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Effective gluon mass and the determination of α_s from J/ψ and Υ branching ratios

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The phase space modification associated with a nonvanishing effective mass for the primary gluons, $M_g=0.66\pm0.08$ GeV for the J/ψ and $M_g=1.17\pm0.08$ GeV for the Υ, is shown to be crucial for a consistent description of the photon spectrum from their radiative decays and for the determination of α_s from the recent, precise quarkonia decay branching ratios. In this approach, the role of the relativistic corrections is marginal and, after applying the M_g-dependent corrections, a good agreement is obtained with the relative perturbative running, $\alpha_s(m_c)\sim0.30\pm0.02$ and $\alpha_s(m_b)\sim0.21\pm0.01$, and with the extrapolation from deep inelastic scattering. On the other hand, for $M_g=0$, the analysis of all experimental $c\bar{c}$ and $b\bar{b}$ quarkonia branching ratios is consistent with the same effective value of the strong interaction coupling constant $\alpha_s^{\text{eff}}\sim0.185\pm0.010$. By assuming a "genuine," i.e., process-independent, gluon mass (~1.2 GeV or higher) to be dynamically generated one predicts a strong suppression of the gluon splitting process at the J/ψ and the hadronic final states should be mainly produced via gluon fusion into light $q\bar{q}$ pairs thus effectively reducing the fitted value of M_g from the photon spectrum in $J/\psi\to\gamma+X$. The gluon fusion mechanism allows us to explain the structure of the hadronic final states observed in J/ψ decays and their close similarity to the continuum $e^+e^-\to$hadrons annihilation at comparable center of mass energies.

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I. INTRODUCTION

Quarkonia decay rates can provide, in principle, a clean test of perturbative QCD. Indeed, very precise experimental data are now available for the J/ψ and Υ decay branching ratios. Also, inclusive quantities allow reabsorption of all potentially large perturbative corrections into the running coupling constant at the relevant scale of the experiment (of the order of the heavy quark mass) and relativistic corrections are expected to be small for heavy $Q\bar{Q}$ bound states [1]. Therefore, one expects the simple expression [2,3] ($V=J/\psi,\Upsilon;Q^2=\frac{4}{9},\frac{1}{9}$; $k_V\sim1.6,0.4$, and $l=e,\mu$)

$$\frac{\Gamma(V\to\text{hadrons})}{\Gamma(V\to l^+l^-)}=\frac{10(\pi^2-9)\alpha_s^3}{81\pi Q^2\alpha^2(M_V)}\times\left[1+\frac{\alpha_s}{\pi}k_V+\frac{36Q^2\alpha}{5\alpha_s}+O(\alpha_s^2,\alpha)\right]+R_V \tag{1}$$

to be a good approximation where $\alpha_s=\alpha_s(M_V/2)$ and we estimate from Ref. [4] $\alpha^{-1}(M_{J/\psi})\sim133.7$, $\alpha^{-1}(M_\Upsilon)\sim132.1$. In Eq. (1) R_V represents the contribution from ($V\to\gamma\to$hadrons) annihilation and we estimate [3,5] $R_V=2.4\pm0.2$ for J/ψ and $R_V=3.6\pm0.2$ for the Υ system. The term $36Q^2\alpha/5\alpha_s$ accounts for the contribution of the γgg final state which is included in $V\to$hadrons. In this sense, a test of perturbative QCD through Eq. (1) has the advantage of requiring only the experimental determination of the leptonic branching ratios and of the fraction ($V\to\gamma\to$hadrons).

The experimental data [5]

$$B(J/\psi\to e^+e^-)=(6.27\pm0.20)\times10^{-2}$$

and

$$B(J/\psi\to\mu^+\mu^-)=(5.97\pm0.25)\times10^{-2}$$

allow us to deduce the average value

$$B(J/\psi\to l^+l^-)=(6.15\pm0.15)\times10^{-2}$$

and

$$B(J/\psi\to\text{hadrons})=(87.76\pm0.45)\times10^{-2}\ .$$

Also the corresponding quantities for Υ decays [5]

$$B(\Upsilon\to e^+e^-)=(2.52\pm0.17)\times10^{-2}\ ,$$

$$B(\Upsilon\to\mu^+\mu^-)=(2.48\pm0.06)\times10^{-2}\ ,$$

and

$$B(\Upsilon\to\tau^+\tau^-)=(2.97\pm0.35)\times10^{-2}$$

give

$$B(\Upsilon\to l^+l^-)=(2.48\pm0.06)\times10^{-2}$$

and

$$B(\Upsilon\to\text{hadrons})=(92.03\pm0.58)\times10^{-2}\ .$$

By using Eq. (1) we obtain (in the J/ψ case we have taken into account the small correction due to the $\gamma\eta_c$

decay mode)

$$\alpha_s(M_{J/\psi}/2)=0.191\pm0.004 \quad (2)$$

and

$$\alpha_s(M_\Upsilon/2)=0.181\pm0.004 \ . \quad (3)$$

We can investigate the consistency of Eqs. (2) and (3) with the perturbative running of α_s by using the exact two-loop relation

$$\alpha_s(\mu_0)=\frac{\alpha_s(\mu)}{1-\frac{b_0}{6\pi}t\alpha_s(\mu)+\frac{b_1}{2\pi b_0}\alpha_s(\mu)\ln\frac{\alpha_s(\mu)\left[1+\frac{b_1}{2\pi b_0}\alpha_s(\mu_0)\right]}{\alpha_s(\mu_0)\left[1+\frac{b_1}{2\pi b_0}\alpha_s(\mu)\right]}} \ , \quad (4)$$

where $t=\ln(\mu/\mu_0)$, $b_0=33-2n_f$, and $b_1=153-19n_f$. By demanding $t=\ln(M_\Upsilon/M_{J/\psi})$, $n_f=4$, and

$$\alpha_s(M_\Upsilon/2)=0.181\pm0.004$$

from Eq. (3), we obtain from Eq. (4) the theoretical prediction

$$\alpha_s^{\text{th}}(M_{J/\psi}/2)=0.258\pm0.008 \ ,$$

an 8σ discrepancy with respect to Eq. (2). Analogously, we can compare with deep inelastic scattering. In this case, by using the relation of the modified minimal subtraction ($\overline{\text{MS}}$) scheme $[L=2\ln(\mu/\Lambda_{\overline{\text{MS}}}^{(n_f)})]$,

$$\alpha_s(\mu)=\frac{12\pi}{b_0L}\frac{1}{1+6b_1\ln L/b_0^2L} \ , \quad (5)$$

which sums up the next-to-leading contributions, and the estimate from deep inelastic scattering,

$$\Lambda_{\overline{\text{MS}}}^{(4)}=248\pm25 \text{ MeV} \ , \quad (6)$$

which represents the average of the six different experimental values reported in Ref. [6], we obtain the theoretical predictions

$$\alpha_s^{\text{th}}(M_{J/\psi}/2)=0.326\pm0.017 \quad (7)$$

and

$$\alpha_s^{\text{th}}(M_\Upsilon/2)=0.209\pm0.006 \ , \quad (8)$$

a 7σ discrepancy in the J/Ψ case with respect to Eq. (2), and a 4σ discrepancy for the Υ as compared to Eq. (3).

Notice that uncalculated, perturbative higher-order corrections to Eq. (1) may, very well, involve large coefficients, for example, $\sim10-20$, to

$$O\left[\left[\frac{\alpha_s(m_Q)}{\pi}\right]^2\right] ,$$

with $m_Q\sim(M_V/2)$ [to $O(\alpha_s/\pi)$ the coefficients are of order 1 for both J/ψ and Υ if one uses the scale $\mu=m_Q$]. However, to $O(\alpha_s/\pi)$, one finds (for both J/ψ and Υ)

$$[\alpha_s(m_Q)/\pi]^2\sim4\times10^{-3}$$

so that, even with large coefficients $\sim10-20$, the final effect on the determination of α_s from Eq. (1), due to uncalculated $O(\alpha_s^2)$ perturbative effects, should be small (a change of 10% in the correction, i.e., as large as the J/ψ $O(\alpha_s/\pi)$ term, produces a change of only $\sim3\%$ in the extracted value of α_s. Truly enough, this is only valid in the sense of an asymptotic expansion and the question of the *convergence* of the perturbative series for expressions as Eq. (1) is, usually, not even addressed in the literature. Therefore, within the usual approach, an inclusive quantity such as the branching ratio in Eq. (1) should represent a very good candidate to test the running of the strong interaction coupling constant up to the heavy quark mass scale. One may, very well, consider the possibility that a different choice of the scale (for example, $\mu=xm_Q$ with $x\sim1$) can lead to an *improved* convergence of the perturbation series with respect to $x=1$ as suggested by Grunberg [7], Brodsky, Lepage, and Mackenzie [8] (BLM), and Stevenson [9]. However, as pointed out in Ref. [8], this is not the case for the branching ratio of Eq. (1), with $V=\Upsilon$. Using the BLM [8] optimization ansatz, a large negative coefficient (-14) of α_s/π is found for the optimized scale $x=0.314$, requiring large positive higher-order terms in order to fit the experiment. On the other hand, due to the similar structure of the Born amplitudes, for the two decay channels in the branching ratio,

$$\frac{\Gamma(\Upsilon\to\gamma gg)}{\Gamma(\Upsilon\to ggg)} ,$$

the corresponding coefficient is small both for the optimized scale (2.2) and for the physical scale $x=1(-2.6)$. The x dependence of α_s as derived from this branching ratio is then expected to be very weak, making it a particularly good candidate for the determination of α_s. As discussed below [see Eq. (17)], we obtain for $x=1$ the perturbative estimate $\alpha_s(m_b)=0.173^{+0.010}_{-0.009}$ in complete agreement with Eq. (3). Thus, it is very hard to imagine that any optimization procedure can explain the 7–8 σ discrepancy associated with the *relative* magnitude of α_s from J/ψ and Υ extracted from the experimental branching ratios and Eq. (1) since (apart from electric charge of the heavy quark) the same physical quantity is considered in each case.

Just for this reason, the usual explanation for the discrepancy has been searched for elsewhere, namely, in the presence of large relativistic corrections which, changing *asymmetrically* the coefficient in front of α_s^3 in Eq. (1), can lead to consistency with the expected perturbative running. In fact, analogous results were obtained in Ref. [3] (see their Table IV) $\alpha_s(m_c)=0.175\pm0.008$ and $\alpha_s(m_b)=0.173\pm0.005$ by using for the J/ψ branching ratios the experimental results available at that time. However, alternative values are reported in the abstract of Ref. [3], namely, $\alpha_s(m_c)=0.29\pm0.02$ and $\alpha_s(m_b)=0.189\pm0.008$. They were obtained by assuming the existence of relativistic corrections of the form $(1+Cv^2/c^2)$ to the ratio given in Eq. (1) (C being a constant which is assigned the value $C\sim-3$) and $v^2/c^2\sim0.24$ for the J/ψ and $v^2/c^2\sim0.08$ for the Υ system. On the other hand, in a consistent calculation, the coefficient C should be computed or derived independently by other means rather than being chosen simply so that the values of $\alpha_s(m_c)$ and $\alpha_s(m_b)$ are consistent with the expected QCD perturbative evolution. However, by admission of the same authors, "their estimate is at best a qualitative one."

Actually, in Ref. [10] a detailed comparison between fully relativistic calculations and nonrelativistic approximations is presented in the framework of a constituent quark-antiquark potential including spin-dependent terms whose parameters are determined from a fit to the heavy meson mass spectrum. By inspection of Table 4 of Ref. [10], in the case of the J/ψ leptonic width, one finds only a 7% difference between the fully relativistic calculation

$$\Gamma^{\rm rel}(J/\psi\to l^+l^-)=5.33\ \text{keV}$$

and its nonrelativistic approximation

$$\Gamma^{\rm nonrel}(J/\psi\to l^+l^-)=5.72\ \text{keV}$$

(to compare with the experimental result [5]

$$\Gamma^{\rm expt}(J/\psi\to e^+e^-)=5.36^{+0.21}_{-0.28}\ \text{keV})\ .$$

Note that for the Υ the effect is only 1% smaller, namely,

$$\Gamma^{\rm rel}(\Upsilon\to l^+l^-)=1.24\ \text{keV}\ ,$$

$$\Gamma^{\rm nonrel}\ (\Upsilon\to l^+l^-)=1.32\ \text{keV}$$

(to compare with the experimental result [5]

$$\Gamma^{\rm exp}(\Upsilon\to e^+e^-)=1.34\pm0.04\ \text{keV})\ .$$

Therefore, the assumption of Ref. [3], where strongly asymmetric corrections to Eq. (1) of $\sim-72\%$ and $\sim-24\%$ were respectively postulated, does not find any justification in the light of actual calculations. Indeed, since the relativistic corrections to $\Gamma(J/\psi\to l^+l^-)$ and $\Gamma(\Upsilon\to l^+l^-)$ are modest and the same within 1% the explanation presented in Ref. [3] requires that all the very large relativistic corrections be contained in $\Gamma(J/\psi\to ggg)$ and $\Gamma(\Upsilon\to ggg)$. There is no obvious reason why the relativistic effects should depend strongly on the specific annihilation channel. Also, it would be even more difficult to understand why the relativistic corrections would be so close for the J/ψ and Υ leptonic widths and so different for their ggg final states. Finally, the ratio

$$\frac{\Gamma(V\to\gamma gg)}{\Gamma(V\to ggg)}$$

should be almost free of relativistic corrections [11]. However, as discussed in Sec. II, the experimental data and the perturbative prediction lead to an even larger discrepancy. Within the approach where the relativistic effects are responsible for the failure of the perturbative QCD predictions, one would be forced to introduce a $\sim60\%$ relativistic correction for the J/ψ, even in this case.

Here we shall not insist on the relativistic effects any further and adopt the conservative hypothesis that the relativistic corrections to

$$\frac{\Gamma(V\to ggg)}{\Gamma(V\to l^+l^-)}$$

may be as large as those calculated in Ref. [10] for $\Gamma(V\to l^+l^-)$ so that we expect a theoretical uncertainty in α_s;

$$\left.\frac{\Delta\alpha_s(m_c)}{\alpha_s(m_c)}\right|_{\rm rel}\sim\left.\frac{\Delta\alpha_s(m_b)}{\alpha_s(m_b)}\right|_{\rm rel}\sim\pm0.02\ ,$$

such as to modify Eqs. (2) and (3) into

$$\alpha_s(M_{J/\psi}/2)=0.191\pm0.004(\text{expt})\pm0.004(\text{rel})\ , \tag{9}$$

$$\alpha_s(M_\Upsilon/2)=0.181\pm0.004(\text{expt})\pm0.004(\text{rel})\ . \tag{10}$$

The estimated uncertainty due to relativistic corrections is therefore too small, by an order of magnitude, to explain the observed discrepancy.

II. EFFECTIVE GLUON MASS CORRECTIONS

In the following we shall propose a simple modification of Eq. (1) based on the phase space suppression associated with a nonvanishing effective gluon mass. In fact, we argue that the discrepancy in the inclusive quarkonia branching ratios is closely related to similar effects present in the photon spectrum of the radiative decays $V\to\gamma+X$. As we shall see, by relating the two aspects of the problem, a good agreement with the perturbative running of α_s will be obtained without introducing any adjustable parameters other than the effective values of the gluon mass M_g for the J/ψ and Υ radiative decays.

The idea of a nonvanishing gluon mass to account for the anomalously small value of α_s extracted from the J/ψ leptonic branching ratio is not new and was originally proposed in Ref. [12]. The introduction of a gluon mass can be understood as a direct consequence of dimensional transmutation (see Ref. [13] for a gauge-invariant truncation of the Schwinger-Dyson equations of QCD and the natural introduction of a massive gluon propagator at low-energy scales). Also, independent evidence for a gluon mass in the range 1–2 GeV, from the analysis of the strong interaction correction to $R(e^+e^-\to\text{hadrons})$, the Z hadronic width and several low-energy processes, is discussed in Ref. [14]. In this approach, the strong interaction coupling constant is frozen at a typical "on shell" value at scales $\sim 1-2$ GeV and, to detect in this region the *perturbative* running associated with a QCD scale parameter $\sim 100-200$ MeV, one has to modify *nonperturbatively* the theoretical predictions of the type in Eq. (1) by taking into account explicitly the phase space modification associated with the nonvanishing, dynamically generated gluon mass. As a consequence, the $O(\alpha_s)$ correction in Eq. (1) associated with the term k_V, and evaluated at $M_g=0$, becomes largely redundant and from now on we shall set $k_V=0$ but treat the value of the $O(\alpha_s)$ correction at $M_g=0$ as an estimate of the theoretical error (in addition to that estimated above from relativistic effects). Moreover, to take full advantage of the experimental data for the radiative process $V\to\gamma+X$, we shall separate the pure $V\to ggg$ contribution, which provides the most stringent constraints, by replacing Eq. (1) with

$$\frac{\Gamma(V\to ggg)}{\Gamma(V\to l^+l^-)}=F_3(\eta)\frac{10(\pi^2-9)\alpha_s^3}{81\pi Q^2\alpha^2(M_V)}\ , \tag{11}$$

where $\eta=2M_g/M_V$ and

$$F_3(\eta)=\frac{\Gamma(J/\psi\to ggg)|M_g}{\Gamma(J/\psi\to ggg)|_{M_g=0}}$$

is the modification accounting for a massive three-gluon final state which, *a priori*, can be identified with the function $f_3(\eta)$ introduced in Ref. [12] (which includes the effect of the longitudinal polarization states) or with the pure phase space suppression factor $f_3^{\rm PS}(\eta)$ (i.e., without longitudinal degrees of freedom). The functions f_3 and $f_3^{\rm PS}$ are shown in Fig. 1 as solid and dashed lines, respectively.

In order to determine M_g we have used the results of Ref. [15] where an excellent fit of the photon spectrum in the decays $J/\psi\to\gamma+X$ as measured by the Mark II Collaboration [16] and $\Upsilon\to\gamma+X$ as measured by the ARGUS [17] and Crystal Ball [18] Collaborations has been obtained by introducing explicitly a nonvanishing gluon mass in the theoretical prediction for $V\to\gamma gg$. Unlike in Ref. [12], however, for simplicity, only the phase space modification was included and the possible contribution of longitudinal polarization states was neglected. For this reason, the analysis of the massive gluon corrections in the ggg final state should be performed consistently, i.e., by using the corresponding function $f_3^{\rm PS}$. However, to give an idea of the difference in the two cases, we shall also quote in the end the numerical results obtained with f_3.

A gluon mass M_g modifies the Dalitz plot of the γgg final state ($x_i=2E_i/M_V$; $i=1,2,$; $z=2E_\gamma/M_V$; E_i and E_γ being, respectively, the gluon and photon energies) so that

$$x_1+x_2+z=2\ ,$$

$$0\le z\le 1-\eta^2\ ,$$

$$x_1^{\min}\le x_1\le x_1^{\max}$$

and

$$x_1^{\max}=1-\frac{z}{2}\left[1-\left[1-\frac{\eta^2}{1-z}\right]^{1/2}\right]\ ,$$

$$x_1^{\min}=1-\frac{z}{2}\left[1+\left[1-\frac{\eta^2}{1-z}\right]^{1/2}\right]\ .$$

The results of the fit to the photon spectrum are

$$J/\psi\to\gamma+X\ ,\quad M_g=0.66\pm0.08\ \text{GeV}\ , \tag{12}$$

$$\Upsilon\to\gamma+X\ ,\quad M_g=1.17\pm0.08\ \text{GeV} \tag{13}$$

(a Gaussian energy smearing consistent with the energy resolution of the various experiments has been introduced in the fitting function). From the above experimental data we also estimate the contribution of the J/ψ and Υ radiative decays which are needed to extract $\Gamma(V\to ggg)$, namely,

$$B(J/\psi\to\gamma+X)=(11.3^{+3.7}_{-2.8})\times10^{-2}\ ,$$

which leads to

$$B(J/\psi\to ggg)=(60.3^{+3.1}_{-3.9})\times10^{-2}$$

and

$$\frac{\Gamma(J/\psi\to\gamma gg)}{\Gamma(J/\Psi\to ggg)}=(18.7^{+7.9}_{-5.3})\times10^{-2}\ ,$$

and

$$\frac{\Gamma(\Upsilon\to\gamma gg)}{\Gamma(\Upsilon\to ggg)}=(2.9\pm0.2)\times10^{-2}\ .$$

In the J/ψ case we have used the data of Ref. [16] for $z>0.4$ and extrapolated down to $z=0$ with the corresponding value of M_g which provides a very good description of the Mark II data ($\chi^2/N_{\rm DF}=\frac{18}{26}$). In this sense, our determination of the branching ratio $B(J/\psi\to\gamma+X)$ does not come entirely from direct measurements and there is some model dependence in our estimate (the error on the fitted value of M_g is, however, included in our estimate of the total branching ratio). Extrapolating down to $z=0$ with $M_g=0$ is not a sensible procedure since the perturbative curve does *not* describe the data at all (see Fig. 9 of Ref. [16]). Note that, for $\eta\sim0.4$, as in the J/ψ case, the integrated branching ratio $B(J/\psi\to\gamma+X)$ obtains the main contribution from intermediate values of z, in agreement with the experimental result. On the other hand, for $M_g=0$ the photon spectrum is nearly linear in z and peaked at $z=1$ so that one would predict

$$B(J/\psi\to\gamma+X)|_{z>z_c}\sim(1-z_c^2)B(J/\psi\to\gamma+X)\ .$$

Just for this reason, it must be stressed that the reported [16] agreement for $z>0.6$ between the experimental integrated branching ratio

$$B(J/\psi\to\gamma+X)|_{z>0.6}=(4.1\pm0.8)\times10^{-2}$$

and the perturbative QCD prediction

$$B(J/\psi\to\gamma+X)|_{z>0.6}\sim0.05$$

is totally accidental (see Fig. 9 of Ref. [16]) and based on assuming $\alpha_s(m_c)\sim 0.18$, in contrast with the expected value $\alpha_s(m_c)\sim 0.30$.

As stated above, the branching ratio

$$\frac{\Gamma(V\to\gamma gg)}{\Gamma(V\to ggg)},$$

in which the two decay processes differ only by the exchange of a gluon with a photon, should provide a measure of α_s largely independent of relativistic corrections [11]. However, on the basis of the perturbative relation [3] and of our extrapolation down to $z=0$,

$$\frac{\Gamma(J/\psi\to\gamma gg)}{\Gamma(J/\psi\to ggg)}=\frac{16\alpha}{5\alpha_s(m_c)}\left[1-2.9\frac{\alpha_s(m_c)}{\pi}\right]$$
$$=(18.7^{+7.8}_{-5.3})\times 10^{-2},$$

one obtains the value $\alpha_s(m_c)\sim 0.11^{+0.04}_{-0.03}$ so that a $\sim 66\%$ effect would be needed to obtain consistency with the deep inelastic scattering result in Eq. (7) in clear contradiction with Ref. [11]. On the other hand, by adopting the point of view of Ref. [11] and using the expected value $\alpha_s(m_c)\sim 0.30$ one would predict perturbatively $B(J/\psi\to\gamma+X)\sim 0.04$ to be the result of integrating over the whole range of z and not just for $z>0.6$.

Equations (12) and (13) are in qualitative agreement with the parton shower model of Ref. [19] where the average invariant mass of the gluon jets, $\langle M_g\rangle$, is found ~ 0.54 GeV for the J/ψ and ~ 1.5 GeV for the Υ. In Ref. [19] an effective gluon mass is generated through the splitting of the massless gluons into gluons or quark pairs which, by forcing the primary gluons to be off mass shell, modifies the population of the Dalitz plot of the γgg (and also ggg) final state and, therefore, the energy spectrum of the photon. In this approach, the different values of $\langle M_g\rangle$ for J/ψ and Υ can be understood due to the very different phase space available in the two cases. Notice that the results of Ref. [19] are in contradiction to the all orders calculation of Ref. [20], where the logarithmic terms that control the shape of the spectrum near $z=1$ are summed to all orders in α_s. Indeed, Ref. [20] predicts a very modest suppression of the photon spectrum near $z=1$ relative to the lowest-order QCD result, whereas Ref. [19] gives a spectrum which vanishes identically at $z=1$ in agreement with the experimental result. This is a consequence of the fact that the parton shower model predicts (see Fig. 4 of Ref. [19]) an invariant mass distribution for each of the initiating gluons which is sharply peaked around $\langle M_g\rangle$ and continuous down to $\delta=M_g/M_V=0$. On the other hand, conventional perturbative QCD, assuming massless gluons in the final states, predicts an invariant mass distribution which increases down to the infrared cutoff (0.45 GeV in Ref. [19]) thus generating a photon spectrum which is a monotonically increasing function of z. Therefore, in spite of this incompatibility, one might be tempted to interpret Eqs. (12) and (13) as consequences of the gluon cascade process without introducing any nonperturbative gluon mass. However, (a) as discussed in Ref. [15], Ref. [19] predicts a too hard photon spectrum in J/ψ decays, (b) our values of M_g in Eqs. (12) and (13) do not seem to scale with the mass of the heavy vector meson as in Ref. [19], and (c) the gluon splitting correction to $\Gamma(V\to ggg)$, corresponding to the lowest order of the parton shower model of Ref. [19], is already included through the coefficient k_V in Eq. (1). This correction is negligibly small and, actually, goes in the wrong direction; i.e., it tends to lower rather than increase the fitted value of α_s. Therefore, by *assuming* that the parton shower agrees with perturbative QCD in the predictions of *inclusive* quantities, i.e., that the coefficient k_V correctly reproduces [up to negligible $O(\alpha_s^2)$ terms] the lowest-order gluon splitting process for $M_g=0$, no explanation is given of the discrepancy in the observed values of α_s (d) Finally, by accepting that a "genuine" but rather large gluon mass (for example, ~ 1.2 GeV or higher) is dynamically generated one can also understand the smaller value (~ 0.66 GeV) obtained in J/ψ radiative decays. In fact, since the phase space is severely limited in this case, the virtual, *massive* gluons would mainly produce hadrons via their fusion into light $q\bar{q}$ pairs, as in the diagram shown in Fig. 16 of Ref. [16], their independent fragmentation being suppressed by one additional massive gluon propagator. However, by fitting the photon spectrum from an analytical expression in which the two gluons are treated as external, on shell particles the fitted value of M_g would be effectively reduced with respect to the corresponding Υ decays. As suggested in Ref. [16], the gluon fusion mechanism would explain why the multiplicity of charged particles measured in the hadronic state X of the decay $J/\psi\to\gamma+X$ is the same as in the continuum e^+e^- annihilation at the same center of mass energy (in the case of the K_s, multiplicity, however, no such simple diagrammatic interpretation seems to be possible [16]). A similar annihilation of the three-gluon system into light $q\bar{q}$ pairs would also allow to understand the similarity of the inclusive hadronic final state in e^+e^- annihilation on and just below the J/ψ resonance [16]. Such effects should, however, be much smaller in the Υ radiative decay where gluons of mass 1.2 GeV are allowed up to $z=0.94$ as compared to J/ψ decays where the corresponding number for $M_g=1.2$ GeV would only be $z=0.43$. Thus one may expect the effective gluon mass in Υ radiative decays to be nearer to an "on-shell" value [13,14] than in J/ψ decays.

In any case, in view of the similarity between the γgg and ggg phase spaces, it is natural to assume [12] that the mass parameter determined from the photon spectrum of the radiative decays can also be used to describe the ggg final state. By using Eqs. (12) and (13) for M_g we obtain $f_3(\eta)\sim 0.4$, $f_3^{\rm PS}(\eta)\sim 0.25$ for J/ψ, and $f_3(\eta)\sim 0.71$, $f_3^{\rm PS}(\eta)\sim 0.62$ for the Υ system. Therefore, the determinations of α_s (9) and (10) are modified as shown in Tables I and II, depending on the choice adopted for the function $F_3(\eta)$ in Eq. (11) [the corresponding quantities for $\psi(2S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$ are also included]. As might be expected, on the basis of our fitting procedure of M_g from the photon spectrum, the use of the function $f_3^{\rm PS}$ produces a better consistency in the relative running of α_s from the b- to the c-quark mass scale and, in the case of Table II, the determinations of α_s are in very good

TABLE I. The experimental results for the various branching ratios are shown together with the different determinations of $\alpha_s(m_Q)$. The first column is evaluated at $M_g=0$ while the second column takes into account the effect of M_g as given by Eqs. (12) and (13). The first error in $\alpha_s(m_Q)$ is experimental and the second reflects the theoretical uncertainty due to the relativistic effects. In the case $M_g \neq 0$ we have added linearly to this theoretical error an estimate of the neglected $O(\alpha_s)$ corrections as discussed in the text.

	Expt.	$\alpha_s(m_Q)$	f_3	$\alpha_s(m_Q)$
$\frac{\Gamma(J/\Psi\to ggg)}{\Gamma(J/\psi\to l^+l^-)}$	9.8±0.6	0.187±0.004±0.004	0.4	0.261±0.005±0.016
$\frac{\Gamma(\psi(2S)\to ggg)}{\Gamma(\psi(2S)\to l^+l^-)}$	14.7±5.1	0.213±0.027±0.004	0.5	0.28±0.03±0.02
$\frac{\Gamma(\Upsilon\to ggg)}{\Gamma(\Upsilon\to l^+l^-)}$	32.56±0.83	0.181±0.003±0.004	0.71	0.203±0.003±0.006
$\frac{\Gamma(\Upsilon(2S)\to ggg)}{\Gamma(\Upsilon(2S)\to l^+l^-)}$	34.8±5.9	0.185±0.009±0.004	0.73	0.206±0.010±0.006
$\frac{\Gamma(\Upsilon(3S)\to ggg)}{\Gamma(\Upsilon(3S)\to l^+l^-)}$	26.2±2.6	0.169±0.006±0.004	0.74	0.187±0.007±0.006

agreement with each other and with deep inelastic scattering [see Eqs. (7) and (8)]. Also, the role of the relativistic corrections is marginal, as expected.

A similar analysis can be applied to the ratio

$$\frac{\Gamma(\eta_c\to gg)}{\Gamma(\eta_c\to\gamma\gamma)} .$$

By using the perturbative relation [3] for $M_g=0$ and the experimental data from Ref. [5],

$$\frac{\Gamma(\eta_c\to gg)}{\Gamma(\eta_c\to\gamma\gamma)}=\frac{9\alpha_s^2(m_c)}{8\alpha^2}\left[1+8.2\frac{\alpha_2(m_c)}{\pi}\right]$$
$$\sim(1.43\pm0.69)\times10^3 \tag{14}$$

one obtains $\alpha_s(m_c)\sim0.21^{+0.04}_{-0.05}$. On the other hand, by taking into account the corresponding functions $f_2(\eta)$ introduced in Ref. [12] (see Fig. 1) one obtains

$$\frac{\Gamma(\eta_c\to gg)}{\Gamma(\eta_c\to\gamma\gamma)}=\frac{9\alpha_s^2}{8\alpha^2}f_2(\eta)$$

and, for $f_2(\eta)\sim0.7$, $\alpha_s\sim0.31\pm0.07$. Also, in the case of the branching ratio

$$\frac{\Gamma(\chi_{c2}\to gg)}{\Gamma(\chi_{c2}\to\gamma\gamma)}$$

we can use the experimental results

$$\Gamma(\chi_{c2}\to gg)=1.73\pm0.18 \text{ MeV [5]} ,$$

$\Gamma(\chi_{c2}\to\gamma\gamma)=1.28\pm0.28\pm0.26$ keV [21–26] and the perturbative relation [3]

$$\frac{\Gamma(\chi_{c2}\to gg)}{\Gamma(\chi_{c2}\to\gamma\gamma)}=\frac{9\alpha_s^2(m_c)}{8\alpha^2}\left[1+3.2\frac{\alpha_s(m_c)}{\pi}\right]$$
$$\sim(1.35\pm0.39)\times10^3 \tag{15}$$

to obtain the value $\alpha_s(m_c)\sim0.21^{+0.03}_{-0.04}$ or the relation

$$\frac{\Gamma(\chi_{c2}\to gg)}{\Gamma(\chi_{c2}\to\gamma\gamma)}=\frac{9\alpha_s^2(m_c)}{8\alpha^2}f_2(\eta)$$

to obtain $\alpha_s\sim0.29\pm0.05$ for $f_2\sim0.75$.

Finally, in the case of the branching ratios

$$\frac{\Gamma(V\to\gamma gg)}{\Gamma(V\to ggg)} ,$$

TABLE II. The same as in Table I but for $F_3=f_3^{\rm PS}$.

	Expt.	$\alpha_s(m_Q)$	$f_3^{\rm PS}$	$\alpha_s(m_Q)$
$\frac{\Gamma(J/\Psi\to ggg)}{\Gamma(J/\psi\to l^+l^-)}$	9.8±0.6	0.187±0.004±0.004	0.25	0.306±0.006±0.016
$\frac{\Gamma(\psi(2S)\to ggg)}{\Gamma(\psi(2S)\to l^+l^-)}$	14.7±5.1	0.213±0.027±0.004	0.38	0.30±0.03±0.02
$\frac{\Gamma(\Upsilon\to ggg)}{\Gamma(\Upsilon\to l^+l^-)}$	32.56±0.83	0.181±0.003±0.004	0.62	0.214±0.003±0.006
$\frac{\Gamma(\Upsilon(2S)\to ggg)}{\Gamma(\Upsilon(2S)\to l^+l^-)}$	34.8±5.9	0.185±0.009±0.004	0.65	0.215±0.010±0.006
$\frac{\Gamma(\Upsilon(3S)\to ggg)}{\Gamma(\Upsilon(3S)\to l^+l^-)}$	26.2±2.6	0.169±0.006±0.004	0.67	0.194±0.007±0.006

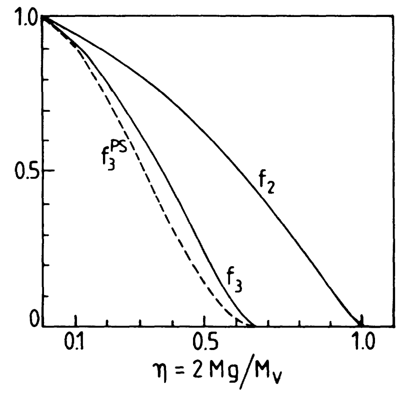

FIG. 1. The functions $f_3(\eta)$ and $f_2(\eta)$ of Ref. [12]. In the case of f_3 the pure phase space contribution $f_3^{\rm PS}$ is also shown.

one can compute for $M_g=0.66$ GeV the suppression factor

$$f_3^{gg\gamma}(\eta)=\frac{\Gamma(J/\psi\to\gamma gg)|M_g}{\Gamma(J/\psi\to\gamma gg)|M_g=0}\sim 0.46$$

so that from

$$\frac{\Gamma(J/\psi\to\gamma gg)}{\Gamma(J/\psi\to ggg)}\sim\frac{f_3^{gg\gamma}}{f_3^{\rm PS}}\frac{16\alpha}{5\alpha_s} \tag{16}$$

we obtain $\alpha_s\sim 0.23^{+0.09}_{-0.07}$, consistent (within its very large error) with the expected value $\alpha_s(m_c)\sim 0.30$. Analogously, for the Υ one obtains from [3]

$$\frac{\Gamma(\Upsilon\to\gamma gg)}{\Gamma(\Upsilon\to ggg)}=\frac{4\alpha}{5\alpha_s(m_b)}\left[1-2.6\frac{\alpha_s(m_b)}{\pi}\right]$$
$$=(2.9\pm 0.2)\times 10^{-2} \tag{17}$$

the value $\alpha_s(m_b)=0.173^{+0.010}_{-0.009}$ or, for $M_g=1.17$ GeV, by using the values $f_3^{\rm PS}\sim 0.62$ and $f_3^{gg\gamma}\sim 0.74$ in the relation corresponding to Eq. (16), the determination $\alpha_s=0.240\pm 0.017$.

Summarizing, by taking into account the effective gluon mass corrections, one can attempt an average of the three experimental determinations of $\alpha_s(m_b)$ shown in Table II (where $f_3^{\rm PS}$ is consistently used as in the photon spectrum) with the value obtained above from the radiative process $\Upsilon\to\gamma+X$ with the result

$$\alpha_s(m_b)\sim 0.21\pm 0.01\ , \tag{18}$$

where the error is dominated by theoretical uncertainties. Analogously, in the case of the $c\bar{c}$ branching ratios, by averaging the two values in Table II with the results from the η_c, χ_{c2}, and the radiative decay $J/\psi\to\gamma+X$, the gluon mass corrections produce

$$\alpha_s(m_c)\sim 0.30\pm 0.02\ . \tag{19}$$

In spite of the many approximations involved in our approach, Eqs. (18) and (19) derive from a sample of statistically consistent determinations and are in good agreement with each other and with the extrapolation from deep inelastic scattering in Eqs. (7) and (8).

III. CONCLUSIONS AND OUTLOOK

We have shown in this paper that the phase space modification associated with assuming a nonvanishing effective mass for the primary gluons in J/ψ and Υ decays *can* play a very important role in the determination of α_s from the now very precise experimental quarkonia branching ratios. Contrary to the presently accepted point of view, we have produced many arguments that the relativistic corrections *cannot* be responsible for the very serious discrepancy with respect to the perturbative calculation with massless gluons in Eq. (1) by using the running coupling constant at the heavy quark mass scale. The evidence for M_g is indirect (since gluons are not directly observed) but the use of the mass parameter obtained from the radiative decays produces consistent results in the integrated branching ratios. The change of the gluon mass from $M_g=0.66\pm 0.08$ GeV for the J/ψ to the Υ result $M_g=1.17\pm 0.08$ GeV effectively describes the energy transfer from the primary gluons to the physical hadronic states available at the two center of mass energies. By *interpreting* the value $M_g\sim 1.2$ GeV (or higher) as a "genuine," process-independent gluon mass one predicts a strong suppression of the gluon splitting process at the J/ψ so that the hadronic final states should be mainly produced via gluon fusion into light $q\bar{q}$ pairs, in agreement with the experimental result.

The "parton shower" interpretation of Eqs. (12) and (13) (that is, the identification of our M_g with the $\langle M_g\rangle$ of Ref. [19]) is unable to explain the structure of the hadronic final states at the J/ψ since, the gluon mass being generated by the splitting process itself, there is no reason to forbid the primary gluons to split into other gluons. At the same time, *by interpreting* Eqs. (12) and (13) as in the model of Ref. [19] and replacing their values in Eq. (11) one is forced to deduce an inner contradiction between conventional perturbative QCD and the parton shower. In fact, the same average gluon mass $\langle M_g\rangle$ generated for the primary gluons via the gluon cascade would actually be needed to explain the relative ratio of the integrated ggg to l^+l^- decay rate so that no finite-order truncation in perturbation theory would be possible, even for inclusive quantities, regardless of the existence of any dynamically generated gluon mass, and whatever the magnitude of the heavy meson mass M_V [since $\langle M_g\rangle\sim\alpha_s(M_V)M_V$].

Still, we feel that our interpretation of Eqs. (12) and (13) is not unique and for this reason one should investigate other phenomenological consequences of a nonvanishing gluon mass in quarkonia decays. For instance, the χ_{c1} state is forbidden by the Landau-Pomeranchuk-Yang (LPY) [27] theorem to decay into two massless gluons. In this case, all allowed decays of the χ_{c1} are $O(\alpha_s^3)$. For massive gluons, however, the $O(\alpha_s^2)$ two gluon decay is allowed. One may note that the ratio [28]

$$\frac{\Gamma_{\rm had}(\chi_{c1})}{\Gamma_{\rm had}(\chi_{c2})}=0.37\pm 0.08$$

is substantial [the $O(\alpha_s^2)$ two-gluon decay is allowed for χ_{c2}]. A recent analysis [29] of χ decays where, on the basis of the factorization theorems, a $\bar{Q}Qg$ component was introduced in the χ_{c1} wave function, should therefore also take into account a possible $O(\alpha_s^2)$ massive two-gluon contribution to the width. A closely related effect occurs in the radiative decays $J/\psi \to \gamma +$ axial-vector meson. Such decays, mediated by two on shell massless gluons, are also forbidden by the LPY theorem. Recent measurements by the Mark III Collaboration [30] indicate, however, a non-negligible (0.1%) branching ratio into such states also, perhaps, indicative of a massive gluon contribution.

Also, one may remark that, as first pointed out in Ref. [12], gluons with a mass ~ 1 GeV provide a simple explanation of several other puzzling features of low-energy QCD phenomenology. More recently an effective cut off at a distance $\sim (1\ \text{GeV})^{-1}$ for the gluon exchange force has been proposed [31] to explain the additive quark rule [32] and Pomeron exchange within QCD.

Finally, a sizable violation of hadronic helicity conservation (HHC) is observed [33] in the hadronic decays of the J/ψ. A simple argument based on Lorentz invariance and helicity conservation at vertices where massless gluons couple to quarks suggests that the processes

$$e^+e^- \to J/\psi \to \rho\pi \ ,$$

$$e^+e^- \to J/\psi \to K^*K$$

should be forbidden. The experimental branching ratios [5]

$$B(J/\psi \to \rho\pi) = (1.28 \pm 0.1) \times 10^{-2} \ ,$$

$$B(J/\psi \to K^*K) = (5.0 \pm 0.4) \times 10^{-3} \ ,$$

$$B(J/\psi \to K^+K^-) = (2.37 \pm 0.31) \times 10^{-4}$$

(the last process, which is HHC allowed, is given for comparison) show no evidence for suppression, but rather a considerable *enhancement*. The contribution of longitudinal helicity states of massive gluons, permitting vertices with quark helicity flip, would be a natural explanation of the failure of HHC, even though some other physical mechanism might be required to fully explain the observed enhancement of the HHC suppressed processes. In the $\psi(2S)$ case, the experimental value [5]

$$B(\psi(2S) \to \rho\pi) < 8.3 \times 10^{-5}$$

at the 90% C.L. gives no positive evidence for suppression relative to the corresponding quantity for K^+K^- decay, namely

$$B(\psi(2S) \to K^+K^-) = (1.0 \pm 0.7) \times 10^{-4}$$

[equivalent to

$$B(\psi(2S) \to K^+K^-) > 1 \times 10^{-5}$$

at the 90% C.L.]. Since the phase space factors for $\rho\pi$ and K^+K^- are very similar, the very different measured values of the ratio,

$$\frac{B(V \to \rho\pi)}{B(V \to K^+K^-)} \ ,$$

$=54 \pm 8$ for J/ψ and $< 0.8^{+2.7}_{-3}$ at the 90% C.L. for $\Psi(2S)$, indicate that the solution of the puzzle should be sought not in a suppression for $\psi(2S)$ decays but rather, as in Ref. [34], in an enhancement for J/ψ decays. In summary, although longitudinal helicity states may contribute to the violation of HHC in J/ψ decays, it seems some other mechanism must lie behind the factor "50" enhancement seen relative to the HHC K^+K^- channel. For the $\psi(2S)$, violation of HHC due to massive gluons may be the dominant mechanism but a more precise measurement of $B(\psi(2S) \to \rho\pi)$ (rather than the present upper limit) is needed to verify this. In any case, the possible contribution of the longitudinal helicity states associated with massive gluons should be taken into account. This has not been done in any published analysis to date.

To conclude, it may be interesting to note that the values of $\alpha_s(m_c)$ and $\alpha_s(m_b)$ found in the analysis of the data with $M_g=0$ are all consistent (see Eqs. (2) and (3), the tables, the results of Eqs. (14), (15), and (17) and also Table IV of Ref. [3]) with the same effective value of the strong interaction coupling constant, for example, $\alpha_s^{\text{eff}} \sim 0.185 \pm 0.010$, which is experimentally observed since a long time [35]. Thus, one may be driven to the conclusion that the running is already "frozen" at the b-quark mass scale and restored by gluon mass corrections which, however, have a much more dramatic effect at the c-quark scale ($\sim +60\%$ as compared to $+\sim 13\%$) if $M_g^2 \sim m_c^2 \ll m_b^2$. This is, probably, an oversimplification of the actual experimental situation in the full range of scales below the Z mass. In fact, both deep inelastic data [6] and our values after the M_g corrections, $\alpha_s(m_c) \sim 0.30 \pm 0.02$ and $\alpha_s(m_b) \sim 0.21 \pm 0.01$, lie below the downward evolution of the CERN e^+e^- collider LEP value [36] $\alpha_s(M_z) = 0.125 \pm 0.005$, which would rather predict $\alpha_s^{Th}(m_c) \sim 0.45^{+0.10}_{-0.05}$ and $\alpha_s^{Th}(m_b) \sim 0.24 \pm 0.02$. This remark suggests that α_s may instead saturate at a value ~ 0.3, even accounting for gluon mass corrections [14], and that the evidence for a "freezing" of the strong interaction coupling constant deserves further detailed study beyond our present analysis. On the experimental side, we want to stress the importance of more precise data on charmonium decays where the sensitivity to the gluon mass effects is greatest. Improved measurements of the photon spectrum in the process $J/\psi \to \gamma + X$, down to lower values of z, would be essential to reduce the model dependence in the evaluation of $B(J/\psi \to \gamma + X)$, as we have pointed out. On the theoretical side, a set of calculations, similar to the pioneering work of Parisi and Petronzio [12], explicitly taking into account the massive gluon degrees of freedom are needed. For instance, the resummed but perturbative calculation of Ref. [20] should be generalized to include the effect of massive gluon propagators in the gluon fusion diagram shown in Fig. 16 of Ref. [16]. In this way the simple model used here, where different "effective" gluon masses are obtained from the J/ψ and Υ radiative decays, would be replaced by a more refined calculation with only one free parameter. The unified description obtained in this way will enable more stringent test of the massive gluon hypothesis [12–14] and a definitive experimental confirmation would have far reaching consequences for our understanding of QCD.

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