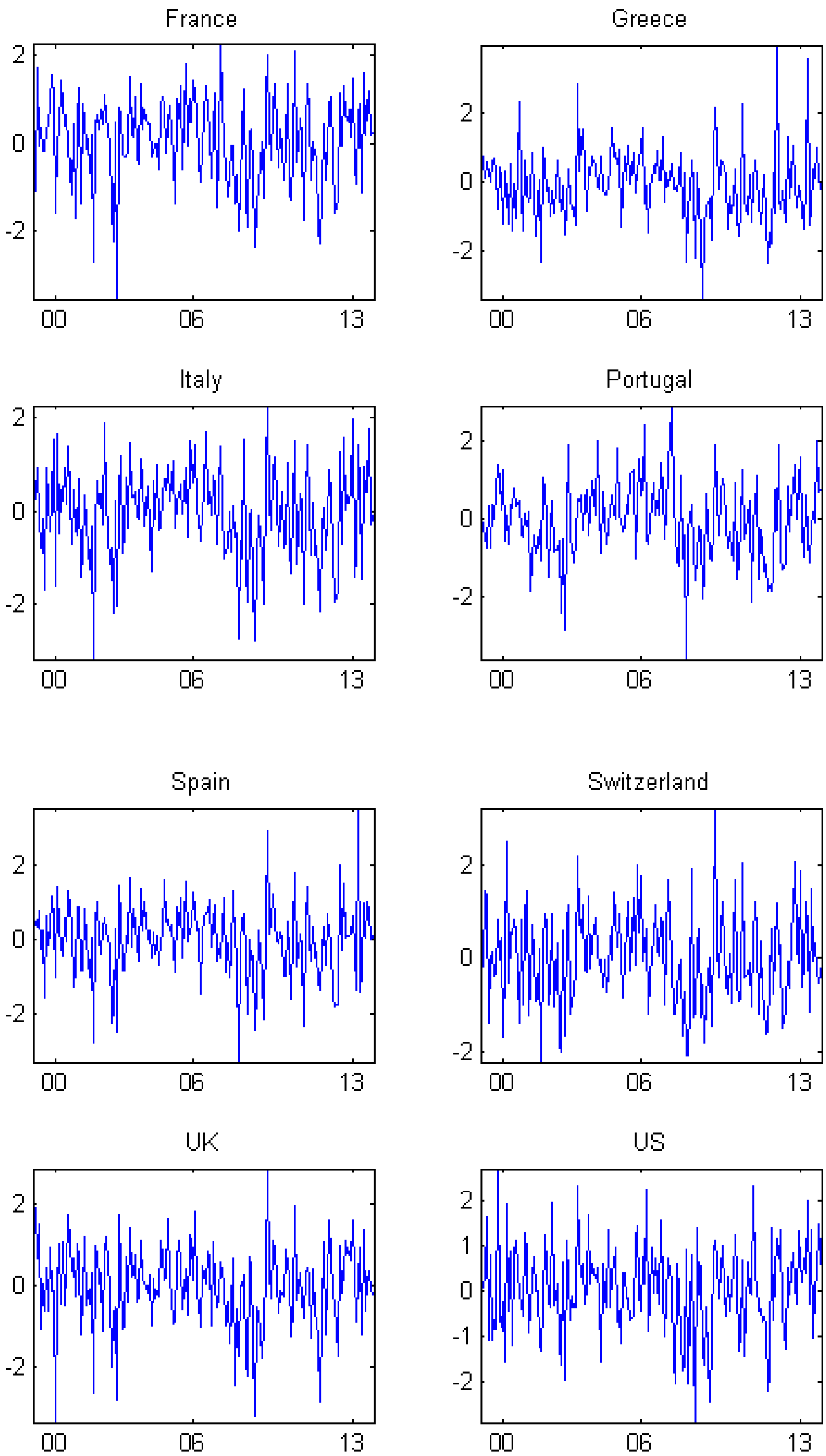


Figure C.IX
Sovereign sector monthly excess returns for each country



Notes: The monthly excess returns of the sovereign sector of each country are computed in deviation with respect to the German sovereign bond returns.

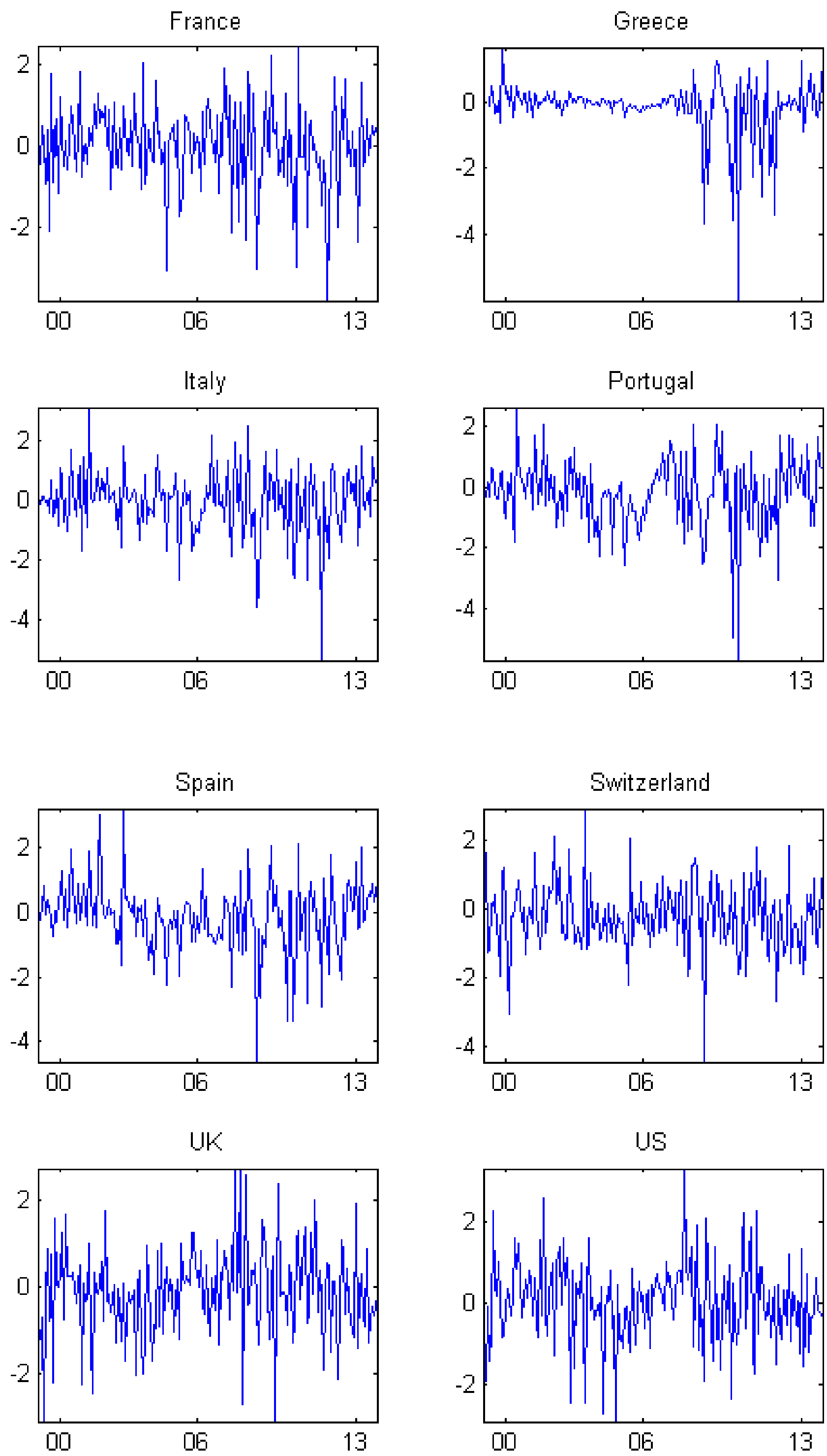
Figure C.X
Banking sector monthly returns for each country – GARCH(1,1)



Notes: The monthly bank returns are filtered by a GARCH(1,1) process.

Figure C.XI

Sovereign sector monthly excess returns for each country – GARCH(1,1)



Notes: The monthly excess returns of the sovereign bonds are filtered using a GARCH(1,1) process.

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