A Convergent Expression for the Magnetic Moment of the Neutron

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Letters to the Editor

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A Convergent Expression for the Magnetic Moment of the Neutron

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CALCULATION of the magnetic moment of the neutron has always yielded a divergent result.[1] We have avoided this difficulty by the use of an invariant perturbation method based upon the S matrix theory.[2] The magnetic moment remains finite but no definite numerical value comes out of the calculations. Our method is quite general and can be applied to all diverging expressions arising from field quantization.

The diverging terms are due to the Dirac δ function appearing in

$$D_\kappa{}^s(r,t)=(t/2|t|)D_\kappa{}^0(r,t)=(1/4\pi)[\delta(T^2)-\tfrac{1}{2}\theta(T^2)(\kappa/T)J_1(\kappa T)]$$

with

$$T^2=t^2=r^2 \quad\text{and}\quad \theta(T^2)=0 \quad\text{for}\quad T^2 \gtreqless 0,$$

where $D_\kappa{}^0(r,t)$ is the antisymmetric invariant function appearing in the commutation relations of a field of quanta of restmass κ. If we define $\delta(T^2)$ by the following limiting process:

$$\delta(T^2)=\operatorname*{Lim}_{\kappa_i\to\infty}-\tfrac{1}{2}\theta(T^2)\sum_1^N c_i(\kappa_i/T)J_1(\kappa_i T),$$

with

$$\sum_1^N c_i=-1 \qquad \operatorname*{Lim}_{\kappa_i\to\infty}\sum_1^N (c_i/\kappa_i{}^2)=0,$$

a calculation then shows that it is always possible to find a set of $c_i=c_i(\kappa_1{}^2,\cdots\kappa_N{}^2)$, identical for each individual diverging term, such that each such term gives a finite contribution to the magnetic moment. Our calculations could be described as the generation of the S matrix by the usual Hamiltonian and its correction[3] by (2). Thus corrected our S matrix might not satisfy causality anymore. That it still does can be seen in the following way. All the terms of the S matrix contain space-time integrations upon invariant $D_\kappa(r,t)$ functions. In our calculations these appear only in the form

$$D_\kappa{}^c=D_\kappa{}^s+(i/2)D_\kappa{}^1$$

where $D_\kappa{}^1$ is the symmetric invariant D function involving a T^{-2} singularity. This is exactly the condition that expresses causality[4] or this complex function contains only incoming waves in the past and outgoing waves in the future. This condition reduces to that introduced by Wentzel[5] in his non-relativistic wave optics of the S matrix.

[1] J. M. Jauch, Phys. Rev. **63**, 334 (1943), does obtain a finite value but it seems to us that he has discussed only one of many terms.
[2] E. C. G. Stueckelberg, Helv. Phys. Acta **17**, 3 (1944); *ibid.* **18**, 3 (1945).
[3] In this interpretation (2) could be considered as an "invariant substraction."
[4] E. C. G. Stueckelberg, Helv. Phys. Acta **19**, 242 (1946); Phys. Soc. Cambridge Conference Report 199 (1947).
[5] Wentzel, Helv. Phys. Acta **21**, 49 (1948).

Comments on the Theories Interpreting the Magnetism of Celestial Bodies

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THE recent works of Babcock[1] and Blackett[2] have directed a new general interest toward the proportionality between magnetic and angular momenta of celestial bodies; accordingly several attempts[3] have been made to explain this phenomenon with the help of a general field theory. All these theories have in common is that they set out from one or two postulates stating explicitly or implicitly a connection between a moving charge (magnetic field) and an impulse-like quantity and arrive finally at the experimental connection of Wilson:[4] $e\sim M(G)^{\frac{1}{2}}$ i.e., to its formulation given by Blackett: $P=\beta(G^{\frac{1}{2}}/c)U$. These considerations involve, perforce, that not only to every mass an electric charge should be associated, but conversely that every moving (rotating) charge must be endowed with a momentum, i.e., with a mass: $M=e/(G)^{\frac{1}{2}}$. This second inference is, however, obviously not true as it may be proved by a simple laboratory experiment: taking a metallic globe of 10 cm diameter and loading it electrically to 1500 volt potential, will you observe when moving it an inert mass corresponding to 100 kg?

It is not sufficient to deduce an empirical expression merely formally in a Lorentz invariant form. From Wilson's observations it follows only that we arrive in the case of earth and sun to a right order of magnitude for their magnetic momenta, if we assume that celestial bodies have electric charges equal to their mass multiplied by the square root of the gravitational constant, but it does not make any statement regarding the inverse relationship. The difficulty in interpreting Wilson's relation arises actually not so much in finding a field theory which can account for the proportionality between mass and charge, but in explaining the fact that this relationship is not reversible.

A field theory can be constructed for any material if two requirements are fulfilled: 1) the prevailing of a re-