Evolution of stellar clusters and associations

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EVOLUTION OF STELLAR CLUSTERS AND ASSOCIATIONS

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1. Introduction

Star clusters are usually defined as agglomerations of stars in space, whose average projected densities appear to exceed the density of background stars. Cluster members are identified either by parallax measurements or by common motion or also with the help of the color-luminosity diagram established from known members.

The *globular* clusters are distributed within a quasi-spherical subsystem of our Galaxy and seem to follow mostly elongated orbits around the galactic center (von Hoerner, 1955). They number about 120 on the whole and fairly complete data are available for 70 of them (Haffner, 1965). The linear diameter of such a cluster is generally of the order of 30–50 pc and the total number of its member stars lies somewhere between 5×10^4 and 5×10^7; on account of the crowding effect, star counts become impossible in the central region and must be replaced by surface brightness measurements. The photographic image of a globular cluster is generally circular, although some slight ellipticities occur in certain cases. It has sometimes proved useful to divide the globular clusters into a dozen classes according to their apparent degree of star concentration.

Galactic or *open* clusters are smaller objects of various apparent forms varying from nearly circular to very irregular. Average figures of linear diameters and masses for 128 open clusters are 6 to 10 pc and 1500 solar masses.

In contrast to the globulars, open clusters are narrowly concentrated toward the galactic plane and this, together with color-luminosity diagrams, evolutionary tracks of individual stars, and the type of variable stars occurring in a cluster, leads us to place the globular clusters among the oldest population of our Galaxy and most of the open clusters among the disk population. In other words, whereas the age of a globular cluster exceeds 10 galactic years (or 2×10^9 years), the spread of ages for open clusters, according to a sample of two dozen of these objects (Haffner, 1965), extend from 10^6 to 5×10^9 years.

Stellar *associations* are groups of stars having definite physical types (generally O-type or T Tauri stars); their average star density exceeds the background density for stars of that precise type, but is well below the overall background stellar density. Therefore, such systems are loosely bound. The members of O-associations are young bright stars which often disclose motions corresponding to a linear expansion of the system.

Stellar associations belong to the younger disk population. Their age is generally less than 10^7 years and, together with young galactic clusters and H II regions, they are useful in delineating the structure of spiral arms.

L. N. Mavridis (ed.), Structure and Evolution of the Galaxy, 250–261.

We shall not go further here into the problems of spatial distribution and external motions of clusters and associations, but rather turn to what is known about their structure and dynamical evolution.

2. Description of a Star Cluster; Relaxation Time and Mean Free Path

In considering a stellar cluster, we are essentially faced with the n-body problem of classical non-relativistic mechanics, and since we now have at our disposal fast and powerful computing machines, one line of attack will certainly pass through the numerical integration of the $6n$ first order equations describing the motion of anyone of the n members of the system in the resultant gravitational field due to all the other members. On the other hand, a star cluster can also be visualized as a mass of 'stellar gas' held together by its own gravitation and this would suggest taking advantage of gas kinetic considerations. We shall adopt this second point of view first, not so much on account of previous historical developments as because it demonstrates more readily several useful physical concepts.

The equilibrium state of a gas in a box is characterized by a definite pressure, due to the repeated impacts of molecules on the surrounding walls of the box and the inner interactions of the gas essentially consisting of collisions between the molecules. How far can we extend this classical gas picture to the 'gas of stars' making up a cluster? Confinement through inner gravitation forces is indeed very different to confining a gas by outer solid walls and we must estimate at first the importance of collisions or encounters within a self-gravitating system.

In an ordinary neutral gas, the particles (molecules or atoms) interact through short-range forces so that we may unambiguously define a mean collision time; such is not the case in a plasma or in a self-gravitating system which are both dominated by long-range interaction forces. If we define a smoothed-out gravitational potential at every point of the system and at every instant, we may try to describe the motion of a particular or test-star among the other members or field-stars of the cluster producing the smoothed-out potential; we shall thus obtain, from given initial conditions, a well determined 'regular' trajectory. But the smoothed-out potential does not take the granular structure of the system into account and when the test-star passes very near to a field star, it undergoes a binary encounter producing a deviation from the regular trajectory. We then call 'relaxation time' T_{rel} the time after which the position and velocity of the test-star are no longer correlated with the initial position and velocity.

To make this definition more precise, we may say that time T_{rel} for a given star has elapsed when the cumulated fluctuations of energy becomes comparable to the star's initial energy or alternately when the sum of squared deviations is of the order of unity. A crude argument shows that, in a system having stars of equal masses m, the relaxation time for a star of velocity v is proportional to v^3/nm^2 and this statement is only little modified in the more sophisticated calculations of T_{rel} (Chandrasekhar, 1942). Let us only recall here that the treatment is based exclusively on binary collision cross-sections; consequently, we meet the difficulty of having to cut off the impact parameters at a certain critical value which is usually chosen, somewhat arbitrarily, as the mean interstellar distance within the cluster. In a plasma, a natural cut-off is afforded by the Debye length.

Let us now denote by A the age of the system; then the ratio

$$\eta = T_{\mathrm{rel}}/A$$

characterizes the relative importance of encounters. Wherever we have $\eta \ll 1$, the system is relaxed in the sense that it is dominated by the effect of stellar encounters; such is the case in the central region of globular clusters. If on the contrary $\eta \gg 1$, as in the outer part of any cluster and within elliptical galaxies, relaxation plays no role because the encounter effect has not yet influenced the regular trajectories in any appreciable amount.

A mean free path λ can formally be associated to the time T_{rel} in multiplying the latter, averaged over the cluster by the mean velocity of the test star. According to former considerations, λ is approximately proportional to the square of the star's kinetic energy and for stars of high energy, λ surpasses by far the dimensions of the system.

Therefore the fluid model of a gas, given in terms of pressure, is inadequate for star clusters and a description nearer to free particle physics, based on the Boltzmann equation, should be adopted.

3. Basic Equations

Denoting by $f_m(\mathbf{x}, \mathbf{v}, t)$ the distribution function of the position, velocity at time t of the stars of mass m and by $\phi(\mathbf{x}, t)$ the smoothed-out gravitational potential, we shall write the Boltzmann equation for these stars

$$\frac{\partial f_m}{\partial t} + \mathbf{v} \cdot \frac{\partial f_m}{\partial \mathbf{x}} - \frac{\partial \phi}{\partial \mathbf{x}} \cdot \frac{\partial f_m}{\partial \mathbf{v}} = \sum_{m'} \left(\frac{\partial_c f_m}{\partial t} \right)_{m'} . \tag{1}$$

The right hand side represents the variation per unit time of f_m, due to encounters of stars of mass m with stars of mass m' in the cluster. If the cluster is unrelaxed, as it could be in an early evolutionary phase, the right hand side of (1) vanishes; otherwise we will replace it by the Fokker-Planck expression

$$\left(\frac{\partial_c f_m}{\partial t} \right)_{m'} = - \frac{\partial}{\partial \mathbf{v}} \cdot (f_m \langle \Delta \mathbf{v} \rangle_{m'}) + \frac{1}{2} \frac{\partial}{\partial \mathbf{v}} \frac{\partial}{\partial \mathbf{v}} (f_m \langle \Delta \mathbf{v} \, \Delta \mathbf{v} \rangle_{m'}), \tag{2}$$

where $\langle \Delta \mathbf{v} \rangle_{m'}$ is the average velocity change per unit time of a star of mass m after collisions with stars of mass m'. Equation (2) is connected to the fact that for long-range forces such as gravitational attraction, the cumulative effect of distant encounter outweighs the influence of the very rare close encounters. The process is therefore akin to brownian movement.

Now the effect of collisions is twofold: there will be a mixing of the individual energies and angular momenta and some of the most energetic stars may escape from the cluster. Both effects tend to alter the structure of the system and cause its dynamical evolution.

For an isolated cluster, Φ is derived from the Poisson equation

$$\nabla^2 \Phi = 4\pi G \sum_m m \int f_m \, \mathrm{d}^3 v \tag{3}$$

considering, for simplicity, a discrete mass spectrum.

The integrodifferential system of Equations (1), (2), (3), nonlinear in f_m even without collisions, is too formidable to be solved in general. On the other hand, the mechanism of a quasi-continuous piling-up of energy for causing a star to escape, such as described by (2), does not work for an isolated cluster (Hénon, 1960), since a star having an energy near to that of escape spends most of its time in the outer region of the cluster and thus experiences no encounters. For the problem to bear a more definite meaning, we should include here the tidal force of the Galaxy; the simplest but rather unsatisfactory way to do so is to introduce a radius of stability r_s defining the point where the gravitational pulls of the cluster and the Galaxy balance each other. Any star which has sufficient energy to reach and go beyond the distance r_s from the cluster's center is lost to the cluster.

Let an open cluster of center C describe a circular orbit in the galactic plane, around the galactic center O; let a cluster star S lie on the line OC, at a distance r from C and with $OC=R$, M and M_g being the respective masses of the cluster and of the Galaxy reduced to a central mass-point. We can express the balance of forces in the frame of reference rotating around O with the angular velocity ω at point C:

$$-G\frac{M_g}{(R-r)^2} + G\frac{M}{r^2} + \omega^2 (R-r) = 0$$

and since

$$\omega^2 = G(M_g/R^3)$$

then

$$r_s = (M/3M_g)^{1/3}\,. \tag{4}$$

For globular clusters, we may consider an elliptic orbit of eccentricity e; the factor 3 in the denominator should then be multiplied by $1+e$ (King, 1962).

In several preliminary investigations a given distribution function f has been assumed, followed by an attempt to solve (3) in order to obtain the corresponding dynamical model. Conversely, assuming a given potential, one could attempt to derive f from (1). Most of the references to papers of that kind can be found in R. Michie's review of cluster dynamics (1964).

The problem is somewhat simplified by the assumption of spherical symmetry, which is justified for globulars and many open clusters. Among other simplifying assumptions, one often considers stars of equal masses for all the cluster, which is convenient but not very realistic as regards energy exchange and evaporation. One also assumes frequently an isotropic velocity distribution, which appears fairly adequate in the central part of a cluster, but much less in the outer part. When this last hypothesis is made, the very complicated expression (2) can then be converted into a relatively tractable form, which has been used by several authors.

4. Dynamical Cluster Models

Starting with a spherical cluster of equal masses and isotropic velocity distribution, Hénon (1961) computed the structure determined by the basic Equations (1) and (3), as well as the evolution of the cluster assumed to take place while keeping the model similar to itself (*modèle homologique*). During evolution, the central density immediately appeared to increase rapidly, producing a sharp cusp which compelled Hénon to start with an infinite central density. Further, the central region absorbed negative energy, which was accounted for by close binary star formation. The external radius increases while the total mass decreases linearly with time, at a rate of order 2 solar masses per million years. The density profile, when projected on the sky, agrees in a satisfactory way with that of the globular cluster 47 Tuc.

The same kind of agreement is claimed by Michie (1961), who started by adopting a distribution function of the form

$$f(E, J) = A \exp(-\alpha E - \beta J^2)\, Q(E) \tag{5}$$

in terms of stellar energy E and angular momentum J, A, α, β are constant parameters and we notice that (5) becomes a Maxwellian distribution at the center of the cluster, exhibiting a depopulation of stars possessing a high angular momentum and including a cut-off factor having its major effect at large distances from the center. Since account is taken of the velocity anisotropy, the expression (2) becomes very complicated and in order to avoid solving the Boltzmann Equation (1), Michie seeks to describe the cluster's evolution by a sequence of equilibrium states all characterized by the invariant form (5). The changes in the three parameters A, α, β are determined in first approximation by taking successive moments of (1). The evolution thus obtained reveals, like Hénon's model, a contraction of the central core and expansion of the outer region. The velocity anisotropy increases from the center outwards, in qualitative agreement with a former study of von Hoerner (1957) who regarded the outer zone of a globular cluster as populated by stars thrown out from a relaxed core, into orbits of high E and low J.

Neglecting velocity anisotropy, King (1966) computed a whole set of models using as distribution function the approximate steady-state solution (Michie, 1963a) of Equations (1), (2) which reduces to a Maxwellian distribution minus a constant,

$$f(r, v) = f_0 \left[\exp(-j^2 v^2) - \exp(-j^2 v_e^2)\right], \tag{6}$$

where j is constant, $v_e = -2\Phi(r)$, $\Phi(r_s) = 0$. In other words, the escape velocity v_e at distance r from the center allows any star to reach the radius of stability r_s given by (4). From (6) one calculates the mass density $\varrho(r)$ which reads

$$\varrho(r) = 4\pi m \int f(r, v)\, v^2\, \mathrm{d}v$$

for a cluster of equal stellar masses m, whence the projected density which is the quantity suited for comparison with observations.

Close examination of Palomar-Schmidt plates enabled King (1962) to follow out the projected density far enough to define a limiting radius r_t which appeared to agree satisfactorily with (4). The projected density of many globular clusters is well represented by the empirical formula

$$\varrho_p(r) = k\left\{\left(1 + \frac{r^2}{r_c^2}\right)^{-1/2} - \left(1 + \frac{r_t^2}{r_c^2}\right)^{-1/2}\right\}^2, \tag{7}$$

in which k is a number factor, r_c the 'core' radius and r_t the limiting radius, three parameters corresponding to the three quantities which fix the quasistationary state of the cluster, viz. its total mass, total energy and the tidal field.

Within the core, relaxation should be attained or at least well under way, while the outer layers are mostly under tidal influence. When $r \ll r_t$, expression (7) reduces to its first term which describes correctly the central region of the globular cluster 47 Tuc up to 5′ from the center (Gascoigne and Burr, 1956) and also the main part of the open cluster Praesepe up to 60′ from its center (Bouvier, 1961). With suitable values of k and r_c, we may represent the projected density in globular clusters of high or low concentration ratio as well as the distribution of both bright and faint stars in a given cluster.

The basic structural properties of clusters, i.e. the core and the outer halo or corona, have been lately discussed by Kholopov (1969) and in particular, for the Pleiades open cluster, Artyukhina (1969) has shown that the core extends to 1° from the center and the corona to 3°.25

All globular clusters in our Galaxy disclose a very similar structure which probably illustrates a trend toward a common final state; the regularizing mechanism seems to reside in the stellar encounters within the central core. Furthermore, according to their light distribution, elliptical galaxies also possess projected densities which can be made to fit models based on (6) and described by (7) with adequate k, r_c. This similarity in aspect cannot be induced here by collisions, because the mean relaxation time of ellipticals is larger than their age ($\eta \gg 1$), so the regularizing mechanism is to be found in the mixing of orbits of stars having different periods and moving in a time-dependent potential. Such a process must have occurred very quickly, in a time of the order of the mean period P of a star in the system, whereas the slow relaxation due to star-star encounters has a time scale $T_{\rm rel}$ much larger than P, by a factor proportional to $n/\log n$ where n is the total number of stars in the system. Consequently, the initial phase of dynamical evolution of a stellar system is entirely dominated by the orbital mixing. Moreover, the interaction of a star with the changing potential does not depend on the star's mass, in contrast to the encounter effect which leads to equipartition and therefore to mass segregation inside the system.

When encounters can be neglected, the basic dynamical problem is to obtain a stationary solution of Equations (1) and (3), for a vanishing right hand side of (1). Such a solution is a function of the integrals of motion E and J, i.e. the total energy and angular momentum, both per unit mass, and it could in principle be deduced from the observed star density. Assuming all stars to have equal masses, we would first transform the observed projected density into the space density

$$\varrho(r) = \frac{4\pi}{r} m \int_{\Phi}^{0} \mathrm{d}E \int_{0}^{J_{\max}} \frac{J f(E, J)}{\sqrt{J_m^2 - J^2}} \mathrm{d}J \tag{8}$$

where $J_{\max}^2 = 2r^2(E - \Phi)$, and then solve this double integral equation for $f(E, J)$. This problem has been dealt with independently by Veltmann (1961) and Bouvier (1962, 1966), who developed $f(E, J)$ in a power series in J^2, retaining a finite number of terms. They thus gained some insight into the various possible types of velocity anisotropy.

Now anisotropy entails a depletion of the higher angular momenta and therefore pulls down the density of stars in the outer regions of a cluster; the tidal part of the drop-off is correspondingly less severe and a larger tidal radius r_t will be inferred. In fact, r_t is too badly known to help us determine the degree of anisotropy; one may fit star counts with a model based on velocity isotropy but the same counts could also be fitted to models of rather marked anisotropy and with a larger tidal radius.

Insofar as we consider clusters of equal stellar masses, the rate of escape remains comparatively very small; the selective evaporation rate estimated by several authors under many simplifying assumptions, had shown a monotonous increase with decreasing mass of the escaping stars. As a consequence, the fainter end of the initial luminosity function will be depopulated, while the bright end also becomes depleted on account of the rapid evolution of the more massive stars. But, apart from the uncertainties in the calculated escape rates, comparison with observations is hampered by the probable lack of a universal initial luminosity function for star clusters (Michie, 1963b; Martinet 1966).

The essential cause of escape lies in energy gained by a star in close encounters with other cluster stars; a correct calculation must start with the expression giving the probability that a certain star suffers a close encounter at time t, which will change its velocity by a finite amount Δv. Following this method, Hénon calculated the rate of escape from an isolated cluster, first with equal masses (1965) then with an arbitrary mass distribution (1969) and adopting an initial isotropic velocity distribution corresponding to the polytropic model of index 5, in order to simplify the computation. The rate of escape is quite sensitive to the mass spectrum; it is multiplied by a factor of 30 or more when equal masses are replaced by unequal ones. This result had already been obtained in numerical experiments (Wielen, 1968) and we shall now turn to this alternative approach to cluster dynamics.

5. Numerical Experiments

The great advantage of the method of numerical experiments, initiated for stellar dynamics by von Hoerner (1960), resides in its being free from starting assumptions.

Given arbitrary initial conditions, we just have to solve numerically the $6n$ first order equations of motion pertaining to the n point-stars building the cluster: if x_{ik}, v_{ik} denote the respective position and velocity coordinates ($i=1, 2,\ldots, n$; $k=1, 2, 3$) we then write

$$\mathrm{d}x_{ik}/\mathrm{d}t = v_{ik}\,, \tag{9}$$

$$\frac{\mathrm{d}v_{ik}}{\mathrm{d}t} = -\,G \sum_{j+i} \frac{m_j}{\sum_k (x_{ik} - x_{jk})^2}\,. \tag{10}$$

The expression in the right hand side of (10) consumes most of the computing time and the divergence occurring when $x_{ik}\to x_{jk}$ is in itself a problem. Even with the help of the fast computers now at our disposal, it becomes difficult, on account of prohibitive computing time and limited memory capacities, to deal with a very large number of stars; the method is therefore limited to multiple stars and small clusters. Von Hoerner had begun with 16, then 25 stars, van Albada worked on a few dozen (1967, 1968), Wielen 100 stars (1968), and Aarseth (1968) managed to cope with the 250-body problem, which represents an upper limit at the present time.

Numerical integrations show that the cluster exhibits a central density cusp after a finite time, as well as an extended halo. This agrees with results obtained by models described in Section 4 as well as with a recent study of von Hoerner (1968) on the high central densities in stellar systems, assuming local virial equilibrium and restrictive properties of energy exchange. Later on, the cluster seems to approach some sort of final state, but the numerical experiments performed with different initial conditions give no precise information as to whether the overall properties tend to become increasingly similar or not (Wielen, 1967).

In such computations, it is advisable to choose, for each star, a variable integration time-step.

Now errors tend to accumulate so that one is never completely assured to find again the initial conditions when time is reversed (Miller, 1964); the accuracy with which the constants of motion do indeed remain constant is thus to be as high as possible.

A comparative study of eleven numerical integrations of the same gravitational 25-body problem showed good agreement of the runs during the initial phase, but when encounters became important, the experiments could no longer be reproduced in detail (Lecar, 1968).

The initial evolutionary phase, dominated by the dynamical mixing of orbits, can also be studied by means of numerical experiments (Hénon, 1964). As stated before, the analytical problem would consist in trying to solve the simultaneous Equations (1) without right hand side and (3), which is a hopeless task in general. But we can avoid solving (1) in the very peculiar case where the initial distribution function $f(\mathbf{x}, \mathbf{v}, 0)$ is equal to a constant positive value C inside and vanishes outside a finite phase domain D_0.

We still have $f=C$ in the domain D_t occupied by the system at any later time and $f=0$ outside D_t (Liouville's theorem of statistical mechanics).

But the state calculated by merely expressing the conservation of the fine-grained phase density f has no physical reality; we may call it a virtual final state. Indeed, as evolution proceeds, the phase domain D_t develops a filamentary structure and the filaments become always longer and thinner so that f finally loses its meaning. The real final state of orbital mixing has to be described in terms of a coarse-grained phase density $\bar{f}$, which is the average of f over cells of finite size. The virtual final state, which in case of a spherical system corresponds to a polytropic model of index 1.5, may nevertheless be useful as a reference state (Bouvier and Janin, 1970a).

6. Complementary Remarks on Stellar Clusters

The evaporation of stars from a cluster, which is certainly favoured by the tidal action of the Galaxy (Wielen, 1968; Hayli, 1970), entails a loss of mass and of energy for the cluster. On the other hand, the cumulative influence of passing clouds of interstellar matter, which is undoubtedly of importance for open clusters since they remain in the disk region, brings over an increase of energy and a tendency of these clusters to expand and gradually disrupt. The effect was examined some time ago by Spitzer (1958) and could partly explain the relatively low occurrence of very old open clusters.

Recently, Bouvier and Janin (1970b) performed some numerical experiments on the disruption of a small stellar cluster by passing interstellar clouds; the effect is sensitive to the initial state of the cluster and a more complete treatment of the problem should include both the continuous action of the galactic field and the stochastic influence of the clouds.

For the galactic clusters, it has proved strikingly illustrative to construct a composite color-luminosity diagram by fitting the respective main sequences to the so-called 'zero age main sequence', which cannot however be defined in a completely unambiguous manner since the physical conditions prevailing at the birth of the cluster, in particular the chemical composition of the protocluster gas cloud, should be different from one cluster to another.

The theory of stellar evolution tells us that, in general, a main-sequence star after having exhausted its hydrogen fuel within the central core, evolves to the right of the main sequence toward the red giant state. The larger the star's mass, the earlier this happens, so we expect the upper main sequence of any cluster diagram to become depleted in course of time. The present age determination will then follow from the absolute magnitude at the turn-off point for the particular cluster considered.

But such a determination is most sensitive to the metal content and cannot claim to be accurate to better than a few billion years (Iben and Rood, 1970).

The oldest open clusters, like M67 and NGC 188, having respectively around five and ten billion years, lack all their upper main sequence but possess well-developed giant branches; their color-luminosity diagram resembles that of globular clusters.

For the latter objects, we have no direct observational evidence about their ages, but they contain no observable interstellar gas and are highly stable and therefore long-lived stellar systems. According to their space distribution and kinematic properties, they belong to the oldest of the stellar populations and their age may go up to 20 billion years. Stellar evolution may account for the red color of the brightest stars within globular clusters of our Galaxy.

In comparing color-luminosity diagrams with theoretical evolutionary tracks, one implicitly assumes that all the stars of the same cluster are coeval; this is justified only if the age spread is noticeably less than the average age of all the members. Such is the case for the great majority of clusters except for the younger one where a large spread of ages is present and where the rate of star formation, still under way, seems even to increase with time (Iben and Talbot, 1966).

We had noticed earlier (Section 4) the similar appearance revealed by all globular clusters; this does not prevent them from differing widely in chemical composition. Some, like M15, M5 or M92 are made of anomalously metal-poor stars while others, as NGC 6356 or 47 Tuc hardly differ in metal content from typical rather young open clusters such as Hyades and Pleiades.

The metal content Z of a globular cluster appears to be generally correlated to the structure of its horizontal branch (Faulkner, 1966) but not always in the same way (Sandage and Wildey, 1967; Rood and Iben, 1968). One more parameter at least, in addition to Z, must vary from one cluster to another; according to Castellani *et al.* (1970), globular clusters form a two-parametric family of objects.

Moreover, globular clusters have been observed in extragalactic systems like M31 (Andromeda) and the Magellanic Clouds, where two types of globular clusters occur, the red and the blue. We first have the same kind of globular clusters as those of our Galaxy, but with ages which do not seem to exceed 5×10^9 years and which correspond to the youngest of the globular clusters of the Galaxy (Bok, 1969); some other globular clusters form a sub-group of intermediate-age clusters, and are only 10^9 years old. Secondly, a number of globular clusters in the Large Magellanic Cloud appear much bluer in integrated light than those of the Galaxy and their color-luminosity array, after having been matched to the Hyades main sequence with allowance for differences in chemical composition and evolutionary effects, resemble those of very young galactic clusters such as h and χ Persei. Thus, the birth of globular clusters in the Magellanic Clouds presumably took place considerably later than in our Galaxy.

These facts, together with the indications of the theory of stellar evolution, led Kholopov (1966) to postulate that shortly after cluster formation, all color-luminosity diagrams of globular clusters were similar to those of young open clusters today and only with evolution did they develop into the presently observed diagrams, typical of the globular clusters in our Galaxy. In contrast to the Magellanic Clouds, clusters formed today in the Galaxy will no longer be globular.

7. Stellar Associations

The O-associations are spherical and since, as mentioned in the introduction, their densities are well below the necessary limit for stability against tidal disruption, Ambarzumian (1955) argues that the O-associations are expanding systems of positive total energy. The expansion motions have been effectively found by Blaauw (1964) and others; the total positive energy is to be understood after deducting the negative binding energy of the numerous multiple stars generally present in associations. According to Ambarzumian, associations contain a nucleus formed by one or more open clusters and in case they are very young, multiple stars of Trapezium type and sometimes star chains. If linear expansion prevails, which means that the member stars have left a common center of formation simultaneously, each of them with a particular velocity remaining constant in time, and if r is the distance gone through from the center during time t, by a star of velocity v, constant with respect to matter remaining at rest in the medium of formation, then the ratio v/r, called the rate of expansion, is characteristic for the association and is equal to the inverse of the time elapsed since the stars left their origin, i.e. of the kinematical age of the system. O-associations are generally found because O stars occur nearly always in groups in the Galaxy and in extragalactic systems, as for instance in the central section of the Larger Magellanic Cloud.

The numbers of stars of an O-association may be of the order of several hundreds and the diameters of the groups range from 30 to 200 pc (Haffner, 1965). With increasing size however, the boundaries become increasingly vague because the association gradually merges into the general stellar field; furthermore several of the larger associations appear to be divided into subgroups with different evolutionary stages. The properties of O-associations have been discussed by Blaauw in his (1964) review of this topic. As a by-product of O-associations, we shall note here the so-called 'runaway' stars; they possess velocities of up to 200 km/sec and their mass distribution differs from that of ordinary stars by a high relative frequency of large masses. Their origin, for the few known cases, in the O-associations is well established. Blaauw (1961) proposed to account for the process causing the runaway stars, which are always single, by assuming that they were former companions of massive stars which underwent rapid disintegration. Another cause of runaway stars could also reside in the dynamical behavior of collapsing stellar clusters (Poveda *et al.*, 1967) or perhaps also in the instability of multiple systems (Worrall, 1967).

T-associations are likewise groups of T Tauri stars, irregular variables with emission lines; such associations, which are also unstable aggregates of an age less than 10^7 years, are connected with strong clouds of obscuring matter and sometimes with bright galactic nebulae (Herbig, 1962).

The study of associations leads to the idea that all the massive stars of flat subsystems of the Galaxy, as well as the T Tauri stars, are born in groups; this cannot be ascertained for every class of stars; data are too sparse for small masses. Further, the existence of subgroups within many associations indicates that star formation develops in irregular fashion through complexes of interstellar clouds. We have emphasized the youth of associations; let it be said that after 10^7 years or so, O stars will have evolved and about simultaneously, the O-association will have dispersed in the general stellar field.

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