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ON THE BROAD BAND TRANSMITTANCE OF DIRECT IRRADIANCE IN A CLOUDLESS SKY AND ITS APPLICATION TO THE PARAMETERIZATION OF ATMOSPHERIC TURBIDITY

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Abstract—The total atmospheric attenuation of direct irradiance is often expressed as the product of independent broad band transmittances pertaining to the different depleting materials in the atmosphere. In theory, it is clear however that the broad band transmittances of the two most important attenuating factors, aerosols and water vapour, cannot be considered independent of each other. This is most relevant when trying to isolate the effect of atmospheric aerosols from the measurement of broad band direct irradiance. These questions are addressed in an analytical approach. Several broad band direct transmittance equations and corresponding turbidity parameters are discussed with respect to the analytical results. Simulations made with the SMARTS2 spectral radiative transfer code are used to illustrate the broad band effects. Copyright © 1996 Elsevier Science Ltd.

1. INTRODUCTION

The higher availability of broad band measurements as opposed to spectral measurements of direct irradiance has motivated the development of numerous parametric broad band atmospheric transmittance models. It is shown here that the principal attenuators of direct irradiance in a cloudless sky and average conditions are, in order of importance, aerosols, water vapour and molecular scattering. Since atmospheric aerosol and water vapour contents are also highly variable in space and time, it is the way in which their effects can be estimated which will largely determine the precision of the considered models.

The possibility of estimating the atmosphere's aerosol content from the measurement of broad band direct irradiance is also highly prized. This is a very delicate task, because of the fact that depletion of direct solar irradiance by aerosol scattering and absorption shows a high degree of spectral dependence. Broad band turbidity indices are not a new theme, but the challenge is now to relate quantitatively broad band and spectral indices, as witnessed by the articles of Pinazo *et al.* (1995) and Grenier *et al.* (1994).

In this article we start with a study of the relations between broad band and spectral transmittance equations. We show that, in theory, the effects of different atmospheric attenuating factors are not independent in a broad band equation as is assumed in the spectral equation. The fact that the assumption of independent broad band transmittances is often presented in standard texts (e.g. Iqbal, 1983) as a practical and widely used approximation has largely motivated us to publish this discussion. In Section 2 it is shown that this assumption is by no means necessary, and a more thorough approach is proposed. In Section 3 an interpretation of our analytical results is made with the aim of lifting the veil from a number of questions pertaining to the parametric modelling of broad band direct irradiance. In Section 4, the implications of the previous results are discussed with respect to Linke's and Unsworth Monteith's broad band turbidity coefficients.

2. FROM SPECTRAL TO BROAD BAND TRANSMITTANCE EQUATIONS

The spectral attenuation of a monochromatic beam passing through the atmosphere and depleted by n attenuating factors can be written:

$$I_\lambda = I_{0\lambda} \prod_{i=1}^{n} \tau_{i\lambda} \tag{1}$$

where I_λ and $I_{0\lambda}$ are the attenuated and extraterrestrial normal beam irradiances of wavelength λ. The spectral transmittance for factor i, $\tau_{i\lambda}$, is given by Bouguer's law:

$$\tau_{i\lambda} = \exp(-m_i \delta_{i\lambda}) \tag{2}$$

where $\delta_{i\lambda}$ is the total (vertical) optical depth for

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attenuating factor i and wavelength λ, which is an integral over the vertical atmosphere. The relative optical atmospheric mass, m_i, is the ratio of the actual path length of the Sun's rays to the length of a corresponding vertical path above sea level for factor i. This corrective factor should take into account the distribution of the attenuating factor i within the atmosphere and the site's altitude and pressure. In this article, we will refer mostly to m or air mass, as the relative optical atmospheric mass for the Rayleigh scattering term. The spectral dependence of the refractive properties of the atmosphere are neglected here, as in Kondratyev (1969).

With respect to the above, the attenuated direct normal irradiance at the bottom of the atmosphere can be written:

$$I = \int_0^\infty I_{0\lambda} \prod_{i=1}^{n} \tau_{i\lambda} \, d\lambda \tag{3}$$

In the following, eqn (3) will be referred to as the spectral transmittance equation and the equivalent broad band equation can be written as often seen in the literature:

$$I = I_0 \prod_{i=1}^{n} T_i \tag{4}$$

where

$$I_0 = \int_0^\infty I_{0\lambda} \, d\lambda.$$

The broad band (also called 'spectrally weighted') transmittances are usually given as in Iqbal (1983),

$$T_i = \frac{\int_0^\infty I_{0\lambda} \tau_{i\lambda} \, d\lambda}{\int_0^\infty I_{0\lambda} \, d\lambda} \tag{5}$$

representing the equivalent total atmospheric transmittance for an atmosphere comprising only a single depleting factor i.

If we take, for illustrative purposes, an atmosphere comprising only two attenuating factors and T_1 is given by eqn (5), then for eqns (3) and (4) to be valid, we see that T_2 must be:

$$T_2 = \frac{\int_0^\infty I_{0\lambda} \tau_{1\lambda} \tau_{2\lambda} \, d\lambda}{\int_0^\infty I_{0\lambda} \tau_{1\lambda} \, d\lambda} \tag{6}$$

Hence T_2 cannot be considered independent of the attenuating process accounted for by T_1.

The above results can be extended to an atmosphere comprising n attenuating factors, whence the broad band transmittance for the ith attenuating factor can be written:

$$T_i = \frac{\int_0^\infty I_{0\lambda} \prod_{j=1}^{i} \tau_{j\lambda} \, d\lambda}{\int_0^\infty I_{0\lambda} \prod_{j=1}^{i-1} \tau_{j\lambda} \, d\lambda} \tag{7}$$

and it is easy to show that

$$\prod_{i=1}^{n} T_i = \frac{\int_0^\infty I_{0\lambda} \prod_{i=1}^{n} \tau_{i\lambda} \, d\lambda}{\int_0^\infty I_{0\lambda} \, d\lambda} = \frac{I}{I_0}$$

thus eqn (7) is in perfect agreement with both the broad band and spectral transmittance eqns (3) and (4). In the subsequent discussions, eqns (5) and (7) will be referred to as, respectively, the independent and interdependent broad band transmittances. For $i = 1$, we see that both eqns (5) and (7) yield the same result and for $i > 1$, eqn (7) has the disadvantage that each new T_i will be dependent upon all the $\tau_{j\lambda}$ for $j \leq i$. It is also deceptive that the definitions of each broad band transmittance are not symmetrical and depend on the order in which they are defined, i.e. each T_i is weighted by an extraterrestrial spectrum which has been depleted by the $i-1$ attenuating factors defined before T_i.

A physical interpretation can be superposed to the above effect if the depleting factors affect the beam in consecutive layers of the atmosphere and T_1 pertains to the first layer. Equation (7) can then be interpreted as the broad band transmittance due to process i, but takes into account the modification of the spectrum due to the $i-1$ processes which have occurred in higher layers of the atmosphere. In the troposphere however, a number of effects occur simultaneously. If this is the case, eqn (7) loses some of its physical meaning but not its validity. Indeed, the order in which the T_is are defined has no influence on their product. Therefore the order in which the depleting factors are encountered need not be known beforehand and eqn (7) is valid for two or more atmospheric processes occurring in the same or separate atmospheric layers.

It also speaks in favour of eqn (7) that, if the atmospheric depleting factors for $j \leq i$ are invari-

ant with time, then T_i will only be dependent on atmospheric mass and this parameterization entails no loss in practicality from eqn (5). The dependence on atmospheric mass is inevitable for all the broad band transmittances unless the spectral transmittance can be considered constant over the whole spectrum. This can be seen from eqn (2) which must be integrated over the whole spectrum and is illustrated in the next section.

If the attenuating factors affect distinct and separate parts of the spectrum however, the broad band transmittances can truly be considered independent. As above, we start with an atmosphere in which only two attenuating factors are present but this time each affects a separate part of the spectrum, symbolised by $\Delta\lambda_1$ and $\Delta\lambda_2$, with $0 \leq \tau_{k\lambda} \leq 1$ if $\lambda \in \Delta\lambda_k$ and $\tau_{k\lambda} = 1$ if $\lambda \notin \Delta\lambda_k$, whereby the spectral eqn (3) is respected. If we also assume that $\Delta\lambda_1$ and $\Delta\lambda_2$ cover the whole spectrum, we can divide correspondingly the extraterrestrial and attenuated normal beam irradiance: $I_0 = I_{01} + I_{02}$ and

$$I = I_1 + I_2 = \sum_{k=1}^{2} \int_{\Delta\lambda_k} I_{0\lambda}\tau_{k\lambda}\,\mathrm{d}\lambda$$

We can then write:

$$I = \int_0^\infty I_{0\lambda}\tau_{1\lambda}\,\mathrm{d}\lambda - I_{02} + \int_0^\infty I_{0\lambda}\tau_{2\lambda}\,\mathrm{d}\lambda - I_{01}$$

$$= I_0(T_1 + T_2 - 1) = I_0(T_1 - \alpha_2) \qquad (8)$$

where $\alpha_2 = 1 - T_2$ is the broad band absorptance for process 2 and T_1, T_2 are considered independent, as defined per eqn (5). This result can be extended to a spectrum divided into n separate bands, with

$$T_k = \frac{\int_0^\infty I_{0\lambda}\tau_{k\lambda}\,\mathrm{d}\lambda}{I_0}$$

$$= \frac{\int_{\Delta\lambda_k} I_{0\lambda}\tau_{k\lambda}\,\mathrm{d}\lambda + I_0 - \int_{\Delta\lambda_k} I_{0\lambda}\,\mathrm{d}\lambda}{I_0} \qquad (9)$$

Note that k represents the spectral band $\Delta\lambda_k$ and not a single process as i in eqn (7). If $\tau_{k\lambda}$ is the result of a combination of different factors affecting the same band, then it can be written as the product of different transmittances as in eqn (3). For a spectrum divided into n spectral bands, it can be shown from eqn (9):

$$I = I_0\left(\sum_{k=1}^{n} T_k + 1 - n\right) \qquad (10)$$

Note also that it is not necessary in this result for the wavebands to cover the whole spectrum, neither is it necessary for the bands to be continuous.

This simple result must be differentiated from eqn (4) in that the broad band transmittances T_k are this time symmetrical and independent, as defined per eqn (5), but relate to a spectral transmittance which is equal to unity for all wavelengths outside the interval $\Delta\lambda_k$ and which covers all the attenuating processes affecting the waveband $\Delta\lambda_k$. If these conditions are respected, eqn (10) is a rigorous result which can be extremely convenient when considering two processes which affect different parts of the spectrum, such as can be assumed for molecular scattering and adsorption by water vapour. This result is valid for any number of processes as long as there is no overlapping of the wavebands affected by the different factors. In case of two processes occurring over the same bands, the first approach and eqn (4) should be favoured. In a real atmosphere it is possible to combine the two results as shown in the next sections.

3. INTERPRETATION OF SOME EXISTING BROAD BAND EQUATIONS

In this section we give a short discussion on possible implications of the above results, with respect to existing parameterized direct irradiance transmittance models. We limit our study to the three models, A, B and C presented in Iqbal (1983), which give, in our opinion, a good general view of the subject. For further reading, please consult the review papers of Bird and Hulstrom (1981), Louche *et al.* (1988), Davies *et al.* (1989) and Gueymard (1993).

The most straightforward broad band transmittance equation (model C in Iqbal, 1983), is to simply rewrite the spectral equation with broad band transmittances, as in eqn (4) above:

$$I = I_0 \cdot T_o T_R T_g T_w T_a \qquad (11)$$

where the subscripts of the broad band transmittances refer to: ozone absorption, Rayleigh scattering, gaseous absorption (by the 'permanent mixed gases' such as CO_2), water vapour absorption and aerosol extinction, respectively. This method has been used extensively with different transmittance equations, usually fitted to detailed spectral models.

Such a model can be found in Bird and Hulstrom (1981), hereafter B&H's model. In this article, we use the term B&H's model

whenever eqn (11) is used with the independent broad band transmittances given as per eqn (5). It should be said that a number of other broad band transmittance models cited in the above mentioned review articles also use eqn (11). When specified in the corresponding texts however, these transmittances are always calculated using the independent eqn (5), as suggested in Iqbal (1983). The effects of calculating the transmittances according to eqn (7) instead of eqn (5) are discussed below.

Paltridge and Platt (1976) proposed the following broad band transmittance equation (model A in Iqbal, 1983):

$$I = I_0[T_o T_R - \alpha_w]T_a \qquad (12a)$$

where $\alpha_w = 1 - T_w$ is the broad band water vapour absorptance. The small term pertaining to absorption by the permanent gases is missing in this equation, as noted by Iqbal (1983). This equation is in fact a combination of eqns (10) and (4) and has also proved to be very popular, as in Davies and Hay (1980).

Paltridge and Platt (1976) suggest that the independent eqn (5) be used for all the broad band transmittances and do not mention the possibility of using interdependent transmittances. However, they justify, on p. 121, the separation of the water vapour absorptance from ozone and Rayleigh attenuation by stating that these effects occur in different parts of the spectrum and were hence well aware of the result expressed in eqn (10). In the comparisons illustrated below, we have added the missing term accounting for the permanent gases:

$$I = I_0[T_o T_R + T_g T_w - 1]T_a \qquad (12b)$$

This relation, used in combination with eqn (5) to compute the broad band transmittances, will be hereafter named P&P's model. We have chosen to add the missing term for the sake of coherence and because it seems unlikely that Paltridge and Platt (1976) have omitted it purposefully.

Figure 1 gives an illustration of the broad band optical depth of the different atmospheric attenuating factors versus air mass, as calculated by three different methods: (i) from eqns (11)

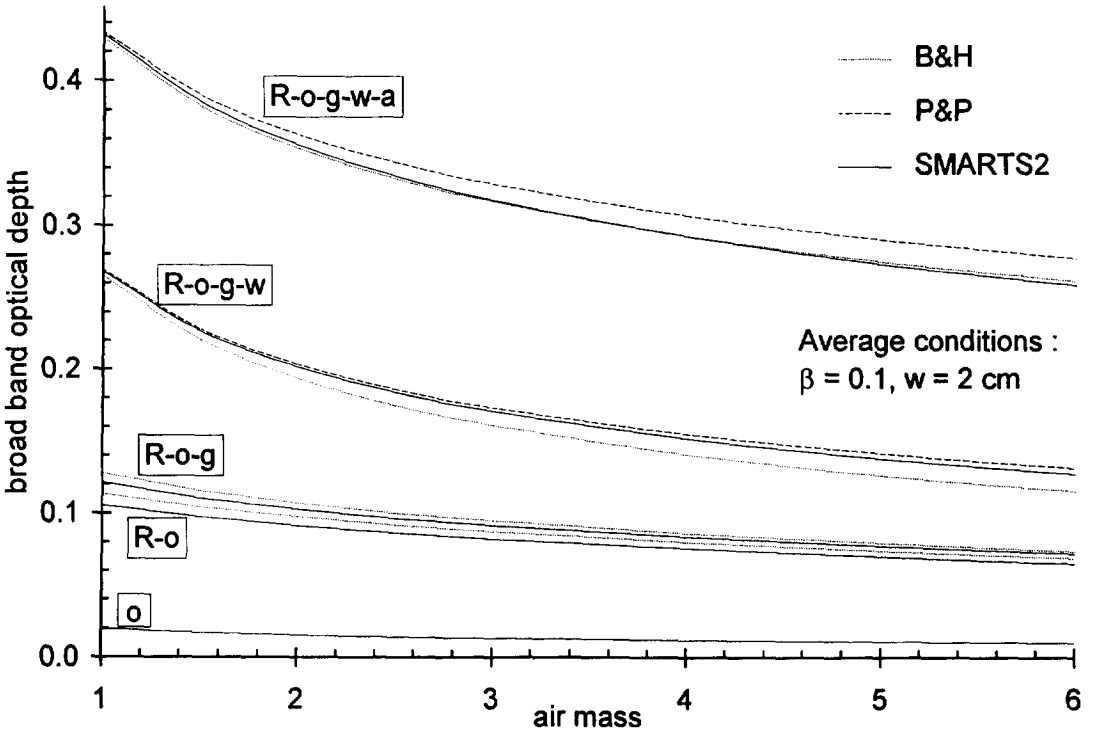

Fig. 1. Broad band optical depth versus air mass when considering the effects of the different atmospheric constituents as computed from two broad band and one spectral model (see text). Were considered, in order: (a) stratospheric ozone only (o); (b) the same + Rayleigh scattering (R-o); the same + uniformly mixed gases (R-o-g); (c) the same + water vapour (R-o-g-w) and (d) the same + aerosols (R-o-g-w-a). The spectral results from SMARTS2 can be considered here as the benchmark towards which B&H's and P&P's broad band models should tend. Average atmospheric conditions in a temperate climate were chosen: height of precipitable water content: $w = 2$ cm, aerosol optical depth at 1 μm: $\beta = 0.1$, Ångström's wavelength exponent: $\alpha = 1.3$ and stratospheric ozone thickness: $O_3 = 0.34$ atm-cm.

and (5), corresponding to B&H's model and Iqbal's model C; (ii) from eqns (12b) and (5), corresponding to P&P's model and Iqbal's model A and (iii) from eqns (11) and (7), which corresponds to the full spectral calculation and was estimated from the spectral code SMARTS2, see Gueymard (1995).

The broad band optical depths, δ, are calculated from $T = \exp(-m\delta)$ where $T = I/I_0$ is the resulting broad band transmittance of the attenuating factors being considered. The integrals in eqns (5) and (7) were obtained here from SMARTS2 and not from approximate formulae as proposed for example in Bird and Hulstrom (1981).

The resolution of SMARTS2 is of 1 nm for wavelengths smaller than 1700 nm and 5 nm for $\lambda > 1700$ nm. The predictions of this code are shown in Gueymard (1995) to be in very good agreement with the more rigorous MODTRAN spectral code. Nitrogen dioxide, NO_2, which exists in both the stratosphere and troposphere, absorbs in the visible wavelengths and is also taken into account by SMARTS2. Only a stratospheric component (as for ozone) was considered here. Its thickness was considered constant throughout (0.0017 atm-cm) and its effect was always computed simultaneously with that of ozone. A diffuse circumsolar effect due to the fact that current pyrheliometric instruments have an opening of 5.7° in comparison to the 0.5° covered by the Sun's disk, can also be modelled by SMARTS2. This effect is however small in the present illustrations and was not considered further.

The atmospheric aerosol content is symbolised in Figs 1 and 2 by Ångström's spectral turbidity factor β. This factor is defined in Ångström's formula: $\delta_{a\lambda} = \beta\lambda^{-\alpha}$, where $\delta_{a\lambda}$ is the spectral aerosol optical depth and λ is in μm. This equation has been derived empirically, see Ångström (1961), and confirmed theoretically in the special case of a Junge spherical polydispersion, see Junge (1963); α is correlated to the size of the aerosols (see for example Shifrin, 1995) while β is the equivalent aerosol optical depth at 1 μm. Although α and β may vary considerably over the solar spectrum in real conditions, Ångström's formula nevertheless gives a good general representation of the spectral behaviour of atmospheric aerosol attenuation (see for example Cachorro *et al.*, 1989). In all the illustrations, α and β were kept constant over the whole spectrum with $\alpha = 1.3$, the global average value suggested by Ångström (1961), while β was made to vary, as specified for each figure.

It can be seen from Fig. 1 that water vapour

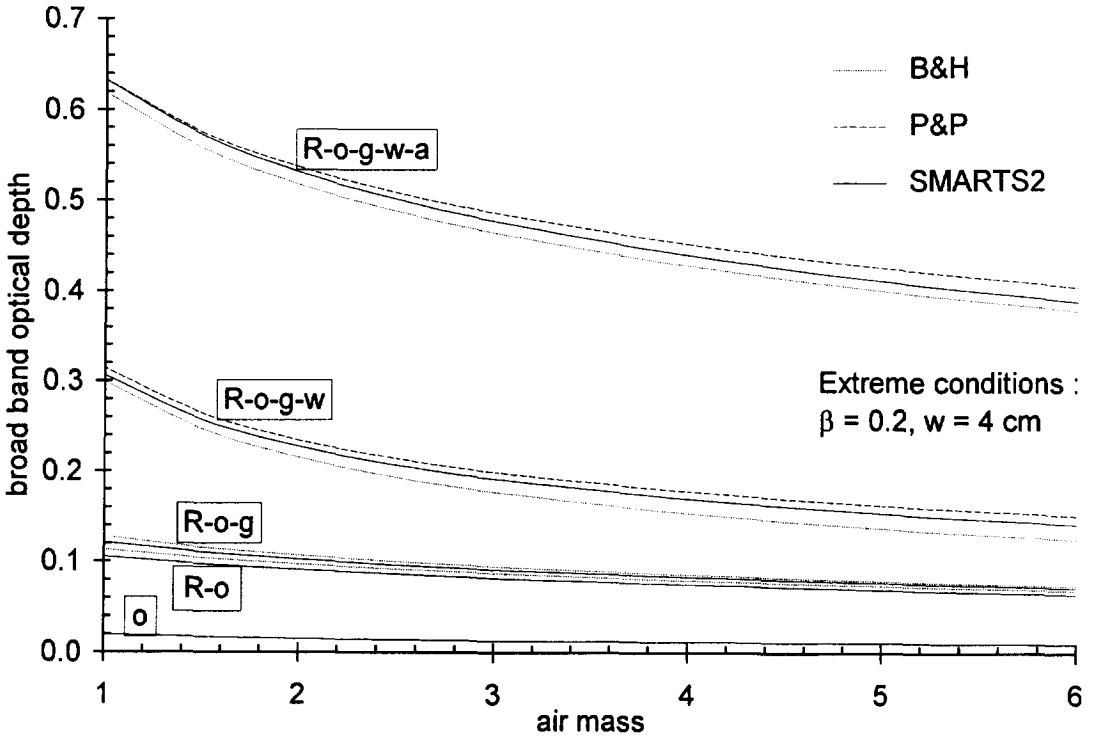

Fig. 2. Same as Fig. 1 for extreme conditions of water vapour and aerosol contents: $w = 4$ cm and $\beta = 0.2$.

and aerosols contribute overwhelmingly to the total broad band optical depth in these cloudless sky conditions. The height of precipitable water vapour content, w, was set at 2 cm in Fig. 1 and Ångström's spectral turbidity factor, β, was set at 0.1. These are the average conditions we have observed in two northern mid latitude sites, see Molineaux *et al.* (1995). The air mass dependence is important and cannot be neglected, for all the attenuating factors taken either together or separately. If the atmosphere is free of aerosols, then it is clear that P&P's model is the most precise. If all constituents are considered together however, then B&H's model is close to perfect for all values of air mass. This appears to be a fortunate coincidence. In fact the aerosol broad band optical depths computed from both B&H's and P&P's models are the same [both are computed from eqn (5)], and overestimated in both cases, except that in B&H's model this error compensates the underestimation of the effect of water vapour.

If extreme rather than average conditions are taken however, such as aerosol optical depth at 1 μm = 0.2 and height of precipitable water vapour = 4 cm as illustrated in Fig. 2, the errors do not compensate so well and P&P's model is better for small values of air mass.

It is also interesting to see from Figs 1 and 2 that doubling the amount of water vapour from 2–4 cm only increases the water vapour broad band optical depth by about 30%. This is because the spectral absorption bands of water vapour rapidly saturate and is illustrated further in the next section. However, doubling Ångström's turbidity factor β from 0.1 to 0.2 also doubles the aerosol broad band optical depth. In real conditions, hygroscopic aerosols will absorb humidity and rapidly increase in size as the relative humidity increases above a certain threshold, see for example Tang *et al.* (1981). This effect, which was not taken into account in the present simulations, will have an influence on the above results in these particular conditions.

The third parameterization method we have chosen (model B in Iqbal, 1983) is taken from Hoyt (1978), but can be attributed to others before him, and is written:

$$I = I_0(1 - \alpha_o - \alpha_g - \alpha_w - \alpha_a)T_R T_{as} \quad (13)$$

where α_a represents the broad band absorptance by aerosols and T_{as} is the transmittance resulting from aerosol scattering. It is not justified in Hoyt's paper how he obtained this relation. Iqbal's interpretation is that eqn (13) is simply a replica of eqn (11) in which the transmittances of the terms related to gaseous absorption have been replaced by 1 minus their corresponding absorptances. Equation (13) can be written thus if the second and higher order terms are all neglected in front of the first order terms. This is justified because the absorptances are small. We propose another possible interpretation. Indeed, if the absorption processes are assumed to occur in different bands, then these terms can be added as in eqn (10), yielding exactly the same result. This means that the two assumptions, either of small absorptances or of separate spectral intervals, are perfectly equivalent. The results obtained with eqn (13) are very close to those of eqn (12) because the two most important effects, i.e. aerosol scattering and water vapour absorptance, are considered in the same manner. This model is therefore not illustrated here.

More complicated models also exist and we would like to attract the reader's attention to the physical method proposed by King and Buckius (1979), which is one of the rare studies in which the interdependence is respected. Another interesting approach was taken up by Gueymard (1989), in which he divided the spectrum into two bands, and used eqn (11) with independent transmittances weighted over each of the two bands. The resulting equations are however too complicated to isolate the effects of aerosols alone, which is the subject of the next section.

4. ASSESSING THE ATMOSPHERIC AEROSOL CONTENT FROM BROAD BAND PYRHELIOMETRIC MEASUREMENTS

The above results show clearly that broad band direct irradiance is very sensitive to atmospheric aerosol content. It is therefore of special interest to investigate the possibility of estimating atmospheric turbidity from broad band measurements of direct irradiance. Since aerosol extinction is a highly spectral process, only a spectral turbidity factor can give an unambiguous description of the atmospheric aerosol content. Thus the keystone of the following discussion is the relation between broad band turbidity factors and Ångström's spectral turbidity factor β. In this context, we will study the implications of the above results with respect to the much referred Linke's and Unsworth–Monteith's turbidity factors.

Linke's turbidity factor, T_L

Linke's (1922) turbidity equation can be written:

$$I = I_0 \exp(-\delta m) = I_0 \exp(-\delta_{CDA} T_L m) \tag{14}$$

where the total broad band optical thickness of the vertical atmosphere, δ, is written as the product of two terms: δ_{CDA}, the corresponding theoretical optical thickness of a Clean Dry Atmosphere (hereafter CDA, real atmosphere free of water vapour, aerosols and any pollutants) and T_L which thus represents the number of CDAs needed to obtain the observed attenuation as a result of the additional effects of water vapour, natural and artificial aerosols and eventual gaseous pollutants.

With the knowledge of δ_{CDA}, T_L can be computed from the measurement of I, the direct irradiance, and the relation:

$$T_L = \frac{1}{m\delta_{CDA}} \ln\left(\frac{I_0}{I}\right) \tag{15}$$

as in Kasten (1980). It is interesting to note that Kasten (1980) in his widely used equation: $\delta_{CDA} = (9.4 + 0.9m)^{-1}$, considered only the effects of Rayleigh scattering and ozone absorption. Absorption by the uniformly mixed gases should also be included in δ_{CDA}, as discussed in Molineaux *et al.* (1995).

Equation (14) can be written: $I = I_0 \cdot T_{CDA}^{T_L}$ with $T_{CDA} = \exp(-m\delta_{CDA})$. In agreement with the above results, we can then write:

$$T_{CDA} = \frac{\int_0^\infty I_{0\lambda}\tau_{o\lambda}\tau_{R\lambda}\tau_{g\lambda}\,d\lambda}{\int_0^\infty I_{0\lambda}\,d\lambda} \tag{16}$$

Thus T_{CDA} and δ_{CDA} are dependent only on air mass in the assumption that the effects of ozone absorption, Rayleigh scattering and absorption by the permanent mixed gases are considered invariant.

From eqn (11), we can write:

$$I = I_0 \cdot T_{CDA} T_{wa}$$

with

$$T_{wa} = \frac{\int_0^\infty I_{0\lambda}\tau_{o\lambda}\tau_{R\lambda}\tau_{g\lambda}\tau_{w\lambda}\tau_{a\lambda}\,d\lambda}{\int_0^\infty I_{0\lambda}\tau_{o\lambda}\tau_{R\lambda}\tau_{g\lambda}\,d\lambda} \tag{17}$$

where the broad band transmittance T_{wa} is defined as per eqn (7). Eventual gaseous pollutants are neglected here.

From eqns (14) and (17) we find:

$$T_L = \frac{\ln(T_{wa})}{\ln(T_{CDA})} + 1 \tag{18}$$

From the above relations, it can be seen that T_L will be dependent on atmospheric mass, water vapour and aerosol content. The dependence of T_L on atmospheric mass for constant turbidity has been seen by many authors as a serious hindrance to the estimation of turbidity with this parameter, see for example Robinson (1966). This has lead to a number of rather unsuccessful attempts, starting with Linke himself (1942), in order to circumvent this difficulty.

Grenier *et al.* (1994) illustrate the dependence of T_L with air mass and propose a Linke turbidity index normalised at air mass 2 which they call T_{LAM2} and which is therefore independent of air mass. They also investigate the dependence of T_{LAM2} on water vapour and Ångström's spectral turbidity factor and give these dependencies in the form of various tables and polynomial fits. Thus Grenier *et al.* (1994) give a possible method for estimating Ångström's spectral turbidity factor from the measurement of direct irradiance and the knowledge of w, the precipitable water content.

The dependence of T_L with air mass is illustrated in Fig. 3, as would be estimated from two different approaches. In this illustration, we have used the relation $\delta_{CDA} = T_o T_R T_g$ where the transmittances are computed from either eqns (5) or (7). We see that the effect of using interdependent rather than independent [eqn (7) rather than eqn (5)] broad band transmittances for computing δ_{CDA}, entails a greater difference than the air mass varying from 1 to 6. In the

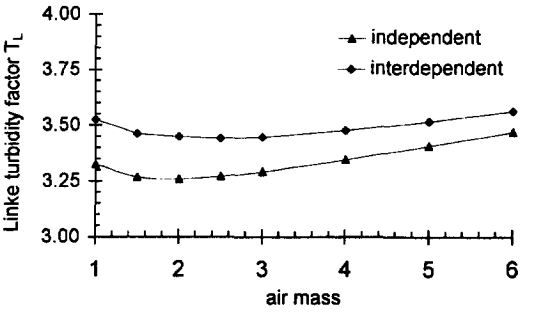

Fig. 3. Linke's broad band turbidity factor versus air mass, as computed by SMARTS2 with two different approaches: using either independent or interdependent broad band transmittances to compute δ_{CDA}. $w = 2$ cm, $\beta = 0.1$, $\alpha = 1.3$ and $O_3 = 0.34$ atm-cm.

approach taken up by Grenier *et al.* (1994), the interdependence is implicit in their formulation of δ_{CDA}. These authors show that the amplitude of the variations illustrated in Fig. 3 does not change much with different conditions of water vapour and atmospheric turbidity.

The dependence of T_L with Ångström's spectral turbidity factor, β, for different values of precipitable water vapour, w, and constant air mass = 2 is depicted in Fig. 4. From the comparison of Figs 3 and 4, it is clear that the variation of T_L with air mass and water vapour is small in front of the variations of T_L with atmospheric aerosol content. The variation of T_L with w for an invariable aerosol content however appears to be at least as much a problem for the estimation of β as the variation of T_L with air mass. The computations from which these figures are drawn are all made from Gueymard's (1995) SMARTS2 spectral code.

It can be seen from Fig. 4 that the dependence of T_L with β is close to linear. Katz *et al.* (1982) have found experimentally that $T_L = 2.0 + 19\beta$ but they used Kasten's (1980) definition of δ_{CDA}, which includes only Rayleigh scattering and ozone absorption. Considering that attenuation by the mixed gases should also be accounted for in δ_{CDA}, Grenier *et al.* (1994) have adapted Katz *et al.*'s results to yield: $T_L = 1.74 + 15.4\beta$, which agrees well with the predictions of SMARTS2 and Fig. 4 for $0.5 < w < 4$ cm.

Also apparent from Fig. 4 is the fact that most of the effect of water vapour occurs between 0 and 0.5 cm. The fact that the amount of atmospheric water vapour is difficult to estimate from ground based measurements is therefore not a serious hindrance in this application, since values of $w < 0.5$ cm are a rare climatic event. Indeed, it is only in cold climates and during the winter time that the height of precipitable water vapour reaches such small values (Gueymard, 1994). The effect of particle growth with high levels of relative humidity has not been considered here and may considerably influence the above results in certain specific conditions.

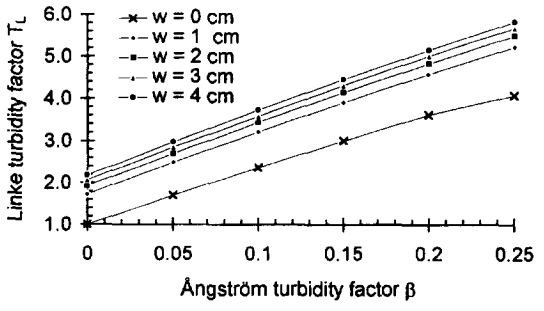

Fig. 4. Linke's broad band turbidity factor versus Ångström's spectral turbidity factor for different heights of precipitable water vapour, as predicted by SMARTS2. $m = 2$, $\alpha = 1.3$ and $O_3 = 0.34$ atm-cm.

Unsworth–Monteith's turbidity parameter, τ_{UM}

This parameter was introduced by Unsworth and Monteith (1972) and can be defined from the following relation:

$$T_a = \exp(-m\tau_{UM}) = \frac{I(\tau_{UM})}{I(\tau_{UM} = 0)} \qquad (19)$$

where $I(\tau_{UM} = 0)$ is the theoretical broad band direct irradiance which would be measured if the atmosphere was free of aerosols, given by:

$$I(\tau_{UM} = 0) = \int_0^\infty I_{0\lambda}\tau_{o\lambda}\tau_{R\lambda}\tau_{g\lambda}\tau_{w\lambda}\,d\lambda = I_0 \cdot T_o T_R T_g T_w \qquad (20)$$

where the broad band transmittances should be computed according to eqn (7). T_a is given by:

$$T_a = \frac{\int_0^\infty I_{0\lambda}\tau_{o\lambda}\tau_{R\lambda}\tau_{g\lambda}\tau_{w\lambda}\tau_{a\lambda}\,d\lambda}{\int_0^\infty I_{0\lambda}\tau_{o\lambda}\tau_{R\lambda}\tau_{g\lambda}\tau_{w\lambda}\,d\lambda} \qquad (21)$$

Thus τ_{UM} is dependent on all the attenuating processes in the atmosphere, including aerosol extinction and water vapour absorption; τ_{UM} is a spectrally weighted optical depth and not an atmospheric transmittance. The 'τ' symbol was chosen in the original article (Unsworth and Monteith, 1972), but corresponds to the (spectrally weighted) optical depth 'δ' symbol in this paper and not the spectral transmittance, also symbolised by 'τ' in the preceding equations.

Unsworth and Monteith (1972) do not give the equations with which they compute $I(\tau_{UM} = 0)$ but it can be seen from eqns (19)–(21) that all the other atmospheric effects have been accounted for before computing T_a. From the literature, both eqns (11) and (12) have been used to compute $I(\tau_{UM} = 0)$. For example, Louche *et al.* (1987) and Kambezidis *et al.* (1993) make use of eqn (11) and Bird and Hulstrom's (1981) model whereas Uboegbulam and Davies (1983) make use of eqn (12) and Davies and Hay's (1980) model. Freund (1983) and later Hay and Darby (1984) use exactly the same approach as Uboegbulam and Davies

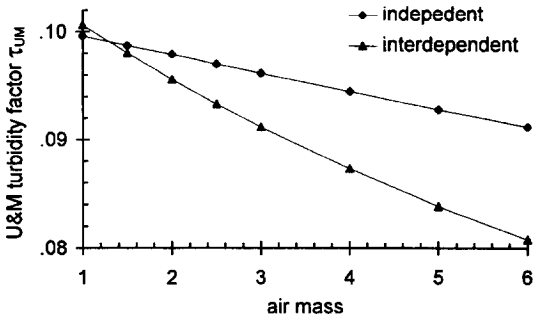

Fig. 5. Unsworth and Monteith's broad band turbidity factor versus air mass, as computed by SMARTS2 with two different appoaches: using either independent or interdependent broad band transmittances to compute $I(\tau_{UM}=0)$ as in eqn (20). $w=2$ cm, $\beta=0.05$, $\alpha=1.3$ and $O_3=0.34$ atm-cm.

(1983) but call their index Aerosol Optical Depth, AOD, instead of τ_{UM}.

From Fig. 1 it can be seen that the total broad band optical depth of a cloudless atmosphere free of aerosols will be better estimated using eqn (12). Thus P&P's model should be preferred over B&H's model when computing $I(\tau_{UM}=0)$. Figure 5 illustrates the difference between the values of τ_{UM} which would be obtained by these two different approaches. The independent approach for calculating $I(\tau_{UM}=0)$ is based on B&H's model. However, P&P's model will yield much closer results to the interdependent approach which is also illustrated in Fig. 5.

Comparing T_L and τ_{UM}

Figure 6 illustrates the linear variation of τ_{UM} with β for different values of w, as was illustrated for T_L in Fig. 4. It is clear that, even if the dependence of τ_{UM} on water vapour cannot be eliminated completely, it is relatively much smaller than in the case of T_L. Similarly and over the same range of β, it can be seen from Figs 3–6 that, relatively speaking, the variation

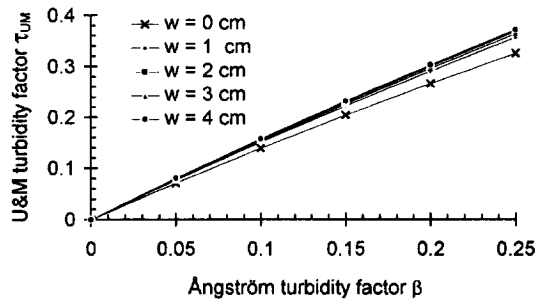

Fig. 6. Unsworth and Monteith's broad band turbidity factor versus Ångström's spectral turbidity factor for different heights of precipitable water vapour, as predicted by SMARTS2. $m=2$, $\alpha=1.3$ and $O_3=0.34$ atm-cm.

of τ_{UM} with air mass is about half of the variation of T_L with air mass (approximately $\pm 1.5\%$ for τ_{UM} and $\pm 3\%$ for T_L). Also important considering Figs 3 and 5, is the fact that the dependence on air mass will be easier to express in analytical form for τ_{UM} than it will be for T_L. Choosing another value than 1.3 for Ångström's wavelength exponent α essentially changes the slope of the curves in Figs 4 and 6, with little influence on the above results.

In this theoretical comparison, it has been assumed that both δ_{CDA} and $I(\tau_{UM}=0)$ have been estimated correctly. In practice however it will be much easier to obtain a reliable value of δ_{CDA} than of $I(\tau_{UM}=0)$ because the latter also depends on the highly variable and inaccessible value of atmospheric precipitable water content.

Kambezidis *et al.* (1993) present a ten year experimental comparison of τ_{UM} and T_L estimated in the city of Athens. It is shown that on average τ_{UM} and T_L are linearly correlated. Both parameters exhibit a significant diurnal variation with a maximum at noon [which may be partly explained by the daily trend in air pollution, see Junge (1963), p. 362]. The amplitude of τ_{UM}'s measured variation is however about one third of that of T_L which partly agrees with the above simulations.

The analytical relation between these two broad band turbidity parameters can be shown. We first write eqn (14) with τ_{UM} instead of T_L:

$$I = I_0 \exp(-m(\delta_{CDA} + \delta_w + \tau_{UM})) \quad (22)$$

with $T_w = \exp(-m\delta_w)$, and the relation between τ_{UM} and T_L can be written:

$$T_L = 1 + \frac{\delta_w + \tau_{UM}}{\delta_{CDA}} \quad (23)$$

In this relation it is assumed that T_w is defined as in eqn (7). Since δ_w depends on w and both δ_w and δ_{CDA} are intricate functions of atmospheric mass, we see that the relation between T_L and τ_{UM} is not straightforward, theoretically speaking.

5. CONCLUSIONS

From this analytical study of the atmospheric transmittance of direct irradiance in a cloudless sky, we have come to the following conclusions:

- the assumption that atmospheric broad band transmittances are independent of each other is neither true nor is it a necessary condition in order to compute these transmittances;
- this assumption yields however very good

results when predicting the total broad band optical depth of the atmosphere in average conditions, due to a favourable compensation of the errors made on the broad band optical depths of water vapour and aerosols;

- if two attenuating processes affect two different parts of the direct solar spectrum, their corresponding broad band transmittances are this time independent, and should be added instead of multiplied;
- this last result is used by Paltridge and Platt (1976) and their model should be preferred to Bird and Hulstrom's (1981) when estimating Unsworth and Monteith's turbidity index;
- the turbidity index proposed by Hay & Darby (1984) is perfectly equivalent to Unsworth & Monteith's (1972);
- this broad band turbidity index may be more adapted than Linke's turbidity factor when trying to isolate the effect of aerosols from the measurement of attenuated direct irradiance, since the latter exhibits a greater dependence on both air mass and water vapour.

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NOMENCLATURE

λ wavelength, [μm] or [nm]
I_λ and I monochromatic [$Wm^{-2}\,\mu m^{-1}$] and broad band normal beam irradiance [Wm^{-2}]
$I_{o\lambda}$ and I_o monochromatic [$Wm^{-2}\,\mu m^{-1}$] and extraterrestrial beam irradiance [Wm^{-2}]
$\tau_{i\lambda}$ atmospheric spectral transmittance for wavelength λ and process i
$\delta_{i\lambda}$ atmospheric spectral vertical optical depth for wavelength λ and process i
$m_{i\lambda}$ relative optical atmospheric mass for wavelength λ and process i
T_i atmospheric broad band transmittance for process i
α_i atmospheric broad band absorptance $= 1 - T_i$

The subscript i may represent one or a combination of the following atmospheric attentuation processes:

o ozone absorption
R molecular or Rayleigh scattering
g absorption by the permanent mixed gases such as CO_2
w absorption by water vapour
a aerosol extinction
CDA all processes existing in a clean dry atmosphere (o, R and g)

$\Delta\lambda_k$ bandwidth No. k, continuous or not
$\tau_{k\lambda}$ atmospheric spectral transmittance for the waveband $\Delta\lambda_k$
m relative optical air mass
δ total broad band optical thickness of the atmosphere for all processes
δ_i broad band optical thickness of the atmosphere for process i
α Ångstrom's wavelength exponent
β Ångstrom's turbidity index, equivalent to the aerosol optical depth at 1 μm
w atmosphere's precipitable water content in a vertical column of unit area [cm]
O_3 ozone thickness in [atm-cm] corresponding to height of gaseous ozone in a vertical column of unit area at normal temperature and surface pressure
T_L Linke's broad band turbidity factor
τ_{UM} Unsworth–Monteith's broad band turbidity factor
AOD Hay & Darby's broad band aerosol optical depth, equivalent to τ_{UM}
B&H symbolising Bird and Hulstrom's (1981) model, combining eqns (5) and (11)
P&P symbolising Paltridge and Platt's (1976) model, combining eqns (5) and (12)

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